Spring 2017

Using Pre-Calculus and Calculus Student Work to Examine Student Problem Solving Abilities in Online and Face-to-Face Mathematics Courses

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USING PRE-CALCULUS AND CALCULUS STUDENT WORK TO EXAMINE STUDENT PROBLEM SOLVING ABILITIES IN ONLINE AND FACE-TO-FACE MATHEMATICS COURSES

by

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A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

CURRICULUM AND INSTRUCTION

OLD DOMINION UNIVERSITY
May 2017

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ABSTRACT

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Sarah Catherine Ferguson
Old Dominion University, 2017
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This study compares the outcomes of student learning between two pairs of courses. Each pair of courses consists of an online section and a face-to-face section. One pair of courses focuses on pre-calculus content while the second pair focuses on calculus content. Both pairs of courses are taught by the same instructor using the same course appropriate materials. Participants for this study include 9 online and 14 face-to-face pre-calculus students and 14 online and 23 face-to-face calculus students from an urban community college in the southeastern portion of the U.S. Written responses from the subjects to a collection of problems focusing on solving systems of equations and inequalities (pre-calculus) and integration (calculus) serve as the study data.

Adopting a mixed method design, student work was reviewed quantitatively and qualitatively. ANOVA calculations were used to quantitatively compare scores and values earned on each question to look for differences in scores between the online and face-to-face groups. Qualitative reviews were used to analyze closely the work to evaluate problem solving approaches utilized by the students. The study revealed limited differences between the online and face-to-face groups relative to their overall score, their problem solving abilities, and their common errors. The findings of this study are consistent with findings from existing literature while offering more insights into the learning outcomes of solving systems of equations and inequalities and integration in the two different learning environments.

Keywords: Online teaching, learning outcome, pre-calculus, systems of equations and inequalities, calculus, integration, problem solving

ACKNOWLEDGMENTS

Without a team of individuals to provide guidance and support, my efforts to complete this dissertation and Ph.D. would have been futile. I would like to thank my committee for their unwavering guidance and support. Dr. Enderson, your willingness to step in as my advisor and dissertation chair late in my program progression is greatly appreciated. Your guidance and patience never falter and I can always count on you giving your honest opinion without any "sugar coating". Dr. Crompton, you have encouraged me to look at my writing differently and truly reflect on each word and how it expresses my intended thoughts. Your research in mobile learning shares similarities with my research in online learning as both are non-traditional but emerging learning fields; reading your work has been so helpful as I review and reflect upon my own writing. Dr. Yen, time in your courses introduced me to analyzing data through ANOVA. Your instruction and patience through learning SPSS and navigating ANOVA tables was appreciated while I was your student and your guidance using these skills to analyze my dissertation data is equally valued. In addition to honoring the support garnered by my advisor and committee members, I would be remiss to not acknowledge four additional individuals without whom my Ph.D. journey would not have been possible. Mom, you have been my proof-reader since the very first sentence I ever wrote. Even when you could not understand what I was trying to say, you encouraged me to get my thoughts down on paper and helped me make sense of the mess. You are my biggest cheerleader and closest friend; I cannot possibly thank you enough. Dad, you are a true lover of learning and have always encouraged me to reach farther than I thought I could, climb the next mountain, and strive for greater goals. You listen to my crazy ideas, help me evaluate plans, and always make sure I know I have your 100% support.

You are an inspiration and you have taught me to never give up. Matthew, you are my husband, my confidant, my partner in life. Through long nights of writing and studying, evenings away at class, and mindless rambling about my current research or projects, your support knew no bounds. You support our family, maintain equilibrium when I get flustered, and are always my rock. Without you I would not be me and this journey would not have been possible. And, Joseph, my son, you are my source of constant joy. Your laugh, your smile and your hugs are exactly what I need to push though difficult days. An ever-present source of joy, you add more to our lives than we could have ever imagined and provide a strong calming focus amidst chaos. Thank you all. Your help, guidance and presence through this journey made this dream a reality.

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CHAPTER 1

Introduction

The delivery system through which knowledge is presented has no more impact on instruction than the type of grocery truck used to deliver food impacts nutritional value (Clark, 1983). Online learning courses often function as distribution sources for knowledge and “rely on the same teach-and-test ontology that has dominated K-12 and university education” (Jonassen, 2007, p. 185). Bold statements in support of diverse instructional modalities, such as online learning, are met with both enthusiasm and critique. With technology becoming increasingly prevalent and learning online common place, educators seek to understand the impact of online based instruction on student learning. “Online learning is but a subset of learning in general” and how students learn online shares similar characteristic to how students learn in a traditional classroom (Anderson, 2008, p. 46).

Once seen as a trend, online learning has now become mainstream as communication technologies are changing the way people live, work, play, and learn (Jonassen, 2007). With smart phones, tablets, laptops, and other devices continuously at our fingertips and constantly connected to the internet, the transfer rate of information is astounding. Technology has had a profound impact on education; from computers in classrooms, to computers that serve as classrooms, technology has brought about an education reform, making online education possible. As Garrison (2011) states, “We are just beginning to discover and understand the extent to which these technologies will transform expectations for, and approaches to, learning” (p. 5). With the influx of technological advances, online learning techniques have gained notoriety; but, as Garrison (2011) cautions: “surfing the Internet is not an educational experience, any more than wandering through a library is” and merely being online does not constitute an online learning experience (p. 4).

Currently, 77% of colleges in the United States offer online learning options for students (Education Reform, 2012). With the multitude of online courses available, many students are turning to online education to achieve their higher education needs, goals, and desires. Allen and Seaman (2011) found that 31% of all students enrolled in higher education take at least one online course and that 65% of chief academic officers acknowledge online learning is a critical component of long-term higher education strategies. These data show online learning is a vested educational practice. Colleges and universities increasingly offer online learning opportunities, and students are taking more online courses. As Martin Luther King Jr. (1948) said, “Education must enable a man to become more efficient, to achieve with increasing facility the legitimate goals of his life” (p. 1). Online learning options are meeting the need to make education available to all who desire to learn (Hrastinski, 2007).

Research Problem

In 2002, online courses were taken by 1.6 million U.S. students. By the fall of 2012 online enrollment had increased to over 7.1 million students (Allen & Seaman, 2014). These statistics show online education is a growing field. With several studies citing the need for in-depth, content rich analysis of online courses, a wave of research relative to online education which focuses on content pedagogy is needed. With studies indicating mathematics is troublesome to convert to a successful online learning experience (Zavarella, & Ignash, 2009) and few online programs with affordances to support problem solving, additional research is needed to study pedagogical methods which will enhance online mathematics courses while maintaining rigor and upholding course integrity (Jonassen, 2007). This line of research will directly impact online mathematics education and the delivery of courses in the higher education arenas.

With multiple studies showcasing the background (Akdemir, 2010, Garrison, 2011), advantages and disadvantages (Hrastinski, 2007, 2008) and the need for online education methods (Hrastinski, 2007), the next logical step for this line of research is to delve deeply into content rich studies that examine the pedagogical implication of learning online. This focused study looks specifically at online mathematics education at the collegiate level with a focus on how well online courses promote problem solving for demonstration of content mastery. In alignment with this research void, this study seeks to examine student content mastery in an online pre-calculus or calculus course in comparison to face-to-face counterparts. The research questions for this study are:

1. In what ways do students' work and scores on their final assessment relative to solving systems of equations and inequalities compare between an online and a face-to-face pre-calculus course?
2. In what ways do students' work and scores on their final assessment relative to solving integrals compare between an online and a face-to-face calculus course?

Theoretical Framework

Problem solving is a "tool for learning" that is an essential skill for students to master (Jonassen, 2007, p. 186). Jonassen (2007) claims "problem solving is the most authentic and therefore the most relevant learning activity in which students can engage" (p. 186). Furthermore, Jonassen (2007) reports problem solving leads to better comprehension and retention while promoting conceptual understanding and clear articulation of thought. Data regarding student work will be evaluated through a problem solving theoretical framework.

Frameworks of problem solving are often tied to the stages of problem solving out lined by Polya (Wilson, Fernandez, & Hadaway, 1993). When working through solving a problem, Polya (1971) explains students' work through four phases: understand, plan, carry out, and look back. While progressing through solving a problem, students may need to adjust their point of view and reorient their thinking to assure full understanding of the problem. Once students clearly identify the goal and requirements, they are able to move forward in problem solving to explore the interconnectedness of their knowledge and the items at hand. After carrying out a solution attempt, students finalize their problem solving experience by looking back to review their completed solution. Wilson, et al. expands on Polya's cycle to introduce a cyclic orientation of constant framework of review, as shown in Figure 1 below.

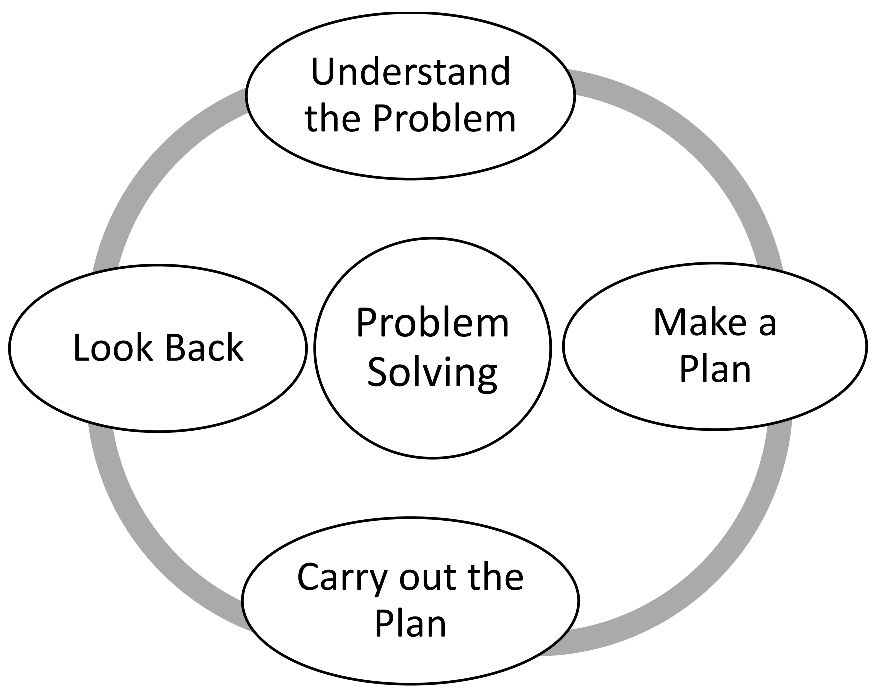


Figure 1:: Cyclic framework of problem solving

This cyclic structure emphasizes the non-linear orientation of many problem solving situations and reaffirms the constant need to re-evaluate solutions relative to the initial problem at hand. Szetela and Nicol (1992) also proposed a problem solving framework which necessitates students move through multiple actions to successfully solve a problem. Szetela and Nicol emphasize that students must:

1. Obtain appropriate representation
2. Consider potentially appropriate strategies
3. Select and implement a promising solution strategy
4. Monitor the implementation
5. Obtain and communicate the desired goal
6. Evaluate the adequacy and reasonableness of the solution
7. If the solution is judged faulty or inadequate, refine the problem representation and proceed with a new strategy (p. 42).

The two frameworks presented by Wilson et al. (1993) and Szetela and Nicol (1992) come together to formulate the framework adopted for this study. Wilson's stages of problem solving are reviewed through a detailed analysis of student work. While analyzing student work, evidence of understanding the problem, initiating and carrying out a solution plan, and reflection were reviewed. These elements are reviewed through coding distinctions outlined in accordance to the problem solving steps presented by Szetela and Nicol (1992).

Conceptual Framework

There are three vital components of problem solving (Jonassen 2007). First, there must be a problem involving an unknown which is worthy of investigation and solution seeking. Second, there must be the ability for the person solving the problem to create a representation of the problem. Jonassen (2007) refers to the creation of a mental representation of the problem as developing the "problem space" (p. 186). The third and final component of problem solving is there must be a way to manipulate the problem space. Manipulation of the problem space often involves activity or cognitive processes to explore, test, and reflect upon solutions (Jonassen, 2007). As problems vary in complexity, they can also vary according to complexity and dynamicity (Jonassen, 2007).

In an online setting, learners must have intellectual and social support to properly engage in learning problem solving skills (Jonassen, 2007). This means, the online course structure needs to convey clearly the content to be covered while leading students through problem solving activities (Jonassen, 2007). For this study, I am seeking to explore evidence of problem solving in an online setting. From a conceptual framework standpoint, this study looks at four systems of equations and inequalities questions and four integration questions to analyze student's abilities to use their problem solving skills to complete each question. The four questions from each category vary in complexity. Through analyzing each set of problems, this study aims at investigating if there is a difference in student's portrayal of understanding and problem solving abilities between online and face-to-face courses. Each question will be evaluated for evidence of student's ability to conquer each of the three vital components of problem solving, embarking on a solution technique, interpreting the problem, and manipulating the problem space.

Purpose

It is important that online education programs for mathematics support and foster students' evolving problem solving skills (Jonassen, 2007). With the growing presence of online learning altering the higher-education landscape, the quality of online learning programs should be evaluated (Larreamendy-Joerns & Leinhardt, 2006). Online education is expanding rapidly and encasing all content domains (McBrien, Jones and Cheng, 2009; Allen and Seaman, 2014). Butner, Murray and Smith (1999) report online education provides convenience, flexibility, and increased opportunities so colleges and universities are expanding their online courses and degree programs to facilitate ease of accessing higher learning opportunities. With increased online learning opportunities and the importance of fostering problem solving skills, there is a need for focused research regarding analyzing online mathematics students' problem solving abilities in comparison to their face-to-face peers in a similar learning environment. The purpose of this study is to examine student work on systems of equations and inequalities and integration questions to evaluate conveyance of content mastery in accordance to a problem solving framework.

Methods

To explore student problem solving, a selection of four systems of equations and inequalities and four integration questions are reviewed from three perspectives. The questions selected seek to scaffold student knowledge and problem solving ability demonstrations. Each set of four questions begins with a basic question and then advances to more involved questions. This progression of questions was selected to gauge students' problem solving endeavors at different levels.

Student work is first evaluated in a qualitative manner to assess statistical differences in students' scores on each question. Next, a second tier of examination is conducted to code student work in accordance with problem solving demonstration coding provided by Szetela and Nicol (1992). The third tier of problem solving analysis is conducted through a detailed evaluation of student work seeking to divulge where errors occurred and problem solving strategies faltered.

Each question reviewed was graded by the course instructor, independent of this research study. The instructor assigned scores were evaluated using Levene's Test, Welch test, Brown-

Forsythe test, and *F*-test to determine the presence of any statistically significant differences in scores between the online and face-to-face sections. Levene's test was first implemented to assess the validity of the homogeneity of variances assumption. When no violation of the homogeneity of variance assumption was found, an *F*-test was used to evaluate between-group differences pertaining to the test scores. If the homogeneity of variances assumption was violated, a corrected *F*-test (i.e, Welch test and Brown-Forsythe test) was used to evaluate between-group differences pertaining to the test scores. After a quantitative analysis of scores, a qualitative analysis of each question commenced. Codes were initially assigned and reviewed by the researcher and two co-coders. Each co-coder is an experienced mathematics educator with familiarity in teaching pre-calculus and calculus. The co-coders were selected based on their understanding of the content covered and their proficiency in analyzing student work. Once alignment of coding was achieved, codes were analyzed for trends.

Definitions of Terms

The key components of this study include a focus on online learning, synchronous learning, and asynchronous learning. *Online learning* is defined as courses "in which at least 80 percent of the course content is delivered online" (Allen & Seaman, 2010, 2011). *Synchronous learning* refers to learning when interaction between teachers and students occurs in real time (Hrastinski, 2008). *Asynchronous learning* refers to a self-paced learning structure where communication is not in real time and often occurs through email or other web mediums (Hrastinski).

Conclusion

This study is aimed at evaluating pre-calculus and calculus students' written work to explore differences that may exist in problem solving and course achievement. Chapter 2 includes a detailed examination of literature relative to online education, technology in the mathematics classroom, problem solving, systems of equations and integration. After reviewing the existing literature, chapter 3 moves to a discussion of the methods used to collect, review, and interpret the data for this study. Chapter 4 showcases the results of this study and includes examples of student work to support the results. The final chapter includes a discussion of the findings, limitations of this study as well as areas for continued research.

CHAPTER 2

Literature Review

Chapter 2 offers a review of literature pertaining to online learning. The goal of this literature review is to explore the background of online learning and then focus on the limited research which is available regarding online mathematics courses in an effort to ascertain the benefits and challenges relative to teaching mathematics online. Once the benefits and challenges are identified, additional research can be fueled to explore enhancements to online mathematics courses which will promote student content mastery through quality online mathematics learning experiences.

Seminal research and scholarly reports were selected for this literature review based on their alignment to the topic of focus and their currency in the field of online education. To begin the process of gathering key literature, works by scholars prominent in the field (Smith & Ferguson 2003, 2005, Hrastinski 2007, 2008, and Allen & Seaman 2010, 2011) were first selected. Next, additional works cited by these authors and in which these authors were cited were reviewed. These authors were deemed prominent because of the frequency of which their work is cited in publications as well as their contributions to the field of online education.

Outside of providing historical reference regarding the emergence of online education, studies or related literature published prior to 2000 were not referenced and studies conducted prior to 2005 were only utilized if they contained significantly cited content directly applicable to the topic or provided necessary background information. Once all relevant searches were exhausted and a body of literature identified, the studies were broken into categories relative to emerging themes: background of online instruction, online learning structures, and difficulties with online mathematics instruction. Each theme contains two to four subthemes which will be discussed in the following narrative.

Literature Review

In a traditional classroom setting, each content area has unique needs. The same holds true for an online environment. As online education expands, researchers suggest the need for focused content studies which illuminate pedagogical practices relative to teaching content-specific online courses (Hrastinski, 2007). Online education plays a pivotal role in increasing education levels and availability (Hrastinski). Fostering student achievement and education in mathematics and science content is a current educational focus as Science, Technology, Engineering and Mathematics (STEM) careers expand and the United States falls behind other countries in mathematics and science testing (Gningue, Peach, & Schroder, 2013).

In an ongoing study of the growth and perception of online education in the United States, Allen and Seaman (2014) have been tracking online enrollments and academic leaders' perceptions of online educational opportunities yearly since 2002. Their study reports that, over 1.6 million students were enrolled in an online course in the fall of 2002, and the number increased, with a 16.1 percent compounded annual growth rate, to over 7.1 million in the fall of 2012. During the same period of time, higher education enrollments grew from 16.6 million to 21.3 million, with a 2.5 percent annual growth rate. Approximately 33.5 percent of all higher education students in 2012, compared to 9.6 percent in 2002, were enrolled in at least one online course (see Figure 2).

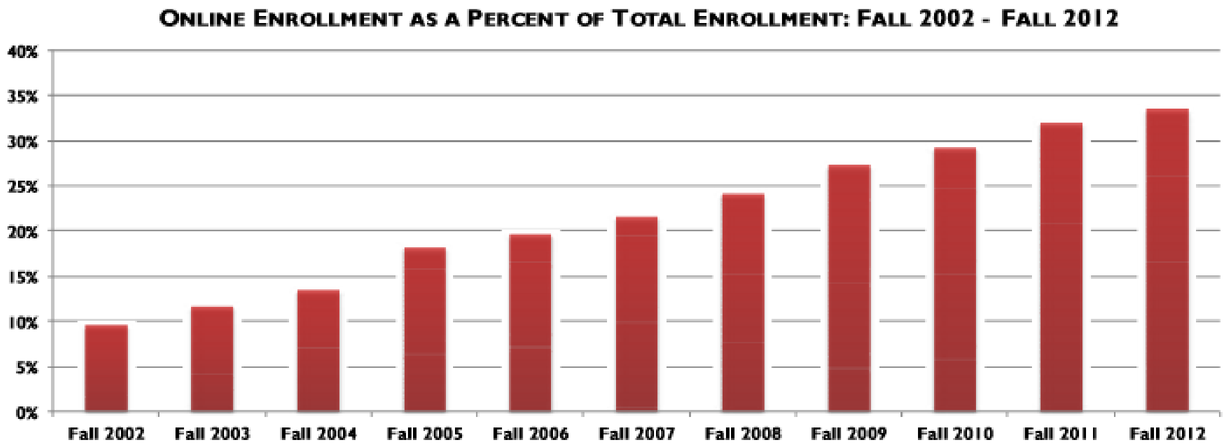


Figure 2. Online enrollment between Fall 2002 and Fall 2012 (Allen & Seaman, 2014).

Making learning available in a flexible format is a central draw of online education opportunities (Sitzmann, et al., 2006). An online course can bring learning to students regardless of time, situation, location and circumstance, and hence allows all types of learners to study at an individualized pace (Johnson, Aragon, Shaik, & Palma-Rivas, 2000). With more and more students turning to online means for educational opportunities, it is critical for the educational research community to examine and understand the extent to which these technologies will transform expectations for, and approaches to, learning, and whether the quality of learning achieved in an online setting matches that of traditional venues of instruction (Allen & Seaman, 2013; Garrison, 2011; Sitzmann, Kraiger, Stewart, & Wisher, 2006).

History and background of online education. Online learning is a rapidly growing educational method being embraced across all levels of education. Hrastinski (2007) comments that online education is serving as a powerful response to the growing need for educational flexibility. Online learning, also called e-learning, shares similarities with distance education but has evolved its own educational theories and practices (Garrison, 2011). Akdemir (2010) defines distance education as “formal education where the learning group is separated and where interactive telecommunication systems are used to connect learners, resources and instructors”

(p. 47). Unlike traditional distance education, which historically focuses solely on content delivery and independent learning (Garrison 2011), online learning uses the internet to assist teachers and students with the transfer of information in a more interactive manner (Moore & Kearsley, 2012). There are several types of learning that occur using the internet as an educational medium. Online learning is distinguishable from self-study, tutoring, and traditional distance education by its influence from educational organizations and its avenues for continuous communication between teachers and students (Hrastinski, 2007).

Traditional undergraduate courses often follow objectivist learning practices consisting of lectures by teachers, note taking by students, and regurgitation of information to complete exams, assessments, or projects (Jonassen, 1999). This transmission model of the traditional classroom structure has been significantly transformed through new online technologies (Harrison & Stephen, 1996). Harrison and Stephen explain online education has created a paradigm shift to focus on building knowledge rather than the passage of knowledge between teachers and students. Even though Harrison and Stephen's findings were published at the forefront of online education, they argued "the successful societies in the next century will be those that find ways to convert educational instructions into knowledge-building instructions" (p. 206).

In the early 1990's, a wave of educational reform began and transmission models of learning gave way to constructivist theories (Hrastinski, 2007). Jonassen (1999) describes constructivist learning as a process through which "knowledge is individually constructed" through interpretations of experiences (p. 217). Each learning theory provides new techniques and philosophies while inviting critiques and constructive criticisms. O'Loughlin (1992) argues "constructivism is flawed because of its inability to come to grips with the essential issues of culture, power, and discourse in the classroom" (p. 791). Constructivist theories gave way to sociocultural educational approaches which highlight learning as a subjective and influenced by context and perspectives (O'Loughlin, 1992).

With a focus on interactive education, learners play an active role in their learning. Computers and online course technologies have erased traditional classroom boarders and expanded interactive opportunities for students (Harrison & Stephens, 1996). Interaction is an important aspect of teaching and learning (Zhu, 2006). Removing location barriers and opening educational resources through online courses is a very attractive opportunity for students who are unable to attend classroom based, face-to-face courses due to location, family, work, health, or additional life related reasons (Allen & Seaman, 2010).

In addition to removing the boundaries of physical location based learning, supporters for online education practices advocate online education has a profound impact on aspects of classroom participation. Students who are traditionally shy and reluctant to participate in classroom discussions often become more active in an online classroom (Zhu, 2006). Online courses also allow students to review material to the depth they need. Pausing online lectures, reviewing and replaying online material, and alleviation of peer pressure to determine pacing, are valuable components of online education opportunities (Braude & Merrill, 2013).

Unlike a traditional classroom setting in which teachers move forward at a dictated lesson pace, online learning promotes individualized learning, content mastery, and provides students an opportunity to focus their attention on the content they need to more deeply examine (Kennedy, Ellis, & Oien, 2007). Encouraging student guided pacing can enhance student learning, but can also potentially be detrimental to students who are not self-motivated to structure their time in their online course environment (Wadswroth, Husman, Duggan, &

Pennington, 2007). In a qualitative study reviewing student learning strategies and motivation in an online developmental mathematics course, Wadsworth et al. (2007) found time management was predictive to their course performance. Wadsworth et al. (2007) reviewed three question surveys from a set of 89 developmental mathematics students and concluded success in an online developmental mathematics course is partially dependent on “the learning strategies and self-efficacy of the students” (p. 12).

Also studying developmental mathematics, Ashby, Sadera and McNary, 2011 sought to evaluate differences in student success rates, which they defined as achieving a 70% or greater final course grade, between online, blended and face-to-face course modalities. Ashby et al. used ANOVA calculations to compare course averages between the online, blended and face-to-face courses and conclude “learning environment has an impact on success for the developmental math student” (p. 137). Ashby et al. found online “and blended students performed worse than the traditional face-to-face developmental math students when not taking attrition into account, however considering only students who completed the course, face-to-face students performed worse” (p. 138).

Allen and Seaman (2010) have been tracking online education growth in higher education institutions in the United States since 2002. Each year Allen and Seaman conduct a study and report on the extent of online education offerings. Reports from years 7 and 8 of the study (2009 and 2010) are included in this review. The 2010 report is titled *Class Differences: Online Education in the United States* and the 2011 report is titled *Going the Distance*. Both reports are part of the Sloan Consortium. In each report, online learning is defined as courses “in which at least 80 percent of the course content is delivered online” (Allen & Seaman, 2010, 2011). Allen and Seaman send a survey to all higher education institutions in the United States that are categorized as public institutions. Out of 4,511 institutions, 2,583 responses were received in 2010 (Allen & Seaman, 2010) and out of 4,523 institutions, 2,512 responses were received in 2011 (Allen & Seaman, 2011). By 2011, 65% of survey respondents indicated online education is a critical component to their institution's strategic plan (Allen & Seaman, 2011). Each year, Allen and Seaman have reported online enrollments are growing at a faster rate than higher education enrollments, signifying a rapid increase in student interest towards completing courses online (2011). Over 5.6 million students took at least one online course in 2010 (Allen & Seaman, 2010). In 2011 over 6.1 million students took at least one online course (Allen & Seaman, 2011). Allen and Seaman also found that while student acceptance of online learning has increased, faculty acceptance of online learning has remained constant, at a rate of 1/3 of faculty accepting online courses as legitimate educational experiences, since initial data collections in 2003 (2010, 2011). Based on eight years of research, Allen and Seaman predict online learning will continue to grow (Allen & Seaman, 2010, 2011).

Online learning structures. There are two main structures for online classrooms, asynchronous and synchronous (Hrastinski, 2008). To create a successful online course experience, it is crucial that educational institutions recognize the strengths and limitations of each structure. Asynchronous and synchronous techniques should each be used as appropriate in course design to maximize learning potential (Hrastinski). At its origin, online education initiatives relied heavily on asynchronous course structures (Hrastinski). Asynchronous learning takes place when teachers and students are not actively participating in learning together at the same time. The ability to log into online courses from anywhere and at times convenient to the student's schedule is a benefit of asynchronous learning (Hrastinski). In an asynchronous model, students are able to download documents, draft and publish discussion board responses, review lesson materials and watch recorded teaching sessions at various times.

There are both advantages and disadvantages to an independent, asynchronous course design. Asynchronous structures allow learners the convenience of scheduling and additional reflection time, but also promote isolation and time delays between question submission and answer reception (Hrastinski, 2007, 2008). Many students find asynchronous structure desirable due to the flexible study times and the ability to fully reflect and analyze their discussion points prior to publicizing to their teacher and classmates (Trenholm, 2006). In an asynchronous classroom, students log into the course, view the required materials, and complete assigned tasks independently. Students are often required to share their thoughts and learning reflections through participation in discussion boards, but their thoughts are not hindered by an environment necessitating an immediate response (Hrastinski, 2008). Students who are shy, private, or in need of additional response processing time benefit from asynchronously formatted discussions and private opportunities to submit questions to the course instructor (Smith, Ferguson & Caris, 2003).

Critics of online learning warn technology centered education models devalue the practice of real time decision making, stifle real time oral discourse, necessitate new forms of student monitoring practices, and foster a digital divide amongst students (Anderson, 2008). To circumvent these obstacles, often synchronous components or a blend of synchronous and asynchronous components are used in online courses. As a complement to asynchronous styles, synchronous learning techniques increase student participation, task support, and motivation (Hrastinski, 2007).

Conversely to asynchronous learning, synchronous learning occurs when online interaction takes place in real time between students and teachers. Video streaming, web conferencing, and chat or instant message applications are commonly used during synchronous sessions (Hrastinski, 2008). Because synchronous sessions occur in real time, students are able to ask questions and get immediate answers, obtain interactive assistance with course material, and feel included in a classroom atmosphere where participation is expected and a critical component to the learning process (Hrastinski, 2007, 2008). Synchronous sessions are beneficial for quick response type scenarios, such as clarifying expectations or directions (Hrastinski, 2008). Synchronous learning requires a common meeting time, making it potentially more difficult for students to coordinate, but interaction with peers also increases motivation while decreasing feelings of isolation (Hrastinski, 2008). With a desire to keep conversations flowing in a synchronous environment, students often respond quickly to question prompts; promoting a quantity over quality focus as students attempt to respond before similar thoughts are shared by their peers (Hrastinski, 2008).

While reviewing a series of undergraduate and graduate level online, synchronous courses, Hrastinski (2007) used qualitative and quantitative data gathering techniques to assess student participation. After analyzing the data, Hrastinski (2007) concludes student participation in online courses is potentially enhanced through synchronous communications. This conclusion is a result of data showcasing synchronous participation improves student motivation, provides social interaction, and ease of information exchange (Hrastinski, 2007). In a study regarding the quality of student responses to questions posed in a synchronous or asynchronous course environment, Hrastinski (2008) found students participating in an asynchronous discussion tended to focus more heavily on course content, than their synchronous counter-parts in discussion content. Hrastinski (2008) reported between 93% and 99% of the sentences submitted in the asynchronous discussion were content related while 57%-58% of the synchronous sentences focused on content. The remaining 43%-44% of synchronous content focused on planning or social related discussions. This difference in sentence content can be attributed to higher content focus and greater levels of information processing time (Hrastinski, 2008).

A study was conducted by Weems (2002) in which a comparison was made between an online developmental mathematics course and a congruent face-to-face section of the course. Weems (2002) used a sample size of 38 students, 20 online and 18 face-to-face, to conduct his study. The online and face-to-face sections of the course utilized the same textbook, supplemental materials, schedule, exams and assignments (Weems, 2002). After analyzing student achievement in the two courses, Weems concluded there was not a significant difference in academic achievement (2007). This study was conducted over the course of one semester. With a small sample size and a lack of repetitiveness in findings, this study alone is not sufficient to provide definitive conclusions regarding a comparison of student achievement in an online versus face-to-face course setting.

Additional research promotes use of both synchronous and asynchronous learning styles in online settings. Hrastinski (2008) concludes by noting “asynchronous and synchronous e-learning complement each other” and should be used as appropriate “for different learning activities” (p. 55). Means, Toyama, Murphy, and Baki (2013) contend that since online learning has become a prominent trend in education, efficacy of practice needs to be established. To study the effectiveness of online learning, Means et al. (2013) conducted a meta-analysis designed to provide statistical analysis of the data available relative to learning outcomes in online courses and blended courses. For the purpose of their meta-analysis, blended courses referred to courses with both online and face-to-face components. Forty-five studies of K-12 and higher education classrooms were utilized. Studies were not limited to a specific content area or classroom orientation. Through their meta-analysis, Means et al. (2013) found the students who participated in online course experiences outperformed the students who attended face-to-face courses. A statistically significant positive difference was found between students in blended course structures while only a moderate difference was found between fully online and fully face-to-face student performances (Means et al., 2013). At the conclusion of their report, Means et al. (2013) encourage additional research and meta-analysis work be conducted on “only those studies of online mathematics learning” (p. 37), highlighting the need for focused research on online mathematics courses.

Similarly, Larson and Sung (2009) performed a three way comparison to investigate differences in student success between online, face-to-face and blended course models. Final grades and were examined to gauge success and an Analysis of Variance test was used to explore significance. All three course in Larson and Sung’s study were taught by the same instructor with the same course resources. Larson and Sung reported no significant difference in student performance was found between the online, face-to-face or blended courses. Furthermore, the blended and online students reported comparable satisfaction levels to their course experience as their face-to-face peers.

Online learning challenges. Online learning presents a unique set of challenges for teachers and students. Merely being online does not constitute an education experience. Teachers are finding students need different types of support and assistance with various issues and problems in an online environment (Burden, 2008; Taylor & Galligan, 2006). Courses that are taught through an online platform are organized differently than traditional face-to-face courses and provide different opportunities for students (Moore & Kearsley, 2012). In an online environment, content delivery and access to information concerns give way to blending connectivity and personal learning freedom (Garrison, 2011). From the students' perspective, online learning requires a change in learning traditions. In online courses, students must learn to use technology to communicate and fully express their ideas, often using only written text. Characteristics of successful online students are self-discipline, ability to focus on their coursework, self-starters, and comfort with online interaction. Students who possess these characteristics are more likely to complete their online course endeavors (Smith & Ferguson, 2005).

From the teacher perspective, online courses necessitate different teaching practices than face-to-face courses. Online teachers must be willing to revise their teaching techniques to adapt to online course properties and needs. While Allen and Seaman (2010, 2010, 2011) report faculty and administration support of online learning is growing, a reluctance to revise teaching practices remains present among faculty members (Beaudoin, 2002). To utilize online learning to its fullest potential, it is necessary that both students and teachers take advantage of the unique opportunities available in an online classroom such as collaboration tools, internet housed resources and simulations, and the elimination of time and location divisions for participation.

An increase in the demand for online courses is not unique to the United States. In Kenya, e-learning is rapidly gaining popularity and Kenyan universities are instituting e-learning protocols as they seek to expand their online and blended course offerings (Muuro, Wagacha, Oboko, & Kihoro, 2014). Muuro et al. conducted a study to investigate student perceptions of challenges pertaining to online learning, particularly relative to collaboration in an online setting. Muuro et al. used a purposive sampling technique to deploy a descriptive survey to students at two private universities. Students voluntarily responded to surveys, consisting of 30 questions, through an online platform and 183 responses were received, correlating to an 87% response rate for the population. (Muuro et al.). Using a quantitative analysis, survey results were analyzed and key challenges of limited group member participation, lack of instructor feedback, lack of time for quality participation, and off topic discussion posts surfaced. Of the 183 participants, 75 indicated little or no online collaboration engagement citing a lack of instructor provided collaboration activities as the main reason for collaboration absence. Muuro et al. cite a need for increased instructor training so engaging online collaboration becomes an integrated part of all online courses. Additionally, Muuro et al. conclude lacking instructor feedback is a major challenge in online environments and urge the importance of improving instructor motivation and training relative to online learning pedagogy.

Supplementary studies, such as Kim, Liu, and Bonk (2005), also found lack of or delays in receiving feedback to be a challenge of online learning. Kim et al. (2005) conducted a study of online MBA programs for which they cite a rapid increase in online enrollments due to the flexible nature of an online course structure. A case study was used to explore student experiences in their online MBA courses and both qualitative and quantitative analyses were conducted. From a qualitative perspective, 10 study participants were interviewed. A Likert scale survey was completed by 102 students to collect data for a quantitative analysis. Once the data was analyzed, Kim et al. reported that students saw their online MBA course as a positive experience citing their coursework as challenging, eye-opening, and enlightening. Students cited flexibility as their favorite aspect of completing their MBA course online, while also indicating opportunities to interact with instructors and peers as beneficial aspects of their online experience. While mainly satisfied with their online MBA experience, students also noted challenges regarding communicating with peers and lacking real-time feedback opportunities (Kim et al.). To improve the quality of their online experience, students requested additional instructor interaction and additional training relative to an online environment.

Structure. Currently, there is limited research available relative to pedagogical approaches to online instruction (Wadsworth et al., 2007). Each academic discipline adapts to online education differently. Smith and Ferguson (2005) postulate, mathematics is poorly supported by online learning systems. Many undergraduate mathematics courses follow an asynchronous course model. Asynchronous courses which rely heavily on discussion boards are not conducive to mathematical figures and procedure explanations (Smith & Ferguson, 2004). Support for mathematical notation and mathematics diagrams is a crucial building block to successful mathematics processes and understandings. Programs are being developed and tested to rectify this issue, but this issue has not yet been resolved. In addition to the lack of support for mathematics notation and mathematics diagrams, online asynchronous courses do not account for what Smith and Ferguson describe as student "panic" when faced with troublesome mathematics problems (2005). In an asynchronous model, the time delay between when teachers respond and when students initially reach out for assistance in panic situations is too great, resulting in students suffering or surrender (Smith & Ferguson, 2005). Mathematics concepts builds rapidly. If undergraduate students begin to struggle and do not perceive a way to find clarity and assistance, confusion on one topic can snowball rapidly. Engelbrecht and Harding (2005) explain there is a need for pedagogy relative to learning mathematics online. Applying traditional face-to-face classroom pedagogy in an online setting will inhibit teachers and students from maximizing the knowledge exploration potential present in online learning structures (Kanuka, 2002).

In online courses, consideration must be made for providing technical help, relaying student orientation information, and promoting learning management system familiarity (Akdemir, 2010). Additionally, Akdemir found teaching mathematics courses online is challenging for instructors due to the application based content of mathematics and the frequent limitations imposed by learning management structures. Akdemir also found there are limitations to the types of student assessments which can be offered in a virtual setting. With enrollments of over 300,000, grading pencil and paper assessments proves to be a daunting task; therefore, multiple choice assessments which are easy to grade are frequently utilized. Professors in this study have identified self-assessment conducted by the students and individual student projects are valuable assessment measures in online settings. Enrollment rates were discovered to be the driving force behind the frequency and type of assessments used (Akdemir).

Relative to the final theme of effectiveness, Akdemir (2010) reports faculty members found ease of student activity monitoring through the learning management tools and flexibility of remote access to be the biggest advantages of online courses. The largest disadvantages were reported to be the significant amount of time required to design and create an online course and overall course design. Akdemir concludes teaching online is more difficult than teaching face-to-face and that element of difficulty is enhanced when teaching mathematics online. Mathematics is very application based and detail oriented. Without considerable time spent designing mathematics courses and attending to student needs, Akdemir explains the difficulty of experiencing quality mathematics courses in an online environment is compounded. This study looks at online mathematics course instruction from the perspective of faculty members. Akdemir recommends additional research be conducted to investigate learning experiences from the student perspective.

Attrition. Mathematics is difficult to manipulate in an online setting due partially to the intricacies of entering mathematics notation and graphs electronically and the ease of finding applications allowing students to embrace cheating. Difficulty manipulating mathematics online can be seen through higher attrition rates in mathematics when compared to other online course experiences or face-to-face classroom experiences (Smith & Ferguson, 2005).

Several studies have been conducted to explore attrition rates in relation to student demographics and online course structure, but research relative to content specific attrition rates is lacking (Smith & Ferguson, 2005). Studies have shown mathematics courses have higher rates of attrition than other content area courses in a face-to-face setting. In a quantitative study, Smith and Ferguson (2005) look at attrition rates as a measure of a student's perception of the course difficulty level citing "higher attrition rates indicate problems from the student point of view" (p. 326). In a study of over 3,000 asynchronous online courses offered through the State University of New York (SUNY) system, the mean attrition rate in mathematics courses vs non-mathematics courses was found to be statistically significant at the 0.001 level with a mean attrition rate for mathematics courses as 0.31 and non-mathematics course as 0.18 (Smith & Ferguson, 2005). From this, Smith and Ferguson (2005) conclude mathematics is more problematic than other content areas online as evidenced by the higher attrition rates. When expending their study to face-to-face course experience, no significant difference was found between mathematics and non-mathematics course attrition (Smith & Ferguson, 2005). Smith and Ferguson (2005) speculate higher attrition rates are due to more non-traditional students embarking in online courses after longer absences from mathematics study. Wadsworth et al., (2007) argues that appropriately implemented strategies to emphasize student self-efficacy will enhance student achievement in online developmental mathematics courses.

Mathematics notation and graphing. While studying student attrition, Smith and Ferguson (2005) also found "online environments are not well adapted to mathematics" (p. 331). The learning management systems widely available do not directly support complex mathematics notation or diagrams to be embedded in student responses and discussions (Smith & Ferguson). With keyboard notations serving as a hindering factor, online mathematics instructors and students are often forced to communicate in code, or through scanned and emailed free writing, rather than utilizing precise typed mathematical notation (Smith & Ferguson). With threaded discussions and email being a key component of asynchronous online courses, limited notation ability compounds student challenges with notation and notation interpretation (Smith & Ferguson).

In addition to accommodating mathematics notation, online learning must also accommodate graphing. Graphing is a pivotal component of many mathematics courses and tends to be a problematic concept for many students. Involving strategic competence, conceptual understanding and relational observations, graphing is a representation activity which requires students to make meaning from abstract concepts through the use of anticipatory thinking (Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2008). Cavanaugh et al. conducted a study to investigate the effectiveness of interactive graphing tools in online Algebra courses; seeking to evaluate the effectiveness of the graphing tools relative to students' ability to successfully graph linear equations. For this study, observations, pre-tests and post-tests were conducted for 101 participants, 30 in a control group which did not receive interactive tools and 71 in an experimental group which did receive interactive tools. Cavanaugh et al. (2008) concluded the main effect for test type is statistically significant and signifies a difference between pre and post-test scores is present in the population, but the ANOVA between subjects comparison of the mean pre-test and post-test scores is not statistically significant, meaning a difference between the control group and experimental group was not justified based on the implementation of the interactive graphing tool. Cavanaugh et al. (2008) recommend additional research be conducted regarding this topic citing the need for online mathematics instruction to incorporate enhancements for effectively teaching students to analyze and interpret data.

Cheating. As mentioned previously, in 2011, 65% of institutions offering online courses perceive online education is a critical component to their institution's strategic plan (Allen & Seaman, 2011). With online courses serving as a critical infrastructure for the institutions, the need to maintain rigor and ensure courses integrity is upheld are growing concerns (Trenholm, 2007). To review cheating in an online environment, Trenholm separates online course into two categories, "Writing-Based" (WB) and "Mathematics or Fact-Based" (MFB) (Trenholm, p. 281). For the purpose of this review, cheating is defined as "the act or action of fraudulently deceiving or violating rules" (Trenholm, p. 284). Trenholm argues cheating is a critical issue in WB as well as MFB courses, but WB courses have programs, such as *Turn-It-In,* which assist with monitoring cheating and plagiarism. MFB courses have fewer such programs and need methodologies instituted to limit and prevent academic dishonesty (Trenholm). Currently, many asynchronous online courses rely on the honor system as a proctor for cheating (Trenholm). Campbell (2006) argues cheating in online courses is easier than face-to-face courses and suggests online courses must be closely monitored. Harmon and Lambrinos (2006) found cheating to be prevalent among online students but the requirement of proctors for exams help deter students from cheating. Due to the prevalence of cheating, Campbell suggests courses taught entirely online should be ban as all cheating cannot be circumvented. When surveying two-year college faculty, Cotton (2002) found 25% of faculty members did not require any form of formal proctoring for their online mathematics course assessments. As online learning continues to grow rapidly, higher education continues to be a competitive environment, and students continue to harbor mathematics phobias, cheating will remain an issue in online mathematics courses (Trenholm).

Technology in mathematics teaching. Technology and mathematics classrooms are not mutually exclusive entities. When thinking about technology in a mathematics classroom, calculators are typically one of the first tools that come to mind. A mathematician without a calculator could be compared to an artist without his paint brush. Technology in a mathematics classroom extends beyond just calculator usage. Homework resources, communication tools, websites, blogs, simulation activities, and wikis are additional examples of technologies commonly integrated into mathematics classrooms (Tuttle, 2008). Marshall McLuhan is famous for his claim: “The medium is the message” (Kelly, 2003, p. 1037). As technology advances and society and education react, Kelly (2003) suggests McLuhan’s statement should be revised to “The tool defines the skill”, alluding to the impact technology has on all aspects of society.

Technology changes how information is passed, the role of students and the role of teachers (Kelly, 2003). Kelly states, “A characteristic of the information age is that knowledge is more widely held, openly shared, and easily accessed” (p. 1038). Technology in the mathematics classroom came in the form of slide rules, calculators, computers, and graphing calculators. The first technological advances in the mathematics classroom came in 1942 with the mainframe computer (Kelly). In 1967, the first basic four function calculators were introduced and used to assist with mathematical calculations. Just over a decade later, in 1978, the personal computer became a valuable tool for mathematics study which forced a reevaluation of the school curriculum (Kelly). In 1985, the graphing calculator began to emerge and transform the mathematics classroom (Kelly).

With the emergence of the calculator came the debate as to the calculator's place in the classroom and the extent to which they should be used as a tool but not as a replacement for computational skills (Kelly, 2003). In 1995 a Wisconsin Mathematics Council meeting speaker boasted he was among the first to use graphing calculators, but then proclaimed he has since doubted the use of calculators as students do not have enough background knowledge to make calculators a useful tool (Askey, 1997). After evaluation of the calculus content covered in the time allotted, a committee studying the coverage of calculus content found "that with heavy use of computers it would take more time to teach the same material rather than less" (p. 738). Admitting the use of technology in mathematics instruction might have drawbacks, Askey contended that technology can contribute to education and argues technology has a place in the mathematics classroom. With all new teaching approaches, there will be skepticism. In this case, Askey's hesitation enhances the need for studies which closely evaluate and explore the outcomes of student learning through technology.

Tuttle (2008) discusses elementary students learning to problem solve, manipulate money, and make practical connections with mathematics while playing simulations such as "Lemonade Stand Game" by ClassBrain, and data tools such as "Illuminations Bar Grapher" by the National Council of Teachers of Mathematics (p. 30). Middle school students continue to use technology in the mathematics classroom as they seek to expand their mathematical abilities. Tuttle describes middle school mathematics technologies as tools for students to use as they explore real-life math, share data, compare analysis, and utilize problem solving strategies. When students move to high school level mathematics, Tuttle claims "Math(ematics) becomes physical" and students expand to technologies such as motion-detectors, internet tools, and even TV shows. Tuttle comments "the online site becomes an extension of the class," indicating the vast resources available to students outside the traditional classroom walls when embarking on utilizing online technologies (p. 30).

Lu (2011) shares her experience teaching an online business mathematics course. In past experience, Lu commented that generally students in online courses perform slightly better than their face-to-face peers. But, Lu acknowledges the effect of online instruction is difficult to measure solely by core grade and thus surveyed students at the conclusion of the course. Lu found 89 % of her online business mathematics students agreed or strongly agreed that their online learning experience was valuable and online offerings for mathematics should continue. Students commented on their appreciation of the availability of their online classroom, they felt comfortable with the resources provided in their online course environment, and the convenience of online office hours and learning opportunities.

In addition to traditional online courses, massive open online classes (MOOCs) are gaining enrollments. MOOCs are typically free courses offered by institutions or outside of intuitions which offer students an opportunity to explore different course content. MOOCs are typically not awarded credit and students are able to openly enroll, study, and then move on from the course experience. Designed for unlimited participation, MOOCs typically contain large numbers of students and permit students to move freely through course materials (Allen & Seaman, 2014). Allen and Seaman have found MOOCs draw a lot of media attention because of their uniqueness in both design and structure. MOOCs do not comprise a large percentage of online course offerings, approximately 5%, but something institutions are considering implementing to "increase visibility of the institution" (Allen & Seaman, 2014, p. 25).

Online resources can be an extension of a face-to-face mathematics learning environment, or an online course can become its own learning environment. When embarking on developing an online calculus course for Engineering students, Allen (2001) was challenged by a comment by Allen, Stecher, & Yasskin (1998):

> At a minimum the complete online course must do everything a book does. To succeed, it must do very much more. Developers should look for computer-assisted teaching devices that the classroom teacher cannot match. (p. 62)

This statement is quite profound; at a time when online learning opportunities were just moving from conceptualization to legitimate offerings, Allen not only found replicating a face-to-face experience in an online environment presented more hurdles than just conquering the necessary technology, but also required formatting a fully inclusive learning opportunity indicative of a traditional classroom experience. After an extensive study of online course pedagogy, Allen developed an online calculus course. The course was presented to community college students and Allen surveyed students to gauge their impression of their learning experiences. Allen found students adapted to the technology needed for an online learning environment, students created their own study groups to foster collaboration and social connections, and students were able to efficiently use their time. Allen also noted the importance of conducting future follow-up surveys of the students to gauge their knowledge retention and their performance in future mathematics courses.

Anders (2014) discusses the creation and implementation of a MOOC for calculus. Anders explains the inaugural launch of a MOOC for calculus, created by Jim Fowler from Ohio State, which attracted over 35,000 enrollments. This first course was offered over the span of 6 weeks and covered 23 hours of content. After embracing over 110,000 enrollments, the course became available for continuous enrollment; allowing students to begin the course whenever they wanted, regardless of semester intervals. Anders (2014) found students embark on this MOOC calculus experience because it keeps their brain engaged on scholarly mathematics material and assists with clarifying calculus content. Extensive numbers of students enrolling in Fowler's MOOC for calculus reinforces the desire of students to learn at their own pace in a manner convenient to their lifestyles, an educational niche being filled by online learning opportunities.

Like Allen (2001), Jungic and Mulholland (2011) also embarked on creating an online calculus course with the preconception that a successful online course would need to be as similar as possible to the traditional face-to-face calculus course offering. Looking to create materials that enhance their face-to-face course and experiment with technology available, Jungic and Mulholland saw creating an online calculus course as a challenge which would provide a good foothold for the early offerings of online mathematics courses at their institution. Video lectures, online homework assignments and discussion boards were used to help students progress though the online calculus course. Jungic and Mulholland compared student achievement in the online course to face-to-face courses and found results to be congruent between the groups, but were surprised to find the high level of instructor to student interaction experienced in a face-to-face course was not indicative of their online course experiences. Since their initial development of the online course and resources, Jungic and Mulholland (2011) have found success in blending their online resources into their face-to-face course and have begun sharing their recorded lectures with their face-to-face students.

Problem solving in mathematics learning. Problem solving is a foundational element in the study of mathematics. "A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems" (Wilson, Fernandez, & Hadaway, 1993, p. 57). Word problems, figure interpretation, constructions, proofs, explorations of patterns, these are skills central to the mathematics field of study (Wilson et al.).

Problem solving is a process that students must work through while approaching and solving a mathematics question. While different perspectives are present regarding the exact process undertaken, commonality exists in the belief that students need to work through a process to appropriately display their ability to think through a problem and arrive at a systematic solution (Szetela & Nicol, 1992).

Assessing student understanding of mathematical processes is difficult because it requires students to clearly communicate their thought processes (Szetela, & Nicol, 1992). Szetela and Nicol (1992) contend that the best way to assess students' problem solving performance is to review student work relative to a devised scale which rates student work and responses. An analytic scale is a method to use which functions as a ranking system and allows teachers to focus on each stage as desired (Szetela, & Nicol, 1992).

Traditionally, assessments in the mathematics classroom are primarily recall questions, lacking feasibility for students to display their depth of understanding through portrayal of work (Rosli, Goldsby, & Capraro, 2013). To enhance evaluation of depth of work, many instructors instituted problem solving rubrics designed to focus on work students provided. Problem solving rubrics can be used to assess students' mathematical understanding. Rosli, Goldsby, and Capraro review problem solving rubrics from Charles, Lester, & O'Daffer, (1987) and Kulm, (1994) while discussing the benefits of having a rubric for analyzing students' abilities; providing "teachers with valid and reliable scores in order to monitor and to provide feedbacks on students' progress" relative to specific criteria (Rosli, Goldsby, & Capraro, p. 58).

Systems of Equations

This study focuses on students' approaches to solving systems of equations, which is a critical topic that spans across Algebra 1, Algebra 2, and pre-calculus courses. Typically, students learn to solve systems of equations using substitution, elimination, graphs, or matrices (Carley, 2014). Different curricula may place different emphasis on which method to use. For example, Der-Ching and Yung-Chi (2015) sought to compare the way the study of Systems of Equations was presented in different textbooks in Finland and Taiwan. Der-Ching and Yung-Chi found the main difference was the approach used to solve systems; graphical techniques were emphasized in Finnish textbooks while Taiwanese textbooks encouraged Algebraic approaches. Traditionally in the U.S., students are introduced to the process of solving systems of equations through the use of graphs before being introduced to algebraic procedures (Proulx, Beisiegel, Miranda, & Simmt, 2009). However, greater focus is placed on algebraic solution techniques over graphing techniques. Sfard and Linchevski (1994) contended that students who solely depend on algebraic solution methods understand how to manipulate the algebraic process but lack comprehension of their solution meaning. After posing questions to a collection of students, Sfard, & Linchevski, found students frequently manipulate symbols while executing a routine rather than applying meaning or understanding beyond a procedural level. Without conceptual understanding, "students may easily become addicted to the automatic symbolic manipulations" currently employed (p. 121).

Being able to algebraically manipulate equations simultaneously allows students to calculate an answer, but "robs them of seeing some of the beauty of mathematics by denying them the experience of understanding the geometry of what they are doing" (Gannon, & Shultz, 2006). Through evaluating the techniques several textbooks used to teach students how to solve systems of equations, Proulx, Beisiegel, Miranda, & Simmt (2009) found traditionally algebraic manipulation garners more longevity of focus than graphical interpretations, but contends students should be exposed to multiple solution methods. Proulx, et al., (2009) presented and discussed a concept map, shown in Figure 3, detailing different solution strategies and techniques which can be deployed to solve and analyze systems of equations and encourage teachers to utilize multiple methods to enhance relevance and solidify concept meaning for students.

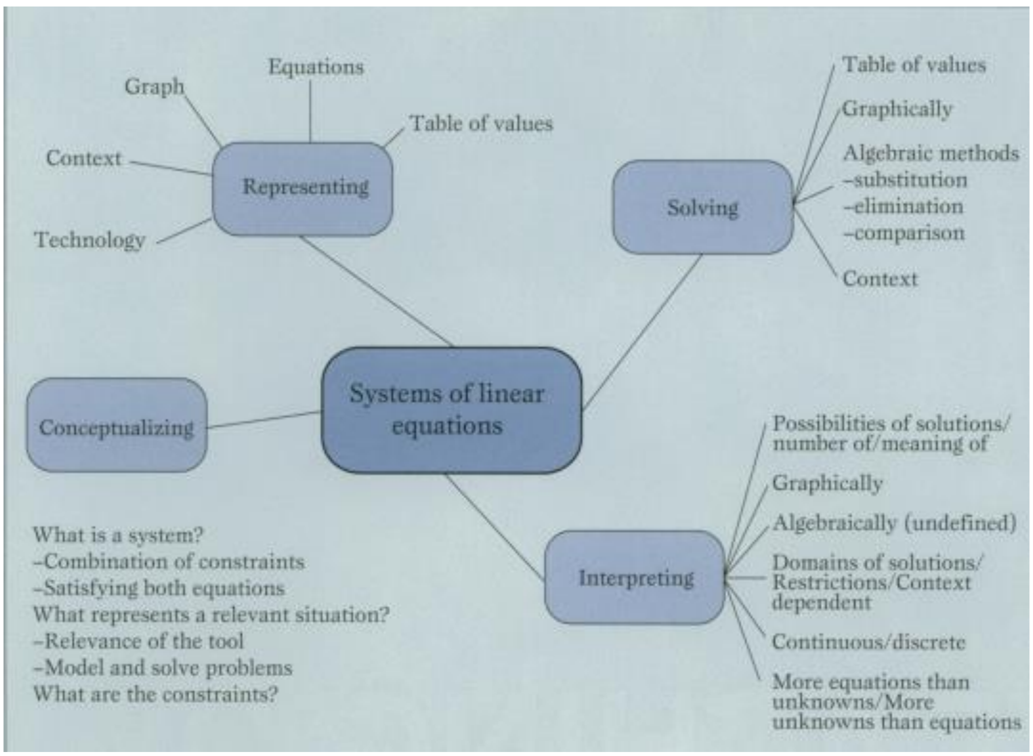


Figure 3: Systems of equations concept map

Solving systems of equations algebraically, graphically, through the use of technology, and through context analysis are strategies suggested by this concept map in connection with simultaneously solving, interpreting, conceptualizing, and representing a system of linear equations (Proulx, et al., 2009).

Integration

How students learn calculus is a growing field of research (Jones, 2014). Differential equations, proofs, statistics, derivatives, Riemann sums, limits, and integrals are the key concepts heavily focused upon with greater attention being focused on the concept of limits (Jones). Jones argues limits are important, but attention also needs to be focused on how students learn derivatives and integrals. With a stronghold in application to real-world concepts, understanding integration is valuable, especially for students studying physics and engineering (Jones).

Looking to appreciate students' understanding of the integral, Jones (2013) interviewed calculus students and presented them with problems to solve. After observing the students working mostly in pairs to complete the provided problems, Jones discovered students had a working knowledge of the integral and were able to manipulate the provided integration problems. Jones argues the idea that students' difficulties surrounding integration stem from students not being able to appropriately decipher which integration interpretation they should utilize in practical settings. Understanding that area under the curve is a crucial component of integration is valuable, but Jones emphasizes that area under the curve is not the only context for which integration should be viewed. Reviewing common textbooks, Jones comments that integration relevant to Riemann sums is a small instructional component relative to integration. Looking at integration as the anti-derivative is a much more prominent focus in textbooks, leaving Jones to recommend further focus and emphasis on accumulation and "adding up pieces" aspects of integration (p. 138).

Bezuidenhout and Olivier (2000) review student work to analyze student procedural and conceptual understanding of integration. Using a pre-test and post-test structure, Bezuidenhout and Olivier analyzed students' work to unveil misconceptions and errors in understanding surrounding key calculus concepts, such as integration. While commonly referenced as area under the curve, this is not the only interpretation for integration, and an area in which students struggle with understanding. Bezuidenhout and Olivier found students have misconceptions regarding integral usage and urge teachers to develop concept images to assist students with constructing solid integration techniques.

Literature Review Summary

Recently, there has been a dramatic increase in online course offerings (Smith & Ferguson, 2005). This literature review has presented research and related literature regarding the background of online learning, online learning structures, and online learning challenges. With online education emerging as a field, there is a large body of research available regarding basic attributes of online learning including structure, need for pedagogical practices, teacher impressions, and benefits. There is not a vast amount of research presently available regarding specific attributes of online learning as associated with learning mathematics online. As evident in this literature review, the research that is available regarding learning mathematics online contends online platforms have room to grow relative to their support for a quality mathematics learning experience.

As presented, the literature reviewed suggests mathematics is not well suited for online learning environments (Smith and Ferguson, 2005). Higher attrition rates (Smith and Ferguson, 2005), difficulties with learning management systems support for mathematics notation (Cavanaugh, Gillan, Bosnick, Hess, & Scott, 2008; Akdemir, 2010; Gningue, Peach & Schroder, 2013), and the ease of cheating options available (Trenholm, 2007) are contributing factors to the demise of perceived online learning quality for mathematics courses.

An asynchronous course format is common for undergraduate level online mathematics courses (Hrastinski, 2007). Test scores, placement exams, and other quantitative data is summarized through research regarding the educational value of online school programs, but, upon an initial review, limited research is available relative to the quality of asynchronous online learning as perceived by the undergraduate mathematics student. Smith & Ferguson (2005) used quantitative research practices to investigate the external problems surrounding online learning relative to mathematics courses, but did not focus on student perceptions regarding the quality of their educational experience. Many empirical articles relative to online learning research focus on online as a learning medium, but the studies pertaining to theory building and specific content areas appear to be lacking (Hrastinski). Through analyzing student's work on systems of equations and integration questions in an effort to see if differences are present between demonstrations of student understanding in online and face-to-face courses, this study will focus on students' problems solving skills relative to their course instruction modality. There is a void of research relative to exploring student learning in online pre-calculus and calculus courses. This study proposes to fill the void of content focused online mathematics research.

CHAPTER 3

Methodology

The purpose of this study is to review student ability to demonstrate mathematics understanding through problem solving by examining online versus face-to-face pre-calculus and calculus student work. Student work will be evaluated in accordance to a problem solving framework which provides analysis structure. All student work is anonymous and provided by instructor of the mathematics courses.

This study is composed of two parts, pre-calculus and calculus. For each part, four questions were selected from the students' cumulative final exams and then analyzed to investigate if there were any performance differences between students who took an online course and those who took a face-to-face session of the same course taught by the same instructor, using the same materials. While the style of analysis was the same for both the pre-calculus and calculus parts of this study, different questions were used. The pre-calculus part of this study used four systems of equations and inequalities questions and the calculus part of this study used four integration questions. These content topics were selected because they are foundational concepts covered in algebra, pre-calculus and calculus.

Since systems of equations are studied and assessed through multiple modalities, this topic was selected to give an opportunity to review student work on a variety of systems of equations and inequalities questions. Integration was selected as the focus topic for calculus because integration is vital to studies of physics and engineering and provides a stronghold in application to real-world concepts (Jones, 2014).

Through analyzing the gathered data, the researcher seeks to reveal understandings relative to the research questions:

1. In what ways do work and scores on the final assessment relative to solving systems of equations and inequalities compare between online and face-to-face pre-calculus students?
2. In what ways do work and scores on the final assessment relative to solving integrals compare between online and face-to-face calculus students?

Research Design

A mixed methods design was used for this study. Each element of the study is reviewed from a qualitative and quantitative perspective; qualitative and quantitative data were collected concurrently and are equally weighted in the analysis (Gay & Airasian, 2003). While often regarded as polar opposite research designs, qualitative and quantitative analysis can be combined harmoniously to provide rich categorical and numerical inferences (Ercikan & Roth, 2006). Qualitative research is used to answer the question "Why?" and provide rich insight into the phenomenon being studied (Ercikan & Roth). In this study, the phenomenon being studied is student demonstration of problem solving abilities. Student work was coded and analyzed qualitatively in an effort to search for themes and commonalities amongst deployed problem solving techniques. To substantiate the qualitative review of students' comments, test scores were quantitatively analyzed. Quantitative research seeks to provide statistical evidence to substantiate phenomenon descriptions realized through collected data (Ercikan & Roth). Ercikan and Roth claim each perception realized through research has a qualitative and a quantitative aspect. It is the goal of this mixed method research design to realize both the qualitative trends within the data collected and the quantitative substantiation of gathered perceptions.

Population

The population of this study consists of community college students in Southern Virginia enrolled in an online pre-calculus an online calculus courses, a face-to-face pre-calculus course or a face-to-face calculus course. Fall 2015 enrollment for pre-calculus and calculus are shown in Table 1.

Table 1

Fall 2015 Enrollments

Course	Total	Online	Face-to-face
pre-calculus	1240	210	1030
calculus	272	50	222

Participants

The participants for this study were pre-calculus and calculus students who attend a select campus of a community college in Southern Virginia for the fall semester of 2015. The course instructor provided student work void of identifying information, and provided her comments and grading notes for each question. The cooperating community college submitted an agreement for research partnership and human subjects procurement was obtained through the affiliated university with a ruling of this study being exempt. The human subjects approval letter is located in Appendix A.

Each student self-enrolled in either the online or the face-to-face section of pre-calculus or calculus. At the beginning of the study, 40 students were enrolled in pre-calculus and 49 students were enrolled in calculus. Table 1 shows the online and face-to-face enrollment numbers for pre-calculus and calculus.

Over the course of the semester, 7 students withdrew from the online pre-calculus course, 3 students elected not to complete the final exam and one student was exempt from the final exam because the testing center administered the wrong exam. Six students withdrew from the face-to-face pre-calculus course. Fourteen students completed the online calculus final exam and 21 students completed face-to-face calculus final exam during the fall semester of 2015. Four students withdrew from the online calculus course and three neglected to take the final exam. Five students withdrew from the face-to-face calculus course and two did not take the final exam. Nine students completed the online pre-calculus final exam; 14 students completed the face-to-face pre-calculus final exam during the fall semester of 2015. An enrollment summary of both courses is provided in Table 2.

Table 2

Enrollment Summary

	Initial Enrollments	Final Exam	No Final Exam	Withdraw	Final Enrollment
Online pre-calculus	20	9	4	7	13
Face-to-Face pre-calculus	20	14	0	6	14
Online calculus	21	14	3	4	17
Face-to-Face calculus	28	21	2	5	23

The pre-requisite requirement for each section were identical and could be achieved one of three ways: 1) place into the course through a satisfactory score on the college’s mathematics placement assessment, 2) successfully complete the preceding mathematics course in the college’s course sequence, or 3) successfully complete an equivalent AP mathematics assessment at the high school level to satisfy a pre-requisite requirement.

Course Structures

All four courses were taught by the same instructor, who volunteered to participate in this research once learning of the study by the mathematics department chairperson. It was important to the researcher that the selected instructor taught both the online and face-to-face courses. Seeking to maintain course alignment, the researcher also insisted the online and face-to-face pre-calculus courses used the same textbook, had the same learning objectives, and were each worth three credits. Similarly, the online and face-to-face calculus courses were required to use the same textbook, the same learning objectives, and each be worth four credit values. All pre-calculus students had equivalent access to the same course resources through Pearson's MyMathLab learning suite; video lectures, calculation examples, worked solutions, and problem solving guidelines were available for each section of content covered in the course textbook and through the course objective. All calculus students had access to the same lesson resources through WebAssign. Like MyMathLab, WebAssign provides students with examples, videos, and learning resources to assist with asynchronously moving through course content. The online courses were structured as asynchronous course experiences. Students were provided with a schedule of topics that correlated to the schedule of lectures in the face-to-face course. The online pre-calculus students were encouraged to watch pre-loaded video lessons provided by MyMathLab, read the corresponding textbook pages, and work through the example problems provided by MyMathLab for each content section. Likewise, the online calculus students were encouraged to watch pre-loaded video lessons provided by WebAssign, read the corresponding textbook pages, and work through the example problems provided by WebAssign for each content section. When asked what alterations were made to teach pre-calculus and calculus online, the instructor replied "activities that I assign for students to do in class or as take-home paper assignments are put into the discussion (board) for online". The video lectures are not recorded by the course instructor but are instead provided by MyMathLab or WebAssign.

The face-to-face pre-calculus and calculus courses were conducted in a lecture format with opportunities for students to ask questions and engage in dialogue with the instructor and classmates. A traditional lecture for the face-to-face course consisted of the teacher working sample problems and explaining solution techniques from the whiteboard located in the front of the classroom. The same topics were covered each week in the face-to-face course as the online course. The face-to-face pre-calculus and calculus courses each met three times a week for one hour per meeting and consisted of a teacher led lecture in which students were shown each problem technique, sample problems, and problem solving strategies. Integrated with the lecture was be time for students to practice problems on their own or in small groups within the classroom. Students in the face-to-face courses were also given handouts and opportunities to work collaboratively with their peers on select assignments. These selected assignments were posted to the discussion board in the online courses and students were encouraged to interact with their peers through the discussion board tool of the online course. The online sections did not have scheduled meetings. Students were expected to use the pre-recorded video lectures provided in place of scheduled class meetings and were given a calendar to follow which outlined which concepts to review each week to maintain accurate pacing through the course.

In all four courses, the teacher encouraged students to review the textbook examples, work through the textbook practice problems, and utilize the course resources provided by MyMathLab or WebAssign. The instructor was available to both online and face-to-face students through email or during designated office hours. All office hours were on campus, but online students were encouraged to email or call if they wanted to schedule synchronous, online meetings with the instructor. Figure 4 provides a visual comparison of the online and face-to-face course structures.

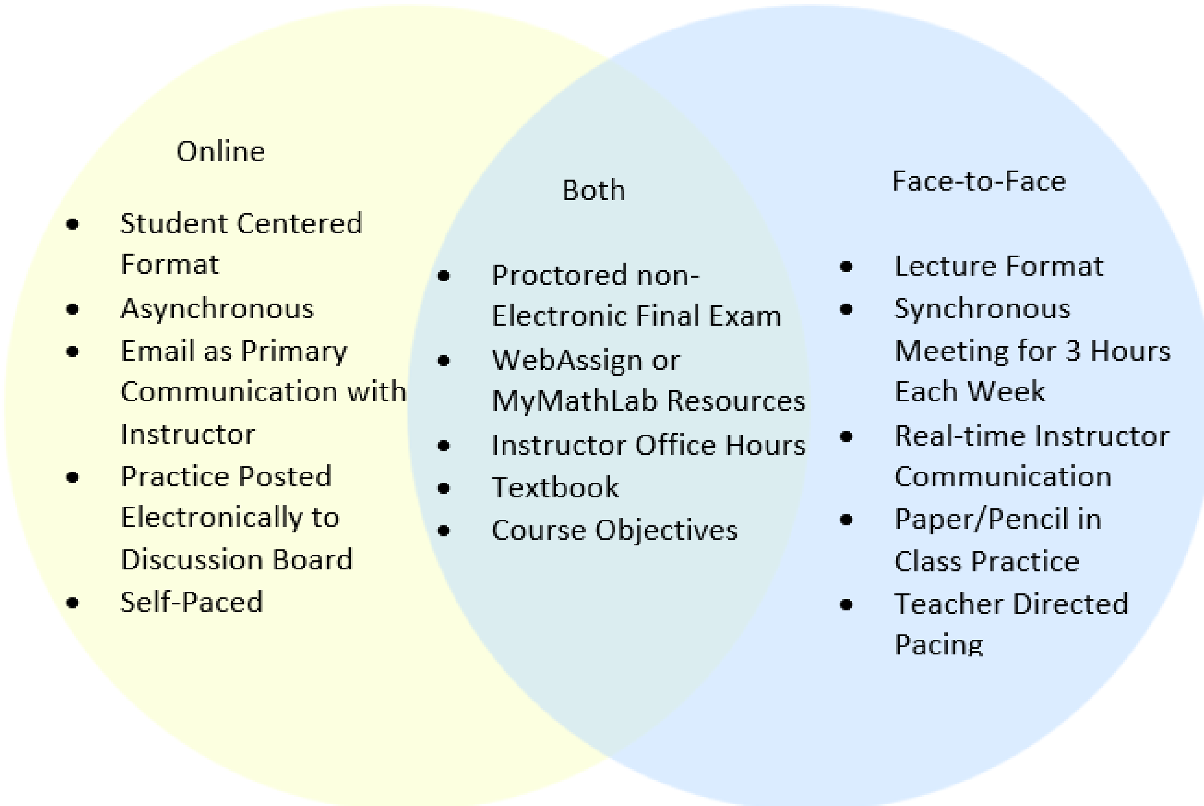


Figure 4: Course Comparison

Instrumentation

Data were collected through analyzing student work for select questions from their course exam. All online and face-to-face students were required to take the final exam in person, on the provided test paper, at the testing center or in a proctored classroom during one of the scheduled testing sessions. Testing centers were located on multiple campuses throughout Southern Virginia and at approved locations outside Virginia for students outside the area.

Multiple versions of the assessment were utilized to deter students from cheating and discussing their answers as students were able to complete their assessment at various times. The instructor wanted to minimize the possibility of students who completed the assessment early discussing specific questions with students who had not yet completed the assessment.

Although multiple versions of the assessment were used, the questions on the different versions aligned and provided a snapshot of student understanding of a specific skill, solving systems of equations and inequalities. Each version of the assessment was created by the course instructor in alignment with the course objectives.

Instrument pre-calculus. Both the online and face-to-face final exam contained four questions evaluating student's ability to solve systems of equations and inequalities. The four questions on the online and face-to-face assessments align with each other, as shown in Table 3. In particular, the four questions assess students' ability to solve a system of two linear equations involving two variables, to use substitution to solve a system comprised of a linear equation and a quadratic equation, to graph the solution of a system of inequalities, and to apply knowledge of matrices to solve a system of three equations involving three variables.

Table 3

Pre-Calculus Questions

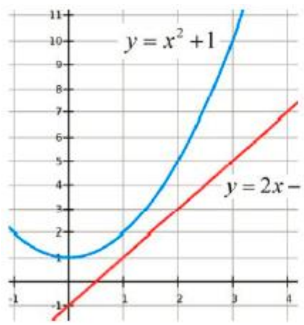
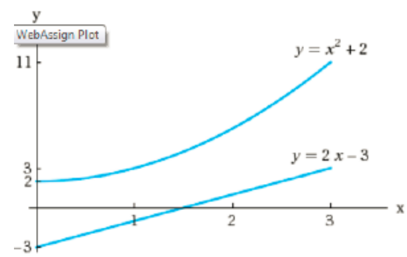
Description	Face-to-Face	Online
1. Students are asked to solve a system of linear equations using a method of their choice and notating their final answer as an ordered pair.	1. Solve the system of linear equations. State your final answer as an ordered pair. $3x+2y=2$ $4x-y=-23$	1. Solve the system using the method of your choice. State final solution as an ordered pair. $7x+9y=-10$ $3x-y=20$
2. Students are asked to solve a system of equations comprised of one linear equation and one quadratic equation using the substitution method.	2. Solve the system of nonlinear equations by using the substitution method. $3x+y=-4$ $y=x^2-2x-10$	2. Solve the nonlinear system by the substitution method. $x+y=3$ $y=x^2-5x+6$

3. Students are asked to graphically solve a system of inequalities.	3. Graph the system of inequalities. $\begin{cases} y < \frac{1}{2}x + 3 \\ y \geq x^2 - 5 \end{cases}$	3. Graph the following system of inequalities, shading to show the solution set of the system. $\begin{cases} y > x^2 - 7 \\ x + y \leq 2 \end{cases}$
4. Students are asked to solve a system of equations in three variables using the matrix method, Gaussian elimination.	4. Solve and state the solution as an ordered triple, using the MATRIX method. $x + y - z = -5$ $2x - y + z = -1$ $-x + 5y - 4z = 1$	4. Solve the system using the matrix method of Gaussian elimination. $x - 7y - z = -16$ $x + y + 7z = 24$ $x - y + z = 2$

Instrument calculus. The online and face-to-face integration portion of the final exam both contained four questions seeking to evaluate student's ability to integrate and apply integration to calculate area under a curve and to evaluate demand and supply functions. The four questions on the online and face-to-face are comparable, as shown in Table 4. In particular, the four questions assess students' ability to analyze a graph and calculate the area between two curves, to calculate consumer and product surplus at market demand, to evaluate an indefinite integral using substitution, and to calculate an indefinite integral using integration by parts. Table 4, shows each question and provides a brief description of the questions.

Table 4

Calculus Questions

Description	Face-to-Face	Online
1. Given a graph, two equations, and boundary points, students are asked to calculate the area between two curves.	1. 	1. 
2. In this multi-part question, students are given demand and supply functions and are asked to find market demand, market price, consumer surplus at market demand, and producer's surplus at market demand.	2. $d(x) = 32 - 0.05x^2, \ s(x) = 0.03x^2$ a) Find the market demand b) Find the market price c) Find the consumers' surplus at market demand d) Find the producers' surplus at market demand.	2. $d(x) = 11.25 - 0.04x^2, \ s(x) = 0.01x^2$ a) Find the market demand b) Find the market price c) Find the consumers' surplus at market demand d) Find the producers' surplus at market demand.
3. Students are asked to use integration by substitution to calculate an indefinite integral.	3. Integrate by substitution. $\int (x^3 + 6)^8 x^2 dx$	3. Integrate by substitution. $\int (x^2 + 16)^7 xdx$
4. Students are asked to use integration by parts to calculate an indefinite integral.	4. Use integration by parts to integrate. $\int 3x^{-4} \ln xdx$	4. Use integration by parts to integrate. $\int x^6 \ln xdx$

Data Collection Procedure

Data for this study were collected throughout the fall semester of 2015. Preparations for data collection and meetings with the classroom instructor took place through the summer of 2015. Prior to embarking on data collection, human subjects review was completed and a research agreement was signed with the cooperating community college. Once the appropriate approval had been gathered, research began through identifying online and face-to-face courses taught by the same instructor during the fall semester of 2015. After an instructor and courses were procured, a journey of developing instruments and completing field tests commenced. The GANTT chart in Figure 5 shows the timeline used for this data collection.

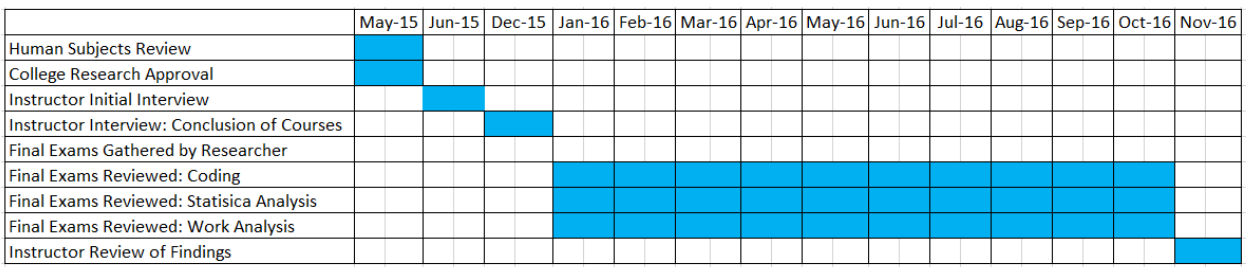


Figure 5: GANTT Chart

Data Analysis Procedure

Multiple components of data analysis were conducted to review student work and interpret problem solving trends. Separate analysis for qualitative and quantitative procedures were used to explore data relative to each research question or foci.

Table 5

Data Analysis Procedures

Research Question	Instrument	Data Analysis Procedure
1. In what ways do work and scores on the final assessment relative to solving systems of equations and inequalities compare between online	pre-calculus questions	Qualitative: Framework for Coding Qualitative: Student Work Analysis Quantitative: *F*-Tests in One-way ANOVA

and face-to-face pre-calculus students?		
2. In what ways do work and scores on the final assessment relative to solving integrals compare between online and face-to-face calculus students?	calculus questions	Qualitative: Framework for Coding Qualitative: Student Work Analysis Quantitative: *F*-Tests in One-way ANOVA

Table 5 summarizes the data analysis procedures used in this study to evaluate each research question.

Student work. Research questions for this study pertain to a qualitative and quantitative analysis of student work on the final exam. To conduct this analysis, a three-tier process was utilized to examine student work for the outlined pre-calculus and calculus problem sets relative to solving systems of equations and inequalities and integration. First, a one-way analysis of variance (ANOVA) (Vijayvargiya, 2009) was conducted which included Levene's test and a *F*-test on each question to establish homogeneity of variance between the groups and compare group means for student scores on each question. In addition, the eta squared (η^2) was computed as the effect size index. Once the statistical analysis was complete, a framework for analysis was devised and utilized to code student work on each question. Codes were developed to identify patterns in solution techniques and to look for similarities of approaches used between the online and face-to-face sections. The third level of analysis used was a close examination of student work to extract evidence of student understanding and evaluate utilized solution techniques.

Statistical Analysis. The first level of review conducted pertained to statistical analysis of the scores students received on each question. Prior to embarking on a statistical analysis of student scores, question averages were calculated for each problem based on the number of points it was worth and the number of points the student received on it by the course instructor.

All point value assignments and question grading criteria were completed by the course instructor. For example, the question presented in Figure 6 was worth 15 points. The course instructor graded this question as “-12”, which means the student earned a 20% on this question.

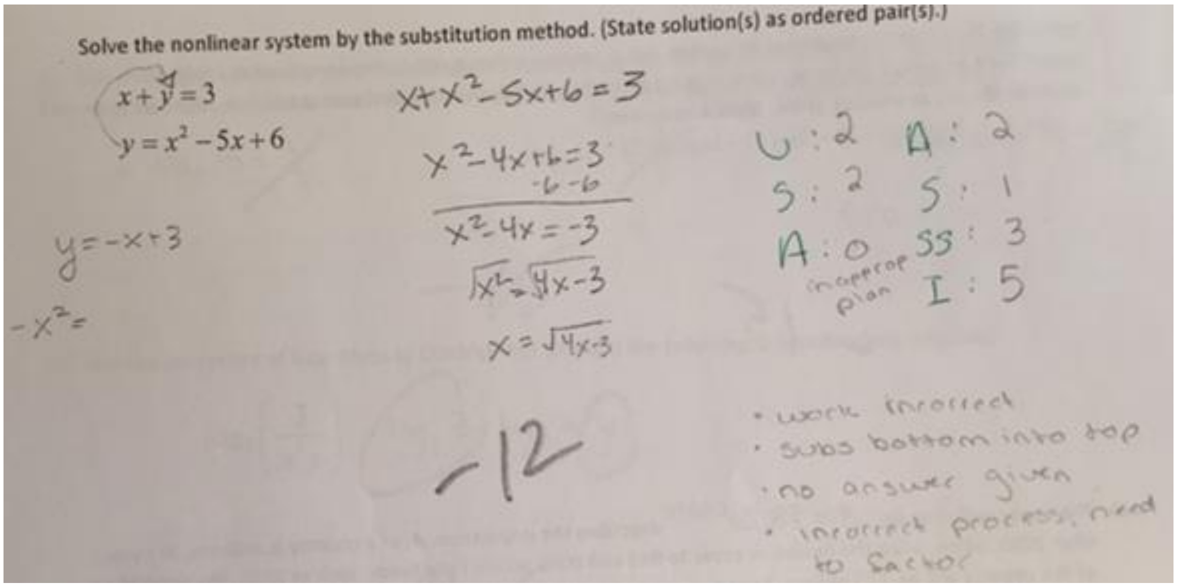


Figure 6: Grading example

At the instructor’s choosing, the online course and the face-to-face course questions were not awarded the same point values. Partial credit was determined by the instructor based on the value assigned to each question. Possible point values for each course are shown in Table 6.

Table 6

Possible Point for each Question

Test	Online	Face-to-Face
Pre-Calculus	15	4
Calculus	4	4

After identifying point value grades for each question, SPSS was used to run Levene’s test and *F*-Tests for each question. Levene’s test was used to test the null hypothesis that the variance in scores between the online and face-to-face sections was equivalent. Once homogeneity of variances was established, ANOVA *F*-tests were used to explore between group differences relative to mean scores.

Framework and coding. Seeking to evaluate student's problem solving abilities as an indicator of understanding, a problem solving scale was sought to assist with evaluating the stages of problem solving a student goes through when solving a question. Students work through a series of stages during the problem solving process (Szetela & Nicol, 1992). The problem solving scale proposed by Szetela and Nicol provides a solid framework for evaluating student understanding at each stage of the problem solving process and serves as the base framework for this data analysis (see Figure 7).

ANALYTIC SCALE FOR PROBLEM SOLVING

Understanding the problem
0 - No Attempt
1 – Completely misinterprets the problem
2 – Misinterprets major parts of the problem
3 – Misinterprets minor parts of the problem
4 – Complete understanding of the problem

Solving the problem
0 – No attempt
1 – Totally inappropriate plan
2 – Partially correct procedure but with major fault
3 – Substantially correct procedure with minor omission or procedural error
4 – A plan that could lead to a correct solution with no arithmetic errors

Answering the problem
0 – No answer or wrong answer based upon an inappropriate plan
1 – Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly
2 – Correct solution

CATEGORIES OF RESPONSES IN SOLUTIONS TO PROBLEMS

Answer
1. Blank
2. Undetermined
3. Incorrect
4. Correct

Statement
1. No statement
2. No context
3. No units
4. None required
5. Complete

Strategy Selected
1. Number sentence
2. Select operations and calculate
3. Algebraic
4. Non-systematic list
5. Systematic list
6. Guess and test
7. Draw diagram
8. Look for pattern
9. Logical reasoning
10. use simpler case
11. Work backwards
12. Undetermined

Implementation
1. No work shown
2. Identifies data only
3. Problem misinterpreted
4. Strategy not clear
5. Strategy initiated (table, graph, list) but incomplete or poorly implemented
6. Conditions or possibilities overlooked
7. Multiple secondary errors
8. A single secondary error
9. Appropriate and complete

Figure 7: Problem Solving Scale (Szetela, & Nicol, 1992).

This scale is comprised of two components, an analytic scale and categories of responses. The analytic scale was used to determine student understanding of each problem. Student's work was reviewed to verify the level at which understanding of the context of the problem, the solution procedure, and the answer requirements were evident. The category of responses delineations was used to evaluate student work. Student answers, accuracy of statement of solution, solution strategy, and strategy implementation were each evaluated.

The second level of analysis involved coding of student approaches based on Szetela, & Nicol's (1992) framework. Each problem received two sets of codes. One code set was determined by Szetela, & Nicol's "Analytic Scale for Problem Solving," where a student's work on each problem receive a code of U (Understanding the problem), S (Solving the problem) and A (Answering the problem). Student's work on each problem also received codes determined by the "Categories of Responses in Solutions to Problems" scale, for which a student's work on each problem received a code of a (Answer), s (Statement), ss (Strategy Selected), and I (Implication). While evaluating student papers, it was determined that the scales proposed by Szetela and Nicol were not precisely aligned to the students' work on solving system of equations and inequality questions. To accommodate the specific content topic of this study, the scales were slightly modified. A code of five was added to the the Understanding the Problem category to accommodate questions with insufficient work provided by the student to convey understanding of the problem. Similarly, the Solving the Problem category also gained a code of five to represent an unclear procedure. A small edit was made to the description of the zero code in the Answering the Problem category to account for unclear plan interpretation. In the Answer category, accommodation for correct answers in parts of questions with multiple steps was added as code number 5. Statement category needed a code for an incomplete statement and thus incomplete was added as number six. Additionally, under the Strategy Selected category, the undetermined listing was enhanced to also include blank and the algebraic listing was enhanced to also include computational. Figure 8 shows the finalized codes used to evaluate students work, with minor revisions highlighted.

ANALYTIC SCALE FOR PROBLEM SOLVING

Understanding the problem
0 - No Attempt
1 – Completely misinterprets the problem
2 – Misinterprets major parts of the problem
3 – Misinterprets minor parts of the problem
4 – Complete understanding of the problem
5 – Not able to be determined

Solving the problem
0 – No attempt
1 – Totally inappropriate plan
2 – Partially correct procedure but with major fault
3 – Substantially correct procedure with minor omission or procedural error
4 – A plan that could lead to a correct solution with no arithmetic errors
5 – Procedure unclear

Answering the problem
0 – No answer or wrong answer based upon an inappropriate or unclear plan
1 – Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly
2 – Correct solution

CATEGORIES OF RESPONSES IN SOLUTIONS TO PROBLEMS

Answer

1. Blank
2. Undetermined
3. Incorrect
4. Correct
5. Parts of question correct

Statement

1. No statement
2. No context
3. No units
4. None required
5. Complete
6. Incomplete

Strategy Selected

1. Number sentence
2. Select operations and calculate
3. Algebraic/Computational
4. Non-systematic list
5. Systematic list
6. Guess and test
7. Draw diagram
8. Look for pattern
9. Logical reasoning
10. use simpler case
11. Work backwards
12. Undetermined/blank

Implementation

1. No work shown
2. Identifies data only
3. Problem misinterpreted
4. Strategy not clear
5. Strategy initiated (table, graph, list) but incomplete or poorly implemented
6. Conditions or possibilities overlooked
7. Multiple secondary errors
8. A single secondary error
9. Appropriate and complete

Figure 8: Final framework for coding student work

In addition to updating the scale categories, several category standards were needed. What constitutes a major error? What constitutes a minor error? What is a secondary error? These distinctions needed to be expounded upon for coding accuracy. For the purpose of this study and the systems of equations and inequalities analyzed, a major error constituted procedural errors where key points of understanding were lacking. For example, when factoring to find quadratic solution points, if a student failed to set the quadratic equal to zero prior to initiating their solution strategy of factoring, this was coded as a major error. Minor errors were coded as smaller breaks in understanding, such as using a solid line instead of a dashed line to graph less than or greater than boundary lines. Secondary errors took the form of calculation errors, inaccurate handling of negatives, and computational inaccuracies independent of the main solution process. Coding category labels of U, S, A, a, s, ss, and I were recorded on each question of individual student's work and values were assigned based on the work present, as shown in *Figure 6*.

The work in *Figure 6* was coded with a 2 for U and S on the Analytic Scale for Problem Solving. The student demonstrated a partial understanding of the problem by initiating proper substitution techniques. They first solved the top equation for y = and then substituted into the bottom equation. After substituting, the student then embarked on an algebraic simplification process to solve for x. Amidst solving, the student incorrectly tried to take the square root of both sides instead of setting the equation equal to zero and factoring. This error showcased an inappropriate solution plan and reflected the student's non-understanding of solving quadratic equations. Therefore, the student did not correctly solve this question and thus received a code of 0 for A. On the Categories of Response in Solutions to Problems scales, this work received a code of 2 for "a" because the answer is undetermined. The student did not reach a final value for x and did not attempt to solve for y; therefore, this answer was undetermined. Following similar reasoning from the "a" category, for the "s" category this work received a score of 1 because no final solution was reached. This student embarked on an algebraic solution strategy and thus received a code of 3 for ss. For the final category of I this work received a score of 5 because the student recognized the need to solve for one equation for y = and substitute, but their error relative to solving a quadratic equation hindered their solution process. The substitution strategy was implemented, but a poor plan for solving without factoring caused the student to not be able to successfully complete this question.

After updating the scales and clearly expanding upon each category classifications, each systems of equations and inequalities or integration question was evaluated according to the aforementioned codes. A single coder went through each question multiple times to assign codes and review codes. Each question was coded upon a first review, and then coded a second time upon a second review. A third review was conducted to review assigned codes. If the codes from the first two reviews aligned, the third review was used to verify accuracy. If the codes from the first two reviews did not align for a particular question, third and sometimes fourth reviews were conducted to solidify coding for each question. Once initial coding was complete, codes for each question were then reviewed and critiqued by a second reviewer. The second reviewer was a mathematics colleague familiar with the content covered in this study but was not connected with the cooperating college, students, or research in any way.

Student work analysis. The final level of the three tiers of analysis used was a close examination of student work. Student work was reviewed to look for patterns and see if differences were present in how online or face-to-face students approached each question. Each question was reviewed and comments recorded regarding the process students used to solve. For the work sample shown in figure 13, it was recorded that this student approached the solution by first solving the top equation for y = and then substituting into the bottom equation. Similar procedural comments were noted for all student work and patterns of techniques, errors, and processes were analyzed.

Validity

Several measures have been taken to ensure validity of findings. Coding techniques were evaluated and selected with the assistance of an experienced researcher. Two co-coders were used to review codes and examine student work to develop interrater reliability. Theory triangulation was also used to explore findings from multiple viewpoints both qualitatively and quantitatively. To further validate findings, a copy of all findings was submitted to the course instructor for review and to assure all course aspects were properly represented.

Limitations

As with all studies, limitations are present within this study. The first limitation is the small sample size. Seeking to only look at online and face-to-face courses which were taught by the same instructor during the same semester, the available participant pool was shallow. But, this small sample size allowed for detailed examination into student work and using the same instructor across course modalities enhanced control group and treatment group alignment. A second limitation is this study focuses only on one section of material from pre-calculus and one section of material from calculus. With the limited content covered, this study provides a pedestal from which to launch a collection of content specific studies examining student achievement in online courses. A third limitation is the timeframe of this study. Looking at only one semester, this study was conducted over a relative short timeframe. Repeating this study over subsequent semesters will provide opportunities for replication and repetition of findings. A final limitation of this study is the use of a solo researcher. While co-coders were used and interrater reliability was established, this study was the work of one researcher well versed in the fields of mathematics and online education and unintended bias could be present.

Summary

The purpose of this study was to examine student work on systems of equations and inequalities and integration to evaluate conveyance of content mastery in accordance to a problem solving framework. To accomplish the purposes of this study four questions from the pre-calculus final exam and four questions from the calculus final exam were reviewed to examine student work in both online and face-to-face courses, and course rosters were analyzed to extract attrition details. Student work was reviewed qualitatively while quantitative data were used to substantiate score comparison of the work samples. Despite limitations, this study provides much needed insight into content specific online course experiences and encourages other similar content rich investigations.

CHAPTER 4

Results

The following two research questions are the focus of this study:

1. In what ways do work and scores on the final assessment relative to solving systems of equations and inequalities compare between online and face-to-face pre-calculus students?
2. In what ways do work and scores on the final assessment relative to solving integrals compare between online and face-to-face calculus students?

The results relative to each research question are presented in this chapter.

In what ways do work and scores on the final assessment relative to solving systems of equations and inequalities compare between online and face-to-face pre-calculus students?

The following analysis presents in detail the differences observed between the online and face-to-face pre-calculus courses. The scores received by students on the four assessment problems were analyzed statistically, through coding relative to the described final framework (refer to figure 6) and through detailed examination of student work. Initial statistical reviews found a statistically nonsignificant difference between student scores on each question. Table 7 shows class averages, in percent, for each question answered correctly or completely. The final average of all four questions is shown in the last column.

Table 7

Averages in Percent

Test	Q1	Q2	Q3	Q4	Final Avg
Online	81.48	72.6	65.19	61.48	70.19
Face-to-Face	92.86	72.32	61.61	54.46	70.31

Examining the average scores on each of the four questions, the online course average scores were higher on questions 2, 3 and 4 but lower on question 1. Averaging overall scores on questions 1 through 4 reveal almost identical scores for the online and face-to-face courses with the online average being only slightly less. The final four question average score for the online course was 70.19% and the overall score for the face-to-face course was 70.31%.

F-tests were conducted to investigate if the between group differences of the average scores for each question was statistically significant. In addition, analysis of coding according to the final framework (refer to figure 6) was conducted to see if themes developed regarding differences in how students approached and performed on each question, followed by an examination of solutions and common errors made by the students. Detailed findings in students' scores and responses aligned with each problem are presented.

Pre-calculus question 1. Question 1 asked students to solve a system of linear equations using a method of choice. Students elected methods of substitution or elimination and needed to carefully execute their solution strategy to identify the solution.

Pre-calculus question 1 statistical analysis. Class averages for question 1 differed by 11.38%. The online class question 1 average was 81.48% while the face-to-face class average was 92.86%. As shown in Table 8, Levene's test did not reflect a statistically significant difference in the variances garnered by these scores, suggesting no violation of the assumption of homogeneity of variances. An *F*-test was used to explore between group differences in student scores. The *F*-test, shown in Table 9, suggested a statistically nonsignificant difference in question 1 scores, $F(1, 21) = 1.19$, $p > .05$, $\eta^2 = 0.05$.

Table 8

Pre-Calculus Question 1 Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
3.99	1	21	0.06

Table 9

Pre-Calculus Question 1 ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	708.83	1	708.83	1.19	0.29
Within Groups	12483.82	21	594.47		
Total	13192.65	22			

Pre-calculus question 1 coding analysis. Upon concluding the statistical analysis, a deeper analysis of coded student work was used for further examination. Detailed review of the codes assigned to student work reveals differences in averages could be attributed to 11 out of 14 face-to-face students, which equates to 78.57% of students in the face-to-face class, receiving full credit while 66.67% of the online students, which equates to six out of nine students, received full credit. The face-to-face course also did not have any students leave this question blank, but the online class average was impacted by one blank answer.

Using Excel to organize coding data, tables housing coding details were created for each question. Tables 10 and 11 show the coded data for question 1. Students A 1 through A 9 represent the online section and students B 1 through B 14 represent the face-to-face section.

The first column of tables 10 and 11 show the test number. The second through eighth columns, show the codes assigned to the work components according to the revised final framework, presented in Figure 8. In accordance with the framework adapted from Szetela, & Nicol's (1992) "Analytic Scale for Problem Solving", columns 2 through 4 correlate to Understanding the problem (U), Solving the problem (S), and Answering the problem (A). Similarly, codes for Szetela, & Nicol's (1992) "Categories of Responses in Solutions to Problems" scale are found in columns 5 through 8. Student work on each problem received a code for a (Answer), s (Statement), ss (Strategy Selected), and I (Implication). Notation of "O 1" denotes the first question of the online assessment while "F 1" denotes the first question of the face-to-face assessment. The final column represents the score earned on each specific question, as scored by the course instructor.

Table 10

Student Question Evaluation O 1

Test	O 1 U	O 1 S	O 1 A	O 1 a	O 1 s	O 1 ss	O 1 I	%
A 1	4	4	2	4	5	3	9	100.00
A 2	4	4	2	4	5	3	9	100.00
A 3	4	4	2	4	5	3	9	100.00
A 4	4	4	2	4	5	3	9	100.00
A 5	0	0	0	1	1	12	1	0.00
A 6	4	4	2	4	5	3	9	100.00
A 7	4	4	2	4	5	3	9	100.00
A 8	3	2	0	2	1	3	7	66.67
A 9	4	4	1	3	5	3	8	66.67

Table 11

Student Question Evaluation F 1

Test	F 1 U	F 1 S	F 1A	F 1 a	F 1 s	F 1 ss	F 1 I	%
B 1	4	4	2	4	5	3	9	100
B 2	4	4	2	4	5	3	9	100
B 3	4	4	2	4	5	3	9	100
B 4	4	4	2	4	5	3	9	100
B 5	4	4	2	4	5	3	9	100
B 6	4	4	2	4	5	3	9	100
B 7	4	4	2	4	5	3	9	100
B 8	4	4	2	4	5	3	9	100
B 9	4	4	2	4	5	3	9	100
B 10	4	4	1	3	3	3	8	50
B 11	4	4	2	4	5	3	9	100
B 12	4	4	2	4	5	3	9	100
B 13	4	3	4	3	3	3	7	62.5
B 14	4	4	1	3	3	3	8	87.5

The tables created for the codes from each question enabled themes to be explored and connections to be made regarding student scores and work. Reviewing the codes for each question in one common location allowed for a snapshot view of each student's work, enabling general and specific observations to be made.

Table 10 shows seven of the nine students in the online section were able to convey an understanding of this question and embark on a proper solution process, as evidenced by scores of 4 for U and S, and scores of 1 or 2 for A. Eight students in the online course properly embarked on an algebraic solution strategy, as shown by a code of 3 in the ss category. In addition, they either got the solution correct or made secondary errors, as noted by codes of 9, 8 or 7 for I.

Table 11 shows all students in the face-to-face section were able to convey an understanding of this question and embark on a proper solution process, as evidenced by scores of 4 for U, scores of 3 and 4 for S, and scores of 1 or 2 for A. All students in the face-to-face course also properly embarked on an algebraic solution strategy, as noted by a code of 3 in the ss category, and had the solution correct or made secondary errors, as shown by codes of 9, 8 or 7 for, I.

Pre-calculus question 1 student work analysis. In the online group, six students correctly answered this question, one student left this question blank, and two made algebraic mistakes when multiplying by a fraction or evaluating with a negative sign. Three students elected to solve through the substitution method, five students used the elimination method. All students who used substitution correctly solved the system of equations by solving the bottom equation for y = and substituted into the top equation. Each student then distributed, combined like terms, simplified, solved for x, solved for y, and notated their solution point, as shown in Figure 9

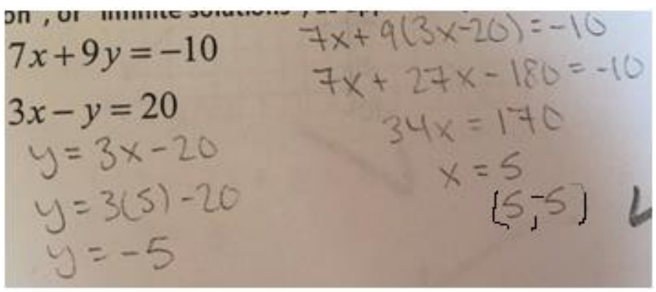


Figure 9: Online student substitution work question 1

The five students who solved by elimination each multiplied the bottom equation by 9 and subtracted the equations to eliminate the y variable and solve for x. After solving for x, the students then solved for y and listed their ordered pair solution, as shown in Figure 10.

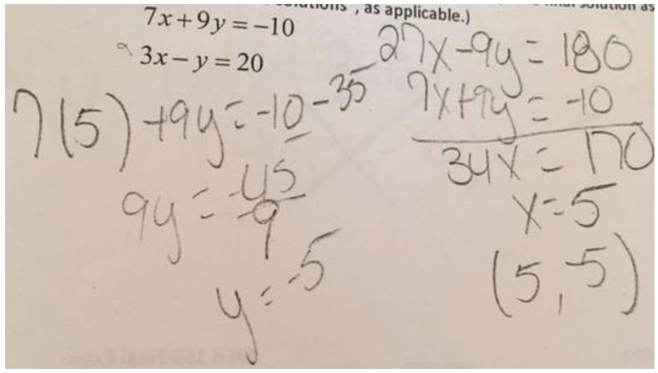


Figure 10: Online student elimination work for Question 1

Like the online students, most face-to-face students elected to solve question 1 through substitution. Eleven face-to-face students utilized the substitution method, while three used the elimination method. Of the students who used substitution, eight elected to solve for y = in the bottom equation and substitute into the top equation. Three students tried to solve the top equation for y = and substitute into the bottom, but this left a negative fractional coefficient of x. Two students were able to still successfully solve, but the negative fractional coefficient of x lead one student to calculation errors. As shown in Figure 11, like their online peers, the students who solved the bottom equation for y= substituted into the top equation, distributed, combined like terms, simplified, and solved for x, also solved for y and wrote their solution as an ordered pair.

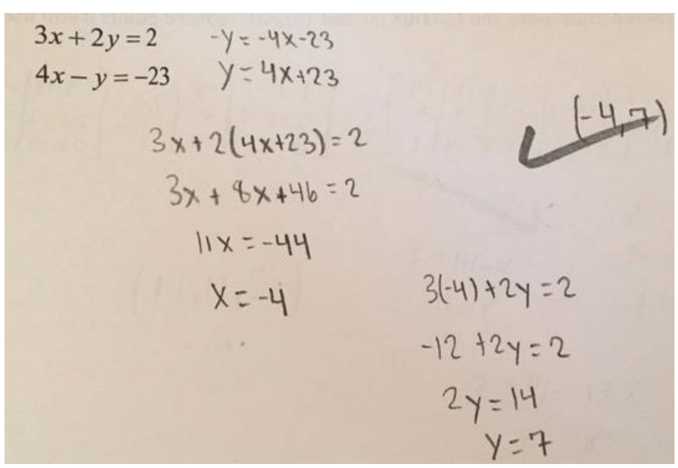


Figure 11: Face-to-Face Student Substitution Work for Question 1

In the face-to-face group, three students solved using the elimination method. All three students elected to multiply the bottom equation by two and subtract the equations. Two of the three students were successful with this computation and one student incorrectly distributed and thus the remainder of his/her calculations were misaligned. The students who used elimination followed the process shown in Figure 12. After distributing and subtracting the equations, the students simplified to solve for x and then solved for y before writing their solution as an order pair.

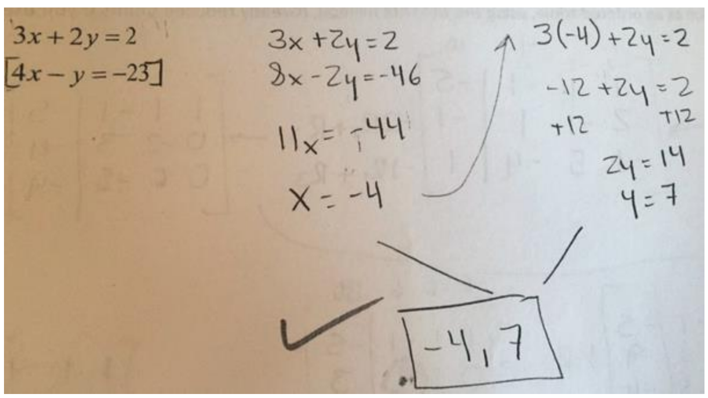


Figure 12: Face-to-Face Student Elimination Work Question 1

The work executed to solve this system of linear equations by substitution or elimination shows little difference between the online and face-to-face courses. As shown in figures 5, 6, 7 and 8, the work provided by the students is comparable in nature and execution of the systematic solution processes assessed by this question. The students taking the face-to-face course were not found to be more proficient in or to chose different strategies than their peers in the online course. Procedural differences could not be identified through this item analysis for solving systems of linear equations using substitution or elimination.

Pre-calculus question 2. The second question asks students to again solve a system of equations, but this system contains one linear equation and one quadratic equation. To successfully solve this system of equations using substitution, as directed by the question instructions shown in figure 4, students must understand how to set a quadratic equation equal to 0, factor, and look for multiple solution points. After substituting and simplifying, students had to set the resulting quadratic equation equal to 0 and factor to find two x solution values.

Pre-calculus question 2 statistical analysis. Class averages for question 2 were very close. The online class average was 72.6% and the face-to-face class average was 72.32%. As shown in Table 12, in accordance with the Levene's test, the violation of the assumption of homogeneity of variances was not suggested. An *F* test, shown in Table 13, further revealed that there was not a statistically significant difference between the group means of question 2 scores, $F(1, 21) < .01$, $p > .05$, $\eta^2 = 0.000018$.

Table 12

Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
0.23	1	21	0.63

Table 13

ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.41	1	0.41	.000	0.99
Within Groups	22854.95	21	1088.33		
Total	22855.36	22			

Pre-calculus question 2 coding analysis. The closeness of the question 2 score averages is echoed by the congruency of responses discovered through the analysis of the codes assigned to student work. Tables 14 and 15 identify the codes assigned to student work for question 2.

Table 14

Student Question Evaluation O 2

Test	O 2 U	O 2 S	O 2A	O 2 a	O 2 s	O 2 ss	O 2 I	points lost
A 1	4	4	2	4	5	3	9	0
A 2	4	4	2	4	5	3	9	0
A 3	4	4	2	4	5	3	9	0
A 4	4	4	2	4	5	3	9	0
A 5	0	0	0	1	1	12	1	-15
A 6	3	3	1	3	6	3	7	-5
A 7	4	4	2	4	5	3	9	0
A 8	3	3	1	3	6	3	5	-5
A 9	2	2	0	2	1	3	5	-12

Table 15

Student Question Evaluation F 2

Test	F 2 U	F 2 S	F 2A	F 2 a	F 2 s	F 2 ss	F 2 I	points lost
B 1	4	4	1	3	5	3	8	0
B 2	4	4	2	4	5	3	9	0
B 3	4	4	2	4	5	3	9	0
B 4	4	4	2	4	5	3	9	0
B 5	4	4	2	4	5	3	9	0
B 6	4	2	0	1	2	3	5	-2.5
B 7	4	4	2	4	5	3	9	0
B 8	4	2	0	1	2	3	5	-2.5
B 9	4	3	1	3	3	3	7	-2
B 10	4	4	2	4	5	3	9	0
B 11	4	2	0	1	2	3	5	-2.5
B 12	4	3	1	3	5	3	7	-2
B 13	4	3	1	3	5	3	8	-2
B 14	4	2	0	1	2	3	5	-2

Deeper analysis of codes assigned to student work revealed five students in the online course and all students in the face-to-face course successfully demonstrated an understanding of this problem by initiating an appropriate solution approach. All students in the face-to-face section received a value of 4 for U, 3 or 4 for S, and 1 or 2 for A. These scored codes indicate students were able to successfully embark on a proper solution process and had their calculations been devoid of secondary errors, they would have successfully calculated an accurate solution. In the online section, five students received values of 4 for U, 4 for S and 2 for A, signifying a complete understanding of the question and an ability to correctly navigate to an accurate solution. Three students were able to convey a partial understanding and received values of 2 or 3 for U and S. Partial understandings showed students were able to successfully complete the substitution step, but not able to continue with the algebraic steps necessary to complete the solution process. An example of this understanding is shown in figure 12. Seven face-to-face students successfully completed the question while the remaining seven did not successfully navigate the factoring component, or did not correctly interpret the factors to complete the solution. The five students in the online course who conveyed complete understanding of the question were all able to solve it successfully. The remaining four students who did not successfully calculate the solution points either left the question blank or had a factoring error.

Pre-calculus question 2 student work Analysis. In both the online and face-to-face courses, factoring was detected to give students difficulty. Figure 13 provides work of a face-to-face student and Figure 14an online student, who both did not successfully complete the factoring step to solve this system of equations.

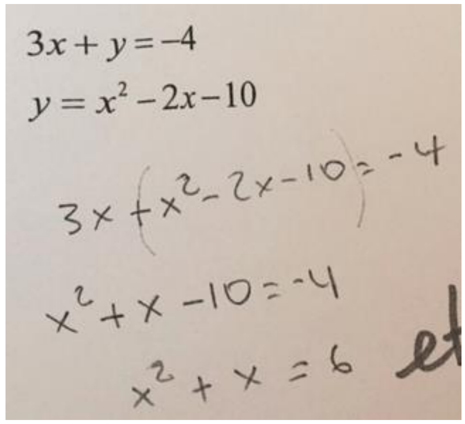


Figure 13: Face-to-face student factoring error

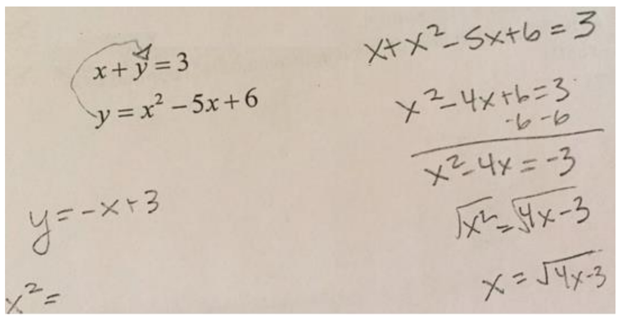


Figure 14: Online student factoring error

The factoring errors shown in Figure 13 and Figure 14 are different in that the online student incorrectly tried to take the square root of each side to progress with solving while the face-to-face student stopped when he/she arrived at the quadratic equation $x^2 + x = 6$. The work displayed by both students indicates a misunderstanding surrounding the process of factoring to complete solving the problem.

Pre-calculus question 3. For both questions 1 and 2, all students who answered the questions relied on an algebraic calculation process to arrive at their solution. Question 3 required students to move from an algebraic interpretation to a graphical interpretation. As previously shown in figure 4, question 3 requires students to graph the solution region generated by a linear inequality and a quadratic inequality.

Pre-calculus question 3 statistical analysis. Class averages for question 3 differed by 3.58%. The online average was 65.19% and the face-to-face average was 61.61%. Levene's test, as revealed in Table 16, did not reflect a statistically significant difference in the variances of these scores across groups, suggesting no violation of the assumption of homogeneity of variances. The *F* test shown in Table 17, suggested a statistically nonsignificant difference in question 3 scores, $F(1, 21) = .04$, $p > .05$, $\eta^2 = 0.0021$.

Table 16

Pre-Calculus Question 3 Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
.000	1	21	1.00

Table 17

Pre-Calculus Question 3 ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	70.15	1	70.15	.04	0.84
Within Groups	33949.05	21	1616.62		
Total	34019.2	22			

Pre-calculus question 3 coding analysis. Compared to questions 1 and 2, students in both courses seemed to struggle with question 3. In questions 1 and 2, most students were able to successfully demonstrate an understanding of the problem and initiate a proper solution technique. Tables 18 and 19, present the numeric values assigned to the student work for question 3.

Table 18

Student Question Evaluation O 3

Test	O 3 U	O 3 S	O 3A	O 3 a	O 3 s	O 3 ss	O 3 I	%
A 1	4	4	2	4	5	7	9	100.00
A 2	4	4	2	4	5	7	8	100.00
A 3	4	4	2	4	5	7	9	100.00
A 4	4	4	2	4	5	7	9	100.00
A 5	2	2	0	2	1	7	4	0.00
A 6	0	0	0	1	1	12	1	0.00
A 7	4	2	1	3	6	7	5	66.67
A 8	2	1	0	3	1	7	5	66.67
A 9	4	3	1	3	5	7	7	53.33

Table 19

Student Question Evaluation F 3

Test	F 3 U	F 3 S	F 3A	F 3 a	F 3 s	F 3 ss	F 3 I	%
B 1	4	4	2	4	5	7	9	100.00
B 2	4	4	2	4	5	7	9	100.00
B 3	4	4	2	4	5	7	9	100.00
B 4	4	4	2	4	5	7	9	100.00
B 5	3	4	1	2	6	7	5	62.50
B 6	2	2	0	2	1	7	5	25.00
B 7	4	4	1	3	5	7	5	75.00
B 8	4	3	1	3	5	7	8	62.50
B 9	4	4	2	4	5	7	9	100.00
B 10	4	4	1	3	5	7	8	75.00
B 11	2	2	1	2	1	7	5	62.50
B 12	2	2	0	3	2	7	7	0.00
B 13	0	0	0	1	1	12	1	0.00
B 14	0	0	0	1	1	12	1	0.00

The work analyzed for question 3 reveals only 14 of the 23 students were able to successfully display a complete understanding of the problem. Five students in the online section and nine students in the face-to-face section received codes of 4 for U. One face-to-face student received a 3, and two online and three face-to-face students received 2, signifying a partial understanding of this question. Three students, one online and two face-to-face, left this question blank and received a0 for U, S and A. Six students, two online and four face-to-face received a 5 for I, signifying poor implementation of graphing techniques.

Pre-calculus question 3 student work analysis. In both groups, the most common error was with shading the appropriate region on the graph. The second most common error was incorrectly graphing the equations. Seven students from the face-to-face course used test points to determine solution regions. One example of this case is shown in Figure 15.

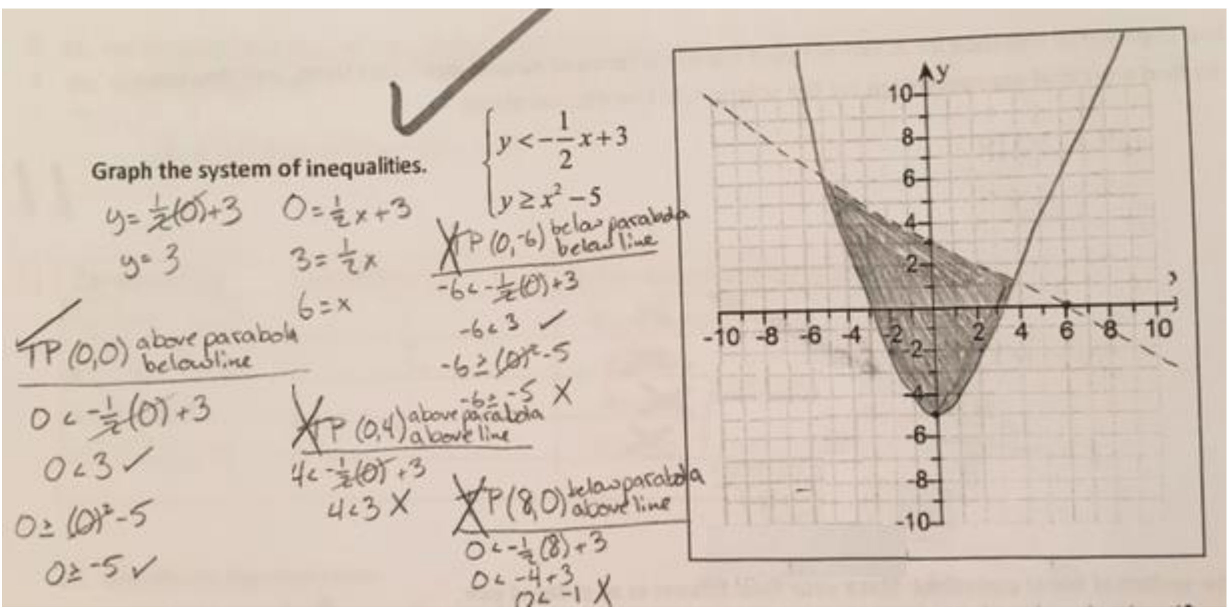


Figure 15: Test points as an approach

The students in the face-to-face course who used test points were able to accurately shade the solution region on their graph. In the online course, four students also attempted to use test points, but only one was able to successfully translate their test points to accurate shading.

Figure 16 shows an example of errors discovered while reviewing question 4 of one online exam.

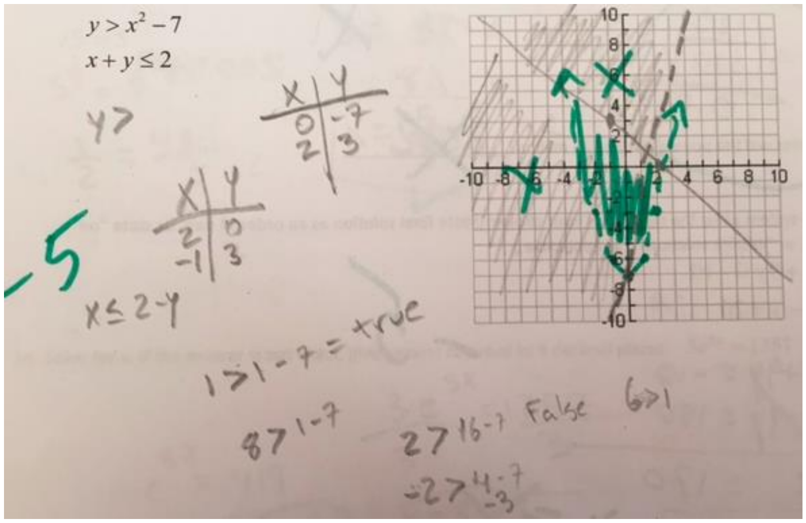


Figure 16: Online errors for one student*t*

In Figure 16, the instructor's corrections are shown in green while the student's original work is shown in pencil. The student incorrectly interpreted the parabolic equation as a line, which he/she graphed as a dotted line through quadrants 1, 3 and 4. The student correctly interpreted the solid line to denote less than or equal to for the quadratic inequality, but the test points did not lead him/her to a correctly shaded solution.

Question 3 also has the least amount of work present as many students elected to draw the graphs without any supporting work. Students were permitted to use graphing calculators on this assessment, so it is hypothesized that some used their calculators to generate graphs which they then translated to their response.

Pre-calculus question 4. Question 4 asked students to use matrix reduction techniques to solve a system of three equations in three variables. The teacher specifies matrix row reduction must be shown for full credit to be awarded. While 19 of the 23 students analyzed were able to convey an understanding of what the question asked through initiating setting up their matrix and embarking on the row reduction process, only 6 students were able to successfully navigate to reduced echelon form and solve this system of equations

Pre-calculus question 4 statistical analysis. Of the four questions reviewed, question 4 had the lowest average in both the online and face-to-face groups, 61.48% and 54.46% respectively. As shown by the Levene's test, in *Table 20,* there was not a statistically significant difference in the variances garnered by these scores across groups, suggesting no violation of the assumption of homogeneity of variances. The *F* test shown in *Table 21*, suggested a non-statistically significant difference in question 4 scores, $F(1, 21) = .17$, $p > .05$, $\eta^2 = 0.0081$.

Table 20

Pre-Calculus Question 4Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
1.06	1	21	0.32

Table 21

Pre-Calculus Question 4 ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	269.81	1	269.81	.171	0.68
Within Groups	33112.84	21	1576.80		
Total	33382.65	22			

Pre-calculus question 4 coding analysis. Tables 22 and 23 show the codes assigned to student work for question 4.

Table 22

Student Question Evaluation O 4

Test	O 4 U	O 4 S	O 4A	O 4 a	O 4 s	O 4 ss	O 4 I	%
A 1	4	2	2	4	5	3	5	46.67
A 2	4	4	2	4	5	3	9	100.00
A 3	4	4	2	4	5	3	9	100.00
A 4	4	1	2	4	5	3	5	46.67
A 5	4	0	0	1	1	12	1	0.00
A 6	4	4	1	2	5	3	8	80.00
A 7	4	4	2	4	5	3	9	100.00
A 8	4	4	1	2	5	3	8	60.00
A 9	4	2	1	3	1	3	5	20.00

Table 23

Student Question Evaluation F 4

Test	F 4 U	F 4 S	F 4A	F 4 a	F 4 s	F 4 ss	F 4 I	%
B 1	4	4	2	4	5	3	9	100.00
B 2	4	4	2	4	5	3	9	100.00
B 3	4	4	1	3	5	2	8	75.00
B 4	4	4	1	3	5	3	8	75.00
B 5	4	4	2	4	5	3	9	100.00
B 6	0	0	0	0	1	12	1	0.00
B 7	4	4	1	3	5	2	8	75.00
B 8	4	4	1	3	5	2	8	75.00
B 9	4	4	1	3	5	2	8	75.00
B 10	4	4	1	3	5	2	8	75.00
B 11	4	0	0	1	1	12	1	12.50
B 12	0	0	0	1	1	12	1	0.00
B 13	0	0	0	1	1	12	1	0.00
B 14	0	0	0	1	1	12	1	0.00

All nine students in the online course were able to convey they understood what this question was asking and received a code of 4 for the U category. Only three students were able to successfully row reduce the matrix and arrive at an accurate solution. Two students were able to solve using substitution and elimination techniques, four students started matrix calculations but were not able to successfully complete the process. Ten students from the face-to-face course attempted to solve question 4 and received a code of 4 for U. Nine of these 10 students were able to successfully convey an understanding of the question's requirements and received a code of 4 for S, only 3 face-to-face students were able to successfully solve the matrix.

Pre-calculus question 4 student work analysis. Computational errors in the row reduction process caused both online and face-to-face students to not successfully complete the solution process. No one from the face-to-face course tried to solve using substitution and elimination techniques. Four students from the face-to-face group left this question blank. One example of a properly initiated, but incorrect solution is shown in *Figure 17*. The step circled in green is where seven students made an error, resulting in an incorrect solution for their calculations.

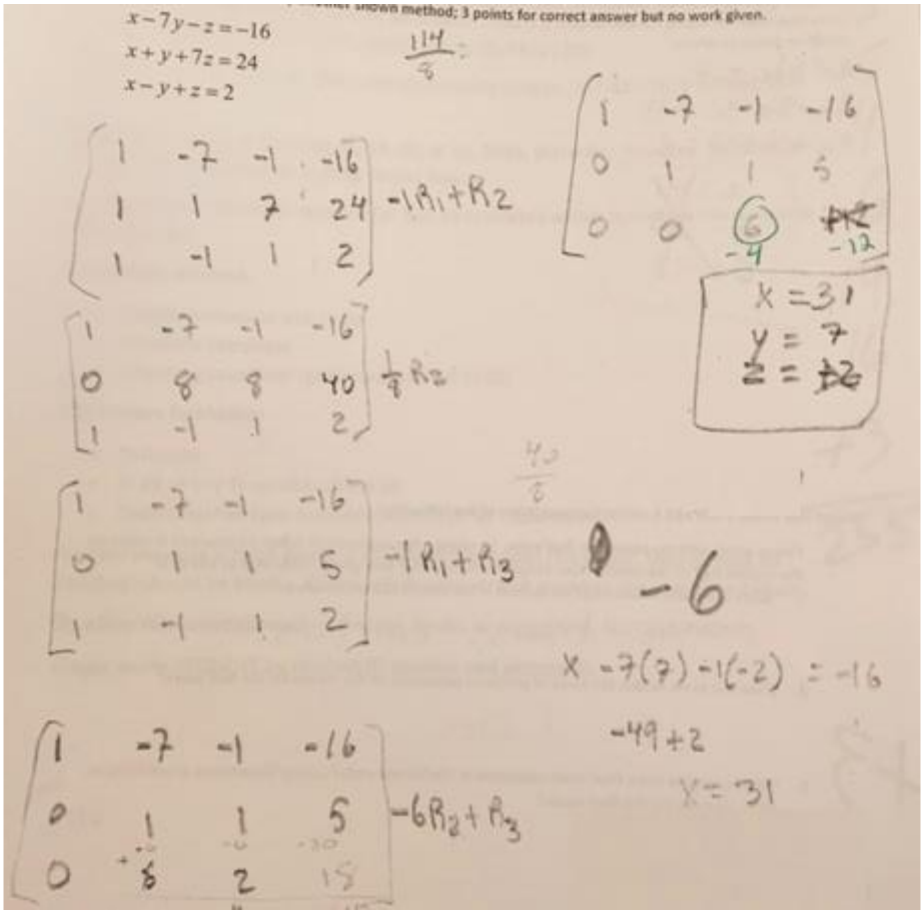


Figure 17: Online Matrix errors

As shown in Figure 17, this student was able to correctly set up the matrix and begin the row reduction process; but, computational errors caused the student to be unsuccessful with solving.

In what ways do work and scores on the final assessment relative to solving integrals compare between online and face-to-face calculus students?

The following analysis presents in detail the differences observed between an online and a face-to-face calculus course. The scores received by students on the assessment were analyzed statistically, through coding relative to the described final framework, presented in figure 6, and through detailed examination of student work. Initial statistical reviews found a non-statistically significant difference between student scores on questions 1 and 2. Statistically significant differences in student scores were found on questions 3 and 4. Table 24 shows class averages, in percent, for each question. The final average of all four questions is shown in the last column

Table 24

Calculus Averages in Percent

Test	Q1	Q2	Q3	Q4	Final Avg
Online	68.75	64.29	60.27	46.43	59.94
Face-to-Face	59.52	83.63	88.1	38.99	67.56

An initial statistical review of overall scores on the integration assessment, which consisted of four specific problems, was conducted using one-way ANOVA. Levene's test, as shown in Table 25, reveals the assumption of homogeneity of variance was violated. Due to violation of the homogeneity of variances assumption, the Welch test and Brown-Forsythe test were implemented as the corrected *F*-tests and, as shown in Table 26, revealed there was not a statistically significant difference in the mean scores between the online and face-to-face groups, $F(1, 99.54) = 1.50$, $p > .05$, $\eta^2 = 0.01$, suggesting that overall student performance on this Integration assessment was similar between the online and face-to-face groups.

Table 25

Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
10.39	1	33	.00

Table 26

Robust Test of Equality of Means

	Statistic[a]	df1	df2	Sig.

Welch	1.50	1	99.54	.22
Brown-Forsythe	1.50	1	99.54	.22

Calculus question 1. Question one of the calculus integration question asked students to calculate the area of a bounded region between two curves. To successfully solve this problem, students needed to create an integration equation to model the bound region, integrate, and simplify.

Calculus question 1 statistical analysis. The class averages for question 1 differed by 9.23%. For question 1, the online class average was 68.75% and the face-to-face class average was 59.52%. As shown in Table 27, the assumption of homogeneity of variances was not violated, according to Levene's test. To further explore the between group differences, the *F*-test was conducted. As shown in Table 28, this test suggested a statistically nonsignificant difference in question 1 scores, $F(1,33) = 0.63$, $p >. 05$, $\eta^2 = 0.019$.

Table 27

Calculus Question 1 Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
2.04	1	33	.16

Table 28

Calculus Question 1 ANOVA

	Sum of Squares	df	Mean Square	F	Sig.

Between Groups	715.03	1	715.03	.63	.43
Within Groups	37392.11	33	1133.09		
Total	38107.14	34			

Calculus question 1 coding analysis. As substantiated by the 9.23% difference in class averages for the first calculus question, the face-to-face students did not perform as well on the first question as the online students. Tables 29 and 30 show the codes assigned to student's work on the first calculus question.

Table 29

Student Question Evaluation O 1

Test	O 1 U	O 1 S	O 1 A	O 1 a	O 1 s	O 1 ss	O 1 I	points lost	points earned	%
D1	0	0	0	1	1	12	1	4	0	0
D2	1	1	0	3	5	3	3	3	1	25
D3	4	4	1	3	5	3	8	1	3	75
D4	4	4	2	4	5	3	9	0	4	100
D5	4	4	2	4	5	3	9	0	4	100
D6	0	0	0	1	1	12	1	4	0	0
D7	4	4	2	4	5	3	9	0	4	100
D8	4	4	2	4	5	3	9	0	4	100
D9	4	3	1	3	5	3	7	1.5	2.5	62.5
D10	4	4	2	4	5	3	9	0	4	100
D11	1	1	0	3	5	3	3	3	1	25
D12	4	4	1	3	5	3	8	1	3	75
D13	4	4	2	4	5	3	9	0	4	100
D14	4	4	2	4	5	3	9	0	4	100

Table 30

Student Question Evaluation F 1

Test	F 1 U	F 1 S	F 1 A	F 1 a	F 1 s	F 1 ss	F 1 I	points lost	points earned	%
C1	1	1	0	2	1	3	3	3.5	0.5	12.5
C2	4	4	1	3	5	3	8	1	3	75
C3	4	4	1	3	5	3	5	2	2	50

C4	4	4	1	4	5	3	8	1	3	75
C5	5	5	2	4	5	12	4	3	1	25
C6	4	2	0	3	5	3	5	3	1	25
C7	1	1	0	3	1	3	3	3	1	25
C8	4	3	1	3	5	3	7	1.5	2.5	62.5
C9	4	3	1	3	5	3	7	2	2	50
C10	4	4	1	3	5	3	8	1	3	75
C11	4	4	1	3	5	3	8	1.5	2.5	62.5
C12	1	1	0	3	1	3	3	3.5	0.5	12.5
C13	2	2	0	3	5	3	5	3	1	25
C14	4	4	1	3	5	3	8	0.5	3.5	87.5
C15	4	4	2	4	5	3	9	0	4	100
C16	4	4	2	4	5	3	9	0	4	100
C17	4	4	2	4	5	3	9	0	4	100
C18	3	3	1	3	5	3	8	2	2	50
C19	4	4	1	3	5	3	8	1	3	75
C20	4	4	1	3	5	3	8	0.5	3.5	87.5
C21	3	3	2	4	5	3	8	1	3	75

Two students in the online section left question 1 blank. Of these two students, one appeared to have guessed at a numerical solution, but provide no work or evidence of embarking upon problem solving strategies, and the other provided no markings on this question at all. No students in the face-to-face section failed to attempt this question. Three face-to-face and two online students misinterpreted this question, as shown by a code of 1 for the U category. Each student who received a score of 1 for the U category attempted to set the boundary curve equations equal to each other and solve for x, demonstrating a misinterpretation of the process used to integrate and find the bounded area. Fourteen students form the face-to-face course and 10 students from the online course were able to convey a complete and accurate understanding of question 1. Of these 24 students, three face-to-face and seven online students were able to successfully navigate their problem solving procedures and arrive at a complete solution. Computational errors, as indicated by a 1 in the A category, accounted for 10 face-to-face students not successfully completing question 1. As indicated by a coding of 5 for the “a” category, 18 face-to-face students and 12 online students provided partially correct statements to accompany their solutions. One student in the face-to-face course and two students in the online course did not provide any work for question 1. The remaining 32 students each provide work suggesting use of an algebraic solution procedure to solve question 1, as indicated by a code of 3 in the "ss" category. As indicated by codes of 7 and 8 in the "I" category, secondary errors caused many students to not successfully solve question 1. Secondary errors for question 1 will be discussed further but for the present time, it is noted that they took the form of integration or arithmetic errors.

Calculus question 1 student work analysis. In the face-to-face course, of the four students successfully solved question 1, three of these students provided substantial work to showcase their integration techniques and one student provided no work to accompany his/her accurate solution. Each of the three students who provided detailed evidence of his/her solution procedures first drafted an integral equation consisting of subtracting the bounded regions to evaluate the area of the desired space between the curves before proceeding to calculating and simplifying the area of the region, as shown in Figure 18.

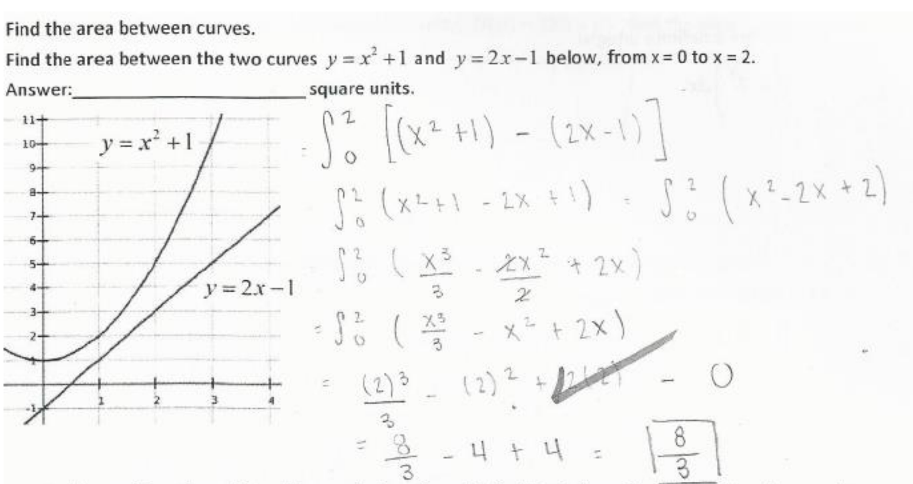


Figure 18:. Calculus Face-to-Face Question 1

Six students in the online course were able to successfully complete this question. Each of these six students provided detailed work to illustrate their generation of the integration question, integration procedure, and simplification calculations. The procedures used by the online and face-to-face students who were able to successfully navigate this question are comparable. Like the work shown previously in Figure 18, the work in Figure 19 demonstrates a similar solution technique.

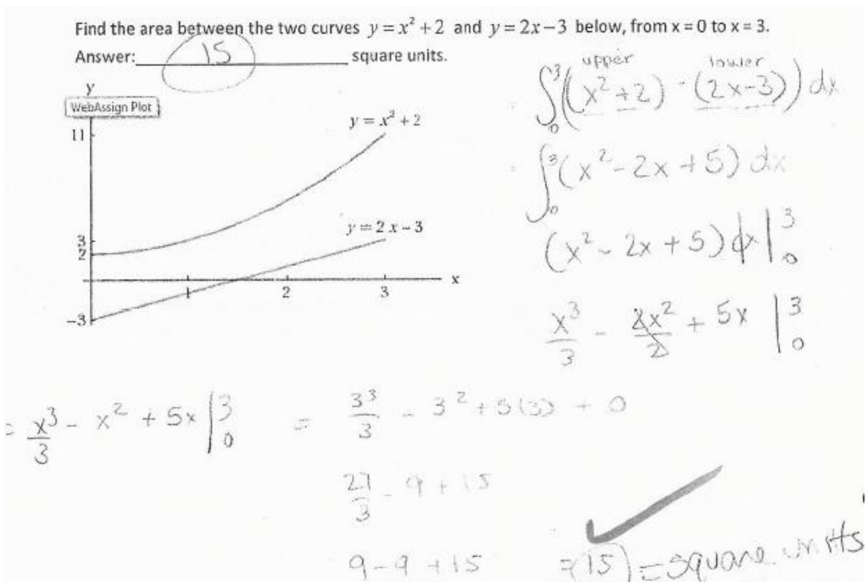


Figure 19: Calculus Online question 1

The most common errors in both the online and face-to-face sections for question 1 were relative to generating the integral equation or arithmetic errors. Three students in the face-to-face course and two students in the online course tried to solve question 1 as a system of equations. Each of these students began by setting the two equations equal to each other and attempting to solve for x algebraically. In four out of these five cases, students recognized the process embarked upon was not going to generate a successful solution and stopped without reaching a conclusion. One student tried to substitute the given boundary x values into their incorrect algebraic equation and simplify, demonstrating a partial understanding of the procedure surrounding utilizing the end points of the interval.

Arithmetic errors prohibited six students in the face-to-face course and three students in the online course from correctly navigating question one. In the final step of their calculations, one student in the face-to-face course and one student in the online course each incorrectly simplified a fraction, resulting in a skewed solution. Additional arithmetic errors surrounding simplifying after integrating and evaluating the boundary values prohibited accurate solutions for four students in the face-to-face course. Two students in the online section correctly set up the initial integration equation but incorrectly simplified their equation prior to integrating. In both cases, students made errors when subtracting the upper and lower curves to generate one simplified integration equation, as shown in Figure 20.

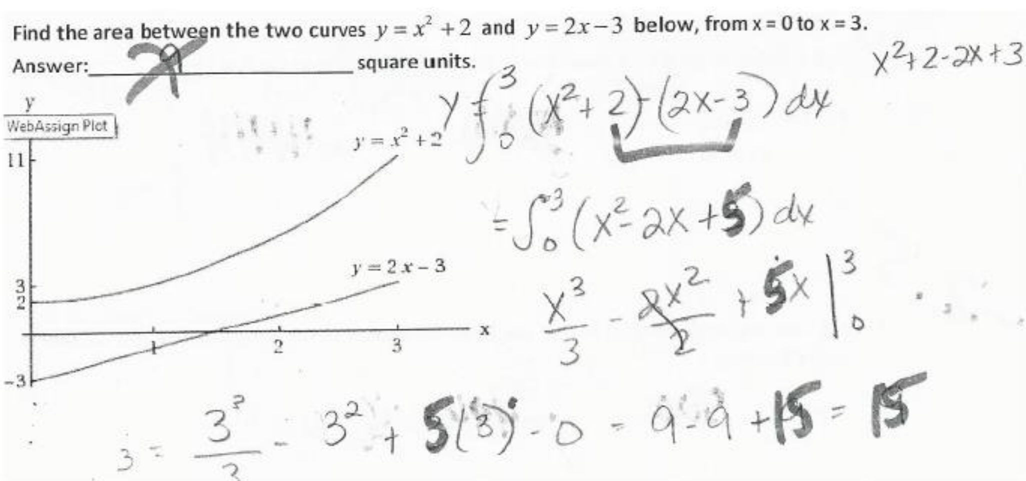


Figure 20: Calculus Question 1 equation error

Simplifying the integrand also caused trouble in the face-to-face course. Four face-to-face students incorrectly simplified the integrand while four other students did not integrate before evaluating the integral end points. Additionally, one student in the face-to-face course set up the integral equation correctly but then did not proceed to integrate or finalize the solution process.

Calculus question 2. Question 2 is a four-part question asking students to demonstrate their understanding of market demand, market price, consumers' surplus at market demand, and producers' surplus at market demand. In this question, students are given a demand function *d(x)* and supply function *s(x)* to use while navigating each part of question 2. As question 2 progresses, students will be required to use their answers from one portion of question 2 while calculating subsequent portions of questions 2. Student's incorrect answers are considered when evaluating their demonstration of understanding of subsequent components. The instructor provided formulas for calculating consumers' surplus at market demand and producers' surplus at market demand for all students as part of their question resources. Theses formulas were provided on an instructor prepared formula sheet which was stapled to the exam.

Calculus question 2 statistical analysis. The class averages for question 2 were different by 19.34%. For question 2, the online class average was 64.29% and the face-to-face class average was 83.63%. As shown in Table 31, the assumption of homogeneity of variances is not rejected by the Levene's test as a statistically significant difference in the variances is not garnered by these scores. With no violation of the homogeneity of variance assumption, the regular F-test was used to evaluate the between-group differences. The *F*-test, shown in Table 32, conveys a statistically nonsignificant difference at the $\alpha = .05$ level with a conclusion of $F(1, 33) = 3.86$, p >.05, $\eta^2 = 0.10$.

Table 31

Calculus Question 2Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.

4.11	1	33	.05

Table 32

Calculus Question 2 ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3143.60	1	3143.60	3.86	.06
Within Groups	26867.56	33	814.17		
Total	30011.16	34			

Calculus question 2 coding analysis. Question 2 showcased the second highest class average for both the online and face-to-face sections, 64.29% and 83.63% respectfully. The face-to-face class average was bolstered by 10 students being able to successfully initiate and carry out an accurate problem solving approach for all components of question 2. Four online students were also able to successfully navigate question 2 while in both the online and face-to-face section one student received no credit for question 2 due to no provided answer or no evaluateable work, as shown by codes of 5 for the U and S categories and a code of 12 for the ss category. As shown by codes of 4 for the U category of Table 34, 15 face-to-face students and seven online students were able to successfully convey an understanding of question 2. Each student who completed question 2 in both the face-to-face section and the online section received a code of 3 in the ss category, conveying an understanding of the computational process necessary for utilizing the market price and demand formulas. As indicated by codes of 7, and 8 in the I category, both online and face-to-face students who were not able to successfully complete question 2 were hindered by secondary errors, not errors demonstrating a complete lack of understanding. Codes for the online and face-to-face student work for question 2 are shown in Tables 33 and 34.

Table 33

Student Question Evaluation O 2

Test	O 2 U	O 2 S	O 2 A	O 2 a	O 2 s	O 2 ss	O 2 I	points lost	points earned	%
D1	5	5	0	5	6	12	4	2	2	50
D2	2	2	1	5	5	3	7	3	1	25
D3	2	2	0	3	5	3	5	1.5	2.5	62.5
D4	4	4	2	4	5	3	9	0	4	100
D5	4	4	1	5	5	3	7	1	3	75
D6	0	0	0	1	1	12	1	4	0	0
D7	4	4	2	4	5	3	9	0	4	100
D8	4	4	2	4	5	3	9	0	4	100
D9	0	0	0	1	1	12	1	4	0	0
D10	4	4	1	5	5	3	8	0.5	3.5	87.5
D11	4	4	1	5	5	3	7	1	3	75
D12	2	2	1	5	6	3	8	2	2	50
D13	3	3	1	5	5	3	8	1	3	75
D14	4	4	2	4	5	3	9	0	4	100

Table 34

Calculus Student Question Evaluation F 2

Test	F 2 U	F 2 S	F 2 A	F 2 a	F 2 s	F 2 ss	F 2 I	points lost	points earned	%
C1	4	4	2	4	5	3	9	0	4	100
C2	4	4	1	5	5	3	7	0.75	3.25	81.25
C3	4	4	2	4	5	3	9	0	4	100
C4	4	4	2	4	5	3	9	0	4	100
C5	2	3	1	5	5	3	5	1	3	75
C6	4	3	1	5	5	3	8	1	3	75
C7	4	3	1	5	5	3	7	1	3	75
C8	4	3	1	5	5	3	7	1	3	75
C9	0	0	0	1	1	12	1	4	0	0
C10	4	4	2	4	5	3	9	0	4	100
C11	4	4	2	4	5	3	9	0	4	100
C12	2	2	0	5	5	3	7	1	3	75
C13	2	2	0	5	5	3	7	1	3	75
C14	4	4	2	4	5	3	9	0	4	100

C15	4	4	2	4	5	3	9	0	4	100
C16	4	4	2	4	5	3	9	0	4	100
C17	4	4	1	5	5	3	8	0.5	3.5	87.5
C18	4	4	2	4	5	3	9	0	4	100
C19	4	4	2	4	5	3	9	0	4	100
C20	2	2	0	5	5	3	7	1.5	2.5	62.5
C21	3	3	1	5	5	3	8	1	3	75

Calculus question 2 student work analysis. Fourteen students, 10 face-to-face and four online, demonstrated a complete understanding of the processes used to evaluate market demand, market price, consumers' surplus at market demand, and producers' surplus at market demand through question 2. One student in the online course switched their market demand and market price calculations, which lead to inaccuracies when calculating consumers' and producers' surplus at market demand values; all other students who attempted question 2, both online and face-to-face, were able to successfully calculate the market demand value. One student provided no work to justify their solutions for question 2 but did supply accurate market demand and market price values; their values for surplus at market demand were incorrect and without their work, their understanding of the surplus at market demand calculations could not be ascertained.

Integration and computational errors hindered seven students from successfully completing question 2. Integration errors, as shown in Figure 21, caused one online and four face-to-face students to inaccurately complete their problem solving plan while calculating the surplus at market demand values.

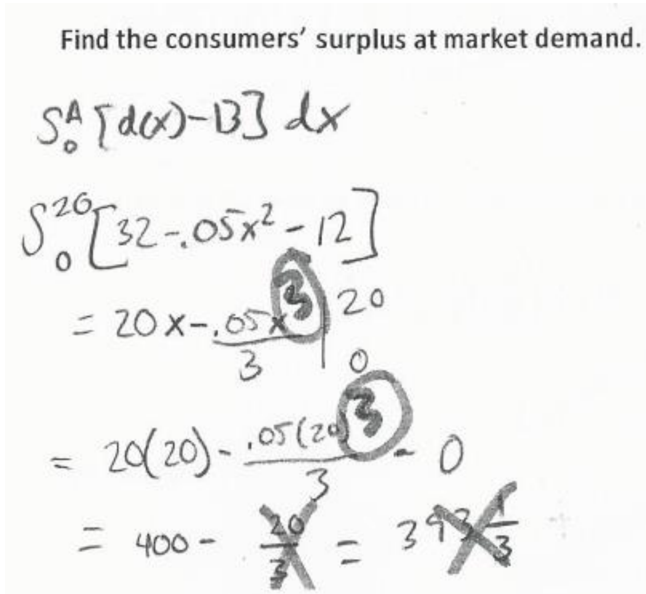


Figure 21: Integration error

Computational errors in the forms of subtraction inaccuracy and rounding errors, were demonstrated by one online and one face-to-face student. Additional errors regarding limits of integration and correctly substituting the market price value into the integration equation for consumers' surplus at market demand caused three face-to-face students and one online student to provide incorrect solutions.

Of the students who completed question 2, four students, two face-to-face and three online students were not able to successfully calculate the market price component of question 2. One face-to-face student did not supply any work for calculating the market price while one used the demand function instead of the supply function to complete their market price calculation. These errors in calculating the market price also lead to errors calculating the consumers' surplus at market demand.

One online student correctly solved for market demand but failed to calculate market price and instead used their market demand value for each component of their surplus at market demand calculations.

One face-to-face and two online students left question 2 blank while one online student was able to calculate market demand but not able to complete the remaining portions of question 2. This student attempted to write the equation for market price, incorrectly, but then did not attempt calculating consumers; or producers' surplus at market demand.

Calculus question 3. Question 3 asks students to evaluate an indefinite integral using the process of substitution. To successfully complete this question, students must identify an appropriate u and du value, use u, du substitution, integrate, substitute back, and simplify.

Calculus question 3 statistical analysis. The class averages for question 3 were different by 27.83%, the greatest difference in averages for all four calculus questions. For question 3, the online class average was 60.27% and the face-to-face class average was 88.1%. Question 3 received the highest average of all the face-to-face questions but the second lowest average of all the online questions. As shown in Table 35, the violation of the assumption of homogeneity of variances was suggested by Levene's test. The Welch test and Brown-Forsythe test, shown in table 36, also supported a statistically significant difference in question 3 scores, $F(1, 15.59) = 5.67$, $p < .05$, $\eta^2 = 0.19$.

Table 35

Calculus Question 3 Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
34.69	1	33	.00

Table 36

Calculus Question 3 Robust Test of Equality of Means

	Statistic[a]	df1	df2	Sig.
Welch	5.67	1	15.59	.03
Brown-Forsythe	5.67	1	15.59	.03

Calculus question 3 coding analysis. A 27.83% difference was realized in the scores between the online and face-to-face students for question 3. The online average for question 3 was a 60.27% while the face-to-face average for question 3 was an 88.1%, suggesting the online students found question 3 to be more troublesome than their face-to-face peers. To explore these score differences, Tables 37 and\38, respectfully, were developed, showcasing the codes assigned for the online and face-to-face sections for question 3.

Table 37

Calculus Student Question Evaluation O 3

Test	O 3 U	O 3 S	O 3 A	O 3 a	O 3 s	O 3 ss	O 3 I	points lost	points earned	%
D1	4	4	2	4	5	3	9	0	4	100
D2	5	5	0	3	6	12	1	4	0	0
D3	4	4	1	3	5	3	5	2	2	50
D4	4	4	2	4	5	3	9	0	4	100
D5	4	4	2	4	5	3	9	0	4	100
D6	4	4	2	4	5	3	9	0	4	100
D7	4	4	1	3	5	3	8	1	3	75
D8	4	4	1	5	5	3	8	0.25	3.75	93.75
D9	0	0	0	1	1	12	1	4	0	0
D10	2	2	0	3	6	3	5	3	1	25
D11	2	2	0	3	6	3	5	3	1	25
D12	4	4	1	3	5	3	8	0.5	3.5	87.5
D13	4	4	1	3	5	3	8	0.5	3.5	87.5
D14	5	5	0	3	1	12	1	4	0	0

Table 38

Calculus Student Question Evaluation F 3

Test	F 3 U	F 3 S	F 3 A	F 3 a	F 3 s	F 3 ss	F 3 I	points lost	points earned	%
C1	4	4	1	3	5	3	8	0.5	3.5	87.5
C2	4	4	2	4	5	3	9	0	4	100
C3	4	4	2	4	5	3	9	0	4	100
C4	4	4	2	4	5	3	9	0	4	100
C5	4	4	2	4	5	3	9	0	4	100
C6	3	3	1	3	5	3	8	1	3	75
C7	4	4	1	3	5	3	8	1	3	75
C8	5	5	0	3	5	12	1	2	2	50
C9	4	4	2	4	5	3	9	0	4	100
C10	4	4	1	3	5	3	8	0.5	3.5	87.5
C11	4	4	2	4	5	3	9	0	4	100
C12	4	4	1	3	5	3	8	0.5	3.5	87.5
C13	4	4	2	4	5	3	9	0	4	100
C14	4	4	2	4	5	3	9	0	4	100
C15	4	4	2	4	5	3	9	0	4	100
C16	4	3	5	3	5	3	8	1	3	75
C17	4	4	2	4	5	3	9	0	4	100
C18	4	3	1	3	5	3	8	0.5	3.5	87.5
C19	4	3	1	3	5	3	8	1	3	75
C20	4	4	2	4	5	3	9	0	4	100
C21	4	3	1	3	5	3	8	2	2	50

As shown by a code of 4 in the U category, nine online and 19 face-to-face students were able to successfully demonstrate an understanding of the integration by substitution process. This demonstration of understanding included correctly assigning u and du value and initiating the integration calculations. The codes of 3 in the S category show three face-to-face students encountered issues while solving but iterates procedural understanding. Codes of 1 in the A category show five online and eight face-to-face students encountered computational errors while working through their integration process. As shown by codes of 3 in the ss category, all students who attempted question 3 utilized an appropriate computational strategy, and the accrued errors comprised of minor calculation issues for both the online and face-to-face students, as shown by codes of 8 in the I category.

Calculus question 3 student work analysis. Fifteen students, 11 from the face-to-face section and four from the online section, successfully navigated their way through question 3. Eleven of these 15 students initiated their work by writing an accurate "let" statement, identifying their *u* value and calculating their *du* value. Each student then rewrote the integral in term of *u* and *du* before integrating, back substituting, and simplifying, an example of student work is show below in Figure 22.

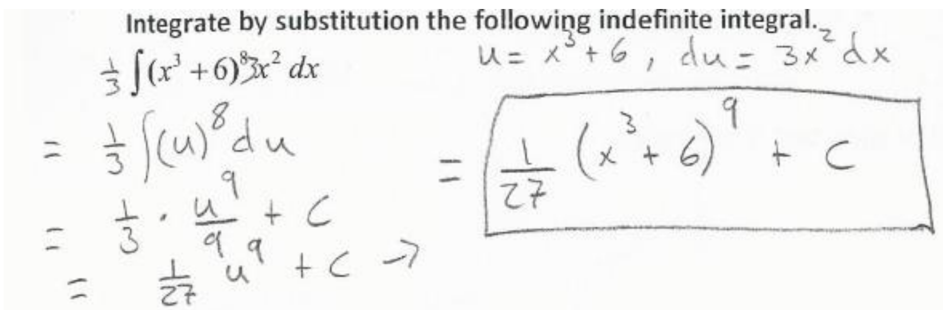


Figure 22: Integration by Substitution Student Work

Two online and one face-to-face student did not provide any work, but arrived at the correct solution. One other face-to-face student also correctly solved the question but did not show their initial *u*, *du* definitions or their pre-integration steps and only included back substitution and a solution.

The face-to-face students who did not successfully complete question 3 made one of four errors; they either did not correctly integrate, did not back substitute after they integrated, did not correctly account for the constant of integration, or did not accurately simplify after completing their integration. One student took the derivative instead of integrating, two others correctly integrated but did not correctly back substitute while simplifying their solution.

In both the online and face-to-face sections, students did not correctly account for the constant which results from the *du* calculation. Two students in the face-to-face section and three students in the online section correctly identified their *u* value and correctly calculated their *du* value, but did not correctly account for the constant revealed by their *du* when substituting and integrating.

Four students in the face-to-face section made notational errors in their solutions. Each student was able to correctly initiate their problem solving strategy but did not successfully conclude, due to minor errors. One student correctly solved, but wrote an integration symbol in his/her final answer, showing an incomplete understanding of when to include the integral symbol. Another student correctly embarked on this question with accurately defined *u* and *du* values, but did not accurately integrate while two students successfully completed the integration and back substitution work but made a calculation error when simplifying.

The errors in the online student's work were more varied. Like the face-to-face course, the online course had one student who accurately completed their calculation but left the integral symbol in their final answer. Four additional students successfully identified the *u* value and accurately calculated the *du* value, but were not able accurately translate these values into the integral equation or were not able to accurately integrate after substituting. One student in the online section left this question completely blank, one tried to use a *ln* to integrate, and one tried to embark on a technique using integration by parts.

The vast spread of averages in question 3 between the online and face-to-face sections is due to 11 face-to-face students receiving perfect scores on question 3, compared to four online students who received perfect scores. Additionally, mistakes made in the calculations of online students resulted in two students receiving 25% credit while one student received 50% credit and another 75% credit. In the face-to-face course, minor errors were less frequent, no students received 25% credit while two students received 50% credit and four students received 75% credit.

Calculus Question 4. Question 4 on the calculus integration asks students to utilize integration by parts to calculate an indefinite integral. The integration by parts formula is provided to students in the question. To complete this question, students need to accurately define their *u* and *dv* values, solve for the *du* and *v* values, and properly deploy the integration by parts process.

Calculus question 4 statistical analysis. The class averages for question 4 differed by 7.44%. For question 4, the online class average was 46.43% and the face-to-face class average was 38.99%. As shown in Table 39, the assumption of homogeneity of variances was violated, according to the Levene's test. The Welch test and Brown-Forsythe test, shown in Table 40, suggested a statistically nonsignificant difference in question 4 scores, $F(1, 20.8) = .37$, $p > .05$, $\eta^2 = 0.013$.

Table 39

Calculus Question 4 Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
5.64	1	33	.02

Table 40

Calculus Question 4Robust Test of Equality of Means

	Statistic[a]	df1	df2	Sig.

Welch	0.37	1	20.80	.55
Brown-Forsythe	0.37	1	20.80	.55

Calculus question 4 coding analysis. As displayed by class averages of 46.43% and 38.99%, online and face-to-face students respectfully, illustrated difficulty with question 4 more than with questions 1 through 3. A 7.44% score difference reveals a greater ability to navigate question 4 by the online students than their face-to-face peers. Tables 41 and 42 show the codes assigned to student's work on the fourth calculus question.

Table 41

Student Question Evaluation O 4

Test	O 4 U	O 4 S	O 4 A	O 4 a	O 4 s	O 4 ss	O 4 I	points lost	points earned	%
D1	0	0	0	1	1	12	1	4	0	0
D2	2	5	0	3	6	12	1	4	0	0
D3	4	4	2	4	5	3	9	0	4	100
D4	4	4	2	4	5	3	9	0	4	100
D5	4	4	2	4	5	3	9	0	4	100
D6	0	0	0	1	1	12	1	4	0	0
D7	4	4	2	4	5	3	9	0	4	100
D8	2	2	0	3	6	3	5	3	1	25
D9	3	4	1	3	6	3	7	1	3	75
D10	2	2	0	3	6	3	5	3	1	25
D11	2	2	0	3	6	3	5	3	1	25
D12	3	2	0	3	6	3	5	2	2	50
D13	2	2	0	3	6	3	5	3	1	25
D14	2	2	0	3	6	3	5	3	1	25

Table 42

Student Question Evaluation F 4

Test	F 4 U	F 4 S	F 4 A	F 4 a	F 4 s	F 4 ss	F 4 I	points lost	points earned	%
C1	2	2	1	3	6	3	5	3	1	25
C2	2	2	1	3	6	3	7	3	1	25

C3	4	3	1	3	6	3	7	3	1	25
C4	4	3	1	3	5	3	7	2	2	50
C5	2	2	0	3	6	3	5	3	1	25
C6	3	3	1	3	6	3	5	3	1	25
C7	2	2	1	3	5	3	5	3	1	25
C8	1	2	1	3	6	3	5	4	0	0
C9	5	5	0	3	6	12	1	2	2	50
C10	2	2	1	3	6	3	5	3	1	25
C11	2	2	1	3	6	3	7	3	1	25
C12	2	2	0	2	6	3	5	3	1	25
C13	5	5	0	3	5	12	1	4	0	0
C14	4	3	1	3	5	3	7	1	3	75
C15	4	3	1	3	5	3	8	1	3	75
C16	4	4	1	3	6	3	7	1.5	2.5	62.5
C17	4	3	1	3	6	3	7	1	3	75
C18	4	3	1	3	5	3	8	0.25	3.75	93.75
C19	0	0	0	1	1	12	1	4	0	0
C20	4	3	1	3	5	3	7	1.5	2.5	62.5
C21	3	2	1	3	6	3	5	2	2	50

In the online section, four students were able to convey a complete understanding of question 4, as shown by a code of 4 in the U category. All four of these students were able to successfully solve question 4, as shown by a code of 4 in the S category and 2 in the A category. Eight students in the face-to-face course were able to convey an understanding of solving using integration by parts, as evident from their code of 4 in the U category, but seven of these students encountered a procedural error, as shown by a code of 3 in the S category, while one student would have been able to successfully solve without the presence of a computational error. Codes of 0 and 1 for all the face-to-face students reveal inappropriate plans, computational errors, and insufficient evidence to interpret student's planned solution strategies. With the exception of the three online students who successfully completed question 4 and the one student who successfully initiated their solution plan for question 4, all other online students received a code of 0 for category A, signifying inappropriate or unclear plans with little or no work justification.

Everyone in both the online and face-to-face sections who provided some work for interpretation, embarked on an Algebraic computational path for solving, as shown by codes of 3 for the ss category. No students who attempted to solve this question used any approaches other than the integration by parts procedure outlined in the question directions. As evidenced by codes of 1, 5, 7, and 8 in category I, implementation of problem solving plans frequently contained no supporting work or work which demonstrated multiple errors.

Calculus question 4 student work analysis. Student work on question 4 demonstrates significant levels of misunderstanding regarding systematically using the integration by parts formula to calculate an indefinite integral. No students from the face-to-face course were able to successfully complete question 4. Four students from the online course were able to successfully navigate a complete problem solving process and arrive at an accurate solution for question 4.

The most common issues in both the online and face-to-face courses was a perceived inability to successfully identify the *u, v, du,* and *dv* values or an inability to successfully place the defined *u, v, du,* and *dv* values into the integration by parts formula. Seven students in the face-to-face course and three students in the online course incorrectly defined their *u, v, du* and *dv* values. The most common errors in *u, v, du,* and *dv* values assignments included students not assigning ln(x) to *u*, which was an error by all three online students and occurred five times in the face-to-face section, or students incorrectly simplifying the dv value, which occurred twice in the face-to-face section. While their parts values were incorrect, four students from the face-to-face course tried to continue the solution process by correctly placing their parts values for *u, v, du,* and *dv* into the integration by parts formula, demonstrating a partial understanding of the integration by parts problem solving strategy.

Ten face-to-face students and one online student who were able to successfully identify the u, v, du and dv parts values demonstrated an understanding of the integration by parts problem solving process, but were unable to successfully complete question 4 due to incorrectly utilizing the integration by parts formula or encountering algebraic issues when simplifying. Three face-to-face students accurately defined the integration parts values but did not carry through their work to successfully place the values into the integration by parts formula. Seven face-to-face students and one online student were able to successfully identify their integration by parts values and accurately set up the integration by parts formula, but encountered calculation errors while integrating or simplifying their calculations. An example of such algebraic errors is shown in Figure 23.

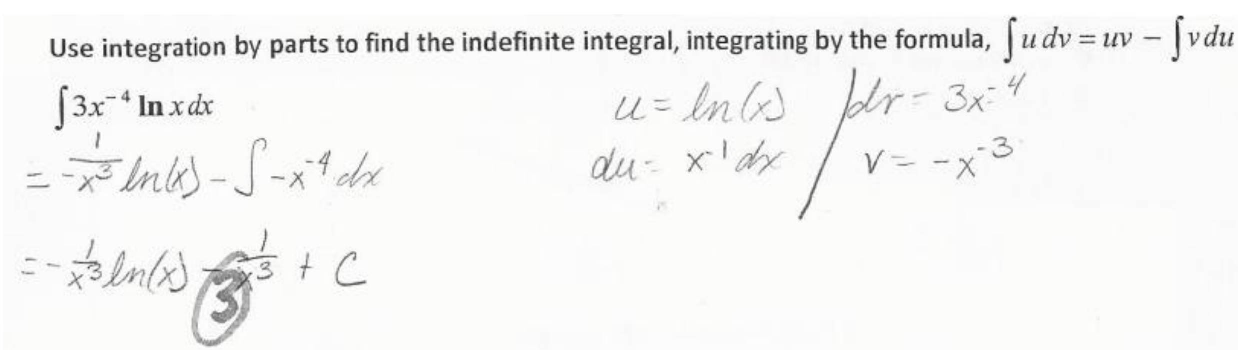


Figure 23: Algebraic Error Student Work Sample for Question 4

The work in *Figure 23* shows the student was able to successfully identify the u, v, du and dv values, substitute the values into the integration by parts formula, and convey an understanding of the process used to solve this integration by parts question. The challenge appeared to emerge in one's inability to successfully solve due to an error when evaluating the integral.

Two online students and one face-to-face student left question 4 completely blank while two additional face-to-face students wrote an incorrect answer with insufficient evidence for an understanding of their work to be interpreted. One online student provided just a definition of v in their integration by parts formula and an incorrect integration statement, signifying minimal understanding of the process for using integration by parts. Two online students and one face-to-face attempted to utilize the integration by parts formula but did not provide a definition of their utilized u, v, du and dv values while the four remaining online students were able to successfully navigate through question 4 and arrive at an accurate solution. During the solution process, one face-to-face student did not demonstrate an understanding of an indefinite integral, as outlined in question 4, and continued to try to calculate a definite value, as shown in Figure 24.

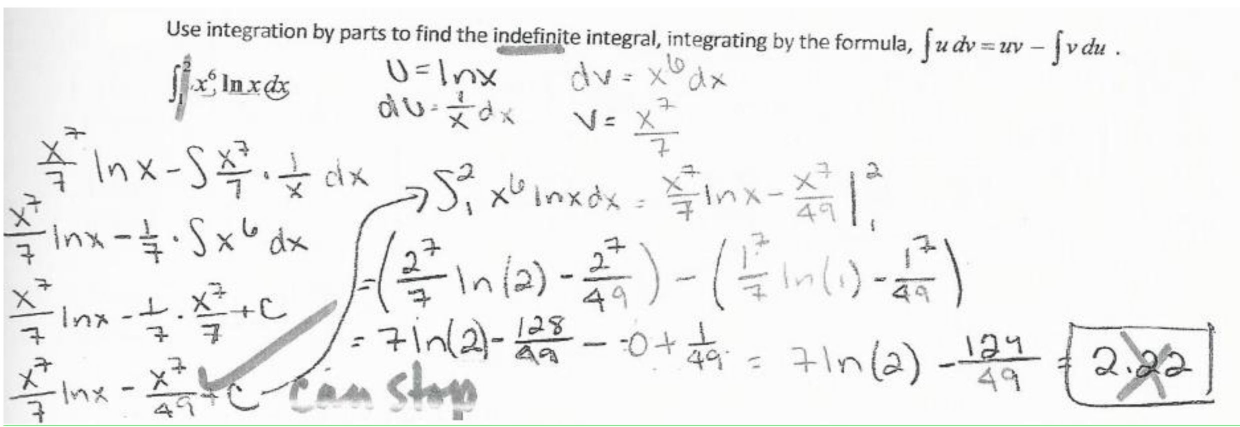


Figure 24: Online Student Work Sample Question 4

Summary of Findings

This study did not reveal any obvious differences between the outcome of the online and face-to-face learning based on student responses to the pre-calculus assessment on solving systems of equations and inequalities using a variety of techniques. Looking at the average scores on each pre-calculus question, the online student's average scores were higher on questions 2, 3 and 4 but lower on question 1. Averaging overall scores on questions 1 through 4 reveal almost identical scores for the online and face-to-face courses with the online average being only slightly less. The final average score for the online course was 70.19% and the face-to-face course was 70.31%.

This study does reveal differences between the outcome of online and face-to-face learning based on student responses to the calculus integration assessment. The online calculus students scored higher on questions 1 and 4 while the face-to-face students scored higher on questions 2 and 3. Questions 1 and 2 were standalone integration concepts, but question 4, regarding integration by parts, arguably builds on question 3, regarding u substitution, and both revolve around student's ability to accurately represent the integral using assigned components. It is interesting that the scores for questions 3 and 4 are not more congruent within the online and face-to-face groups. Both the online and face-to-face groups showcased more understanding of integration by substitution than integration by parts and the margin of difference between questions 3 and 4 was large for both the online and face-to-face sections.

After further analysis using Levene's test and an *F*-test, differences in the online and face-to-face scores are not statistically significant for the pre-calculus assessment. Levene's tests and *F*-tests for scores on questions 1 through 4 reveal non-statistically significant differences between means or the variances for the courses. After conducting a statistical analysis, further analysis was conducted through coding student work using a variation to Szetela and Nicol's (1992) Analytic Scale for Problem Solving and Categories of Responses in Solutions to Problems framework.

The *F*-Tests, Welch tests and Brown-Forsythe tests performed for the calculus scores reveals no statistically significant difference for questions 1, 2 and 4, but suggests there is a statistically significant difference between the scores on questions 3 of the calculus assessment.

The overall final averages on questions 1 through 4 were also deemed statistically nonsignificant. As shown in Table 43, the violation of the assumption of homogeneity of variances was suggested by the Levene's test. The Welch test and Brown-Forsythe test supported a statistically nonsignificant difference in overall scores, $F(1, 99.54) = 1.50$, $p > .05$, $\eta^2 = 0.01$ as shown in Table 44.

Table 43

Overall Calculus Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
10.393	1	138	.00

Table 44

Overall Calculus Robust Test of Equality of Means

	Statistic[a]	df1	df2	Sig.
Welch	1.50	1	99.54	.22
Brown-Forsythe	1.50	1	99.54	.22

After conducting a statistical analysis, further analysis was conducted through coding student work using a variation to Szetela and Nicol's (1992) Analytic Scale for Problem Solving and Categories of Responses in Solutions to Problems framework was adopted for the this study.

Analyzing the codes assigned to student work did not reveal any significant differences in student's conveyance of their understanding or in their approaches to solving each pre-calculus question. Students in both the online and face-to-face courses were able to convey their understanding of each concept, made similar calculation errors, approached each question in similar manners, and overall performed in the same way.

Reviewing student's pre-calculus work to gain a detailed understanding of the approaches used to solve each question involved looking at the work provided and using the work to interpret how students approached each question and the trajectories embarked upon during the solution process. Analysis of student work on each question revealed similar characteristics relative to techniques used and mistakes made.

For question 1, pre-calculus students in both courses elected to use substitution or elimination techniques to solve the systems of equations. In both courses, students made errors with their substitution or elimination techniques while other students from both courses were able to successfully complete the calculation and arrive at an accurate solution.

When solving question 2, pre-calculus students embarked on a solution path through substitution. In both courses, students were able to accurately complete this substitution process and then factor to find the two solution points. Also in both courses, students made similar factoring errors and failed to accurately factor to complete their solution calculations.

The graphing component of question 3 caused similar issues for pre-calculus students in the online and face-to-face courses. In both sections students made errors graphing the linear and quadratic equations. Students also made errors with their test point calculations and with shading the solution region. Students who were able to successfully complete this question used similar techniques in both courses and displayed similar levels of understanding.

Question 4 on the pre-calculus assessment had the lowest average in both the online and face-to-face sections and displayed similar tendencies for students to struggle with successfully row reducing a matrix to arrive at the solution point. In both courses, some students were able to successfully solve using row reduction techniques, while others elected to solve through other means; and in both courses students began but were not able to complete the matrix calculations.

For question 1 on the calculus assessment, students were asked to calculate the area between two curves. Question 1 received the highest overall scores for the online section and the second lowest overall scores for the face-to-face section. Common errors in both the online and face-to-face sections included misinterpreting the solution process by trying to set the curve equations equal to each other and solve using a system of equations. Additional errors with integration techniques were see throughout both the online and face-to-face sections as students incorrectly constructed and evaluated their integral equation.

The second question on the calculus Integration assessment was composed of four interrelated parts. Students in both the online and face-to-face courses demonstrated understanding of completing market demand and market price calculations through the first two parts of question 2. Question 2 received the second highest average for both the online and face-to-face sections. Both online and face-to-face students misinterpreted equation components while calculating consumers' and producers' surplus market demands and in each course a couple students incorrectly utilized their previously calculated values inappropriately through the subsequent components of question 2.

The third calculus integration question asked students to use integration by substitution to evaluate an indefinite integral. Question 3 received the highest average for the face-to-face section and the second lowest average for the online section. Most students in the face-to-face section were able to successfully identify their u and du values and initiate an appropriate solution plan. Students in the online section were more likely to misinterpret large components of questions 3 and less likely to accurately identify the u and du values to initiate a successful solution plan.

Question 4 on the calculus assessment asked students to use integration by parts to evaluate an indefinite integral and received the lowest scores for both the online and face-to-face sections. Students did not convey an understanding of integration by parts and did not showcase abilities to accurately identify the correct u, v, du, and dv components needed to successfully complete the integration by parts procedure. Four online students were able to successfully solve question 4, no face-to-face students were able to successfully complete question 4.

The only notable difference revealed is students in the online pre-calculus course demonstrated a more frequent tendency to try a question if they were not fully sure how to complete the question and arrive at an accurate solution; while the face-to-face students demonstrated a more frequent tendency to leave a question blank. Students in the online class made common arithmetic errors, struggled with factoring techniques, made graphing mistakes, and were not thrown off course by calculation errors while performing row reduction calculations in a matrix. These mistakes were also observed in the face-to-face; no clear evidence was observed that one group has more of a tendency to make algebraic or computational mistakes.

Unlike the pre-calculus students, several notable differences were revealed regarding the online and face-to-face calculus students. Converse of the pre-calculus students, the online calculus students showed a greater tendency to leave a question blank. Ten integration questions were left blank by online calculus students while only four were left blank by face-to-face calculus students, meaning 17.86% of the online and 4.76% of the face-to-face questions were left blank. Also notable, the 24 questions answered by the face-to-face students and 19 questions answered by the online students were completed with complete accuracy and received a score of 100%, meaning 28.57% of the face-to-face questions were answered with complete accuracy while 33.93% of the online questions were answered with complete accuracy. Additionally, the face-to-face students received greater components of partial credit than their online peers, potentially contributing to differences in class averages.

CHAPTER 5

Conclusion

The way people live, work, learn and play is impacted by technology. Laptops, tablets, smartphones and the internet are enhancing the rate at which knowledge can be accessed and transferred. With over 7.1 million students turning to online means for educational opportunities (Allen & Seaman, 2014), "we are just beginning to discover and understand the extent to which these technologies will transform expectations for, and approaches to, learning" (Garrison, 2011, p. 5). As students increasingly embark on online learning experiences, it is important to ensure quality of learning is not being negatively impacted.

Through this study the work of 23 pre-calculus students, nine online and 14 face-to-face, and 35 calculus students, 14 online and 21 face-to-face, was analyzed statistically, methodically, and analytically. Statistical reviews were conducted to determine if statistically significant differences were present in test scores between the online and face-to-face course sections. Methodical analyses, using a modified version of Szetela and Nicol's Problem Solving Scale, as shown in *Figure 8,* were conducted to look for coding trends relative to student's use of problem solving strategies. A final analytical review of student work was conducted to examine approaches students used to complete each problem and to investigate similarities and differences in problem solving strategies used by online and face-to-face students. Through these qualitative and quantitative means, this study explored the research questions:

1. In what ways do work and scores on the final assessment relative to solving systems of equations and inequalities compare between online and face-to-face pre-calculus students?

2. In what ways do work and scores on the final assessment relative to solving integrals compare between online and face-to-face calculus students?

Each course which participated in this study was taught by the same instructor. The online and face-to-face pre-calculus courses used the same textbook, had the same course objectives, and provided the same resources to students. Similarly, the online and face-to-face calculus courses used the same textbook, had the same course objectives, and provided the same resources to students. The instructor strived to make the only difference between the courses the modality of instruction. Face-to-face students participated in weekly lectures covering courses content while online students relied on pre-recorded video lectures provided by MyMathLab or WebAssign.

This study joins a limited body of works that focus on comparing student acquisition of mathematics knowledge in an online setting to a face-to-face setting. Like Weems (2002), Jungic and Mulholland (2011), Larson and Sung (2009), no statistical differences were found between the online and face-to-face pre-calculus groups in this study, but more variance was discovered between the online and face-to-face calculus students' scores.

Analysis of student work revealed similarities between errors, misconceptions, and accurate solution techniques in both the online and face-to-face sections for both pre-calculus and calculus. It is interesting to note inconsistencies between the pre-calculus and calculus students were revealed, implying analysis of student learning could differentiate between course content as well as between online or face-to-face delivery modalities. In the pre-calculus courses, online students demonstrated a reduced tendency to leave questions blank while in the calculus courses face-to-face students demonstrated a reduced tendency to leave questions blank.

Greater variances in scores were realized between the online and face-to-face calculus courses than the online and face-to-face pre-calculus courses. While the face-to-face pre-calculus overall average scores were slightly higher, the difference was not found to be statistically significant. A greater difference was found between the scores for the online and face-to-face calculus courses, but again the differences in the overall averages for the calculus courses was not found to be statistically significant. The lack of statistically significant difference between the online and face-to-face pre-calculus scores for this collection of systems of equations and inequalities questions cannot be generalized to all pre-calculus content. Similarly, the variation in scores between the online and face-to-face calculus courses relative to the four selected integration questions does not convey a similar difference will be present among all calculus topics. This result leads to the conclusion that the results of online and face-to-face content studies will vary based on the content analyzed.

Future Research

Additional studies should be conducted to compare online and face-to-face mathematics students' work in other courses, at other grade levels, and with other instructors and programs to establish transferability and replicability of findings. Student attrition, student perceptions of their online experiences, and future mathematics course experiences should also be evaluated to better understand a holistic view of students' online mathematics course experiences and how their experience impacts the broad spectrum of their mathematics learning endeavors. To gain a deeper understanding of students' experiences in their online and face-to-face courses, interviews and round table discussions would be recommended. A snapshot of student work is valuable to examine, but elements of students' voices to further explain their solutions would provide a much deeper level of insight into student content mastery and utilization of problem solving strategies. Having a face-to-face conversation with students to review their systems of equations and inequalities or integration assessment and talking through the decisions represented by their work would be a valuable means of adding depth to this study.

In addition to the techniques listed to enhance similar studies, inquisitions raised while conducting and reviewing data for this study foretell multiple areas for potential research. Since the online and face-to-face courses performed similarly, it could be argued the teacher did not substantially impact student learning in the face-to-face course. This argument raises the questions;

1. What unique attributes should an online educator possess to enhance online student learning experiences?
2. What aspects of a face-to-face classroom setting should be accentuated to maximize face-to-face student learning?
3. From a pedagogical perspective, how should online and face-to-face course attributes differ?

From an instructional design perspective, the online courses analyzed in this study sought to replicate the face-to-face course experience relative to resources, expectations and course sequencing. This course structure was utilized intentionally under the instructor's assumption that unequal resources would provide unequal learning opportunities. This raises questions related to,

1. With the differences present between the online and face-to-face mediums, would varying resources be beneficial?
2. What unique resources would online pre-calculus and calculus students desire to enhance their online learning experience?

Similar to the questions raised regarding course structure, this study raised questions regarding online and face-to-face pre-calculus and calculus student learning needs. In what ways do online pre-calculus and calculus student learning needs differ from their face-to-face peers? To analyze this question, it would be beneficial to utilize case studies highlighting student experiences and student voice while gaining an understanding of the unique learning needs of online pre-calculus and calculus students.

High attrition is an area of concern for online mathematics students (Smith & Ferguson, 2005). Future research should be conducted to study the actions of students who withdraw after partially completing an online mathematics courses. Do the students register for the same course online during a different semester, register for a different level mathematics course online, register for the same course face-to-face, or register for a different level mathematics course face-to-face?

The data gathered from these suggested future research endeavors could be used to expand the knowledge base regarding online mathematics course structure, design and implementation. This data could also be used to train perspective online mathematics educators and to expand the practices of educators currently teaching online mathematics courses. Additionally, this data could be used to look deeply into case studies of students who used specific problem solving techniques and further evaluate connections between problem solving techniques and course delivery modalities.

Limitations

The major limitations present in this study include a small sample size, using two content areas, and following a defined problem solving framework. A small sample size provided detailed evaluation of student work from three perspectives, statistically, methodologically, and analytically. While a larger sample size would provide more data, a larger sample size might also impact the depth of analysis that could be conducted. Assuring the same instructor taught both the online and face-to-face sections limited the available sample of students, but ensured less variation within confounding variables.

Selecting two content areas, pre-calculus and calculus, provided focus but also limited breadth. Further exploration into different pre-calculus and calculus concepts as well as into additional Mathematics content courses would provide greater insight into problem solving strategies used by students. Expanding the realm of content covered would also restrict the depth to which each element of student work could be reviewed. Additional opportunities for continued research through expanded courses and content selections abound and provide exciting research initiatives to tackle.

The problem solving framework used provides a detailed view of analyzing student work. While providing detailed structure for analysis, this framework also limits alternative foci from being applicable. The utilized problem solving framework does not account for presence of student voice or longevity or impact. Understanding from students why the selected different techniques or hearing their justification of their provided work is not included in the problem solving framework. Additionally, following students to subsequent courses and looking at the longevity of their content knowledge gained through the online or face-to-face course is not a component of the utilized problem solving framework. The utilized problem solving framework does focus attention to student's ability to convey understanding of the posed problem and to carry out a problem solving strategy. Detailed analysis of each portion of the problem solving process provides great insight into student understanding of the selected topics and allows for comparisons to be made between the online and face-to-face sections.

Conclusion

This study evaluated pre-calculus and calculus students' written work to explore difference that may exist in problem solving and course achievement. Chapter 1 contained an introduction the research problem and a review of the framework used to encapsulate this study. Chapter 2 included a detailed examination of literature relative to online education, technology in the mathematics classroom, problem solving, systems of equations and integration. After reviewing the existing literature, chapter 3 then moved to a discussion of the methods used to collect, review, and interpret the data for this study. Chapter 4 showcased the results of this study and included examples of student work to support the results. The final chapter includes a discussion of the findings, limitations of this study as well as areas for continued research.

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APPENDIX A

Human Subjects Review

OLD DOMINION UNIVERSITY

OFFICE OF THE VICE PRESIDENT FOR RESEARCH

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DATE:	May 29, 2015
TO:	Yating Liu
FROM:	Old Dominion University Education Human Subjects Review Committee
PROJECT TITLE:	[758736-1] Comparing Online and Face-to-Face Mathematics Instruction
REFERENCE #:	
SUBMISSION TYPE:	New Project
ACTION:	DETERMINATION OF EXEMPT STATUS
DECISION DATE:	May 29, 2015
REVIEW CATEGORY:	Exemption category # *6.1 & 6.2*

Thank you for your submission of New Project materials for this project. The Old Dominion University Education Human Subjects Review Committee has determined this project is EXEMPT FROM IRB REVIEW according to federal regulations.

We will retain a copy of this correspondence within our records.

If you have any questions, please contact Ed Gomez at 757-683-6309 or egomez@odu.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within Old Dominion University Education Human Subjects Review Committee's records.

VITA

Sarah Ferguson

Education

Doctor of Philosophy: Old Dominion University, Norfolk, VA — May 2017
Curriculum and Instruction
Focus on Mathematics Education and Online Learning

Masters of Science: Towson University, Towson, MD — December 2008
Mathematics Education

Bachelor of Science: Shippensburg University, Shippensburg, PA — May 2005
Mathematics
Summa Cum Laude

Certifications/Credits

Secondary Mathematics Teaching Certification
Pennsylvania, Virginia, Utah, and Washington

21+ Graduate Level Mathematics Credits

Professional Collegiate Experience

Old Dominion University, Norfolk, VA — August 2016 - Present

STEM Department: MonarchTeach Master Teacher
Teaching Assignments

- STEM 101: Step 1: Inquiry Approaches to Teaching STEM
- STEM 401: Project-Based Instruction

Tidewater Community College, Virginia Beach, VA — Fall 2014 and Fall 2016

Math Adjunct Professor
Teaching Assignments

- MTH 121: Fundamentals of Mathematics 1
- MTH 152: Thinking Mathematically

Adjunct Faculty Olympic College, Bremerton, WA — September 2010 – April 2013

Math Adjunct Professor
Teaching assignments

- MTH 90B: Pre-Algebra
- MTH 94: Algebra 1

Navy Shipyard Apprenticeship Program

- MTH 100: Practical Math Emphasizing Concept Applicability and Hands-On Applications

Professional K-12 Teaching Experience

Math Content Manager: K12, Cyber School August 2010 – August 2016

- Conduct observations of online classrooms
- Conduct Professional Development Sessions
- Monitor Mathematics Curriculum and implement necessary changes
- Update Curricula
- Design math courses to align with client desires

Synchronous Instruction Manager: K12, Cyber School August 2010 – October 2016

- Develop Synchronous Instruction Program
- Organize Synchronous sessions
- Observe Synchronous teacher
- Review and Approve Synchronous Session Curricula

High School Math Teacher, Dallastown Area School District, Dallastown, PA August 2005 – July 2010

- Teaching assignments: AP Calculus AB, Pre-Calculus, Geometry, Algebra 1, Collaborative Algebra 1
- Non-teaching assignments
    - Classrooms For the Future Grant
    - Algebra 1 level 1, Geometry and Pre-Calculus Curriculum Writing
    - Facilitate Mathematics Remediation Lab
    - Math Tutoring

Publications

Ferguson, S. (2016). An AP Calculus Classroom Amusement Park. *Mathematics Teacher*, *109*(7), 514-519.

Ferguson, S, Enderson, M. and Liu, Y. (in press). Student understanding of system of equation and inequalities: A comparison between online and face-to-face learning. *International Journal of Research in Education and Science.*

Ferguson, S. (in press). Constructing the Unit Circle. *Mathematics in School*.

Technical Reports

Damrose-Mahlmann, C., **Ferguson, S.**, McDowell, K.A., Necessary, J. & Pribesh, S. (December 2013). *Reflections on interprofessional education and practice.* Norfolk, VA. Old Dominion University.

Conference Presentations

2012 iSTEM conference Huntsville, AL

- Presentation on using virtual course to enhance student learning.