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Improved Ballistic Wind Prediction Using Projectile Tracking Data

William Arthur Kenney
Old Dominion University

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IMPROVED BALLISTIC WIND PREDICTION USING PROJECTILE TRACKING DATA

by

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B.S. May 2005, University of Mary Washington

A Thesis Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirements for the Degree of

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ABSTRACT

IMPROVED BALLISTIC WIND PREDICTION USING PROJECTILE TRACKING DATA

William Arthur Kenney
Old Dominion University, 2017
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The United States Air Force AC-130 gunships have been in operation since the Vietnam War and have seen frequent use during recent conflicts. They are able to employ gun weapon systems from above a target in a way that maximizes possible time on target. When firing, the gun operators must deal with miss distances caused by winds acting on the projectile in flight. Operators currently perform a “tweak” to predict a ballistic wind affecting fired rounds which is then used in the fire-control to correct for the real winds and bring shots onto target. This correction, a single-point wind prediction, is made using only the initial state of the gun and aircraft and the final impact location. This thesis explores the possibility of using a round tracking sensor to track a projectile as it falls and produce a multipoint ballistic wind which would be better at correcting for true winds than a single-point ballistic wind.

An algorithm for a multipoint wind prediction method is described and simulation are run using it and a single-point prediction method against measured wind profiles. The results of the single-point and multipoint ballistic winds are compared to the measured winds to test for a goodness of fit. The results are also tested for stability that when used the ballistic wind remains valid even if the aircraft and gun change state from the initial state when the ballistic wind was predicted. The results show that a multipoint ballistic wind that is a better fit and more stable ballistic wind than a single-point ballistic wind is possible using the algorithm presented. Also,
the multipoint ballistic wind can be produced with very few data points along the trajectory of the projectile.
This thesis is dedicated to all AC-130 operators, pilots, gunners, maintainers, and anyone else who puts their lives in danger to protect and save our guys on the ground.

I hope research like this helps. Keep doing bad things to bad people.
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And to Jess. Thank you. Always.
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CHAPTER 1
INTRODUCTION

When firing a gun, winds tend to be the largest uncontrollable error contributor to final impact miss distance [1], [2]. Most other errors, such as aiming and accounting for projectile physical parameters, can be minimized prior to firing. The winds and their effects on the round throughout its flight cannot be known before firing [1]. This is true regardless of the type of gunfire, be it stationary and ground based or in motion on an orbiting aircraft.\textsuperscript{1}

For a stationary gunner, winds and other errors can be corrected for by applying an offset to the pointing angles of the gun, called “Kentucky Windage” [3]. This type of correction assumes that all errors observed on one shot will act the same on the next shot. For example, if wind and other errors combine to force a round to impact high and to the right of a target, then a stationary gunner can Kentucky Windage the shot by aiming low and to the left of the target.

For moving gunners this type of correction does not apply, especially for an orbiting gunship such as the United States Air Force (USAF) AC-130 gunships. When circling a target error effects which manifest themselves in different frames of reference will mix in such a way that Kentucky Windage cannot be used to correct the errors [4]. A method of separating the errors into their specific reference frames and accounting for each error source individually is needed.

Correcting the wind error when firing from an orbiting gunship is a problem that has been solved in each iteration of the AC-130 gunship’s gun fire-control (FC) system [5], [6]. Each model’s operators has had a method of correcting the observed wind induced miss distance suited to their specific method of FC, be it changing the orbit center or using a “tweak”.

\textsuperscript{1} IEEE Transactions and Journals style is used in this thesis for formatting figure, tables, and references.
Annoyingly little literature exists on these methods, however. The Technical Orders (TO) for past gunships describe in general terms either the method of correction via changing the orbit center or the intent of the correction via a tweak. Research into the exact methods of predicting a ballistic wind has not been published in a publicly accessible database. Whether this is due to protection of intellectual property or classification of the method is not clear.

Current methods attempt to predict a ballistic wind using only the initial firing conditions and the final impact of the round. This can be done to correct for the wind effects on the round, though the ballistic wind predicted can lose validity over time and as the aircraft changes state. A single-point ballistic wind is computationally easy to calculate. The prediction requires no more hardware than would already be available for normal operations of a gun FC system: a method of measuring the aircraft and gun state and a sensor to detect and locate the round’s impact.

The single-point ballistic wind has been in use for years on USAF AC-130 gunships. The method is trusted and has been shown to be effective. The limitations are well known. The ballistic wind values may not be valid if the aircraft changes state from the time of the original calculation to the time of fire even if only changing the altitude of the aircraft. A more flexible and stable method of modeling the winds would improve overall gun accuracy.

A multipoint ballistic wind prediction is possible, though not with technology currently in use on the AC-130 gunships. In order to create a multipoint ballistic wind, the location and speed of the round must be known at various locations along the projectile’s flight path. Round tracking sensors exist and could be used to provide this telemetry data to a FC system.

Could a round tracking system be implemented to allow for the calculation of a more stable ballistic wind?
This research tests the hypothesis that a more stable ballistic wind profile can be calculated using data from a round tracking sensor. The multipoint ballistic wind prediction can be made with very few data points and can be done in a way that is suited to a tactical application of the algorithm.

The primary result of this research is to demonstrate the possible benefits of a multipoint ballistic wind. A secondary result is a demonstration of a workable algorithm to solve for a multipoint ballistic wind. Both results are measured against the current single-point ballistic wind method. In the end, the goal is to assess whether a projectile tracking sensor and a multipoint ballistic wind offer enough of a reduction in wind induced miss distance to be a viable technology.

If a tactically usable algorithm can be developed to predict more dynamic ballistic wind profiles, it would increase the accuracy of the gun weapon system. Assuming that the system would track each round fired, winds could be predicted for each round individually. Using the winds from the most recently fired round the FC system could improve the firing solution for the subsequent round. This does not lead to first round accuracy but does introduce the possibility of greatly improved accuracy for all following rounds.

This thesis is divided into seven chapters, including this Introduction. Chapter 2 explains the current state of the systems to be modeled for this research. The current state-of-the-art for aircraft flight, FC system, wind correction method, and projectile tracking systems are described. Chapter 3 describes the models designed and implemented to recreate the relevant parts of the real-world systems described in Chapter 2. The modeling assumptions and limitations are presented along with the expected input and output. Validation of the individual models is discussed, though the validation criteria and results are not presented. Two factors controlling
the performance of the wind prediction model are tuned and the results are discussed in Chapter 4.

Chapter 5 uses the models to simulate the current wind prediction method, a single-point wind correction. Real measured winds are used in this chapter and the wind prediction model finds a single value ballistic wind to account for the effects of the measured winds. Chapter 6 uses the same measured winds and initial conditions used in Chapter 5 to predict a ballistic wind based on multiple points along the flight path of the round. The multipoint wind prediction method is described and the results of the simulation runs are presented.

If the above hypothesis is correct, then the wind predictions from Chapter 6 should prove to be more stable than the wind predictions made in Chapter 5. Chapter 6 investigates the closeness of the predicted winds to the true winds to indicate which ballistic wind method performs better. The ballistic winds are also tested as the state of the aircraft and gun are changed to see which ballistic wind method performs better, allowing less error into the impact prediction.

Chapter 7 presents the conclusion to the research. Along with summing up the results presented, the chapter includes recommendations for future experiments or analysis and a discussion of some of the remaining limitations on a FC system using the multipoint wind prediction method described in Chapter 6.
CHAPTER 2
CURRENT STATE OF THE ART

This research is focused on determining if increased knowledge of a ballistic projectile’s location in flight can be used to make ballistic wind predictions which closely match the true winds acting on the projectile. Specifically, this study will look at a weapons platform which relies on wind predictions to improve weapon effectiveness on USAF AC-130 gunships.

In order to establish a framework for the models and simulation which are developed in Chapter 3, this chapter reviews the current state of technology of the systems and subsystems which will be modeled. This description is by no means exhaustive, but gives enough details and background data to allow for the design and implementation of models to recreate the system of interest.

A brief description of USAF fixed-wing gunships is presented, describing the theory of operations and the flight profile used during a weapons engagement. Gun weapon systems require a FC system to properly point the gun so that rounds fired will impact the desired target. Features of a FC are detailed and errors common to FC systems are discussed.

One of the most common and largest errors experienced by FCs is the effect of wind on the projectile. Existing methods to predict the effects of wind and account for them to improve impact accuracy are described. Finally, various round tracking systems and their configurations are detailed.

2.1 Side-Firing Gunships

After the first flight by the Wright brothers in 1903 [7], it did not take long for airplanes to be adopted for military use. In 1909, the US Army Signal Corps purchased and used the first
military aircraft. Early uses included both combat and non-combat roles. The first recorded deployment of a gun on a military airplane was in 1915 when French pilot, Roland Garros, used a forward firing machine-gun to engage enemy aircraft. For engaging ground targets, some early aviators carried rifles in flight which they would fire sideways out of the cockpit. In the 1920s both the Americans and the French mounted side-firing guns on various aircraft [8], though there was no specific tactic developed to employ such weapons.

One of the problems faced with all air-to-ground engagements is that the aircraft generally has a short time to engage the target [9]. Strafing a target or engaging in a fly-by attack allows for a short period of time where weapons can be brought to bear on a target. A pilot then must turn the aircraft and reacquire the target before they can reengage.

Pilots both military and civilian had developed a maneuver called the “pylon turn” by the 1920s [9]. The pylon turn is a maneuver where the pilot turns the aircraft at a constant bank angle. This has the effect of pulling the aircraft into a roughly circular turn around a stationary ground location. Pilots developed the pylon turn maneuver for airplane racing. Military aviators saw the advantage of combining side-firing weapons with a coordinated pylon turn. The tactic was initially tested in 1926 by the US Army and developed from there into the side-firing fixed wing gunships used today [8].

A pylon turn is defined by the bank angle of the aircraft, the aircraft’s speed, and the altitude of flight [6]. There values are called the “nominals” and they control the geometry of the pylon turn. With a given set of nominals the total range from the gun to the target, the slant range, can be calculated. Nominals can be chosen to achieve a specific slant range.

There are many advantages to using side-firing weapons in a pylon turn. From a combat perspective the primary advantage is that it increases weapon time on target [9]. A pylon turn
can be executed around a specific target or target area which allows the weapon to be trained on the target for the duration of the orbit. Side-firing weapons employed without using a pylon turn and forward firing weapons have a limited time to engage before the aircraft has passed the target and must turn to reengage.

Along with increasing the time available to fire at the target, the pylon turn reduces the apparent target motion relative to the aircraft. If the pylon turn is properly executed, a target can be placed at the center of the orbit [8]. From the perspective of an observer on the aircraft, a target at the center of the orbit appears stationary. Even though the aircraft is in motion the target appears stationary relative to the aircraft making it easier to engage the target with weapons.

The idea of combining side-firing guns with aircraft executing a pylon turn was first tested in 1926 but was not pursued by the US military at that time. During World War II, the US military proposed using a side-firing gun on an aircraft to engage submarines, but again the tactic was not pursued. It wasn’t until the Vietnam War that a true side-firing gunship executing a pylon turn was used by the US military [10].

Pylon turns are the standard flight profile for modern USAF AC-130 gunships [5]. The side-firing guns can be trained on targets throughout the orbit and engage for extended periods without losing sight of the target. Pilots select nominals to fly in order to hold a specific slant range around a target. The nominals determine the geometry of the orbit and the target-to-gun system. The selection of the nominals will vary based on pilot preference and mission needs.
2.2 Gun Fire-Control

When a gunship engages a target with its guns, a firing solution must be calculated. The firing-solution is a set of gun pointing angles (azimuth and elevation) which will allow a round fired by the gun to impact the intended target [11]. The firing-solution takes into account the current state of the aircraft and target location. For modern gun weapon systems, the calculating of the firing-solution is accomplished by the gun FC system.

The FC ties together different data sources available on the gunship and uses those data to compute the firing-solution. The specific operations and functions of a given FC may vary based on hardware and software design considerations, but the common functions are as follows:

1. Get target location data from a sensor system
2. Convert the target location from a sensor-relative frame of reference to a gun-relative frame of reference
3. Use ballistic model to predict a set of azimuth and elevation gun angles which will allow a ballistic projectile to impact the target location
4. Move gun into position to match firing-solution
5. Fire gun

Each of the above steps involves many hardware components providing input data on the state of the gun, target, and aircraft as well as software algorithms to calculate the required pointing angles and control the gun weapon system. A full discussion of such FCs is beyond the scope of this research.

Pertinent to this research are the possible errors in the firing-solution generated by the FC. An error in the firing-solution is determined to exist if the round fails to impact the intended target. The error is judged by the characteristics of how the round missed the target. There are
many sources of possible error in a FC and the firing-solution it generates. A full list of the error sources would depend on the specific configuration and design of the system, but some common error sources are poor ballistic modeling, mechanical errors controlling the pointing of the gun, incorrect targeting data, winds, and production tolerances for the ammunition. During the development of the FC system, all efforts are made to reduce or remove any errors which can be eliminated \textit{a priori} based on gaining more knowledge of the system.

For example, the initial velocity of the round is found through testing and is treated as an input to the system. Each round has a different initial velocity which cannot be known before firing. The initial velocities measured during testing result in a distribution of possible values. The average initial velocity value is used in the FC, thus accounting for an epistemic error \cite{12} which would exist if the initial velocity had not been measured at all. The variability in the initial velocity still exists as an aleatory error which cannot be corrected.

All errors in the system can be described as causing either a bias \cite{13} or dispersion on the round impacts. A bias error causes round impacts to be offset from the intended target in a repeatable and predictable way. Dispersion errors cause the rounds to impact within a “cloud” or region but not a single repeatable location.

When firing from an orbiting aircraft, impacts can be tracked in two frames of reference: a platform relative frame and a world relative frame. Biasing effects manifest in one of these two frames as a roughly static offset. Because the aircraft is orbiting a bias in one frame will appear to drift in the other frame in a predictable way based on the heading of the aircraft at time of fire.

The platform relative frame of reference is fixed to the aircraft \cite{4}. Regardless of the aircraft orientation, the Y-axis of the platform relative frame is oriented with the positive
direction pointing vertical and parallel to the gravity vector at the aircraft. The X-axis, called the down-range (DR) direction, points with the positive direction to the left side of the aircraft orthogonal to the Y-axis. The Z-axis, called the cross-range (CR) direction, completes the right-handed system and points with the positive direction to the nose of the aircraft. When discussing errors in round impacts off of target, the origin of the platform relative frame is assumed to be at the intended target.

Platform relative biases are roughly static as observed from the aircraft. These bias errors can be corrected by applying a static offset to the gun pointing angles. This correction can be held through the entire orbit. Examples of platform relative bias include misalignments of the gun, poor ballistic modeling, and inaccuracies in the body description and physical properties of the round being fired.

The world relative reference frame is a local East-North-Up reference frame [4]. When discussing errors in impacts, the origin of the world relative frame is at the target. The X-axis points positive to the East, the Y-axis points positive to the North, and the Z-axis completes the orthogonal system pointing positive upwards parallel to the gravity vector.

World relative biases are static as observed from the ground. A world relative bias will cause all shots to fall in roughly the same direction in East and North relative to the target. These biases can also be corrected by applying an offset to the gun pointing angles. The offset is not static and will change as the aircraft orbits the target location. Winds account for the world relative bias affecting the flight of ballistic projectiles.

Dispersion effects also manifest in specific frames depending on the cause of the error. Given the nature of dispersive errors, they cannot be separated into a specific frame of reference.
The dispersion will appear as “noise” on the impacts regardless of the frame of reference in which they are rendered.

In flight, attempts can be made to correct for biases that could not be corrected for on the ground. To detect and remove biases in both the platform and world reference frame multiple shots must be taken at headings around the orbit [14]. This is required to decouple world relative bias from platform relative bias. Once the shot data are collected and decoupled, the appropriate corrections can be made to the pointing angles of the gun to remove any platform or world relative bias.

### 2.3 Correcting For Winds

Winds affect the flight of a projectile in two ways, as a bias and as a dispersion in the observed impacts.

The wind’s average effect on the projectile causes a world relative bias, moving the impact of the round to a roughly constant location as measured in meters East and North of the target [6]. While there is no such thing as a true average wind, there is a component of the wind column which changes very slowly over time which is generally regarded as the average wind.

The average wind speed column, if known, does not capture all of the wind effects. The wind’s dispersive effect on the round is due to the variability of the winds over time and unpredictable gusts which occur after the round is fired [2]. Gusts and variability in wind speeds close to the ground will always cause dispersion on the impacts which cannot be accounted for \textit{a priori}.

It is possible to account for the offset in the impacts due to the average wind column. Historically, two different ways have been used to correct for wind effects with weapon systems.
Each method relies on knowing only two points, the initial firing conditions and the final impact, to correct the impacts.

When firing from a pylon turn, the target is usually at the center of the orbit to maximize weapon time on target and minimize the need to change gun pointing angles to fire on a target. If winds are present and causing the impacts to fall in a roughly constant location East and North relative to the intended target, the orbit can be offset to correct for this miss [6]. Adjusting the center point of the orbit by the same magnitude as the average wind induced miss distance in the opposite direction will cause shots fired at the center of the orbit to impact on the original target, Fig 1. The target is no longer in the center of the orbit but the gun is still aimed as though the target was centered.

Offsetting the orbit center was commonly used in older gunships because it did not require extensive ballistic calculations or fully trainable gun systems. Modern FC and gun weapon systems are capable of recalculating ballistic solutions and training the guns automatically to account for offsets required to bring missed impacts back on target. This method, referred to as a “tweak”, is defined as,

“A computation performed either manually or by fire control computer to correct for errors in weapon or sensor alignment and to solve for the ballistic wind. The purpose of performing a tweak is to cause ordnance to impact on target. [15]”

Note that the definition of tweak also encompasses correcting for alignment offsets as well as the winds. The wind correction result of the tweak algorithm is a “ballistic wind”. The ballistic wind is not a measure of the true winds affecting the round in flight. Ballistic winds are an approximation of the winds from the tip of the barrel to the ground level that would account
for the observed wind induced miss distance. A ballistic wind is a single wind speed and direction value which is assumed to apply for the entire flight path of the projectile.

The tweak process finds the ballistic wind which best accounts for the observed world relative bias in any impact data. Multiple shots are taken and the miss distances are recorded. Using the impact data, a search algorithm is used to iterate over a search space of possible wind vectors. The wind model is then applied to the ballistics model in the FC. Applying the winds in the ballistics model under the initial firing conditions, an impact is predicted. The algorithm

Fig 1. Wind Corrected Orbit.
varies the parameters of the wind model to reduce the difference in the observed impacts the predicted impacts with ballistic winds applied.

The ballistic winds predicted by the tweak are valid only for a period of time. This period varies based on the wind itself; no clear time limit exists. If the winds are calm and slow to change when the ballistic wind is calculated, then the tweak results may be valid for a long period. If the true winds are highly dynamic and changing rapidly, then the tweak results may become “stale” in a short period.

2.4 Tracking Projectiles

Technology exists to track a projectile in flight. Such round tracking technologies fall broadly into two categories: internal trackers and external trackers.

Internal trackers, also known as telemetry rounds, contain hardware to allow them to detect or measure their location and relay that data back to a base station [16]. Telemetry rounds contain some form of GPS or Inertial Navigation Unit used to measure the location of the round in flight. The round then transmits that information to a base station which records the information.

Telemetry rounds require changes to the projectile itself to allow for the inclusion of the necessary hardware. It is common for telemetry rounds to be inert, any explosive warhead being removed to allow for the inclusion of the tracking hardware. These rounds are often used in experiments where the terminal effects of the round are not under study. Because of the changes, telemetry rounds may not be representative of the rounds intended for tactical use.

External trackers are sensors that track the round in flight without needing the round itself to transmit a signal to the tracker system. There are a variety of methods used to track projectiles
in flight. A rigid body system measures a direct range and pointing angles to the projectile from a known sensor location [17]. A Doppler system pings the projectile with a radio or microwave signal and finds the round’s velocity based on the Doppler shift of the return signal [18]. The round’s velocity is integrated over time to predict the position of the projectile. Sensor array systems exist which rely on the pointing angles of multiple sensors pointing at the round in flight and triangulation to find the location of the round [19].

For each of the external systems, some form of sensor must be used. These all rely on reflected electromagnetic radiation to detect and locate the round. The specific sensor configuration used depends on what material the round is composed of. LIDAR systems can be used to track a round if a portion of the round is painted in such a way as to reflect LIDAR signals. Radar tracking will work with any round in current use as they are all metal jacketed, though round size is a limitation. Tracer rounds, those with base-burners, can be tracked with infrared or electro-optical sensors.

Regardless of the method of tracking the round used, the tracker itself must measure or calculate the location of the round in some reference frame relative to some origin point. The frame and the point are arbitrary. The only firm requirement is that the data be of such a form that it can be translated into a frame which is relevant to the weapon system.
CHAPTER 3
MODELS

In Chapter 2, the system of interest was described. This research investigates the use of a round tracking system to predict a ballistic wind to reduce wind induced bias errors on projectiles fired from an AC-130 gunship. In order to simulate firing from an AC-130 gunship and attempt to correct for the wind effects on a projectile, a series of models were developed to recreate the systems described in Chapter 2.

A model is required to simulate the flight conditions of the aircraft at the time of fire. Chapter 3 describes the simplifying assumptions made in developing the model. The chapter also details the equations used and the required input to the model.

It was decided early in the simulation design that modeling the entire FC would greatly increase the complexity of the system, introducing more chances for errors without increasing the level of fidelity of the simulation. Instead of modeling the entire FC a ballistics model, which would be used by the FC, was developed to simulate the flight of a projectile. Section 3.2 presents the design consideration made, the assumption inherent to the model, and the required input parameters.

Modeling the wind is described in Section 3.3. A method is required which will model a consistent wind both for developmental testing and for simulation of the ballistic wind predictions.

Along with the winds, a model is developed to simulate the data supplied by a round tracking sensor. The modeling assumptions are fairly broad; the resultant model described in Section 3.4 is designed to give the proper output expected from a round tracking system.
Finally, in Section 3.5 a method of wind prediction is described and a model is detailed. The internal algorithm is described along with the expected inputs and outputs to allow the wind prediction model to interact with the other models and their data.

The high-level architecture of the resulting simulation software is shown in Fig 2. From Fig 2 one can see what the expected inputs into each of the sub-models is and what data are being sent to the other models. All messages are sent via multicast network messages. This design decision was made to ensure that the method of communication is as close as possible to that which would be used in a real tactical application of these systems. Also, by limiting the interactions of the various models to only those inputs and outputs shown in Fig 2, it was possible to ensure that the wind prediction model would only have access to that data which a hardware round tracking sensor would provide. This control of network messages prevents the chance of the wind prediction model having knowledge of the underlying winds which would not truly be available to a wind prediction system.

3.1 Aircraft State

This research focuses on projectiles fired from aircraft executing a pylon turn. The aircraft motion in a pylon turn is a direct contributor to the state of the projectile at time of fire. The orientation of the aircraft and the speed of the aircraft are factors that must be accounted for when attempting to predict the motion of a projectile fired from the aircraft.
A fully descriptive model of the aircraft’s motion in flight is not needed for this analysis. The firing of a gun is an almost instantaneous event from the moment of trigger to the time the round exits the barrel. The motion of the aircraft after the time of fire has no effect on the flight of the round. The motion of the aircraft before the round exits the barrel is only important in that it imparts a velocity to the round. This allows for a simplified model of the aircraft’s motion and state to be used.

When modeling the ballistics of a projectile fired from the aircraft very few factors of the aircraft’s state need to be considered. The model used here is as simple as possible to model an aircraft in a pylon turn and supply the needed inputs to the ballistics model.
3.1.1 Assumptions

The model used assumes that the acceleration due to gravity is constant at all altitudes and latitudes. This is not strictly true (see Eq. 10 and Eq. 11). For the purpose of modeling the flight of the aircraft, the small changes in gravity due to changes in latitude or altitude will change the geometry of the orbit only slightly. This change does not affect the quality of the ballistics model or the applicability of winds to the flight of the projectile. As such, it is safe to ignore the dynamic nature of the gravitational acceleration.

This model further assumes that the geometry of the orbit is controlled only by those forces acting normal to the direction of travel of the aircraft. The forward motion of the aircraft is only used to apply a velocity to the system. Any forces acting in that direction, such as drag on the aircraft, are ignored.

Similarly, any orientation of the aircraft off of the ideal nominals is assumed to be zero. The aircraft in this model experiences no pitching and no yawing between the velocity vector and the heading vector.

It is assumed that there are no winds aloft that are affecting the flight of the aircraft. This is not realistic, but the aircraft dynamics in a winded orbit do not directly affect the applicability of the winds to the ballistic prediction.

3.1.2 Model Description

With the assumptions applied, the geometry of the orbit is controlled by few factors. A free-body diagram of the remaining forces, Fig 3, can be used to illustrate the system. The two most consequential forces acting on an aircraft are lift and gravity [6]. Gravity constantlypulls the aircraft downward relative to the local geographic reference frame. Lift constantly pulls the
When the aircraft is banked the lift vector can be decomposed into a vertical and horizontal force. Fig 3. Free-Body Diagram of Simplified Force of Flight.

To keep the aircraft flying at a constant altitude, the vertical component of lift must equal the force of gravity acting on the aircraft, such that

\[ \vec{F}_{\text{lift},y} = \vec{F}_{\text{gravity}} , \]  

(Eq. 1)

Newton’s second law states

\[ \vec{F}_{\text{gravity}} = mg , \]  

(Eq. 2)

where \( g \) is the acceleration due to gravity in meters per second-squared and \( m \) is the mass of the aircraft in kilograms.

If the aircraft flight is flat and level, the forces are balanced and no horizontal component exists. If the aircraft is banked, the airspeed over the wings must be high enough that the lift force’s vertical component can balance out the gravity force. There is a remaining horizontal component to the lift force when banked

\[ \vec{F}_{\text{lift},x} = mg \tan(\beta) , \]  

(Eq. 3)
where $\beta$ is the bank angle of the aircraft. This horizontal component of lift acts a centripetal force on the aircraft. To hold a constant turn radius, this force must balance with a centrifugal force. Substituting, we get

$$\vec{F}_{\text{lift},x} = mg \tan(\beta) = \frac{mv^2}{R}, \quad \text{(Eq. 4)}$$

where $v$ is the airspeed of the aircraft in meters per second and $R$ is the turn radius of the orbit in meters.

Rearranging we can find an equation to determine the turn radius of the orbit

$$R = \frac{v^2}{g \tan(\beta)}, \quad \text{(Eq. 5)}$$

This equation matches the pilot guidance for choosing flight nominals used by AC-130 pilots [6].

3.1.3 Model Factors and Parameters

Inputs into the flight model are limited to the nominals, as discussed in Chapter 2. Pilots will select a desired turn radius (Eq. 5) to the intended target for an engagement. Based on this desired range to target a set of flight nominals are chosen. The variables from Eq. 5 are the flight nominals, which along with the altitude of the aircraft control the shape of the orbit and the slant range to target. The derivation above serves to demonstrate that the only state variables needed to describe the aircraft for this simulation are the list of nominals, Table 1.

<table>
<thead>
<tr>
<th>Nominal</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Speed [m/s]</td>
<td></td>
</tr>
<tr>
<td>Bank Angle [deg]</td>
<td></td>
</tr>
<tr>
<td>Altitude [m]</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Flight Nominals
3.1.4 Model Verification and Validation

The implementation of the model was verified through code inspection and unit testing. Code inspection was performed to make sure that the equations were properly coded and that the inputs and outputs were of the proper form. Unit testing checked that known inputs produced expected outputs from the code.

Similarly, the inputs and outputs were validated against an independently generated table of nominals. This table of nominals is used to select effective nominals for weapon use in tactical situations [20]. The nominal tables were generated for use in tactical operations. The tables allow a pilot to select a desired turn radius and slant range and show the required nominals to achieve those range values. The results of the model used in this simulation matched the expected results from the independently generated table.

3.2 Ballistics

The forces acting on a projectile in flight are well known and studied in the fields of physics and aeronautical engineering. When implementing a ballistic model to describe the motion of a spinning projectile in flight one must choose the number of degrees-of-freedom (DOF) for the model [21].

The number of DOF chosen will control the complexity of the model. When dealing with exterior ballistics, the DOF refer only to those possible motions of the round which are physically modeled [22]. The maximum DOF in a ballistics model is six. This 6-DOF model would account for motion in all three spatial directions (as determined by the frame of reference chosen) and rotation about all three orientation angles (roll, pitch, and yaw). 6-DOF ballistics
models are generally high-fidelity models that are used to study the body orientation of the round in flight or to model flight control and guidance on a round.

It is possible to simplify an exterior ballistics problem to a model with four DOF (4-DOF) [21]. A 4-DOF model describes the motion of the round in all three spatial dimensions and allows for the rotation of the round around its central body axis. A 4-DOF model does not model the yawing and pitching motion of a projectile in flight as a true physical moment acting on the round’s body. Instead, a 4-DOF ballistic model simplifies the yawing and pitching motions into a single term, the yaw of repose.

The yaw of repose approximation assumes that the precession and nutation of the round early in its flight are very small magnitude and have no effect on the trajectory. After the precession and nutation have settled out, the spinning of the round will cause a yawing and pitching of the central axis of rotation for the round off of the velocity vector of the round. The Modified Point-Mass (MPM) model [21], a type of 4-DOF ballistic model, assumes that the yawing and pitching angles between these vectors can be combined into a single angular offset. This total angular offset is the yaw of repose, a steady state yawing and pitching of a gyroscopically stable round.

For this analysis, the exterior ballistics model designed and implemented is a version of the MPM 4-DOF model. The 4-DOF model was chosen as a basis for this research due to ease of coding and the general popularity of the model in both academic and defense applications.

3.2.1 Assumptions

The ballistics of the round is modeled with a 4-DOF model. It is assumed that drag, lift, Magnus, and gravity are the only physical forces that need to be modeled.
The model will terminate when the round is predicted to impact the ground. This implementation of the ballistic model assumes a flat Earth. The purpose of the analysis is to study the effects of winds on the trajectory of the projectile. The curvature of the Earth, whether spherical, ellipsoidal, or flat would have no effect on the predicted trajectory of the round.

The atmosphere is modeled using the International Civilian Aviation Organization (ICAO) standard atmosphere [23]. The ICAO atmospheric model is used to find the air density and speed of sound at varying altitudes. The ICAO atmosphere model assumes that any variations in air density or speed of sound due to variations in wind speed will be small and have little effect on the trajectory of the round when compared to the effect of the wind itself.

The implemented model assumes that there are winds acting on the round. The winds act in a horizontal plane, specifically the DR/CR plane of the gun frame. Vertical winds are assumed to be nonexistent. The actual model generating the wind values is separate from the modeling of the ballistics and is described in Section 3.3.

The model does not include the Coriolis force on the round as the total effect is assumed to be small [22].

3.2.2 Equations of Motion

The model used in this research is based on the ballistic model used in the NATO Armaments Ballistic Kernel [23]. This model is a 4-DOF MPM which models the forces acting on the round in a frame of reference aligned to the gun.

A common term appears in many of the equations of motion. For ease of notation and computation, this term is simplified by the following equation:
\[
Q = \left(\frac{\pi \rho d^2}{8m}\right),
\]  
(Eq. 6)

where \(d\) is the diameter of the projectile in meters, \(m\) is the mass of the projectile in kilograms, \(\rho\) is the density of the atmosphere in kilograms per meter-cubed.

The drag force is modeled by the following:

\[
\vec{D} = -Q \left( C_{D_0} + C_{D_a^2} \alpha_e^2 \right) v \vec{v},
\]  
(Eq. 7)

where \(C_{D_0}\) is the dimensionless zero-yaw drag coefficient, \(C_{D_a^2}\) is the dimensionless quadratic drag force coefficient, \(\alpha_e\) is the magnitude of the yaw of repose of the projectile in radians, \(v\) is the magnitude of the velocity vector relative to the air in meters per second, and \(\vec{v}\) is the velocity vector of the projectile relative to the air in meters per second.

The lift force is modeled by the following:

\[
\vec{L} = Q \left( C_{L_\alpha} + C_{L_{\alpha^3}} \alpha_e^2 \right) |v|^2 \vec{\alpha_e},
\]  
(Eq. 8)

where \(C_{L_\alpha}\) is the dimensionless lift force coefficient, \(C_{L_{\alpha^3}}\) is the dimensionless cubic lift force coefficient, and \(\vec{\alpha_e}\) is the yaw of repose vector for the projectile in radians.

The Magnus force is modeled by the following:

\[
\vec{M} = -Q dp C_{mag-f} (\vec{\alpha_e} \times \vec{v}),
\]  
(Eq. 9)

where \(p\) is the axial spin rate of the projectile around the body axis of symmetry in radians per second, \(C_{mag-f}\) is the dimensionless Magnus force coefficient.

The gravity force is modeled by the following:

\[
\vec{g} = -g_0 \begin{bmatrix} \frac{X_1}{R} \\ 1 - \frac{2X_2}{R} \\ \frac{X_3}{R} \end{bmatrix},
\]  
(Eq. 10)
where $R$ is the radius of the Earth assuming a spherical model ($R = 6.356766 \times 10^6$ m) and $g_0$ is the strength of the gravity vector at the origin of the gun frame,

$$g_0 = 9.80665(1 - 0.0026\cos(2\phi)), \quad \text{(Eq. 11)}$$

where $\phi$ is the geodetic latitude of the origin if the gun frame.

The total acceleration acting on the projectile at any given time is calculated using the following:

$$\ddot{u} = \vec{D} + \vec{L} + \vec{M} + \vec{g}, \quad \text{(Eq. 12)}$$

where $\ddot{u}$ is the total acceleration of the projectile with respect to the gun frame, $\vec{D}$ is the acceleration due to the drag force (Eq. 7), $\vec{L}$ is the acceleration due to the lift force (Eq. 8), $\vec{M}$ is the acceleration due to the Magnus force (Eq. 9), and $\vec{g}$ is the acceleration due to gravity (Eq. 10).

The spin of the projectile around its centerline of symmetry is the only rotational motion physically modeled in the 4-DOF model. The change in spin acceleration is modeled by the following:

$$\dot{p} = \frac{\pi \rho d^4 pv C_{spin}}{8I_x}, \quad \text{(Eq. 13)}$$

where $C_{spin}$ is the dimensionless spin damping moment coefficient and $I_x$ is the axial moment of inertia in kilogram meters-squared.

The yaw of repose is modeled by the following:

$$\overline{\alpha_e} = \frac{8I_x p (\vec{v} \times \vec{u})}{\pi \rho d^3 \left( C_{Ma} + C_{Ma3} \alpha \right) v^4}, \quad \text{(Eq. 14)}$$
where $p$ is the current axial spin rate of the round in radians per second, $\dot{\vec{u}}$ is the current acceleration vector in meters per second-cubed, $C_{M\alpha}$ is the dimensionless overturning moment coefficient, and $C_{M\alpha^3}$ is the dimensionless cubic overturning moment coefficient.

In Eq. 7 and Eq. 8 the higher order terms that depend on the yaw of repose are dropped. For example, the equation for drag can be expanded to include a quartic drag force effect due to the yaw of the round [23]. This and other similar contributions from higher power terms of the yaw of repose are assumed to be zero. Earlier study has determined that the Modified Point-Mass model is able to predict the flight path of a round accurately if the yaw of repose predicted in flight is 0.6 mrad or less [24]. A yaw of repose with such a small magnitude will have a negligible effect given the form of the quartic drag force term [23],

$$C_{D\alpha^4\alpha_e^4}.$$  \hspace{1cm} (Eq. 15)

3.2.3 Model Factors and Parameters

The 4-DOF model used requires input parameters to model a specific ammunition type. For this analysis, the PGU-13 A/B round type is used for all simulated shots. This round type is used in many air-to-ground systems [25]. The round description, including the aeroballistic coefficients and the physical constants, are taken from the Projectile Design and Analysis System (PRODAS) software suite.

Each round type has a set of physically measurable properties that do not change relative to the air mass the round is traveling through. These values are:
As the round travels through the air it will interact with the mass of air differently depending on the speed of the round relative to the speed of sound in the air mass. Each of the equations of motion above includes dimensionless coefficients that tune the equations to the round type selected. The values of these coefficients are the aeroballistic coefficients of the round. The aeroballistic coefficients are indexed by Mach value which is solved for in each iterative step as part of the ballistics model.

To simulate the flight of the projectile the state of the gun at time of fire is needed. These inputs include the altitude of the gun, the latitude of the gun, the current speed of the gun, and the gun’s inertial pointing angles. For this simulation, the altitude, latitude, and speed of the gun are taken as inputs from the aircraft model (Chapter 3).

3.2.4 Model Verification and Validation

The ballistic model implemented for this research was verified and validated to ensure accuracy. The model was verified via code review and unit testing. Code inspection verified that the ballistics model in the code matched the documented model.
A feature added to the model allows for a user to turn on or off individual forces and moments. This allows for unit testing of the model in a “build-up” manner; adding forces into the system and confirming that they act as expected. Testing confirmed that each force was acting as expected resulting in the motion associated with that force. Where possible, the results were verified against theoretical results (such as when gravity is the only acting force). Testing verified that the model is correct to within the limits of the documented model and the algorithms used in its implementation.

The flight path predictions made by the model were validated by comparison to other validated models. The PRODAS software has a built-in 4-DOF ballistics model and support for many ammunition types. Both PRODAS and the 4-DOF model developed for this research were used to produce surface-fire range tables with the same ammunition. The predicted DR and CR impact locations matched between the PRODAS table and one generated using the 4-DOF developed for this analysis. PRODAS is considered valid due to extensive testing and wide acceptance of the modeling suite for ballistics analysis.

The research model was similarly validated against the ballistics model used in tactical code for AC-130 gunships. The predicted final state of the round produced by the models were compared over 2000 random starting conditions. The model developed for this research produced predicted impacts which match the tactical code’s predicted impacts to within machine truncation limitations. The tactical code is considered valid due to years of successful use engaging hostile forces in combat situations and validation during testing at Dahlgren.
3.3 Wind Modeling

Two different wind models were used in this research: a static wind model and a measured wind model. During simulation, the static wind model was used both for code development and validation and to simulate the ballistic wind which results from the current method of wind prediction in AC-130 tactical systems (Section 2.3). The measured wind model was used to introduce dynamic winds which are closer to reality than the static wind model. The wind models were applied to the ballistics model (Section 3.2) in separate simulations and used to modify the velocity of the round relative to the air stream in the equations of motion.

3.3.1 Assumptions

For both models, it is assumed that the vertical wind speed is 0.0 m/s. The vertical winds tend to be very low so this assumption does not cause any large errors. It is common for wind measuring systems to use vertical winds as a validation; low to nonexistent vertical winds are considered an indication that the measuring system is functioning as expected [26].

Both models also assume that the winds do not change over time. Again, this is not strictly true, but for the sake of analysis the winds are held constant.

3.3.2 Model Description

Static winds can be generated with speed up to 100.0 m/s in any direction. The 100.0 m/s limit is close to the highest observed wind speed [27]. This highest observed value was chosen as the limit to test the system in as broad a range as possible. The static wind column generated by the model will have the same wind speed and direction at all altitudes.
Measured winds are produced off of meteorological balloon data. This met balloon data are actual data that was recorded during previous testing at the Naval Surface Warfare Center, Dahlgren VA. The wind speed and direction at altitudes are modified only to add a wind speed of 0.0 m/s at the ground.

For both models, the vertical winds are 0.0 m/s.

3.3.3 Model Factors and Parameters

The wind speed and direction of the static wind column can be set either programmatically or using configuration settings. Measured wind columns are chosen based on which set of met balloon data are to be used. Once chosen, no other user input to the wind model is required.

3.3.4 Model Verification and Validation

The wind models were validated by inspecting the results of the applied winds on the impact predicted by the ballistic model. When a static wind was applied, the predicted final impact of the round moved in the direction expected and by the rough magnitude expected. There is no way to directly predict how far a given wind will push a round without using a ballistic model. The validation tests confirmed that larger wind magnitudes moved the round farther than smaller magnitude winds.

The format of the data output by the wind model for the measured winds was verified to match the format used by the static model. The measured winds can be applied to the ballistics model and testing confirmed that the final impact was moved by the winds. Given the dynamic
nature of the measured winds, it is not possible to validate based on direction or magnitude of the induced impact miss distance.

3.4 Tracker Model

The technology to track a round in flight exists. Different methods and devices exist to track the round. Regardless of the method the expected output data from a tracking system is the same. A tracking system must detect the round and provide relevant position and velocity data about the round in a relevant reference frame.

The exact method of detection and measurement is not relevant to this research. The ability to use the resulting positional and velocity data is what matters. Given this, the model developed for this research ignores the specific methods and any idiosyncrasies they may have and focuses on the production of valid tracking data for the projectile in flight.

3.4.1 Assumptions

The tracker model assumes that any round tracking device used in a tactical application would report the position and velocity of the round.

It is assumed that a real-world application of the tracker would be a separate piece of hardware from the rest of the gun FC system. As a separate configuration item, any model meant to recreate the tracker output must be a separate software process. This controls the availability of data in the system. All data coming into or out of the tracker model are controlled by defined network messages.

The messages sent by the tracker model are limited. Any real tracking hardware would have to share network bandwidth with other devices. This limits the size of the message that can
be sent by the tracker to the wind prediction model. Attempting to send a flight path for a projectile which consists of 1000s of data points may bog down a network and prevent other traffic from getting where it is needed.

The tracker model is further assumed to base its data on the full predicted ballistic flight path with winds applied. The tracker model must know the entire path and then down-select the data points to produce a track that is smaller.

3.4.2 Model Description

The tracker model uses the predicted flight path of the round produced by the ballistics model with winds generated by the wind model applied. The trajectory of the round is produced by the ballistics model to a granularity controlled only by the integration time step chosen.

The tracker model uses the full trajectory to generate a “tracked” flight path. The number of data points in the track is user configurable. The data are then used to populate a message which is sent over a multicast network. The messages generated by the tracker model contain the positions and velocities of the round in flight and the initial gun state. The initial gun state data includes the ammo type, initial geographic position, aircraft speed, aircraft course, and the inertial azimuth and elevation of the barrel of the gun. Additionally, a value is included to indicate the number of tracked positions in the message. The tracker positions are included as an array of latitude, longitude, and altitude values for the number of selected data points.
3.4.3 Model Factors and Parameters

For the purposes of all simulation in this research the number of data points produced by the tracker model was set to 10. This number was selected to test the possible improvement seen when tracking comparatively few data points.

The tracker model relies on the ballistics model and the wind model. The ballistics and wind models each have their own inputs and controls. The tracker model itself does not control the parameters of these other models.

Network messages can be sent to the tracker model to make it produce a track and send a track.

3.4.4 Model Verification and Validation

Model verification was performed to ensure that the tracker model would run as expected and send the network message expected. Testing confirmed that the tracker model produced an array of positions on command and sent those points in a message of the expected size to the wind prediction model.

The tracker model’s output was validated by inspection. Multiple ballistic flyouts were generated with random initial conditions and wind column applied. The resulting full trajectory was recorded. The trajectory was then processed with the tracker model which produced an array of points simulating the tracker results.

The tracker model produced the proper number of positions as selected for each run. The positions in the tracker data were compared to the full trajectory. The tracker values matched the full trajectory values.
3.5 Wind Prediction Model

Wind effects on the round result in both an epistemic and aleatory error in the predicted flight path and final impact of the round. Winds pushing on the round will cause the round to miss the intended target. This error is not accounted for in the initial pointing angles of the gun. If the winds from the starting point of the round in flight to the ground were perfectly known, then they could be input in the ballistics model and their effect could be accounted for when predicting the pointing angles needed to get a round to impact a target.

The epistemic nature of the error caused by winds arises from the fact that winds are slow to change. The wind column will vary over time, but the ballistic effect of the wind is generally the same over short periods of time. This has allowed for successful prediction of ballistic winds in tactical applications in the past.

3.5.1 Assumptions

The wind column can be predicted based on the observed location and velocity of the round in flight. The model in this research assumes that there are no errors other than unaccounted for winds affecting the flight of the round. In the real world this is not true, but the other errors tend to manifest themselves in the platform relative frame of reference whereas the wind errors manifest themselves in the world relative reference frame. Methods exist to separate the platform relative errors from the world relative errors. Here, it is assumed that all platform relative errors have been accounted for, leaving only the wind induced errors.

This model is not intended to solve for the true winds. The model will solve for ballistic winds between the initial point and the final location used. This final location can be anywhere along the trajectory of the round including the final impact on the ground. The smaller the
distance between the initial and final points, the closer the predicted wind should be to the actual winds acting on the round.

3.5.2 Model Description

The wind prediction model predicts winds using a two dimensional bisecting search algorithm [28]. Using a set of initial conditions for the round and a final winded location winds are iteratively applied to the ballistics model to find a set of East and North winds that push the predicted final location of the round towards the winded location. The model is said to have predicted the correct ballistic winds when the distance between the predicted final location and the winded location is smaller than some specified distance, the closure tolerance.

In various locations in this thesis, the successful termination of this search algorithm is referred to as “closure” or “closing” on the solution. This is used to mean that the search algorithm has converged on to the correct answer.

The search algorithm was modified for this application from its standard form. A standard bisecting search converges on the correct solution poorly when the axes of the search space are not fully aligned with the axes of the metric being closed on. Here, the search space is defined over a range of possible East and North winds. The model searches through that space to minimize a DR and CR miss distance in the gun reference frame. The East/North winds can be rotated into the gun frame to act on the rounds as a combination of headwind and crosswind.

If the headwind/crosswind effects on the round were perfectly aligned, then a headwind would only affect the DR portion of the projectile’s flight and the crosswind would only affect the CR portion of the projectile’s flight. The total DR and CR motion of the round are not independent, however. They are cross correlated; each depending on the total time of flight of
the round. For example, a round in flight experiencing a headwind will have more drag applied to it resulting in a reduced time of flight. This reduced time of flight gives the CR forces (Magnus and lift) less time to act on the round, reducing the total CR deflection even though there is no cross-wind [22].

A bisecting search does not account for this cross-correlation. The search algorithm was modified for this application to account for the cross-correlation. The standard form of the bisecting search limits each search axis by one-half on each iteration through the search. The modified method applies a multiplicative increase onto the resulting limited search space. This has the effect of “bumping out” the limited search space each iteration and reduces the chance that the winded location ends up outside of the search space due to cross correlation.

The wind prediction model yields a ballistic wind valid for that range of altitudes between the initial and final points supplied to the model. To be used, the closure tolerance and cross correlation correction coefficient (CCCC) values must be set appropriately (see Chapter 4).

3.5.3 Model Factors and Parameters

In order to predict a wind vector, the wind prediction model requires the initial state of the projectile and a final location for the projectile. The initial state of the projectile includes the following,
Table 3. Projectile State Data

<table>
<thead>
<tr>
<th>Initial State Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Velocity Vector</td>
</tr>
<tr>
<td>Accelerations</td>
</tr>
<tr>
<td>Orientation</td>
</tr>
<tr>
<td>Spin</td>
</tr>
</tbody>
</table>

The search space is limited to ±100.0 m/s of wind speed in both the East and North directions. This speed is likely excessive for this analysis and any practical application. It was chosen here because it should prove significantly higher than almost any true winds that would be encountered. At worst, starting a search space wider than needed increases the number of iterations needed to close on the ballistic winds. In a practical application of this wind prediction model, the search space can be set narrower to reduce the number of calculations performed.

There is the possibility of the search algorithm failing to find a ballistic wind that can account for the observed location of the round. This could happen if the required ballistic wind exceeds the limits of the search space or if the search fails to account for the cross-correlation of the data as discussed above. It is best to prevent such a failure from occurring by properly tuning the model parameters. To further ensure that the model as coded does not continue to search for a solution when it cannot possibly close, a hard 50 iteration limit is placed on the search algorithm.
3.5.4 Model Verification and Validation

The wind prediction model was both verified and validated through extensive testing. The model was run with single-point impact data with random winds to calculate a ballistic wind for the entire wind column.

Testing confirmed that the wind prediction model was able to consistently predict the winds based on an input tolerance to the tweak closure. Adjusting this tolerance to require that the predicted wind-induced impact to be closer to the observed sample impact forced the wind prediction to be closer to the actual applied winds. The reverse was also observed; increasing the tolerance allowed the predicted wind to be less accurate when compared to the applied winds. This verified that the tweak process not only found the correct wind values, but that the tolerance control applied to the tweak closure performed as expected.

For some test conditions it was noted that the tweak model failed to predict winds correctly. This is due to the cross-correlation of the data. Increasing the CCCC value would allow the search algorithm to account for the cross-correlation between headwind/crosswinds and the DR/CR effect on the final impact. The details of the setting of the CCCC for the tweak closure is detailed in Chapter 4.
CHAPTER 4

TUNING WIND PREDICTION MODEL PARAMETERS

After coding the models described above, values had to be chosen for the CCCC on the wind prediction search algorithm and the wind prediction closure tolerance.

The CCCC must be tuned to allow the wind prediction model to correctly predict the wind speed values. As described above, a headwind acting on a round has both a DR and CR effect on the final impact. Similarly, a crosswind acting on a round affects both direction of travel. This cross-correlation can cause the wind prediction model to converge to an incorrect set of wind speeds.

It is possible to correct for this cross-correlation by rotating the impact data from the DR/CR frame to a headwind/crosswind frame. The exact nature of the rotation and value needed depends on the entire state of the round and the winds at time of fire. This calculation is complex and assumes a knowledge of the winds that the modeler would not have. A simpler solution is to increase the size of the search space enough on each iteration to cover the cross-correlation.

The exact value of the CCCC was chosen to allow the wind prediction to solve for the correct wind values while still collapsing the search space quickly. Small values for the CCCC may still allow the cross-correlation to prevent the wind prediction model from converging on the correct values. Large values for the CCCC may eliminate the problems caused by cross-correlation but require more iterations of the search algorithm to complete the search due to the size of the search space after each iteration.

The wind prediction tolerance controls when the wind prediction model will terminate its search. This setting is a distance; if the predicted winds allow a round to fall within the specified
distance of the measured impact, then the wind prediction is said to be good and the search terminates.

The accuracy of the wind prediction is controlled by the tolerance chosen. Using a large distance for the tolerance will allow the predicted winds to be farther off of the actual winds. Using a small distance for the tolerance will force the predicted winds to be closer to the actual winds. But, choosing a tolerance too small will cause the wind prediction model to take longer to converge on the correct solution, sacrificing speed for accuracy.

Additionally, setting the tolerance to a very small value may not be practical. There is a limitation to the ability to measure the impact location of a round. If the precision on the measured impact location is ±0.1 m, then closing to a tolerance less than ±0.1 m is attempting to converge to a location that is not be accurate. The round itself has a specific diameter of 0.03 m for this research. Closing with a tolerance of 0.015 m is sufficient to insure that the round would hit the target.

Conversely, setting the tolerance to a very small value may force the wind prediction to be more stable and less susceptible to changes in state. As shown below, at shorter times of flight the tolerance has a strong effect on the accuracy of the wind prediction and its validity when used at different slant ranges. This may force the tolerance to be a smaller value than practical considerations would suggest.

4.1 Simulation Description

The same type of simulation was used to tune both the wind prediction closure tolerance and the CCCC values. A stochastic simulation with 500 runs was performed. A stochastic simulation was used in this simulation to give better coverage of the possible range of flight
nominals and gun state. Given the possible variations in initial state over all of the initial state variables using a stochastic method which randomly generates the state variable values at each run helps to ensure that the testing better covers the total range of possible values. In a real-world setting, a wind prediction system like the one modeled here would have to be able to perform under any initial conditions within an expected range. A stochastic simulation is the easiest way to recreate that type of environment.

The simulation used a static wind column. The static wind column has the same wind speed and direction for all altitudes. This is not a realistic model of the wind but it is useful for testing and tuning the models. Additionally, the static wind column is a common model used to correct wind errors applied to guns. This type of wind model when applied to a specific gun and round type is referred to as ballistic winds, an averaging of the effects of the real winds into a single set of wind values.

Wind speeds are randomly generated for each simulation run. The possible value for the east and north winds is taken from a uniform distribution of ±100.0 m/s. This speed limitation is based on the highest measured surface wind speed. The highest possible wind speed in this simulation is 141.4 m/s which would be applied as a constant wind over the flight of the round. This ballistic wind is not realistic. Category 5 hurricanes have a sustained wind speed of at least 70.0 m/s [29]. The upper limit for the wind speed in this simulation is specifically set to exceed the maximum possible to ensure that the models are stable and valid at higher speeds. A good wind prediction model should be capable of calculating wind speeds even if they fall outside of the expected range of real wind speeds.

The flight nominals of the aircraft were varied randomly as well. The flight model of the aircraft and the modeling of the gun pointing reduce the number of settable variables to the
following table. The values for each variable were selected from a uniform random distribution between the values shown in Table 4.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>6000 to 20000</td>
</tr>
<tr>
<td>Gun Quadrant Elevation</td>
<td>-60 to -10</td>
</tr>
<tr>
<td>Aircraft speed</td>
<td>100 to 250</td>
</tr>
</tbody>
</table>

The simulation was run as a stochastic simulation to ensure that the possible combinations of initial state were covered as well as possible. The ranges selected for all of the variables include but are not limited to those possible values seen in actual gunfire missions. At each set of randomly generated initial conditions and winds a single ballistic flyout is run, creating a winded impact location. This winded impact location and the initial gun and aircraft state are then used by the wind prediction model to predict a ballistic wind which accounts for the observed offset of the impact from the expected no-wind impact location. It is expected that the predicted winds will closely match the static wind model values used in each run.

4.2 Initial Tuning of the Cross-Correlation Correction Coefficient

Multiple sets of simulation data were collected to analyze the effect of changing the CCCC value on the wind prediction model’s ability to close on the proper wind speeds. For each of the data sets the wind prediction closure tolerance was set to $10^{-16}$ m. This value was selected because it would force the wind prediction model to terminate on maximum number of iterations, or 50 iterations, through the wind search space. This reduced the possibility that any
errors in the wind prediction are from the closure tolerance. Any errors that are outliers from the rest of the data set are due to the cross-correlation as described above.

The initial run set the CCCC equal to 1.0, meaning the wind prediction model was not trying to account for the cross-correlation. The east and north winds predicted for each run are then compared to the applied winds for that run and a radial wind error is computed. That radial wind error is plotted against the time of flight for the round in each run, as shown in Fig 4.

![Fig 4. Radial Wind Error with CCCC=1.0.](image)

Most of the runs have a very low error, so low that the scale of the plot obscures the exact magnitude. Of note are the few data points which show data runs with higher radial wind errors. These errors remain after the wind predictor model had completed 50 iterations through the bisecting search algorithm. They are due to the unaccounted for cross-correlation.
More data were generated increasing the CCCC value by 0.01 for each up to CCCC=1.1. Data sets for CCCC values up to 1.07 are shown in Fig 5. Note that for CCCC values of 1.05 and higher, the outliers have been eliminated from the radial wind errors. This indicates that the CCCC value is sufficiently high to account for the observed cross-correlation between the DR/CR impacts of the round and the headwind/crosswind effects on the round.
Fig 5. Wind Errors versus Time of Flight at Varying Values of CCCC.
4.3 Tuning the Wind Prediction Closure Tolerance

The wind prediction model’s closure tolerance was investigated next. The CCCC value used during the data generation for this portion was 1.1. This value is higher than the apparent lower possible value of 1.05 found in Section 4.2. The higher CCCC value was chosen for this portion of the research to ensure that the cross-correlation problem would not affect the results as the number of data points generated in each set increased.

The number of data points generated in each run was increased from 500 to 5,000. The simulation was run as described above and radial wind errors were calculated. An initial data set was generated with a closure tolerance of 0.01 m. The results are plotted in Fig 6.

![Fig 6. Radial Wind Errors over Time of Flight with Closure Tolerance of 0.01 m.](image)

Note the shape of the curve to the data. This curve is expected. Consider the situation where a round is fired from a very short distance. With a low time of flight the winds would have very
little time to affect the round and changes its trajectory. A round with a longer time of flight will have a longer time for the winds to affect the trajectory. This means predicting winds for rounds with a longer time of flight requires more accuracy to meet a closure tolerance than such predictions would require for rounds with shorter times of flight.

This relationship established a baseline for the validity of the wind correction as the time of flight of the round changes. A prediction made based on firing at a lower time of flight can have more error in it than one made at a longer time of flight and still fall within tolerance. If a prediction is made at a lower time of flight, then the aircraft ascends and attempts to fire accurately using the prior wind prediction, there is a chance that the round will fall outside of the tolerance based only on the effects of the wind prediction errors playing out over a longer time of flight.

This simulation was repeated with varying closure tolerances and a pattern was observed. All of the data sets showed the same curved pattern as above, Fig 6. The edges of the curves for each data set were isolated and trendlines calculated to fit the maximum edge of each of the curves. The best fit was achieved with a power curve.

\[ y = ax^b. \]  

(Eq. 16)

The values for the fitting constants, \( a \) and \( b \), found at varying wind prediction tolerances showed a clear relationship to each other as shown in Table 5.
Table 5. Curve Fitting Constants at Varying Closure Tolerances

<table>
<thead>
<tr>
<th>$d_{tol}$ [m]</th>
<th>$a$</th>
<th>$b$</th>
<th>$c = \frac{a}{d_{tol}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.331439579</td>
<td>-1.357146875</td>
<td>4.331439579</td>
</tr>
<tr>
<td>0.5</td>
<td>2.216156149</td>
<td>-1.364725952</td>
<td>4.32312298</td>
</tr>
<tr>
<td>0.1</td>
<td>0.437442152</td>
<td>-1.359738326</td>
<td>4.37441522</td>
</tr>
<tr>
<td>0.05</td>
<td>0.221737097</td>
<td>-1.366364296</td>
<td>4.33741948</td>
</tr>
<tr>
<td>0.01</td>
<td>0.04373935</td>
<td>-1.359060356</td>
<td>4.373934976</td>
</tr>
<tr>
<td>0.005</td>
<td>0.021994055</td>
<td>-1.362480741</td>
<td>4.373934976</td>
</tr>
<tr>
<td>0.001</td>
<td>0.004424346</td>
<td>-1.363123512</td>
<td>4.42346263</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.002193684</td>
<td>-1.362639354</td>
<td>4.387367296</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.000429472</td>
<td>-1.353061278</td>
<td>4.294720087</td>
</tr>
<tr>
<td>0.00001</td>
<td>4.29354E-05</td>
<td>-1.352526276</td>
<td>4.293535625</td>
</tr>
</tbody>
</table>

Based on the pattern in the fitting constants a generalized equation was made to describe the relationship between the maximum possible wind error and the time of flight of the round which would fall within a specified closure tolerance

$$
\varepsilon_{wind} \leq 4.374563 \cdot d_{tol} \cdot \tau^{-1.36009}, \quad (Eq. 17)
$$

where $\varepsilon_{wind}$ is the radial wind error in meters per second, $d_{tol}$ is the wind prediction tolerance in meters, $\tau$ is the time of flight in seconds, and the fitting parameters are set based on the data in Table 5.

The closure tolerance was then set using Eq. 17. For practical considerations, the closure never needs to predict a wind that would move the round any closer to the measured impact than one-half of the width of a man-sized target. This would ensure a direct hit onto the target assuming that all other errors were accounted for at the time of fire. Based on small-arms target practice standards [30], the width of a man-sized target is 0.45 m.

The longest predicted time of flight, 40.0 s, was used. Knowing the time of flight and the closure tolerance the above equation can be used to find an upper bound to the wind prediction error.
This upper bound can then be used at a lower time of flight, in this case 2.5 s, to calculate a closure

tolerance, given

\[ a d_1 \tau_1^b \geq \epsilon_{\text{wind}} \leq a d_2 \tau_2^b, \]

\[ d_1 \tau_1^b = d_2 \tau_2^b, \]

\[ \frac{d_1 \tau_1^b}{\tau_2^b} = d_2, \]

(Eq. 18)

\[
\frac{(0.225)40.0^{-1.36009}}{2.5^{-1.36009}} = d_2 = 5.182 \times 10^{-3} \text{ m}.
\]

This closure tolerance, 5.182 \times 10^{-3} \text{ m}, is enough to ensure that a wind prediction calculated
based on shots with a time of flight of 2.5 s will still result in rounds impacting within 0.225 m if
fired with a time of flight of 40.0 s.

### 4.4 Final Tuning of Tolerance and CCCC

In Section 4.2 the CCCC was investigated and a range of possible values was determined. That data showed that a CCCC value of 1.05 or larger was sufficient to remove the outliers due to cross-correlation between the head/crosswinds and the DR/CR impacts of the round when a sample
of 500 data points is used. The wind prediction closure tolerance was set to a small value, 10^{-16} \text{ m},
to ensure that the wind prediction went through as many cycles as possible.

Based on the closure tolerance tuning in Chapter 4.3, the CCCC was reexamined. The sample size was increased from 500 shots to 5,000 shots to cover more initial states of the gun and aircraft. It is possible that there are states that were not covered with 500 sample shots that would show the same outliers seen with lower CCCC values. The stochastic nature of the data generation was controlled to ensure that the same states were generated for each CCCC value and that the first
500 states tested matched the states in the prior analyses. New data sets were generated with CCCC values ranging from 1.05 to 1.1 in steps of 0.01.

The goal of the analysis was to find that CCCC setting which eliminates the cross-correlation outliers while minimizing the number of iterations the wind prediction model executes to close to within the specified tolerance. At each CCCC value, the number of iterations to close was recorded and compared to subsequent runs. The expectation is that larger CCCC values open the search space and lead to more iterations overall.

Runs with CCCC = 1.05 revealed outliers in runs beyond number 500 (Fig 7). These three data points indicate that a larger CCCC value is required to reduce the chances of seeing a failure to close properly due to cross-correlation.

Fig 7. Outliers at CCCC=1.05 with 5,000 Samples.
At CCCC = 1.06 no outliers were apparent from the data. This held true for all CCCC values larger than 1.05 investigated.

The numbers of iterations required for the wind prediction to close to within the specified tolerance at a given CCCC were compared against the number of iterations required at CCCC = 1.06 (Table 6).

<table>
<thead>
<tr>
<th>CCCC value</th>
<th>Change in average iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.07</td>
<td>+0.2528</td>
</tr>
<tr>
<td>1.08</td>
<td>+0.4664</td>
</tr>
<tr>
<td>1.09</td>
<td>+0.695</td>
</tr>
<tr>
<td>1.1</td>
<td>+0.8958</td>
</tr>
</tbody>
</table>

No benefit was observed with CCCC values greater than 1.06. The number of iterations required by the wind prediction model increased on average as the CCCC value increased, though the increase was small.

4.5 Conclusion

Initial runs of the simulation confirmed that the wind prediction model performed as expected. It was able to close on a single-point wind value to within the specified tolerance, verifying the model’s functionality.

Tuning tests were performed to find and set the values of the cross-correlation correction coefficient and the wind prediction closure tolerance. The CCCC was set to 1.06. This setting was sufficient to eliminate all outliers in a 5,000 sample data set.
The closure tolerance was set to $5.182 \times 10^{-3}$ m. This value was selected based on the behavior of the data showing the relationship between radial errors in the wind prediction based on the time of flight of the projectile. This tolerance was selected to ensure that the effects of changes in state which would affect the time of flight of a round would induce no more than 0.225 m of possible miss distance so to poor wind prediction.
CHAPTER 5

SINGLE-POINT PREDICTION OF BALLISTIC WINDS

The single-point wind prediction model can be used with a constant value wind model, as shown in Chapter 4, or it can be used against a measured wind model. When tested with a constant wind model it was shown that the wind prediction model generates a wind speed and direction (or East and North wind speeds) which match the constant wind model speed and direction to within an error tolerance based on the closure tolerance distance used in the wind prediction model as described by Eq. 17.

In this chapter, single-point wind predictions are made at varying initial states using multiple measured wind models. This data is used as a baseline of current wind prediction capabilities. Later chapters will use these single-point ballistic wind predictions to compare to wind predictions made using data from a round tracking sensor.

5.1 Simulation Description

For this simulation sixteen measured winds were used as the winds applied to the round in flight. These winds were measured using a radiosonde meteorological balloon. The measurements were taken on different four different days at the Naval Surface Warfare Center at Dahlgren, Virginia. A sample wind profile showing the East and North wind speeds at altitude is shown in Fig 8.
Fig 8. Wind Profile 1.

A stochastic simulation was run with each of the 16 wind profiles. Each simulation consisted of ballistic impact predictions made with 5000 different initial conditions. The gun altitude, aircraft speed, and total gun depression angle were generated for each of the runs from a uniform distribution with the limits shown in Table 7.

<table>
<thead>
<tr>
<th>Variables and Ranges</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude [m]</td>
<td>6000 to 20000</td>
</tr>
<tr>
<td>Gun Quadrant Elevation [deg]</td>
<td>-60 to -10</td>
</tr>
<tr>
<td>Aircraft Speed [m/s]</td>
<td>100 to 250</td>
</tr>
</tbody>
</table>

The random number generator seed was controlled to ensure that the same 5000 states were used with each wind profile. The 5,000 states also matched the states used in the analysis in Chapter 4.
At each of the 5,000 initial states, a ballistic flyout was performed with measured winds applied. The winded impact location of the round was recorded. The initial state and the winded impact location were used by the wind prediction model to generate ballistic wind that would account for the observed offset of the winded impact location from the expected no-wind impact location. The closure tolerance used for all simulation runs was $5.182 \times 10^{-3}$ m, as used in Chapter 4. The result is a ballistic wind which holds the same speed and direction from the ground up to the altitude of the gun at time of fire.

5.2 Results

The wind prediction model was able to solve for a ballistic wind on all 5,000 runs for each of the 16 measured wind profiles. This was verified in two ways.

First, the total number of iterations required to close on a ballistic wind to within the closure tolerance was recorded for each run. The maximum possible number of iterations allowed by the model for each attempt at finding a ballistic wind was 50. The minimum number of iterations used for any of the runs was 10; the maximum was 22. These values are well below the maximum of 50 runs allowed. If the wind prediction model had failed to close to within the closure tolerance distance specified, then the model would have continued to iterate through the search space until the search reached the maximum number of iterations. The fact that no run ever required close to 50 iterations to complete indicates that the wind prediction model successfully closed on a ballistic wind.

Second, each ballistic wind prediction was tested to ensure that the resulting impact fell within the closure tolerance of the initial winded impact used as input to the wind prediction model. The ballistic wind result was applied to the round in flight and another ballistic flyout was
performed. The new impact location was compared with the initially generated winded impact location and the distance between them was calculated. For all 80,000 data runs the distance between the new impact location and the original winded impact location was within the closure tolerance.

Displaying the results of all 80,000 runs is difficult. Even limiting the data to a single wind profile out of the 16 tested profiles does not help as each profile was used to test 5,000 different initial states. To better investigate the data, it is simpler to select a few ballistic wind profiles based on states at varying altitudes and plot those against the measured wind profiles. An example of the wind prediction made for three sample states out of the set of 5000 for Wind Profile 1 is presented in Fig 9. The black lines represent the East and North wind speeds as measured at varying altitudes. The red lines and points represent the ballistic wind predicted for a given simulated firing event. Each prediction was made based on a shot taken at the altitude of the top red dot for a given line. The East and North wind speeds are constant through the entire wind profile from the initial altitude to the ground for these ballistic winds.

An inspection of the results shows that the ballistic wind profile tends to fall close to the average of the wind speeds from the starting altitude to the ground at 0.0 m. For example, the highest altitude ballistic wind profile for the North wind speed has a value of $-4.41 \text{ m/s}$. The average wind speed from that same altitude to the ground is $-5.41 \text{ m/s}$. The values are close but not exact. This is expected due to the physical effects of the wind on the round, which will change based on the speed of the round. The state of the round, such as the air speed of the round, changes as the altitude decreases. The change in air speed relative to the speed of sound will make the ballistic wind diverge from the average wind speed due to the increased drag force experience in the transonic region of flight [21].
five8

Fig 9. Three Representative Ballistic Winds with Wind Profile 1.

A visual inspection [31] also shows that the values make intuitive sense. The measured winds have regions where wind speeds fall on either side of the predicted ballistic wind speed. This indicates that the ballistic wind profile is an attempt at balancing out the effects of the dynamic measured wind profile with a single value. The East wind speed graph has all three wind predictions grouped closely together. Visual inspection of the measured East wind shows that the wind speeds at almost all altitudes were between 5 m/s and 10 m/s. It is expected that the predicted values would fall in that band of wind speeds, which is what the results show.

Plotting similar data for all 5,000 ballistic winds for a given measured wind profile would do little more than fill the graph with red lines. A graph which shows only the top of the ballistic wind
profile is more readable. The points on such a graph, Fig 10, represent the entire ballistic wind but are only shown at the initial altitude where the prediction was made.

The ballistic winds are expected to change as the altitude changes. Changing the altitude of the initial fire changes the amount of atmosphere that the round flies through. The ballistic wind will necessarily change based on certain parts of the measured wind profile being included or excluded by the starting altitude.

What is more interesting is that the ballistic winds are not the same at the same altitude. The spread of the points at a given altitude indicates that some factor other than altitude is causing a change in the expected wind effects on the round in flight. The gun elevation, which was also allowed to vary for the data points shown, and the dynamics of the measured wind profile itself are the factors which cause the spread in the ballistic winds at a given altitude.

Gun elevation will change the slant range to the impact location and the time of flight of the round. The measured winds have a different effect on a round which takes longer to reach the ground than on one with a shorter time of flight. If fired from the same altitude, the measured winds affecting the round are the same but the state of the round varies in other ways. Rounds with a longer time of flight will have a lower airspeed at each altitude than rounds with a lower time of flight. The equations of motion used to model the flight of the round depend on airspeed to calculate the forces acting on the round. Thus, even though the air column is the same for both steep and shallow shots, the round will experience those winds differently, which leads to a different prediction of the ballistic wind.
The dynamics of the measured wind also affect the spread in ballistic wind predictions. The East winds in Fig 10 show little variation from about 5,500 m to 250 m of altitude. This leads to a very narrow spread in the speeds of the predicted ballistic winds in that band of altitudes. The measured North wind speeds show more variation which leads to a greater spread in the ballistic winds at a given altitude. This same feature hold for all 16 tested wind profiles, as can be seen in Fig 11 through Fig 26.

5.3 Conclusion

From the data detailed above, it is clear that the wind prediction model is capable of finding a single-point ballistic wind that accounts for the miss distance when a measured wind is applied to the round. In Chapter 4, the wind prediction model was tested using a static wind model. Here,
dynamic winds based on real winds as measured by a meteorological balloon were used to induce a
miss distance in the final impact. The miss distance and the initial state were used to predict a
ballistic wind to correct for the cumulative effect of the measured winds.

In Chapter 4, it was expected that the ballistic winds would match the randomly generated
winds to within some error metric. In this chapter, the ballistic winds do not match the input winds
due to the nature of the wind model used to generate the ballistic wind. Features of the ballistic wind
were used to confirm that the results were correct.

The predicted speeds of the ballistic winds are mostly controlled by the measured wind
speeds used as inputs. The speeds of the ballistic wind also vary based on the state of the gun at the
time of fire. The initial altitude is a strong controller. This is evident from the East and North speed
predictions changing as the altitude changes.

The initial altitude is not the only controller, though. The spread in ballistic wind values at a
given altitude indicate that something else beyond the altitude is affecting the ballistic wind. The
gun elevation, which controls the time of flight of the round, changes the state of the round at a
given altitude. This difference in state leads to different interactions with the atmosphere. The
ballistic wind prediction will change based on the time of flight and the variability of the
atmosphere.

Based on the variations in the ballistic wind values seen in the graphs it is expected that
predictions will only be valid if the gun does not change state greatly. This is not a reasonable
expectation in flight. Any state change which causes a round to have a different time of flight than
the firing event used to make the wind prediction may render the ballistic wind invalid, or at the very
least less valid.
The data generated in this chapter will be used as a point of comparison in later chapters. The results of a multipoint ballistic wind prediction method will be compared to this single-point data to determine which method better models the winds and which method is less prone to errors induced by changes in state.
Fig 11. Wind Profile 1.

Fig 12. Wind Profile 2.

Fig 13. Wind Profile 3.

Fig 14. Wind Profile 4.
Fig 15. Wind Profile 5.
Fig 16. Wind Profile 6.
Fig 17. Wind Profile 7.
Fig 18. Wind Profile 8.
Fig 19. Wind Profile 9.

Fig 20. Wind Profile 10.

Fig 21. Wind Profile 11.

Fig 22. Wind Profile 12.
Fig 23. Wind Profile 13.

Fig 24. Wind Profile 14.

Fig 25. Wind Profile 15.

Fig 26. Wind Profile 16.
CHAPTER 6
MULTIPOINT WIND PREDICTION

The single-point wind prediction method was shown to work as expected and make ballistic wind predictions which account for the observed wind induced miss distance to within the closure tolerance. The method can be used to predict winds under varying initial gun and aircraft states. Results from the previous chapter show that the ballistic wind speeds vary based on the initial conditions at the time of fire even when fired through the same wind column.

During a live-fire event, the state of the gun and aircraft is constantly changing. This change in state may reduce the ability of the single-point ballistic wind speeds to correct for the actual wind effects. This possibility is due to the limited number of data points being used to predict the winds, using only the initial and final locations of the projectile. A method which uses more data, if available, is expected to generate a predicted wind that better matches the true winds acting on the round.

This chapter proposes a method to model winds accurately based on increased information about the round in flight. A round tracking sensor is modeled to produce location and velocity data about the projectile. This information is used to generate a prediction of the wind speeds acting on the round.

The closeness of the multipoint wind predictions are compared to the measured wind profiles. The metrics derived are then compared to similar metrics calculated using the single-point wind prediction. It is shown that based only on closeness of fit the multipoint wind prediction method produces wind predictions which are a much closer match to the true winds than the single-point wind prediction method.
6.1 Multipoint Data Generation

The previous analysis of the single-point wind prediction only used the initial firing state and the final impact location to make a wind prediction. For the multipoint wind prediction, a round tracking sensor is modeled to provide data for the path of the round in flight. This track sensor model runs as a separate process for the simulation. This process uses the ballistics model, applying the measured winds to produce an offset impact and a full trajectory of the round in flight.

Based on user configuration settings, the track sensor model produces a data set with a specified number of locations and velocities for the round in flight. These data points are sent via a network message to the wind prediction model. The design and execution of the track sensor model is intended to isolate any possible information about the measured winds being applied to the ballistic model. The wind prediction model has no information about the underlying winds in the system.

6.2 Determining Wind Prediction Parameters

For this research, the track sensor model was configured to generate data for 11 points along the flight path of the projectile. The first point is always the initial location of the round as it exits the barrel. The last point is always the impact location. The other 9 data points are evenly spaced along the flight path of the round. The spacing is based on the time of flight of the round, not the distance traveled or the altitude of the round at a given point. This leads to 10 intervals bounded by 11 points with the same time of flight in each interval.

The number of data points chosen for the track sensor is purposefully set to a low number. The intent is to show that even with fairly sparse data, only 11 points, the wind
prediction can be improved when compared to the single-point method. There is nothing to prevent further investigation with progressively larger numbers of tracked locations. This investigation will show that improvements are seen with few data points; any extra data will only further improve the wind predictions increase the overall reliability of the prediction.

The multipoint method makes a wind prediction within each interval in the track data. The time of flight of the round in each interval has the potential to be much shorter than the shortest time of flight simulated with the single-point wind prediction method. As was shown in Chapter 4, the closure tolerance for the wind prediction and the time of flight of the round control the maximum possible radial wind error. This relationship is expected to hold for each interval of the multipoint wind prediction. This reduced time of flight increases the possible wind prediction error. To reduce the possible maximum wind error, the closure tolerance was reduced to 0.00001 m for all of the runs.
6.3 Multipoint Wind Prediction Method

The multipoint wind prediction model uses the same wind prediction closure method as the single-point wind prediction. The single-point wind prediction model takes into account only initial state of the gun and the final impact location to predict a ballistic wind that accounts for the wind induced miss distance. The multipoint model performs the same ballistic wind prediction but between measured points along the flight path of the round. A diagram of the algorithm used in this analysis is presented in Fig 27.

Fig 27. Multipoint Wind Prediction Model Architecture.

The multipoint wind prediction model uses the same wind prediction closure method as the single-point wind prediction. The single-point wind prediction model takes into account only initial state of the gun and the final impact location to predict a ballistic wind that accounts for the wind induced miss distance. The multipoint model performs the same ballistic wind prediction but between measured points along the flight path of the round. A diagram of the algorithm used in this analysis is presented in Fig 27.
The wind prediction model receives information about the position and velocity of the round at various points along its flight path ordered by the altitude of the round from the track sensor model. Starting with the initial state of the round and gun and the first measured position of the round along its flight path, the wind prediction model find a ballistic wind which accounts of the observed difference between the round location and the predicted location had there been no wind. This ballistic wind is considered to be valid only between the two points for which it was calculated. The wind prediction is recorded at the given altitudes.

The predicted state of the round at the first measured location is used in the next iteration. The round tracking model assumes that the position and the velocity of the round are measured, but the accelerations of the round are not known and must be predicted using the ballistics model. The position, orientation, spin rate, and accelerations of the round are taken from the ballistics model prediction at the end of the wind prediction model search. The velocity of the round is set to the velocity measured by the track sensor for the round at that location.

The process continues by finding a ballistic wind which would account for the measured location between the next two points in the track data to the end of the tracked data list. The resulting data are raw (Fig 28) and require further processing. The predicted North wind speeds fit fairly well to the real winds. The East wind speeds do not appear to fit well at all. This was seen in many of the wind predictions when the applied measured winds were comparatively static. Note that the measured wind speed data has a roughly constant overall trend from 4000 m almost until the ground. There are small oscillations in the data off of a roughly constant value but there is no large-scale trend to the data when compared to the North wind speed data.

The wide oscillations seen in the raw ballistic winds in the East direction are an artifact of the prediction error expected based on Eq. 17. The time of flight between the data points is small,
allowing for the wind prediction model to have a high error in the predicted ballistic winds in a given interval. This wind error will change the accelerations in the state of the round at the end of that interval. The error in the accelerations and slight error in position allowed for by the closure tolerance with both affect the wind prediction in the next interval. If the actual winds acting on the round do not change largely in the following interval, the wind prediction model will “chase” the errors in the acceleration and position and overcompensate for the effects of the wind in the wrong direction. This compounds over time leading to the large oscillations observed.

Fig 28. Initial Raw Wind Speed Predictions.
Once the entire path of the round has been processed for raw ballistic wind predictions, the data are filtered. In this research, the data were put through a running average filter with a sliding window of 2 data points. This filter eliminated the oscillation seen in the predicted values for the East wind speed, which can be seen in Fig 29.

![Filtered Wind Speed Predictions](image)

Fig 29. Filtered Wind Speed Predictions.

The wind speed at ground level was set to 0.0 m/s. Though the winds immediately above the ground level may be non-zero, at the ground there is no wind [26].

The last step in processing the raw ballistic winds into final form is to assume that the wind speeds are linearly interpolated between the actual data points. In the graphs above it is
assumed that the wind speed is constant from the initial point in the interval to the end of the interval. The ballistic wind then immediately jumps to the single value of the next interval. Instead, it is assumed that the ballistic wind speed predicted only applies at the start of an interval. The wind speed at the end of each interval is assume to be the wind speed at the start of the next interval. Any values between these points are modeled using a linear interpolation between the points as shown in Fig 30.

![Diagram of wind speed prediction](image)

Fig 30. Final Multipoint Wind Prediction.

### 6.4 Simulation Description

The simulation was performed similar to the previous chapters. A set of 5,000 random initial states were generated and used. The random seed for these 5,000 states was controlled to
ensure that the states would match previous runs and would be the same for each of the wind profiles used. For each of the 5,000 initial states, a measured wind profile was applied and the ballistic model was then used to generate an impact location. This was repeated with all 16 measured wind profiles.

The Monte Carlo nature of the simulation, with 5,000 randomly generated initial states, was selected to ensure that the possible range of states was covered with a reduced chance of biasing results based on selection of initial state. To limit the initial states to a possible subset of states or to do a parametric search through the allowed ranges of the initial state variables may cause the analysis to miss some aspect of the system. By performing a stochastic analysis the chances of missing an effect due to excluding a combination of initial state values via a strictly controlled selection process is reduced.

For each initial state, the full track of the projectile was recorded from the ballistics model and input to the track sensor model. From these track data, 10 evenly spaced points along the path are selected which, with the initial location of the round at time of fire, form the 11 points used to make the ballistic wind prediction. The spacing of these points was controlled by the total time of flight of the round, dividing the total time into 10 evenly spaced segments with the tracked points making up the end points of those segments. The points were not selected based on altitude or position.

For each of the 5,000 random runs with a given measured wind profile, a multipoint ballistic wind profile was generated using the setting referenced in Section 6.2 and using the method described in Section 6.3.
6.5 Results

With 5,000 initial states and 16 different wind profiles, 80,000 individual runs were completed. All 80,000 runs completed successfully, producing ballistic wind profiles which account for the measured winds and correct the impact miss distance to within the specified closure tolerance.

6.6 Analysis of Results

The goal of this research is to investigate the efficacy of wind predictions made using multiple measured locations along the flight path of the round. The best way to judge a predicted ballistic wind is to apply it in a simulated ballistic flyout to determine whether or not the ballistic winds correct for the observed impact miss distance. The wind prediction model already accounts for this kind of analysis. The ballistic wind prediction is controlled by the closure tolerance. Wind predictions are checked at time of calculation to ensure that they generate an impact within the closure tolerance when applied to a ballistic flyout.

As a check on the multipoint wind prediction compared to the single-point, the fit of the wind model to the measured winds can be used as an analog to the correctness of the wind prediction. A perfect wind prediction model would match the measured winds exactly. It is not practical to expect a modeled wind profile to match the measured winds perfectly. It is reasonable to expect that a good ballistic wind model will match the true winds closely. The closeness of fit is measured by looking at the standard deviation of the predicted wind model off of the measured wind speeds at all altitudes. The standard deviation metric was calculated for both the single-point results and the multipoint model results for all 5,000 initial states. The results for each of the 16 different wind profiles were kept separate.
Using the above wind prediction as an example and comparing to the single-point wind prediction, the differences and quality of fit are visually apparent (Fig 31).

Fig 31. Comparison of Single-point and Multipoint Models.

The green line in both plots represents the single-point wind predictions and the red line represents the multipoint wind prediction. The multipoint is following the blue line, the measured wind speeds applied to the round in flight, more closely than the single-point values. There are variations in the measured winds which are not captured by either of the wind prediction methods. This is a limitation caused by the use of only 10 data points along the trajectory of the round.
To give a quantitative measure of the closeness of the predicted data to the actual data, the difference between the measured wind speed and the predicted wind speed was calculated at each included altitude in the measured wind speed for both wind prediction methods and in both the East and North directions.

The standard deviations of the residuals shown in Fig 32 were calculated to test the goodness of the fit of the predicted winds to the measured winds. For the East winds, this single-point wind prediction had a standard deviation of 2.44 m/s and the multipoint wind prediction had a standard deviation of 1.38 m/s. For the North winds, the single-point wind prediction had a standard deviation of 3.28 m/s and the multipoint wind prediction had a standard deviation of 0.973 m/s. The multipoint wind prediction has a lower standard deviation that the single-point wind prediction, indicating that the data multipoint prediction move closely matches the measured winds.

This same metric was calculated for all 5,000 wind predictions made with all 16 measured wind sets. The results are summarized in Table 8 and Table 9. For all 16 measured wind profiles, the multipoint wind predictions had a lower standard deviation off of the measured winds than the single-point wind predictions. This indicates that the multipoint wind prediction is giving results as expected; that the winds predicted by the multipoint method more closely match the true underlying winds. It is expected that a wind prediction which more closely matches the true winds will be more stable for use in predicting impact locations as the initial state of the system changes.
Fig 32. Modeled Wind Errors Off of Measured Winds.
Table 8. East Wind Prediction Standard Deviations.

<table>
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<th>Wind Profile</th>
<th>Single-Point Winds</th>
<th>Multipoint Winds</th>
</tr>
</thead>
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<td></td>
<td>Minimum Standard Deviation</td>
<td>Mean Standard Deviation</td>
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6.7 Changing State

An additional simulation was performed to compare the results of the single-point ballistic wind prediction to the results of the multipoint ballistic wind prediction as the state of the aircraft and gun are changed from the state in which the prediction was made. A random set of 50 initial states for the aircraft and gun were chosen. A single-point and multipoint ballistic wind profile was predicted using those 50 initial states with all 16 measured wind profiles.

The initial state of the gun was then changed and a ballistic flyout was simulated. The aircraft altitude, speed, and total gun depression were allowed to vary based on a uniform random distribution with bounds detailed in Table 10. A uniform continuous distribution was
selected for these variations because there is no reason to assume that changes in the state of the aircraft and gun will tend to cluster around the initial state. The uniform distribution gives an equal probability of occurrence to all values in the range specified and does not favor values closer to the initial state. A Monte Carlo simulation was selected over parametrically stepping through the ranges for each state variable because the effects of coupling between the state variable and the ballistic winds are not known. A parametric search could miss an effect due to selecting incorrect values.

<table>
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<th>State Variable</th>
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<tbody>
<tr>
<td>Altitude [m]</td>
<td>U(-50.0, 50.0)</td>
</tr>
<tr>
<td>Aircraft Speed [m/s]</td>
<td>U(-25.0, 25.0)</td>
</tr>
<tr>
<td>Gun Elevation [deg]</td>
<td>U(-5.0, 5.0)</td>
</tr>
</tbody>
</table>

The measured winds were applied and an impact location was generated. This impact was considered to be “truth” data. Similar impacts were generated using both the single-point and the multipoint ballistic wind model.

The state of the gun and aircraft was then changed and the data generation repeated to collect a total of 100 impacts around the original state where the ballistic winds were calculated. After all 100 variations off of the original state had been used, a new original state was selected along with the single-point and multipoint ballistic wind profiles for that state. The process was repeated for each original state, generating 100 variations off of the original state.
6.7.1 Results

To analyze the usefulness of a ballistic wind as the state changes, the total magnitude of the impact miss distance was calculated. The impact location predicted using the measured winds was compared to the impact location predicted with the ballistic wind and a difference was calculated in the DR and CR directions to find a miss distance.

The DR component of the miss distance was converted to be normal to the line-of-sight from the gun to the target [32]. This eliminates the skewing of the impact data in the DR direction due to conic projection to the simulated surface of the Earth. The DR and CR miss distances were then converted to an angular miss instead of a linear miss distance. The resulting data for one initial state and wind profile is shown in Fig 33. The red impacts are based on the multipoint ballistic wind and the blue impacts are based on the single-point.

The DR and CR angular miss distances were then combined into a single radial miss distance. For this analysis, the direction of the miss is less important than the total distance. For a given original state the maximum radial miss distance for a given ballistic wind method out of the 100 varied states was found. The radial miss distance for the single-point and multipoint ballistic winds were compared to find which method had the lowest radial miss distance under the same change in state.

It was expected that the multipoint ballistic wind, with its closer fit to the measured wind, would have less miss distance induced by a changing state than the single-point ballistic wind.
Fig 33. Example Impact Dispersion under Varying States.

6.7.2 Analysis of Results

Testing 100 changes in initial state for each of the 50 initial states using all 16 wind profiles resulted in 800 different maximum radial miss distance for the single-point and multipoint ballistic wind profiles. A histogram was generated to see what the predicted distribution of miss distances was for each ballistic wind prediction method.
Fig 34. Single-point Radial Miss Varying all State Variables.

Fig 35. Multipoint Radial Miss Varying all State Variables.
For the single-point ballistic wind the radial miss distances are low (Fig 34) but the greatest number of data points are not at 0.0 mrad. The maximum single-point radial miss distance predicted was 2.1068 mrad. The multipoint radial miss distance (Fig 35) were also not clustered at 0.0 mrad. The maximum multipoint radial miss distance predicted was 0.3069 mrad.

The changing of the aircraft and gun state from that state where the ballistic wind was predicted induces less error if a multipoint ballistic wind is used compared to a single-point ballistic wind. This is expected based on the results above in Section 6.6.

A comparison of the radial miss distances under the same conditions is needed to judge whether one ballistic wind method is always better than the other. Even though the multipoint ballistic wind appears to have a much lower radial miss distance, it may not always be better than the single-point method. The maximum radial miss distances for the multipoint ballistic wind was subtracted from the maximum radial miss distances for the single-point ballistic wind.

Very few negative points exist in the comparison data (Fig 36). This means that the multipoint ballistic wind was more often more stable relative to changes in all three state variables when compared to the single-point method. There are 29 negative data points, instances where the single-point ballistic wind appears to be more stable than the multipoint ballistic wind. The largest negative magnitude was -0.1165 mrad.
Fig 36. Differences between Single-point and Multipoint Stability Varying all State Variables.

Fig 37. Instances where Single-point Method Appears more Stable Varying all State Variables.
The largest number of instances where the single-point ballistic wind appears more stable occurred for wind profiles 14 and 16 (Fig 37). Looking at Table 8 and Table 9, it is not surprising that wind profiles 14 and 16 have some instances where the single-point ballistic wind is slightly better than the multipoint method. Note that the minimum, mean, and maximum standard deviations of the single-point ballistic wind profiles for wind profiles 14 and 16 are all low in comparison to the other wind profiles. This indicates that the single-point method did better at fitting wind profiles 14 and 16 than the others.

6.8 Conclusion

In this chapter, a method of multipoint wind predictions was proposed and tested. It was shown that with very few data points the multipoint method can generate a wind prediction which closely match the measured winds applied to the round.

By analyzing the standard deviation of the differences between the measured winds and the two ballistic wind profiles, the closeness of the ballistic wind to the actual winds can be calculated. The results indicate that the multipoint ballistic wind more closely fit the measured wind profiles than the single-point ballistic wind.

The two ballistic wind methods were also tested under changing initial state of the aircraft and gun. This ballistic wind, whether a single-point and multipoint ballistic wind, is tuned based on the state of the gun and aircraft at the time of fire. Anything that changes the state of the system may invalidate the ballistic wind profile. Using the ballistic wind in a different state may lead the ballistic model to predict an impact which does not match the impact using the true winds. Ideally, a ballistic wind would be insensitive to changes in state.
A simulation was run to test the radial miss distance induced by changing the state of the aircraft and gun from the state when the ballistic wind was generated. The results showed that the multipoint ballistic wind was able to accept a change in the aircraft and gun state and maintain a lower maximum radial miss distance than the single-point ballistic wind. The multipoint ballistic wind did not always have the lower radial miss distance, however. There were instances where the single-point ballistic wind appeared to perform better under changing states, though the difference in the maximum radial miss distance between the two methods in these few instances was small.

Overall, the multipoint ballistic wind performed better than the single-point ballistic wind. Given the data collected, the largest multipoint miss distance induced was 0.3069 mrad. The largest single-point miss distance induced was 2.1068 mrad.

The data indicate that a multipoint ballistic wind based only on 10 tracked points of the round in flight allows for a more consistent impact prediction as the aircraft and gun state changes than the single-point ballistic wind.
CHAPTER 7

CONCLUSION

A successful method of making multipoint ballistic wind predictions was developed and tested as part of this research. The multipoint prediction method presented in this thesis is based on a repetition of the single-point wind prediction between all available tracked locations of the round. The single-point wind prediction method is itself based on a bisecting search, a relatively simple search algorithm used to find an optimal value to minimize an error metric. The multipoint wind prediction method being a series of bisecting searches makes the programming of the algorithm easier and less prone to errors, indicating that it could be used for tactical applications. The multipoint prediction method was able to predict ballistic winds which closely fitted the true measured winds using few data points for the tracked round, only 10 points along the flight path and the initial firing conditions.

The multipoint wind predictions are all much closer to the measured winds applied the round than the same single-point wind predictions. This result may seem trivial, but recall that the use of a ballistic wind does not require that it match the underlying real winds acting on the tracked round. The ballistic wind only has to cover for the physical effects on the round. It was assumed and hoped for that the multipoint ballistic winds would closely match the underlying measured winds. The analysis of the fit of both ballistic wind models to the true winds showed that the multipoint more closely matched the true winds in all cases.

Testing the stability of the single-point and multipoint wind models showed that the multipoint wind was almost always the more stable method. Changing the aircraft and gun state had less of an effect on the accuracy of the predicted impacts when a multipoint ballistic wind was used than seen when a single-point ballistic wind was used. The highest error caused by
changing state was just over 2.1068 mrad using a single-point ballistic wind. The highest using a multipoint ballistic wind was just over 0.3069 mrad. This is within the manufacturers stated dispersion of the ammunition used in this simulation, meaning that this extra miss distance due to changing state is not likely to be discernable given the imprecision of the round itself.

For some of the wind profiles used, a few simulation runs indicated that the single-point ballistic wind would be more stable than the multipoint ballistic wind. Out of 800 runs, only 29 showed that the single-point ballistic wind was more stable. The slight improvement on the stability metric with the single-point, 0.1165 mrad better than the multipoint, is also well below the nominal dispersion of the round type.

Further investigation of the instances where the single-point method was more stable revealed that the stability was due to the almost static nature of the measured wind profiles being tested. Wind Profiles 14 and 16 had very low wind speeds in both the East and North directions and the wind speeds in one of the directions had a clear average trend with small variations off of it. This is the ideal case for the single-point ballistic wind. Looking at the standard deviation values calculated as a closeness of fit of the single-point ballistic wind to the true winds, Table 8 and Table 9, wind profiles 14 and 16 have a very low standard deviation when compared to the other wind profiles, meaning that the single-point ballistic wind model was able to fit those winds more closely than the other wind profiles.

None of this invalidates or reduces the usefulness of the multipoint ballistic wind. The slight improvement using the single-point ballistic wind is within the dispersion of the round. The results point to the fact that under a roughly static set of wind speeds, both the single-point and multipoint methods should converge towards each other.
7.1 Secondary Results

The relationship between the radial wind error and the time of flight was unexpected, though it makes sense on further review. As seen in Eq. 17 the lower the time of flight the higher the maximum radial error in the wind prediction can be off of the true wind. The predicted ballistic wind is still expected to correct the round’s impact to be within the closure tolerance on the wind prediction model’s search, but the actual value of the predicted wind can be wrong. At lower times of flight the error can be larger because the wind does not have as much time to affect the flight of the round. At longer times of flight, the radial error must be lower to achieve the same closure tolerance because the wind has a longer time to act on the round.

Another secondary result of note is that changes to the total gun elevation is a strong contributor to the instability of the ballistic wind predictions. In light of the relationship shown in Eq. 17 this result is not surprising. Changing the elevation has a large effect on the time of flight of the round. Small errors in the ballistic winds can lead to large miss distances by simply changing the elevation of the gun.

Also of note is that the multipoint wind prediction was able to do so well with only 10 points along the path of the projectile. Even at higher altitudes where the distance between the data points was the greatest, the multipoint wind prediction model was able to generate a ballistic wind which proved to be more stable than the single-point method.

7.2 Future Research

The research in this thesis shows the possible benefits to be gained by using a round racking sensor as part of a FC system. The data can be used to model the winds accurately and
in a way that is stable as the aircraft state changes. To complete this analysis, some limitations were imposed on the modeled parts of the system which offer chances for future research into this topic. Increasing the fidelity of the models and simulation could give better indications of the total possible improvements which could be seen from using a round tracking sensor to predict the ballistic winds.

7.2.1 Full fire-control simulation

One of the modeling decisions made for this analysis was that a full simulation of a FC was not needed and that a ballistics model would suffice. This is a valid assumption to limit the complexity of the system for simulation but it leaves some questions unanswered. Most importantly, what effect does a multipoint wind prediction model have on the commanded gun pointing angles?

This analysis had a target determined by randomly selected gun pointing angle and aircraft state. It was assumed that the winds would cause a round to miss a target and that modeling the winds would allow for the round to hit the target. In reality, the target exists external to the FC and is not determined by the gun or aircraft state. The gun and aircraft state are calculated by the FC to engage that target. Winds are used as part of the calculation of the gun pointing angles by the FC. By predicting and using a ballistic wind in the FC and by changing the state of the aircraft relative to the target, the gun pointing angles will change to bring the round back on target.
7.2.2 Full pylon turn orbits

It was assumed for this analysis that the orbit of the aircraft was sufficiently modeled by a stationary aircraft at the time of fire. This makes the target static relative to the aircraft, which isn’t always the case. It also forces the gun to fire the same way into the winds for each shot simulated.

In reality, the aircraft is orbiting. This would make targets change location relative to the aircraft unless they were perfectly centered in the orbit path. Changing target location will force the gun elevation to change over time. As was seen in Chapter 6, changing the state of the aircraft and gun can have an effect on the possible errors in impacts that result from using a ballistic wind predictions.

Investigating the effect of full pylon turns combined with a full model of a FC would give a good indication of whether the multipoint ballistic wind model introduces any instabilities to the gun pointing angles at the time of flight of the round changes in different parts of the orbit.

7.2.3 More tracked data points

The analysis above assumed that the round tracking sensor would provide 10 data points along the flight path of the round. This is a very low value. What are the benefits of adding more values? Or, conversely, what is the effect of having less values?

A parametric analysis of the number of data points required to achieve a certain level of stability would help to inform future work into developing the necessary hardware and software to integrate a round tracking sensor.
7.2.4 Combined errors

There are other errors which are assumed to either not exist or not contribute for this analysis. In a real system these errors would manifest themselves and complicate the wind prediction. These errors would have to be sorted and dealt with in their specific frames of reference to allow for the correction of the world relative errors with a ballistic wind prediction.

A fuller simulation which accounts for the platform relative errors such as misalignment of the sensor and gun, errors in the ammunition description, and limitations in ballistics modeling could reveal possible complications that might exist if a round tracking sensor was integrated into the FC of a gunship. Any method used to try and decouple the errors into their proper frames of reference will contain uncertainties which may affect the ability of the multipoint wind prediction model to properly close on the ballistic wind.

7.2.5 Wind vector field

This thesis shows that it is possible to correctly predict a ballistic wind profile which closely matches the underlying winds. These ballistic wind profiles can be used to correct wind errors in subsequent firings. These winds are only valid for the round that was used to predict them, however, and may not be the best ballistic wind to apply to later rounds.

The validity of the ballistic wind will depend on the variations of the true winds over both time and space. The winds which are acting at one location in the orbit may not be representative of the winds acting at other locations. Further, the true winds are expected to vary over time, possibly reducing the usefulness of the winds predicted at any location in the orbit.

This research presents an opportunity to research the creation of a model of a wind vector field which covers the entire orbit. It may be possible to combine the individual ballistic winds
to describe not only the winds at a single location in the orbit but around the entire orbit. Such a model could allow for accurate predictions of the ballistic winds as they change over time. A change in the ballistic winds at one location in the orbit from an earlier ballistic wind could be used to predict a change in the ballistic winds at other locations in the orbit.

7.2.6 Tuning ballistic model

The prediction of a multipoint ballistic wind allows for the tuning of the ballistics model for different round types. It is possible for a ballistics model to be poorly calibrated for the round type being fired and still allow for a usable prediction of the round’s flight. Calibrating, or tuning, the model requires a source of truth data to compare the model against. A multipoint ballistic wind can be used as the truth data, allowing for better calibration of the ballistics model for all round types.

The process of calibrating would require making multipoint ballistic wind predictions for multiple round types at the same time. One can then be selected as the correct wind prediction and the form factors and aeroballistic coefficients of the other rounds could be adjusted to make the ballistic wind predictions match the correct wind. If a separate device was capable of measuring the true winds, then the ballistics model could be tuned for each round type using the true winds as the truth data.

The tuning of the ballistics model made possible by this thesis’ result is required if the ballistic wind prediction for a given round type is to be applied to other ammunition. If not tuned, it is possible that the ballistic winds predicted for each round will vary from the true winds due to poor modeling. The result of this thesis coupled with a way of measuring the true winds may allow for the tuning of the form factors on the ballistic model. Better tuning of the ballistics
model will allow for more accurate prediction of the flight path of the round, which may improve overall accuracy of the FC.

7.2.7 Tactical application

Perhaps the most obvious research opportunity for the results of this thesis is to apply it in a FC in a representative tactical environment. At this point, a viable algorithm has been identified and indications are that a ballistic wind which closely fits the true winds can be predicted. A practical demonstration is possible as long as the hardware is available to support the data required, namely a round tracking sensor.

The other research ideas presented above are all interesting modeling questions and topics which should be investigated to better understand the capabilities and limitations of a multipoint ballistic wind. A practical implementation may reveal that the benefits gained through the above research is not worth the effort of the research itself.
REFERENCES


## APPENDIX A

### GLOSSARY OF TERMS

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<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>Aleatory error</td>
<td>Those errors in a calculation or simulation which result from factors and effects that could not possibly be known at the time of calculation. An example would be the initial velocity of a projectile. An average initial velocity is used in the modeling but the exact velocity cannot be known until the round is fired, at which point it is too late to account for the actual initial velocity.</td>
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<tr>
<td>Epistemic error</td>
<td>Those errors in a calculation or simulation result which are caused by a lack of knowledge of a factor which could have been known and better measured before the calculation and accounted for. An example would be accounting for the exact mass of a projectile. It is possible to measure each round, but in practice a single mass is assumed to be correct for all rounds of a given type.</td>
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<tr>
<td>Firing-solution</td>
<td>That set of gun azimuth and elevation pointing angles the gun must be pointed at to cause a round fired by the gun to impact an intended target.</td>
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<td>Flyout</td>
<td>A ballistic flyout, or just flyout, is the result of a single run of the ballistics model.</td>
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<td>Nominals</td>
<td>Flight parameters which determine the geometry of a pylon turn. Parameters include altitude above target, bank angle, and airspeed.</td>
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<td>Term</td>
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<tr>
<td>No-wind impact</td>
<td>That predicted impact location generated by the ballistics model which has no winds applied to the round in flight.</td>
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<tr>
<td>Pylon turn</td>
<td>A flight maneuver wherein a pilot holds a constant bank angle and airspeed causing the aircraft to fly in a circle of constant altitude around a specified center location</td>
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<td>Round</td>
<td>A single projectile or type of projectile. Used interchangeably with ammo, ammunition, and projectile.</td>
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<td>Slant range</td>
<td>The total linear distance between the initial location of a projectile and its final impact location.</td>
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<td>Tweak</td>
<td>A set of calculated values used to correct for unknown factors causing shots fired to impact off the intended target. Also known as “Kentucky Windage”.</td>
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<td>Wind column</td>
<td>A measure of the East and North wind speeds indexed by the altitude. Also called the wind profile.</td>
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<tr>
<td>Winded impact</td>
<td>That predicted impact location generated by the ballistics model which has a wind model applied to the round in flight. Expected to show a wind induced offset from the no-wind impact.</td>
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William Arthur “Art” Kenney from Fredericksburg, Virginia, received his bachelor’s degree in physics from the University of Mary Washington in 2005. After completing his degree, Art taught high school physics for four years in Spotsylvania County, Virginia. He started working at the Naval Surface Warfare Center in Dahlgren, Virginia in 2009 and continues to work there to this day. Art will receive his master’s degree in the Summer of 2017 after which he plans to continue working to point big guns at bad guys.