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Dynamic Elasto-Plastic Behavior of Steel Building Sub-Assemblage Including CFRP Retrofitting Under Impact Load

Al Mohammed Salih Al-Aloosi

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DYNAMIC ELASTO-PLASTIC BEHAVIOR OF STEEL BUILDING
SUB-ASSEMBLAGE INCLUDING CFRP RETROFITTING UNDER IMPACT LOAD

by

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ABSTRACT

DYNAMIC ELASTO-PLASTIC BEHAVIOR OF STEEL BUILDING SUB-ASSEMBLAGE INCLUDING CFRP RETROFITTING UNDER IMPACT LOAD

Ali Mohammed Salih Al-Aloosi
Old Dominion University, 2016
Director: Dr. Zia Razzaq

This dissertation presents the outcome of a theoretical and experimental study of the dynamic as well as the quasi-static elasto-plastic behavior of a steel building sub-assemblage including carbon fiber reinforced polymer (CFRP) retrofitting. The steel sub-assemblage consists of a beam-column attached to a pair of beams at its top end while its bottom end is fixed. An apparatus is constructed and used for conducting a series of experiments by applying a lateral impact load on the beam-column in the presence or absence of a static axial load. An innovative procedure for determining the shape of the forcing function caused by the impact load is then developed. A mathematical prediction model based on a partial differential equation of flexural dynamic equilibrium is formulated including new nonlinear terms to account for the elasto-plastic behavior of both a steel cantilever as well as a steel building sub-assemblage. To solve the materially nonlinear partial differential equation of equilibrium, an iterative finite-difference solution algorithm is formulated and is coupled with a tangent stiffness scheme to enforce cross-sectional axial force and flexural equilibrium conditions. The experimental results are found to be in good agreement with the predicted behavior for both non-retrofitted and CFRP-retrofitted steel building sub-assemblages. It is found that the maximum impact load to develop a dynamic plastic hinge in the beam-column of the sub-assemblage decreases in the presence of an axial load for non-retrofitted sub-assemblage. Also, for the CFRP-retrofitted sub-assemblage, the increase in the maximum impact load capacity is less than that found for the case of quasi-static loading.
This research is dedicated to my parents

For their endless love, support, and encouragement
ACKNOWLEDGMENTS

Sincere appreciation goes to **Dr. Zia Razzaq** for his advice, comments and criticism. His patience, motivation, guidance, and inspiration have provided me with infinite motivation throughout the work and over the years. He has set an example of excellence as a researcher, mentor, instructor, and role model. I could not have imagined having a better advisor and mentor for my Ph.D. study.

A special thanks also goes to my committee members Dr. Duc T. Nguyen, Dr. Julie Z. Hao and Dr. Mojtaba B. Sirjani for their willingness to serve on my doctoral committee and provide valuable feedback.

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I can never thank my parents enough for shaping me into the person I am today. I owe so much to the effort and support they have provided me, ever since I came to this life up until this hour. I wish I could pay back even a very small portion of their kindness.

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NOMENCLATURE

$A_e$ Elastic area of the cross-section

$a_i$ Impactor acceleration

$a_b$ Member acceleration

$B_e$ Rate of change of the moment over the rate of change of the curvature

$c$ Damping coefficient

$D$ width of the specimen cross-section

$dA$ Element Area

$dt$ Time increment

$E$ Modulus of Elasticity for Steel

$E_T$ Tangent modulus

$\varepsilon$ Normal strain

$\varepsilon_o$ Axial strain

$\dot{\varepsilon}$ Strain rate

$F(t)$ Forcing function

$g$ Gravitational acceleration

$h$ Longitudinal panel length

$h_i$ Height of impactor

$h_e$ Effective height from the CG of impactor to top face of specimen

$h_c$ Clear height from bottom face of impactor to top face of specimen

$I$ Moment of inertia

$k_T$ and $k_B$ Stiffness of the rotational springs

$K_{spr}$ Stiffness of the linear spring at end T.

$L$ Length of long member

$L_s$ Length of short member

$L_c$ Length of cantilever

$M_{xp}$ Summation of the bending moment from the plastic elements.

$M_x$ Applied moment about x-axis

$\dot{M}_x$ Change in applied moment

$m$ Mass per length of the member
N  Number of longitudinal elements
P  Axial Load
P_p  Axial force from the plastic elements
\dot{P}  Change in applied axial load
R_T  Reaction from the transitional spring at end T
S_{x_e}  First moment of elastic area
t  Time
v  Deflection due to applied load
v_T  Deflection at end T.
W  Applied static load
y  Distance of element from centroidal axis
\omega  Deflection at the second time increment
\sigma  Normal Stress
\sigma_y  Yield stress of Steel
\sigma_e  Element stress
\dot{\sigma}  Stress rate
\phi_x  Curvature about the x axis
\dot{\phi}_x  Curvature rate
\theta_T and \theta_B  End rotations of the sub-assemblage
\int_A  Cross-sectional integration
\{f\}  Cross-sectional load vector
\{\delta\}  Cross-sectional deformation vector
\{\dot{f}\}  Cross-sectional load rate vector
\{\dot{\delta}\}  Cross-sectional deformation rate vector
[K]  Member global stiffness matrix
[N]  Coefficient matrix of the impact analysis
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CHAPTER 1
INTRODUCTION

1.1 Preliminary Remarks

The main structural framing of a typical steel building consists of a series of interconnected columns and beams. When a column is subjected to a horizontal impact load, it is expected to be primarily resisted by the column itself and by the attached beams. A basic building structural sub-assemblage, under such conditions, consists of a column with beams framing in at the boundaries. The primary focus of this dissertation is to study the behavior of such a structural sub-assemblage when the column is subjected to a concentrated lateral impact bending load.

The 9/11 events and other types of terrorist attacks on buildings have created a real need for dealing with shock or impact loads on building structures. The so-called UFC or Uniform Facilities Criteria [24] requires that each government building be made safe against progressive collapse due to a single explosion or impact loading. Studies have been conducted on this issue using simple elastic dynamic analysis. Rigorous theoretical prediction models verified experimentally to account for materially nonlinear or elasto-plastic behavior of such structural sub-assemblages are missing, though, and need to be developed.

The steel building sub-assemblage studied herein comprises a beam-column attached to a pair of beams at its top end, while its bottom end is fixed. The beam-column is then subjected to an impact load and its inelastic or elasto-plastic response and formation of plastic hinges are studied. The behavior of the sub-assemblage is also studied under a gradually increasing quasi-static bending load in the presence or absence of an axial load. In real life structures, dead and live loads also act axially on the beam-columns.

In order to reduce the structural damage under excessive loads and to increase the structure’s load-carrying capacity, some retrofitting methods have been used in the past. The present study explores an innovative carbon fiber reinforced polymer (CFRP) retrofitting scheme for strengthening the sub-assemblage.

The flexural impact load is applied by means of a solid steel cylinder-shaped impactor dropped from different heights above the sub-assemblage. The different weights and drop heights of the impactors result in different forcing functions. The forcing functions are used in the analysis
presented in this dissertation and a comparison is made of the predicted and observed behavior of the structural sub-assemblage.

The study presents an experimentally verified and materially nonlinear mathematical behavior prediction model for a steel building sub-assemblage under impact loading.

1.2 Literature Review

A number of studies on isolated beams and frames experiencing the sudden removal of a column can be found in the literature, as outlined herein.

Kima and Kim [1] investigated the capacity of steel moment resisting frames using alternate path methods recommended in the GSA and DOD guidelines. The results from linear static and nonlinear dynamic analyses were compared. The comparison between linear static and nonlinear dynamic analyses showed that even though the maximum vertical deflections estimated by linear analysis were smaller than those obtained by nonlinear dynamic analysis, the linear procedure provided more conservative results for progressive collapse potential. Also, the dynamic analysis results varied more significantly depending on the variables such as applied load, location of column removal, or number of building stories.

A field experiment and numerical simulations were performed by Song and Sezen [2] to investigate the progressive collapse potential of an existing four-story steel moment frame building. In this study, an existing steel frame building was tested by physically removing four first-story columns. The building was also modeled and analyzed following the requirements of the current progressive collapse evaluation and design guidelines. According to their results, the strain values calculated from the nonlinear dynamic analysis were smaller than those from the linear static analysis and were closer to the measured strains, and a linear static analysis showed higher Demand-to-Capacity ratio (DCR) values and vertical displacements than nonlinear dynamic analysis for both 2-D and 3-D models.

Khandelwal and El-Tawil [3] investigated the behavior of seismically designed moment resisting frames that have suffered loss of a critical member as a result of an extreme loading scenario. In these systems, lateral load resistance is concentrated in a few perimeter frames, while the remainder of the gravity bearing structural system derives its stability by ‘leaning’ onto the perimeter frames. The main conclusion from this simulation is that loss of an interior gravity column will lead to the collapse of the entire system. The lost column initially precipitated mostly
in plane motion of the gravity system, which started pulling on the perimeter system, deforming it out of plane. The conclusion was that most of the building weight is supported by a gravity system that leans on a few perimeter frames that have limited out-of-plane strength and stiffness.

Determining dynamic forces on structures is of a great importance when designing a building. There are many forms of dynamic forces that act on buildings such as seismic, bombs, impact, etc. Ozturk and Catal [4] studied the dynamic response of semi-rigid frames using the stiffness matrix with a computer program. The semi-rigid frame was modeled by rotational springs. The response characteristics of five different multistory frames are compared with reference to their modal attributes. The study indicates that the connection flexibility influences the dynamic characteristics of frames. Also, the connection flexibility tends to increase periods, especially in lower modes, while it tends to decrease the frequency. Therefore, it was concluded that the dynamic behavior of a semi-rigid frame is different from the dynamic behavior of a rigidly connected frame.

Chen et al. [5] developed two finite-element models with and without concrete slabs. Also, a full-scale two-story steel moment frame was built to study the progressive collapse resistance after the sudden removal of a perimeter column in the first floor. The experimental results showed that the concrete slabs had a great influence on the progressive collapse resistance of the test steel frame. After the removal of the column, the partial loads previously carried by the removed column were transferred to its adjacent columns by the slabs.

Kim and Park [6] checked the applicability of the plastic design method to estimate the size of girders required to prevent the progressive collapse of steel special moment resisting frames. The results showed that the increase of only the girder size may result in weak story when the structure is subjected to seismic load. The formation of weak story can be prevented by increasing the column size in such a way that the strong column-weak beam requirement is satisfied. The nonlinear dynamic analyses results showed that the structures that were designed without considering progressive collapse did not satisfy the failure criterion required by the General Service Administration (GSA) guidelines; on the other hand, the structures redesigned by the plastic design method to prevent progressive collapse turned out to satisfy the given failure criterion in most of the model structures.

Hadi et al. [7] proposed a retrofitting scheme that is based on the concept of increasing the redundancy of a building, to bridge over the potential failed columns. To achieve this goal, a hat-
braced steel frame is placed on the top of the building, and vertical steel cables are placed parallel to the columns to provide an alternate path over the potential failed column. In the case of a column collapse, the cables transfer the residual loads above the failed column to the hat braced frame which, in turn, redistributes these loads to the adjacent columns. The results showed that the building example setup with the proposed retrofitting scheme successfully absorbed the different column failure scenarios without spreading the failure. It can be concluded from the numerical results that the proposed scheme is efficient in resisting the potential progressive collapse of the sample building used in this study in the event of a first floor column failure.

Hamburger and Whittaker [8] studied the design of steel structures for blast related progressive collapse resistance. Moment-resisting steel frames are ideally suited to the provision of this continuity and in avoiding progressive collapse. A structure with a regular 30-foot grid pattern was reviewed. The beams and girders along the column lines were assumed to be provided with moment-resistance. An evaluation of the structure for its ability to resist instantaneous removal of a single interior column was performed using the federal progressive collapse guidelines. It was determined that collapse resistance, in steel moment frame structures, can be provided without weight increase. Also, there is a significant cost premium associated with the provision of moment connections between every beam, girder and column. It was determined that by using W36x300 sections as the beams and girders at one floor level, it would be possible to provide progressive collapse protection for as many as 15 supported stories.

Szyndlewski and Krauthammer [9] provided the methodology for an energy-based progressive collapse assessment of multistory buildings. The progressive collapse analysis, based on an energy flow perspective, of steel-framed building was performed, and compared with conventional force and deformation approaches. This study discovered that a building can arrest the collapse, and achieve its stable configuration only if the kinetic energy is completely dissipated by the structure. Otherwise, the remaining kinetic energy will cause the collapse to continue. In the arrested collapse, columns contributed very little to the dissipation of the released gravity work. On the other hand, the observations of the engineering community confirmed that beams and connections play a significant role in arresting collapse propagation.

Razzaq et al. [10] conducted a theoretical and experimental study of slender tubular columns with partial rotational end restraints in the presence of initial imperfections. New explicit formulas and finite-difference formulation were derived for predicting the elastic buckling load and
predicting the natural frequency. The results showed good agreements between the methods and the exact analysis.

Pathak and Alemdar [11] followed a two-step procedure to apply the Virtual Work Method for the analysis of Steel Buildings. An arbitrary column at the base of the structure is removed to initiate the collapse. The first step addresses internal strain energy stored in the system under gravity loads only. The second step continues from the previous step and it involves an application of a virtual load at the location of the column removed. The proposed method is an attempt to quantify each member contribution to a collapse initiated upon a member removal. It was observed that connections play a significant role in the overall response, since the major contribution to vertical displacement is obtained from connection yielding. The examples presented show how various member contributions change when loading is increased, due to shifting from linear to nonlinear response of the system.

Different Retrofitting schemes of structures were suggested by many researchers to mitigate the effect of dynamic loads. Galal and El-Sawy [12] investigated the effect of three retrofitting strategies on enhancing the response of existing steel moment resisting frames designed for gravity loads. The studied steel frames were damaged by being subjected to six scenarios of sudden removal of one column in the ground floor. The study was based on the idea that progressive failure in steel buildings occurs due to insufficient strength in the beams that are needed to carry over the load from the removed column location to the adjacent columns. The response of the damaged frames was evaluated when retrofitted with Fibre-Reinforced Polymer (FRP) composites. It was found that the level of tie force exerted in the beams of the existing building calculated from nonlinear dynamic analysis using ELS software was more than three times of the limits stated by the DoD guideline for all studied buildings. It was also found that, for all studied buildings, chord rotation, tie force, and displacement ductility demand in cases of loss of Internal and Edge Long Column scenarios were more than those arising from the cases of First Internal and First Edge Long Column removal scenarios.

Tsai [13] proposed a performance-based design approach for retrofitting regular building frames with steel braces against sudden column loss. The added braces are applied to the structural bays adjacent to the failed column. The proposed retrofit design approach is intended for the building frames that fail to pass the progressive collapse evaluation per GSA or DoD guidelines.
Nonlinear dynamic analysis results indicate that the column-loss response of the braced frames is approximated to the performance target and thus the proposed retrofit design approach is feasible for practical applications.

It was found that the mechanism of a hinge formation for the structural frame may be altered due to the added diagonal tensile braces, especially for larger increase ratios. Without the added braces, the failure mechanism under column loss is governed by the formation of beam-end plastic hinges at the beam-column faces. Since the proposed approach is applied to multi-story building frames, conservative design results are obtained. From the response of three frame models retrofitted with varied strength parameters, it was realized that the extent of the conservativeness increases with the increment of collapse resistance.

Adaros and Tanali [14] proposed a solution where the existing columns are used as hangers and are connected to a new transfer girder at the roof level. The columns that behave as hangers will be subjected to tension. The new girder is designed to span two bays to the nearest columns, transferring the load back to the structure and into the foundation. Although the ACI 318 code does not allow columns to act as hangers, it was found that the distribution of forces was very sensitive to the tensile capacity of the concrete. It was concluded that it is conservative to design the cables ignoring the presence of the column and its reinforcement but it is not conservative to neglect them for the analysis of load distribution through the height of the building. A brittle shear failure can occur if a connection is not provided at the roof and lower level where high forces must get transferred from the column to the cables.

In their study, Sideri et al. [15] presents the progressive collapse analysis of a steel moment frame, with long spans undergoing a beam-type yielding collapse, and the difference of the response throughout the height of the building. The progressive collapse analyses included two separate cases: the removal of the corner column of the ground floor and the removal of the column at the middle of the ground floor. The analysis results have shown that the beams which belong to floors close to the removal behave as a single span beam, with a length equal to twice the column to column span while the beams which are close to the top of the building behave as continuous beams experiencing negative moments at their supports.

Connections play a critical role in structures. Tan and Yang [16] pointed out the shortcomings of the current design approaches. The study provides the behavior and failure modes
of different types of connections, including their resistances and rotational capacities in catenary action.

Some practical design implications have been drawn up from the experimental tests and from the parametric study. A new tying resistance expression is proposed to consider the effect of large rotation of connections. In addition, four new connection acceptance criteria of rotational capacities have been proposed to incorporate catenary action under a middle column removal scenario.

Türker et al. [17] presented an investigation into the determination of the quality of the semi-rigid connections when considering changes in the dynamic characteristics of steel structures. However, the accepted notion is that the connections of members of a steel structure that exhibit semi-rigid characteristics cause changes in the dynamic characteristics of the structures. Experimental measurements on the models were performed and compared with data for theoretical fully rigid and elastic rotational springs modal analyses. An approach was improved, depending on the rotational spring stiffness to determine the connection percentages of both support and beam-to-column connections.

Yu et al. [18] provided the details of three test specimens and described the test observations of the sway frames under the combined actions of gravity and lateral loads. It was discovered that full-strength end-plate connections may resist moments larger than the capacity of the beam. It is, therefore, necessary to stiffen the column web in order to improve the resistance of compressive forces transferring from the beam to the column. Otherwise, the maximum compressive resistance would be limited to a maximum value given by buckling and bearing of the column web which will limit the rotation capacity of the connections. It was also found that the analysis gives reasonably good prediction of the response behavior of sway frames, provided that the “real” behavior of the connections is carefully modeled. The use of partial strength connections leads to a well-defined failure mode, since the connection strength is weaker than the adjoining members. The column base fixity effect is an important consideration factor for the design of a multi-story frame.

Yang and Tan [19] presented numerical results for six tests on different beam–column connections. The beam-column joint considered for these tests is located above the story with an internal perimeter column removed. Both static and explicit dynamic solvers were employed to conduct numerical simulations. The simulation results demonstrate that under a middle column
removal scenario, increasing the number of bolt rows increases the load-carrying capacity and rotational stiffness. However, it was also observed that connection ductility is adversely affected by the increase of bolt rows, which means this effect is not always beneficial. In addition, the work shows that current acceptance criteria for the rotation capacities of steel joints are probably too conservative since they only consider pure flexural resistance.

Lim et al. [20] performed a study on 2D frames including second-order and material nonlinearity effects with various connection stiffnesses and different combinations of spans and stories. It was observed that the collapse modes of 2D frame structures depended on various factors, such as the geometries, material properties, loads, and especially the connection’s rigidity. Rigid frames were assumed in many previous studies for simplicity. This study showed that rigid frame structures can survive better with more bays and fewer stories. However, the analyses of the semi-rigid frames showed different results. Semi-rigid connections can act as circuit breakers. They can also cause total destruction of the failed span. Therefore, it is possible to prevent the whole frame collapse with an ordinary semi-rigid joint design and to sacrifice only the bay with the failed span.

A structure is likely to be subjected to various types of hazards during its life time. These hazards can be subdivided into two general categories: man-made (blast) and natural (earthquakes, wind, etc.). For a successful approach to any system design, it is essential to understand the nature of the hazard. Varghese and Ajith [21] investigated finite element tools for the simulation of blast load effects on a bridge, and the performance of a bridge under various blast load generation methods. They also discussed other factors of blast loads, such as blast loads characteristics, standoff distance, height of burst, and material behavior under high strain rate.

Asprone et al. [22] investigated the behavior of reinforcing steel under dynamic loading rates. Tensile failure tests were performed on steel specimens at different strain rates using a modified Hopkinson bar device. The steel was from a reinforced concrete arch bridge. The results were compared with the existing formulations, providing the dynamic properties of reinforcing steel. The conclusions to be drawn is that the reinforcing steel was found to be strain-rate sensitive in terms of yield stress, ultimate stress, and ultimate strain. Also, as the strain rate increases, yield stress increases more than ultimate stress. Indeed, yield stress assumes a maximum Dynamic Increase Factor (DIF) value of 1.62, while ultimate stress reaches a DIF of 1.17.
The US Department of Defense provided guidelines, called UFC [25] or Unified Facilities Criteria, for the design of buildings to resist progressive collapse in new and existing structures. UFC states that buildings should be designed to withstand local damages and to limit their effects within the structural system. The current UFC provides varied levels of resistance to progressive collapse. These levels include: Tie forces (TF), specifying minimum tensile forces that must be used to enhance the continuity and ductility of the structure. There are three horizontal ties that must be provided: longitudinal, transverse, and peripheral. If a vertical structural member cannot provide the required vertical tie force strength, either the member is re-designed or the Alternate Path (AP) method is used to prove that the structure can bridge over the element when it is removed. The AP method cannot be used as an alternative for inadequate horizontal ties. In general, the procedure divides the floor plan into sub-areas. Each sub-area has its own longitudinal and transverse ties and peripheral ties; the (AP method) requires that the structure be capable of bridging over a missing structural element. The detailed procedures and general requirements for the different methods are provided as separate sections in UFC.

UFC provides the details for the three approaches which are different for different material types. UFC is considered the main reference for the design against progressive collapse. It provides qualified solutions that were proven to be adequate and serve the purpose. The practicality and easiness of application distinguish the UFC from other guidelines and make it more acceptable by the public. Researchers should not stop at these guidelines, however, more experiments and analyses should be conducted to arrive at new methods or to enhance existing ones.

Galambos [27] explained the stub column test procedure to obtain an accurate stress-strain relationship for the cross section. The stub column test also helps in estimating cross-sectional residual stress distribution.

Ballio and Campanini [28] suggested a residual stress distribution for rectangular tubular members. They concluded that it is difficult to establish better methods for design than those adopted by various codes.

Santathadaporn and Chen [29] described a method for computing moment-thrust curvature relationships for metal column sections under biaxial bending. They expressed the problem as a system of simultaneous linear equations by considering the rate of change of material properties. They used the tangent stiffness matrix to predict the incremental deformations and to estimate the correction vector for the unbalanced forces.
Alvesa and Jones [30] presented a simple theoretical method to predict the failure of beams made from a perfectly plastic material and subjected to impact loads. They showed that the strains can be estimated by defining a hinge length. By comparing the results with numerical calculations and experimental data, the definition led to reasonable predictions for the plastic strains and the strain rate. They developed new definitions for estimating the bending, membrane, and shear hinge lengths in the beam theory. These results were in reasonable agreement with the corresponding experimental results and the numerical data for aluminum and mild steel beams.

Jones [31] studied the behavior of fully clamped beams when struck at the mid-span by a rigid mass and compared it with the corresponding exact theoretical predictions of dynamic rigid-plastic analyses. Also, comparisons and observations were made between the quasi-static theoretical predictions and experimental results. He discovered that quasi-static analyses gave a good agreement with experimental results in which the striker masses were much larger than the corresponding structural mass.

Shen and Jones [32] developed dynamic plastic analysis of a grillage to predict its permanent displacements when struck by a mass. They showed that the dynamic plastic response of the grillage progresses through five phases of motion in a certain sequence. These phases relate to the behavior of plastic hinges including their development, movement and disappearance.

A series of experimental tests on clamped metal beams struck by a mass at various impact points was reported by Liu and Jones [33]. Two types of beam failures were observed and classified as tensile tearing and shear failure. Good agreement was found between the permanent transverse deformations of the beams and the theoretical rigid-plastic predictions. It was observed that the energy absorbing capability of the beams decreased sharply when a beam was struck close to a support.

Nassr et al. [34] used a Single-Degree-of-Freedom (SDOF) model to determine the effect of axial load on column strength and stability during a blast event. The SDOF model was validated with the results of the finite element software LS-DYNA. Theoretical models were compared with experimental data from blast tests on full scale steel columns. In the SDOF model, the real system was replaced by an equivalent spring-mass system. The FEM software LS-DYNA was used in this study to analyze steel columns and to compare its results with those obtained from the simplified SDOF model. The accuracy of the nominal static yield strength of steel and the resistance function used in the SDOF model were investigated by conducting static tests on six wide-flange steel
sections, and the following conclusions were stated. First, the P-δ effect in the steel columns bending in single curvature due to blast load could be modeled accurately using the SDOF model. Second, it was determined that the use of beam column interaction formulas commonly used in design for static loads may overestimate the actual column capacity under blast load.

Al-Thairy and Wang [35] presents the development of a simplified analytical method to predict the critical velocity of transverse impact by a rigid body on a steel column under axial load. This method is based on energy balance with a quasi-static approximation of the column behavior. The main emphasis of their analytical method was to obtain the column axial load critical impact velocity relationship. The critical impact velocity is defined as the minimum velocity of the impact body that causes the column to lose its stability. The accuracy of the proposed method was checked against a range of ABAQUS software simulation results. Excellent agreement could be seen between the two sets of results for all levels of axial load, for different impact locations.

Wen et al. [36] proposed a quasi-static procedure estimating the dynamic plastic response and failure of clamped metal beams subjected to a low velocity impact at any point on the span by a heavy mass. It has been shown that a good agreement is obtained between the theoretical predictions and the experimental observations in terms of the maximum permanent transverse displacements and the failure modes given by the failure maps. Based on the principle of virtual work, load-displacement relationships have been obtained and used to derive equations which represent the transitional curve between transverse shear failure and tensile tearing failure of a beam by assuming that axial tensile and shear strains are independent parameters. Wen et al. also determined that the beam broke in a shear failure mode when its critical shear strength was reached and that it would fail in a tensile tearing mode when the rupture strain of its material was attained.

The paper by Zeinoddini et al. [37] described experimental studies in which axially pre-loaded tubes were examined under lateral dynamic impact loads. The tubes were impacted by a dropped object with a velocity of about 7 m/s at their mid-span. The axial pre-loading varied within a range of 0%, 25%, 50%, 60%, 65%, 70%, and 75% of the specimen squash load (Py) while the striker mass and the drop height were kept constant. The experimental tests carried out and reported have shown that pre-loading has a substantial effect on the level of damage in tubes subjected to lateral impacts.
Leissa et al. [38] also studied, numerically, the behavior of axially pre-loaded steel tubes subjected to dynamic lateral impacts. A non-linear finite element implicit time domain dynamic approach, using ABAQUS software, was used for the simulation for those impact experiments. The numerical models employed were found to efficiently simulate the failure sequences in the tubes which were subjected to a dynamic instability.

Leissa and Sonalla [39] presented theoretical results for the vibration of undamped cantilever. The cantilever beam was deflected from rest and released in four different loading cases. For two of the examples, plots of the resulting periodic motions were made for various points taken along the beam. Displaced shapes of the beam at subsequent times were also shown. In the first example the initial deflection was caused by an end moment. In the second example, a concentrated end load was used. In the third example, a uniformly distributed load was causing the initial deflection. The last case, the initial cubic displacement function was studied and was shown to contribute strongly. Although most points along the beam executed “almost periodic” motion, the displacements of points near the fixed end were less regular.

Hulbert [40] presented a time finite element formulation of structural dynamics. The author employed a time-discontinuous Galerkin method and incorporated stabilizing terms having least-squares form on the second degree elastic dynamic equation.

Shen and Jones [41] suggested an energy criterion to predict the inelastic failure behavior modes of beams under dynamic loading. The numerical results gave some insight to the characteristics of the different failure modes. Failure modes were examined by the theoretical analysis of the second order equation of motion for the dynamic response of a clamped beam under impulsive loading. The analysis considered the simultaneous influence of the transverse shear and axial membrane forces and bending moment.

Lruf and Jones [42] presented theoretical rigid plastic analyses which examine the transverse shear and bending responses and the influence of finite deflections on the behavior of clamped beams struck transversely by a mass at any point on the span. The experimental tests were conducted later and comparisons with theoretical predictions were made as part of a wider study.

Dibold et al. [43] studied biaxial vibrations of elasto-plastic beams with a prescribed rigid-body motion. The theoretical analysis was based on the equation of motion derived using Hamilton’s principle. The authors used Legendre polynomials to discretize the biaxial deflections. The authors also discretized the plastic strains over length, height and width. Gelerkins procedure
was used for the integration of the equations of motion. An analogy was utilized to replace the thermal expansion strain by the plastic strain.

Based on the literature review conducted, and to the best of the author’s knowledge, the outcome of the investigation presented in this dissertation has not previously been published.

1.3. Problem Definition

Figure 1 shows the outline of a steel building frame with a typical sub-assemblage RS-B. Figure 2 shows the schematic of a sub-assemblage used in this research. The sub-assemblage consists of a beam-column connected to a pair of beams. The sub-assemblage is assumed to have semi-rigid supports. The axial load, P_{axial}, applied on the sub-assemblage represents the dead and live loads from the building floors. In order to verify the usage of the procedure used for the sub-assemblage, a more basic case of the cantilever was studied. A schematic of the cantilever is shown in Figure 3. The forcing function used on both, the sub-assemblage and the cantilever, was applied by a solid steel cylinder dropped vertically above the member.

The specific problems addressed in this dissertation are as follows:

1. Development of an apparatus for the cantilever under impact bending load and the entire sub-assemblage under axial static load combined with an impact bending load.
2. Establishment of an experimental procedure to define a forcing function, F(t), for use in theoretical prediction of the structures tested.
3. Conduct a detailed experimental study of the dynamic elasto-plastic behavior of the steel building sub-assemblage.
4. Formulating materially nonlinear partial differential equations and appropriate boundary conditions representing the dynamic response of the sub-assemblage.
5. Development of a finite-difference based nonlinear algorithm involving inelastic tangent stiffness of the structure to solve the governing partial differential equations and to predict experimental behavior.
6. The devising of a CFRP retrofitting scheme for the sub-assemblage and study its effectiveness, both experimentally and theoretically.

The theoretical prediction model should be applicable to sub-assemblages in a full-scale steel building.
1.4 Objectives and Scope

The principal objectives of this study are to:

1. Develop a mathematical prediction model based on a partial differential equation of inelastic dynamic equilibrium to account for the elasto-plastic behavior of both a steel cantilever as well as a steel building sub-assemblage.

2. Formulate a nonlinear finite-difference solution algorithm to capture the elasto-plastic behavior of a steel sub-assemblage under quasi-static and impact loading, with associated boundary and initial conditions.

3. Establish an experimental approach to define forcing functions generated by an impact load on a cantilever and a steel building sub-assemblage.

4. Build a new testing apparatus in the laboratory for conducting quasi-static and impact loading experiments in the presence or absence of a static axial load.

5. Investigate both quasi-static and dynamic plastic hinges development phenomena.

6. Explore the effectiveness of a carbon fiber-reinforced polymer (CFRP) retrofitting scheme for the sub-assemblage for increasing the stiffness and strength under both quasi-static and impact loading, in the presence or absence of an axial load.

In this study, Grade 60 steel tubes with cross-sectional dimension of 2x2x1/8 inch were used for all specimens. Steel building sub-assemblages with flexible end conditions were subjected to quasi-static and impact loading. As a retrofitting material, CFRP strips, commercially known as Aslan 500, were used.

1.5 Assumptions and Conditions

The following assumptions and conditions are adopted in this study:

1. Small deflection theory is applicable.

2. The Specimen material has an elastic perfectly-plastic relationship with the elastic unloading of the material.

3. The normal stress-strain relation of CFRP is elastic up to the point of material cracking.

4. The compression and tension stress-strain relationships for both steel and CFRP are the same.

5. The axial load on the beam-column portion of the sub-assemblage is applied quasi-statically and then held constant prior to the application of the lateral or flexural impact load.
6. Applied quasi-static and dynamic loads are considered as concentrated point loads.
7. Member shear deformation and axial shortening are negligible.
8. The Bernoulli-Navier plane sections hypothesis is applicable under both quasi-static and dynamic loading conditions.
CHAPTER 2

EXPERIMENTAL INVESTIGATION

This chapter presents the outcome of an experimental study of both a cantilever and a building sub-assemblage. Quasi-static load tests were conducted on steel sub-assemblies. Also, a number of impact load tests were conducted on steel cantilevers and sub-assemblies.

2.1 Material Properties and Specimens

Tension tests were conducted to determine the stress-strain curve for both steel and CFRP strips. This section presents the mechanical properties for each material. These properties were used as input data for the analysis presented in this dissertation.

2.1.1 Steel tensile tests

Steel tension tests were performed in accordance with ASTM E8-04 (30), *Standard test Methods for Tension Testing of Metallic Materials*, to determine the stress-strain curve. Four tension test samples, from the four sides of hollow rectangular section member were machined to the dimensions shown in Figure 4. All four curves were similar in behavior, as shown in Figures 5-8. The average yield stress $\sigma_Y$ and Young’s modulus $E$ of the four sides were found to be 62 ksi and 30,000 ksi, respectively.

2.1.2 CFRP strips

Although the manufacturer, Hughes Brothers Inc., had provided stress-strain relationships for the CFRP strips, samples were tested in the lab for comparison purposes. The data used in this report for the stress-strain diagram was selected according to ODU test results, which were found to be close to the manufacturers’ results. The cross-sectional dimensions of the CFRP strip were 0.68 X 0.079 in. The rupture occurred approximately at a stress of 255 ksi and the corresponding strain was 0.013 in./in. The modulus of elasticity was found to be 20,000 ksi. The stress strain relationship is shown in Figure 9.
2.1.3 Sikadur 30 epoxy paste adhesive

Sikadur 30 is a two-component, moisture-insensitive, high-modulus, high-strength, structural epoxy paste adhesive. The epoxy was used to bond the CFRP strips to the exterior face of the steel specimens. The manufacturer’s instructions were followed to prepare and apply the epoxy. It has the consistency of peanut butter and is well-suited for interior and exterior applications. Sikadur 30 requires a curing time of seven days at room temperature.

2.2 Test Specimens

2.2.1 Cantilever

Figure 10 shows the overall specimen dimensions and the X, Y, Z coordinates. In this figure, \( L_C \) represents the overall length of the cantilever, which is 33 inches, while Q and B represent the end sections of the cantilever. Figure 11 shows an elastic-plastic moment-rotation relationship for the rotational spring at B. For any rotation greater than or equal to \( \theta_{Bpc} \), the connection is considered fully plastic and supporting a moment of \( M_{Bpc} \). The slope of the elastic range of this curve is the rotational spring constant \( K_B \) of the connection at B.

Four tests were used in the experimental investigations for impact loading on the cantilever. Table 1 summarizes the designation, type of member and type of loading for impact tests.

The name of each test is a combination of letters and numbers. The letter C stands for cantilever. The number before the dash represents the specimen number, while the number after the dash refers to the test number on a specific specimen.

2.2.2 Sub-assemblages

Figure 12 shows the overall specimen dimensions and the X, Y, Z coordinates with T as the origin. In the figure, L and \( L_S \) represent the overall lengths of the long and short members, respectively. T, Q, and B represent sections along the length of the beam-column on the sub-assemblage. The elastic-plastic moment-rotation relationship for the rotational spring at B is the same as that used at end B for the cantilever. The rotational spring \( K_T \) at end T represents the torsional resistance of beam R-S as shown in Figures 13. Figures 14 and 15 show the elastic moment-rotation and the load-deflection relationships for the rotational and translational springs at end T, respectively. The rotational and translational springs at end T remained elastic during the
experiments. The slopes of the elastic range of these curves represent the rotational spring constant $K_T$ and the translational spring constant $K_{spr}$ at T. Four tests were performed in the experimental investigations for quasi-static loading. Table 2 summarizes the designation, type of member, type of loading, and the presence or absence of axial loading and retrofitting for quasi-static tests.

A total of twenty-four tests were performed in the experimental investigations for impact loading on the sub-assemblage. The first sub-assemblage was used to find the drop height that would cause a plastic hinge, plastic moment, on the specimen. Eight impact tests were performed on the first sub-assemblage. Table 3 summarizes the designation, type of member, type of loading for these tests. The other sub-assemblages were used to study the effect of CFRP retrofitting and axial loading on the development of plastic moment at Q and B. Table 4 summarizes the designation, type of member, type of loading, and presence or absence of axial loading and retrofitting for the rest of the impact tests.

2.2.2.1 Non-Retrofitted Sub-assemblages

All specimens were constructed by welding a long member, beam-column, to the middle of a shorter member, beam. The length of the beam-column L was 66 inches and the length for the beam $L_S$ was 30 inches. The name of each test is a combination of letters and numbers. The letters SA, in the test name, stands for sub-assemblage. The number before the dash represents the specimen number while the number after the dash refers to the test number on a specific specimen. Figure 16 shows a schematic for a non-retrofitted sub-assemblage and its cross-section. For some of the tests, a static axial loading applied to it at T.

2.2.2.2 CFRP Retrofitted Sub-assemblages

The CFRP retrofitted specimens were similar to the non-retrofitted specimens. In addition, two layers of CFRP strips were added to the bottom face of the long member, as shown in Figure 17. The retrofitting method used was to install four CFRP strips onto the bottom face of the long member of the sub-assemblage and to arrange them in two layers. Each layer was of two CFRP strips, 66 inch long, installed side by side and along the full length of the beam-column. The strips were glued to the steel tube using Sikadur 30 epoxy. In order to make sure that the CFRP strips did not detach from the specimen during the experiment and to avoid unexpected failure in the epoxy, clamps made up of flat and screw bars were placed around the tube and the CFRP as shown.
in Figure 18. These built-up rings were placed every 11 inches. For some of the CFRP-retrofitted specimens, an axial loading was applied to it at point T.

2.3 Quasi-static Test

Quasi-static Tests were conducted on four different specimens. The following sub-sections will discuss these tests.

2.3.1 Test Set-up

The test set-up is shown in Figures 19 and 20. As described above, the sub-assemblage consisted of two square tubes of unequal lengths welded together to form a T shape frame. At end B, the sub-assemblage was welded to a 7x7x0.75 inch base plate. The base plate was connected with 8 no. ½ inch bolts to a second plate that had the same dimensions as the first base plate. The second base plate was welded to a 7x4x0.5 inches channel. The channel was stiffened by a 0.25 inch stiffener in the middle. The stiffener connected the flanges of the channel from the inside. There was also a 12x7x1 inch plate welded to the web of the channel from the outside to increase its resistance against rotation during the experiment. The channel was connected to two big columns from both sides using 2 no. of 1 inch diameter bolts on each side.

Each of the other two ends of the short member, beam, of the sub-assemblage was connected to the sides of a built-up steel pedestals using four angles that were 2 inches long. The four angles were arranged around the four sides of the tube as shown in Figure 21. A bolt ran through the side angles and went through the tube. This was a flexible connection with rotational spring constants Kt. The steel pedestal was 1.5 ft. high and was made of I section, with one plate welded at each end. The two pedestals were welded, from the bottom, to an 8x8x1/2 inch square tube. The square tube was attached to two giant columns by two bolts at each end of it.

A vertical quasi-static loading W was applied using a hydraulic jack, ENERPAC, through solid steel plate. The load W applied a lateral force at Q on the beam-column of the sub-assemblage. The hydraulic jack was fixed on a heavy-duty steel frame. A manual hydraulic pump was used to control the applied load during the test. The applied force of the pump was measured using an electronic load cell.

Tests with axial static loading P_{axial} had another horizontal ENERPAC hydraulic jack applying an axial load on the long member of the sub-assemblage at T. Figure 22 shows the test
setup with the hydraulic jacks for lateral load $W$ and axial load $P_{axial}$. The electronic cell displayed the voltages that could be converted to the corresponding applied force using a mathematical formula.

Deflections, of the specimen, were measured using a dial gauge with a least count of 0.001 in. The dial gauge was placed at section Q on the beam-column. The readings were recorded at each load increment. The rotational stiffnesses for the rotational springs, $K_T$ and $K_B$, and the linear transitional spring, $K_{spr}$, were $1.5 \times 10^6$, $6 \times 10^6$ and 34000, respectively.

There were a total of six strain gauges mounted on each sub-assemblage. They were denoted as SG1, SG2, SG3, SG4, SG5, and SG6. The strain gauges were installed on the long member of the sub-assemblage and at two sections, Q and B. Figures 23 - 28 show the strain gauges and the dial gauge.

SG1, SG2, and SG3 were located at section B on the sub-assemblage. SG1 and SG2 were on the side face of the steel tube. SG1 and SG2 were installed at a distance 0.35 inch from the top and bottom fibers of the tube, respectively. SG3 was on the bottom face of the steel tube. SG1, SG2, and SG3 were on the same section along the sub-assemblage.

SG4, SG5, and SG6 are located at section Q on the sub-assemblage, at the middle length of the long member in the sub-assemblage. SG4 and SG5 were on the side face of the steel tube. SG4 and SG5 were installed at a distance 0.35 inch from the top and bottom fibers of the tube, respectively. SG6 was on the bottom face of the steel tube. SG4, SG5, and SG6 were located at section Q on the sub-assemblage.

The strain gauges were attached to a Vishay strain acquisition system to collect data from the strain gauges. Figure 29 shows the Vishay strain acquisition system.

### 2.3.2 Test Procedure

A quasi-static load was applied using the hydraulic jack at section Q, the middle length of the long member of the sub-assemblage. The quasi-static load was applied through a solid rectangular steel block. The force was increased gradually to cause bending of the beam-column of the sub-assemblage and the effect of loading lateral on the specimen was observed. The output from the load cell was recorded for each load level. The dial gauge at section Q, the middle of the long member, was used to measure the deflection at that section. Six strain gauges were used to measure the strains at both sections B and Q on the sub-assemblage for each load reading. The
loading was gradually increased until reaching the maximum load-carrying capacity of the sub-assemblage. Figure 30 shows a sub-assemblage during quasi-static test.

For the tests with axial loading $P_{\text{axial}}$, a static axial load of 15 kip was applied on the long member at T. During the test and while the lateral loading was increased, the axial load was monitored and kept at a constant value of 15 kip throughout the experiment.

2.3.3 Experimental Results

Load, deflection and strains data were collected using a load meter, a dial gauge and strain gages, respectively. The data was analyzed using M.S. EXCEL and MATLAB. Load-deflection curves were developed in order to find the maximum load capacity of the sub-assemblage. Moment-curvature graphs were established for all specimens to find the maximum moment at sections Q and B. The test results for each of the test specimens were observed and recorded.

Figure 31 shows the load deflection curve for specimen SA3. The maximum applied load and deflection were 4770 lb. and 2.665 inches, respectively. The maximum strain values for the test from SG1, SG2, SG3, SG4, and SG5 at points B and Q on the sub-assemblage were 0.008276, -0.00826, -0.01274, -0.00949 and 0.009278 in./in., respectively. The strain values at B and Q are shown in Figures 32 and 33, respectively. The strain values were used to find experimental moment-curvature relationship at B and Q, as shown in Figures 34 and 35, respectively. The maximum moment values at B and Q of the specimen were 40.18 kip-in and 40.2 kip-in, respectively. It was noticed that the full moment capacity, plastic moment, was reached at both sections B and Q.

Specimen SA5 was subjected to an axial static loading of 15 kip at T during the experiment. The maximum applied load and deflection were 3610 lb. and 1.96 inches, respectively. The load-deflection curve was generated and shown in Figure 36. It can be noticed that the load capacity of specimen SA5 was dropped by 25% compared to that from SA3. The maximum strain values for the test from SG1, SG2, SG3, SG4 and SG5 at sections B and Q on the sub-assemblage were 0.00453, -0.01127, -0.01395, -0.01617 and 0.006035 in./in., respectively. The strain values at sections B and Q are shown in Figures 37 and 38, respectively. The strain values were used to find the experimental moment-curvature relationship at B and Q, as shown in Figures 39 and 40, respectively. For this specimen, an axial static load of 15 kip was applied at the beginning of the test, causing the member to bend slightly. This initial bending, plus any pre-existing strains in the
strain gauges, caused the moment-curvature curve to have a sudden rise at the beginning of the graph. The maximum moment values at sections B and Q of the specimen were 35.8 kip-in and 35.8 kip-in, respectively. It was noticed that the full moment capacity, plastic moment, was reached at both sections B and Q. With the existence of the axial loading, the moment capacity of the cross section was dropped by 12% compared to that from specimen SA3.

In order to study the effect of CFRP retrofitting, two layers of CFRP strips were installed at the bottom face of the sub-assemblage. The load-deflection curve for specimen SA7 was generated, as is shown in Figure 41. The maximum lateral load and deflection values were 6000 lb. and 2.84 inches, respectively. CFRP retrofitting increased the load capacity by 26% compared to that for specimen SA3. The maximum strain values for the test from SG1, SG2, SG3, SG4, SG5 and SG6 at B and Q on the sub-assemblage were 0.015571, -0.00558, -0.00745, -0.01796, 0.004218 and 0.009266 in./in., respectively. The strain values at sections B and Q are shown in Figures 42 and 43, respectively. The strain values were used to find experimental moment-curvature relationship at B and Q as shown in Figures 44 and 45, respectively. The maximum moment values at sections B and Q of the specimen were 48.1 kip-in and 48.5 kip-in, respectively. It was noticed that the full moment capacity, plastic moment, was reached for both sections at sections B and Q. With CFRP retrofitting, the moment capacity of the specimen SA7 was increased by 19% compared to that from specimen SA3.

Specimen SA9 was subjected to axial static loading $P_{\text{axial}}$ and was retrofitted with two layers of CFRP strips at the bottom face of the sub-assemblage. The load-deflection curve for specimen SA9 was generated, as is shown in Figure 46. The maximum applied lateral load and deflection values were 5310 lb. and 2 inches, respectively. The load carrying capacity for specimen SA9 was 11% more than that for SA3, 47% more than that for SA5 and 13% less than that for specimen SA7. The maximum strain values for the test from SG1, SG2, SG3, SG4, SG5 and SG6 at sections B and Q on the sub-assemblage were 0.001798, -0.00194, -0.00214, -0.01128, 0.003015 and 0.007753 in./in., respectively. The strain values at sections B and Q are shown in Figures 47 and 48, respectively. The strain values were used to find experimental moment-curvature relationship at B and Q, as shown in Figures 49 and 50, respectively. For specimen SA9, an axial load of 15 kip was applied at the beginning of the test causing the member to bend slightly. This initial bending, plus any pre-existing strains in the strain gauges, caused the moment-curvature curve to have a sudden rise at the beginning of the graph. The maximum moment values at B and Q of the
specimen were 39 kip-in and 46 kip-in, respectively. The full moment capacity, plastic moment, was reached at section Q on the sub-assemblage. The moment capacity for Specimen SA9 was 15% more than that for Specimen SA3, 29% more than that for Specimen SA5, and 6% less than that for specimen SA7. The moment developed at point B was 85% of the maximum moment capacity.

Figure 51 shows the development of plastic hinges in the sub-assemblage under the quasi-static load in the absence of an axial load. Figure 52 shows the development of plastic hinges in the sub-assemblage under the quasi-static load in the presence of the axial load.

2.4 Impact Test

A series of impact tests were conducted on steel cantilevers and sub-assemblages. The apparatus for the experimental study was in the horizontal plane while the impact load was applied vertically.

2.4.1 Cantilever

Cantilever tests were performed to examine the testing procedure to be later used for the sub-assemblage tests. The same impactors were used for both the cantilever and the sub-assemblages.

2.4.1.1 Test Set-up

The test set-up is shown in Figure 53. The cantilever was made-up of a square tube that was 33 inches long. At end B, the cantilever was welded to a 7x7x0.75 inch base plate. The base plate was connected with eight half-inch bolts to a second plate that had the same dimensions as the first base plate. The second base plate was welded to a 7x4x0.5 inches channel. The channel was stiffened by a 0.25 inch stiffener in the middle that connected the flanges of the channel from inside. There was also a 12x7x1 inch plate welded to the web of the channel from the outside to increase its resistance against rotation during the experiment. The channel was connected to two big columns from both sides using two 1-inch-in-diameter bolts on each side.

An impact loading was applied by a dropping a solid steel cylinder, the impactor, at the free end of the cantilever, section Q. The impact test was performed using three impactors that were of different weights. The weights for Impactors 1, 2, and 3 were 60 lb., 140 lb., and 400 lb., respectively. Table 5 shows the designation for the three impactors.
A 3.5 inch diameter circular solid steel cylinder (steel housing), the same diameter of the impactors, was attached to each of the impactors with four 3/8 in bolts, as shown in Figure 54. A 500 g accelerometer was installed inside the steel housing facing the hitting part of the impactor, from the inside, and was used to read the acceleration signals from the impactor during the impact. Figure 55 shows the three impactors and their weights. Figure 56 shows Impactor 3 with the attached steel housing. At the free end of the cantilever, a chamber was welded at the bottom face of the tube. Another 500g accelerometer was inserted inside the chamber to measure the acceleration from the cantilever, at Q. The readings from both accelerometers were recorded at each test. The rotational stiffness for the rotational spring, \( K_B \), was \( 6 \times 10^6 \).

There were three strain gauges installed on this specimen. They were denoted as SG1, SG2 and SG3. The strain gauges were installed at three locations at a distance of one inch from end B. Figures 23 - 25 shows these strain gauges.

SG1 and SG2 were on the side face of the steel tube. SG1 and SG2 were installed at a distance of 0.35 in from the top and bottom fibers of the tube, respectively. SG3 was installed on the bottom face of the steel tube.

The strain gauges were attached to Vishay strain acquisition system to collect data from the strain gauges, as shown previously in Figure 29.

2.4.1.2 Test Procedure

The impact loading was applied using three different impactors dropped at section Q individually. The impactors were solid steel cylinders of the same diameter but were different in height \( h_i \). Impactor 1 was 6 inches in height and weighed 60 lb. Impactor 2 was 18 inches in height and 140 lb. in weight. Impactor 3 was 60 inches in height and 400 lb. in weight.

The impactors were attached from the top to a quick release device. The impactors were hung to the pulley of a crane with a quick release. The quick release allowed the release and free fall of objects attached to it by pulling a chain on the side of the quick release. When the quick release was opened, the impactor fell freely on the specimen. Starting with Impactor 1 and ending with Impactor 3, each one of the impactors was lifted in each experiment, consecutively, to a 1 inch of clear height \( h_c \) above section Q of the specimen. Each impactor was centered before being released. After running the experiment with the three impactors from 1 inch of clear height, Impactor 3 was dropped from 2 inch of clear height. It is important to mention that the clear height was measured
from the bottom of the impactor to the top face of the specimen, as shown in Figure 57. The data acquisition software for both the accelerometer and the strain gauges was set to start recording the data from the beginning of the test. Then, the impactor was released to fall freely and hit the specimen. The impactor rebounded on the specimen a few times. One should note, here, that this research is interested only in the first cycle of impact. The output from the accelerometers was collected through data acquisition device. The data from the strain gauges were collected by Vishay blue box.

The name of each test is a combination of letters and numbers. The first part of the test name refers to specimen, while the number after the dash refers to the test number using different impactors.

2.4.1.3 Experimental Results

Acceleration and strain data were collected using accelerometers and strain gauges from impact tests. The data was analyzed using EXCEL and MATLAB software. Moment-curvature plots were established for all specimens to find the maximum moments developed. The test results for each of the test specimens were observed and recorded, and are presented later in this section.

When the impactor dropped, it fell freely until it became in contact with the specimen at time $t_i$. The impactor and the specimen traveled together until the specimen reached the maximum deflection. Then, the specimen started to push the impactor upward and the impactor was detached from it at time $t_d$. The specimen vibrated freely until the impactor fell back on the specimen again, for a second impact, at time $t_{si}$. This study is interested only in the first impact. Any data after the second impact, $t_{si}$, were ignored.

The C1 specimen was subjected to impact loading using Impactors 1, 2 and 3 dropped freely from 1 inch in height. Then Impactor 3 was dropped again, for an additional test, but from 2 inches in height.

For C1-1 test, Impactor 1 was dropped from 1 inch of clear height above section Q of the specimen. Figures 58 and 59 show the acceleration-time relation from the impactor $a_i$ and the beam-column $a_b$, respectively. The curve-fitted maximum acceleration values, during the impact, from the impactor $a_i$ and the cantilever $a_b$ were 3.8g and 5.61g, respectively. The collected data for $a_b$ ranged from 513g to -513g, but the quadratic curve-fitted graph needed to show only the data within the range of 30g and -30g. For the $a_i$ data, the collected data ranged from 33g and -33g. The
maximum strain values during the impact for SG1, SG2 and SG3 strain gauges were 0.000557, -0.000654, and -0.000676 in./in., respectively. The experimental strain-time values during the impact for the three strain gauges are shown in Figure 60. Moment-curvature relation at the fixed end, section B, is shown in Figure 61. The maximum moment values at end B on the specimen was 10.8 kip-in.

For the C1-2 test, Impactor 2 was dropped from 1 inch of clear height above section Q of the specimen. Figures 62 and 63 show the acceleration-time relation from the impactor at and the beam-column ab, respectively. The curve-fitted maximum acceleration values, during the impact, from the impactor ai and the cantilever ab were 2.98g and 4.18g, respectively. The collected data for ab had a maximum range of data between 513g to -274g, but the graph for the quadratic curve-fitted graph needed to show only the data within the range of 40g and -40g. The maximum strain values during the impact for SG1, SG2 and SG3 strain gauges are 0.000971, -0.001059, and -0.001230 in./in., respectively. The experimental strain-time values during the impact for the three strain gauges are shown in Figure 64. Moment-curvature relation at the fixed end B is shown in Figure 65. The maximum moment value at end B on the specimen was 20.3 kip-in.

For the C1-3 test, Impactor 3 was dropped from 1 inch of clear height above end Q of the specimen. Figures 66 and 67 show the acceleration-time relation from the impactor ai and the beam-column ab, respectively. The curve-fitted maximum acceleration values, during the impact, from the impactor ai and the cantilever ab were 1.9g and 3g, respectively. Since the maximum value for ab in the curve-fitted graph was 3g, Figure 66 needed to show only the range of data between 40g and -40g, while the range of the original collected data was between 513g and -276g. The maximum strain values during the impact for SG1, SG2, and SG3 strain gauges are 0.002531, -0.002421, and -0.002790 in./in., respectively. The experimental strain-time values during the impact for the three strain gauges are shown in Figure 68. Moment-curvature relation at the fixed end, point B, is shown in Figure 69. The maximum moment value at end B on the specimen was 38.1 kip-in.

For the C1-4 test, Impactor 3 was dropped from 1 inch of clear height above section Q of the specimen. Figures 70 and 71 show the acceleration-time relation from the impactor ai and the cantilever ab, respectively. The curve-fitted maximum acceleration values, during the impact, from the impactor ai and the beam-column ab were 2.43g and 4g, respectively. The collected data for ab ranged from 513g to -282g but the figure only shows the data for a smaller range in order to point
out the shape of the quadratic curve-fitted graph for $a_b$. The maximum strain values during the impact for SG1, SG2, and SG3 strain gauges were 0.004545, -0.004783, and -0.007756 in./in., respectively. The experimental strain-time values during the impact for the three strain gauges are shown in Figure 72. Moment-curvature relation values at the fixed end, section B, are shown in Figure 73. The maximum moment value at end B on the specimen is 39.5 kip-in.

2.4.2 Sub-assemblage

Acceleration-time data were collected and were used to define the forcing function for each test. Strain values were recorded for the assemblages during the tests and were used to find moment-curvature relations for each test.

2.4.2.1 Test Set-up

The test set-up is shown in Figure 74. As described above, the sub-assemblage was made-up of two square tubes of unequal lengths welded together to form a sub-assemblage. At end B, the sub-assemblage was welded to a 7x7x0.75 inch base plate. The base plate was connected with eight half-inch bolts to a second plate that had the same dimensions as the first base plate. The second base plate was welded to a 7x4x0.5 inches channel. The channel was stiffened by a 0.25 inch stiffener in the middle. The stiffener connected the flanges of the channel from the inside. There was also a 12x7x1 inch plate welded to the web of the channel from the outside to increase its resistance to rotation during the experiment. The channel was connected to two big columns from both sides using two 1 inch diameter bolts on each side.

Each of the other two ends of the short member of the sub-assemblage were connected to the sides of a constructed steel pedestal using four angles that were two inches long. The four angles were arranged around the four sides of the tube as shown in Figure 21. A bolt running through the side angles and went through the tube. This type of connection simulated a flexible connection. The steel pedestal was 1.5 ft high and made of an I section with one plate welded at each of the two ends. The two pedestals were welded, from the bottom, to an 8x8x1/2 in. square tube. The tube was attached to two giant columns by two bolts at each end of it.

Impact loading was applied by a dropping a solid steel cylinder at section Q on the sub-assemblage. The impact test was performed using three impactors that were different in weight. The weights for Impactors 1, 2, 3 were 60 lb., 140 lb., and 400 lb., respectively. Table 5 shows the
designation for the three impactors. A circular solid steel cylinder (steel housing) that was 3.5 inch in diameter, the same diameter of the impactors, was attached to each of the impactors with four 3/8 inch bolts, as shown in Figure 54. A 500 g accelerometer was fixed inside the steel housing facing the hitting part of the impactor from the inside, and was used to read the acceleration signals from the impactor during the impact. Figures 55 shows the three impactors and their weights. Figure 56 shows Impactor 3 with the attached steel housing. In the middle of the long member, point Q, of the sub-assemblage, a chamber was welded at the bottom face of the tube. Another 500g accelerometer was inserted inside the chamber to record acceleration from the sub-assemblage at that point. The readings from both accelerometers were recorded at each test.

Tests with axial loading had a horizontal ENERPAC hydraulic jack that applied the static axial load on the long member at T.

A total number of six strain gauges installed on this specimen, same locations as the static test. They were denoted as SG1, SG2, SG3, SG4, SG5, and SG6. The strain gauges are installed on the long member of the sub-assemblage at two locations, Q and B. Figures 23 - 28 shows these strain gauges.

SG1, SG2, and SG3 were located at section B on the sub-assemblage, the fixed end. SG1 and SG2 are on the side face of the steel tube. SG1 and SG2 were installed at a distance 0.35 in from the top and bottom fibers of the tube, respectively. SG3 was on the bottom face of the steel tube.

SG4, SG5, and SG6 were located at section Q on the sub-assemblage, the impact point on the sub-assemblage. SG4 and SG5 were on the side face of the steel tube. SG4 and SG5 were installed at a distance 0.35 inch from the top and bottom fibers of the tube, respectively. SG6 was on the bottom face of the steel tube.

The strain gauges were attached to Vishay strain acquisition system to collect their data, as shown previously in Figure 29.

### 2.4.2.2 Test Procedure

The impact loading was applied using three different impactors dropped at section Q individually. The impactors were solid steel cylinders of the same diameter but were different in height. Impactor 1 was 6 inches in height and weighed 60 lb. Impactor 2 was 18 in height and 140 lb. in weight. Impactor 3 was 60 inches height and 400 lb. in weight.
For the tests with axial loading, an axial load of 15 kip was initially applied on the long member at T. Figure 75 shows a sub-assemblage during impact test.

The impactors were hung to the pulley of a crane with a quick release. The quick release allowed the release and free fall of impactors attached to it by pulling a chain attached to the side of the quick release. Starting with Impactor 1 and the ending with Impactor 3, each one of the impactors was lifted in each experiment, consecutively, to a 1 inch of clear height \( h_c \) above section Q of the specimen. Each impactor was centered before being released. After running the experiment with the three impactors from 1 inch of clear height, Impactor 3 was lifted to 6 inches of clear height for specimens without an axial load and 5 inches of clear height for specimens with an axial load. It is important to mention that the clear height was measured from the bottom of the impactor to the top face of the specimen, as is shown in Figure 57. The data acquisition software for both the accelerometer and the strain gauges was set to start recording the data from the beginning of the test. Then the impactor was released to fall freely and hit the specimen. The impactor rebounded on the specimen a few times. It should be noted, again, that this research is interested only in the first cycle of impact. The output from the accelerometer was collected through data acquisition device. The data from the strain gauges were collected by a Vishay blue box.

The name of each test is a combination of letters and numbers. The first part of the test designations refers to specimen, while the number after the dash refers to the test number using Impactors 1, 2, and 3.

### 2.4.2.3 Experimental Results

Acceleration and strain data were collected using strain gauges and accelerometers from impact tests. The data was analyzed using EXCEL and MATLAB softwares. Moment-curvature plots were established for all specimens to find the maximum moments developed. The test results for each of the test specimen was observed and recorded, and is presented later in this section.

When the impactor dropped, it fell freely until it became in contact with the specimen at time \( t_i \). The impactor and the specimen traveled together until the specimen reached the maximum deflection. Then, the specimen started to push the impactor upward and the impactor detached from it at time \( t_d \). The specimen vibrated freely until the impactor fell back on the specimen again, for a second impact,
at time $t_s$. Again, this study is interested only in the first impact. Any data after the second impact, $t_a$, was ignored.

One sub-assemblage specimen was used for repeated impact tests to identify the drop height to generate a plastic moment at any of the points Q and B. In specimen SA5 the same Impactor 3, 400 lb. was used and it dropped freely from different heights above point Q of the specimen at each drop. The test designations SA1-1, SA 1-2, SA 1-3, SA 1-4, SA 1-5, and SA 1-6 refers to the drop heights of 1, 2, 3, 4, 5, and 6 inches, respectively. The specimen was straitened back to its straight shape after each test.

The accelerometer used in the impact experiments was a uniaxial model with 500g maximum reading. It is known that the higher the accelerometer capacity, the more noise it produces during test. Therefore, the data points collected from the accelerometers were curve-fitted with a second order polynomial that represented the overall path of the graph. To show an example of the amount of noise produced by the accelerometer, the impactor was released on the tube specimen placed on the floor of the lab. The acceleration data from the impactor for 0.5 inch and 1 inch drops are shown in Figures 76 and 77, respectively.

### 2.4.2.3.1 Non-Retrofitted Sub-assemblage Impact Tests to Define Critical Height

For the SA1-1 test, Impactor 3 was dropped from a 1 inch clear height above section Q of the long member of sub-assemblage. Figure 78 shows the acceleration-time response from the impactor $a_i$. The curve-fitted maximum acceleration value, during the impact, from the impactor $a_i$ was 3.5g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000457, -0.000543, -0.001347, and 0.001173 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 79 and 80, respectively. Maximum moment values at sections B and Q on the specimen were 9.72 kip-in and 26.3 kip-in and are shown in Figures 81 and 82, respectively.

For the SA1-2 test, Impactor 3 was dropped from a two inch clear height above section Q of the long member of the sub-assemblage. Figure 83 shows the acceleration-time response from the impactor $a_i$ which originally ranged between 51g and -81g. The curve-fitted maximum acceleration value, during the impact, from the impactor $a_i$ was 5g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000599, -0.000684, - 0.001477, and 0.001517 in./in., respectively. The experimental strain-time values at B and Q on the sub-
assemblage were recorded and are shown in Figures 84 and 85, respectively. Maximum moment values at sections B and Q on the specimen were 14.5 kip-in and 27.2 kip-in and are shown in Figures 86 and 87, respectively.

For the SA1-3 test, Impactor 3 was dropped from three inch clear height above section Q of the long member of sub-assemblage. Figure 88 shows the acceleration-time response from the impactor \( a_i \). The curve-fitted maximum acceleration value, during the impact, from impactor \( a_i \) was 6.3g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000721, -0.000795, -0.001741, and 0.001830 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 89 and 90, respectively. Maximum moment values at sections B and Q on the specimen were 21.2 kip-in and 29 kip-in and are shown in Figures 91 and 92, respectively.

For the SA1-4 test, Impactor 3 was dropped from four inch clear height above section Q of the long member of sub-assemblage. Figure 93 shows the acceleration-time response from the impactor \( a_i \), which originally ranged between 85g and -108g. The curve-fitted maximum acceleration value, during the impact, from impactor \( a_i \) was 6.8g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000824, -0.000862, -0.001825, and 0.002003 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 94 and 95, respectively. Maximum moment values at sections B and Q on the specimen were 23.8 kip-in and 33.9 kip-in and are shown in Figures 96 and 97, respectively.

For the SA1-5 test, Impactor 3 was dropped from a five inch clear height above section Q of the long member of sub-assemblage. Figure 98 shows the acceleration-time response from the impactor \( a_i \), which originally ranged between 84g and -142g. The curve-fitted maximum acceleration value, during the impact, from impactor \( a_i \) was 7.4g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000932, -0.000941, -0.002732, and 0.002686 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 99 and 100, respectively. Maximum moment values at sections B and Q on the specimen were 30.2 kip-in and 38 kip-in and are shown in Figures 101 and 102, respectively.

For the SA1-6 test, Impactor 3 was dropped from six inch clear height above section Q of the long member of sub-assemblage. Figure 103 shows the acceleration-time response from the
impactor \( a_i \), which originally ranged between 186g and -203g. The curve-fitted maximum acceleration value, during the impact, from the impactor \( a_i \) was 7.65g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.001471, -0.001614, -0.006159, and 0.007933 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 104 and 105, respectively. Maximum moment values at sections B and Q on the specimen were 35.1 kip-in and 40 kip-in and are shown in Figures 106 and 107, respectively. At six inches of drop height, the moment at the mid span reached the plastic moment of the section.

For the SA1-7 test, Impactor 3 was dropped from a three inch clear height above section Q of the long member of sub-assemblage. Figure 108 shows the acceleration-time response from the impactor \( a_i \), which originally ranged between 93g and -155g. The curve-fitted maximum acceleration value, during the impact, from the impactor \( a_i \) was 5.9g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000832, -0.001586, -0.003807, and 0.002678 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 109 and 110, respectively. Maximum moment values at sections B and Q on the specimen were 25.4 kip-in and 35.3 kip-in and are shown in Figures 111 and 112, respectively.

For the SA1-8 test, Impactor 3 was dropped from a four inch clear height above section Q of the long member of sub-assemblage. Figure 113 shows the acceleration-time response from the impactor \( a_i \), which originally ranged between 78g and -122g. The curve-fitted maximum acceleration value, during the impact, from the impactor \( a_i \) was 6.1g. The maximum strain values during the impact for the SG1, SG2, SG4, and SG5 strain gauges were 0.000954, -0.001777, -0.006655, and 0.005150 in./in., respectively. The experimental strain-time values at B and Q on the sub-assemblage were recorded and are shown in Figures 114 and 115, respectively. Maximum moment values at sections B and Q on the specimen were 27.8 kip-in and 36.8 kip-in and are shown in Figures 116 and 117, respectively. At five inches drop height, the plastic hinge was developed in the specimen and the moment at the mid span reached the plastic moment.

Based on the results from these experiments, six inches clear drop heights were used for the specimens without axial loading to check the effect of retrofitting and five inches clear drop height was used for the specimens with 15 kip axial loading to observe the development of plastic moment in the specimen.
Figure 118 shows the development of a plastic hinge in the cantilever under impact load. Figure 119 shows the development of a plastic hinge in the sub-assemblage under impact load and without axial load. Figure 120 shows the development of a plastic hinge in the sub-assemblage under impact load and with the presence of the axial load.

2.4.2.3.2 Non-Retrofitted Sub-assemblage Impact Tests without Axial Load

The SA2 specimen was subjected to impact loading using Impactors 1, 2, and 3 dropped freely from a one inch height, respectively. Then Impactor 3 was dropped from a six inch clear height above the specimen. At six inches of clear drop height, the plastic hinge was developed in the specimen and the moment at section Q reached the plastic moment.

For the SA2-1 test, Impactor 1 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 121 and 122 show the acceleration-time response from the impactor a₁ and the beam-column a₀, respectively. The data from a₀ ranged between 209g and -206g. The curve-fitted maximum acceleration values, during the impact, from the impactor a₁ and the beam-column a₀ were 6.1g and 9.54g, respectively. The maximum strain values during the impact for the SG1, SG2, SG4, SG5, and SG6 strain gauges were 0.000279, -0.000207, -0.000219, 0.000326, and 0.000381 in./in., respectively. The experimental strain-time values at points B and Q on the sub-assemblage were recorded, during the impact for the strain gauges, and are shown in Figures 123 and 124, respectively. Moment-curvature relation values at B and Q of the specimen are shown in Figures 125 and 126, respectively. The maximum moment values at end B and section Q of the specimen are 4 kip-in and 5.9 kip-in, respectively. The moment value at Q was higher than the moment value at B.

For the SA2-2 test, Impactor 2 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 127 and 128 show the acceleration-time response from the impactor a₁ and the beam-column a₀, respectively. The data from a₀ ranged between 270g and -129g. The curve-fitted maximum acceleration value, during the impact, from the impactor a₁ and the beam-column a₀ were 5.3g and 7.9g, respectively. The maximum strain values during the impact for the SG1, SG2, SG4, SG5, and SG6 strain gauges were 0.000369, -0.000414, -0.000531, 0.000437, and 0.000637 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded, during the impact for the strain gauges, and are shown in Figures 129 and 130, respectively. Moment-curvature relation values at B and Q of the specimen
are shown in Figures 131 and 132, respectively. The maximum moment values at end B and section Q of the specimen were 7.4 kip-in and 11.4 kip-in, respectively. Section Q developed a higher moment value than section B.

For the SA2-3 test, Impactor 3 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 133 and 134 show the acceleration-time response from the impactor a_i and the beam-column a_b, respectively. The data from a_b ranged between 370g and -149g. The curve-fitted maximum acceleration value, during the impact, from the impactor a_i and the beam-column a_b were 3.3g and 3.72g, respectively. The maximum strain values during the impact for the SG1, SG2, SG4, SG5, and SG6 strain gauges were 0.000743, -0.000731, -0.000976, 0.000863, and 0.001234 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 135 and 136, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 137 and 138, respectively. The maximum moment values end B and section Q of the specimen were 13.8 kip-in and 22 kip-in, respectively. It is clear that Q had higher strain and moment values than B.

For the SA2-4 test, Impactor 3 was dropped from six inches of clear height above section Q of the long member of sub-assemblage. Figures 139 and 140 show the acceleration-time response from impactor a_i and the beam-column a_b, respectively. The data from a_i ranged between 54g and -63g while the data from a_b ranged between 513g and -513g. The curve-fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 8g and 9.99g, respectively. The maximum strain values during the impact for the SG1, SG2, SG4, SG5, and SG6 strain gauges are 0.001923, -0.001670, -0.004621, 0.004044, and 0.005661 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 141 and 142, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 143 and 144, respectively. The maximum moment values at end B and at section Q of the specimen were 36.2 kip-in and 39.3 kip-in, respectively. These moment values mean that only section Q reached a full moment capacity and developed a plastic hinge.
2.4.2.3.3 Non-Retrofitted Sub-assemblage Impact Tests with Axial Load

The SA4 specimen was subjected to impact loading using Impactors 1, 2 and 3 dropped freely from a one inch height, respectively. Then Impactor 3 was dropped from a five inch clear height above the specimen. At a five inches of clear drop height, the plastic hinge was developed in the specimen and the moment at the mid span reached the plastic moment.

For the SA4-1 test, impactor 1 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 145 and 146 show the acceleration-time response from the impactor a_i and the beam-column a_b, respectively. The data from a_b ranged between 179g and -97g. The curve-fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 6.3g and 4.79g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, and SG5 strain gauges were -0.000211, -0.000767, -0.000777, -0.000814, and -0.000161 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 147 and 148, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 149 and 150, respectively. For this specimen, an axial load of 15 kip was applied at the beginning of the test, making the member bend slightly. This initial bending, plus any pre-existing strains in the strain gauges, made the moment-curvature curve show a sudden rise at the beginning of the graph. The maximum moment values at end B and section Q of the specimen were 5 kip-in and 6.2 kip-in, respectively. By comparing the results of this test to those of SA2-1, SA4-1 also had a higher moment value at Q than B.

For the SA4-2 test, Impactor 2 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 151 and 152 show the acceleration-time response from the impactor a_i and the beam-column a_b, respectively. The data from a_b ranged between 291g and -226g. The curve-fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 5.64g and 4.29g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, and SG5 strain gauges were 0.000026, -0.001067, -0.001141, -0.001300, and 0.000091 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 153 and 154, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 155 and 156, respectively. For this specimen, an axial load of 15 kip was applied at the beginning of the test, making the member bend slightly. This initial bending, plus any pre-
existing strains in the strain gauges, made the moment-curvature curve show a sudden rise at the beginning of the graph. The maximum moment values at end B and section Q of the specimen were 9.2 kip-in and 13 kip-in, respectively. Both SA4-2 and SA2-2 had a higher moment value at Q than the moment value at B.

For the SA4-3 test, Impactor 3 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 157 and 158 show the acceleration-time response from the impactor \( a_i \) and the beam-column \( a_b \), respectively. The data from \( a_b \) ranged between 513g and -259g. The curve-fitted maximum acceleration values, during the impact, from the impactor \( a_i \) and the beam-column \( a_b \) were 3.58g and 2.95g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, and SG5 strain gauges were 0.000487, -0.001280, -0.001430, -0.001831, and 0.000809 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 159 and 160, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 161 and 162, respectively. For this specimen, an axial load of 15 kip was applied at the beginning of the test, making the member bend slightly. This initial bending, plus any pre-existing strains in the strain gauges, made the moment-curvature curve show a sudden rise at the beginning of the graph. The maximum moment values at end B and section Q of the specimen are 17.1 kip-in and 25.6 kip-in, respectively. Section Q developed higher moment value than section B.

For the SA4-4 test, Impactor 3 was dropped from five inches of clear height above section Q of the long member of sub-assemblage. Figures 163 and 164 show the acceleration-time response from the impactor \( a_i \) and the beam-column \( a_b \), respectively. The data from \( a_i \) ranged between 46g and -49g, while the data from \( a_b \) ranged between 513g and -513g. The curve-fitted maximum acceleration values, during the impact, from the impactor \( a_i \) and the beam-column \( a_b \) were 6.86g and 5.09g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4 and SG5 strain gauges were 0.001580, -0.002263, -0.002527, -0.006302 and 0.004016 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 165 and 166, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 167 and 168, respectively. The maximum moment values at end B and section Q of the specimen were 34.1 kip-in and 36.4 kip-in, respectively.
2.4.2.3.4 Retrofitted Sub-assemblage Impact Tests without Axial Load

The SA6 specimen was subjected to impact loading using Impactors 1, 2, and 3 dropped freely from a one inch clear height, respectively. Then Impactor 3 was dropped from six inches clear height above the specimen. At six inches of clear drop height, the plastic hinge did not develop in the specimen and the moment at the mid-span did not reach the plastic moment because of the retrofitting. The negative sign of the strain value means compression while the positive sign means tension.

For the SA6-1 test, Impactor 1 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 169 and 170 show the acceleration-time response from the impactor a₁ and the beam-column a₄, respectively. The data from a₄ ranged between 273g and -171g. The curve-fitted maximum acceleration values, during the impact, from the impactor a₁ and the beam-column a₄ were 8.3g and 3.57g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were 0.000266, −0.000236, −0.000185, 0.0000346, 0.000250, and 0.000467 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 171 and 172, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 173 and 174, respectively. The maximum moment values at end B and section Q of the specimen were 6.7 kip-in and 9.5 kip-in, respectively. With CFRP retrofitting, Q also developed a higher moment at Q than at B.

For the SA6-2 test, Impactor 2 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 175 and 176 show the acceleration-time response from the impactor a₁ and the beam-column a₄, respectively. The data from a₄ ranged between 513g and −513g. The curve-fitted maximum acceleration values, during the impact, from the impactor a₁ and the beam-column a₄ were 5.8g and 7.6g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were 0.000361, −0.000319, −0.000260, −0.000514, 0.000419, and 0.000789 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 177 and 178, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 179 and 180, respectively. The maximum moment values at end B and section Q of the specimen were 9.2 kip-in and 15 kip-in, respectively. Both non-retrofitted and
CFRP retrofitted tests developed a higher moment value at Q than the maximum moment value at B.

For the SA6-3 test, Impactor 3 was dropped from a one inch clear height section Q of the long member of sub-assemblage. Figures 181 and 182 show the acceleration-time response from the impactor a_i and the beam a_b, respectively. The data from a_b ranged between 513g and -513g. The curve-fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 3.98g and 8.72g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were 0.000873, -0.000650, -0.000544, -0.001309, 0.000641, and 0.001525 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 183 and 184, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 185 and 186, respectively. The maximum moment values at end B and sections Q of the specimen were 17.6 kip-in and 28 kip-in, respectively. It is clear that Q had higher strain and moment values than B.

For the SA6-4 test, Impactor 3 was dropped from five inches of clear height above section Q of the long member of sub-assemblage. Figures 187 and 188 show the acceleration-time response from the impactor a_i and the beam-column a_b, respectively. The curve fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 8.78g and 10g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were 0.001759, -0.001372, -0.001177, -0.005164, 0.003331, and 0.005140 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 189 and 190, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 191 and 192, respectively. The maximum moment values at end B and section Q of the specimen were 36.6 kip-in and 49 kip-in, respectively.

2.4.2.3.5 Retrofitted Sub-assemblage Impact Tests with Axial Load

The SA8 specimen was subjected to impact loading using Impactors 1, 2 and 3 dropped freely from a one inch clear height, respectively. Then Impactor 3 was dropped from a five inch clear height above the specimen. At five inches of clear drop height, the plastic hinge did not develop in the specimen and the moment at the mid span did not reach the plastic moment because of the retrofitting.
For the SA8-1 test, Impactor 1 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 193 and 194 show the acceleration-time response from the impactor \( a_i \) and the beam-column \( a_{ib} \), respectively. The data from \( a_{ib} \) ranged between 131g and -169g. The curve fitted maximum acceleration values, during the impact, from the impactor \( a_i \) and the beam-column \( a_{ib} \) were 8.7g and 4.47g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were -0.000236, -0.000826, -0.000895, -0.000978, -0.000277, and -0.000070 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 195 and 196, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 197 and 198, respectively. The strain values led to a higher maximum moment value at Q than the maximum moment value at B. The maximum moment values at end B and point Q of the specimen were 7.7 kip-in and 10.9 kip-in, respectively.

For the SA8-2 test, Impactor 2 was dropped from a one inch clear height above section Q of the long member of sub-assemblage. Figures 199 and 200 show the acceleration-time response from the impactor \( a_i \) and the beam-column \( a_{ib} \), respectively. The data from \( a_{ib} \) ranged between 104g and -101g. The curve-fitted maximum acceleration values, during the impact, from the impactor \( a_i \) and the beam-column \( a_{ib} \) were 6.27g and 4.6g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were -0.000064, -0.000860, -0.000979, -0.0001194, -0.000064, and 0.000345 in./in., respectively. The experimental strain-time values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 201 and 202, respectively. Therefore Q developed a higher moment value than B. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 203 and 204, respectively. The maximum moment values at end B and point Q of the specimen were 11.1 kip-in and 16.2 kip-in, respectively.

For the SA8-3 test, Impactor 3 was dropped from 1 inch clear height above section Q of the long member of sub-assemblage. Figures 205 and 206 show the acceleration-time response from the impactor \( a_i \) and the beam-column \( a_{ib} \), respectively. The data from \( a_{ib} \) ranged between 240g and -183g. The curve-fitted maximum acceleration values, during the impact, from the impactor \( a_i \) and the beam-column \( a_{ib} \) were 4.2g and 3.85g, respectively. The maximum strain values during the impact for the SG1, SG2, SG3, SG4, SG5, and SG6 strain gauges were 0.000308, -0.001072, -0.001320, -0.002016, 0.000188, and 0.001107 in./in., respectively. The experimental strain-time
values at sections B and Q on the sub-assemblage were recorded and are shown in Figures 207 and 208, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 209 and 210, respectively. For this specimen, an axial load of 15 kip was applied at the beginning of the test, causing the member to bend slightly. This initial bending, plus any pre-existing strains in the strain gauges, made the moment-curvature curve to jump and show a horizontal line at the beginning of the graph. The maximum moment values at end B and section Q of the specimen were 23.2 kip-in and 35.7 kip-in, respectively. The values showed that Q had a higher maximum moment value than B.

For the SA8-4 test, Impactor 3 was dropped from five inches of clear height above section Q of the long member of sub-assemblage. Figures 211 and 212 show the acceleration-time response from the impactor a_i and the beam-column a_b, respectively. The data from a_i ranged between 46g and -52g, while the data from a_b ranged between 445g and -314g. The curve fitted maximum acceleration values, during the impact, from the impactor a_i and the beam-column a_b were 7.34g and 12.27g, respectively. The maximum Microstrain values during the impact for the SG1, SG2, SG3, SG4, SG5 and SG6 strain gauges were 0.001230, -0.001683, -0.002137, -0.006444, 0.002696 and 0.004742 in./in., respectively. The experimental strain-time values at points B and Q on the sub-assemblage were recorded and are shown in Figures 213 and 214, respectively. Moment-curvature relation values at B and Q of the specimen, during the impact, are shown in Figures 215 and 216, respectively. For this specimen, an axial load of 15 kip was applied at the beginning of the test making the member bend slightly. This initial bending, plus any pre-existing strains in the strain gauges, made the moment-curvature curve jump and show a horizontal line at the beginning of the graph. The maximum moment values at end B and point Q of the specimen were 35.7 kip-in and 45 kip-in, respectively.

2.5 Discussion

Table 6 compares the maximum load capacity and the maximum moments in quasi-static tests. The table also shows the effect of the axial load and CFRP retrofitting. All static tests developed a full moment capacity of the section at the maximum load at both sections B and Q on the sub-assemblage. For Test SA3, the maximum static load was 4770 lb. The existence of the 15 kip axial load decreased the load capacity for Test SA5 to 3610 and by 25% compared to Test SA3. The CFRP retrofitting increased the maximum load for Test SA7 to 6000 and by 26%
compared to that for Test SA3. Test SA9, CFRP retrofitted with axial load, had a maximum load value that was 15% less than that for Test SA7. The Test SA9 maximum load value was 47% more than that for Test SA5 and 12% less than that for Test SA7. For Test SA9, the maximum static load was 5310 lb.

Table 7 compares the maximum load capacity and the maximum moments in impact tests on the sub-assemblage. The table also shows the effect of the axial load and CFRP retrofitting. The load capacity of the sub-assemblage without axial load and without CFRP retrofitting was 3200 lb. For Test SA4-4, the existence of the axial load decreased the load capacity of the sub-assemblage to 2744 lb. and by 16% compared to that for Test SA2-4. For Test SA6-4, the CFRP retrofitting increased the maximum load to 3512 lb. and by 9% compared to that for Test SA2-4. Test SA8-4, CFRP-retrofitted with axial load, had maximum load of 2936 lb. and it was 17% less than that for Test SA6-4. The Test SA8-4 maximum load was 7% less than that for Test SA4-4 and 10% less than that for specimen SA2-4.

Table 8 compares between quasi-static and impact loads to develop a plastic moment at sections Q and B. For the specimens without CFRP retrofitting and without axial load, Test SA3, the maximum static load was 4770 lb. and it created a plastic moment at both sections Q and B. For the same type of specimen, Test SA2-4, the maximum impact load was 3200 lb. and it created a plastic moment at section Q and 90% of plastic moment at section B. When a 15 kip static axial load was applied, the maximum quasi-static load was 3610 lb. and it created a plastic moment at both sections Q and B. For the same specimen, the impact load was 2744 lb. and it created a plastic moment at section Q and 91% of plastic moment at section B. For the CFRP retrofitted specimen, Test SA6, the maximum quasi-static load was 6000 lb. and it created a plastic moment at both sections Q and B. For a similar specimen, the maximum impact load was 3512 lb. and it created a plastic moment at section Q and 74% of the plastic moment at section B. If the effect of both axial load and CFRP was included, the maximum static load was 5310 lb. and it created a plastic moment at section Q and 87% of plastic moment at section B. For a similar specimen the maximum impact load was 2936 lb. and it created a plastic hinge at section Q and 77% of plastic moment at section B.

In general, the static loading created plastic moments and sections Q and B, while the impact loading developed a plastic moment at section Q only.
Table 9 compares the acceleration values from the impactor \( a_i \) and the acceleration values from specimen \( a_b \) with the weight of the impactor. The three impactors were dropped from the same height of a one inch above Section Q on the cantilever. For Test C1-1, Impactor 1 was used. The maximum curve-fitted acceleration value from the impactor was 3.8g and the maximum curve-fitted acceleration value from the cantilever was 5.61g. For Test C1-2, Impactor 2 was used. The maximum curve-fitted acceleration value from the impactor was 2.98g and the maximum curve-fitted acceleration value from the cantilever was 4.18g. For Test C1-3, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 1.9g and the maximum curve-fitted acceleration value from the cantilever was 3g. For Test C1-4, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 2.43g and the maximum curve-fitted acceleration value from the cantilever was 4g. Figure 217 shows the weight of impactor against the impactor acceleration for the cantilever.

Table 10 compares the maximum curve-fitted acceleration value from the impactor \( a_i \) and the maximum curve-fitted acceleration value from specimen \( a_b \) with the weight of the impactor dropped from the same height of a one inch above the sub-assemblage. In Test SA2-1, Impactor 1 was used. The maximum curve-fitted acceleration value from the impactor was 6.1g and the maximum curve-fitted acceleration value from the sub-assemblage was 9.54g. In Test SA2-2, Impactor 2 was used. The maximum curve-fitted acceleration value from the impactor was 5.3g and the maximum curve-fitted acceleration value from the sub-assemblage was 7.9g. In Test SA2-3, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 3.3g and the maximum curve-fitted acceleration value from the sub-assemblage was 3.72g. In Test SA2-4, Impactor 3 was used. The maximum curve fitted acceleration value from the impactor was 8g and the maximum curve-fitted acceleration value from the sub-assemblage was 9.99g. Figure 218 shows the weight of impactor against the impactor curve-fitted acceleration value for the specimen without axial load and without CFRP retrofitting.

Table 11 compares the maximum curve-fitted acceleration value from the impactor \( a_i \) and the maximum curve fitted acceleration value from specimen \( a_b \) with the weight of the impactor dropped from the same height of one inch above the sub-assemblage with the existence of 15 kip axial load. In Test SA4-1, Impactor 1 was used. The maximum curve-fitted acceleration value from the impactor was 6.3g and the maximum curve-fitted acceleration value from the sub-assemblage was 4.79g. In Test SA4-2, Impactor 2 was used. The maximum curve-fitted
acceleration value from the impactor was 5.64g and the maximum curve-fitted acceleration value from the sub-assemblage was 4.29g. In Test SA4-3, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 3.58g and the maximum curve-fitted acceleration value from the sub-assemblage was 2.95g. In Test SA4-4, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 6.86g and the maximum curve-fitted acceleration value from the sub-assemblage was 5.09g. Figure 219 shows the weight of impactor against the impactor acceleration for the specimen with axial load and without CFRP retrofitting.

Table 12 compares the maximum curve-fitted acceleration value from the impactor $a_i$ and the maximum curve-fitted acceleration value from specimen $a_b$ with the weight of the impactor dropped from the same height of one inch above the sub-assemblage. In Test SA6-1, Impactor 1 was used. The maximum curve-fitted acceleration value from the impactor was 8.3g and the maximum curve-fitted acceleration value from the sub-assemblage was 3.57g. In Test SA6-2, Impactor 2 was used. The maximum curve-fitted acceleration value from the impactor was 5.8g and the maximum curve-fitted acceleration value from the sub-assemblage was 7.6g. In Test SA6-3, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 3.98g and the maximum curve-fitted acceleration value from the sub-assemblage was 8.72g. In Test SA6-4, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 8.78g and the maximum curve-fitted acceleration value from the sub-assemblage was 10.1g. The high value of this acceleration was caused because the impactor hit the chamber of the accelerometer. Figure 220 shows the weight of impactor against the impactor acceleration for the specimen without axial load and with CFRP retrofitting.

Table 13 compares the maximum curve-fitted acceleration value from the impactor $a_i$ and the maximum curve-fitted acceleration value from specimen $a_b$ with the weight of the impactor dropped from the same height of one inch above the sub-assemblage. In Test SA8-1, Impactor 1 was used. The maximum curve-fitted acceleration value from the impactor was 8.7g and the maximum curve-fitted acceleration value from the sub-assemblage was 4.47g. In Test SA8-2, Impactor 2 was used. The maximum curve-fitted acceleration value from the impactor was 6.27g and the maximum curve-fitted acceleration value from the sub-assemblage was 4.6g. In Test SA8-3, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was 4.2g and the maximum curve-fitted acceleration value from the sub-assemblage was 3.85g. In Test SA8-4, Impactor 3 was used. The maximum curve-fitted acceleration value from the impactor was
7.34g and the maximum curve-fitted acceleration value from the sub-assemblage was 12.27g. Figure 221 shows the weight of impactor against the impactor acceleration for the specimen with axial load and with CFRP retrofitting.

Table 14 compares the maximum strain values between Tests SA2 and SA6. In all tests the strain values at section Q on the specimen were larger than the strains at section B.

Table 15 compares the maximum strain values between Tests SA4 and SA8. In all tests the strain values at section Q on the specimen were larger than the strains at section B.
CHAPTER 3
THEORETICAL ANALYSIS

Presented in this chapter are analytic formulations and solution procedures used to predict the behavior of the steel building sub-assemblage under quasi-static and impact conditions. An iterative algorithm utilizing finite-difference method has been developed. The solution procedure was first applied to the basic problem of the cantilever to test the effectiveness of the dynamic analysis algorithm. The models of the cantilever and the sub-assemblage used in this study are as shown in Figures 222 and 223, respectively.

3.1 Moment-Curvature Relationship

The determination of the moment-curvature relationship for the tube cross-section was accomplished using the tangent stiffness method described by Santathadaporn and Chen [29]. The normal strain of any element at a cross section subjected to bending moment $M_x$ and axial load $P_{axial}$ can be expressed as:

$$\varepsilon = \varepsilon_o + \phi_x y$$  \hspace{1cm} (1)

where $\varepsilon$ is the normal strain; $\varepsilon_o$ is the average axial strain; $y$ is the distance from the centroidal x-axis; and $\phi_x$ is the curvature about the x-axis. The pre-existing manufacturing residual strains were ignored in this study. The cross-sectional equilibrium equations for the axial thrust and the bending moment about x-axis are:

$$P_{axial} = \int_A \sigma \, dA$$  \hspace{1cm} (2)

$$M_x = \int_A \sigma \, y \, dA$$  \hspace{1cm} (3)

in which $dA$ is an elemental area of the cross section; $M_x$ is the moment about the x-axis; $P_{axial}$ is the axial load; $\sigma$ is the normal stress on that area; and $\int_A$ represents the cross-sectional integration.

Substituting the elastic-perfectly plastic stress-strain relationship for the steel into equations 2 and 3 results in the formulation of the cross-sectional equations for the axial force and bending moment in the inelastic range, which can be expressed as
\[ P_{\text{axial}} = -\int_{A_e} \sigma_e \, dA - \int_{A_p} \sigma_Y \, dA \]  
\[ M_x = \int_{A_e} \sigma \, y \, dA + \int_{A_p} \sigma_Y \, y \, dA \]

in which \( dA \) is an elemental area of the cross section; \( \sigma \) is the normal stress on that area; \( \sigma_Y \) is the yield stress; the \( e \) and \( p \) subscripts refer to the elastic and plastic elements, respectively, of a partially plastified section and \( \int_A \) represents the cross-sectional integration. Just as in equation 1, the strain rate equation will be:

\[ \dot{\varepsilon} = \dot{\varepsilon}_o + \dot{\phi}_x y \]  

where \( \dot{\varepsilon} \) is the normal strain rate; \( \dot{\varepsilon}_o \) is the average axial strain rate; and \( \dot{\phi}_x \) is the curvature rate about the x-axis. The stress-strain rate relationship is

\[ \dot{\sigma} = E_T \dot{\varepsilon} \]  

where \( \dot{\sigma} \) is the normal stress rate and \( E_T \) is the tangent modulus which equals \( E \) in the elastic range and zero in the inelastic range. The vectors for force \( \{f\} \) and deformation \( \{\delta\} \) for uniaxial bending are as follows:

\[ \{f\} = \{P_{\text{axial}} \ M_x\}^T \]  
\[ \{\delta\} = \{\varepsilon_o \ \phi_x\}^T \]

The corresponding load-deformation rate vectors will be:

\[ \{\dot{f}\} = \{\dot{P} \ \dot{M}_x\}^T \]  
\[ \{\dot{\delta}\} = \{\dot{\varepsilon}_o \ \dot{\phi}_x\}^T \]

where \( \dot{P}_{\text{axial}} \) is the axial force rate and \( \dot{M}_x \) is the bending moment rate. The elements of the force \( \{f\} \) rate are:

\[ \dot{P}_{\text{axial}} = -\int_{A_e} \dot{\sigma}_e \, dA - \int_{A_p} \dot{\sigma}_Y \, dA \]  
\[ \dot{M}_x = \int_{A_e} \dot{\sigma} \, y \, dA + \int_{A_p} \dot{\sigma}_Y \, y \, dA \]

The relationship between force \( \{f\} \) and deformation \( \{\delta\} \) rate vectors is:

\[ \{\dot{f}\} = [K] \{\dot{\delta}\} \]  

where \([K]\) is the tangent stiffness matrix which is:
\[ [K] = \begin{bmatrix} - \int E_t \, dA & - \int E_t \, y \, dA \\ \int E_t \, y \, dA & \int E_t \, y^2 \, dA \end{bmatrix} \] (15)

The \([K]\) will remain constant during the elastic range. When some elements plastified, this made a change to the \([K]\) matrix. \(E_t\) for the plastified elements was zero and this contributed to the summation.

The procedure was described by Santathadaporn and Chen [29] for biaxial bending. Figure 224 shows the convergence of this method. For a given axial force \(P_{axial}\) and moment \(M_x\), find the axial strain \(\varepsilon_o\) and curvature \(\phi_x\) using Equation 14. Elements of the stiffness matrix \([K]\) were found by calculating the summation over the cross-section of the tube. Using the strain \(\varepsilon_o\) and curvature \(\phi_x\), the internal resisting force and moment were evaluated by the summation over the cross section. The difference between internal and external forces needed to be within an acceptable range. The procedure continued until the maximum load carrying capacity for the cross-section was reached. At that level, the determinant of the \([K]\) matrix was zero.

In the following sections, the relationship was found for four different cases

### 3.1.1 Moment-curvature relationship for SA3

For Test SA3, the tube was subjected to a bending moment \(M_x\) about the x-axis. The axial load \(P_{axial}\) was set to zero. Figure 225 shows the moment-curvature relationship for Test SA3. The maximum moment was 40.9 kip-in.

### 3.1.2 Thrust moment-curvature relationship for SA5

For Test SA5, the tube was subjected to an axial force \(P_{axial}\) and a bending moment \(M_x\) about the x-axis. The axial load \(P_{axial}\) was held constant to 15 Kip. Figure 226 shows the moment-curvature relationship for Test SA5. The maximum moment was 37.3 kip-in. The effect of axial load decreased the plastic moment of the cross section by 9%, compared to that for Test SA3.

### 3.1.3 Moment-curvature relationship for SA7

For Test SA7, the square steel tube was retrofitted with two layers of CFRP strips. The retrofitted cross-section was subjected to bending moment \(M_x\) about the x-axis. The axial load
\( P_{\text{axial}} \) was set to zero. Figure 227 shows the moment-curvature relationship for Test SA7. The maximum moment was 48.9 kip-in. The effect of retrofitting increased the plastic moment of the cross section by 20%, compared to Test SA3.

3.1.4 Thrust moment-curvature relationship for SA9

For Test SA9, the square tube was retrofitted with two layers of CFRP strips. The CFRP-retrofitted tube was subjected to an axial force \( P_{\text{axial}} \) and a bending moment \( M_x \) about the x-axis. The axial load \( P_{\text{axial}} \) was kept constant and was set to 15 kip. Figure 228 shows the moment-curvature relationship for Test SA9. The maximum moment was 46.8 kip-in. The plastic moment increased of the cross section by 15%, compared to Test SA3.

3.2 Quasi-Static Inelastic Analysis of Sub-assemblage

A gradual increasing concentrated load \( W \) was applied at Section Q of the sub-assemblage. For the case where axial load is applied, a constant axial force \( P_{\text{axial}} \) of 15 kip was applied at the long member of the sub-assemblage.

The sub-assemblage was restrained against vertical movement at its end T with a transitional linear spring stiffness \( K_{\text{spr}} \). End rotations were restrained by pair of rotational springs at ends T and B of stiffnesses \( K_T \) and \( K_B \), respectively.

3.2.1 Governing Differential Equation

Figure 229 shows the discretized square tube cross section with outside dimension D and wall thickness t in which x and y are the centroidal axes. Each wall of the tube was divided into subareas \( dA \). The elastic perfectly plastic stress-strain relationship for the steel material of the tube including the elastic unloading, is shown in Figure 230.

The normal strain \( \varepsilon \) at any point of the cross section subjected to bending moments and about x-axis and axial force \( P_{\text{axial}} \) was expressed in Equation 1. The cross-sectional equations for the axial force and bending moment can be written as in Equations 2 and 3.
Substituting the elastic-perfectly plastic stress-strain relationship, in Equation 1, for the steel into equations 2 and 3 results in the formulation of the cross-sectional equations for the axial force; bending moment in the inelastic range can be expressed as in Equations 4 and 5. Substituting Equation 1 into Equations 4 and 5 gives:

\[ P_{axial} = -E\varepsilon_0 A_e + ES_{xe} \nu'' - P_p \]  
\[ M_x = -E\varepsilon_0 S_{xe} - EI_{xe} \nu'' + M_{xp} \]

where \( S_{xe} \) is the first moment of the area from the cross-section; \( I_{xe} \) is the moment of inertia; \( P_p \) is the summation of the axial load from the plastic elements; \( \nu \) is the deflection at any section along the beam-column; \( \nu'' \) is the second derivative of the deflection; and \( M_{xp} \) is summation of the bending moment from the plastic elements. The terms are defined below:

\[ A_e = \int A_e dA \]  
\[ S_{xe} = \int A_e y dA \]  
\[ I_{xe} = \int A_e y^2 dA \]  
\[ P_p = \int A_p \sigma_y dA \]  
\[ M_{xp} = \int A_p \sigma_y y dA \]

At end T, the linear spring constant value is \( K_{spr} \) and its reaction \( R_T \) can be written as follows:

\[ R_T = \frac{m_T - m_B}{L} + 0.5W - P \frac{\nu_T}{L} \]

where the end restraint moments at T and B are \( m_T \) and \( m_B \), respectively, \( W \) is the applied lateral load, and \( \nu_T \) is the deflection at end T. Then the external applied moment \( M_x \), at any location along the beam-column of sub-assemblage on portion T-Q will be:

\[ M_x = z R_T - m_T + P_{axial} \nu \]

where \( z \) is the distance from the origin at end T. Substituting equation 19 into equation 20 gives the expression for the portion T-Q of the sub-assemblage shown in Figure 223.
\[ M_x = \left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (\theta_T) + \left( \frac{K_B}{2h} \right) (\theta_B) \left( \frac{z}{L} \right) + 0.5W(Z) + P_{\text{axial}}v + P_{\text{axial}}v_T \left( \frac{z}{L} \right) \]  
\[ \text{(21)} \]

where \( \theta_B \) is the rotation of the sub-assemblage at end B. The moment \( M_x \) for the portion Q-B of the sub-assemblage will be:

\[ M_x = z R_T - m_T - W \left( Z - \frac{L}{2} \right) + P_{\text{axial}}v \]  
\[ \text{(22)} \]

Substituting Equation 19 into Equation 22 gives the expression for the portion Q-B of the sub-assemblage shown in Figure 223.

\[ M_x = \left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (\theta_T) + \left( \frac{K_B}{2h} \right) (\theta_B) \left( \frac{z}{L} \right) + 0.5W(L - Z) + P_{\text{axial}}v + P_{\text{axial}}v_T \left( \frac{z}{L} \right) \]  
\[ \text{(23)} \]

Solving Equation 16 for \( \epsilon_0 \) explicitly, then \( \epsilon_0 \) and \( M_x \) are substituted into Equation 17. The new value of \( M_x \) substituted into Equation 21 results into the following materially nonlinear ordinary differential equation for the portion T-Q of the sub-assemblage.

\[ \left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (\theta_T) + \left( \frac{K_B}{2h} \right) (\theta_B) \left( \frac{z}{L} \right) + 0.5W(Z) + P_{\text{axial}}v + P_{\text{axial}}v_T \left( \frac{z}{L} \right) \]

\[ = S_{xe} \left( P_{\text{axial}} + P_p \right) - \left( ES_{xe}^2 - EI_{xe} A_e \right) v'' - M_{xp} A_e \]  
\[ \text{(24)} \]

Solving Equation 16 for \( \epsilon_0 \) explicitly, then \( \epsilon_0 \) and \( M_x \) are substituted into Equation 17. The new value of \( M_x \) substituted into Equation 23 results into the following materially nonlinear ordinary differential equation for the portion Q-B of the sub-assemblage.

\[ \left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (\theta_T) + \left( \frac{K_B}{2h} \right) (\theta_B) \left( \frac{z}{L} \right) + 0.5W(L - Z) + P_{\text{axial}}v + P_{\text{axial}}v_T \left( \frac{z}{L} \right) \]

\[ = S_{xe} \left( P_{\text{axial}} + P_p \right) - \left( ES_{xe}^2 - EI_{xe} A_e \right) v'' - M_{xp} A_e \]  
\[ \text{(25)} \]

Equations 24 and 25 are the governing differential equations for the sub-assemblage at any location on the sub-assemblage.

### 3.2.2 Boundary Conditions

Referring to Figure 223, the relationship between the rotational spring moment \( m_T \) and the rotation \( \theta_T \) at end T can be expressed as:
\[ m_T = K_T \theta_T \quad (26a) \]

where \( K_T \) is the stiffness of the rotational spring at end T. \( \theta_T \) is the rotation of the sub-assembly at end T. The elastic-plastic moment-rotation relationship at B is expressed as:

\[ m_B = K_B \theta_B \quad \text{for} \ |\theta_B| < \theta_{Bpc} \quad (26b) \]
\[ m_B = m_{Bpc} \quad \text{for} \ |\theta_B| \geq \theta_{Bpc} \quad (26c) \]

where \( K_B \) is the stiffness of the rotational spring at end B; \( \theta_B \) is the rotation of the sub-assembly at end B; \( \theta_{Bpc} \) is the rotation when plastification occurs in the rotational spring at B and; \( m_{Bpc} \) is the plastic moment of the section. The existence of the axial load will have an effect on the value of \( m_{Bpc} \) and as:

\[ m_{Bpc} = \begin{cases} m_{pp}, & \text{if} \ P_{axial} = 0 \\ m_{ppa}, & \text{if} \ P_{axial} > 0 \end{cases} \quad (26d) \]

where \( m_{pp} \) is the plastic moment of the section without axial load \( P_{axial} \) and \( m_{ppa} \) is the plastic moment of the section with the axial load \( P_{axial} \). In this study, \( \theta_B \) is assumed to be in the elastic range. The justification for this assumption is that the results of the analysis were good and acceptable. The \( m - \theta \) relationship for both ends was shown and discussed previously in chapter 2.

\( \theta_T \) and \( \theta_B \) are the first derivatives of the deflections at ends T and B. The relationship can be expressed as below

\[ \theta_T = v'(0) \quad (27) \]
\[ \theta_B = -v'(L) \quad (28) \]

The elastic relationship between \( K_{spr} \) and \( v \) is expressed in the equation below:

\[ M = K_{spr} v \quad (29) \]

where \( K_{spr} \) is the stiffness of the linear spring at end T. The load-deflection relationship for the translational spring at end T was shown and discussed previously in chapter 2. The sub-assembly end at B is not allowed to sway and the condition can be expressed as

\[ v(L,t) = v_B = 0 \quad (30) \]

End T of the sub-assembly has the following deflection term:

\[ v(0,t) = v_T \quad (31) \]
The shear relationship at end T is expressed in the following equation:

\[ R_T = K_{spr} v_T \]  \hspace{1cm} (32)

where \( R_T \) is the reaction at end T. \( K_{spr} \) is the stiffness of the linear spring at end T. \( v_T \) is the deflection at end T.

Since \( R_T \) is the shear value at end T, \( R_T \) is a combination of two values. The first part is due to bending and the second part is due to the axial load \( P_{axial} \) acting at end T.

The first part originates from the equation

\[ B_e \frac{\partial^2 v}{\partial z^2} = -M_x \]  \hspace{1cm} (33)

The shear is the first derivative of the moment

\[ \frac{\partial}{\partial z} \left( B_e \frac{\partial^2 v}{\partial z^2} \right) = -\frac{\partial M_x}{\partial z} = -R_T \]  \hspace{1cm} (34)

Therefore, using Equations 33 and 34, the outcome can be expressed in terms of deflections as follows

\[ \frac{\partial B_e}{\partial z} \left( \frac{\partial^2 v}{\partial z^2} \right)_{(0,t)} + \left( B_e \frac{\partial^3 v}{\partial z^3} \right)_{(0,t)} = -K_{spr} v_T_{(0,t)} \]  \hspace{1cm} (35)

Figure 232 shows a schematic of the shear at end T is due to axial load \( P_{axial} \). The shear value at end T due to \( P_{axial} \) can be expressed as follows

\[ P \sin \theta = P_{axial} \frac{\partial v}{\partial z} (0,t) \]  \hspace{1cm} (36)

The final shear condition at end T in terms of deflection is as follows

\[ \frac{\partial B_e}{\partial z} \left( \frac{\partial^2 v}{\partial z^2} \right)_{(0,t)} + \left( B_e \frac{\partial^3 v}{\partial z^3} \right)_{(0,t)} + P_{axial} \frac{\partial v}{\partial z} (0,t) = -K_{spr} v_T (0,t) \]  \hspace{1cm} (37)

The boundary conditions presented above will be used in the static analysis of the sub-assemblage.

### 3.2.3 Finite-Difference Formulation

Central finite-difference expressions were used to solve the equations derived in the previous section. A total of \( N \) panels were used on the sub-assemblage and the supports. Figure 231 shows the longitudinal panels of the sub-assemblage where, Node 1 is at end T and Node \( N+1 \) is at point
B. Nodes 0 and N+2 are fictitious Nodes outside the sub-assemblage. Therefore, Equations 24 and 25 will be in the following finite difference form:

Equation for portion T-Q of the sub-assemblage

\[
\left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (-v_0 + v_2) + \left( \frac{K_B}{2h} \right) (-v_N + v_{N+2}) \left( \frac{z}{L} \right) + 0.5W(Z) + P_{axial}v + P_{axial}v_T \left( \frac{z}{L} \right) = S_{x_e} (P + P_p) - (E S_{x_e}^2 - E I_{x_e} A_e) \left( \frac{1}{h^2} \right) (v_{i-1} - v_i + v_{i+1}) - M_{xp} A_e
\] (38)

Equation for portion Q-B of the sub-assemblage

\[
\left( \frac{z}{L} - 1 \right) \left( \frac{K_T}{2h} \right) (-v_0 + v_2) + \left( \frac{K_B}{2h} \right) (-v_N + v_{N+2}) \left( \frac{z}{L} \right) + 0.5W(L - Z) + P_{axial}v + P_{axial}v_T \left( \frac{z}{L} \right) = S_{x_e} (P + P_p) - (E S_{x_e}^2 - E I_{x_e} A_e) \left( \frac{1}{h^2} \right) (v_{i-1} - v_i + v_{i+1}) - M_{xp} A_e
\] (39)

the subscript i refers to the ith Node point over the domain 0<Z<L.

Applying Equation 38 at Nodes i=1 to i=(0.5N+1) and Equation 39 at Nodes i=(0.5N+1) to i=N+1, with the conditions represented by Equations 26a, 26b, 30 and 37, leads to the following matrix equation:

\[
\{f\} = [K]\{U\} + \{f_p\}
\] (40)

where

\{f\} = load vector

\([K]\) = stiffness matrix of the order N+1

\{U\} = flexural displacement vector

\{f_p\} = plastic 'load' vector which develops in the inelastic range

Solving Equations 38 and 39 repeatedly at different increasing external applied loads allowed the finding of the deflections and other parameters until the collapse load was reached. In the inelastic range, the parameters \(P_p, M_{xp}, I_{x_e}, S_{x_e}\) and \(A_e\) will start to change at the different locations on the sub-assemblage.
3.2.4 Solution Algorithm

Herein, a finite difference iteration algorithm for the nonlinear analysis of the sub-assemblage is presented. The steps of the solution are as follows:

1. For a small external load increment \( W \) in the elastic range, while fixing the axial force \( P_{axial} \), calculate the flexural deflections \( v_0 \) at every node along the sub-assemblage, using equation 40.
2. Compute the moments \( M_x \) using Equations 21 and 23.
3. Compute the change in moment \( M_x \) and axial load \( P_{axial} \).
4. Use the tangent stiffness procedure from in Reference 29 to compute the axial strains and curvatures due to the moment and axial load.
5. Use the curvatures and axial strains found in Step 4 to find the properties for each section along the beam.
6. Find the deflections \( v_2 \) from the section parameters found in Step 5 and using equation 40.
7. Compare the deflections \( v_2 \) from Step 6 with the deflections found in the previous cycle , \( v_1 \).
8. If \( |v_2 - v_1| > \) tolerance, then repeat Steps 1 to 6 for the same applied load until convergence occurs. In the elastic range, deflections will be the same without any iterations. In the inelastic range, there will be few iterations before convergence occur.
9. If \( |v_2 - v_1| < \) tolerance repeat Steps 1 to 6 and increase the load \( W \) with each cycle, until maximum load-carrying capacity of the sub-assemblage is reached.

The procedure was carried out using constant load increments in the elastic range. The load increments were decreased in the inelastic range for better and faster convergence.

3.2.5 Numerical study

In this section, the behavior of a 2”x2”x1/8” square tube sub-assemblage under quasi-static concentrated load, in the middle, will be studied. The length of the specimen is 66” with \( \sigma_y \) and \( E \) values equal 62000psi and 30000000psi. Four cases were studied, sub-assemblage without axial SA3, sub-assemblage with axial load SA5, sub-assemblage without axial load and with retrofitting CFRP SA7, and sub-assemblage with axial load and CFRP retrofitting SA9. The rotational stiffnesses for the rotational springs, \( K_T \) and \( K_B \), and the linear transitional spring, \( K_{spr} \), were 1.5x10^6, 6*10^6 and 34000, respectively.
3.2.5.1 Sub-assemblage without axial load SA3

The finite-difference procedure was used to assess the load carrying capacity for Specimen SA3. The output results also identified the formation of plastic hinges at Q and B of the sub-assemblage. Figure 242 shows the load-deflection curve for specimen SA3. The peak load for this specimen was 4425 lb. At this load, plastic moments formed at both sections Q and B of the sub-assemblage. Figures 243 - 246 show strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.006504, - 0.0065, - 0.00682, and 0.006818 in./in., respectively. Figures 247 and 248 show the moment-curvature relationships at Q and B with maximum moment values of 40.4 and 40.04 kip-in, respectively. Maximum moment capacity, plastic hinges, of the section was reached at Q and B.

The corresponding bending stiffness degradation curve for the specimen is shown in Figure 249. The dimensionless determinant D1 is defined as follows:

\[ D1 = \frac{|[K]|}{|K|_0} \]

where \([K]\) is the determinant of the stiffness matrix for the member at the different stages during the analysis, and \(|[K]|_0\) is the determinant of the stiffness matrix for the member at the zero applied load.

3.2.5.2 Sub-assemblage with axial load SA5

An axial load of 15000 lb. was applied on the sub-assemblage in addition to the lateral load. Figure 250 shows the load-deflection curve for specimen SA5. The peak load for this specimen, in the presence of the axial load, was decreased to 3860 lb. At this load, plastic moments formed at both points Q and B of the sub-assemblage. Figures 251 - 254 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.003395, - 0.00936, - 0.01278, and 0.004408 in./in., respectively. Figures 255 and 256 show the moment-curvature relationships at Q and B with maximum moment values of 36 and 35.8 kip-in, respectively. These moment values show that plastic hinges were developed at both Q and B. The corresponding bending stiffness degradation curve for the specimen is shown in Figure 257.
3.2.5.3 CFRP retrofitted Sub-assemblage without axial load SA7

Two layers of CFRP strips were added at the bottom face of the sub-assemblage as a retrofitting, in order to increase the load capacity of the beam. Figure 258 shows the load-deflection curve for specimen SA7. The peak load for this specimen, in the presence of the axial load, was increased to 5400 lb. At this load, plastic moments formed at both sections Q and B of the sub-assemblage. Figures 259 - 264 show the strain-time relation for SG1, SG2, SG3, SG4, SG5, and SG6 on the sub-assemblage with maximum strain values of 0.014728, -0.00403, -0.00549, -0.01628, 0.00371, and 0.007401 in./in., respectively. Figures 265 and 266 show the moment-curvature relationships at Q and B with maximum moment values of 47.9 and 47.6 kip-in, respectively. Both sections, Q and B, developed plastic moments. The corresponding bending stiffness degradation curve for the specimen is shown in Figure 267.

3.2.5.4 CFRP retrofitted Sub-assemblage with axial load SA9

Two layers of CFRP strips were added at the bottom face of the sub-assemblage as a retrofitting to increase the load capacity of the beam. In order to study the effect of axial loading on retrofitted members, an axial load of 15000 lb. was applied on the sub-assemblage in addition to the lateral load. Figure 268 shows the load deflection curve for specimen SA9. The peak load for this specimen, in the presence of the axial load, increased to 5050 lb. Figures 269 - 274 show the strain-time relation for the locations of SG1, SG2, SG3, SG4, SG5, and SG6 on the sub-assemblage with maximum strain values of 0.001701, -0.00189, -0.00217, -0.00822, 0.002819, and 0.006814 in./in., respectively. Figures 276 and 277 show the moment-curvature relationships at Q and B with maximum moment values of 45.5 and 33 kip-in, respectively. The plastic moment was reached at section Q only. The corresponding bending stiffness degradation curve for the specimen is shown in Figure 275.

3.3 Impact Inelastic Analysis

An impact load was applied at section Q on the specimens. The forcing function found from each experiment was used in the theoretical analysis.
3.3.1 Cantilever

One end of cantilever was restrained against rotation by a rotational spring at end B with stiffness $K_B$ and the other end was free.

3.3.1.1 Governing Differential Equations

Harris et. al.[51] and Razzaq et. al.[10] expressed the elastic partial differential equation of motion of beam flexure as follows:

$$EI \frac{\partial^4 v}{\partial z^4} + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (41)$$

In the elastic range, $EI$ is constant which means that the second partial derivative of it will be zero. In the inelastic range $EI$ is changing with the load. Therefore, the inelastic partial differential equation of motion can be expressed as

$$\frac{\partial^2}{\partial z^2} \left( B_e \frac{\partial^2 v}{\partial z^2} \right) + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (42)$$

where $B_e$ is the elasto-plastic flexural rigidity; $v$ is the deflection of the beam, and $m$ is the unit mass for the member. The damping effect was not considered in this study.

Solving the differentiation of the first term of the equation leads to:

$$B_e \frac{\partial^4 v}{\partial z^4} + 2 \frac{\partial^3}{\partial z^3} \left( \frac{\partial B_e}{\partial z} \right) + \frac{\partial^2 v}{\partial z^2} \left( \frac{\partial^2 B_e}{\partial z^2} \right) + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (43)$$

There is no closed form solution for Equation 43 in the literature. Razzaq et al. [10] solved Equation 41 using the finite difference scheme. An iterative nonlinear finite difference scheme will be utilized for the solution solving Equation 43.

$B_e$ is a function of $z$. In order to find the first and second partial derivatives of $B_e$, Lagrangian polynomial will be fitted. For example, for nodes 2, 3, and 4:

$$L_2(z) = \frac{(z - z_3)(z - z_4)}{(z_2 - z_3)(z_2 - z_4)}$$

$$L_3(z) = \frac{(z - z_2)(z - z_4)}{(z_3 - z_2)(z_3 - z_4)}$$

$$L_4(z) = \frac{(z - z_2)(z - z_3)}{(z_4 - z_2)(z_4 - z_3)}$$

$$f(z_2) = B_{e2}$$

$$f(z_3) = B_{e3}$$
\[ f(z_4) = B_{e4} \]

Lagrangian polynomial formula:

\[ B_e(z) = \sum_{k=2}^{4} L_k(z) \cdot f(z_k) \]

Therefore, the equation for \( B_e \) is:

\[ B_e(z) = L_2(z) \cdot f(z_2) + L_3(z) \cdot f(z_3) + L_4(z) \cdot f(z_4) \]

The equation will be of second order:

\[ B_e(z) = \alpha_1 z^2 + \alpha_2 z + \alpha_3 \quad (44) \]

Where \( \alpha \) is the coefficient for the \( z \) variable. The first and second derivatives of \( B_e \):

\[ \frac{\partial B_e}{\partial z} = 2\alpha_1 z + \alpha_2 \quad (45a) \]

\[ \frac{\partial^2 B_e}{\partial z^2} = 2\alpha_1 \quad (45b) \]

The problem is considered an initial boundary value problem. The terms in the equation are dependent on both \( z \), the location along the beam, and \( t \), the time.

The boundary condition at end B is the same boundary condition that has been used in the quasi-static test for the sub-assemblage.

### 3.3.1.2 Boundary Conditions

Referring to Figure 222, the cantilever is 33 in. long and it represents the length between Q and B. The end Q of the cantilever is a free end and therefore has no bending moment

\[ m_Q = \frac{\partial^2 \nu}{\partial z^2}(0, t) = 0 \quad (46a) \]

The shear force at end Q can be expresses as

\[ V_Q = \frac{\partial^3 \nu}{\partial z^3}(0, t) = -F(t) \quad (46b) \]

At the end B, the cantilever has no vertical movement.

\[ \nu_{(L,t)} = \nu_B = 0 \quad (46c) \]

The elastic-plastic moment-rotation relationship at B is expressed as:
\[ m_B = K_B \theta_B \quad \text{for} \ |\theta_B| < \theta_{Bpc} \] (46d)

\[ m_B = m_{Bpc} \quad \text{for} \ |\theta_B| \geq \theta_{Bpc} \] (46e)

where \( K_B \) is the stiffness of the rotational spring at end B, \( \theta_{Bpc} \) is the rotation when plasticity occurs in the rotational spring at B, \( \theta_B \) is the rotation of the cantilever at end B, and \( m_{Bpc} \) is the plastic moment of the section. The existence of the axial load will have an effect on the value of \( m_{Bpc} \) and as :

\[
m_{Bpc} = \begin{cases} m_{pp}, & \text{if} \quad P_{axial} = 0 \\ m_{ppa}, & \text{if} \quad P_{axial} > 0 \end{cases}
\] (46f)

where \( m_{pp} \) is the plastic moment of the section without axial load \( P_{axial} \) and \( m_{ppa} \) is the plastic moment of the section with the axial load \( P_{axial} \). In this study, \( \theta_B \) is assumed to be in the elastic range. The justification for this assumption is that the results of the analysis were good and acceptable.

\( \theta_B \) is the first derivative of the deflection at end B. The relationship can be expressed as below:

\[ \theta_B = -v'(L) \] (46g)

The boundary conditions presented above were used in the impact analysis of the cantilever.

### 3.3.1.3 Initial Conditions

Equation 45 is also time dependent. Initial conditions are needed in the solution. The initial conditions for the problem are:

\[ v(z, 0) = 0 \] (47a)

\[ \frac{\partial v}{\partial t}(z, 0) = 0 \] (47b)

the initial condition given by equation 47a states that at time \( t \) equal zero, the deflection is zero. Equation 47b states that the initial velocity is zero.

Initial plastification at \( t=0 \) may occur and need to be addressed with the following

\[ M_x = -EI_x \varepsilon'' + M_{xp} \] (48)

Equation 48 need to be enforced for any initially plastified areas in the cross-section.
3.3.1.4 Finite-Difference Formulation

Central finite-difference expressions were used to solve the equations derived in the previous section. A total of N panels were used on the entire cantilever. Figure 233 shows the longitudinal panels of the cantilever. Where, Node 1 is at end Q and Node N+1 is at Node B. Nodes 0 and N+2 are fictitious Nodes outside the cantilever. Using second order finite difference expressions (Reference 2), Equation 43 can be written as:

\[
\frac{B_e}{h^4} (v_{i-2,j} - 4v_{i-1,j} + 6v_{i,j} - 4v_{i+1,j} + v_{i+2,j}) \\
+ \frac{2}{h^3} (-v_{i-2,j} + 2v_{i-1,j} - 2v_{i+1,j} + v_{i+2,j}) \left( \frac{\partial B_e}{\partial z} \right) \\
+ \frac{1}{h^2} (v_{i-1,j} - 2v_{i,j} + v_{i+1,j}) \left( \frac{\partial^2 B_e}{\partial z^2} \right) + \frac{m}{(\Delta t)^2} (v_{i,j-1} - 2v_{i,j} + v_{i,j+1}) = F(t)
\]

in which, \( h \) is the panel length along the z-axis of the cantilever, and \( \Delta t \) is the time interval. The subscript \( i \) refers to the \( i \)th Node point over the domain \( 0 < x < L \), and the subscript \( j \) refers to the number of time increments such that the time at \( j \) is given by the following equation:

\[ t_j = j(\Delta t), \quad \text{for each} \quad j = 0, 1, 2, 3, \ldots \]

Similarly, the boundary conditions 46a, 46b, 46c, and 46d can be expressed in finite difference form as follows:

\[
\frac{1}{h^2} (v_{0,j} - 2v_{1,j} + v_{2,j}) = 0
\]

(50a)

\[
\frac{1}{2h^3} (-v_{-1,j} + 2v_{0,j} - 2v_{2,j} + v_{3,j}) = -F(t)
\]

(50b)

\[ v_{N+1,j} = 0 \]

(51a)

\[
\left( \frac{B_e}{h} + \left( \frac{KB}{2} \right) \right) (v_{N+2,j}) - \left( \frac{2B_e}{h} \right) (v_{N+1,j}) - \left( \frac{-B_e}{h} + \left( \frac{KB}{2} \right) \right) (v_{N,j}) = 0
\]

(51b)

Applying equation 49 at \( i=1, 2, 3 \ldots (N) \) and invoking conditions 50a, 50b, 51a and 51b leads to the following matrix equation:

\[
\begin{align*}
\{v_{i,j+1}\} &= C_1 \{N\} \{v_{i,j}\} + C_2 \{v_{i,j-1}\} - C_1 \{F(t)\}
\end{align*}
\]

(52)

in which

\[
C_1 = -\frac{1}{(b_3)}
\]

(53)

\[
C_2 = b_3 C_1
\]

(54)
\[ b_3 = \frac{m}{(\Delta t)^2} \]  

(55)

\([N]\) is a symmetric coefficient matrix of the order \((N)\) by \((N)\). The terms of the matrix are defined below for \(N=6\), as an example.

\[ \mathbf{N}_{11} = -2b_1 + 2b_3 \]
\[ \mathbf{N}_{12} = 4b_1 \]
\[ \mathbf{N}_{13} = -2b_1 \]
\[ \mathbf{N}_{21} = 2b_1 - b_{44}B_e''(2) \]
\[ \mathbf{N}_{22} = -5b_1 - b_{33}B_e'(2) + 2b_{44}B_e''(2) + 2b_3 \]
\[ \mathbf{N}_{23} = 4b_1 + 2b_{33}B_e'(2) - b_{44}B_e''(2) \]
\[ \mathbf{N}_{24} = -b_1 - b_{33}B_e'(2) \]
\[ \mathbf{N}_{31} = -b_1 + b_{33}B_e'(3) \]
\[ \mathbf{N}_{32} = (4b_1) - 2b_{33}B_e'(3) - b_{44}B_e''(3) \]
\[ \mathbf{N}_{33} = -6b_1 + 2b_{44}B_e''(3) + 2b_3 \]
\[ \mathbf{N}_{34} = (4b_1) + 2b_{33}B_e'(3) - b_{44}B_e''(3) \]
\[ \mathbf{N}_{35} = -b_1 - b_{33}B_e'(3) \]
\[ \mathbf{N}_{41} = -b_1 + b_{33}B_e'(4) \]
\[ \mathbf{N}_{42} = (4b_1) - 2b_{33}B_e'(4) - b_{44}B_e''(4) \]
\[ \mathbf{N}_{43} = -6b_1 + 2b_{44}B_e''(4) + 2b_3 \]
\[ \mathbf{N}_{44} = (4b_1) + 2b_{33}B_e'(4) - b_{44}B_e''(4) \]
\[ \mathbf{N}_{45} = -b_1 - b_{33}B_e'(4) \]
\[ \mathbf{N}_{51} = -b_1 + b_{33}B_e'(5) \]
\[ \mathbf{N}_{52} = (4b_1) - 2b_{33}B_e'(5) - b_{44}B_e''(5) \]
\[ \mathbf{N}_{53} = -6b_1 + 2b_{44}B_e''(5) + 2b_3 \]
\[ \mathbf{N}_{54} = (4b_1) + 2b_{33}B_e'(5) - b_{44}B_e''(5) + b_2 \]
\[ \mathbf{N}_{55} = -b_1 + b_{33}B_e'(6) - b_{44}B_e''(5) \]
\[ \mathbf{N}_{56} = (4b_1) - 2b_{33}B_e'(6) + 2b_{44}B_e''(6) \]
\[ \mathbf{N}_{61} = -7b_1 - B_e'(6)b_{33} - b_{44}B_e''(6) + 2b_3 \]
\[ \mathbf{N}_{14} = \mathbf{N}_{15} = \mathbf{N}_{16} = \mathbf{N}_{25} = \mathbf{N}_{26} = \mathbf{N}_{36} = \mathbf{N}_{41} = \mathbf{N}_{51} = \mathbf{N}_{52} = \mathbf{N}_{61} = \mathbf{N}_{62} = \mathbf{N}_{63} = 0 \]
In which

\[ b_1 = \frac{B_e}{h^4} \]  \hspace{1cm} (56)

\[ b_2 = \frac{p}{h^2} \]  \hspace{1cm} (57)

\[ b_5 = \frac{\left(\frac{B_e}{h} - \frac{K_T}{2}\right)}{\left(\frac{B_e}{h} + \frac{K_T}{2}\right)} \]  \hspace{1cm} (58)

\[ b_9 = \frac{\left(\frac{B_e}{h} - \frac{K_B}{2}\right)}{\left(\frac{B_e}{h} + \frac{K_B}{2}\right)} \]  \hspace{1cm} (59)

\[ b_{10} = \frac{\left(\frac{2B_e}{h}\right)}{\left(\frac{B_e}{h} + \frac{K_T}{2}\right)} \]  \hspace{1cm} (60)

\[ b_{25} = \frac{K_{spr}}{h} \]  \hspace{1cm} (61)

\[ b_{25} = \frac{h}{B_e} \]  \hspace{1cm} (62)

\[ b_{33} = \frac{1}{h^3} \]  \hspace{1cm} (63)

\[ b_{44} = \frac{1}{h^2} \]  \hspace{1cm} (64)

\[ C_3 = \frac{1}{(b_1 + b_2 + b_{44})} \]  \hspace{1cm} (65)

Equation 52 is used to predict the deflections at \( v_{i,j+1} \) if \( v_{i,j} \) and \( v_{i,j-1} \) are known. In order to avoid negative time intervals due to the use of central finite-difference, startup equations are considered. The deflections at the first time interval are elastic deflections. The deflections at the second time interval were derived by Razzaq et al. [10]. The following special forward start-up difference equation \( v_{i,j} \) is derived. Assuming a constant acceleration:

\[
\frac{1}{\Delta t} [v(z_i, t_1) - v(z_i, 0)] = \frac{\partial v}{\partial t} (z_i, 0) + \frac{\Delta t}{2} \frac{\partial^2 v}{\partial t^2} (z_i, 0) + \frac{(\Delta t)^2}{6} \frac{\partial^3 v}{\partial t^3} [z_i, 0](\Delta t) \]  \hspace{1cm} (66)

Equation 67 also holds on the initial line, which is:

\[
EI \frac{\partial^4 v}{\partial z^4} (z_i, 0) + P \left[ \frac{\partial^2 v}{\partial z^2} (z_i, 0) \right] + m \frac{\partial^2 v}{\partial t^2} (z_i, 0) = F(t) \]  \hspace{1cm} (67)

Re-arranging the equation:

\[
\frac{\partial^2 v}{\partial t^2} (z_i, 0) = - \frac{EI}{m} \frac{\partial^4 v}{\partial z^4} (z_i, 0) \]  \hspace{1cm} (68)

Using the initial condition gives the following:

\[
\frac{\partial^2 v}{\partial t^2} (z_i, 0) = - \frac{EI}{m} \frac{\partial^4 v}{\partial z^4} (z_i, 0) \]  \hspace{1cm} (69)
It can be noticed that in the previous equation the partial derivative changed to ordinary derivative. Using the central difference for the second and fourth order ordinary derivatives of initial deflection results:

\[
\frac{\partial^2 v}{\partial t^2}(z_i, 0) = -\frac{E l}{m} - \frac{1}{h^4} (\omega_{i-2} - 4\omega_{i-1} + 6\omega_i - 4\omega_{i+1} + \omega_{i+2})
\] (70)

where \( \omega \) is the deflection at the second time increment. Substituting equation 69 into equation 70, and rearranging gives:

\[
v(z_i, t_1) = v(z_i, 0) - \frac{(\Delta t)^2 E l}{2h^4 m} (\omega_{i-2} - 4\omega_{i-1} + 6\omega_i - 4\omega_{i+1} + \omega_{i+2})
\] (71)

Rearranging the equation gives:

\[
v(z_i, t_1) = b_6 \omega_{i-2} + (-4b_6)\omega_{i-1} + (6b_6 + 1)\omega_i + (-4b_6)\omega_{i+1} + b_6 \omega_{i+2}
\] (72)

in which

\[
b_6 = -\frac{(\Delta t)^2 B_e}{2m h^4}
\] (73)

Using the iterative solution procedure described section 3.3.1.5 and using equation 52, the panel length and the time increment \( \Delta t \) should be kept small to avoid numerical instability.

### 3.3.1.5 Solution Procedure

Herein, a finite-difference iteration algorithm for the nonlinear dynamic analysis of the sub-assemblage is presented. The steps of the solution are as follows:

1. Find the deflection at specified sections along the cantilever for the first time increment using the elastic formula.
2. Find the deflection values at each of the sections for the second time increment using Equation 72.
3. Compute the moment \( (M_x) \) at each section according to the new lateral load value.
4. Compute the moment increments.
5. Use the tangent stiffness procedure from Reference 29 to compute the axial strains and curvatures due to the moment and the axial load.
6. Use the curvatures and axial strains found in step 5 to find the sectional properties for each section along the cantilever.
7. Find the deflections $v_{i,j}$ from the sectional properties found in step 6 and using equation 52.

8. Compare the deflections $v_{i,j}$ from step 6 with the deflections from the previous iteration $v_{i,j-1}$.

9. If $|v_{i,j}-v_{i,j-1}| >$ tolerance then repeat steps 1 to 7 for the same applied load until convergence occurs. In the elastic range, deflections will be the same without any iterations. In the inelastic range, there will be few iterations before convergence occur.

10. If $|v_{i,j}-v_{i,j-1}| <$ tolerance, repeat Steps 1 to 7 with the new value of forcing function $F(t)$ for each time until the end of time $t$.

The procedure was carried out using constant load increments in the elastic range. The load increments were decreased in the inelastic range for better and faster convergence.

### 3.3.2 Sub-assemblage

The impact load was applied at point Q on the sub-assemblage. The forcing function found from each test was used in the theoretical analysis. For the case with axial load applied, a constant axial force $P$ of 15 kip was applied at the long member of the sub-assemblage.

The sub-assemblage was restrained against vertical movement at its end $T$ with a translational spring stiffness $K_{spr}$. End rotations were restrained by pair of rotational springs at ends $T$ and $B$ with stiffness $K_T$ and $K_B$, respectively.

### 3.3.2.1 Governing Differential Equations

Harris et al. [51] and Razzaq et al. [10] expressed the elastic partial differential equation of motion of beam flexure with axial loading as follows:

$$EI \frac{\partial^4 v}{\partial z^4} + P \frac{\partial^2 v}{\partial z^2} + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (74)$$

in the elastic range, $EI$ is constant which means the second partial derivative of it will be zero. In the inelastic range $EI$ is constant and changing with load. Therefore, the inelastic partial differential equation of motion can be expressed as

$$\frac{\partial^2}{\partial z^2} \left( B_e \frac{\partial^2 v}{\partial z^2} \right) + P \frac{\partial^2 v}{\partial z^2} + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (75)$$
where $B_e$ is the elasto-plastic flexural rigidity; $v$ is the deflection of the beam. $P_{axial}$ is the Axial load, and $m$ is the unit mass for the member. The damping effect was not considered in this study.

Solving the differentiation of the first term of the equation leads to:

$$B_e \frac{\partial^4 v}{\partial z^4} + 2 \frac{\partial^3 v}{\partial z^3} \left( \frac{\partial^2 v}{\partial z^2} \right) + \frac{\partial^2 v}{\partial z^2} \left( \frac{\partial^2 B_e}{\partial z^2} \right) + P \frac{\partial^2 v}{\partial z^2} + m \frac{\partial^2 v}{\partial t^2} = F(t) \quad (76)$$

There is no closed form solution for equation 76 in the literature. Razzaq et al. [10] solved Equation 74 using the finite difference scheme. An iterative nonlinear finite difference scheme will be utilized for solving Equation 76.

$B_e$ is a function of $z$. In order to find the first and second partial derivatives of $B_e$, the Lagrangian polynomial will be fitted. For example, for nodes 2, 3, and 4:

$$L_2(z) = \frac{(z - z_3)(z - z_4)}{(z_2 - z_3)(z_2 - z_4)}$$

$$L_3(z) = \frac{(z - z_2)(z - z_4)}{(z_3 - z_2)(z_3 - z_4)}$$

$$L_4(z) = \frac{(z - z_2)(z - z_3)}{(z_4 - z_2)(z_4 - z_3)}$$

$f(z_2) = B_{e2}$

$f(z_3) = B_{e3}$

$f(z_4) = B_{e4}$

Lagrangian polynomial formula:

$$B_e(z) = \sum_{k=2}^{4} L_k(z). f(z_k)$$

Therefore, the equation for $B_e$ is:

$$B_e(z) = L_2(z). f(z_2) + L_3(z). f(z_3) + L_4(z). f(z_4)$$

The equation will be of second order:

$$B_e(z) = \alpha_1 z^2 + \alpha_2 z + \alpha_3 \quad (77)$$

where $\alpha$ is the coefficient for the $z$ variable. The first and second derivatives of $B_e$:

$$\frac{\partial B_e}{\partial z} = 2\alpha_1 z + \alpha_2 \quad (78a)$$

$$\frac{\partial^2 B_e}{\partial z^2} = 2\alpha_1 \quad (78b)$$
The terms in Equation 76 are dependent on both \( z \) and \( t \). The boundary condition at both ends B and T are the same boundary conditions that have been used in the quasi-static test for the sub-assemblage.

### 3.3.2.2 Initial Conditions

Equation 76 is also time dependent. Initial conditions are needed in the solution. The initial conditions for the problem are:

\[
\begin{align*}
v(z, 0) &= 0 \\
\frac{\partial v}{\partial t}(z, 0) &= 0
\end{align*}
\]

Equation 79 states that at time \( t = 0 \), the deflection is zero. Equation 80 states that the initial velocity is zero.

Initial plastification at \( t = 0 \) may occur and needs to be addressed with the following:

\[
M_x = -EI_{xe}v'' + M_{xp}
\]

Equation 81 needs to be enforced for any initially plastified areas in the cross-section.

### 3.3.2.3 Finite-Difference Formulation

Central finite-difference expressions were used to solve the equations derived in the previous section. A total of \( N+1 \) nodes were used on the sub-assemblage and the supports, where, Node 1 is at end T and Node \( N+1 \) is at Node B. Nodes 0 and \( N+2 \) are fictitious nodes outside the sub-assemblage. Using second order finite difference expressions (Reference 2), Equation 79 can be written as:

\[
\begin{align*}
\frac{B_e}{h^4} &\left( v_{i-2,j} - 4v_{i-1,j} + 6v_{i,j} - 4v_{i+1,j} + v_{i+2,j} \right) \\
+ &\frac{2}{h^3} \left( -v_{i-2,j} + 2v_{i-1,j} - 2v_{i+1,j} + v_{i+2,j} \right) \left( \frac{\partial B_e}{\partial z} \right) \\
+ &\frac{1}{h^2} \left( v_{i-1,j} - 2v_{i,j} + v_{i+1,j} \right) \left( \frac{\partial^2 B_e}{\partial z^2} \right) + \frac{P}{h^2} \left( v_{i-1,j} - 2v_{i,j} + v_{i+1,j} \right) \\
+ &\frac{m}{(\Delta t)^2} \left( v_{i,j-1} - 2v_{i,j} + v_{i,j+1} \right) + \frac{c}{2\Delta t} \left( -v_{i,j-1} + v_{i,j+1} \right) = F(t)
\end{align*}
\]

Equation 82.
In which, \( h \) is the panel length along the z-axis of the sub-assemblage, \( \Delta t \) is the time interval, the subscript \( i \) refers to the \( i \)th node point over the domain \( 0 < x < L \), and the subscript \( j \) refers to the number of time increments such that the time at \( j \) is given by the following equation:

\[
t_j = j(\Delta t), \quad \text{for each} \quad j = 0, 1, 2, 3, ...
\]

Similarly, the boundary conditions 26a, 26b, 30 and 37 can be expressed in finite difference form as follows:

\[
v(L, t) = 0 \quad \text{(83)}
\]

\[
\frac{B_e}{2h^4}(v_{0,j}) - \left( \frac{-P}{2h^2} + \frac{B_e}{h^4} + \left( \frac{\partial B_e}{\partial z} \right) \left( \frac{1}{h^3} \right) \right) (v_{1,j})
\]

\[
- \left( \frac{P}{2h^2} + \frac{B_e}{h^4} + \left( \frac{\partial B_e}{\partial z} \right) \left( \frac{1}{h^3} \right) \right) (v_{3,j}) - \frac{B_e}{2h^4} (v_{4,j})
\]

\[
- \left( \frac{K_{spr}}{h} - \left( \frac{\partial B_e}{\partial z} \right) \left( \frac{2}{h^3} \right) \right) (v_{2,j}) = 0 \quad \text{(84)}
\]

\[
\left( \frac{B_e + \left( \frac{K_T}{2} \right)}{h} \right) (v_{1,j}) - \left( \frac{2B_e}{h} \right) (v_{2,j}) - \left( \frac{-B_e}{h} + \left( \frac{K_T}{2} \right) \right) (v_{3,j}) = 0 \quad \text{(85)}
\]

\[
\left( \frac{B_e + \left( \frac{KB}{2} \right)}{h} \right) (v_{N+2,j}) - \left( \frac{2B_e}{h} \right) (v_{N+1,j}) - \left( \frac{-B_e}{h} + \left( \frac{KB}{2} \right) \right) (v_{N,j}) = 0 \quad \text{(86)}
\]

Applying Equation 82 at \( i=1, 2, 3 \ldots \) (N) and invoking conditions 83 through 86 leads to the following matrix equation

\[
\{v_{i,j+1}\} = C_1 [\mathbf{N}] \{v_{i,j}\} + C_2 \{v_{i,j-1}\} - C_1 \{F(t)\} \quad \text{(87)}
\]

in which

\[
C_1 = -\frac{1}{(b_3 + b_4)} \quad \text{(88a)}
\]

\[
C_2 = b_3 C_1 \quad \text{(88b)}
\]

\[
b_3 = \frac{m}{(\Delta t)^2} \quad \text{(88c)}
\]

\[
b_4 = \frac{c}{2\Delta t} \quad \text{(88d)}
\]

\([\mathbf{N}]\) is a symmetric coefficient matrix of the order (N) by (N). The terms of the matrix are defined below for \( N=6 \) as an example.
\begin{align*}
\kappa_{11} &= (1 - b_{11} B'_e(1)) \ast (-b_2 + 2b_1 + B'_e(1)2b_{33})b_{10} + (1 - b_{11} B'_e(1)) \\
&\ast (2b_{25} - 4B'_e(1)b_{33}) + (-4b_1b_{10}) + 6b_1 + 2b_{33}b_{10}B'_e(1) \\
&+ (-2b_{44} B''_e(1)) + b_2b_{10} + (-2b_2) + (-2b_3) + b_{44}b_{10}BP2(1) \\
\kappa_{12} &= (1 - b_{11} B'_e(1)) \ast (-b_2 + 2b_1 + 2B'_e(1)b_{33})(-b_5) + (1 - b_{11} B'_e(1)) \\
&\ast (b_2 - 2b_1 - 2B'_e(1)b_{33}) + (-4b_1)(-b_5) - 4b_1 - 2b_{33}b_5 B'_e(1) \\
&+ (-2b_{33} B'_e(1)) + b_{44}(-b_5)B''_e(1) + b_{44}B''_e(1) + (b_2)(-b_5) + b_2 \\
\kappa_{13} &= (-b_{33} B'_e(1)) + 2b_1 + b_{33} B'_e(1) \\
\kappa_{21} &= b_1b_{10} + (-b_{33}b_{10} B'_e(2)) + (-4b_1) + 2b_{33} B'_e(2) + b_{44}B''_e(2) + b_2 \\
\kappa_{22} &= b_1(-b_5) - b_{33}(-b_5)B'_e(1) + 6b_1 - 2b_{44} B''_e(2) - 2b_2 - 2b_3 \\
\kappa_{23} &= -4b_1 - 2b_{33} B'_e(1) + b_{44}B''_e(2) + 2b_2 \\
\kappa_{24} &= b_1 + b_{33} B'_e(2) \\
\kappa_{31} &= b_1 - b_{33} B'_e(3) \\
\kappa_{32} &= (-4b_1) + 2b_{33} B'_e(3) + b_{44}B''_e(3) + b_2 \\
\kappa_{33} &= 6b_1 - 2b_{44} B''_e(3) - 2b_2 - 2b_3 \\
\kappa_{34} &= (-4b_1) - 2b_{33} B'_e(3) + b_{44}B''_e(3) + b_2 \\
\kappa_{35} &= b_1 + b_{33} B'_e(3) \\
\kappa_{42} &= b_1 - b_{33} B'_e(4) \\
\kappa_{43} &= (-4b_1) + 2b_{33} B'_e(4) + b_{44}B''_e(4) + b_2 \\
\kappa_{44} &= 6b_1 - 2b_{44} B''_e(4) - 2b_2 - 2b_3 \\
\kappa_{45} &= (-4b_1) - 2b_{33} B'_e(4) + b_{44}B''_e(4) + b_2 \\
\kappa_{46} &= b_1 + b_{33} B'_e(4) \\
\kappa_{53} &= b_1 - b_{33} B'_e(5) \\
\kappa_{54} &= (-4b_1) + 2b_{33} B'_e(5) + b_{44}B''_e(5) + b_2 \\
\kappa_{55} &= 6b_1 - 2b_{44} B''_e(5) - 2b_2 - 2b_3 \\
\kappa_{56} &= (-4b_1) - 2b_{33} B'_e(5) + b_{44}B''_e(5) + b_2 \\
\kappa_{64} &= b_1 - b_{33} B'_e(6) \\
\kappa_{65} &= (-4b_1) + 2b_{33} B'_e(6) + b_{44}B''_e(6) + b_2
\end{align*}
\[ \kappa_{66} = -b_1 b_9 - b_2 B_e'(6) b_{33} + 6 b_1 - 2 b_{44} B_e''(6) - 2 b_2 - 2 b_3 \]

\[ \kappa_{14} = \kappa_{15} = \kappa_{16} = \kappa_{25} = \kappa_{26} = \kappa_{36} = \kappa_{41} = \kappa_{51} = \kappa_{52} = \kappa_{61} = \kappa_{62} = \kappa_{63} = 0 \]

In which

\[ b_1 = \frac{B_e}{h^4} \] (89a)

\[ b_2 = \frac{p}{h^2} \] (89b)

\[ b_5 = \left( \frac{B_e}{h} - \frac{K_T}{2} \right) / \left( \frac{B_e}{h} + \frac{K_T}{2} \right) \] (89c)

\[ b_9 = \left( \frac{B_e}{h} - \frac{K_B}{2} \right) / \left( \frac{B_e}{h} + \frac{K_B}{2} \right) \] (89d)

\[ b_{10} = \left( 2 \frac{B_e}{h} \right) / \left( \frac{B_e}{h} + \frac{K_T}{2} \right) \] (89e)

\[ b_{25} = \frac{K_{spr}}{h} \] (89f)

\[ b_{25} = \frac{h}{B_e} \] (89g)

\[ b_{33} = \frac{1}{h^3} \] (89h)

\[ b_{44} = \frac{1}{h^2} \] (89i)

\[ C_3 = \frac{1}{(b_1 + b_2 + b_{44})} \] (89j)

Equation 87 is used to predict the deflections at \( v_{i,j+1} \) if \( v_{i,j} \) and \( v_{i,j-1} \) are known. In order to avoid negative time intervals due to the use of central finite-difference, a startup equation is considered. The deflections at the first time interval are elastic deflections. The deflections at the second time interval was derived at Razzaq et. al. [10]. The following special forward start-up difference equation \( v_{i,j} \) is derived. Starting with the assuming a constant acceleration:

\[ \frac{1}{\Delta t} [v(z_i, t_1) - v(z_i, 0)] = \frac{\partial v}{\partial t} (z_i, 0) + \frac{\Delta t}{2} \frac{\partial^2 v}{\partial t^2} (z_i, 0) + \frac{(\Delta t)^2}{6} \frac{\partial^3 v}{\partial t^3} (z_i, 0) \] (90)

Equation 100 also holds on the initial line, which is:

\[ EI \frac{\partial^4 v}{\partial z^4} (z_i, 0) + P \left[ \frac{\partial^2 v}{\partial z^2} (z_i, 0) \right] + m \frac{\partial^2 v}{\partial t^2} (z_i, 0) + c \frac{\partial v}{\partial t} (z_i, 0) = F(t) \] (91)

Re-arranging the equation:

\[ \frac{\partial^2 v}{\partial t^2} (z_i, 0) = - \frac{EI}{m} \frac{\partial^4 v}{\partial z^4} (z_i, 0) - \frac{P}{m} \left[ \frac{\partial^2 v}{\partial z^2} (z_i, 0) \right] - c \frac{\partial v}{m} (z_i, 0) \] (92)

Using the initial condition gives the following:
\[
\frac{\partial^2 v}{\partial t^2}(z_i, 0) = -\frac{EI}{m} \frac{d^4 v}{dz^4}(z_i, 0) - \frac{p}{m} \left[ \frac{d^2 v}{dz^2}(z_i, 0) \right]
\]  

(93)

It can be noticed that in the previous equation the partial derivative changed to an ordinary derivative. Using the central difference for the second and fourth order ordinary derivatives of initial deflection results:

\[
\frac{\partial^2 v}{\partial t^2}(z_i, 0) = -\frac{EI}{m} \frac{1}{h^4} (\omega_{i-2} - 4\omega_{i-1} + 6\omega_i - 4\omega_{i+1} + \omega_{i+2})
\]

\[-\frac{p}{m} \left[ \frac{1}{h^2} (\omega_{i-1} - 2\omega_i + \omega_{i+1}) \right]
\]  

(94)

where \(\omega\) is the deflection at the second time increment. Substituting equation 94 into equation 90, and rearranging gives:

\[
v(z_i, t_1) = v(z_i, 0) - \frac{(\Delta t)^2 EI}{2h^4} (\omega_{i-2} - 4\omega_{i-1} + 6\omega_i - 4\omega_{i+1} + \omega_{i+2})
\]

\[-\frac{(\Delta t)^2 p}{2} \left[ \frac{1}{h^2} (\omega_{i-1} - 2\omega_i + \omega_{i+1}) \right]
\]

(95)

which takes the form below:

\[
v(z_i, t_1) = b_6 \omega_{i-2} + (-4b_6 + b_7)\omega_{i-1} + (6b_6 - 2b_7 + 1)\omega_i
\]

\[+(-4b_6 + b_7)\omega_{i+1} + b_6\omega_{i+2}
\]  

(96)

in which

\[
b_6 = -\frac{(\Delta t)^2 B_e}{2mh^4}
\]  

(97)

\[
b_7 = -\frac{(\Delta t)^2 p}{2mh^2}
\]  

(98)

\[
b_8 = -\frac{(\Delta t)^2 p}{2m}
\]  

(99)

Using the iterative solution procedure described section 3.3.2.4 and using Equation 87, the panel length and the time increment \(\Delta t\) should be kept small to avoid numerical instability.

### 3.3.2.4 Solution Algorithm

Herein a finite difference iteration algorithm for the nonlinear dynamic analysis of the sub-assemblage is presented. The step of the solution are as follows:

1. Find the deflection at specified sections along the cantilever for the first time increment using elastic formula.
2. Find the deflection values at each of the sections for the second time increment using Equation 96.

3. Compute the moment \( (M_x) \) at each section according to the new lateral load value using Equations 21 and 23.

4. Compute the change in moment \( M_x \) and axial load \( P_{\text{axial}} \).

5. Use the tangent stiffness procedure from Reference 29 to compute the axial strains and curvatures due to the moment and axial load.

6. Use the curvatures and axial strains found in step 5 to find the sectional forces and deformations for each section along the sub-assemblage.

7. Find the deflections \( v_{i,2} \) from the sectional forces and deformations found in step 6 and using Equation 87.

8. Compare the deflections \( v_{i,j} \) from step 6 with the deflections from the previous iteration \( v_{i,j-1} \).

9. If \( |v_{i,j} - v_{i,j-1}| \) tolerance then repeat steps 1 to 7 for the same applied load until convergence occurs. In the elastic range, deflections will be the same without any iterations. In the inelastic range, there will be few iterations before convergence occur.

10. If \( |v_{i,j} - v_{i,j-1}| \) tolerance Repeat steps 1 to 7 with the new value of forcing function \( F(t) \) for each time until the end of time \( t \).

The procedure was carried out using constant load increments in the elastic range. Those load increments were decreased in the inelastic range for better and faster convergence.

### 3.3.3 Numerical study

In this section, the outcome of the inelastic procedure described above will be discussed for the different tests. The analysis procedure was applied to both elastic and inelastic ranges. The forcing function from the experimental results were used in the theoretical analysis. The following subsections present both experimental as well as theoretical values. In the following figures of this chapter, ‘Theo’ stands for theoretical and ‘Exp’ stands for experimental.

#### 3.3.3.1 Cantilever

For the C1-1 test analysis, Figures 278 and 279 show the strain-time relation for SG1 and SG2 on the cantilever with maximum strain values of 0.000557 and -0.000557 in./in., respectively.
The strain values indicate that the cantilever at B is in the elastic range. Figure 280 shows the moment-curvature relationship at B with a maximum moment value of 9.4 kip-in.

For the C1-2 test analysis, Figures 281 and 282 show the strain-time relation for SG1 and SG2 on the cantilever with maximum strain values of 0.001046 and -0.001046 in./in., respectively. The strain values indicate that the cantilever at B is in the elastic range. Figure 283 shows the moment-curvature relationship at B with a maximum moment value of 17.8 kip-in.

For the C1-3 test analysis, Figures 284 and 285 show the strain-time relation for SG1 and SG2 on the cantilever with maximum strain values of 0.002817 and -0.002817 in./in., respectively. The strain values indicate that the cantilever at B is in the inelastic range. Figure 286 shows the moment-curvature relationship at B with a maximum moment value of 37.7 kip-in.

For the C1-4 test analysis, Figures 287 and 288 show the strain-time relation for SG1 and SG2 on the cantilever with maximum strain values of 0.004512 and -0.004512 in./in., respectively. The strain values indicate that the cantilever at B is in the inelastic range. Figure 289 shows the moment-curvature relationship at B with a maximum moment value of 39.8 kip-in. This moment value means that section B on the cantilever has reached a full moment capacity and developed a plastic hinge.

### 3.3.3.2 Non-Retrofitted Sub-assemblage Impact tests without axial load

For analysis of these tests, there was no axial load applied. The forcing function was applied at Q. The strain and moment values were presented and maximum values were identified for each test.

For the SA2-1 test analysis, Figures 290 - 293 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000154, -0.000154, -0.000263, and 0.000263 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 294 and 295 show the moment-curvature relationships at Q and B with maximum moment values of 6.5 and 3.5 kip-in, respectively. The moment value at Q was higher than the moment value at B.

For the SA2-2 test analysis, Figures 296 - 299 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000362, -0.000362, -0.000495, and 0.000495 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 300 and 301 show the moment-curvature relationships at
Q and B with maximum moment values of 13.8 and 7.1 kip-in, respectively. Section Q developed higher moment value than section B.

For the SA2-3 test analysis, Figures 302 - 305 show the strain-time relation for the locations of SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000661, -0.000661, -0.001118, and 0.001118 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range for steel. Figures 306 and 307 show the moment-curvature relationships at Q and B with maximum moment values of 23.7 and 12 kip-in, respectively. It is clear that Q had higher strain and moment values than B.

For the SA2-4 test analysis, Figures 308 - 311 show the strain-time relation for the locations of SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.001880, -0.001880, -0.004339, and 0.004339 in./in., respectively. The strain values indicate that the sub-assemblage is in the elastic range at B and in the inelastic range at Q. Figures 312 and 313 show the moment-curvature relationships at Q and B with maximum moment values of 39.8 and 36.2 kip-in, respectively. These moment values mean that only section Q has reached a full moment capacity and developed a plastic hinge. Section B reached 90% of the moment capacity for the section. Figure 238 shows the spread of plasticity in the cross sections at sections Q and B.

3.3.3.3 Non-Retrofitted Sub-assemblage Impact tests with axial load

For analysis of these tests, there was a 15 kip axial load applied during the impact. Axial strain values from the axial load $P_{axial}$ have were to the flexural strains from the impact load $F(x)$. The forcing function has been applied at Q. The strain and moment values were presented and maximum values were identified for each test.

For the SA4-1 test analysis, Figures 314 - 317 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000365, -0.000850, -0.000819, and -0.000192 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 318 and 319 show the moment-curvature relationships at Q and B with maximum moment values of 7.6 and 5.4 kip-in, respectively. By comparing this test to SA2-1, SA4-1 also had a higher moment value at Q than B.

For the SA4-2 test analysis, Figures 320 - 323 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000009, -0.001182, -0.001110, and 0.000044 in./in., respectively. The strain values indicate that the sub-assemblage at
B and Q is in the elastic range. Figures 324 and 325 show the moment-curvature relationships at Q and B with maximum moment values of 15.3 and 9.9 kip-in, respectively. Both SA4-2 and SA2-2 had a higher moment value at Q than the moment value at B.

For the SA4-3 test analysis, Figures 326 - 329 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000365, -0.001468, -0.001716, and 0.000649 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 330 and 331 show the moment-curvature relationships at Q and B with maximum moment values of 26.3 and 18 kip-in, respectively. Section Q had developed higher moment value than section B.

For the SA4-4 test analysis, Figures 332 - 335 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.001012, -0.002185, -0.012247, and 0.005111 in./in., respectively. The strain values indicate that the sub-assemblage is in the elastic range at B and in the inelastic range at Q. Figures 336 and 337 show the moment-curvature relationships at Q and B with maximum moment values of 37.1 and 27.3 kip-in, respectively. These moment values mean that only section Q reached a full moment capacity and developed a plastic hinge. Section B reached 73% of the moment capacity for the section. Figure 235 shows the spread of plasticity in the cross sections at sections Q and B.

### 3.3.3.4 CFRP Retrofitted Sub-assemblage Impact tests without axial load

For analysis of these tests, there was no axial load applied. Two layers of CFRP strips were added to the bottom of the long member of the sub-assemblage. The forcing function were applied at Q. The strain and moment values have been presented and maximum values were identified for each test.

For the SA6-1 test analysis, Figures 338 - 343 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000260, -0.000203, -0.000273, -0.000431, 0.000234, and 0.000542 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 344 and 345 show the moment-curvature relationships at Q and B with maximum moment values of 8.8 and 6 kip-in, respectively. With CFRP retrofitting, Q also developed a higher moment at Q than at B.

For SA6-2 test analysis, Figures 346 - 351 show strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000449, -0.000314, -0.000444, -
0.000689, 0.000564, and 0.000903 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Figures 352 and 353 show the moment-curvature relationships at Q and B with maximum moment values of 15.6 and 10 kip-in, respectively. Both non-retrofitted and CFRP retrofitted tests developed a higher moment value at Q than the maximum moment value at B.

For the SA6-3 test analysis, Figures 354 - 359 show the strain-time relation for the locations of SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000692, - 0.000513, - 0.000662, - 0.001216, 0.000751, and 0.001352 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range for steel. Figures 360 and 361 show the moment-curvature relationships at Q and B with maximum moment values of 32.8 and 20.6 kip-in, respectively. It is clear that Q had higher strain and moment values than B.

For the SA6-4 test analysis, Figures 362 - 367 show the strain-time relation for SG1, SG2, SG4 and SG5 on the sub-assemblage with maximum strain values of 0.001556, - 0.001146, - 0.001381, - 0.004341, and 0.004310 in./in., respectively. The strain values indicate that the sub-assemblage is in the elastic range at B and in the inelastic range at Q. Figures 368 and 369 show the moment-curvature relationships at Q and B with maximum moment values of 49 and 38.7 kip-in, respectively. These moment values mean that only section Q has reached a full moment capacity and developed a plastic hinge. Section B has reached 79% of the moment capacity for the section. Figure 240 shows the spread of plasticity in the cross sections at sections Q and B.

### 3.3.3.5 CFRP Retrofitted Sub-assemblage Impact tests with axial load

For analysis of these tests, there was a 15 kip axial load applied before the impact. Axial strain values from the axial load $P_{axial}$ were added to the flexural strains from the impact load $F(x)$. Two layers of CFRP strips were installed to the bottom of the long member of the sub-assemblage. The forcing function was applied at Q. The strain and moment values were presented and maximum values were identified for each test.

For the SA8-1 test analysis, Figures 370 - 375 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of - 0.000128, - 0.000793, - 0.000827, - 0.000959, - 0.000055, and 0.000063 in./in., respectively. The strain values indicate
that the sub-assemblage at B and Q is in the elastic range. The strain values lead to a higher maximum moment value at Q than the maximum moment value at B. Figures 376 and 377 show the moment-curvature relationships at Q and B with maximum moment values of 11.2 and 7.2 kip-in, respectively.

For the SA8-2 test analysis, Figures 378 - 383 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of - 0.000013, - 0.000904, - 0.000949, - 0.0001204, 0.000147, and 0.000325 in./in., respectively. The strain values indicate that the sub-assemblage at B and Q is in the elastic range. Therefore Q developed a higher moment value than B. Figures 384 and 385 show the moment-curvature relationships at Q and B with maximum moment values of 17.1 and 10.1 kip-in, respectively.

For the SA8-3 test analysis, Figures 386 - 391 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000510, - 0.001413, - 0.001511, - 0.002329, 0.000426, and 0.001195 in./in., respectively. The strain values indicate that the sub-assemblage is in the elastic range at B and in the inelastic range at Q. Figures 392 and 393 show the moment-curvature relationships at Q and B with maximum moment values of 33.8 and 24 kip-in, respectively. The values showed that Q had a higher maximum moment value than B.

For the SA8-4 test analysis, Figures 394 - 399 show the strain-time relation for SG1, SG2, SG4, and SG5 on the sub-assemblage with maximum strain values of 0.000866, - 0.001675, - 0.001799, - 0.010826, 0.002561, and 0.004370 in./in., respectively. The strain values indicate that the sub-assemblage is in the elastic range at B and in the inelastic range at Q. Figures 400 and 401 show the moment-curvature relationships at Q and B with maximum moment values of 46 and 31 kip-in, respectively. These moment values mean that only section Q reached a full moment capacity and developed a plastic hinge. Section B has reached 67% of the moment capacity for the section. Figure 237 shows the spread of plasticity in the cross sections at sections Q and B.

3.4 Convergence Study

Typically the finer the mesh or increments is the more accurate the solution will be. However, it will require more calculation time and a large amount of memory. It is best to find the minimum number of elements that will give a converged solution. This section describes a study about the optimum number of sections, elements, load increments and time increments that was performed
to arrive at a satisfactory solution. The results from this convergence study were used in the analysis.

3.4.1 Cross Sectional Elements

Results from the moment-curvature curve for different cross sectional mesh size were examined. First, 15 elements were used, and the maximum moment was 23.26 kip-in. For 30 elements, the maximum moment was 27.13 kip-in. For 60 elements, the maximum moment was 40.93 kip-in. For 120 elements, it was 40.93 kip-in. For 120 elements and more, the results were more stable. Figure 238 shows the maximum moment using different number of elements.

3.4.2 Longitudinal sections along the sub-assemblage

Several tests were performed, and each test considered a different length of equally-sized portions of the sub-assemblage. If 60 sections were used, the maximum load was 3728 lb. For 80 sections, the maximum load was 3807 lb. For 100 sections, the maximum load was 3853 lb. If 132 sections were used, the maximum load was 3860 lb. Figure 239 shows the maximum load capacity using different elements.

3.4.3 Load increments

Results from the inelastic load-deflection curve for different load increments were examined. A 160 lb. load increment was used and the maximum load was 4120 lb. If a 120 lb. load increment was used, the result was 4320 lb. If 100 lb. load increment was used, the maximum load was 4425 lb. If a 60 lb. load increment was used, the maximum load was 4425 lb. Figure 240 shows the maximum load capacity using different load increments.

3.4.4 Time increments

A study was performed to select the time increments test suited for stable results. It was found that for any time increments greater than 0.00001 sec., the results diverged and the maximum strain for SG1 was 10^217 in./in. when the time increment was decreased to 0.0000127 sec., the results became stable and the maximum strain for SG1 was 0.003306 in./in. If a 0.00001 sec. time
increment was used, the maximum strain from SG1 was 0.00306 in./in. Figure 241 shows the time maximum strain from SG1 for different time increments.
CHAPTER 4
COMPARISON OF RESULTS AND DISCUSSION

This chapter presents a comparison between the experimental and the theoretical results. The theory presented in the previous chapter showed a good agreement with the experimental results.

4.1 Sub-assemblage Behavior under Quasi-Static Loading

Table 16 compares the maximum load carrying capacity for quasi-static tests based on the experimental results and the inelastic finite-difference analysis used. Table 17 compares the maximum moments from sections Q and B, on the sub-assemblage, for quasi-static tests based on the experimental results and the inelastic finite-difference analysis used.

For specimen SA3, the experimental maximum load was 4770 lb. and the theoretical maximum load was 4425 lb. The theoretical result was less than the experimental by 8%. Figure 242 shows the comparison between the theoretical and experimental load-deflection curves. Figures 243 - 246 show theoretical and experimental load-strain curves for SG1, SG2, SG4 and SG5, respectively. There is an overall similarity in the shapes of all the load-strain curves. At section Q, the experimental maximum moment was 40.2 kip-in and the theoretical value was 40.4 kip-in. Figure 247 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. At section B, the experimental maximum moment was 40.18 kip-in and the theoretical value was 40.04 kip-in. Figure 248 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Good agreement was achieved between the experimental results and the theoretical results.

For specimen SA5, the experimental maximum load was 3610 lb. and the theoretical maximum load was 3860 lb. The theoretical result was more than the experimental by 7%. Figure 250 shows the comparison between the theoretical and experimental load-deflection curves. The shapes of the two curves was very similar but there was a small difference in the deflection. The experimental deflection values didn’t start from zero because the beam-column had a little deflection when the axial load was applied which was not the case for the theoretical results. Figures 251 - 254 show the theoretical and experimental strain-time curve for SG1, SG2, SG4 and
SG5, respectively. The pattern of all of the load-strain curves was the same. At section Q, the experimental maximum moment was 35.8 kip-in and the theoretical value was 36 kip-in. The difference between theoretical and the experimental results was 1%. Figure 255 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. At point B, the experimental maximum moment was 35.8 kip-in and the theoretical value was 35.8 kip-in. Figure 256 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The peak values from the moments for both curves was the same. However, the experimental curves showed a sudden rise at the beginning because of the bending in the beam caused by the axial load.

For specimen SA7, the experimental maximum load was 6000 lb. and the theoretical maximum load was 5400 lb. The difference between the theoretical and the experimental results was 11% which is considered a good percentage of difference. Figure 258 shows the comparison between the theoretical and experimental load-deflection curves. Figures 259 - 264 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. At section Q, the experimental maximum moment was 48.5 kip-in and the theoretical value was 47.9 kip-in. The difference between the theoretical and the experimental results was 2%. Figure 265 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. At section B, the experimental maximum moment was 48.1 kip-in and the theoretical value was 47.6 kip-in. The difference between theoretical and the experimental results was 2%. Figure 266 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. It was found that good agreement was reached between the tested and the predicted results for moment-curvature curves.

For specimen SA9, the experimental maximum load was 5310 lb. and the theoretical maximum load was 5050 lb. The difference between theoretical and the experimental results was 5%. Figure 268 shows the comparison between the theoretical and experimental load-deflection curves. The theoretical values showed a very good agreement to the experimental results except for the initial experimental deflection value, which was caused by the axial load. Figures 269 - 274 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. All load-strain curves showed a similar pattern. They started with an initial negative axial strain due to compression, and bending strains were added as the load increased. At section Q, the experimental maximum moment was 46 kip-in and the theoretical value was 45.5
kip-in. The difference between the theoretical and the experimental results was 1%. Figure 276 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. At section B, the experimental maximum moment was 39 kip-in and the theoretical value was 33 kip-in. The difference between the theoretical and the experimental results is 18%. Figure 277 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Generally, good agreement was achieved between the experimental and the theoretical results. The moment values from experimental strains showed a sudden rise due to the initial bending from the axial load, as in test SA5.

4.2 Cantilever Behavior under Impact Loading

Table 18 compares the maximum moments from sections Q and B, on the sub-assemblage for the impact tests based on the experimental results and the inelastic finite difference analysis used for tests C1-1, C1-2, C1-3, and C1-4.

For test C1-1, Figures 278 and 279 show theoretical and experimental strain-time curve for SG1 and SG2, respectively. Both figures showed the same trending and the peak values were very close. Table 19 shows the ratios of experimental to theoretical values for these strain gauges. The ratios between the tested to the predicted strain results ranged from 0.99 to 1.17. For the same test, the experimental maximum moment at section B was 10.8 kip-in and the theoretical value was 9.4 kip-in. The difference between theoretical and the experimental results was 15%. Figure 280 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The experimental and the theoretical moment values were in good agreement and they were in the elastic range.

For test C1-2, Figures 281 and 282 show the theoretical and the experimental strain-time curve for SG1 and SG2, respectively. Table 20 shows the ratios of the experimental to the theoretical values for these strain gauges. The ratios between the tested to the predicted strain results ranged from 0.93 to 1.01. For the same test, the experimental maximum moment at section B was 20.3 kip-in and the theoretical value was 17.8 kip-in. The difference between the theoretical and the experimental results was 14%. Figure 283 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. It was found that good agreement was reached between the tested and the predicted results.
For test C1-3, Figures 284 and 285 show the theoretical and the experimental strain-time curve for SG1 and SG2, respectively. Table 21 shows the ratios of the experimental to the theoretical values for these strain gauges. The ratios between the tested to the predicted strain results ranged from 0.89 to 0.86, which are considered to be reasonable results. There was an overall good agreement in the shape of all the load-strain curves. For the same test, the experimental maximum moment at section B was 38.1 kip-in and the theoretical value was 37.7 kip-in. The difference between theoretical and the experimental results was 2%. Figure 286 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Both the experimental and the theoretical curves were very similar and their peak values were very close.

For test C1-4, Figures 287 and 288 show the theoretical and the experimental strain-time curve for SG1 and SG2, respectively. Table 22 shows the ratios of experimental to theoretical values for these strain gauges. The ratios between the tested to the predicted strain results ranged from 0.94 to 0.99. For the same test, the experimental maximum moment at section B was 39.5 kip-in and the theoretical value was 39.2 kip-in. Both the theoretical and the experimental results formed a plastic hinges. Figure 289 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. It can be seen that there was good agreement between the predicted values and the experimental values for the strains and the moments. A plastic hinge was developed theoretically and experimentally.

4.3 Sub-assemblage Behavior under Impact Loading

Table 23 compares the maximum moments from sections Q and B on the sub-assemblage, for impact tests based on the experimental results and the inelastic finite difference analysis used for tests SA2-1, SA2-2, SA2-3 and SA2-4.

For test SA2-1, Figures 290 - 293 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 24 shows the ratios of experimental to theoretical values for these strain gauges. The overall shapes of the graphs are in reasonable agreement. The ratios between the tested to the predicted strain results ranged from 0.83 to 1.8. For the same test, the experimental maximum moment at section Q was 5.9 kip-in and the theoretical value was 6.5 kip-in. The difference between the theoretical and the experimental results was 9%. Figure 294 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 4 kip-
in and the theoretical value was 3.5 kip-in. The difference between the theoretical and the experimental results was 14%. Figure 295 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The theoretical and experimental maximum moment values for both B and Q were in satisfactory agreement.

For test SA2-2, Figures 296 - 299 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 25 shows the ratios of the experimental to the theoretical values for these strain gauges. The tested results and predicted results show good agreement. The ratios between the tested to the predicted strain results ranged from 0.88 to 1.14. For the same test, the experimental maximum moment at section Q was 11.4 kip-in and the theoretical value was 13.8 kip-in. The difference between the theoretical and the experimental results was 17%. Figure 300 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 7.4 kip-in and the theoretical value was 7.1 kip-in. The difference between theoretical and the experimental results was 4%. Figure 301 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. The experimental and the theoretical moment values were in good agreement and they are in the elastic range.

For test SA2-3, Figures 302 - 305 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 26 shows the ratios of experimental to theoretical values for these strain gauges. The predicted and theoretical load versus rotation curves has good correlation. The ratios between the tested to the predicted strain results ranged from 0.77 to 1.12. For the same test, the experimental maximum moment at section Q was 22 kip-in and the theoretical value was 23.7 kip-in. The difference between the theoretical and the experimental results was 7%. Figure 306 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 13.8 kip-in and the theoretical value was 12 kip-in. The difference between theoretical and the experimental results was 15%. Figure 307 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Both experimental and theoretical curves were very similar and their peak values were very close.

For test SA2-4, Figures 308 -311 show theoretical and experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. At the peak load, the predicted and theoretical load versus rotation curves showed good correlation, but at the vibration stage, the two curves were slightly away from
each other. Table 27 shows the ratios of experimental to theoretical values for these strain gauges. The ratios between the tested to the predicted strain results ranged from 0.88 to 1.06. For the same test, the experimental maximum moment at section Q was 39.3 kip-in and the theoretical value was 39.8 kip-in. There was no difference between the theoretical and the experimental results. Figure 312 shows the comparison between the theoretical and the experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 36.2 kip-in and the theoretical value was 36.2 kip-in. There was no difference between theoretical and the experimental results. Figure 313 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. It was found that good agreement was reached between the tested and the predicted results.

Table 28 compares the maximum moments from sections Q and B, on the sub-assemblage, for impact tests based on the experimental results and the inelastic finite-difference analysis used for tests SA4-1, SA4-2, SA4-3 and SA4-4.

For test SA4-1, Figures 314 - 317 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 29 shows the ratios of the experimental to the theoretical values for these strain gauges. There was an overall similarity in the shape of all the strain-time curves. The ratios between the tested to the predicted strain results ranged from 0.58 to 0.99. For the same test, the experimental maximum moment at section Q was 6.2 kip-in and the theoretical value was 7.6 kip-in. The difference between the theoretical and the experimental results is 19%. Figure 318 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 5 kip-in and the theoretical value was 5.4 kip-in. The difference between theoretical and the experimental results was 8%. Figure 319 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The experimental and the theoretical moment values were in good agreement and they were both in the elastic range.

For test SA4-2, Figures 320 - 323 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 30 shows the ratios of the experimental to theoretical values for these strain gauges. Both figures had the same trending and the peak values were very close. The similarity between the figures also extended to the free vibration part of the graph. The ratios between the tested to the predicted strain results ranged from 0.9 to 2.8. For the same test, the experimental maximum moment at section Q was 13 kip-in and the theoretical value was
15.3 kip-in. The difference between theoretical and the experimental results was 16%. Figure 324 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 9.2 kip-in and the theoretical value was 9.9 kip-in. The difference between theoretical and the experimental results was 8%. Figure 325 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Both the experimental and the theoretical curves were very similar and their peak values were very close.

For test SA4-3, Figures 326 - 329 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 31 shows the ratios of the experimental to the theoretical values for these strain gauges. The tested results and predicted results show good agreement. The ratios between the tested to the predicted strain results ranged from 0.87 to 1.33. For the same test, the experimental maximum moment at section Q was 25.6 kip-in and the theoretical value was 26.3 kip-in. The difference between the theoretical and the experimental results was 3%. Figure 330 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 17.1 kip-in and the theoretical value was 18 kip-in. The difference between theoretical and the experimental results was 5%. Figure 331 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. The theoretical and experimental maximum moment values for both B and Q were in satisfactory agreement.

For test SA4-4, Figures 332 - 335 show the theoretical and the experimental strain-time curve for SG1, SG2, SG4, and SG5, respectively. Table 32 shows the ratios of experimental to theoretical values for these strain gauges. The overall shape of the curves was the same but there was a difference in the peak values. This was due to the big difference between the experimental and theoretical time resolution. This resolution was due to the fact that the maximum possible speed at which the machine could collect experimental strain readings was 2000 reading/sec. In theory, the minimum converged rate of reading in the theoretical analysis was 100,000 reading/sec. This effect of this difference was noticeable when some peak points were missing from the collected data. The ratios between the tested to the predicted strain results ranged from 0.42 to 1.56. For the same test, the experimental maximum moment at section Q was 36.4 kip-in and the theoretical value was 37.1 kip-in. The difference between the theoretical and the experimental results was 3%. Figure 336 shows the comparison between the theoretical and experimental moment-curvature graphs at
section Q. For the same test, the experimental maximum moment at point B was 34.1 kip-in and the theoretical value was 27.3 kip-in. The difference between the theoretical and the experimental results was 24%. Figure 337 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. It was found that good agreement was reached between the tested and the predicted results.

Table 33 compares the maximum moments from sections Q and B, on the sub-assemblage, for impact tests based on the experimental results and the inelastic finite-difference analysis used for tests SA6-1, SA6-2, SA6-3, and SA6-4.

For test SA6-1, Figures 338 - 343 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 34 shows the ratios of the experimental to the theoretical values for these strain gauges. There were overall similarities in the shapes of all the load-strain curves. The ratios between the tested to the predicted strain results ranged from 0.67 to 1.16. For the same test, the experimental maximum moment at section Q was 9.5 kip-in and the theoretical value was 8.8 kip-in. The difference between the theoretical and the experimental results was 8%. Figure 344 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 6.7 kip-in and the theoretical value was 6 kip-in. The difference between the theoretical and the experimental results was 11%. Figure 345 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. Experimental and theoretical moment values were in good agreement and they were in the elastic range.

For test SA6-2, Figures 346 - 351 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 35 shows the ratios of the experimental to the theoretical values for these strain gauges. Both figures had the same trending, and the peak values were very close. The peak strain value for SG3, CFRP, showed a bigger difference. The ratios between the tested to the predicted strain results ranged from 0.58 to 1.02. For the same test, the experimental maximum moment at section Q was 15 kip-in and the theoretical value was 15.6 kip-in. The difference between the theoretical and the experimental results was 4%. Figure 352 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 9.2 kip-in and the theoretical value was 10 kip-in. The difference between theoretical and the experimental results was 8%. Figure 353 shows the comparison between the theoretical and
experimental moment-curvature graphs at section B. The experimental and the theoretical curves showed slightly different slopes but their peak values were very close.

For test SA6-3, Figures 354 - 359 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 36 shows the ratios of experimental to theoretical values for these strain gauges. The overall shape of the curves is the same and there is a close agreement in the peak values. The ratios between the tested to the predicted strain results ranged from 0.82 to 1.26. For the same test, the experimental maximum moment at section Q was 28 kip-in and the theoretical value was 32.8 kip-in. The difference between the theoretical and the experimental results is 15%. Figure 360 shows the comparison between the theoretical and the experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 17.6 kip-in and the theoretical value was 20.6 kip-in. The difference between theoretical and the experimental results was 15%. Figure 361 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B.

For test SA6-4, Figures 362 - 367 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 37 shows the ratios of the experimental to the theoretical values for these strain gauges. At the peak load, the predicted and the theoretical load versus the rotation curves showed good correlation, but at the vibration stage, the two curves were slightly away from each other. The ratios between the tested to the predicted strain results ranged from 0.75 to 1.19. For the same test, the experimental maximum moment at section Q was 49 kip-in and the theoretical value was 49 kip-in. Both the theoretical and the experimental results developed a plastic hinge at section Q. Figure 368 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at point B was 36.6 kip-in and the theoretical value was 38.7 kip-in. The difference between the theoretical and the experimental results was 6%. Figure 369 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. It has been found that a good agreement was reached between the tested and predicted results.

Table 38 compares the maximum moments from sections Q and B on the sub-assemblage for impact tests based on the experimental results and the inelastic finite-difference analysis used for tests SA6-1, SA6-2, SA6-3, and SA6-4.
For test SA8-1, Figures 370 - 375 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 39 shows the ratios of the experimental to the theoretical values for these strain gauges. The overall shape of the curves was the same but there was a small difference in the peak values. The ratios between the tested to the predicted strain results ranged from 1.04 to 5.0. For the same test, the experimental maximum moment at section Q was 10.9 kip-in and the theoretical value was 11.2 kip-in. The difference between the theoretical and the experimental results was 3%. Figure 376 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 7.7 kip-in and the theoretical value was 7.2 kip-in. The difference between the theoretical and the experimental results is 7%. Figure 377 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The theoretical and experimental maximum moment values for both B and Q were in satisfactory agreement.

For test SA8-2, Figures 378 - 383 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 40 shows the ratios of the experimental to the theoretical values for these strain gauges. The tested results and the predicted results have a good agreement. The ratios between the tested to the predicted strain results ranged from 0.95 to 4.5. For the same test, the experimental maximum moment at section Q was 16.2 kip-in and the theoretical value was 17.1 kip-in. The difference between the theoretical and the experimental results was 6%. Figure 384 shows the comparison between the theoretical and the experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 11.1 kip-in and the theoretical value was 10.1 kip-in. The difference between the theoretical and the experimental results was 10%. Figure 385 shows the comparison between the theoretical and the experimental moment-curvature graphs at section B. The experimental and the theoretical moment values were in good agreement and they were in the elastic range.

For test SA8-3, Figures 386 - 391 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 41 shows the ratios of the experimental to the theoretical values for these strain gauges. The theoretical values showed very good agreement with the experimental results. The ratios between the tested to the predicted strain results ranged from 0.44 to 0.92. For the same test, the experimental maximum moment at section
Q was 35.7 kip-in and the theoretical value was 33.8 kip-in. The difference between theoretical and the experimental results was 6%. Figure 392 shows the comparison between the theoretical and experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at section B was 23.2 kip-in and the theoretical value was 24 kip-in. The difference between the theoretical and the experimental results was 5%. Figure 393 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The theoretical and experimental maximum moment values for both B and Q were in satisfactory agreement.

For test SA8-4, Figures 394 - 399 show the theoretical and the experimental strain-time curve for SG1, SG2, SG3, SG4, SG5, and SG6, respectively. Table 42 shows the ratios of the experimental to the theoretical values for these strain gauges. The overall shape of the curves was the same but there was a difference in the peak values for SG4. This was due to the big difference between the experimental and the theoretical time resolution. This resolution was due to the fact that the maximum possible speed at which the machine could collect experimental strain readings was 2000 reading/sec. In theory, the minimum converged rate of reading in the theoretical analysis was 100,000 reading/sec. The ratios between the tested to the predicted strain results ranged from 0.46 to 1.41. For the same test, the experimental maximum moment at section Q was 45 kip-in and the theoretical value was 46 kip-in. There was no difference between theoretical and the experimental results. Figure 400 shows the comparison between the theoretical and the experimental moment-curvature graphs at section Q. For the same test, the experimental maximum moment at point B was 35.7 kip-in and the theoretical value was 31 kip-in. The difference between the theoretical and the experimental results was 15%. Figure 401 shows the comparison between the theoretical and experimental moment-curvature graphs at section B. The theoretical and experimental maximum moment values for both B and Q were in satisfactory agreement.

4.4 Discussion

The Experimental results from quasi-static and impact loading were compared to the theoretical analysis. The ratio of the experimental to the theoretical results for all quasi-static tests were shown in the tables in the previous section in this chapter. For specimen SA3, the experimental to theoretical ratio for the load carrying capacity of the section was 7%. The maximum load reached created static plastic hinges at both sections B and Q. Therefore, the
experimental and theoretical moments on these sections reached maximum moments for the section. Test SA5 also showed a difference between its experimental and its theoretical load carrying capacity of 7%. The addition of the axial load to the system was well captured and calculated by the theoretical procedure. The development of static plastic hinges at B and Q in both experimental and theoretical results showed good agreement. The CFRP retrofitting used in test SA7 increased the load carrying capacity of the sub-assemblage by 25% experimentally and by 22% theoretically. The difference between experimental and theoretical load carrying capacity was 11%. The difference was little higher due to epoxy and the rings that were used to install the CFRP strips to the cross-section. For test SA7, the experimental and the theoretical results showed that the full moment capacity of the section was reached at Q and B. Although the experimental and theoretical maximum load showed good agreement for specimen SA9, the behavior of that specimen was different from other quasi-static specimens. The SA9 test results showed the development of a full moment capacity at Q and 70% of the moment capacity at B. This specimen developed only one plastic hinge under the load, while the other quasi-static specimens developed two static plastic hinges.

An experimental method for finding the forcing function was presented. The accelerometer that was embedded in the impactors was used to record the necessary data to define the forcing function. The data collected from the accelerometer was curve fitted with a second order polynomial using EXCEL software. This defined forcing function was used as an input for the theoretical analysis. The behavior similarity and the good agreement between its theoretical and experimental results verified this method to predict the forcing function from the impactor.

The basic case of the cantilever justified the usage of the developed dynamic analysis for other members with different boundary conditions. The maximum curve-fitted acceleration for the forcing function increased with the increase of the weight of the impactor. In addition, there was also an increase in the developed moment at B on the cantilever. The same behavior was proved theoretically. Experimental and theoretical results from dynamic testing of the cantilever showed good agreement that was clear from the shape of the moment-curvature curves and their maximum values. In tests C1-1 through C1-2, and with applying the defined forcing function from each test, the experimental to theoretical ratio was about 1.14, which is considered a good ratio for dynamic testing. For the C1-4 test both experimental and theoretical results showed the
development of a plastic hinge at B. There was no CFRP retrofitting or axial loading case for the cantilever experimental tests; therefore, these cases were excluded from the theoretical analysis. The quasi-static study for the cantilever was considered in this research. The results for such a study have already been covered and are very well recognized by researchers.

It is very important to note that there was a difference in time resolution between the experimental and the theoretical strain results. This resolution was due to the fact that the maximum possible speed at which the machine could collect experimental strain readings was 2000 reading/sec. In theory, the minimum converged rate of reading in the theoretical analysis was 100,000 reading/sec. The effect of this difference was noticeable when some peak point were missing from the collected data. The developed dynamic analysis procedure was used to confirm the experimental results for the sub-assemblage. The forcing function used in the analysis was found experimentally, as explained earlier. Tests SA2-1 through SA2-4 were for the sub-assemblage without CFRP retrofitting and without axial load. The experimental to theoretical ratios of the maximum moments at Q and B showed good agreement in both the elastic and the inelastic ranges. Experimental results showed that the moment at Q was always larger than the moment at B. The theoretical analysis supported that statement with all of the cases. This behavior lead to the development of a full plastic moment at Q and of 91% of the plastic moment at B. It is very interesting to notice that this behavior was different from that seen in the quasi-static test, which developed a full plastic moment at Q and B.

The developed theoretical procedure was used to capture the behavior of the sub-assemblage with the existence of static axial loading. Tests SA4-1 through SA4-4 had the axial load applied before the impact took place. In the theoretical analysis, the effect of the axial load was added to the procedure. The results from these tests also showed a difference between the moments developed at Q and B. The moment at Q was larger than the moment at B in both the experimental and the theoretical results. There was good agreement between the theoretical and the experimental moments for tests SA4-1 through SA4-4 and at both locations Q and B except for test SA4-4. The experimental to theoretical ratio for the moment at B was 1.24. An increase in the maximum acceleration was noticed when compared with the sub-assemblage without axial load and using the same impactor and clear drop height.
Tests SA6-1 through SA6-4 were retrofitted with CFRP strips. The theoretical algorithm was modified to account for the CFRP strips. While the strips were installed to the sub-assemblage using epoxy and small rings, these were not accounted for in the analysis. Just as in the previous impact tests, the experimental results showed that the maximum moment at Q was larger than the maximum moment at B. Therefore, for SA6-4 test, the moment at Q reached a full plastic moment for the CFRP retrofitted section, while the moment at B reached 75% of the moment capacity for the section. In addition to increasing the moment capacity for the section, CFRP strips strengthened the end section at B and decreased the effect of the impact on B. For all CFRP retrofitted tests, the theory showed good agreement and behavior with the experimental data. Also, the CFRP retrofitted tests had a higher acceleration values than the non-retrofitted specimens and the specimens with the axial load.

Tests SA8-1 through SA8-4 were CFRP retrofitted and had the axial load was applied before the impact. The acceleration values for the forcing functions for these tests were the highest compared to other cases using the same impactor and same clear drop height. This resulted in their having higher moments at Q and B compared with other tests. The experimental to theoretical ratios of the maximum moments at Q and B showed a good agreement in both the elastic and the inelastic ranges. Experimental results showed that the moment at Q was always larger than the moment at B. The theoretical analysis confirmed the statement that adding CFRP strengthened the end section at B and decreased the effect of the impact on B. For the SA8-4 test, the moment at Q reached a full plastic moment for the CFRP retrofitted section while the moment at B reached 79% of the moment capacity for the section.

In general, strain-time and moment-curvature curves in both experiments and theory had the same shape, and the maximum values were within acceptable range. Different from the quasi-static loading, the impact loading for all cases developed a full plastic hinge only at Q, while the moment at B was less than a plastic moment.
CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

The following conclusions can be drawn from the study presented in this dissertation:

1. A mathematical model based on partial differential equations of inelastic dynamic equilibrium was successfully developed including new terms to account for inelastic or elasto-plastic behavior of a steel cantilever as well as a building sub-assemblage.

2. Materially nonlinear finite-difference algorithms were developed for both quasi-static and impact loading conditions involving a coupling of the governing flexural partial differential equation of equilibrium with an iterative tangent stiffness procedure to enforce inelastic cross-sectional equilibrium conditions.

3. The apparatus conceived, designed and built in the laboratory for testing steel cantilever and sub-assemblage specimens functioned well for both quasi-static and impact loading experiments.

4. An innovative method for defining a forcing function due to an impact load was developed by using an accelerometer attached to the impactor itself.

5. The maximum acceleration of the forcing function for the sub-assemblage was found to be inversely related to the mass of the impactor and directly related to the drop height.

6. In comparison to the non-retrofitted sub-assemblage, the maximum acceleration of the forcing function increased when CFRP retrofitting was used in the presence or absence of an axial load.

7. The maximum acceleration of the forcing function increased in the presence of an axial load for the sub-assemblage, as compared with that for the case of zero axial load.

8. When CFRP retrofitting was used in the absence of an axial load, the maximum impact load to develop a dynamic plastic hinge in the sub-assemblage increased by 10%. Its collapse load under quasi-static loading, however, increased by 26% when CFRP retrofitting was used. Thus, CFRP retrofitting was found to be more effective under quasi-static than for impact loading.
9. The maximum impact load to develop a dynamic plastic hinge decreased by 16% in the presence of an axial load for non-retrofitted sub-assemblage; however, the maximum quasi-static load to develop a plastic hinge decreased by 24%.

10. The maximum impact load to develop a dynamic plastic hinge in a CFRP-retrofitted sub-assemblage was reduced by 16% in the presence of an axial load; however, for a similar specimen the maximum static load to develop a plastic hinge was reduced by 11%.

11. The impact load caused the development of a dynamic plastic hinge in the sub-assemblage directly under the load itself while the quasi-static load caused a plastic hinge not only under the load point but also at the fixed end.

In summary, the experimental results agreed well with those predicted based on the theory developed and programmed. The theory can be utilized for predicting the elasto-plastic behavior of prototype or full scale sub-assemblages found in steel buildings.

**5.2 Future Research**

Further studies can focus on combined fire and impact loading on steel building sub-assemblages. In addition, the behavior of connections including the influence of CFRP retrofitting should be investigated.
REFERENCES


25. Unified Facilities Criteria (UFC), design of buildings to resist progressive collapse, Department of Defense, June 2013


### TABLES

Table 1. Clear drop height and weight of impactor for tests C1-1 through C1-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in.)</th>
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Table 2. Effect of retrofitting and axial load on quasi-static tests

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Table 3. Clear drop height and axial load values for tests SA1-1 through SA1-8 (Impactor 3; non-retrofitted test sub-assemblage)

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Table 4. Clear drop height, weight of impactor, retrofitting and axial load for impact tests

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<td>1</td>
<td>140</td>
<td>No</td>
<td>15</td>
</tr>
<tr>
<td>SA4-3</td>
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<td>400</td>
<td>No</td>
<td>15</td>
</tr>
<tr>
<td>SA4-4</td>
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<td>400</td>
<td>No</td>
<td>15</td>
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<tr>
<td>SA6-1</td>
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<td>60</td>
<td>Yes</td>
<td>0</td>
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<tr>
<td>SA6-2</td>
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<td>140</td>
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<td>0</td>
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<tr>
<td>SA6-3</td>
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<td>400</td>
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<td>0</td>
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<td>SA6-4</td>
<td>6</td>
<td>400</td>
<td>Yes</td>
<td>0</td>
</tr>
<tr>
<td>SA8-1</td>
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<td>60</td>
<td>Yes</td>
<td>15</td>
</tr>
<tr>
<td>SA8-2</td>
<td>1</td>
<td>140</td>
<td>Yes</td>
<td>15</td>
</tr>
<tr>
<td>SA8-3</td>
<td>1</td>
<td>400</td>
<td>Yes</td>
<td>15</td>
</tr>
<tr>
<td>SA8-4</td>
<td>5</td>
<td>400</td>
<td>Yes</td>
<td>15</td>
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</table>

Table 5. Weights of impactors

<table>
<thead>
<tr>
<th>Impactor</th>
<th>weight of Impactor (lb.)</th>
<th>hi (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impactor 1</td>
<td>60</td>
<td>6</td>
</tr>
<tr>
<td>Impactor 2</td>
<td>140</td>
<td>18</td>
</tr>
<tr>
<td>Impactor 3</td>
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<td>60</td>
</tr>
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</table>
Table 6. Effect of axial load $P_{\text{axial}}$ and CFRP retrofitting on quasi-static load capacity $W$,
Max. Deflection and Max. Moments

<table>
<thead>
<tr>
<th>Test</th>
<th>Axial load $P_{\text{axial}}$ (kip)</th>
<th>Retrofitting</th>
<th>Max quasi-static load $W$ (lb.)</th>
<th>Max. Deflection at Q (in.)</th>
<th>Max. Moment at Q (kip-in.)</th>
<th>Max. Moment at B (kip-in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA3</td>
<td>0</td>
<td>No</td>
<td>4770</td>
<td>2.665</td>
<td>40.2</td>
<td>40.2</td>
</tr>
<tr>
<td>SA5</td>
<td>15</td>
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<td>3610</td>
<td>1.96</td>
<td>35.8</td>
<td>35.8</td>
</tr>
<tr>
<td>SA7</td>
<td>0</td>
<td>Yes</td>
<td>6000</td>
<td>2.84</td>
<td>48.5</td>
<td>48.1</td>
</tr>
<tr>
<td>SA9</td>
<td>15</td>
<td>Yes</td>
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<td>2.00</td>
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<td>39.0</td>
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</table>

Table 7. Effect of axial load $P_{\text{axial}}$ and CFRP retrofitting on Max. Impact load and Max. Moments

<table>
<thead>
<tr>
<th>Test</th>
<th>Max load (lb.)</th>
<th>Max. Moment at Q (kip-in.)</th>
<th>Max. Moment at B (kip-in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA2-4</td>
<td>3200</td>
<td>39.3</td>
<td>36.2</td>
</tr>
<tr>
<td>SA4-4</td>
<td>2744</td>
<td>36.4</td>
<td>34.1</td>
</tr>
<tr>
<td>SA6-4</td>
<td>3512</td>
<td>49.0</td>
<td>36.6</td>
</tr>
<tr>
<td>SA8-4</td>
<td>2936</td>
<td>45.0</td>
<td>35.7</td>
</tr>
</tbody>
</table>
Table 8. Comparison of quasi-static and impact loads for developing a plastic moment at Q and B

<table>
<thead>
<tr>
<th>Test</th>
<th>Quasi-static load (lb.)</th>
<th>Max. Moment at Q (kip-in)</th>
<th>Max. Moment at B (kip-in)</th>
<th>Specimen</th>
<th>Impact load (lb.)</th>
<th>Max. Moment at Q (kip-in)</th>
<th>Max. Moment at B (kip-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA3</td>
<td>4770</td>
<td>40.2</td>
<td>40.2</td>
<td>SA2-4</td>
<td>3200</td>
<td>39.3</td>
<td>36.2</td>
</tr>
<tr>
<td>SA5</td>
<td>3610</td>
<td>35.8</td>
<td>35.8</td>
<td>SA4-4</td>
<td>2744</td>
<td>36.4</td>
<td>34.1</td>
</tr>
<tr>
<td>SA7</td>
<td>6000</td>
<td>48.5</td>
<td>48.1</td>
<td>SA6-4</td>
<td>3512</td>
<td>49.0</td>
<td>36.6</td>
</tr>
<tr>
<td>SA9</td>
<td>5310</td>
<td>46.0</td>
<td>39.0</td>
<td>SA8-4</td>
<td>2936</td>
<td>45.0</td>
<td>35.7</td>
</tr>
</tbody>
</table>

Table 9. Comparison of Max. curve fitted acceleration of impactor $a_i$ and sub-assemblage $a_b$ with weight of impactor for tests C1-1 to C1-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in)</th>
<th>weight of Impactor (lb.)</th>
<th>Max. curve fitted Impactor acceleration $a_i$ (g)</th>
<th>Sub-assemblage acceleration $a_b$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1-1</td>
<td>1</td>
<td>60</td>
<td>3.8</td>
<td>5.61</td>
</tr>
<tr>
<td>C1-2</td>
<td>1</td>
<td>140</td>
<td>2.98</td>
<td>4.18</td>
</tr>
<tr>
<td>C1-3</td>
<td>1</td>
<td>400</td>
<td>1.9</td>
<td>3</td>
</tr>
<tr>
<td>C1-4</td>
<td>2</td>
<td>400</td>
<td>2.43</td>
<td>4</td>
</tr>
</tbody>
</table>
Table 10. Comparison of Max. curve fitted acceleration of impactor $a_i$ and sub-assemblage $a_b$ with weight of impactor for tests SA2-1 to SA2-2

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in)</th>
<th>weight of Impactor (lb.)</th>
<th>Impactor acceleration $a_i$ (g)</th>
<th>Sub-assemblage acceleration $a_b$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA2-1</td>
<td>1</td>
<td>60</td>
<td>6.1</td>
<td>9.54</td>
</tr>
<tr>
<td>SA2-2</td>
<td>1</td>
<td>140</td>
<td>5.3</td>
<td>7.90</td>
</tr>
<tr>
<td>SA2-3</td>
<td>1</td>
<td>400</td>
<td>3.3</td>
<td>3.72</td>
</tr>
<tr>
<td>SA2-4</td>
<td>6</td>
<td>400</td>
<td>8.0</td>
<td>9.99</td>
</tr>
</tbody>
</table>

Table 11. Comparison of Max. curve fitted acceleration of impactor acceleration $a_i$ and beam $a_b$ with weight of impactor for tests SA4-1 to SA4-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Drop Height (in.)</th>
<th>weight of Impactor (kip)</th>
<th>Impactor acceleration $a_i$ (g)</th>
<th>Sub-assemblage acceleration $a_b$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA4-1</td>
<td>1</td>
<td>60</td>
<td>6.30</td>
<td>4.79</td>
</tr>
<tr>
<td>SA4-2</td>
<td>1</td>
<td>140</td>
<td>5.64</td>
<td>4.29</td>
</tr>
<tr>
<td>SA4-3</td>
<td>1</td>
<td>400</td>
<td>3.58</td>
<td>2.95</td>
</tr>
<tr>
<td>SA4-4</td>
<td>5</td>
<td>400</td>
<td>6.86</td>
<td>5.09</td>
</tr>
</tbody>
</table>
Table 12. Comparison of Max. curve fitted acceleration of impactor $a_i$ and beam $a_b$ with weight of impactor for tests SA6-1 to SA6-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in.)</th>
<th>weight of Impactor (lb.)</th>
<th>Impactor acceleration $a_i$ (g)</th>
<th>Sub-assemblage acceleration $a_b$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA6-1</td>
<td>1</td>
<td>60</td>
<td>8.30</td>
<td>3.57</td>
</tr>
<tr>
<td>SA6-2</td>
<td>1</td>
<td>140</td>
<td>5.80</td>
<td>7.60</td>
</tr>
<tr>
<td>SA6-3</td>
<td>1</td>
<td>400</td>
<td>3.98</td>
<td>8.72</td>
</tr>
<tr>
<td>SA6-4</td>
<td>5</td>
<td>400</td>
<td>8.78</td>
<td>10.10</td>
</tr>
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Table 13. Comparison of Max. curve fitted acceleration of impactor $a_i$ and beam $a_b$ with weight of impactor for tests SA8-1 to SA8-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in.)</th>
<th>weight of Impactor (lb.)</th>
<th>Impactor acceleration $a_i$ (g)</th>
<th>Sub-assemblage acceleration $a_b$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA8-1</td>
<td>1</td>
<td>60</td>
<td>8.70</td>
<td>4.47</td>
</tr>
<tr>
<td>SA8-2</td>
<td>1</td>
<td>140</td>
<td>6.27</td>
<td>4.60</td>
</tr>
<tr>
<td>SA8-3</td>
<td>1</td>
<td>400</td>
<td>4.20</td>
<td>3.85</td>
</tr>
<tr>
<td>SA8-4</td>
<td>5</td>
<td>400</td>
<td>7.34</td>
<td>12.27</td>
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</table>
Table 14. Comparison of the maximum strain values for tests

SA2-1 to SA2-4 and SA6-1 to SA2-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in.)</th>
<th>weight of Impactor (lb.)</th>
<th>SG1 10^-6 (in./in.)</th>
<th>SG2 10^-6 (in./in.)</th>
<th>SG3 10^-6 (in./in.)</th>
<th>SG4 10^-6 (in./in.)</th>
<th>SG5 10^-6 (in./in.)</th>
<th>SG6 10^-6 (in./in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA2-1</td>
<td>1</td>
<td>60</td>
<td>279</td>
<td>-207</td>
<td>-</td>
<td>-219</td>
<td>326</td>
<td>381</td>
</tr>
<tr>
<td>SA2-2</td>
<td>2</td>
<td>140</td>
<td>369</td>
<td>-414</td>
<td>-</td>
<td>-531</td>
<td>437</td>
<td>637</td>
</tr>
<tr>
<td>SA2-3</td>
<td>3</td>
<td>400</td>
<td>743</td>
<td>-731</td>
<td>-</td>
<td>-976</td>
<td>863</td>
<td>1234</td>
</tr>
<tr>
<td>SA2-4</td>
<td>6</td>
<td>400</td>
<td>1923</td>
<td>-1670</td>
<td>-</td>
<td>-4621</td>
<td>4044</td>
<td>5661</td>
</tr>
<tr>
<td>SA6-1</td>
<td>1</td>
<td>60</td>
<td>266</td>
<td>-236</td>
<td>-185</td>
<td>-346</td>
<td>250</td>
<td>467</td>
</tr>
<tr>
<td>SA6-2</td>
<td>2</td>
<td>140</td>
<td>361</td>
<td>-319</td>
<td>-260</td>
<td>-514</td>
<td>419</td>
<td>789</td>
</tr>
<tr>
<td>SA6-3</td>
<td>3</td>
<td>400</td>
<td>873</td>
<td>-650</td>
<td>-544</td>
<td>-1309</td>
<td>641</td>
<td>1525</td>
</tr>
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<td>400</td>
<td>1759</td>
<td>-1372</td>
<td>-1177</td>
<td>-5164</td>
<td>3331</td>
<td>5140</td>
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</table>

Table 15. Comparison of the maximum strain values for tests

SA4-1 to SA4-4 and SA8-1 to SA8-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Clear Drop Height (in.)</th>
<th>weight of Impactor (lb.)</th>
<th>SG1 10^-6 (in./in.)</th>
<th>SG2 10^-6 (in./in.)</th>
<th>SG3 10^-6 (in./in.)</th>
<th>SG4 10^-6 (in./in.)</th>
<th>SG5 10^-6 (in./in.)</th>
<th>SG6 10^-6 (in./in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA4-1</td>
<td>1</td>
<td>60</td>
<td>-211</td>
<td>-767</td>
<td>-777</td>
<td>-814</td>
<td>-161</td>
<td>-</td>
</tr>
<tr>
<td>SA4-2</td>
<td>2</td>
<td>140</td>
<td>26</td>
<td>-1067</td>
<td>-1141</td>
<td>-1300</td>
<td>91</td>
<td>-</td>
</tr>
<tr>
<td>SA4-3</td>
<td>3</td>
<td>400</td>
<td>487</td>
<td>-1280</td>
<td>-1430</td>
<td>-1831</td>
<td>809</td>
<td>-</td>
</tr>
<tr>
<td>SA4-4</td>
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<td>400</td>
<td>1580</td>
<td>-2263</td>
<td>-2527</td>
<td>-6302</td>
<td>4016</td>
<td>-</td>
</tr>
<tr>
<td>SA8-1</td>
<td>1</td>
<td>60</td>
<td>-236</td>
<td>-826</td>
<td>-895</td>
<td>-978</td>
<td>-277</td>
<td>-70</td>
</tr>
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<td>SA8-2</td>
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<td>-64</td>
<td>-860</td>
<td>-979</td>
<td>-1194</td>
<td>-64</td>
<td>345</td>
</tr>
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<td>SA8-3</td>
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<td>400</td>
<td>308</td>
<td>-1072</td>
<td>-1320</td>
<td>-2016</td>
<td>188</td>
<td>1107</td>
</tr>
<tr>
<td>SA8-4</td>
<td>5</td>
<td>400</td>
<td>1230</td>
<td>-1683</td>
<td>-2137</td>
<td>-6444</td>
<td>2696</td>
<td>4742</td>
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</table>
Table 16. Experimental and theoretical load capacity for quasi-static tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Experimental Max load (lb.)</th>
<th>Theoretical Max load (lb.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA3</td>
<td>4770</td>
<td>4425</td>
</tr>
<tr>
<td>SA5</td>
<td>3610</td>
<td>3860</td>
</tr>
<tr>
<td>SA7</td>
<td>6000</td>
<td>5400</td>
</tr>
<tr>
<td>SA9</td>
<td>5310</td>
<td>5050</td>
</tr>
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</table>

Table 17. Comparison between theoretical and experimental maximum moments at Q and B for quasi-static tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max. Moment at Q (kip-in.)</td>
<td>Max. Moment at B (kip-in.)</td>
</tr>
<tr>
<td></td>
<td>Max. Moment at Q (kip-in.)</td>
<td>Max. Moment at B (kip-in.)</td>
</tr>
<tr>
<td>SA3</td>
<td>40.4</td>
<td>40.1</td>
</tr>
<tr>
<td></td>
<td>40.2</td>
<td>40.2</td>
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<tr>
<td>SA5</td>
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<td>35.8</td>
</tr>
<tr>
<td></td>
<td>35.8</td>
<td>35.8</td>
</tr>
<tr>
<td>SA7</td>
<td>47.9</td>
<td>47.6</td>
</tr>
<tr>
<td></td>
<td>48.5</td>
<td>48.1</td>
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</tr>
<tr>
<td></td>
<td>46</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 18. Comparison between theoretical and experimental maximum moments at B for the cantilever impact tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max. Moment at B (kip-in.)</td>
<td>Max. Moment at B (kip-in.)</td>
</tr>
<tr>
<td>C1-1</td>
<td>9.4</td>
<td>10.8</td>
</tr>
<tr>
<td>C1-2</td>
<td>17.8</td>
<td>20.3</td>
</tr>
<tr>
<td>C1-3</td>
<td>37.7</td>
<td>38.1</td>
</tr>
<tr>
<td>C1-4</td>
<td>39.8</td>
<td>39.5</td>
</tr>
</tbody>
</table>
### Table 19. Experimental and theoretical strains for test C1-1

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000557</td>
<td>0.000557</td>
<td>0.999477</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00065</td>
<td>-0.00056</td>
<td>1.17443</td>
</tr>
</tbody>
</table>

### Table 20. Experimental and theoretical strains for test C1-2

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000971</td>
<td>0.001046</td>
<td>0.927865</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00106</td>
<td>-0.00105</td>
<td>1.011955</td>
</tr>
</tbody>
</table>

### Table 21. Experimental and theoretical strains for test C1-3

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.002531</td>
<td>0.002817</td>
<td>0.898389</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00242</td>
<td>-0.00282</td>
<td>0.859344</td>
</tr>
</tbody>
</table>

### Table 22. Experimental and theoretical strains for test C1-4

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.004545</td>
<td>0.004512</td>
<td>1.007121</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.004783</td>
<td>-0.004512</td>
<td>1.05997</td>
</tr>
</tbody>
</table>
Table 23. Comparison between experimental and theoretical the maximum moments from Q and B for SA2-1 to SA2-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA2-1</td>
<td>6.5</td>
<td>3.5</td>
</tr>
<tr>
<td>SA2-2</td>
<td>13.8</td>
<td>7.1</td>
</tr>
<tr>
<td>SA2-3</td>
<td>23.7</td>
<td>12</td>
</tr>
<tr>
<td>SA2-4</td>
<td>39.8</td>
<td>36.2</td>
</tr>
</tbody>
</table>

Table 24. Experimental and theoretical strains for test SA2-1

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000279</td>
<td>0.000154</td>
<td>1.809254</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00021</td>
<td>-0.00015</td>
<td>1.34235</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00022</td>
<td>-0.00026</td>
<td>0.831686</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000326</td>
<td>0.000263</td>
<td>1.238034</td>
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</tbody>
</table>
Table 25. Experimental and theoretical strains for test SA2-2

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000369</td>
<td>0.000362</td>
<td>1.018199</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00041</td>
<td>-0.00036</td>
<td>1.14237</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00053</td>
<td>-0.0005</td>
<td>1.072546</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000437</td>
<td>0.000495</td>
<td>0.882679</td>
</tr>
</tbody>
</table>

Table 26. Experimental and theoretical strains for test SA2-3

<table>
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<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000743</td>
<td>0.000662</td>
<td>1.123124</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00073</td>
<td>-0.00066</td>
<td>1.104984</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00098</td>
<td>-0.00112</td>
<td>0.872527</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000863</td>
<td>0.001119</td>
<td>0.771507</td>
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</table>

Table 27. Experimental and theoretical strains for test SA2-4

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.001923</td>
<td>0.001883</td>
<td>1.02098</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00167</td>
<td>-0.00188</td>
<td>0.887839</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.004621</td>
<td>-0.004339</td>
<td>1.065027</td>
</tr>
<tr>
<td>SG5</td>
<td>0.004044</td>
<td>0.004339</td>
<td>0.931942</td>
</tr>
</tbody>
</table>
Table 28. Comparison between experimental and theoretical the maximum moments from Q and B for SA4-1 to SA4-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA4-1</td>
<td>7.6</td>
<td>5.4</td>
</tr>
<tr>
<td>SA4-2</td>
<td>15.3</td>
<td>9.9</td>
</tr>
<tr>
<td>SA4-3</td>
<td>36.3</td>
<td>18</td>
</tr>
<tr>
<td>SA4-4</td>
<td>37.1</td>
<td>27.3</td>
</tr>
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</table>

Table 29. Experimental and theoretical strains for test SA4-1

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000211</td>
<td>0.000365</td>
<td>0.57767</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00077</td>
<td>-0.00085</td>
<td>0.902092</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00081</td>
<td>-0.00082</td>
<td>0.993213</td>
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<tr>
<td>SG5</td>
<td>0.000161</td>
<td>0.000193</td>
<td>0.835995</td>
</tr>
</tbody>
</table>
Table 30. Experimental and theoretical strains for test SA4-2

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000026</td>
<td>9.28E-06</td>
<td>2.80252</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00107</td>
<td>-0.00118</td>
<td>0.902241</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.0013</td>
<td>-0.00111</td>
<td>1.170411</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000091</td>
<td>4.41E-05</td>
<td>2.065614</td>
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Table 31. Experimental and theoretical strains for test SA4-3

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000487</td>
<td>0.000365</td>
<td>1.333405</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00128</td>
<td>-0.00147</td>
<td>0.871903</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00183</td>
<td>-0.00172</td>
<td>1.06692</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000809</td>
<td>0.000649</td>
<td>1.245598</td>
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</table>

Table 32. Experimental and theoretical strains for test SA4-4

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.00158</td>
<td>0.001012</td>
<td>1.561027</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00226</td>
<td>-0.00219</td>
<td>1.035467</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.006302</td>
<td>-0.012247</td>
<td>0.514566</td>
</tr>
<tr>
<td>SG5</td>
<td>0.004016</td>
<td>0.005111</td>
<td>0.785742</td>
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</table>
Table 33. Comparison between experimental and theoretical the maximum moments from Q and B for SA6-1 to SA6-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA6-1</td>
<td>8.8</td>
<td>6</td>
</tr>
<tr>
<td>SA6-2</td>
<td>15.6</td>
<td>10</td>
</tr>
<tr>
<td>SA6-3</td>
<td>32.8</td>
<td>20.6</td>
</tr>
<tr>
<td>SA6-4</td>
<td>49</td>
<td>38.7</td>
</tr>
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</table>

Table 34. Experimental and theoretical strains for test SA6-1

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000266</td>
<td>0.00026</td>
<td>1.022315</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00024</td>
<td>-0.0002</td>
<td>1.161488</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.00019</td>
<td>-0.00027</td>
<td>0.677066</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00035</td>
<td>-0.00043</td>
<td>0.801174</td>
</tr>
<tr>
<td>SG5</td>
<td>0.00025</td>
<td>0.000234</td>
<td>1.066423</td>
</tr>
<tr>
<td>SG6</td>
<td>0.000467</td>
<td>0.000542</td>
<td>0.861487</td>
</tr>
</tbody>
</table>
### Table 35. Experimental and theoretical strains for test SA6-2

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000361</td>
<td>0.000449</td>
<td>0.803241</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00032</td>
<td>-0.00031</td>
<td>1.015122</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.00026</td>
<td>-0.00044</td>
<td>0.585325</td>
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<tr>
<td>SG4</td>
<td>-0.00051</td>
<td>-0.00069</td>
<td>0.745143</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000419</td>
<td>0.000564</td>
<td>0.742573</td>
</tr>
<tr>
<td>SG6</td>
<td>0.000789</td>
<td>0.000903</td>
<td>0.873655</td>
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</tbody>
</table>

### Table 36. Experimental and theoretical strains for test SA6-3

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000873</td>
<td>0.000693</td>
<td>1.260066</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00065</td>
<td>-0.00051</td>
<td>1.265336</td>
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<tr>
<td>SG3</td>
<td>-0.00054</td>
<td>-0.00066</td>
<td>0.821551</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00131</td>
<td>-0.00122</td>
<td>1.075884</td>
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<tr>
<td>SG5</td>
<td>0.000641</td>
<td>0.000751</td>
<td>0.853161</td>
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<tr>
<td>SG6</td>
<td>0.001525</td>
<td>0.001353</td>
<td>1.127271</td>
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</table>
Table 37. Experimental and theoretical strains for test SA6-4

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.001759</td>
<td>0.001556</td>
<td>1.130265</td>
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<tr>
<td>SG2</td>
<td>-0.00137</td>
<td>-0.00115</td>
<td>1.196204</td>
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<tr>
<td>SG3</td>
<td>-0.00118</td>
<td>-0.00138</td>
<td>0.852207</td>
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<td>SG4</td>
<td>-0.005164</td>
<td>-0.004630</td>
<td>1.115397</td>
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<tr>
<td>SG5</td>
<td>0.003331</td>
<td>0.003441</td>
<td>0.968178</td>
</tr>
<tr>
<td>SG6</td>
<td>0.005140</td>
<td>0.004310</td>
<td>1.192472</td>
</tr>
</tbody>
</table>

Table 38. Comparison between experimental and theoretical the maximum moments from Q and B for SA8-1 to SA8-4

<table>
<thead>
<tr>
<th>Test</th>
<th>Theoretical</th>
<th>Experimental</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SA8-1</td>
<td>11.2</td>
<td>7.2</td>
<td>10.9</td>
<td>7.7</td>
</tr>
<tr>
<td>SA8-2</td>
<td>17.11</td>
<td>10.1</td>
<td>16.2</td>
<td>11.1</td>
</tr>
<tr>
<td>SA6-3</td>
<td>33.8</td>
<td>24</td>
<td>35.7</td>
<td>23.2</td>
</tr>
<tr>
<td>SA6-4</td>
<td>46</td>
<td>31</td>
<td>45</td>
<td>35.7</td>
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</table>
Table 39. Experimental and theoretical strains for test SA8-1

<table>
<thead>
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<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>-0.00024</td>
<td>-0.00013</td>
<td>1.837266</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00083</td>
<td>-0.00079</td>
<td>1.040795</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.0009</td>
<td>-0.00083</td>
<td>1.081832</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00098</td>
<td>-0.00096</td>
<td>1.019054</td>
</tr>
<tr>
<td>SG5</td>
<td>-0.00028</td>
<td>-5.6E-05</td>
<td>4.987027</td>
</tr>
<tr>
<td>SG6</td>
<td>-0.00007</td>
<td>6.33E-05</td>
<td>1.837266</td>
</tr>
</tbody>
</table>

Table 40. Experimental and theoretical strains for test SA8-2

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>-6.4E-05</td>
<td>-1.4E-05</td>
<td>4.588363</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00086</td>
<td>-0.0009</td>
<td>0.95041</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.00098</td>
<td>-0.00095</td>
<td>1.03055</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00119</td>
<td>-0.0012</td>
<td>0.991684</td>
</tr>
<tr>
<td>SG5</td>
<td>-6.4E-05</td>
<td>0.000147</td>
<td>-0.43418</td>
</tr>
<tr>
<td>SG6</td>
<td>0.000345</td>
<td>0.000325</td>
<td>4.588363</td>
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</table>
Table 41. Experimental and theoretical strains for test SA8-3

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.000308</td>
<td>0.00051</td>
<td>0.603902</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00107</td>
<td>-0.00141</td>
<td>0.758164</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.00132</td>
<td>-0.00151</td>
<td>0.873392</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.00202</td>
<td>-0.00233</td>
<td>0.8653</td>
</tr>
<tr>
<td>SG5</td>
<td>0.000188</td>
<td>0.000427</td>
<td>0.440643</td>
</tr>
<tr>
<td>SG6</td>
<td>0.001107</td>
<td>0.001196</td>
<td>0.925759</td>
</tr>
</tbody>
</table>

Table 42. Experimental and theoretical strains for test SA8-4

<table>
<thead>
<tr>
<th>Strain gauge</th>
<th>Experimental strain (in./in.)</th>
<th>Theoretical strain (in./in.)</th>
<th>Experimental / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG1</td>
<td>0.00123</td>
<td>0.000867</td>
<td>1.419124</td>
</tr>
<tr>
<td>SG2</td>
<td>-0.00168</td>
<td>-0.00168</td>
<td>1.004565</td>
</tr>
<tr>
<td>SG3</td>
<td>-0.00214</td>
<td>-0.0018</td>
<td>1.187477</td>
</tr>
<tr>
<td>SG4</td>
<td>-0.006444</td>
<td>-0.010826</td>
<td>0.595239</td>
</tr>
<tr>
<td>SG5</td>
<td>0.002696</td>
<td>0.002561</td>
<td>1.052604</td>
</tr>
<tr>
<td>SG6</td>
<td>0.004742</td>
<td>0.004370</td>
<td>1.085148</td>
</tr>
</tbody>
</table>
Figure 1. Building frame with sub-assemblage
Figure 2. Schematic of steel sub-assemblage and loading

Figure 3. Schematic of steel cantilever and loading
Figure 4. Typical tensile test specimen

Figure 5. Stress-strain curves for tensile tests for specimen 1
Figure 6. Stress-strain curves for tensile tests for specimen 2

Figure 7. Stress-strain curves for tensile tests for specimen 3
$y = 2E+07x + 2731.7$

**Figure 8.** Stress-strain curves for tensile tests for specimen 1

**Figure 9.** Stress-strain relationship obtained for CFRP strips
Figure 10. Schematic for loaded cantilever and its cross-section

Figure 11. Moment-rotation relationship for rotational spring at end B
Figure 12. Coordinate system for sub-assemblage

Figure 13. Torsional resistance of beam R-S
Figure 14. Moment-rotation relationship for rotational spring at end T

Figure 15. Load-deflection relationship for translational spring at end T
Figure 16. Schematic for non-retrofitted sub-assemblage and its cross-section
Figure 17. Schematic for CFRP retrofitted sub-assemblage and its cross-section
Figure 18. Sub-assemblage specimens with CFRP retrofitting

Figure 19. Test Set-up for sub-assemblage quasi-static test
Figure 20. Schematic of Test Set-up for sub-assemblage
Figure 21. Connection of one end of the short member of the sub-assemblage

Figure 22. Test setup with hydraulic jacks for vertical and axial loads
Figure 23. Strain gauges at point B

Figure 24. Strain gauges and at point B for CFRP retrofitted specimens
Figure 25. Location of strain gauges and dial gauge at section B

Figure 26. Strain gauges and the dial gauge at section Q
Figure 27. Strain gauges at section Q for CFRP retrofitted specimens

Figure 28. Locations of the strain gauges at section Q
Figure 29. Vishay strain acquisition system

Figure 30. Sub-assemblage during quasi-static test
Figure 31. Experimental load-deflection curve for specimen SA3

Figure 32. Experimental load-strain values for specimen SA3 at B
Figure 33. Experimental load-strain values for specimen SA3 at Q

Figure 34. Moment-curvature relation based on experimental strains for test SA3 at B
Figure 35. Moment-curvature relation based on experimental strains for test SA3 at Q

Figure 36. Experimental load-deflection curve for specimen SA5
Figure 37. Experimental load-strain values for specimen SA5 at section B

Figure 38. Experimental load-strain values for specimen SA5 at section Q
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Figure 63. Acceleration-time relation from beam \(a_b\) for C1-2 test

Figure 64. Experimental strain-time values from for C1-2 test at B
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Figure 66. Acceleration-time relations from Impactor \( a_i \) for C1-3 test

\[ y = -628.55x^2 + 91.886x - 1.4533 \]
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Figure 68. Experimental strain-time values from for C1-3 test at B
y = -804.63x^2 + 111.12x - 1.4058

Figure 69. Moment-curvature relation based on experimental strains for C1-3 test at B

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Figure 96. Moment-curvature relation based on experimental strains for SA1-4 test at B

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\[ y = -8913.9x^2 + 670.01x - 5.2046 \]

Figure 98. Acceleration-time relation from Impactor \( a_i \) for SA1-5 test

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\[ y = -5347.8x^2 + 409.39x - 4.0217 \]

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\[ y = -8175.8x^2 + 608.78x - 4.4768 \]

Figure 163. Acceleration-time relation from Impactor $a_i$ for SA4-4 test

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Figure 180. Moment-curvature relation based on experimental strains for SA6-2 test at Q
\[ y = -6359.6x^2 + 464.52x - 4.4965 \]

Figure 181. Acceleration-time relation from Impactor $a_i$ for SA6-3 test

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Figure 190. Experimental strain-time values from for SA6-4 test at Q
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Figure 194. Acceleration-time relation from beam $a_b$ for SA8-1 test

The equation provided is:

$$y = -41155x^2 + 1559.6x - 7.8903$$
Figure 195. Experimental strain-time values from for SA8-1 test at B

Figure 196. Experimental strain-time values from for SA8-1 test at Q
Figure 197. Moment-curvature relation based on experimental strains for SA8-1 test at B

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Figure 199. Acceleration-time relation from Impactor $a_i$ for SA8-2 test

Figure 200. Acceleration-time relation from beam $a_b$ for SA8-2 test
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Figure 202. Experimental strain-time values from for SA8-2 test at Q
Figure 203. Moment-curvature relation based on experimental strains for SA8-2 test at B

Figure 204. Moment-curvature relation based on experimental strains for SA8-2 test at Q
$y = -4689.9x^2 + 335.14x - 2.4983$

Figure 205. Acceleration-time relation from Impactor $a_i$ for SA8-3 test

Figure 206. Acceleration-time relation from beam $a_b$ for SA8-3 test
Figure 207. Experimental strain-time values from for SA8-3 test at B

Figure 208. Experimental strain-time values from for SA8-3 test at Q
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Figure 210. Moment-curvature relation based on experimental strains for SA8-3 test at Q
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Figure 212. Acceleration-time relation from beam $a_b$ for SA8-4 test
Figure 213. Experimental strain-time values from for SA8-4 test at B

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Figure 320. Comparison of theoretical and experimental strain-time relations of SG1 for test SA4-2

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Figure 323. Comparison of theoretical and experimental strain-time relations of SG5 for test SA4-2
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Figure 325. Comparison of theoretical and experimental moment-curvature relations at B for test SA4-2
Figure 326. Comparison of theoretical and experimental strain-time relations of SG1 for test SA4-3

Figure 327. Comparison of theoretical and experimental strain-time relations of SG2 for test SA4-3
Figure 328. Comparison of theoretical and experimental strain-time relations of SG4 for test SA4-3

Figure 329. Comparison of theoretical and experimental strain-time relations of SG5 for test SA4-3
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Figure 348. Comparison of theoretical and experimental strain-time relations of SG3 for test SA6-2

Figure 349. Comparison of theoretical and experimental strain-time relations of SG4 for test SA6-2
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Figure 359. Comparison of theoretical and experimental strain-time relations of SG6 for test SA6-3
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Figure 362. Comparison of theoretical and experimental strain-time relations of SG1 for test SA6-4

Figure 363. Comparison of theoretical and experimental strain-time relations of SG2 for test SA6-4
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Figure 392. Comparison of theoretical and experimental moment-curvature relations at Q for test SA8-3

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Figure 396. Comparison of theoretical and experimental strain-time relations of SG3 for test SA8-4

Figure 397. Comparison of theoretical and experimental strain-time relations of SG4 for test SA8-4
Figure 398. Comparison of theoretical and experimental strain-time relations of SG5 for test SA8-4

Figure 399. Comparison of theoretical and experimental strain-time relations of SG6 for test SA8-4
Figure 400. Comparison of theoretical and experimental moment-curvature relations at Q for test SA8-4

Figure 401. Comparison of theoretical and experimental moment-curvature relations at B for test SA8-4
APPENDIX A: COMPUTER PROGRAMS

This appendix presents the computer programs based on the theory presented in this dissertation

STATIC ANALYSIS

clc
clear

B=2;
t=0.125;
I=(B^4)-(B-2*t)^4)/12;
Zmodulus=((B^3)/4)-(B-2*t)*(0.5*B-t)^2;
Area=(B^2)-((B-2*0.125)^2);
E=30000000;
SigmaY=62000;
EI=E*I;
Mp=SigmaY*Zmodulus;
he=.00625;
we=.0625;
HEN=2*t/he+(B-2*t)/he;
WEn=2*t/we+(B-2*t)/we;
HoleHEN=(B-2*t)/he;
HoleWEn=(B-2*t)/we;
NO=HEN*WEn-HoleHEN*HoleWEn;
Ae=Area/(NO);
Kspr=34000;
Kt=-1.5*10^6;
KB=6.0*10^6;
K=[1,0;0,1];
L=66;
NE=132;
h=L/NE;
STaxial=zeros(NE+1);
EpsilonY=SigmaY/E;
dvd=170;
AEL=Area*ones(NE+2,1);
Sx=zeros(NE+2,1);
Mpp=zeros(NE+2,1);
Ix=I*ones(NE+2,1);
Pp=zeros(NE+2,1);
E_sum=zeros(NE+2,1);
Ee=zeros(HEn,WEn,NE+2);
MINT=zeros(NE+2,1);
PINT=zeros(NE+2,1);
phi=zeros(NE+1);
he_ts=.0625;
we_ts=.0625;
HENs=2*t/hs+(B-2*t)/hs;
WEnTs=2*t/we_ts+(B-2*t)/we_ts;
HoleHENTs=(B-2*t)/hs;
HoleWEnTs=(B-2*t)/we_ts;
NOTS=HENs*WEnTs-HoleHENTs*HoleWEnTs;
Area_{ts}=(B^2)-(B-2t)^2;
Elem_Area_{ts}=Area/(NOTS);
EtTS=zeros(HE_{nts},WE_{nts},NE+2);
YElemTS=zeros(HE_{nts},WE_{nts});
SIGMAElemTS=zeros(HE_{nts},WE_{nts});
epsilon_{ts}=zeros(HE_{nts},WE_{nts});
EtotalTS=0;
phi_{ts}=zeros(NE+2);
M=zeros(NE+2,1);
dint=zeros(1,2,NE+2);

for HETS=1:HE_{nts};
TPTS=HETS*he_{ts};
if TPTS<=t
for WETS=1:WE_{nts}
EtTS(HETS,WETS,:)=E;
YElemTS(HETS,WETS)=((0.5*B)-(he_{ts})*(HETS-0.5));
SIGMAElemTS(HETS,WETS)=0;
end
end
if TPTS>(B-t)
for WETS=1:WE_{nts}
EtTS(HETS,WETS,:)=E;
YElemTS(HETS,WETS)=((0.5*B)-(he_{ts})*(HETS-0.5));
SIGMAElemTS(HETS,WETS)=0;
end
end
if TPTS>(B-t) & & TPTS<=t
RWEn_{ts}=t/we_{ts};
for WETS=1:RWEn_{ts}
EtTS(HETS,WETS,:)=E;
YElemTS(HETS,WETS)=((0.5*B)-(he_{ts})*(HETS-0.5));
SIGMAElemTS(HETS,WETS)=0;
end
end
LWEn_{ts}=t/we_{ts};
for WETS=WE_{nts}-LWEn_{ts}+1:WE_{nts}
EtTS(HETS,WETS,:)=E;
YElemTS(HETS,WETS)=((0.5*B)-(he_{ts})*(HETS-0.5));
SIGMAElemTS(HETS,WETS)=0;
end
end
end

for HE=1:HE
TP=HE*he;

if TP<=t
for WE=1:WE
del(HE,WE)=((0.5*B)-(B/HE)*HE-0.5));
end
end
if TP>(B-t)
for WE=1:WE
del(HE,WE)=((0.5*B)-(B/HE)*HE-0.5));
end
if TP>t && TP<=(B-t)  
RWe=t/te;
for WE=1:RWe
dei(HE,WE)=((0.5*B)-(B/HEn)*(HE-0.5));
end
LWe=t/te;
for WE=WWe-LWEn+1:WWe
dei(HE,WE)=((0.5*B)-(B/HEn)*(HE-0.5));
end
end
end

for it=1:dvd

if it==56  
disp('CHECK')
end

if it==1
w(it)=0;
elseif it<40
w(it)=w(it-1)-100;
else
w(it)=w(it-1)-5;
end

total=1;
mtotal(it)=0;
while total>0

for iji=1:NE+3
if iji==1
x=(iji-1)*h;
S1=AEL(iji+1,1)*Kt/(2*h);
S2=-AEL(iji+1,1)*KB/(2*h);
S3=(-(E*Sx(iji+1,1)^2)+(E*Ix(iji+1,1)*AEL(iji+1,1)))/h^2;
S7=(Sx(iji+1,1)*(Pp(iji+1,1)))+Mpp(iji+1,1)*AEL(iji+1,1);
RM(1,NE+1)=-S2*(x/L);
RM(1,NE+2)=0;
RM(1,NE+3)=S2*(x/L);
RM(iji,iji)=-S3-S1*(x/L-1);
RM(iji,iji+1)=2*S3;
RM(iji,iji+2)=-S3+S1*(x/L-1);

end
Constant(iji, it) = S7 - 0.5*w(it)*x*AEL(iji+1,1);

elseif iji==2
S1=Kt/(2*h);
S2=KB/(2*h);
S3=-(E*Sx(iji,1)^2)+(E*ix(iji,1)*AEL(iji,1)))/h^2;
S7=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM(iji,NE+1)=S2*(1/L);
RM(iji,NE+2)=0;
RM(iji,NE+3)=-S2*(1/L);
RM(iji,1)=S1*(1/L);
RM(iji,2)=-Kspr;
RM(iji,3)=-S1*(1/L);
Constant(iji, it)=0.5*w(it);

elseif iji==3
x=(iji-2)*h;
S1=AEL(iji,1)*Kt/(2*h);
S2=-AEL(iji,1)*KB/(2*h);
S3=-(E*Sx(iji,1)^2)+(E*ix(iji,1)*AEL(iji,1)))/h^2;
S7=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM(iji,1)=-S1*(x/L-1);
RM(iji,2)=0;
RM(iji,NE+1)=-S2*(x/L);
RM(iji,NE+2)=0;
RM(iji,NE+3)=S2*(x/L);
RM(iji,iji-1)=-S3;
RM(iji,iji)=2*S3+S1*(x/L-1);
RM(iji,iji+1)=-S3;
Constant(iji, it)=S7-0.5*w(it)*x*AEL(iji,1);

elseif iji==4
x=(iji-2)*h;
S1=AEL(iji,1)*Kt/(2*h);
S2=-AEL(iji,1)*KB/(2*h);
S3=-(E*Sx(iji,1)^2)+(E*ix(iji,1)*AEL(iji,1)))/h^2;
S7=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM(iji,1)=-S1*(x/L-1);
RM(iji,2)=0;
RM(iji,3)=S1*(x/L-1);
RM(iji,NE+1)=-S2*(x/L);
RM(iji,NE+2)=0;
RM(iji,NE+3)=S2*(x/L);
RM(iji,iji-1)=-S3+S1*(x/L-1);
RM(iji,iji)=2*S3;
RM(iji,iji+1)=-S3;
Constant(iji, it)=S7-0.5*w(it)*x*AEL(iji,1);

elseif iji>4 && iji<0.5*NE+1
x=(iji-2)*h;
S1=AEL(iji,1)*Kt/(2*h);
S2=-AEL(iji,1)*KB/(2*h);
S3=-(E*Sx(iji,1)^2)+(E*ix(iji,1)*AEL(iji,1)))/h^2;
S7=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM(iji,1)=-S1*(x/L-1);
RM(iji,2)=0;
RM(iji,3)=S1*(x/L-1);
RM(iji,NE+1)=-S2*(x/L);
RM(iji,NE+2)=0;
RM(iji,NE+3)=-S2*(x/L);
RM(iji,iji-1)=-S3+S1*(x/L-1);
RM(iji,iji)=2*S3;
RM(iji,iji+1)=-S3;
Constant(iji, it)=S7-0.5*w(it)*x*AEL(iji,1);
RM(iji, NE+2) = 0;
RM(iji, NE+3) = S2*(x/L);
RM(iji, iji-1) = -S3;
RM(iji, iji) = 2*S3;
RM(iji, iji+1) = -S3;
Constant(iji, it) = S7 - 0.5*w(it)*x*AEL(iji, 1);

elseif iji >= 0.5*NE+1 & it < NE
x = (iji-2)*h;
S1 = AEL(iji, 1)*Kt/(2*h);
S2 = -AEL(iji, 1)*KB/(2*h);
S3 = -(E*Sx(iji, 1)^2)/(E*Ix(iji, 1)*AEL(iji, 1)) + Mpp(iji, 1)*AEL(iji, 1);
RM(iji, 1) = -S1*(x/L-1);
RM(iji, 2) = 0;
RM(iji, 3) = S1*(x/L-1);
RM(iji, NE+1) = -S2*(x/L);
RM(iji, NE+2) = 0;
RM(iji, NE+3) = S2*(x/L);
RM(iji, iji-1) = -S3;
RM(iji, iji) = 2*S3;
RM(iji, iji+1) = -S3;
Constant(iji, it) = S7 - 0.5*w(it)*(L-x)*AEL(iji, 1);

elseif iji == NE
x = (iji-2)*h;
S1 = AEL(iji, 1)*Kt/(2*h);
S2 = -AEL(iji, 1)*KB/(2*h);
S3 = -(E*Sx(iji, 1)^2)/(E*Ix(iji, 1)*AEL(iji, 1)) + Mpp(iji, 1)*AEL(iji, 1);
RM(iji, 1) = -S1*(x/L-1);
RM(iji, 2) = 0;
RM(iji, 3) = S1*(x/L-1);
RM(iji, NE+2) = 0;
RM(iji, NE+3) = S2*(x/L);
RM(iji, iji-1) = -S3;
RM(iji, iji) = 2*S3;
RM(iji, iji+1) = -S3 - S2*(x/L);
Constant(iji, it) = S7 - 0.5*w(it)*(L-x)*AEL(iji, 1);

elseif iji == NE+1
x = (iji-2)*h;
S1 = AEL(iji, 1)*Kt/(2*h);
S2 = -AEL(iji, 1)*KB/(2*h);
S3 = -(E*Sx(iji, 1)^2)/(E*Ix(iji, 1)*AEL(iji, 1)) + Mpp(iji, 1)*AEL(iji, 1);
RM(iji, 1) = -S1*(x/L-1);
RM(iji, 2) = 0;
RM(iji, 3) = S1*(x/L-1);
RM(iji, NE+3) = S2*(x/L);
RM(iji, iji-1) = -S3;
RM(iji, iji) = 2*S3 - S2*(x/L);
RM(iji, iji+1) = -S3;
Constant(iji, it) = S7 - 0.5*w(it)*(L-x)*AEL(iji, 1);

elseif iji == NE+2

\begin{verbatim}
RM(iji,NE+1)=0;
RM(iji,NE+2)=Area;
RM(iji,NE+3)=0;
Constant(iji,it)=0;
elseif iji==NE+3
  x=(iji-3)*h;
  S1=AEL(iji-2,1)*Kt/(2*h);
  S2=-AEL(iji-2,1)*KB/(2*h);
  S3=(-(E*Sx(iji-2,1)^2)+(E*Ix(iji-2,1)*AEL(iji-2,1))/h^2);
  S7=(Sx(iji-2,1)*(Pp(iji-2,1)))+Mpp(iji-2,1)*AEL(iji-2,1);
  RM(iji,1)=-S1*(x/L-1);
  RM(iji,2)=0;
  RM(iji,3)=S1*(x/L-1);
  RM(iji,iji-2)=-S3-S2*(x/L);
  RM(iji,iji-1)=2*S3;
  RM(iji,iji)=-S3+S2*(x/L);
  Constant(iji,it)=S7-0.5*w(it)*(L-x)*AEL(iji-2,1);
end
end
INVrm=inv(RM);
Deflect(:,it)=INVrm*Constant(:,it);
for jj=1:NE+1
  Deflections(jj,it)=Deflect(jj+1,it);
end
MB(it)=(-KB/(2*h))*(-Deflect(NE+1,it)+Deflect(NE+3,it));
MT(it)=(Kt/(2*h))*(-Deflect(1,it)+Deflect(3,it));
for jj=1:NE+1
  x=(jj-1)*h;
  if jj>=1 && jj<=0.5*NE+1
    MEXT(jj,it)=MT(it)*(x/L-1)+MB(it)*(x/L)+0.5*w(it)*x;
    PEXT(jj,it)=0;
  else
    MEXT(jj,it)=MT(it)*(x/L-1)+MB(it)*(x/L)+0.5*w(it)*(L-x);
    PEXT(jj,it)=0;
  end
  if it==1
    dM(jj,it)=MEXT(jj,it);
    dP(jj,it)=1;
  else
    dM(jj,it)=MEXT(jj,it)-MEXT(jj,it-1);
    dP(jj,it)=PEXT(jj,it)-PEXT(jj,it-1);
  end
end
for jj=1:NE+1
  MS(jj,1)=dM(jj,it);
  M(jj,1)=M(jj,1)+MS(jj,1);
  MS_diff=MS(jj,1);
  P_diff=0;
  sosa=0;
  while abs(MS_diff) >= 0.01 || abs(P_diff)>= 0.01
  end
end
\end{verbatim}
K=zeros(2,2);
for HETS=1:HEnTS
TPTS=HETS*he_ts;
if TPTS<=t
for WETS=1:WEnTS
K(1,1)=EtTS(HETS,WETS,jj)*(Elem_Area_ts)+K(1,1);
K(1,2)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(1,2);
K(2,1)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,1);
K(2,2)=((YElemTS(HETS,WETS)^2)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,2);
end
end
if TPTS>(B-t)
for WETS=1:WEnTS
K(1,1)=EtTS(HETS,WETS,jj)*(Elem_Area_ts)+K(1,1);
K(1,2)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(1,2);
K(2,1)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,1);
K(2,2)=((YElemTS(HETS,WETS)^2)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,2);
end
end
if TPTS>t && TPTS<(B-t)
RWEn_ts=t/we_ts;
for WETS=1:RWEn_ts
K(1,1)=EtTS(HETS,WETS,jj)*(Elem_Area_ts)+K(1,1);
K(1,2)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(1,2);
K(2,1)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,1);
K(2,2)=((YElemTS(HETS,WETS)^2)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,2);
end
LWE_tsn=t/we_ts;
for WETS=LWE_tsn:WEnTS-LWE_tsn+1:WEnTS
K(1,1)=EtTS(HETS,WETS,jj)*(Elem_Area_ts)+K(1,1);
K(1,2)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(1,2);
K(2,1)=(YElemTS(HETS,WETS)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,1);
K(2,2)=((YElemTS(HETS,WETS)^2)*EtTS(HETS,WETS,jj))*(Elem_Area_ts)+K(2,2);
end
end
end
if it==2
dint(1,1,:)=0;
end
if det(K)==0
break;
else
F=[P_diff,MS_diff];
d=F/K;
dint(1,1,jj)=d(1,1)+dint(1,1,jj);
dint(1,2,jj)=d(1,2)+dint(1,2,jj);
pint_ts=0;
mint_ts=0;
end
end
end
if it==2
dint(1,1,:)=0;
end
if det(K)==0
break;
else
F=[P_diff,MS_diff];
d=F/K;
dint(1,1,jj)=d(1,1)+dint(1,1,jj);
dint(1,2,jj)=d(1,2)+dint(1,2,jj);
pint_ts=0;
mint_ts=0;
end
end
end
end
end
for WETS=1:WEnTS
epsilon_ts(HETS,WETS)=-dint(1,1,jj)-dint(1,2,jj)*YElemTS(HETS,WETS);
SIGMAElemTS(HETS,WETS)=E*epsilon_ts(HETS,WETS);
if SIGMAElemTS(HETS,WETS)>=SigmaY
SIGMAElemTS(HETS,WETS)=SigmaY;
EtTS(HETS,WETS,jj)=0;
elseif SIGMAElemTS(HETS,WETS)<=-SigmaY
SIGMAElemTS(HETS,WETS)=-SigmaY;
EtTS(HETS,WETS,jj)=0;
else
EtTS(HETS,WETS,jj)=E;
end
EtotalTS=EtotalTS+EtTS(HETS,WETS,jj)/NOTS;
pint_ts=pint_ts+SIGMAElemTS(HETS,WETS)*(Elem_Area_ts);
mint_ts=mint_ts+SIGMAElemTS(HETS,WETS)*YElemTS(HETS,WETS)*(Elem_Area_ts);
end
end
if TPTS>(B-t)
for WETS=1:WEnTS
epsilon_ts(HETS,WETS)=-dint(1,1,jj)-dint(1,2,jj)*YElemTS(HETS,WETS);
SIGMAElemTS(HETS,WETS)=E*epsilon_ts(HETS,WETS);
if SIGMAElemTS(HETS,WETS)>=SigmaY
SIGMAElemTS(HETS,WETS)=SigmaY;
EtTS(HETS,WETS,jj)=0;
elseif SIGMAElemTS(HETS,WETS)<=-SigmaY
SIGMAElemTS(HETS,WETS)=-SigmaY;
EtTS(HETS,WETS,jj)=0;
else
EtTS(HETS,WETS,jj)=E;
end
EtotalTS=EtotalTS+EtTS(HETS,WETS,jj)/NOTS;
pint_ts=pint_ts+SIGMAElemTS(HETS,WETS)*(Elem_Area_ts);
mint_ts=mint_ts+SIGMAElemTS(HETS,WETS)*YElemTS(HETS,WETS)*(Elem_Area_ts);
end
end
if TPTS>t && TPTS<=(B-t)
RWEn_ts=t/we_ts;
for WETS=1:RWEnTS
epsilon_ts(HETS,WETS)=-dint(1,1,jj)-dint(1,2,jj)*YElemTS(HETS,WETS);
SIGMAElemTS(HETS,WETS)=E*epsilon_ts(HETS,WETS);
if SIGMAElemTS(HETS,WETS)>=SigmaY
SIGMAElemTS(HETS,WETS)=SigmaY;
EtTS(HETS,WETS,jj)=0;
elseif SIGMAElemTS(HETS,WETS)<=-SigmaY
SIGMAElemTS(HETS,WETS)=-SigmaY;
EtTS(HETS,WETS,jj)=0;
else
EtTS(HETS,WETS,jj)=E;
end
EtotalTS=EtotalTS+EtTS(HETS,WETS,jj)/NOTS;
pint_ts=pint_ts+SIGMAElemTS(HETS,WETS)*(Elem_Area_ts);
mint_ts=mint_ts+SIGMAElemTS(HETS,WETS)*YElemTS(HETS,WETS)*(Elem_Area_ts);
end
end
LWEn_ts=t/we_ts;
for WETS=WEnTS-LWEn_ts+1:WEnTS
epsilon_ts(HETS,WETS)=-dint(1,1,jj)-dint(1,2,jj)*YElemTS(HETS,WETS);
SIGMAElemTS(HETS,WETS)=E*epsilon_ts(HETS,WETS);
if SIGMAElemTS(HETS,WETS) >= SigmaY
SIGMAElemTS(HETS,WETS) = SigmaY;
EtTS(HETS,WETS, jj) = 0;
elseif SIGMAElemTS(HETS,WETS) <= -SigmaY
SIGMAElemTS(HETS,WETS) = -SigmaY;
EtTS(HETS,WETS, jj) = 0;
else
EtTS(HETS,WETS, jj) = E;
end
EtotalTS = EtotalTS + EtTS(HETS,WETS, jj)/NOTS;
pint_ts = pint_ts + SIGMAElemTS(HETS,WETS) * (Elem_Area_ts);
mint_ts = mint_ts + SIGMAElemTS(HETS,WETS) * YElemTS(HETS,WETS) * (Elem_Area_ts);
end
end
end
P_diff = pint_ts;
MS_diff = M(jj, 1) + mint_ts;
MMy(jj) = M(jj, 1);
MMp(jj) = M(jj, 1)/1.1965;
phi(jj) = dint(1, 2, jj);
STaxial(jj) = dint(1, 1, jj);
phi_ts(jj) = phi(jj) * SigmaY / E;
end
sosa = 0;
if MS_diff >= 0.0001 || P_diff >= 0.0001
sosa = sosa + 1;
end
end
end
PHI(:, 1) = phi(:);
if it == 1
for ii = 1:NE+1
Epsilon_axial(ii, 1) = 0;
end
else
for ii = 1:NE+1
Epsilon_axial(ii, 1) = -STaxial(ii);
end
end
PHIt ime(:, it) = PHI(:, 1);
M time(:, it) = M(:, 1);
for ii = 1:NE+1
AxialSTRAIN(ii, 1) = 0;
AEL(ii, 1) = 0;
Sx(ii, 1) = 0;
Mpp(ii, 1) = 0;
E_sum(ii, 1) = 0;
Ix(ii, 1) = 0;
MINT(ii, 1) = 0;
PINT(ii, 1) = 0;
Pp(ii, 1) = 0;
for HE=1:HEn
TP=HE*he;
if TP<=t
for WE=1:WEn
EPSILONe(HE,WE)=Epsilon_axial(ii,1)*DEL(HE,WE);
Ee(HE,WE,ii)=E;
SIGMAe(HE,WE)=E*EPSILONe(HE,WE);
ELASTICAe(HE,WE)=Ae;
PLASTICAe(HE,WE)=0;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
if SIGMAe(HE,WE)>=SigmaY
SIGMAe(HE,WE)=SigmaY;
Ee(HE,WE,ii)=0;
ELASTICAe(HE,WE)=0;
PLASTICAe(HE,WE)=Ae;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
end
if SIGMAe(HE,WE)<=-SigmaY
SIGMAe(HE,WE)=-SigmaY;
Ee(HE,WE,ii)=0;
ELASTICAe(HE,WE)=0;
PLASTICAe(HE,WE)=Ae;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
end
E_sum(ii,1)=Ee(HE,WE,ii)/NO+E_sum(ii,1);
AEL(ii,1)=ELASTICAe(HE,WE)+AEL(ii,1);
Sx(ii,1)=ELASTICAe(HE,WE)*DEL(HE,WE)+Sx(ii,1);
Mpp(ii,1)=SIGMAe(HE,WE)*PLASTICAe(HE,WE)*DEL(HE,WE)+Mpp(ii,1);
Ix(ii,1)=ELASTICAe(HE,WE)*DEL(HE,WE)^2/12+ELASTICAe(HE,WE)*(DEL(HE,WE)^2)+Ix(ii,1);
MINT(ii,1)=SIGMAe(HE,WE)*Ae*DEL(HE,WE)+MINT(ii,1);
PINT(ii,1)=SIGMAe(HE,WE)*Ae+PINT(ii,1);
Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);
AxialSTRAIN(ii,1)=EPSILONe(HE,WE)+AxialSTRAIN(ii,1);
end
end
if TP>(B-t)
for WE=1:WEn
EPSILONe(HE,WE)=Epsilon_axial(ii,1)*DEL(HE,WE);
Ee(HE,WE,ii)=E;
SIGMAe(HE,WE)=E*EPSILONe(HE,WE);
ELASTICAe(HE,WE)=Ae;
PLASTICAe(HE,WE)=0;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
if SIGMAe(HE,WE)>=SigmaY
SIGMAe(HE,WE)=SigmaY;
Ee(HE,WE,ii)=0;
ELASTICAe(HE,WE)=0;
PLASTICAe(HE,WE)=Ae;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
end
if SIGMAe(HE,WE)<=-SigmaY
SIGMAe(HE,WE)=-SigmaY;
Ee(HE,WE,ii)=0;
ELASTICAe(HE,WE)=0;
PLASTICAe(HE,WE)=Ae;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
E_sum(ii,1) = Ee(HE, WE, ii)/NO + E_sum(ii, 1);
AEL(ii, 1) = ELASTICAe(HE, WE) + AEL(ii, 1);
Sx(ii, 1) = ELASTICAe(HE, WE) * del(HE, WE) + Sx(ii, 1);
Mpp(ii, 1) = SIGMAe(HE, WE) * PLASTICAe(HE, WE) * del(HE, WE) + Mpp(ii, 1);
Ix(ii, 1) = (ELASTICAe(HE, WE)^2)/12 + ELASTICAe(HE, WE) * (del(HE, WE)^2) + Ix(ii, 1);
MINT(ii, 1) = SIGMAe(HE, WE) * Ae * del(HE, WE) + MINT(ii, 1);
PINT(ii, 1) = SIGMAe(HE, WE) * Ae + PINT(ii, 1);
Pp(ii, 1) = Pp_e(HE, WE) + Pp(ii, 1);
AxialSTRAIN(ii, 1) = EPSILONe(HE, WE) + AxialSTRAIN(ii, 1);

if TP > t && TP <= (B - t)

RWin = t/we;
for WE = 1:RWin
EPSILONe(HE, WE) = Epsilon_axial(ii, 1) + PHI(ii, 1) * del(HE, WE);
Ee(HE, WE, ii) = E;
SIGMAe(HE, WE) = E * EPSILONe(HE, WE);
ELASTICAe(HE, WE) = Ae;
PLASTICAe(HE, WE) = 0;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
if SIGMAe(HE, WE) >= SigmaY
SIGMAe(HE, WE) = SigmaY;
Ee(HE, WE, ii) = 0;
ELASTICAe(HE, WE) = 0;
PLASTICAe(HE, WE) = Ae;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
end
if SIGMAe(HE, WE) <= -SigmaY
SIGMAe(HE, WE) = -SigmaY;
Ee(HE, WE, ii) = 0;
ELASTICAe(HE, WE) = 0;
PLASTICAe(HE, WE) = Ae;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
end
E_sum(ii, 1) = Ee(HE, WE, ii)/NO + E_sum(ii, 1);
AEL(ii, 1) = ELASTICAe(HE, WE) + AEL(ii, 1);
Sx(ii, 1) = ELASTICAe(HE, WE) * del(HE, WE) + Sx(ii, 1);
Mpp(ii, 1) = SIGMAe(HE, WE) * PLASTICAe(HE, WE) * del(HE, WE) + Mpp(ii, 1);
Ix(ii, 1) = (ELASTICAe(HE, WE)^2)/12 + ELASTICAe(HE, WE) * (del(HE, WE)^2) + Ix(ii, 1);
MINT(ii, 1) = SIGMAe(HE, WE) * Ae * del(HE, WE) + MINT(ii, 1);
PINT(ii, 1) = SIGMAe(HE, WE) * Ae + PINT(ii, 1);
Pp(ii, 1) = Pp_e(HE, WE) + Pp(ii, 1);
AxialSTRAIN(ii, 1) = EPSILONe(HE, WE) + AxialSTRAIN(ii, 1);

end

if TP > t & & TP <= (B - t)

RWin = t/we;
for WE = 1:RWin
EPSILONe(HE, WE) = Epsilon_axial(ii, 1) + PHI(ii, 1) * del(HE, WE);
Ee(HE, WE, ii) = E;
SIGMAe(HE, WE) = E * EPSILONe(HE, WE);
ELASTICAe(HE, WE) = Ae;
PLASTICAe(HE, WE) = 0;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
if SIGMAe(HE, WE) >= SigmaY
SIGMAe(HE, WE) = SigmaY;
Ee(HE, WE, ii) = 0;
ELASTICAe(HE, WE) = 0;
PLASTICAe(HE, WE) = Ae;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
end
if SIGMAe(HE, WE) <= -SigmaY
SIGMAe(HE, WE) = -SigmaY;
Ee(HE, WE, ii) = 0;
ELASTICAe(HE, WE) = 0;
PLASTICAe(HE, WE) = Ae;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
end
E_sum(ii, 1) = Ee(HE, WE, ii)/NO + E_sum(ii, 1);
AEL(ii, 1) = ELASTICAe(HE, WE) + AEL(ii, 1);
Sx(ii, 1) = ELASTICAe(HE, WE) * del(HE, WE) + Sx(ii, 1);
Mpp(ii, 1) = SIGMAe(HE, WE) * PLASTICAe(HE, WE) * del(HE, WE) + Mpp(ii, 1);
Ix(ii, 1) = (ELASTICAe(HE, WE)^2)/12 + ELASTICAe(HE, WE) * (del(HE, WE)^2) + Ix(ii, 1);
MINT(ii, 1) = SIGMAe(HE, WE) * Ae * del(HE, WE) + MINT(ii, 1);
PINT(ii, 1) = SIGMAe(HE, WE) * Ae + PINT(ii, 1);
Pp(ii, 1) = Pp_e(HE, WE) + Pp(ii, 1);
AxialSTRAIN(ii, 1) = EPSILONe(HE, WE) + AxialSTRAIN(ii, 1);
end

LWin = t/we;
for WE = 1:LWin
EPSILONe(HE, WE) = Epsilon_axial(ii, 1) + PHI(ii, 1) * del(HE, WE);
Ee(HE, WE, ii) = E;
SIGMAe(HE, WE) = E * EPSILONe(HE, WE);
ELASTICAe(HE, WE) = Ae;
PLASTICAe(HE, WE) = 0;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
if SIGMAe(HE, WE) >= SigmaY
SIGMAe(HE, WE) = SigmaY;
Ee(HE, WE, ii) = 0;
ELASTICAe(HE, WE) = 0;
PLASTICAe(HE, WE) = Ae;
Pp_e(HE, WE) = PLASTICAe(HE, WE) * SIGMAe(HE, WE);
end
E_sum(ii, 1) = Ee(HE, WE, ii)/NO + E_sum(ii, 1);
AEL(ii, 1) = ELASTICAe(HE, WE) + AEL(ii, 1);
Sx(ii, 1) = ELASTICAe(HE, WE) * del(HE, WE) + Sx(ii, 1);
Mpp(ii, 1) = SIGMAe(HE, WE) * PLASTICAe(HE, WE) * del(HE, WE) + Mpp(ii, 1);
Ix(ii, 1) = (ELASTICAe(HE, WE)^2)/12 + ELASTICAe(HE, WE) * (del(HE, WE)^2) + Ix(ii, 1);
MINT(ii, 1) = SIGMAe(HE, WE) * Ae * del(HE, WE) + MINT(ii, 1);
PINT(ii, 1) = SIGMAe(HE, WE) * Ae + PINT(ii, 1);
Pp(ii, 1) = Pp_e(HE, WE) + Pp(ii, 1);
AxialSTRAIN(ii, 1) = EPSILONe(HE, WE) + AxialSTRAIN(ii, 1);
end
\begin{verbatim}
PLASTICAe(HE,WE)=Ae;
Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);

if SIGMAe(HE,WE) <= -SigmaY
  SIGMAe(HE,WE) = -SigmaY;
  Ee(HE,WE,ii)=0;
  ELASTICAe(HE,WE)=0;
  PLASTICAe(HE,WE)=Ae;
  Pp_e(HE,WE)=PLASTICAe(HE,WE)*SIGMAe(HE,WE);
end

E_sum(ii,1)=Ee(HE,WE,ii)/NO+E_sum(ii,1);
AEL(ii,1)=ELASTICAe(HE,WE)+AEL(ii,1);
Sx(ii,1)=ELASTICAe(HE,WE)*del(HE,WE)+Sx(ii,1);
Mpp(ii,1)=SIGMAe(HE,WE)*PLASTICAe(HE,WE)*del(HE,WE)+Mpp(ii,1);
Ix(ii,1)=(ELASTICAe(HE,WE)^2)/12+ELASTICAe(HE,WE)*(del(HE,WE)^2)+Ix(ii,1);
MINT(ii,1)=SIGMAe(HE,WE)*Ae*del(HE,WE)+MINT(ii,1);
PINT(ii,1)=SIGMAe(HE,WE)*Ae+PINT(ii,1);
AxialSTRAIN(ii,1)=EPSILONe(HE,WE)+AxialSTRAIN(ii,1);
end
end

for iji=1:NE+3

if iji==1
  x=(iji-1)*h;
  S11=AEL(iji+1,1)*Kt/(2*h);
  S22=AEL(iji+1,1)*KB/(2*h);
  S33=(-(E*Sx(iji+1,1)^2)+(E*Ix(iji+1,1)*AEL(iji+1,1)))/h^2;
  S77=(Sx(iji+1,1)*(Pp(iji+1,1)))+Mpp(iji+1,1)*AEL(iji+1,1);
  RM2(1,NE+1)=-S22*(x/L);
  RM2(1,NE+2)=0;
  RM2(1,NE+3)=S22*(x/L);
  RM2(iji,NE+1)=-S33-S11*(x/L-1);
  RM2(iji,iji+1)=2*S33;
  RM2(iji,iji+2)=-S33+S11*(x/L-1);
  Constant2(iji,it)=S77-0.5*w(it)*x*AEL(iji+1,1);

elseif iji==2
  S11=Kt/(2*h);
  S22=KB/(2*h);
  S33=(-(E*Sx(iji,1)^2)+(E*Ix(iji,1)*AEL(iji,1)))/h^2;
  S77=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
  RM2(iji,NE+1)=S22*(1/L);
  RM2(iji,NE+2)=0;
  RM2(iji,NE+3)=-S22*(1/L);
  RM2(iji,1)=S11*(1/L);
  RM2(iji,2)=-Kspr;

end
\end{verbatim}
\[ RM2(iji, 3) = -S11 \times (1/L); \]
\[ \text{Constant2}(iji, it) = 0.5 \times w(it); \]

elseif iji == 3
\[ x = (iji - 2) \times h; \]
\[ S11 = AEL(iji, 1) \times Kt / (2 \times h); \]
\[ S22 = -AEL(iji, 1) \times KB / (2 \times h); \]
\[ S33 = \left( -\left( E \times Sx(iji, 1)^2 + (E \times Ix(iji, 1) \times AEL(iji, 1)) \right) / h^2 \right); \]
\[ S77 = \left( Sx(iji, 1) \times (Pp(iji, 1)) \right) + \left( Mpp(iji, 1) \times AEL(iji, 1) \right); \]
\[ RM2(iji, 1) = -S11 \times (x/L - 1); \]
\[ RM2(iji, NE+1) = -S22 \times (x/L); \]
\[ RM2(iji, NE+2) = 0; \]
\[ RM2(iji, NE+3) = S22 \times (x/L); \]
\[ RM2(iji, iji-1) = -S33; \]
\[ RM2(iji, iji) = 2 \times S33 + S11 \times (x/L - 1); \]
\[ RM2(iji, iji+1) = -S33; \]
\[ \text{Constant2}(iji, it) = S77 - 0.5 \times w(it) \times x \times AEL(iji, 1); \]

elseif iji == 4
\[ x = (iji - 2) \times h; \]
\[ S11 = AEL(iji, 1) \times Kt / (2 \times h); \]
\[ S22 = -AEL(iji, 1) \times KB / (2 \times h); \]
\[ S33 = \left( -\left( E \times Sx(iji, 1)^2 + (E \times Ix(iji, 1) \times AEL(iji, 1)) \right) / h^2 \right); \]
\[ S77 = \left( Sx(iji, 1) \times (Pp(iji, 1)) \right) + \left( Mpp(iji, 1) \times AEL(iji, 1) \right); \]
\[ RM2(iji, 1) = -S11 \times (x/L - 1); \]
\[ RM2(iji, 2) = 0; \]
\[ RM2(iji, NE+1) = -S22 \times (x/L); \]
\[ RM2(iji, NE+2) = 0; \]
\[ RM2(iji, NE+3) = S22 \times (x/L); \]
\[ RM2(iji, iji-1) = -S33 + S11 \times (x/L - 1); \]
\[ RM2(iji, iji) = 2 \times S33; \]
\[ RM2(iji, iji+1) = -S33; \]
\[ \text{Constant2}(iji, it) = S77 - 0.5 \times w(it) \times x \times AEL(iji, 1); \]

elseif iji > 4 && iji < 0.5*NE+1
\[ x = (iji - 2) \times h; \]
\[ S11 = AEL(iji, 1) \times Kt / (2 \times h); \]
\[ S22 = -AEL(iji, 1) \times KB / (2 \times h); \]
\[ S33 = \left( -\left( E \times Sx(iji, 1)^2 + (E \times Ix(iji, 1) \times AEL(iji, 1)) \right) / h^2 \right); \]
\[ S77 = \left( Sx(iji, 1) \times (Pp(iji, 1)) \right) + \left( Mpp(iji, 1) \times AEL(iji, 1) \right); \]
\[ RM2(iji, 1) = -S11 \times (x/L - 1); \]
\[ RM2(iji, 2) = 0; \]
\[ RM2(iji, 3) = S11 \times (x/L - 1); \]
\[ RM2(iji, NE+1) = -S22 \times (x/L); \]
\[ RM2(iji, NE+2) = 0; \]
\[ RM2(iji, NE+3) = S22 \times (x/L); \]
\[ RM2(iji, iji-1) = -S33; \]
\[ RM2(iji, iji) = 2 \times S33; \]
\[ RM2(iji, iji+1) = -S33; \]
\[ \text{Constant2}(iji, it) = S77 - 0.5 \times w(it) \times x \times AEL(iji, 1); \]

elseif iji > 0.5*NE+1 && iji < NE
\[ x = (iji - 2) \times h; \]
\[ S11 = AEL(iji, 1) \times Kt / (2 \times h); \]
\[ S22 = -AEL(iji, 1) \times KB / (2 \times h); \]
\[ S33 = \left( -\left( E \times Sx(iji, 1)^2 + (E \times Ix(iji, 1) \times AEL(iji, 1)) \right) / h^2 \right); \]
S77=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM2(iji,1)=-S11*(x/L-1);
RM2(iji,2)=0;
RM2(iji,3)=S11*(x/L-1);
RM2(iji,NE+1)=-S22*(x/L);
RM2(iji,NE+2)=0;
RM2(iji,NE+3)=S22*(x/L);
RM2(iji,iji-1)=-S33;
RM2(iji,iji)=2*S33;
RM2(iji,iji+1)=-S33;
Constant2(iji,it)=S77-0.5*w(it)*(L-x)*AEL(iji,1);

elseif iji==NE
x=(iji-2)*h;
S11=AEL(iji,1)*Kt/(2*h);
S22=-AEL(iji,1)*KB/(2*h);
S33=(-(E*Sx(iji,1)^2)+(E*Ix(iji,1)*AEL(iji,1)))/h^2;
S77=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM2(iji,1)=-S11*(x/L-1);
RM2(iji,2)=0;
RM2(iji,3)=S11*(x/L-1);
RM2(iji,NE+2)=0;
RM2(iji,NE+3)=+S22*(x/L);
RM2(iji,iji-1)=-S33;
RM2(iji,iji)=2*S33-S22*(x/L);
RM2(iji,iji+1)=-S33;
Constant2(iji,it)=S77-0.5*w(it)*(L-x)*AEL(iji,1);

elseif iji==NE+1
x=(iji-2)*h;
S11=AEL(iji,1)*Kt/(2*h);
S22=-AEL(iji,1)*KB/(2*h);
S33=(-(E*Sx(iji,1)^2)+(E*Ix(iji,1)*AEL(iji,1)))/h^2;
S77=(Sx(iji,1)*(Pp(iji,1)))+Mpp(iji,1)*AEL(iji,1);
RM2(iji,1)=-S11*(x/L-1);
RM2(iji,2)=0;
RM2(iji,3)=S11*(x/L-1);
RM2(iji,NE+3)=S22*(x/L);
RM2(iji,iji-1)=-S33;
RM2(iji,iji)=2*S33-S22*(x/L);
RM2(iji,iji+1)=-S33;
Constant2(iji,it)=S77-0.5*w(it)*(L-x)*AEL(iji,1);

elseif iji==NE+2
RM2(iji,NE+1)=0;
RM2(iji,NE+2)=Area;
RM2(iji,NE+3)=0;
Constant2(iji,it)=0;

elseif iji==NE+3
x=(iji-3)*h;
S11=AEL(iji-2,1)*Kt/(2*h);
S22=-AEL(iji-2,1)*KB/(2*h);
S33=(-(E*Sx(iji-2,1)^2)+(E*Ix(iji-2,1)*AEL(iji-2,1)))/h^2;
S77=(Sx(iji-2,1)*(Pp(iji-2,1)))+Mpp(iji-2,1)*AEL(iji-2,1);
RM2(iji,1)=-S11*(x/L-1);
RM2(iji,2)=0;
RM2(iji,3)=S11*(x/L-1);
RM2(iji,iji-2)=-S33-S22*(x/L);
RM2(iji,iji-1)=2*S33;
RM2(iji,iji)=-S33+S22*(x/L);
Constant2(iji,it)=S77-0.5*w(it)*(L-x)*AE(iji-2,1);
end
end

INVrm2=inv(RM2);
Deflect2(:,it)=INVrm2*Constant2(:,it);

for jj=1:NE+1
Deflections2(jj,ji)=Deflect2(jj,ji+1);
end

deltaDEFLECTION(:,it)=Deflections2(:,it)-Deflections(:,it);
total=0;
for ij=1:NE+1
if abs(deltaDEFLECTION(ij,ji))>0.012
    total=1+total;
    mtotal(it)=1+mtotal(it);
end
end

for ii=1:NE+1
    if ii==0.5*NE+1
        DEPLEC(1,ji)=Deflections(ii,ji);
    end
    if ii==NE+1
        Strain_BtP1(1,ji)=EPSILONtop(ii,1);
        MOMent_SectionPLOTp1(1,ji)=Mtime(ii,ji);
        CURVent_SectionPLOTp1(1,ji)=PHI(ii,1);
    end
    if ii==0.5*NE+1
        Strain_Midt(1,ji)=EPSILONtop(ii,1);
        MOMent_SectionPLOTmid(1,ji)=Mtime(ii,ji);
        CURVent_SectionPLOTmid(1,ji)=PHI(ii,1);
    end
    if ii==NE+1
        Strain_BbP1(1,ji)=EPSILONbottom(ii,1);
    end
    if ii==0.5*NE+1
        Strain_Midb(1,ji)=EPSILONbottom(ii,1);
    end
end

if it==150
TIME_Strain_top(:,1)=STRAINtop(:,1);
TIME_Strain_bottom(:,1)=STRAINbottom(:,1);
TIME_Deflection(:,1)=Deflections(:,1);
TIME_Sx(:,1)=Sx(:,1);
TIME_Pp(:,1)=Pp(:,1);
TIME_Mpp(:,1)=Mpp(:,1);
TIME_Ael(:,1)=AEL(:,1);
TIME_Ix(:,1)=Ix(:,1);
TIME_Mext(:,1)=-MEXT(:,1);
TIME_Minternal(:,1)=MINT(:,1);
TIME_phi(:,1)=phi(:,1);
end

j=1:dvd;
plot(Deflections(0.5*NE-1,j),-w(j));
grid on
grid minor
xlabel('Deflections (in)')
ylabel('Load (lb.)')
title('Load Vs. Deflections')

IMPACT PROGRAM

clc
clear

NE=28;
B=2;
t=0.125;
Area=(B^2)-((B-2*0.125)^2);
Zmodule=(((B^3)/4)-(B-2*t)*(0.5*B-t)^2;
I=((B^4)-(B-2*t)^4)/12;
E=30000000;
EI=E*I;
Kspr=34000;
Kt=1.5*10^6;
KB=6*10^6;
SigmaY=62000;
EpsilonY=SigmaY/E;
he=.0625;
we=.0625;
HEN=2*t/he+(B-2*t)/he;
WEn=2*t/we+(B-2*t)/we;
HoleHEN=(B-2*t)/he;
HoleWEn=(B-2*t)/we;
NO=HEN*WEn-HoleHEN*HoleWEn;
Elem_Area=Area/(NO);
L=66;
h=L/NE;
dt=.00001;
mass=.000657781;
C=0;
dvd=18000;
r=zeros(NE,NE);
Etotal_ts=0;
Pts=-0;
P=0;
M=0;
dint=zeros(1,2);
K=[1,0;0,1];
m=100;
MMy(1)=0;
phi(1)=0;
ij=1;
ADDb=1;
ADDt=1;
d_Elem=zeros(HEn,WEn);
E_Section=zeros(NE+1,dvd);
Mpp=zeros(1,NE+1);
PHIel=zeros(1,NE+1);
DynStrain_Elem=zeros(HEn,WEn);
DynStress_Elem=zeros(HEn,WEn);
DynE_Elem=zeros(HEn,WEn);
DynE_Elem_sum=0;
DynCurvaturesNE=zeros(NE+1,1);
BP=EI*ones(1,NE);
BP1=zeros(1,NE);
BP2=zeros(1,NE);
Deflection_PERMSETmid=0;
DynStress_SectionPLOT=zeros(NE+1,dvd);
Ftmin=0;
ti=zeros(1,dvd);
DynStrain_PERMSETmid=0;
DynStrain_PERMSETend=0;
b1=EI/h^4;
b2=P/h^2;
b5=(EI/h-Kt/2)/(EI/h+Kt/2);
b9=(EI/h-KB/2)/(EI/h+KB/2);
b10=(2*EI/h)/(EI/h+Kt/2);
b45=(2*EI/h)/(EI/h+KB/2);
b3=mass/(dt^2);
b6=-(dt^2)*EI/(2*mass*h^4);
b7=-(dt^2)*P/(2*mass*h^2);
b8=-(dt^2)*P/(2*mass);
b25=Kspr/h;
b11=h/EI;
b33=1/(h^3);
b44=1/(h^2);
c1=-1/(b3+b4);
c2=(b3-b4)*c1;
c3=1/(b44+b2+b1);
Deflections=zeros(NE,dvd);
Deflectionscheck=zeros(NE,dvd);
DEFLECT=zeros(NE+1,1);
Ft=zeros(NE,dvd);
maxDeflection=zeros(NE,1);
MAXDynStrain=0;
YIELD_STRAIN=SigmaY/30000000;
idxELASTIC=1;
he_ts=.125;
we_ts=.125;
HEn_ts=2*t/he_ts+(B-2*t)/he_ts;
WEn_ts=2*t/we_ts+(B-2*t)/we_ts;
HoleHEn_ts=(B-2*t)/he_ts;
HoleWEn_ts=(B-2*t)/we_ts;
NO_ts=HEn_ts*WEn_ts-HoleHEn_ts*HoleWEn_ts;
Area_ts=(B^2)-(B-2*t)^2;
Elem_Area_ts=Area/(NO_ts);
Et_ts=zeros(HEn_ts,WEn_ts);
Y_Elem_ts=zeros(HEn_ts,WEn_ts);
Sigma_Elem_ts=zeros(HEn_ts,WEn_ts);
epsilon_ts=zeros(HEn_ts,WEn_ts);
DynStrain_ElemMAXFIND=zeros(NE+1,dvd);

for HE_ts=1:HEn_ts;
    TP_ts=HE_ts*he_ts;
    if TP_ts<=t
        for WE_ts=1:WEn_ts
            Et_ts(HE_ts,WE_ts)=E;
            Y_Elem_ts(HE_ts,WE_ts)=((0.5*B)-(he_ts)*(HE_ts-0.5));
            Sigma_Elem_ts(HE_ts,WE_ts)=0;
        end
    end
    if TP_ts>(B-t)
        for WE_ts=1:WEn_ts
            Et_ts(HE_ts,WE_ts)=E;
            Y_Elem_ts(HE_ts,WE_ts)=((0.5*B)-(he_ts)*(HE_ts-0.5));
            Sigma_Elem_ts(HE_ts,WE_ts)=0;
        end
    end
    if TP_ts>t && TP_ts<=(B-t)
        RWEn_ts=t/we_ts;
        for WE_ts=1:RWEn_ts
            Et_ts(HE_ts,WE_ts)=E;
            Y_Elem_ts(HE_ts,WE_ts)=((0.5*B)-(he_ts)*(HE_ts-0.5));
            Sigma_Elem_ts(HE_ts,WE_ts)=0;
        end
        LWEn_ts=t/we_ts;
        for WE_ts=WE_ts-LWEn_ts+1:WEts
            Et_ts(HE_ts,WE_ts)=E;
            Y_Elem_ts(HE_ts,WE_ts)=((0.5*B)-(he_ts)*(HE_ts-0.5));
            Sigma_Elem_ts(HE_ts,WE_ts)=0;
        end
    end
end
while det(K)~=0
    ij=ij+1;
    M=M+m;
end
m_diff=m;
P_diff=0;
while m_diff >= 0.01 || P_diff>=0.01;
K=zeros(2,2);
for HE_ts=1:HEn_ts
TP_ts=HE_ts*he_ts;
if TP_ts<=t
for WE_ts=1:WEn_ts
K(1,1)=Et_ts(HE_ts,WE_ts)*(Elem_Area_ts)+K(1,1);
K(1,2)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(1,2);
K(2,1)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,1);
K(2,2)=((Y_Elem_ts(HE_ts,WE_ts)^2)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,2);
end
end
if TP_ts>(B-t)
for WE_ts=1:WEn_ts
K(1,1)=Et_ts(HE_ts,WE_ts)*(Elem_Area_ts)+K(1,1);
K(1,2)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(1,2);
K(2,1)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,1);
K(2,2)=((Y_Elem_ts(HE_ts,WE_ts)^2)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,2);
end
end
if TP_ts>t && TP_ts<=(B-t)
RWEn_ts=t/we_ts;
for WE_ts=1:RWEn_ts
K(1,1)=Et_ts(HE_ts,WE_ts)*(Elem_Area_ts)+K(1,1);
K(1,2)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(1,2);
K(2,1)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,1);
K(2,2)=((Y_Elem_ts(HE_ts,WE_ts)^2)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,2);
end
LWE_tsn=t/we_ts;
for WE_ts=WEn_ts-LWE_tsn+1:WEn_ts
K(1,1)=Et_ts(HE_ts,WE_ts)*(Elem_Area_ts)+K(1,1);
K(1,2)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(1,2);
K(2,1)=(Y_Elem_ts(HE_ts,WE_ts)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,1);
K(2,2)=((Y_Elem_ts(HE_ts,WE_ts)^2)*Et_ts(HE_ts,WE_ts))*(Elem_Area_ts)+K(2,2);
end
end
end
F=[P_diff,m_diff];
d=F/K;
if ij==0
dint(1,1)=Pts;
end
if det(K)==0
break;
else

dint(1,1)=d(1,1)+dint(1,1);
dint(1,2)=d(1,2)+dint(1,2);
pint_ts=0;
mint_ts=0;
for HE_ts=1:HEn_ts;
TP_ts=HE_ts*he_ts;
if TP_ts<t
for WE_ts=1:WEn_ts
epsilon_ts(HE_ts,WE_ts)=-dint(1,1)-dint(1,2)*Y_Elem_ts(HE_ts,WE_ts);
if Et_ts(HE_ts,WE_ts)~=0
Sigma_Elem_ts(HE_ts,WE_ts)=Et_ts(HE_ts,WE_ts)*epsilon_ts(HE_ts,WE_ts);
end
if Sigma_Elem_ts(HE_ts,WE_ts)>=SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=SigmaY;
Et_ts(HE_ts,WE_ts)=0;
elseif Sigma_Elem_ts(HE_ts,WE_ts)<=-SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=-SigmaY;
Et_ts(HE_ts,WE_ts)=0;
else
Et_ts(HE_ts,WE_ts)=E;
end
Etotal_ts=Etotal_ts+Et_ts(HE_ts,WE_ts)/NO_ts;
pint_ts=pint_ts+Sigma_Elem_ts(HE_ts,WE_ts)*(Elem.Area_ts);
mint_ts=mint_ts+Sigma_Elem_ts(HE_ts,WE_ts)*Y_Elem_ts(HE_ts,WE_ts)*(Elem.Area_ts);
end
end
if TP_ts>(B-t)
for WE_ts=1:WEn_ts
epsilon_ts(HE_ts,WE_ts)=-dint(1,1)-dint(1,2)*Y_Elem_ts(HE_ts,WE_ts);
if Et_ts(HE_ts,WE_ts)~=0
Sigma_Elem_ts(HE_ts,WE_ts)=Et_ts(HE_ts,WE_ts)*epsilon_ts(HE_ts,WE_ts);
end
if Sigma_Elem_ts(HE_ts,WE_ts)>=SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=SigmaY;
Et_ts(HE_ts,WE_ts)=0;
elseif Sigma_Elem_ts(HE_ts,WE_ts)<=-SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=-SigmaY;
Et_ts(HE_ts,WE_ts)=0;
else
Et_ts(HE_ts,WE_ts)=E;
end
Etotal_ts=Etotal_ts+Et_ts(HE_ts,WE_ts)/NO_ts;
pint_ts=pint_ts+Sigma_Elem_ts(HE_ts,WE_ts)*(Elem.Area_ts);
mint_ts=mint_ts+Sigma_Elem_ts(HE_ts,WE_ts)*Y_Elem_ts(HE_ts,WE_ts)*(Elem.Area_ts);
end
end
if TP_ts>t && TP_ts<(B-t)
RWEen_ts=t/we.ts;
for WE_ts=1:RWEen_ts
epsilon_ts(HE_ts,WE_ts)=-dint(1,1)-dint(1,2)*Y_Elem_ts(HE_ts,WE_ts);
if Et_ts(HE_ts,WE_ts)~=0
Sigma_Elem_ts(HE_ts,WE_ts)=Et_ts(HE_ts,WE_ts)*epsilon_ts(HE_ts,WE_ts);
end
if Sigma_Elem_ts(HE_ts,WE_ts)>=SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=SigmaY;
Et_ts(HE_ts,WE_ts)=0;
elseif Sigma_Elem_ts(HE_ts,WE_ts)<=-SigmaY
Sigma_Elem_ts(HE_ts,WE_ts)=-SigmaY;
Et_ts(HE_ts,WE_ts)=0;
else
    Et_ts(HE_ts, WE_ts) = E;
end
Etotal_ts = Etotal_ts + Et_ts(HE_ts, WE_ts) / NO_ts;
pint_ts = pint_ts + Sigma_Element_ts(HE_ts, WE_ts) * (Elem_Area_ts);
mint_ts = mint_ts + Sigma_Element_ts(HE_ts, WE_ts) * Y_Element_ts(HE_ts, WE_ts) * (Elem_Area_ts);
end

LWEn_ts = t / we_ts;
for WE_ts = WEn_ts - LWEn_ts + 1:WEn_ts
    epsilon_ts(HE_ts, WE_ts) = -dint(1, 1) - dint(1, 2) * Y_Element_ts(HE_ts, WE_ts);
    if Et_ts(HE_ts, WE_ts) ~= 0
        Sigma_Element_ts(HE_ts, WE_ts) = Et_ts(HE_ts, WE_ts) * epsilon_ts(HE_ts, WE_ts);
    end
    if Sigma_Element_ts(HE_ts, WE_ts) >= SigmaY
        Sigma_Element_ts(HE_ts, WE_ts) = SigmaY;
        Et_ts(HE_ts, WE_ts) = 0;
    elseif Sigma_Element_ts(HE_ts, WE_ts) <= -SigmaY
        Sigma_Element_ts(HE_ts, WE_ts) = -SigmaY;
        Et_ts(HE_ts, WE_ts) = 0;
    else
        Et_ts(HE_ts, WE_ts) = E;
    end
end
Etotal_ts = Etotal_ts + Et_ts(HE_ts, WE_ts) / NO_ts;
pint_ts = pint_ts + Sigma_Element_ts(HE_ts, WE_ts) * (Elem_Area_ts);
mint_ts = mint_ts + Sigma_Element_ts(HE_ts, WE_ts) * Y_Element_ts(HE_ts, WE_ts) * (Elem_Area_ts);
end
end
end

P_diff = Pts + pint_ts;
m_diff = M + mint_ts;
MMy(ij) = M;
phi(ij) = dint(1, 2);
STaxial(ij) = dint(1, 1);
STaxial_ts(ij) = STaxial(ij) * SigmaY / E;
end
end
end
end

for it = 1:dvd
    % % SA1-1
    % FFd = 0.058;            % Total Time for Forcing Function
    % KFF = FFd / dt;
    % ADDDeflecion(:, 1) = 0;
    % mst = 4;
    % msb = 28;
    % est = 8;
    % esb = 24;
    % for jj = 1:NE
        % if jj == 0.5 * NE + 1;
            % ti = dt * (it);
% if it>=1 && it<KFF
%     Ft(jj,it)=((-4344*(ti+0.01)^2)+(350.2*(ti+0.01))-3.5709)*420*0.7; % Forcing Function
%     if Ft(jj,it)>Ftmin % Finding Max value of Forcing Function
%         iddxx=it;
%         Ftmax=Ft(jj,it);
%     end
%     Ftmin=Ft(jj,it);
% else
%     Ft(jj,it)=0;
% end
% end

% %     SA1-2
% FFd=0.05; % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=4;
% msb=28;
% est=8;
% esb=24;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<KFF
%             Ft(jj,it)=((-6549*(ti+0.01)^2)+(505.49*(ti+0.01))-4.7487)*420*0.7; % Forcing Function
%             if Ft(jj,it)>Ftmin % Finding Max value of Forcing Function
%                 iddxx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end

% %     SA1-3
% FFd=0.055; % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=4;
% msb=29;
% est=8;
% esb=25;
% for jj=1:NE
\begin{verbatim}
% if jj==0.5*NE+1;
%     ti=dt*(it);
%     if it>=1 && it<=KFF
%         Ft(jj,it)=((-7742.5*(ti+0.01)^2)+(584.64*(ti+0.01))-
% 5.0174)*420*0.68;         % Forcing Function
%         if Ft(jj,it)>Ftmin                   % Finding Max value of
%             Ft(jj,it)=0;
% else
%     Ft(jj,it)=0;
% end

% Fld=0.06;                   % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=4;
% msb=29;
% est=6;
% esb=27;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-8744.3*(ti+0.01)^2)+(655.54*(ti+0.01))-
% 5.4977)*420*0.6;         % Forcing Function
%             if Ft(jj,it)>Ftmin                   % Finding Max value of
%                 Ft(jj,it)=0;
% else
%     Ft(jj,it)=0;
% end
% end

% % SA1-4
% % % SA1-5
% Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=4;
% msb=29;
% est=7;
\end{verbatim}
% esb=26;
% for jj=1:NE
%    if jj==0.5*NE+1;
%        ti=dt*(it);
%        if it>=1 && it<=KFF
%            Ft(jj,it)=((-8913.9*(ti+0.01)^2)+(670.01*(ti+0.01))-5.2046)*420*0.75;          % Forcing Function
%                if Ft(jj,it)>Ftmin                   % Finding Max value of Forcing Function
%                    iddxx=it;
%                    Ftmax=Ft(jj,it);
%                end
%                Ftmin=Ft(jj,it);
%        else
%            Ft(jj,it)=0;
%        end
%    else
%        Ft(jj,it)=0;
%    end
% end

% %     %     SA1-6
% FFd=0.06;                   % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% ADDb=1.5;
% ADDt=1.5;
% mst=2;
% msb=31;
% est=9;
% esb=24;
% for jj=1:NE
%    if jj==0.5*NE+1;
%        ti=dt*(it);
%        if it>=1 && it<=KFF
%            Ft(jj,it)=((-8433.9*(ti+0.01)^2)+(670.07*(ti+0.01))-4.9684)*420;%+2*b1*(h^3)*60/EI;          % Forcing Function
%                if Ft(jj,it)>Ftmin                   % Finding Max value of Forcing Function
%                    iddxx=it;
%                    Ftmax=Ft(jj,it);
%                end
%                Ftmin=Ft(jj,it);
%        else
%            Ft(jj,it)=0;
%        end
%    else
%        Ft(jj,it)=0;
%    end
% end

% %     %     SA1-7
% FFd=0.06;                   % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=3;
% msb=30;
% est=6;
% esb=27;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-7096.1*(ti+0.01)^2)+(551.61*(ti+0.01))-4.6432)*420; % Forcing Function
%             if Ft(jj,it)>Ftmin % Finding Max value of Forcing Function
%                 iddx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end
%
% %     %     SA1-8
% FFd=0.064; % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=[-1.56310239055531;-1.56427525696205;-1.56481417562215;-1.56406834868747;-1.56139671929313;-1.55616805822587;-1.54775853372070;-1.53557034665435;-1.51904496459489;-1.49763771844653;-1.47081585790881;-1.43806854344601;-1.39891042898569;-1.35287558261977;-1.29950894530158;-1.2383763105172;-1.16902406492533;-1.09093913727988;-1.00582722699833;-0.9153194888868077;-0.82103567755969;-0.724609299020352;-0.627679076617836;-0.53189634789482;-0.438929970413437;-0.35046168739706;-0.268175390420027;-0.193758017990184;-0.128924499491514;-0.0754048668055632;-0.0349260320892689;-0.00921503784736732];
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-5875.3*(ti+0.01)^2)+(479.24*(ti+0.01))-3.7764)*420; % Forcing Function
%             if Ft(jj,it)>Ftmin % Finding Max value of Forcing Function
%                 idx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end
% end
%
%     %     %     SA2-1
%     FFd=0.02;                   % Total Time for Forcing Function
%     KFF=FFd/dt;
%     ADDEDfelecion(:,1)=0;
%     mst=6;
%     msb=27;
%     est=5;
%     esb=28;
%     for jj=1:NE
%         if jj==0.5*NE+1;
%             ti=dt*(it);
%             if it>=1 && it<=KFF
%                 Ft(jj,it)=((-56246*(ti+0.01)^2)+(2127.7*(ti+0.01))-14.1)*60*0.6;          % Forcing Function
%                 if Ft(jj,it)>Ftmin                   % Finding Max value of
%                     iddxx=it;
%                     Ftmax=Ft(jj,it);
%                 end
%                 Ftmin=Ft(jj,it);
%             else
%                 Ft(jj,it)=0;
%             end
%         else
%             Ft(jj,it)=0;
%         end
%     end
% %     %     SA2-2
%     FFd=0.034;                   % Total Time for Forcing Function
%     KFF=FFd/dt;
%     ADDEDfelecion(:,1)=0;
%     mst=7;
%     msb=26;
%     est=3;
%     esb=30;
%     for jj=1:NE
%         if jj==0.5*NE+1;
%             ti=dt*(it);
%             if it>=1 && it<=KFF
%                 Ft(jj,it)=((-20830*(ti+0.01)^2)+(1105.6*(ti+0.01))-8.9442)*140*0.67;          % Forcing Function
%                 if Ft(jj,it)>Ftmin                   % Finding Max value of
%                     iddxx=it;
%                     Ftmax=Ft(jj,it);
%                 end
%                 Ftmin=Ft(jj,it);
%             else
%                 Ft(jj,it)=0;
%             end
%         else
%             Ft(jj,it)=0;
%         end
%     end
% %
Ft(jj,it)=0;
end
end

% % % SA2-3
% FFd=0.057;
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% mst=3;
% msb=29;
% est=2;
% esb=31;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-4544.4*(ti+0.01)^2)+(354.42*(ti+0.01))-
%             3.465)*420*0.66;          % Forcing Function
%             if Ft(jj,it)>Ftmin                   % Finding Max value of
%                 iddxx=it;
%                 Ftmax=Ft(jj,it);
%             else
%                 Ftmin=Ft(jj,it);
%             end
%         else
%             Ft(jj,it)=0;
%         end
%     end
%     %     %     SA2-4
%     FFd=0.06;
%     KFF=FFd/dt;
%     ADDDeflecion(:,1)=0;
%     mst=6;
%     msb=27;
%     est=3;
%     esb=30;
%     for jj=1:NE
%         if jj==0.5*NE+1;
%             ti=dt*(it);
%             if it>=1 && it<=KFF
%                 Ft(jj,it)=((-9485.8*(ti+0.01)^2)+(711.39*(ti+0.01))-
%                 5.5229)*420*0.9;%+2*b1*(h^3)*60/EI;          % Forcing Function
%                 if Ft(jj,it)>Ftmin                   % Finding Max value of
%                     iddxx=it;
%                     Ftmax=Ft(jj,it);
%                 end
%             else
%                 Ftmin=Ft(jj,it);
%             end
%         else
%             Ft(jj,it)=0;
%         end
Ft(jj,it)=0;  
end  
else  
Ft(jj,it)=0;  
end  
end  

end  

ii=1;  

for HE=1:HEn  

TP=HE*he;  
if TP<=t  
for WE=1:WEn  
d_Elem(HE,WE)=(0.5*B)-(B/HEn)*(HE-0.5);  
end  
end  
if TP>(B-t)  
for WE=1:WEn  
d_Elem(HE,WE)=(0.5*B)-(B/HEn)*(HE-0.5);  
end  
end  
if TP>t && TP<(B-t)  

RWEEn=t/we;  
for WE=1:RWEEn  
d_Elem(HE,WE)=(0.5*B)-(B/HEn)*(HE-0.5);  
end  
LWEEn=t/we;  
for WE=WEn-LWEEn+1:WEn  
d_Elem(HE,WE)=(0.5*B)-(B/HEn)*(HE-0.5);  
end  
end  
end  

for i=1:NE+1  
if i==NE+1  
r(i,i)=2;  
elseif i==NE  
r(i,i)=1;  
r(i,i+1)=-2;  
else  
r(i,i)=1;  
r(i,i+1)=-2;  
r(i,i+2)=1;  
end  
end  

deflection=zeros(NE,1);  
Fk=zeros(NE,dvd);  
itMAX=0;  
dd=0;
for it=1:dvd

% % % SA1-1
% FFd=0.058; % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-4344*(ti+0.01)^2)+(350.2*(ti+0.01))-
% 3.5709)*420*0.7; % Forcing Function
%             if Ft(jj,it)>Ftmin % Finding Max value of
%                 idx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end

% % % SA1-2
% FFd=0.05; % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-6549*(ti+0.01)^2)+(505.49*(ti+0.01))-
% 4.7487)*420*0.7; % Forcing Function
%             if Ft(jj,it)>Ftmin % Finding Max value of
%                 idx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end
%% SA1-3
 FFd=0.055;            % Total Time for Forcing Function
 KFF=FFd/dt;
 ADDDeflecion(:,1)=0;
 for jj=1:NE
     if jj==0.5*NE+1;
       ti=dt*(it);
       if it>=1 & it<=KFF
         Ft(jj,it)=((-7742.5*(ti+0.01)^2)+(584.64*(ti+0.01))-
 5.0174)*420*0.64;     % Forcing Function
         if Ft(jj,it)>Ftmin                   % Finding Max value of
                         % Forcing Function
             idx=it;
             Ftmax=Ft(jj,it);
         end
         Ftmin=Ft(jj,it);
       else
         Ft(jj,it)=0;
       end
     else
         Ft(jj,it)=0;
     end
 end

%% SA1-4
 FFd=0.06;            % Total Time for Forcing Function
 KFF=FFd/dt;
 ADDDeflecion(:,1)=0;
 for jj=1:NE
     if jj==0.5*NE+1;
       ti=dt*(it);
       if it>=1 & it<=KFF
         Ft(jj,it)=((-8744.3*(ti+0.01)^2)+(655.54*(ti+0.01))-
 5.4977)*420*0.6;       % Forcing Function
         if Ft(jj,it)>Ftmin                   % Finding Max value of
                         % Forcing Function
             idx=it;
             Ftmax=Ft(jj,it);
         end
         Ftmin=Ft(jj,it);
       else
         Ft(jj,it)=0;
       end
     else
         Ft(jj,it)=0;
     end
 end

%% SA1-5
 FFd=0.06;            % Total Time for Forcing Function
 KFF=FFd/dt;
 ADDDeflecion(:,1)=0;
 for jj=1:NE
% if jj==0.5*NE+1;
%     ti=dt*(it);
%     if it>=1 && it<=KFF
%         Ft(jj,it)=((-8913.9*(ti+0.01)^2)+(670.01*(ti+0.01))-
%             5.2046)*420*0.75;          % Forcing Function
%         if Ft(jj,it)>Ftmin
%             idx=it;
%             Ftmax=Ft(jj,it);
%         end
%         Ftmin=Ft(jj,it);
%     else
%         Ft(jj,it)=0;
%     end
% else
%     Ft(jj,it)=0;
% end
%
% %     %     SA1-6
% FFd=0.06;                   % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-8433.9*(ti+0.01)^2)+(670.07*(ti+0.01))-
%             4.9684)*420;          % Forcing Function
%             if Ft(jj,it)>Ftmin
%                 idx=it;
%                 Ftmax=Ft(jj,it);
%             end
%             Ftmin=Ft(jj,it);
%         else
%             Ft(jj,it)=0;
%         end
%     else
%         Ft(jj,it)=0;
%     end
% end
%
% %     %     SA1-7
% FFd=0.06;                   % Total Time for Forcing Function
% KFF=FFd/dt;
% ADDDeflecion(:,1)=0;
% for jj=1:NE
%     if jj==0.5*NE+1;
%         ti=dt*(it);
%         if it>=1 && it<=KFF
%             Ft(jj,it)=((-7096.1*(ti+0.01)^2)+(551.61*(ti+0.01))-
%             4.6432)*420;          % Forcing Function
% if Ft(jj,it)>Ftmin           % Finding Max value of Forcing Function
%    idx=it;                  
%    Ftmpax=Ft(jj,it);        
% end                         
% else                        
%    Ftmin=Ft(jj,it);        
% end                         
% else                        
%    Ft(jj,it)=0;            
% end                         
% end

% FFd=0.064;                   % Total Time for Forcing Function
% KFF=FFd/dt;                 
% ADDDefleccion(:,1)=[-1.56310239055531;-1.56427525696205;-1.56406834868747;-1.56139671929313;-1.55616805822587;-1.54775853372070;-1.53557034665435;-1.51904964594898;-1.49763718446537;-1.47081585790881;-1.43806854346011;-1.39891042898569;-1.35287558261977;-1.29950894530158;-1.23837363105172;-1.16902406492533;-1.09093913727988;-1.00582722698333;-0.915319488868077;-0.821035677855969;-0.724609299020352;-0.6276790766178366;-0.531896347899482;-0.4389299704134375;-0.3504616876397066;-0.2681753904200275;-0.1937580179901846;-0.1289244994915141;-0.07540486680556312-0.0349260320892689-0.00921503784736732];
% for jj=1:NE
%     if jj==0.5*NE+1;
%       ti=dt*(it);
%       if it>=1 & it<=KFF
%         Ft(jj,it)=(-5875.3*(ti+0.01)^2)+(479.24*(ti+0.01))-3.7764)*420;       % Forcing Function
%       if Ft(jj,it)>Ftmin           % Finding Max value of Forcing Function
%         idx=it;
%         Ftmpax=Ft(jj,it);        
%       end                         
%       Ftmpax=Ft(jj,it);        
%     else                        
%       Ft(jj,it)=0;            
%     end                         
% end

% FFd=0.02;                    % Total Time for Forcing Function
% KFF=FFd/dt;                 
% ADDDefleccion(:,1)=0;
% for jj=1:NE
%     if jj==0.5*NE+1;
\[
\text{ti} = \text{dt} \times (\text{it});
\]
\[
\text{if it} > 1 \&\& \text{it} \leq \text{KFF}
\]
\[
\text{Ft(jj, it)} = ((-56246 \times (\text{ti}+0.01)^2) + (2127.7 \times (\text{ti}+0.01)) - 14.1) \times 60 \times 0.6;
\]
\[
\text{if Ft(jj, it)} > \text{Ftmin} \quad \% \text{Finding Max value of Forcing Function}
\]
\[
\text{idx} = \text{it};
\]
\[
\text{Ftmax} = \text{Ft(jj, it)};
\]
\[
\text{end}
\]
\[
\text{Ftmin} = \text{Ft(jj, it)};
\]
\[
\text{else}
\]
\[
\text{Ft(jj, it)} = 0;
\]
\[
\text{end}
\]
\[
\% \text{end}
\]

\[
\% \% \% \text{SA2-2}
\]
\[
\% \text{FFd=0.034;} \quad \% \text{Total Time for Forcing Function}
\]
\[
\% \text{KFF=FFd/dt;}
\]
\[
\% \text{ADDDeflecion(:,1)=0;}
\]
\[
\% \text{for jj=1:NE}
\]
\[
\% \quad \text{if jj=0.5*NE+1;}
\]
\[
\% \quad \text{ti} = \text{dt} \times (\text{it});
\]
\[
\% \quad \text{if it} > 1 \&\& \text{it} \leq \text{KFF}
\]
\[
\% \quad \text{Ft(jj, it)} = ((-20830 \times (\text{ti}+0.01)^2) + (1105.6 \times (\text{ti}+0.01)) - 8.9442) \times 140 \times 0.67;
\]
\[
\% \quad \text{if Ft(jj, it)} > \text{Ftmin} \quad \% \text{Finding Max value of Forcing Function}
\]
\[
\% \quad \text{idx} = \text{it};
\]
\[
\% \quad \text{Ftmax} = \text{Ft(jj, it)};
\]
\[
\% \quad \text{end}
\]
\[
\% \quad \text{Ftmin} = \text{Ft(jj, it)};
\]
\[
\% \quad \text{else}
\]
\[
\% \quad \text{Ft(jj, it)} = 0;
\]
\[
\% \quad \text{end}
\]
\[
\% \quad \text{else}
\]
\[
\% \quad \text{Ft(jj, it)} = 0;
\]
\[
\% \% \% \% \text{end}
\]

\[
\% \% \% \text{SA2-3}
\]
\[
\% \text{FFd=0.056;} \quad \% \text{Total Time for Forcing Function}
\]
\[
\% \text{KFF=FFd/dt;}
\]
\[
\% \text{ADDDeflecion(:,1)=0;}
\]
\[
\% \text{for jj=1:NE}
\]
\[
\% \quad \text{if jj=0.5*NE+1;}
\]
\[
\% \quad \text{ti} = \text{dt} \times (\text{it});
\]
\[
\% \quad \text{if it} > 1 \&\& \text{it} \leq \text{KFF}
\]
\[
\% \quad \text{Ft(jj, it)} = ((-4544.4 \times (\text{ti}+0.01)^2) + (354.42 \times (\text{ti}+0.01)) - 3.465) \times 420 \times 0.66;
\]
\[
\% \quad \text{if Ft(jj, it)} > \text{Ftmin} \quad \% \text{Finding Max value of Forcing Function}
\]
\[
\text{end}
\]
% idx=it;
% Ftmax=Ft(jj,it);
%         end
%                 Ftmin=Ft(jj,it);
%             else
%                 Ft(jj,it)=0;
%             end
%         else
%             Ft(jj,it)=0;
%         end

% SA2-4
PPd=0.06;                   % Total Time for Forcing Function
KFF=PPd/dt;
ADDDeflecion(:,1)=0;
for jj=1:NE
    if jj==0.5*NE+1;
        ti=dt*(it);
        if it>=1 && it<=KFF
            Ft(jj,it)=((-9485.8*(ti+0.01)^2)+(711.39*(ti+0.01))-5.5229)*420*0.9;
            % Forcing Function
            if Ft(jj,it)>Ftmin                   % Finding Max value of Forcing Function
                idx=it;
                Ftmax=Ft(jj,it);
            end
            Ftmin=Ft(jj,it);
        else
            Ft(jj,it)=0;
        end
    else
        Ft(jj,it)=0;
    end
end

d1=[EI/Kspr 0 0 0 0 0 0 0];
d2=[0 -EI/Kt 0 0 0 0 0 0];
d3=[0 0 0 0 (L^3)/6 (L^3)/6 L 1];
d4=[0 0 0 0 EI*L+KB*(L^2)/2 EI*KB*L KB 0];
d5=[(L^3)/48 (L^2)/8 L/2 1 0 -(L^3)/48 -(L^2)/8 -(L^2)/8 -L/2 -1];
d6=[(L^2)/8 L/2 1 0 -(L^2)/8 -(L^2)/8 -L/2 -1 0];
d7=[L/2 1 0 0 -L/2 -1 0 0];
d8=[1 0 0 0 -1 0 0 0];

d=[d1;d2;d3;d4;d5;d6;d7;d8];
 LOAD=zeros(8,1);
 total=1;
counter(it)=0;
 while total>0
     counter(it)=counter(it)+1;
 end
if it==1
LOAD(8)=Ft(0.5*NE+1,it)/EI;
in=inv(rs);
disp(in);
Const=in*LOAD;
for i=1:NE
x=(i-1)*h;
if x<= L/2
Deflections(i,1)=Const(1)*(x^3)/6+Const(2)*(x^2)/2+Const(3)*x+Const(4);
extelse
Deflections(i,1)=Const(5)*(x^3)/6+Const(6)*(x^2)/2+Const(7)*x+Const(8);
end
end
for i=3:NE+2
defl(i,1)=Deflections(i-2,1);
end
defl(NE+3,1)=0;
defl(2,1)=b10*defl(3,1)-b5*defl(4,1);
defl(1,1)=((-0.5*b2+b1)/((0.5*b1)))*defl(2,1)-((-0.5*b2+b1)/((0.5*b1))*defl(4,1)+defl(5,1)+(2*b25/b1)*defl(3,1);
defl(NE+4,1)=b45*defl(NE+3,1)-b9*defl(NE+2,1);
else
for j=1:NE
Deflections(j,2)=b6*defl(j,1)+(-4*b6+b7)*defl(j+1,1)+(6*b6-2*b7+1)*defl(j+2,1)+(-4*b6+b7)*defl(j+3,1)+b6*defl(j+4,1);
end
Fk(:,it)=-Ft(:,it)*c1;
if it==iddxx
maxDeflection(:,1)=Deflections(:,it);
r1=[EI/Kspr 0 0 1 0 0 0 0];
r2=[0 -EI/Kt 1 0 0 0 0 0];
r3=[0 0 0 0 (L^3)/6 (L^2)/2 L 1];
r4=[0 0 0 0 EI*L+KB*(L^2)/2 EI+KB*L KB 0];
r5=[(L^3)/48 (L^2)/8 L/2 1 -(L^3)/48 -(L^2)/8 -L/2 -1];
r6=[(L^2)/8 L/2 1 0 -(L^2)/8 -L/2 -1 0];
r7=[L/2 1 0 0 -L/2 -1 0 0];
r8=[1 0 0 0 -1 0 0 0];
rst=[r1;r2;r3;r4;r5;r6;r7;r8];
LOAD=zeros(8,1);
LOAD(8)=Ft(0.5*NE+1,idxELASTIC)/EI;
in=inv(rst);
disp(in);
Const=in*LOAD;
for i=1:NE
x=(i-1)*h;
if x<= L/2
deflection(i,1)=Const(1)*(x^3)/6+Const(2)*(x^2)/2+Const(3)*x+Const(4);
extelse
deflection(i,1)=Const(5)*(x^3)/6+Const(6)*(x^2)/2+Const(7)*x+Const(8);
end
end
% DeflectionPERMset(:,1)=maxDeflection(:,1)-deflection(:,1)+ADDDeflection(:,1);
DeflectionPERMset(:,1)=ADDDeflection(:,1);
end
rm=zeros(NE,NE);
for iji=1:NE
    EI=BP(iji);
    b1=EI/h^4;
    b2=P/h^2;
    b5=(EI/h-Kt/2)/(EI/h+Kt/2);
    b9=(EI/h-KB/2)/(EI/h+KB/2);
    b10=(2*EI/h)/(EI/h+Kt/2);
    b3=mass/(dt^2);
    b4=C/(2*dt);
    b6=-(dt^2)*EI/(2*mass*h^4);
    b7=-(dt^2)*P/(2*mass*h^2);
    b8=-(dt^2)*P/(2*mass);
    b25=Kspr/h;
    b11=h/EI;
    b33=1/(h^3);
    b44=1/(h^2);
    c1=-(1/(b3+b4));
    c2=(b3-b4)*c1;
    c3=1/(b44+b2+b1);
    if iji==1
        rm(iji,iji)=((1-b11*BP1(iji))*(1-b11*BP1(iji))*b10+(1-b11*BP1(iji))*(2*b25-BP1(iji))*4*b33)+(-4*b1*b10)+6*b1+2*b33*b10*BP1(iji)+(-2*b44*BP2(iji))+b2*b10+(-2*b2)+(-2*b3)*c1;
        rm(iji,iji+1)=((1-b11*BP1(iji))*(1-b11*BP1(iji))*(2*b25-BP1(iji))*2*b33)+(-4*b1*b10)+6*b1+2*b33*b10*BP1(iji)+(-2*b44*BP2(iji))+b2*b10+(-2*b2)+(-2*b3)*c1;
        rm(iji,iji+2)=((1-b11*BP1(iji))*(1-b11*BP1(iji))*b10+(1-b11*BP1(iji))*(2*b25-BP1(iji))*b33)+(-4*b1*b10)+6*b1+2*b33*b10*BP1(iji)+(-2*b44*BP2(iji))+b2*b10+(-2*b2)+(-2*b3)*c1;
        elseif iji==2
        rm(iji,iji-1)=(b1*b10+(-b33*b10*BP1(iji)))+(-4*b1)+(2*b33+BP1(iji))*b33+BP2(iji)*b44+b2)*c1;
        rm(iji,iji)=((b1*(b5)+(-b33*(-b5)*BP1(iji)))+(6*b1)+(-2*b44)*BP2(iji))+(-2*b2)+(-2*b3)))*c1;
        rm(iji,iji+1)=((-4*b1)+(-2*b33*(BP1(iji)))+(b44)*BP2(iji)+(b2)))*c1;
        rm(iji,iji+2)=(b1+33*BP1(iji))*c1;
        elseif iji>1 && iji<NE-1
        rm(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
        rm(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44)b2)*c1;
        rm(iji,iji)=((6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
        rm(iji,iji+1)=(-4*b1-2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
        rm(iji,iji+2)=(b1+33*BP1(iji))*c1;
        elseif iji==NE-1
        rm(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
        rm(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
        rm(iji,iji)=(6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
        rm(iji,iji+1)=(-4*b1-2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
        elseif iji==NE
        rm(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
        rm(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
        rm(iji,iji)=(-b1*b9-b9*BP1(iji)*b33+6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
        end
    end

if it>iddxx
    if abs(MAXDynStrain)<abs(YIELD_STRAIN)
Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
Deflections1(:,it+1)=Deflections(:,it+1);
else
  if it<iddxx
    Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
    Deflections1(:,it+1)=Deflections(:,it+1);
  elseif it>iddxx && it<KFF
    Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
    Deflections1(:,it+1)=Deflections(:,it+1)+DeflectionPERMset(:,1)*((it-iddxx)/(KFF-iddxx));
  else
    Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
    Deflections1(:,it+1)=Deflections(:,it+1)+DeflectionPERMset(:,1);
  end
  end
else
  Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
  Deflections1(:,it+1)=Deflections(:,it+1);
end
end
if it<=iddxx
  for jjii=1:NE
    DEFLECT(jjii+1,1)=Deflections(jjii,it+1);
  end
  DEFLECT(1,1)=b10*DEFLECT(2,1)-b5*DEFLECT(3,1);
else
  for jjii=1:NE
    DEFLECT(jjii+1,1)=Deflections1(jjii,it+1);
  end
  DEFLECT(1,1)=b10*DEFLECT(2,1)-b5*DEFLECT(3,1);
end
DynCurvaturesNE(:,1)=-r*(DEFLECT(:,1))/(h^2);
for ii=1:NE
  DynE_Elem_sum=0;
  Ix(ii,1)=0;
  Minternal(ii,1)=0;
  Pinternal(ii,1)=0;
  AxialSTRAIN(ii,1)=0;
  Ael(ii,1)=0;
  Sx(ii,1)=0;
  Mpp(ii,1)=0;
  E_sum(ii,1)=0;
  Ix(ii,1)=0;
  Minternal(ii,1)=0;
  Pinternal(ii,1)=0;
  Pp(ii,1)=0;
  for HE=1:HEn
    TP=HE*he;
    if TP<=t
      for WE=1:WEn
        DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);
        DynE_Elem(HE,WE)=E;
        DynStress_Elem(HE,WE)=E*DynStrain_Elem(HE,WE);
        ElasticAe(HE,WE)=Elem_Area;
        PlasticAe(HE,WE)=0;
        Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        if DynStress_Elem(HE,WE)>= SigmaY
          Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        end
      end
    end
  end
end
else
  Deflections(:,it+1)=rm*Deflections(:,it)+Deflections(:,it-1)*c2-(Fk(:,it));
  Deflections1(:,it+1)=Deflections(:,it+1);
end
end
DynStress_Elem(HE,WE)=SigmaY;
DynE_Elem(HE,WE)=0;
ElasticAe(HE,WE)=0;
PlasticAe(HE,WE)=Elem_Area;
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
end
if DynStress_Elem(HE,WE)<= -SigmaY
DynStress_Elem(HE,WE)=-SigmaY;
DynE_Elem(HE,WE)=0;
ElasticAe(HE,WE)=0;
PlasticAe(HE,WE)=Elem_Area;
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
end
Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);
DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;
Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);
Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);
Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);
Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);
Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);
Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);
end
end

if TP>(B-t)
for WE=1:WEn
DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);
DynE_Elem(HE,WE)=E;
DynStress_Elem(HE,WE)=E*DynStrain_Elem(HE,WE);
ElasticAe(HE,WE)=Elem_Area;
PlasticAe(HE,WE)=0;
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
if DynStress_Elem(HE,WE)>= SigmaY
DynStress_Elem(HE,WE)=SigmaY;
DynE_Elem(HE,WE)=0;
ElasticAe(HE,WE)=0;
PlasticAe(HE,WE)=Elem_Area;
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
end
if DynStress_Elem(HE,WE)<= -SigmaY
DynStress_Elem(HE,WE)=-SigmaY;
DynE_Elem(HE,WE)=0;
ElasticAe(HE,WE)=0;
PlasticAe(HE,WE)=Elem_Area;
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
end
Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);
DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;
Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);
Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);
Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);
Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);
Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);
end
end
if TP>t && TP<=0.5*B
    RWEn=t/we;
    for WE=1:RWEn
        DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);
        DynE_Elem(HE,WE)=E;
        DynStress_Elem(HE,WE)=E*DynStrain_Elem(HE,WE);
        ElasticAe(HE,WE)=Elem_Area;
        PlasticAe(HE,WE)=0;
        Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        if DynStress_Elem(HE,WE)>= SigmaY
            DynStress_Elem(HE,WE)=SigmaY;
            DynE_Elem(HE,WE)=0;
            PlasticAe(HE,WE)=Elem_Area;
            Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        end
        if DynStress_Elem(HE,WE)<= -SigmaY
            DynStress_Elem(HE,WE)=-SigmaY;
            DynE_Elem(HE,WE)=0;
            PlasticAe(HE,WE)=Elem_Area;
            Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        end
        Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);
        DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;
        Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);
        Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);
        Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);
        Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);
        Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);
        Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);
    end
end
if TP>t && TP<=0.5*B
    LWEn=t/we;
    for WE=WEW-1:LWEn+1:WEW
        DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);
        DynE_Elem(HE,WE)=E;
        DynStress_Elem(HE,WE)=E* DynStrain_Elem(HE,WE);
        ElasticAe(HE,WE)=Elem_Area;
        PlasticAe(HE,WE)=0;
        Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        if DynStress_Elem(HE,WE)>= SigmaY
            DynStress_Elem(HE,WE)=SigmaY;
            DynE_Elem(HE,WE)=0;
            PlasticAe(HE,WE)=Elem_Area;
            Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        end
        if DynStress_Elem(HE,WE)<= -SigmaY
            DynStress_Elem(HE,WE)=-SigmaY;
            DynE_Elem(HE,WE)=0;
            PlasticAe(HE,WE)=Elem_Area;
            Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
        end
        Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);
        DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;
        Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);
        Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);
        Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);
        Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);
        Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);
        Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);
    end
end
ElasticAe(HE,WE)=0;  
PlasticAe(HE,WE)=Elem_Area;  
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);  
end  
Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);  
DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;  
Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);  
Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);  
Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);  
Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);  
Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);  
Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);  
end  
end  
if TP>0.5*B && TP<=(B-t)  
RWEn=t/we;  
for WE=1:RWEn  
DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);  
DynE_Elem(HE,WE)=E;  
DynStress_Elem(HE,WE)=E*DynStrain_Elem(HE,WE);  
ElasticAe(HE,WE)=Elem_Area;  
PlasticAe(HE,WE)=0;  
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);  
if DynStress_Elem(HE,WE)>= SigmaY  
DynStrain_Elem(HE,WE)=SigmaY;  
DynE_Elem(HE,WE)=0;  
ElasticAe(HE,WE)=0;  
PlasticAe(HE,WE)=Elem_Area;  
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);  
end  
if DynStress_Elem(HE,WE)<= -SigmaY  
DynStress_Elem(HE,WE)=-SigmaY;  
DynE_Elem(HE,WE)=0;  
ElasticAe(HE,WE)=0;  
PlasticAe(HE,WE)=Elem_Area;  
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);  
end  
Ix(ii,1)=(ElasticAe(HE,WE)^2)/12+ElasticAe(HE,WE)*(d_Elem(HE,WE)^2)+Ix(ii,1);  
DynE_Elem_sum=DynE_Elem(HE,WE)/NO+DynE_Elem_sum;  
Minternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area*d_Elem(HE,WE)+Minternal(ii,1);  
Pinternal(ii,1)=DynStress_Elem(HE,WE)*Elem_Area+Pinternal(ii,1);  
Ael(ii,1)=ElasticAe(HE,WE)+Ael(ii,1);  
Sx(ii,1)=ElasticAe(HE,WE)*d_Elem(HE,WE)+Sx(ii,1);  
Mpp(ii,1)=DynStrain_Elem(HE,WE)*PlasticAe(HE,WE)*d_Elem(HE,WE)+Mpp(ii,1);  
Pp(ii,1)=Pp_e(HE,WE)+Pp(ii,1);  
end  
end  
LWEn=t/we;  
for WE=WEn-LWEn+1:WEn  
DynStrain_Elem(HE,WE)=DynCurvaturesNE(ii,1)*d_Elem(HE,WE);  
DynE_Elem(HE,WE)=E;  
DynStress_Elem(HE,WE)=E*DynStrain_Elem(HE,WE);  
ElasticAe(HE,WE)=Elem_Area;  
PlasticAe(HE,WE)=0;  
Pp_e(HE,WE)=PlasticAe(HE,WE)*DynStrain_Elem(HE,WE);
if DynStress_Elem(HE,WE) >= SigmaY
DynStress_Elem(HE,WE) = SigmaY;
DynE_ELEM(HE,WE) = 0;
ElasticAe(HE,WE) = 0;
PlasticAe(HE,WE) = Elem_Area;
Pp_e(HE,WE) = PlasticAe(HE,WE) * DynStrain_Elem(HE,WE);
end
if DynStress_Elem(HE,WE) <= -SigmaY
DynStress_Elem(HE,WE) = -SigmaY;
DynE_ELEM(HE,WE) = 0;
ElasticAe(HE,WE) = 0;
PlasticAe(HE,WE) = Elem_Area;
Pp_e(HE,WE) = PlasticAe(HE,WE) * DynStrain_Elem(HE,WE);
end
Ix(ii,1) = (ElasticAe(HE,WE)^2)/12 + ElasticAe(HE,WE) * (d_ELEM(HE,WE)^2) + Ix(ii,1);
DynE_ELEM_sum = DynE_ELEM(HE,WE)/NO + DynE_ELEM_sum;
Minternal(ii,1) = DynStress_Elem(HE,WE) * Elem_Area * d_ELEM(HE,WE) + Minternal(ii,1);
Pinternal(ii,1) = DynStress_Elem(HE,WE) * Elem_Area + Pinternal(ii,1);
Ael(ii,1) = ElasticAe(HE,WE) + Ael(ii,1);
Sx(ii,1) = ElasticAe(HE,WE) * d_ELEM(HE,WE) + Sx(ii,1);
Mpp(ii,1) = DynStrain_Elem(HE,WE) * PlasticAe(HE,WE) * d_ELEM(HE,WE) + Mpp(ii,1);
Pp(ii,1) = Pp_e(HE,WE) + Pp(ii,1);
end
end
end
E_SECTION(ii,it) = DynE_ELEM_sum;
INTMOMENT(ii,it) = Minternal(ii,1);
INTP(ii,it) = Pinternal(ii,1);
Dynstrain_ELEMMAXFIND(ii,it) = DynStrain_ELEM(1,1);
Dynstress_SECTIONPLOT(ii,it) = DynStress_ELEM(1,1);
Curvature(ii,it) = DynCurvaturesNE(ii,1);
STRAINtop(ii,1) = DynStrain_ELEM(1,1)/EpsilonY;
STRAINbottom(ii,1) = DynStrain_ELEM(1,1)/EpsilonY;
if ii == 0.5*NE+1
if it >= iddxx && it < KFF
Dynstrain_SECTIONPLOTtop(1,it) = (DynStrain_ELEM(mst,1) - (0.5*(HEn-mst)/HEn)*DynStrain_PERMSETmid*((it-iddxx)/(KFF-iddxx))) * ADDb;
Dynstrain_SECTIONPLOTbottom(1,it) = (DynStrain_ELEM(msb,1) + (0.5*(msb)/HEn)*DynStrain_PERMSETmid*)((it-iddxx)/(KFF-iddxx))) * ADDb;
else
Dynstrain_SECTIONPLOTtop(1,it) = (DynStrain_ELEM(mst,1) - (0.5*(HEn-mst)/HEn)*DynStrain_PERMSETmid)*ADDb;
Dynstrain_SECTIONPLOTbottom(1,it) = (DynStrain_ELEM(msb,1) + (0.5*(msb)/HEn)*DynStrain_PERMSETmid)*ADDb;
end
MOMENT_SECTIONPLOT(1,it) = Minternal(ii,1);
CURVATURE_SECTIONPLOT(1,it) = DynCurvaturesNE(ii,1);
end
if ii == NE-2
if it >= iddxx && it < KFF
Dynstrain_SECTIONPLOT_Btop(1,it) = (DynStrain_ELEM(est,1) + (0.5*(HEn-est)/HEn)*DynStrain_PERMSETend*)((it-iddxx)/(KFF-iddxx))) * ADDb;
end
DynStrain_SectionPLOT_Bbottom(1,it)=(DynStrain_Elem(esb,1)-(0.5*(est)/HEn)*DynStrain_PERMSETend*((it-iddxx)/(KFF-iddxx)))*ADDb;
elseif it>KFF
DynStrain_SectionPLOT_Btop(1,it)=(DynStrain_Elem(est,1)+(0.5*(HEn-est)/HEn)*DynStrain_PERMSETend)*ADDb;
DynStrain_SectionPLOT_Bbottom(1,it)=(DynStrain_Elem(esb,1)-(0.5*(est)/HEn)*DynStrain_PERMSETend)*ADDb;
else
DynStrain_SectionPLOT_Btop(1,it)=DynStrain_Elem(est,1)*ADDb;
DynStrain_SectionPLOT_Bbottom(1,it)=DynStrain_Elem(esb,1)*ADDb;
end
MOMent_SectionPLOT_B(1,it)=Minternal(ii,1);
CURVenT_SectionPLOT_B(1,it)=DynCurvaturesNE(ii,1);
end
if it==iddxx
if abs(DynStrain_ElemMAXFIND(0.5*NE+1,it))>abs(YIELD_STRAIN)
DynStrain_PERMSETmid=abs(DynStrain_ElemMAXFIND(0.5*NE+1,it))-abs(YIELD_STRAIN);
else
DynStrain_PERMSETmid=0;
end
if abs(DynStrain_ElemMAXFIND(NE-1,it))>abs(YIELD_STRAIN)
DynStrain_PERMSETend=abs(DynStrain_ElemMAXFIND(NE-1,it))-abs(YIELD_STRAIN);
else
DynStrain_PERMSETend=0;
end
end
if abs(DynStrain_ElemMAXFIND(NE-1,it))>abs(YIELD_STRAIN)
if dd<1
dd=dd+1;
idxELASTIC=it;
end
end
if ii==0.5*NE+1
if abs(MAXDynStrain)<abs(DynStrain_ElemMAXFIND(ii,it));
MAXDynStrain=DynStrain_ElemMAXFIND(ii,it);
itMAX=it;
end
end
for i=1:NE
ZO=h*(i-1);
syms Z;
if E_Section(i)>=28999900
BP(i)=EI;
BP1(i)=0;
BP2(i)=0;
else
if i==NE-1
x1=h*(i-1);
x2=h*(i);
x3=h*(i+1);
x4=h*(i+2);
L1=((Z-x2)*(Z-x3)*(Z-x4))/((x1-x2)*(x1-x3)*(x1-x4));
end
end

L2=((Z-x1)*(Z-x3)*(Z-x4))/((x2-x1)*(x2-x3)*(x2-x4));
L3=((Z-x1)*(Z-x2)*(Z-x4))/((x3-x1)*(x3-x2)*(x3-x4));
L4=((Z-x1)*(Z-x2)*(Z-x3))/((x4-x1)*(x4-x2)*(x4-x3));
Modulus=L1*Ix(i-1,1)+L2*Ix(i,1)+L3*Ix(i+1,1)+L4*Ix(i+2,1);
BPs1=diff(Modulus,Z);
BPs2=diff(Modulus,Z,2);
BP(i)=subs(Modulus,Z,ZO);
BP1(i)=subs(BPs1,Z,ZO);
BP2(i)=subs(BPs2,Z,ZO);
elseif i==NE
x1=h*(i-2);
x2=h*(i-1);
x3=h*(i);
x4=h*(i+1);
L1=((Z-x2)*(Z-x3)*(Z-x4))/((x1-x2)*(x1-x3)*(x1-x4));
L2=((Z-x1)*(Z-x3)*(Z-x4))/((x2-x1)*(x2-x3)*(x2-x4));
L3=((Z-x1)*(Z-x2)*(Z-x4))/((x3-x1)*(x3-x2)*(x3-x4));
L4=((Z-x1)*(Z-x2)*(Z-x3))/((x4-x1)*(x4-x2)*(x4-x3));
Modulus=L1*Ix(i-2,1)+L2*Ix(i-1,1)+L3*Ix(i,1)+L4*Ix(i+1,1);
BPs1=diff(Modulus,Z);
BPs2=diff(Modulus,Z,2);
BP(i)=subs(Modulus,Z,ZO);
BP1(i)=subs(BPs1,Z,ZO);
BP2(i)=subs(BPs2,Z,ZO);
else
x1=h*(i-1);
x2=h*(i);
x3=h*(i+1);
x4=h*(i+2);
L1=((Z-x2)*(Z-x3)*(Z-x4))/((x1-x2)*(x1-x3)*(x1-x4));
L2=((Z-x1)*(Z-x3)*(Z-x4))/((x2-x1)*(x2-x3)*(x2-x4));
L3=((Z-x1)*(Z-x2)*(Z-x4))/((x3-x1)*(x3-x2)*(x3-x4));
L4=((Z-x1)*(Z-x2)*(Z-x3))/((x4-x1)*(x4-x2)*(x4-x3));
Modulus=L1*Ix(i,1)+L2*Ix(i+1,1)+L3*Ix(i+2,1)+L4*Ix(i+3,1);
BPs1=diff(Modulus,Z);
BPs2=diff(Modulus,Z,2);
BP(i)=subs(Modulus,Z,ZO);
BP1(i)=subs(BPs1,Z,ZO);
BP2(i)=subs(BPs2,Z,ZO);
end
end
end
if it==1
Deflectionscheck(:,1)=Deflections(:,1);
Deflectionscheck1(:,1)=Deflectionscheck(:,1);
Deflections1(:,1)=Deflections(:,1);
else
Deflectionscheck(:,2)=Deflections(:,2);
Deflectionscheck1(:,2)=Deflectionscheck(:,2);
Deflections1(:,2)=Deflections(:,2);
rm=zeros(NE,NE);
for iji=1:NE
EI=BP(iji);
b1=EI/h^4;
b2=P/h^2;
b5=(EI/h-Kt/2)/(EI/h+Kt/2);
b9=(EI/h-KB/2)/(EI/h+KB/2);
b10=mass/(dt^2);
b4=C/(2*dt);
b6=-(dt^2)*EI/(2*mass*h^4);
b7=-(dt^2)*P/(2*mass*h^2);
b8=-(dt^2)*P/(2*mass);
b25=Kspr/h;
b11=h/EI;
b33=1/(h^3);
b44=1/(h^2);
c1=-(1/(b3+b4));
c2=(b3-b4)*c1;
c3=1/(b44+b2+b1);

if iji==1
rm2(iji,iji)=((1-b11*BP1(iji))*(-b2+2*b1+BP1(iji)*2*b33)*b10+(1-b11*BP1(iji))*2*b25*BP2(iji)+(b2-2*b1+BP1(iji)*2*b33)+(-4*b1)+2*b33*BP1(iji)+b44*BP2(iji)+b2)*c1;
rm2(iji,iji+1)=((-b2+2*b1+BP1(iji)*2*b33)*(-b5)+(1-b11*BP1(iji))*(b2-2*b1+BP1(iji)*2*b33)+(-4*b1)+2*b33*(-b5)*BP1(iji)+(-2*b33)*BP1(iji)+b44*(-b5)*BP2(iji)+b44*BP2(iji)+b2*(-b5)+b2)*c1;
rm2(iji,iji+2)=(-b33*BP1(iji)+2*b1+b33*BP1(iji))*c1;
elseif iji==2
rm2(iji,iji-1)=(b1+(-2*BP1(iji)*b33)+BP2(iji)*b44+b2)*c1;
rm2(iji,iji)=((-4*b1)+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji+1)=((-4*b1)+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji+2)=(-b1+b33*BP1(iji))*c1;
elseif iji>1 && iji<(NE-1)
rm2(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
rm2(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji)=(6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
rm2(iji,iji+1)=(-4*b1-2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji+2)=(b1+BP1(iji)*b33)*c1;
elseif iji==NE-1
rm2(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
rm2(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji)=(6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
rm2(iji,iji+1)=(-4*b1-2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
elseif iji==NE
rm2(iji,iji-2)=(b1-BP1(iji)*b33)*c1;
rm2(iji,iji-1)=(-4*b1+2*BP1(iji)*b33+BP2(iji)*b44+b2)*c1;
rm2(iji,iji)=(-b1*b9-b9*BP1(iji)*b33+6*b1-2*BP2(iji)*b44-2*b2-2*b3)*c1;
end
end

if it>iddxx
if abs(MAXDynStrain)<abs(YIELD_STRAIN)
Deflectionscheck(:,it+1)=rm2*Deflectionscheck(:,it)+Deflectionscheck(:,it-1)*c2-(Fk(:,it));
Deflectionscheck1(:,it+1)=Deflectionscheck(:,it+1);
else
    if it<iddxx
        Deflectionscheck(:,it+1)=rm2*Deflectionscheck(:,it)+Deflectionscheck(:,it-1)*c2-(Fk(:,it));
        Deflectionscheck1(:,it+1)=Deflectionscheck(:,it+1);
    elseif it>=iddxx && it<KFF
        Deflectionscheck(:,it+1)=rm2*Deflectionscheck(:,it)+Deflectionscheck(:,it-1)*c2-(Fk(:,it));
        Deflectionscheck1(:,it+1)=Deflectionscheck(:,it+1)+DeflectionPERMset(:,1)*((it-iddxx)/(KFF-iddxx));
    else
        Deflectionscheck(:,it+1)=rm2*Deflectionscheck(:,it)+Deflectionscheck(:,it-1)*c2-(Fk(:,it));
        Deflectionscheck1(:,it+1)=Deflectionscheck(:,it+1)+DeflectionPERMset(:,1)*((it-iddxx)/(KFF-iddxx));
    end
end
else
    Deflectionscheck(:,it+1)=rm2*Deflectionscheck(:,it)+Deflectionscheck(:,it-1)*c2-(Fk(:,it));  % Dynamic Formula Usit in Finite Difference
    Deflectionscheck1(:,it+1)=Deflectionscheck(:,it+1);
end

deltaDEFLECTION(:,it)=Deflectionscheck(:,it)-Deflections(:,it);
deltaDEFLECTION1(:,it)=Deflectionscheck1(:,it)-Deflections1(:,it);
total=0;
for ij=1:NE-1
    if abs(deltaDEFLECTION(ij,it))>(.012) || abs(deltaDEFLECTION1(ij,it))>(.012)
        total=1+total;
    else
        total=0;
    end
end

if it==2700
    TIME_Strain_top(:,1)=-STRAINtop(:,1);
    TIME_Strain_bottom(:,1)=-STRAINbottom(:,1);
    TIME_Deflection(:,1)=Deflections(:,it);
    TIME_Sx(:,1)=Sx(:,1);
    TIME_Pp(:,1)=Pp(:,1);
    TIME_Mpp(:,1)=Mpp(:,1);
    TIME_Ael(:,1)=Ael(:,1);
    TIME_Ix(:,1)=Ix(:,1);
    TIME_Minternal(:,1)=Minternal(:,1);
    TIME_phi(:,1)=-DynCurvaturesNE(:,1);
end
end

for it=1:dvd
  if it<=iddxx
    Deflections(:,it)=Deflections(:,it);
  else
    Deflections(:,it)=Deflections1(:,it);
  end
end

j=1:dvd;
Tt=j*dt;
plot(Tt-dt,DynStrain_SectionPLOTtop(1,j));hold on;
plot(Tt-dt,DynStrain_SectionPLOTbottom(1,j),'color','g');hold on;
plot(Tt-dt,DynStrain_SectionPLOT_Btop(1,j),'color','r');hold on;
plot(Tt-dt,DynStrain_SectionPLOT_Bbottom(1,j),'color','m');
grid on
grid minor
xlabel('Time (Sec)')
ylabel('MicroStrain (in/in)')
title('Strain Vs. Time (')
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Ali Al Aloosi was born in Baghdad, Iraq in September, 1980. He received his bachelor degree in Civil Engineering from Baghdad University in 2003, and then he worked in several construction projects in Iraq before traveling to the Sultanate of Oman. In Oman, he worked with Petroleum Development of Oman (PDO) for about two years. Then he worked in the ministry of manpower in Oman, on department of construction projects, for three years. After six years of experience in the construction industry, Ali decided to continue his higher education in civil engineering. In January 2009, Ali traveled to United States and earned a M.S. degree in Civil Engineering. In 2011, Ali began his PhD program in Civil Engineering at ODU. He has taught several laboratory classes for the CEE and CET departments, both in the capacity of a teaching assistant and an instructor. He has also served as a teaching assistant for a number of courses in structural analysis and design, and has served as the president of the Old Dominion University chapter of Chi Epsilon, National Civil Engineering Honor Society.