Reconstructing the parameters from the given observables, referred to as the inverse problem, plays a central role in a variety of science and engineering applications. The mapping of parameters to observables is a well-posed problem with unique solutions, and can therefore be solved directly with differential equation solvers or linear algebra solvers. In contrast, the inverse problem requires backward mapping from observable space to parameter space, which is often non-unique. Consequently, solving inverse problems is ill-posed and a far more challenging computational problem.

As shown in Figure 1, VAIM [1] adopts the architecture of an autoencoder composed of two neural networks, a forward mapper \( \Psi(\cdot) \) from parameter space to observable space and a backward mapper \( \Phi(\cdot) \) observable space to parameter space. In between of the forward and backward mappers, A latent layer is restricted to certain well-known distributions, such as a Gaussian or uniform distribution. When the proposed architecture is appropriately trained, sampling the variables in the latent layer allows the inverse mapper to rebuild the posterior parameter distribution, given the observables.

![VAIM Architecture](Image)

**Fig. 1: VAIM architecture**

We first test VAIM on three toy inverse problems with different solution patterns:

1. \( f(x) = x^2, x \in [-3, 3] \),
2. \( f(x) = \sin(x), x \in [-2\pi, 2\pi] \), and
3. \( f(x) = x_1^2 + x_1^3, x_0, x_1 \in [-2, 2] \).

The results for these examples is shown Figure 2. Then, we apply VAIM to a simplified version of an substantial application in fundamental nuclear physics: QCD analysis. For this application, we aim to construct the inverse function mapping the quantum correlation functions to observables. The preliminary results is shown Figure 3.

![Parameter Distributions Generated by VAIM in Four Control Samples](Image)

**Fig. 3: Parameter distributions generated by VAIM in four control samples**

We illustrate that VAIM precisely predicts the parameter solution distributions.

![Predicted Solution Samples with MDN and VAIM](Image)

**Fig. 4: Predicted solution samples with MDN and VAIM**

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### References
