VAIM For Inverse Problems

Manal Almaeen[1], Yasir Alanazi[1], Nobuo Sato[3], W. Melnitchouk[3], Michelle P. Kuchera[2], and Yaohang Li[1]

[1] Old Dominion University, Norfolk, Virginia 23529
[2] Davidson College, Davidson, North Carolina 28035
[3] Jefferson Lab, Newport News, Virginia 23606

Introduction

Reconstructing the parameters from the given observables, referred to as the inverse problem, plays a central role in a variety of science and engineering applications. The mapping of parameters to observables is a well-posed problem with unique solutions, and can therefore be solved directly with differential equation solvers or linear algebra solvers. In contrast, the inverse problem requires backward mapping from observable space to parameter space, which is often non-unique. Consequently, solving inverse problems is ill-posed and a far more challenging computational problem.

Method

As shown in Figure. 1, VAIM [1] adopts the architecture of an autoencoder composed of two neural networks, a forward mapper $\Psi(\cdot)$ from parameter space to observable space and a backward mapper $\Phi(\cdot)$ observable space to parameter space. In between of the forward and backward mappers, A latent layer is incorporated whose purpose is to capture the lost information in the parameter–observable forward mapping. During forward training, the variables in the latent layer are restricted to certain well-known distributions, such as a Gaussian or uniform distribution. When the proposed architecture is appropriately trained, sampling the variables in the latent layer allows the inverse mapper to rebuild the posterior parameter distribution, given the observables.

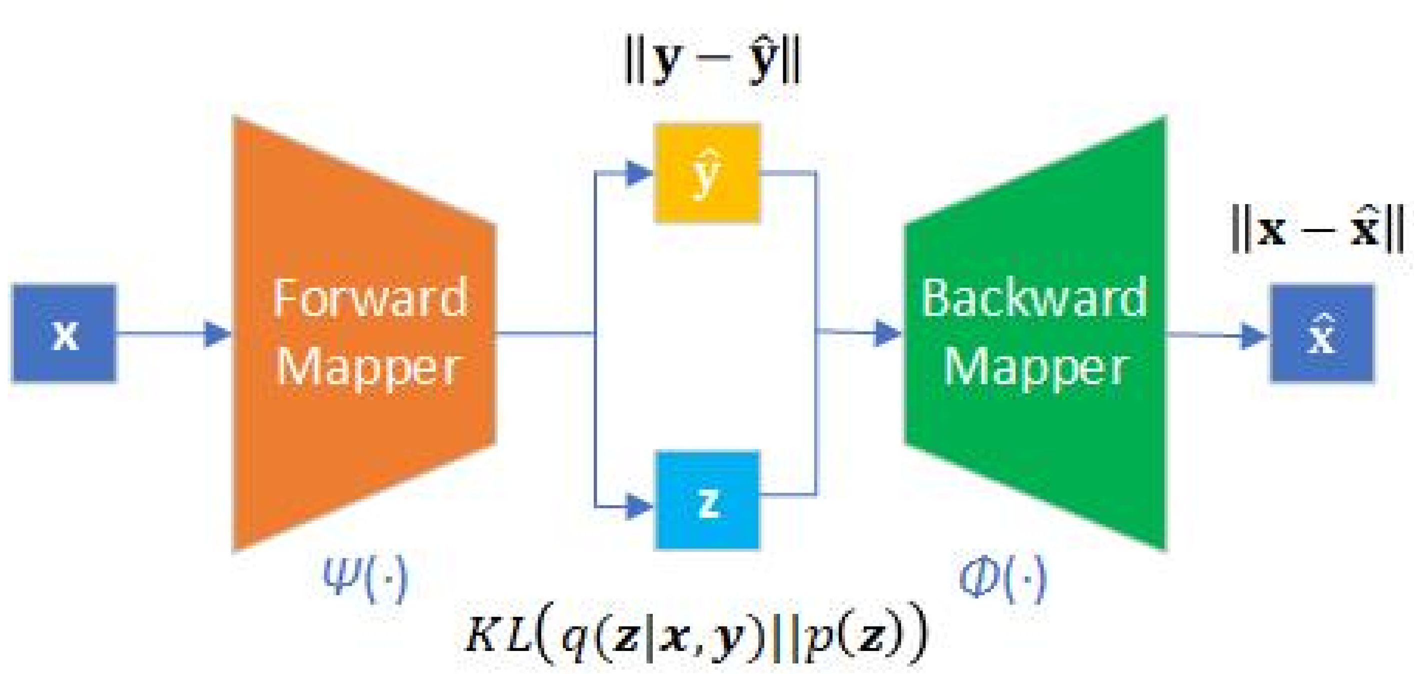

Fig. 1: VAIM architecture

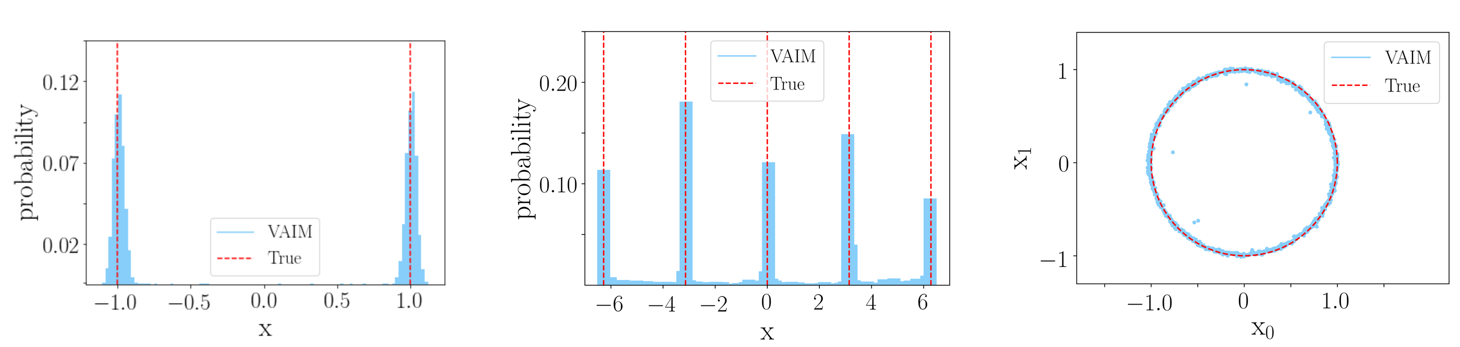

Fig. 2: Left to right, predicted solutions on $f(x)=x^2$ when $f(x)=0.36$, $f(x,y)=x^2+y^2$ when $f(x,y)=1.0$ and $f(x)=sin(x)$ when $f(x)=0$

Results

We first test VAIM on three toy inverse problems with different solution patterns :

1. $f(x)=x^2$, $x\in[-3,3]$,
2. $f(x)=\sin(x)$, $x\in[-2\pi,2\pi]$, and
3. $f(x)=x_0^2+x_1^2$, $x_0,x_1\in[-2,2]$.

The results for these examples is shown figure. 2.
Then, we apply VAIM to a simplified version of an substantial application in fundamental nuclear physics: QCD analysis. For this application, we aim to construct the inverse function mapping the quantum correlation functions to observables. The preliminary results is shown in Figure. 3. one can find that VAIM predicts a few solution clusters for each control sample and the parameter vector of the control sample falls right into one of these clusters. This illustrate that VAIM precisely predicts the parameter solution distributions.

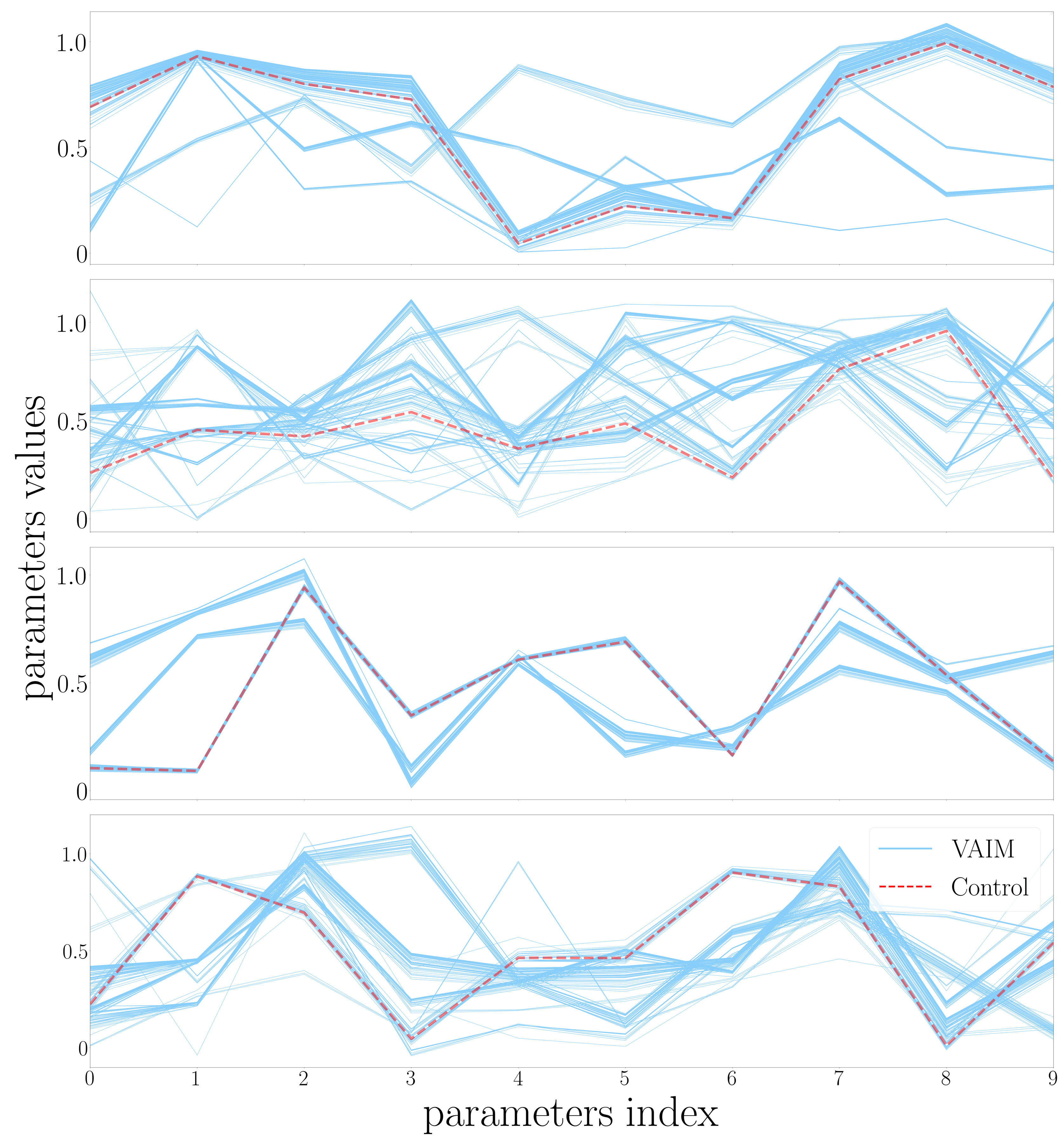

Fig. 3: Parameter distributions generated by VAIM in four control samples

Comparison

1. comparison with Mixture Density Networks (MDN)
   - MDN [2] is often used to solve inverse problems. The goal of the MDN is to construct a conditional probability distribution of the parameters given the observable inputs, which addresses the one-to-many mapping issue in inverse problems.
   - Compared to MDN, VAIM does not need to rely on the assumption of a Gaussian mixture model.

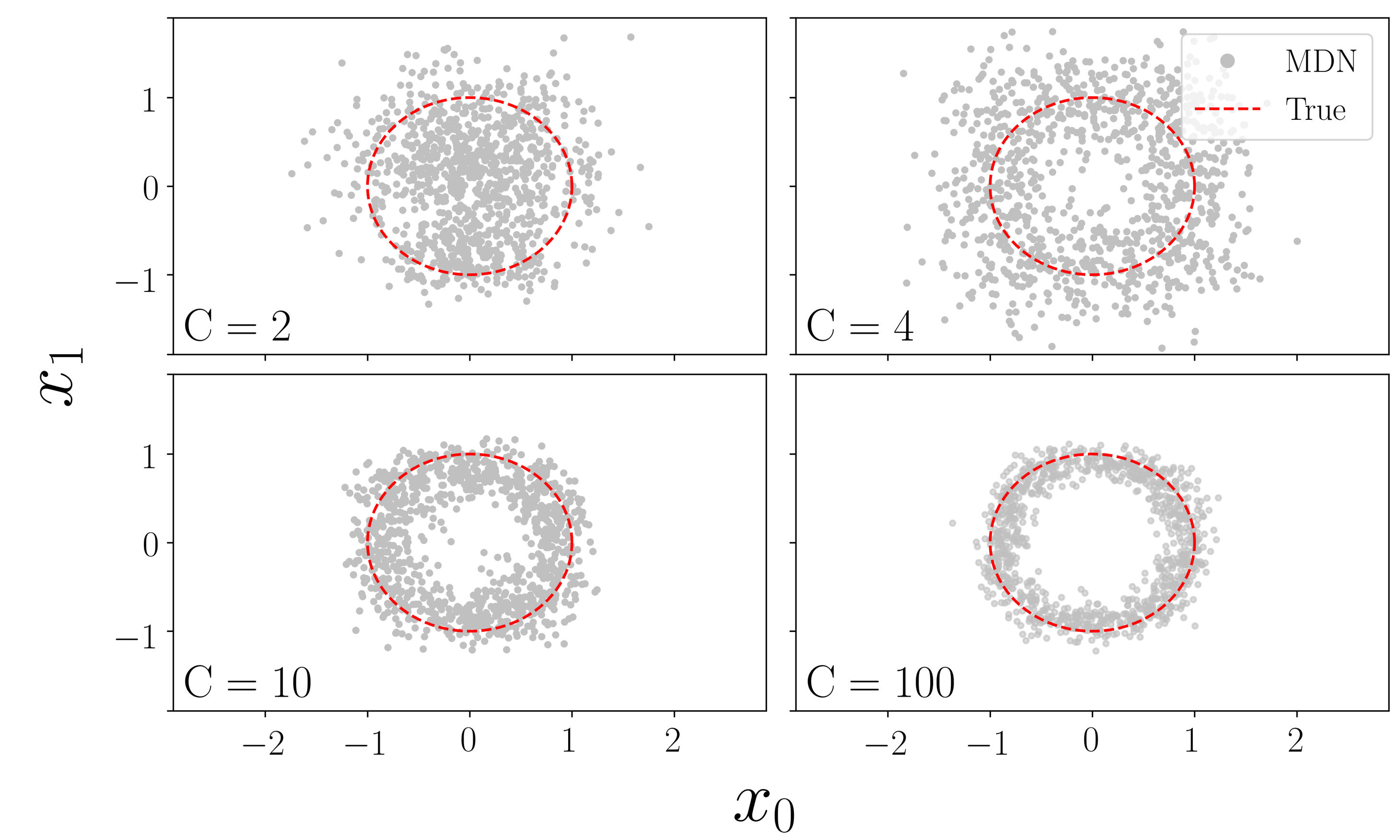

Fig. 4: Predicted solution samples with $C=2$, 4, 10, and 100 mixing components on $f(x)=x_0^2+x_1^2$ with respect to $f(x)=1.0$ using MDN.

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References

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