Conservation According to the Pure Theory of Exhaustible Resources

George Lymbouris
Old Dominion University

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CONSERVATION ACCORDING TO THE PURE THEORY
OF EXHAUSTIBLE RESOURCES

by
George Lymbouris
B.S. August 1975, Old Dominion University

A Thesis submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirements for the Degree of

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ECONOMICS

OLD DOMINION UNIVERSITY
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Carey M. Durden (Director)
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Wayne K. Talley
ABSTRACT

CONSERVATION ACCORDING TO THE PURE THEORY OF EXHAUSTIBLE RESOURCES

George Lymbouris
Old Dominion University, 1978
Director: Dr. Gorey C. Durden

The objective of this thesis is to develop a theory for defining optimal conservation and applying definition to the prevailing rate of domestic petroleum production. Optimal conservation is defined as action designed to achieve or to maintain, from the point of view of society as a whole, the maximum present value of natural resources (or of a natural resource). The proper "action" is defined according to the pure theory of exhaustible resources. The literature survey is a chronological series of articles concerning this theory.

Through building a model which shows that optimal conservation occurs when the marginal internal rate of return is equal to the market rate of interest, adjusted for risk and uncertainty, this paper is able to determine whether the prevailing rate of domestic petroleum fits this definition or not. Testing data for newly discovered United States petroleum reserves for years 1959 through 1975 we concluded that the prevailing rate of discoveries is less than optimal conservation amount.
Finally, we propose the reason for this dilemma and propose a policy of unitization to correct the discovered problem.
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CHAPTER I
INTRODUCTION

The objective of this thesis is to present a comprehensive model which will promote the concept of optimal conservation of a natural resource. The model will be based on the pure theory of exhaustible resources. Through the construction of such a model the optimum level of resource conservation will be determined. After the construction of the model, we will apply it to the production of petroleum. We wish to determine whether the prevailing rate of production coincides with optimal conservation or not.

Optimal conservation is defined as action designed to achieve or to maintain, from the point of view of society as a whole, the maximum present value of natural resources (or of a natural resource).\(^1\) The theory which explains this optimal rate of production is the pure theory of exhaustible resources. The theory deals with adjusting the rate of production of an exhaustible resource in such a manner that marginal sacrifices from giving up

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current satisfactions are equal to marginal gains in future satisfactions. The compensating factor is society's marginal rate of time preference. If postponement of present consumption is adjusted to coincide with society's rate of time preference a situation of intergenerational equity is established. The optimal pattern of resource production is determined according to the pure theory of exhaustible resources.

The model will be constructed in such a manner as to define optimal conservation as the point where the present value obtained from a resource is equal to the present value of the cost of producing it while both are discounted at society's marginal rate of time preference. This maximization condition exists when the marginal internal rate of return in each time period is equal to the market rate of interest adjusted for risk and uncertainty. At this point the producer maximizes his benefits along with optimizing society's intergenerational equity.

After constructing such an optimization model, we will test whether or not the production rate for new petroleum reservoirs in the United States is behaving in an optimal conservationist manner. This will be determined by using a discount cash-flow rate of return method of analysis. We will look at domestic production for the years 1959 through 1975. This analysis may give us some insight as to why current domestic output is falling relative
to total demand and, consequently, we are increasing our dependency on foreign imports.

In conclusion, the objective of this thesis is not to determine why we are in an oil crisis, but to define and implement a model which will describe the behavior of private markets with respect to optimal conservation of a natural resource.

In Chapter II the literature survey will outline the historical development of the pure theory of exhaustible resources. We will begin with the early seminal writings of Lewis C. Grey and Harold Hotelling and progress up to a comprehensive contemporary work, which includes an empirical test concerning future energy resources, by William D. Nordhaus. In Chapter III we will construct the model to explain optimal conservation behavior under a set of market assumptions. An optimal model will be constructed around conditions necessary to acquire an optimal rate of petroleum production. That model will be reinterpreted according to the investment theory of the firm adjusted for a limited time horizon. The investment model will define the conditions under which an optimal conservation would exist. In Chapter IV we will conduct an empirical test on domestic petroleum production based on the investment model constructed in the previous chapter. The objective is to determine whether the production of new petroleum reserves coincides with optimal conservation. To determine
this we will use the discount cash flow rate of return method of analysis for a sample of years, 1959 through 1974, for the entire United States. Finally, in Chapter V we will look at reasons to explain the results obtained from the empirical test. We will analyze the influence of state conservation regulations on the rate of petroleum production.
CHAPTER II
LITERATURE SURVEY

This chapter will review literature pertaining to the "pure theory of exhaustible resources." Papers to be reviewed are by Lewis C. Grey, Harold Hotelling, Anthony Scott, Orris C. Herfindahl, Richard L. Gordon, Oscar Bust and Ronald G. Cummings and William D. Nordhaus. In general, these authors define and interpret the pure theory of exhaustible resources in connection with firms, industries and capital markets. Each paper's objective is to propose a model which will dictate a theory for the optimum production of an exhaustible finite resource. The papers by Grey and Hotelling are theoretical papers which set up the assumptions behind the theory of exhaustion; while Scott, Herfindahl and Gordon apply the theory to the operation of exhaustible firms and industries. The paper by Bust and Cummings applies the theory to a model which indicates an optimal rate of resource production being simulated with an optimal rate of capital investment. The final piece of literature, by William D. Nordhaus, is an empirical study using the theory of exhaustion to show behavior of future energy markets. Since this thesis's objective is to analyze the role of royalty behavior in the petroleum industry, this
review will emphasize the definition and interpretation of royalty by these authors in the theory of exhaustible resources.

The first piece of literature is an early piece, written by Lewis C. Grey. Grey interprets the Ricardian rent theory under the assumption of exhaustibility. Through this interpretation Grey comes up with the concept of royalty. Grey concludes that royalty is a capital fund which must be remunerated in order to induce the owner of an exhaustible resource to employ it productively. By optimizing this royalty, the resource owner will optimally produce his limited resource. Grey sets up his assumptions for an optimal royalty through the concept of the Ricardian rent theory under the assumption of exhaustibility.

To construct his concept of royalty, through the Ricardian rent theory, Grey first shows an illustration of a hypothetical exhaustible coal mine and shows what influences act on its optimal rate of production. This firm, coal mine, is shown in Table 1. Two major factors which influence its optimal rate of output are: diminishing productivity and the influence of the rate of interest. The factor of diminishing productivity, according to Ricardian theory of rent, is that the landowner will find it to his interest to add units of labor and capital to a given surface

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Table 1. Variations in the Net Return in the Removal of Varying Quantities of Coal in a Given Period of Time.

<table>
<thead>
<tr>
<th>Quantity of Coal Removed (Tons)</th>
<th>Value of Coal Removed</th>
<th>Expense of Removal per 100 tons</th>
<th>Total Net Return</th>
<th>Average Net Return Per 100 Tons</th>
<th>Increase in Expense Due to the Removal of Each Additional 100 Tons After the Point of Maximum Net Return Per 100 Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>120</td>
<td>-20</td>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>80</td>
<td>60</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>50</td>
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<td>50</td>
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</tr>
<tr>
<td>500</td>
<td>500</td>
<td>52</td>
<td>240</td>
<td>60</td>
<td>40</td>
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<td>600</td>
<td>600</td>
<td>55</td>
<td>270</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>700</td>
<td>700</td>
<td>54</td>
<td>287</td>
<td>83</td>
<td>17</td>
</tr>
<tr>
<td>800</td>
<td>800</td>
<td>64</td>
<td>288</td>
<td>99</td>
<td>1</td>
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<tr>
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<td>288</td>
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<td>1000</td>
<td>1000</td>
<td>73</td>
<td>270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1100</td>
<td>1100</td>
<td>79</td>
<td>231</td>
<td></td>
<td></td>
</tr>
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</table>
of land up to the point where the last unit applied just equals the product which might be derived from its employment on marginal land. In other words labor and capital is added up to the intensive margin. At this point the landowner will receive a maximum rental from his land. An owner of an exhaustible resource will hesitate to go up to this point. Since he is concerned with the problem of exhaustion, he will wish to produce at the point where he will receive the maximum returns per unit of expense. This is the point where the average net return per ton of coal is a maximum, or where the average expense of removal per ton is at a minimum. Any attempt to appropriate the coal more rapidly results in a diminishing product per unit of expense, thus a diminishing average net return per ton of coal. This being the case, the owner of the mine would wish to postpone for future removal all coal over and above that amount which can be removed at a minimum expense per ton. This figure is shown as four-hundred tons in Table 1. A conventional, non-exhaustible firm would wish to remove eight-hundred tons per period.

The second factor, influence of the interest rate, places an antagonistic condition on the above preferred position. The tendency for the owner of the mine to postpone for future removal all coal which would otherwise have to be removed at an increased average expense per ton is counteracted by the fact that the present value of the return from
future removal is lessened by the discount on the future. The net return from each ton removed in the present, even at an increased expense, may be greater than the present value of the same coal removed at minimum expense in the future. This comparison is shown in Table 2. The table shows the present value of marginal net returns of different quantities of coal. The rate of production till total exhaustion is the one which will maximize the total present value of the resource. For example, if the total quantity of coal is twelve hundred tons, the mine owner would wish to remove four-hundred tons for three future periods. The present value of the last four hundred tons in the third year is $41.66; whereas the removal of an additional one hundred tons in the first year will yield a net return of only forty dollars. If the entire quantity of coal were 3,700 tons, the owner of the mine will find it desirable to remove six hundred tons in the present; for the sixth hundred tons could not be removed at any time in the future so as to yield a greater net return than thirty dollars. If postponed until the eighth year, the present value of the net return is only $29.41.

Through this example Grey shows that a surplus exists at the intensive margin. It is this surplus which is defined as royalty. The rent received for the resource in the ground, royalty, must be optimal in order to optimally produce a given resource. Next Grey defines the optimal
TABLE 2

PRESENT VALUES OF THE NET RETURNS DERIVED FROM THE REMOVAL OF VARIOUS QUANTITIES
OF COAL AT DIFFERENT FUTURE PERIODS
WITH INTEREST RATE AT TEN PERCENT

<table>
<thead>
<tr>
<th>Present Value of</th>
<th>No. Tons</th>
<th>1st Year</th>
<th>2nd Year</th>
<th>3rd Year</th>
<th>4th Year</th>
<th>5th Year</th>
<th>6th Year</th>
<th>7th Year</th>
<th>8th Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>The maximum average net return per 100 tons</td>
<td>400</td>
<td>$50.00</td>
<td>45.50</td>
<td>41.66</td>
<td>38.48</td>
<td>35.71</td>
<td>33.33</td>
<td>31.25</td>
<td>29.41</td>
</tr>
<tr>
<td>Net return of each 100 tons</td>
<td>500</td>
<td>40.00</td>
<td>36.36</td>
<td>33.33</td>
<td>30.76</td>
<td>28.51</td>
<td>26.66</td>
<td>25.00</td>
<td>23.52</td>
</tr>
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<td>600</td>
<td>30.00</td>
<td>27.27</td>
<td>25.00</td>
<td>23.07</td>
<td>21.42</td>
<td>20.00</td>
<td>18.75</td>
<td>17.64</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>17.00</td>
<td>15.45</td>
<td>14.16</td>
<td>13.07</td>
<td>12.14</td>
<td>11.33</td>
<td>10.62</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>1.00</td>
<td>.90</td>
<td>.83</td>
<td>.76</td>
<td>.71</td>
<td>.66</td>
<td>.62</td>
<td>.58</td>
</tr>
<tr>
<td></td>
<td>900</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
amount of royalty. When successive prospective incomes are terminable, the present income is divided into two parts: that is from the entire net income in the present is subtracted the present value of that portion of the income whose accrual is most remote in time. The remainder is interest; the subtrahend is a depreciation fund, or charge. In conclusion, Grey calls this depreciation fund a royalty. Thus the optimum royalty is the product of the process of capitalization. Thus the optimal rate of royalty is equal to the rate of interest. In summary, Grey, through his interpretation of the Ricardian rent theory under the assumption of exhaustion comes up with a definition of royalty. He concluded that royalty is a capital fund which must be remunerated in order to induce the owner of an exhaustible resource to employ it productively at an optimal rate.

The following article is by Harold Hotelling,² who many consider the "father" of the pure theory of exhaustible resources. Hotelling's major contribution to the theory is his interpretation of Grey's model into a mathematical application. By the use of calculus of variations he builds a model which determines the optimal production rate for a finite resource. He builds a model for a free competitive situation and a monopolistic one.

The major assumption concerning both models, free competition and monopoly, is that the owner of an exhaustible resource wishes to maximize the present value of his resource. The rate of interest will be denoted by $\gamma$, so that $e^{-\gamma t}$ is the present value of a unit of profit to be obtained at time $t$, interest rates are assumed to remain unchanged in the mean time.

Hotelling's first model is on the free competition situation. Since it is a matter of indifference to the owner of a mine whether he receives for a unit of his product a price $P_0$ now or a price $P_0 e^{\gamma t}$ at time $t$, it is not unreasonable to expect that the price $P$ will be a function of the time in the form $P=P_0 e^{\gamma t}$. In other words a firm is a price taker, with the same initial price till total exhaustion. The various units of the mineral are then to be thought of as being at any time all equally valuable, excepting for varying costs of placing them upon the market.

Hotelling, from this point on, uses the notation "P" to be interpreted as the net price received after paying the cost of extraction and placing upon the market. Thus, the formula "P"=$P_0 e^{\gamma t}$ fixes the relative prices at different times till exhaustion. But what about the absolute level of $P_0$ at $t=0$? This depends upon demand and upon total supply of the resource. So by denoting total supply by "0", and putting $q=f(P,t)$ for the quantity taken at time $t$ if the price is $P$, Hotelling develops the equation,
\[ \int_0^T q \, dt = \int_0^T f(P_0 e^{\gamma t}, t) \, dt = a \]

with the upper limit \( T \) being the time of final exhaustion. Since \( q \) will then be zero, we shall have the equation \( f(P_0 e^{\gamma t}, T) = 0 \) to determine \( T \). Thus the nature of the solution will depend upon the function \( f(P, t) \) to get \( q \).

After stating that the optimal rate of production is coordinated with the net price per unit of resource, Hotelling next determines the optimal rate of this net price. He defines the net price as the "social value of the resource," and he builds a model which will maximize it over time. The "social value of the resource" for a unit of time is defined as

\[ (1) \quad U(q) = \int_0^q p(q) \, dq, \]

where the integrand is a diminishing function and the upper limit is the quantity actually placed upon the market and consumed. If future enjoyment be discounted at a rate of interest \( \gamma \), the present value is

\[ V = \int_0^T u [q(t)] e^{-\gamma t} \, dt. \]

Since \( \int_0^T q \, dt \) is fixed, the production schedule \( q(t) \) which makes \( V \) a maximum be such that a unit increment in "q" will increase the integrand as much at one time as at another. That is,

\[ \frac{d}{dq} u [q(t)] e^{-\gamma t} \]
which by (1) equals $P \text{e}^{-\gamma t}$, is to be constant. Calling this constant $P_0$, Hotelling concludes that $P=P_0 \text{e}^{\gamma t}$ the result obtained in the original model. Thus the discount rate based on the rate of interest is the amount which controls the growth of the net price in order to optimize the social value of the resource.

After building a competitive model, Hotelling builds a monopoly one. Where a competitive situation the firm was a price-taker, here the firm is a price discriminator. Thus as resources deplete the firm has the individual power to change the price charged for it. The problem is defined as choosing $q$ as a function of $t$, subject to the condition

$$\int_0^\infty q \, dt = a,$$

so as to maximize the present value,

$$J = \int_0^\infty q \, p(q) \, e^{-\gamma t} \, dt,$$

of the profits of the owner of a mine, or exhaustible resource. He does not restrict $q$ to be a continuous function of $t$, though $P$ will be considered a continuous function of $q$ with a continuous first derivative which is negative. The upper limit of the integrals may be taken as $\infty$ even if the exploitation is to take place only for a finite time $T$, for then $q=0$ when $t > T$. This is a problem in the calculus of variations.
Hotelling next simplifies these equations by observing that

\[ qP(q) e^{-\gamma t} - \lambda q \]

where \( \lambda \) is a Lagrange multiplier, is to be a maximum for every value of \( t \). He, therefore, concludes

\[ e^{-\gamma t} \frac{d}{dq} (Pq) - \lambda = 0 \]

and

\[ e^{-\gamma t} \frac{d^2}{dq^2} (Pq) < 0 \]

The constant \( \lambda \) is determined by solving (2) for "q" as a function of \( \lambda \) and \( t \) and substituting in (1). Upon integrating from 0 to \( T \) an equation will then be obtained for \( \lambda \) in terms of \( T \) and of the amount "a" initially in the mine, which is here assumed to be known. The additional equation required to determine \( T \) is obtained by putting \( q=0 \) for \( t=T \).

In summary, Hotelling developed mathematical models of the pure theory of exhaustible resources. He outlined rules for the optimal rate of production for a competitive and monopolistic situation. Through the use of the calculus of variation and different assumptions on competitive and monopolistic price determination he developed models specific for each.

The next piece of literature is by Anthony T. Scott.\(^3\) Scott's major contribution to the pure theory of exhaustible resources.

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of exhaustible resources is his interpretation of Grey's royalty into a user cost concept. That is, the present value of the future profit foregone by a decision to produce a unit of output today. Scott uses the user cost concept to develop a model which determines the optimal rate of output of an exhaustible resource.

Scott builds his model around an exhaustible mine. This mine operates under the following assumptions:
(1) the contents of the mine are completely known;
(2) the owner invests, borrows and lends at the same rate of interest; (3) the owner's objective is to maximize the present value of the asset, both initially and in every period in which he must make decisions; (4) in order to accomplish (1) he may adjust only the rate of extraction, measured in tons per period and, in the long-run, the amount of investment in size (capacity) of the mine; (5) conditions are constant through time. This assumption has three aspects: (a) the reserves are of uniform grade throughout their known total volume, (b) supply prices of all inputs (except the resource itself) delivered at the mine are given, fixed, and unchanging through time. The manager is always a price taker, (c) the owner knows the prices in (b) with complete certainty. A diagram of the mine operating under these assumptions is shown in Fig. 1.
Fig. 1. Diagram of Mine Production under the Condition of Certainty.
Being a finite resource firm it is faced with short-run diminishing returns. That is, up to certain rate of mining, cost will increase in less proportion than the value of the tons extracted. But beyond a certain least-unit-cost rate, the cost will rise more rapidly than the additional value attained from the extracted resource. This being the case the total cost curve is agine-shaped, and the average and marginal cost curves are u-shaped. The upward sloping portion of the latter curves represents the diminishing returns in the short run. Thus in Figure 1, position "A" is the least-unit-cost rate of production; it is here that the maximum current profit per ton extracted is obtained. Also note "B", where the marginal cost curve intersects the marginal revenue line, is the rate of maximum profit per period. The reason why points A and B do not coincide, as in a conventional firm's optimum rate of production point, is because of the influence of interest rate, or discounted future profits. This is why the least-cost-per-unit point of the marginal and average cost curves are below the marginal revenue line. The marginal revenue line must take this added cost of discounting into account, thus this is why it lies above the least-cost-per unit point. By the firm operating under these assumptions Scott is able to determine the optimum rate of output, between points A and B, using his user cost concept.
User cost is defined as the present value of the future profit foregone by a decision to produce a unit of output today. If the unit is small, marginal to present and future operations, the user cost is derived from the addition to total future profits of the opportunity to mine the unit then. If this unit is mined in a number of future periods, user cost is defined as the maximum increment to the mine's present value that could be gained by a decision to allocate the unit of output to a future period. In short, because physical and economic conditions are assumed constant through time, the concept deals with small reallocations between present and future rates of output.

To pick the optimal rate of production it is necessary to look at the relationship of the profits curve to the user cost curve. The optimal rate of production has as its objective to maximize the long-run profits of the mine. Noting the profits curve, its slope measures the extra profit in a given period from an additional unit of output. A conventional manufacturing plant would wish to produce at point B, where the marginal profit from each unit produced is zero. But due to the influence of the discount rate, user cost, an exhaustible resource firm is forced to produce at a rate less than "B" but more than "A", the least-cost-unit rate. If the discounted profits from future production that is prevented
by present production is higher than present profits then the user cost concept can be used to pick the optimal rate of output. The slope of the prevented profits or user cost curve, the marginal user cost, is the determinant for this rate. If it is steeper than the profit curve at a given rate, marginal user cost is already greater than present profit; the rate should be diminished to maximize the present value of the resource, and contrariwise if it is flatter. When the slopes are equal, no reallocation between present and future can increase present value in the short run.

Scott shows the user cost concept through an algebraic solution. Where \( u \) is user cost, \( R \) revenue, \( C \) short run cost, \( r \) the rate of interest, \( m \) a subscript denoting the marginal unit of output, \( t \) a subscript denoting one of \( n \) period; there is the solution where \((R-C)\) denotes current profit and \((R_{mt}-(nt))\) denotes the current profit on the marginal unit in period \( t \). From this the user cost equation for an optimal production rate is

\[
U_{mt} = \max_{t} (R_{mt} - (nt)) (l+r)^{-(t-t_0)},
\]

based on the assumption that the maximum-profit alternative to production today is a small addition to production in all \( t \) future periods.
In summary, Scott, through using the concept of user cost, derives a theory for optimum rate of production for an exhaustible resource. Under the assumption of constant conditions, meaning future profits curves are all the same, the owner of the mine is able to pick an optimal rate of extraction for his limited resource. He accomplishes this by equating his marginal profits per period to his marginal user cost. That is the profit gained by producing an additional unit is equal to the profit foregone by a decision to produce that profit today. As long as the rate of output per period is managed in such a manner the total present value of the deposit is maximized.

The survey of literature continues with an article written by Orris C. Herfindahl. Herfindahl uses the concept of royalty to build a model which theorizes the optimum path of price and quantity of a given exhaustible resource. By explaining a price-quantity concept he is able to theorize the optimum rate of output for an exhaustible resource industry. First he builds a model under the assumption of constant-cost firms and then modifies it under the assumption of non-constant cost firms, that is an industry producing two different grades of resource.

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The models are built on the assumption that demand for a particular mineral product, present and future, is known by all participants in the market. Thus the demand function is assumed to stay constant through time and the production function is assumed unchanged through time. This means the model is based on constant and fully predictable future markets.

The constant-cost resource model is built on the conditions that all deposits are fully known and that their quality is uniform. This means that current cost per unit of metal (resource) recovered is the same for all firms in this industry. This situation is shown in Figure 2. This figure shows the course of the price which will at each point in time induce enough production to satisfy the amount demanded. The optimal amount produced per period is derived by having the (PV) present value of the royalty at different point in time being equal. That is, 

\[ (P_{tj} - C) e^{-rtj} = (P_{tj} - C) e^{-rtj}, \]

where royalty, the price of the metal (resource) in the ground, is equal to the price of the metal \(P_t\) minus the current marginal cost \(C\) of getting it out of the ground and processed. Thus for a maximum present value of profits, the royalty must rise at the rate of interest or discount rate \(r\).

In conclusion, the absolute price for a piece of metal, for any time period, is denoted by the formula 

\[ P_t = (P_0 - C)e^{rt} + C. \]

This formula shows that \(P_t\) does not rise
Fig. 2. Diagram Showing the Relationship Between Growth in Royalty and Demand Function.
at the rate $r$ per unit time. But this relative rate of increase of $P_t$ does approach $r$ as a limit, as $P_t$ becomes large in relation to $C$. This is explained by Figure 2. If the participants in the market foresee the future correctly, an initial price $OA$ will yield production and sales of $OB$ for that year. As price increases, production and sales will decrease, until finally at $P_T=OK$ nothing will be produced or demanded and all deposits will be used up. From $0$ to $T$ is considered the period of exploitation. So if the initial price is too low, the deposits will be used up before price has reached $OK$, the maximum that buyers are willing to pay. And too high an initial price will result in the maximum price being reached before the deposits are exhausted. Thus it is the price of the metals in the ground, royalty, which brings equilibrium in the market for mineral properties which enable the path of price and quantity to play its role of maximizing income over time.

From this basic model Herfindahl expands it to model operating under the assumption of two different grades. That is firms in the industry are divided into two groups, a "good" grade with low current costs and a "poor" grade with high current costs of production. Thus the assumption of constant cost in the first model is nullified. Herfindahl has as his objective to decide which grade would be produced first, and at what point will the grades switch production.
Fig. 3. Resource Use of Two Different Qualities; Highest Cost Produced First.
Both grades are not produced at the same time. This is assumed because the price of the metal must be the same no matter what its source, simultaneous exploitation would require that the absolute growth in price be equal to the absolute increase in the royalties on the two different grades; which are growing at the same relative rate but at different sizes. Suppose both grades are exploited at \( t_0 \), at \( t_1 \) it would be necessary that

\[
Pt = (P_a - C_a)e^{rt} + Ca = (P_0 - (b)e^{rt} + C_b
\]

where \( C_a \) and \( C_b \) designate current costs of the two grades, \( a \) and \( b \). But this is impossible since \((C_b - C_a)e^{rt} \neq (C_b - C_a)\) if \( r \) is greater than zero.

Also the "poorer" grade as higher cost resource will not be produced first. This is shown in Figure 3. This is because the royalty \( C_bA \) rises at a rate of interest and so also does \( EF \), the royalty on the better deposit. \( OC_a \) is current cost per unit for the better deposit where \( OC_b \) is current cost per unit for the poorer. This causes a situation where the owner of the good deposit will receive a higher royalty if he works his deposit earlier. The present value at \( T_0 \) of the royalty earned at \( t_b \) is

\[
(P_{tb} - C_a)e^{-rb} \]

When it is exploited at \( t_0 \), deposit \( b \)'s royalty will be higher since

\[
(P_0 - Ca) = [P_t - (b)e^{-rb} - C_a] > (P_{tb} - C_a)e^{rb} \]

This creates the unstable situation of \((C_b - C_a)e^{-rb} \neq (C_b - C_a)\).
Thus through the process of elimination, Herfindahl proves that the lower cost deposit will be exploited first. He shows this in Figure 4. This resource will be produced up to the point $t_a$, the point where the royalty of the "good" resource is equal to the royalty of the "poor" quality resource.

In summary, Herfindahl traced the path of price and quantity at an optimal rate for an industrial situation. He used the concept of royalty to build models for a constant cost industry and a non-constant cost industry. He came up with the conclusion that a "good quality resource or least cost rate resource will be produced first because this creates an orderly stable condition for the growth of royalty regardless of the quality of the resources produced. The growth rate of royalty is equal for both qualities; whereas if the "poor" were exploited first it would cause an unstable situation, growth of royalty for "good" resource would be greater than "poor" resource.

The following piece of literature is by Richard L. Gordon. Gordon takes the constant cost industrial theory proposed by Herfindahl and builds a mathematical model from it. He builds his industrial model by first setting

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Fig. 4. Resource Use of Two Different Qualities; Lowest Cost Produced First.

$C_b =$ cost of quantity $b$

$C_a =$ cost of quantity $a$

$t_a =$ end of quality $a$

$T =$ end of total $a+b$
Gordon's model for an optimal firm is based on Hotelling's model of pure monopoly. A firm is faced with a finite fixed supply "K". With this assumption, a firm's optimum rate of production, $q(t)$, over the time period, from $t_1$ to $t_2$, in which the firm operates cannot exceed $K$. Thus the constraint on a firm's operation is (1) $\int_{t_1}^{t_2} q(t) \, dt < K$. Exhaustion occurs when cumulative production equals total physical supply. The firm has as its goal to maximize the present value of its finite resource, subject to (1) as a constraint. This is defined as a problem in calculus of variation. The constrained present value problem is to maximize

$\text{(2) } \text{NPV} = \int_{t_1}^{t_2} \Pi[q(t), t] \, e^{-rt} \, dt - \lambda \int_{t_1}^{t_2} q(t) \, dt - K$. 

Where

$\text{NPV} = \text{net present value}$

$\Pi = \text{total profits}$

$t = \text{time}$

$q(t) = \text{output}$

$r = \text{continuous rate of interest}$

$p = \text{average profits}$

$\lambda = \text{Lagrangean multiplier}$
The solution involves selection of the optimal dates for starting and completing output and the optimal output pattern. Also, the differentiation with respect to the Lagrangean multiplier gives the constraint as another equilibrium condition. The optimal production pattern is determined by the Euler equation of the calculus of variations, and the boundary conditions are simply that the function (2) attains a stationary value of \( t_1 \) and \( t_2 \).

By building this calculus model, Gordon is able to simplify it into algebraic terms in order to construct his industrial model. Since the Euler equation requires that \( \partial f / \partial q - (d/dt)(dF/dq) = 0 \), in this case \( F \) is the whole function in equation (2) and \( q = dq/dt \). However, \( q \) does not affect profits so the second term in the Euler equation vanishes. So by appropriate substitution and rearrangement the solution becomes:

\[
(3) \quad M\Pi(t) = MR_t - MC_t = \lambda e^{rt}
\]

The conventional second order condition \( \partial^2 \Pi / \partial q^2 < 0 \) and \( \partial^2 R / \partial q^2 < \partial^2 C / \partial q^2 \) also apply. With pure competition, \( P = MR \) so (3) becomes

\[
(4) \quad (P - MC) = \lambda e^{rt}.
\]
With constant costs, this reduces further to

(5) \( (P - AC) = \lambda e^{rt} \)

The terminal conditions require:

(6) \( \Pi[q(t_1), t_1] = \lambda q(t_1)e^{rt_1} \)

(7) \( \Pi[q(t_2), t_2] = \lambda q(t_2)e^{rt_2} \)

By dividing (7) by \( q(t_2) \) he gets the solution

\( p(t_2) = \lambda e^{rt_2} \)

Since (3) must also hold at \( t_2 \), marginal profits equal average profits at the termination of production. Since \( \Pi = pq \), then \( M\Pi = \partial \Pi / \partial q = P + q(\partial P / \partial q) \). Thus, the equality at \( t_2 \) implies \( q(\partial \Pi / \partial q) = 0 \) which occurs if \( q \) or \( \partial P / \partial q \) is zero; the firm either produces nothing at \( t_2 \) or produces the output that maximizes average profits at \( t_2 \).

From this model, Gordon constructs a general rule for a firm's optimal rate of production. By replacing \( \lambda \) by marginal profits at \( t_2 \), the solution

\( M\Pi(t) = M\Pi(t_2)e^{r(t-t_2)} \)

is established. From this a general rule, that marginal profits must rise of \( r \) percent per year to a maximum at \( t_2 \), in order to optimize the present value of an exhaustible resource for a firm.

A general constant cost industrial model has as its objective to determine a set of market-clearing prices which insure exhaustion and \( r \) percent growth in the marginal profits of operating deposits. The industrial equilibrium is determined by the reaction of firms to an arbitrary set
price. Prices are determined by a constant-cost model where the required r percent profit growth is produced by price increases.

In the case of a firm's reaction to price, assuming cost increase with output, the individual firm itself can produce r percent marginal profit growth. The firm simply lowers output enough to produce cost reductions that provide the required rise in marginal profits. The requirement that terminal marginal profits equal maximum average profits at t₂ means that competitive firms attain minimum average cost at t₂. Thus, the familiar condition of the conventional theory holds true although the usual course—equality of marginal cost to a price that exceeds average cost—no longer operates since marginal costs are kept below price. The behavior results instead from the increase in present value produced by eliminating periods in which output is below the level at which average costs are minimized.

Now that a firm's reaction in a competitive industry has been analyzed, Gordon next builds a model which determines price. The required price rise is shown by transforming equation (5) to the form:

\[ P(t)=[P(t_2)-AC(t_2)]e^{r(t-t_2)}+ AC_t \]

and differentiating the natural logarithm \( \ln P(t) \) with respect to \( t \). This yields:
\[
\frac{d \ln P(t)}{dt} = r \frac{[P(t)-AC]}{P(t)} < r
\]

with the second derivative being
\[
\frac{d^2 \ln P(t)}{dt^2} = r \frac{AC[dP(t)/dt]}{P(t)^2} > 0
\]

This shows that prices are required to rise less rapidly than profits, but at an accelerating rate.

In conclusion, Gordon shows that equation (5), \((P-AC) = e^{rt}\), which applies to constant-cost industries, will satisfy the requirement that marginal profits equal maximum average profit at \(t^2\) for each firm in the industry through the solution
\[
P(t)-AC(t) = P(t)-MC(t) = [P(t^2)-AC(t^2)] e^{r(t-t^2)}.
\]

This solution shows that as long as the industry determines prices at the point where \(AC_t=MC_t\), the industry is operating at an optimal rate of production. The optimal number of firms will be in equilibrium at that point.

The next piece of literature is written by the team of Oscar Burt and Ronald G. Cummings.\(^6\) They construct a comprehensive model for simultaneous optimization in the rate of natural resource extraction along with an optimal investment rate in that industry. Burt and Cummings build a "social benefit" function and define how the optimal point is attained. They conclude that

optimal social benefits are attained when the marginal social benefits attributed from the production of a natural resource is equal to the marginal social cost attributed to the depreciation of capital due to production of that resource.

Through building the production function model, to act as social benefits function, they are able to set up their optimization conditions. The production function is built on the assumption of a finite planning period, where \( t=0 \) and \( t=T \) meaning beginning and end respectively. The following terms are used:

1. \( U_t \) = the rate at which the resource is used at time \( t \).
2. \( X_t \) = the amount of resource in stock at \( t \)
3. \( Y_t \) = the amount of capital stock at \( t \)
4. \( V_t \) = measures the rate of periodic capital investment.

The objective of the model is to optimize simultaneously both \( U_t \), rate of extraction and \( V_t \), rate of capital investment.

The variable \( U_t \) is assumed subject to the constraints

\[
0 \leq U_t \leq h(X_t, Y_t) \quad t = 0 \ldots, T-1
\]

where \( h(X_t, Y_t) \) is assumed concave, that is the larger resource and capital stocks allow larger rates of production. At the beginning of any period \( t+1 \), resource and capital stocks are given by the difference equations,
\[ (2) \ x_{t+1} = x_t + g(U_t, x_t) \quad x_0 = a \]
\[ (3) \ y_{t+1} = y_t - D(U_t, v_t, y_t) \quad y_0 = b \]
\[ t = 0, 1 \ldots, T-1. \]

The functions \( g(U_t, x_t) \) and \( D(U_t, v_t, y_t) \) are assumed concave and convex, respectively, where the function \( g(U_t, x_t) \) is net additions to resource stocks during the period \( t \), and the function \( D(U_t, v_t, y_t) \) is a depreciation function for capital stocks which measure net changes during period \( t \); these conditions must be optimized in order to maximize social benefits.

To optimize these two conditions the following optimization function is set up.

\[ (4) \ \text{Max} \left\{ T-1 \sum_{t=0}^{T-1} B_t (U_t, v_t, y_t) B^T + B^T \psi (x_t, y_t) \right\} \]

subject to

(a) \( x_{t+1} - x_t - G(U_t, x_t) = 0 \)
(b) \( y_{t+1} - y_t + D(U_t, v_t, y_t) = 0 \)
(c) \( u_t - h(x_t, y_t) \leq 0 \)
(d) \( u_t, v_t, x_t, y_t \geq 0 \)
(e) \( x_0 = a \) and \( y_0 = b \)

where the variables of optimization are \( U_t, V_t, t=0, 1 \ldots, t-1, \) and \( X_t, Y_t, t=1, \ldots, T. \)

In order to explain the conditions for optimization, Burt and Cummings use a Kuhn and Tucker expression of (4) and obtain Lagrangian expressions, \( \lambda_t, \mu_t, \) and \( V_t \) as discounting factors. They use the discounting factors to describe the optimal path for social benefits.
The optimal time path for production from a natural resource is given by

\[
(5) \quad \frac{\partial B^0}{\partial u_0} - v_0 = \beta \lambda_1 \left( -\frac{\partial g_0}{\partial u_0} + \beta \mu_1 \frac{\partial D_0}{\partial u_0} \right)
\]

where \( \beta \lambda_1 \) and \( \beta \mu_1 \) are interest rate Lagrangian multipliers. The left-hand side is the measure of marginal social benefits attributed to an increment of resource use at \( t=0 \), less the imputed value of the constraint \( 4 - C \) if strict equality should hold at \( t=0 \). \( v_0 = 0 \) if \( u_0 \neq h(x_0, y_0) \).

On the right-hand side, the partial derivatives, \( \frac{\partial g_0}{\partial u_0} \) and \( \frac{\partial D_0}{\partial u_0} \), measure the marginal influence of \( u_0 \) on resource and capital stocks. These derivatives are multiplied by the discounted values of increments to resource stocks in all future periods associated with an increment to stock in period \( t=1 \). Likewise, \( \beta \mu_1 \) is the discounted value of increments to capital stocks in future periods associated with an increment to capital stocks in period one. In short, the marginal social value of current production is equal to the sum of the discounted marginal value of a unit of resource remained in stocks instead used in current production and the discounted marginal value of capital stocks consumed by an increment to current production.
The other condition for optimization of benefits is the optimal rate of capital investment. This is shown by the following equation,

$$\frac{\partial B_0}{\partial V_0} = \beta \mu_1 \frac{\partial D_0}{\partial V_0}. $$

This is optimal when the marginal social cost of investment in the current period is equal to discounted value of the increment to all future capital stocks associated with an increment to current investment.

By combining these two equations, Burt and Cummings are able to theorize the condition for a simultaneous optimum rate of resource extraction and capital investment which will maximize the social benefit function. By substituting the $\beta \mu$ in (6) into (5), to get

$$\frac{\partial B_0}{\partial U_0} - V_0 = \beta \lambda_1 (- \frac{\partial g_0}{\partial U_0}) - \frac{\partial B_0}{\partial V_0} \frac{\partial V_0}{\partial U_0}. $$

where the last term measures the capital cost associated with an increment to current production. By holding $Y_1$ constant, total adjustment in capital at the margin must take place through variation in $V_0$ in order that the net effect on depreciation will be zero. Thus, for "social benefits" to be optimized the marginal social benefits, which are gotten by the summation of "capitalized value" of an increment to resource stock multiplied by the marginal effect that current production has on next year's stock and "capitalized value" of an increment to capital stocks multiplied by the marginal effect that current production
has on next year's capital stock; is equal to the marginal social cost of investment, which is the depreciation rate of existing capital stock determined by their production of the given resource rate.

The final piece of literature is written by William D. Nordhaus. This is an empirical study concerning energy resources using the pure theory of exhaustible resources. Nordhaus develops a model to determine the efficient allocation of energy resources over time. His model, based on the assumption of future markets, determines the order of use of future energy resources till the period of time when technology produces energy without the assistance of finite resources.

The major assumption of his model is that he uses the concept of prices for energy resources being obtained from future markets. He assumes that there are consumers with initial resources and given preferences, and producers operating with well-defined technical relations. Even though this form of forecasting is generally uncertain about exact demand or supply conditions it does create the condition of convex production and preference sets, and where that markets exist for all goods, services, and contingencies. This means that all goods have future markets along with insurance markets for technological failures,

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and it also means all the costs and benefits of a particular process of production must be internalized to the decision maker. In short, a market under these assumptions produces a general equilibrium of prices and quantities. This equilibrium condition is considered efficient in the sense that there is no way of improving the lot of one consumer without worsening the lot of another. In other words, future prices obtained from the market system are appropriate indicators of social scarcity given the preferences and initial endowments of the society.

By setting up his model to operate in a market institution, Nordhaus then analyzes market equilibrium under the assumption of an exhaustible resource. First he distinguishes two concepts: (1) the marginal cost per unit of output, excluding rents and royalties, is \( Z(t) \); and royalties, \( \&(t) \), which reflects the presumed scarcity of a particular resource. The \( (t) \) refers to the time period.

Assuming a world of certainty and a time horizon of "T" years, where there are \( R(t) \) units of the resource remaining at any point of time, and extraction costs are zero up to the resource limit. The optimal royalty path is determined under the condition that alternative assets yield a rate of return, \( r(t) \). The equilibrium condition for some owners to hold and others to sell the resource is equality between the rate of capital gains on the resource and the interest rate:
where \( Y \) is the change in \( Y \). Thus the resource price, in the ground, rises exponentially at the rate of interest.

There are many market prices which can solve equation (1); each having different levels of "\( Y \)". But the unique solution depends on the terminal condition that all resources are used up at the end of the last period \( (T) \).

(2) \( Y_T \), such that \( R(T) = 0 \).
The set of prices which satisfy these equations is the optimal price path.

Next Nordhaus defines the optimal time path of price under the assumption of positive extraction cost. (Remember the previous paragraph was with zero extraction cost, it just showed the optimal path of royalty). Price, \( P(t) \) is the sum of current extraction cost and royalty:

\[
P(t) = Z(t) + Y(t)
\]

where optimization of production occurs the present value of that resource is maximized. That is, the present value of the profit from selling a unit of the resource at time "\( t \)" when extraction cost is \( Z_0 \) is \([P(t)-Z_0]\exp(-rt)\). This is maximized if "\( t \)" is chosen so that \( \Delta P(t) = r[P(t)-Z_0] \) or when

\[
(4) \quad \frac{\Delta P}{\Delta P} = r\left(\frac{P-Z_0}{P}\right) = \frac{rY}{P}.
\]
In short, if production of a resource with cost $Z_0$ occurs for all $t$, then (4) must hold for all $t$. Moreover, for all periods when sales occur, $(P-Z_0) \exp(-rt)$ is constant; this is the solution to equation (4). Also during these periods equation (1), $\Delta Y/Y=r$, is satisfied. So when cost of extraction is positive, and $Y=P-Z_0$ is the royalty equation, the condition for recovering a resource optimally is that the expected rate of increase in the price of the resource is less than or equal to the interest rate times the share of royalties in the total resource price. This rate is always less than the interest rate.

Now that the optimal time path of exhaustion for energy resource has been defined, the condition for a systematic progression of resource termination is outlined. This condition is based on the "backstop technology" concept. That is, today's technology for energy production is highly dependent on resources that are very cheap to extract but relatively scarce when viewed on a very long-run time horizon. This type of technology professes royalties to play a relatively important role in today's price. As future periods come into play these many low-cost energy resources will be depleted leaving more abundant but also more expensive to produce resources. Ultimately when the economy transfers to an infinite or very plentiful resource, the economic importance of
scarcity of resources will disappear, and capital and labor cost alone will determine prices. This ultimate technology, depending on a very abundant resource base, is labeled as the "backstop technology." So the objective of the whole process of switching from the use of one resource to the next is to eventually come to the "backstop technology."

Note the following example which shows how the "backstop technology" concept operates. Consider two processes for energy production. One uses one unit of petroleum per unit of output; petroleum resources are considered finite in supply (R recoverable units) and free to extract. The second process uses nuclear fuel, which is considered super-abundant and free, and K dollars worth of capital per unit of output. Now assume that the rate of interest is "r", and that demand is inelastic, with "D" units of electricity demanded per year. The petroleum process will be used first, and the switch to the nuclear process (the backstop technology) will take place R/D years out.

At the switch point T, the price for the energy product, P, is given by the cost of the backstop technology, 

\[ P(T) = (r + \delta)K, \]

where \( \delta \) is the depreciation rate and K is the capital requirement of the backstop technology. This implies that the price and therefore the royalty on petroleum at the switch point are also \( P(T) \). Thus the price and the royalty
on petroleum along the efficient path from now to \( T \)
are

\[
Y_t = P(t) = P(T) \exp[-r(T-t)] = (r+)K \exp [-r(T-t)]
\]

The royalty on the scarce resource is simply the switch price, \( P(T) \), discounted back to the present. In sum, when the scarcity price, denoted by royalty in this case, reaches the cost of capital and labor to produce the backstop technology, this is the switch date. As long as the market is assumed a reliable indicator of social scarcity then its prices for energy resources are reliable indicators in the behavior of royalty.

By constructing a model under the above stated format, Nordhaus is able to conduct an empirical study on future energy markets. He uses four energy resources: petroleum, coal, natural gas, and uranium -235. On the demand side he provides five demand categories: electricity, industrial heat, residential heat, and two transport categories. Nordhaus's findings were that in a program with four fifty-year periods, the backstop technology was reached in the fourth period. In that fourth period an electric-hydrogen basis for the linear programming model was reached in the years 2120-70. The expensive shale oil and the most expensive category of U.S. oil were saved for the period 2070. During the next fifty years, 1970-2020, the economy will utilize a small fraction of the world coal resources, low cost U-235 in conventional nuclear
reactors, and all but the high-cost petroleum resources.

In summary, William D. Nordhaus used the pure theory of exhaustible resources to build an empirical model concerning future energy markets. Using the concept of "backstop technology," that is the objective of the economy to produce energy under a technology free of finite resources, he is able to trace the path of resource prices through well defined future markets. When the resource price, based on the summation of marginal cost and royalty, becomes equal to the price of the backstop system, based on just marginal cost and no royalty, the switch is made to this resource technology.

In conclusion, this literature survey took a look at different aspects concerning the pure theory of exhaustible resources. It emphasized the role of royalty in determining the optimal rate of production of a finite resource. By looking at papers which built up the theory, by Grey and Hotelling, the survey was next able to apply the theory to exhaustible firms and industries through the writings of Scott, Herfindahl, Gordon, and Burt and Cummings. The survey was ended by an empirical study pertaining to future energy markets by William D. Nordhaus. Through reading the literature survey the reader should attain an understanding of the role of the pure theory of exhaustible resources in defining an optimal conservation policy.
CHAPTER III

OPTIMAL CONSERVATION MODEL

The purpose of this chapter is to construct an optimal conservation model based on the pure theory of exhaustible resources. The assumptions and conditions under which optimal conservation is practiced will be delineated. The model will be used as a basis for the test (in Chapter IV) to determine whether the petroleum industry is acting in an optimal manner with respect to conservation as earlier defined in Chapter I.

A. Social Time Preference

The rate of social time preference is the major variable underlying the concept of optimal production for a limited exhaustible resource. This is defined as the opportunity cost to society for postponing present satisfactions through consumption for future satisfactions in consumption. This opportunity cost is identified by society through the market rate of interest.

In dealing with an exhaustible resource, for every unit consumed today there is one less unit available for future consumption. So in order to provide for future satisfaction, present consumption must be postponed in such
a manner so as present and future satisfactions will be maximized. The primary inducement for consumers to sacrifice present satisfactions is receiving a greater potential future satisfactions. This net satisfaction is reflected in society's marginal rate of time preference. For example, if consumers view with indifference an increment of X amount of current satisfactions or, alternatively, an increment of 1.05 X amount of satisfactions one year from now, their marginal rate of time preference is 5 percent per annum. Therefore, foregoing X amount of current satisfactions would increase consumer's net satisfactions only by receiving in exchange 1.05 X amount of satisfactions one year from now.1 That is, some quantity of natural resource will be saved for future consumption because it will be more valuable to future consumers. Future consumers are willing to pay a higher price which generates a larger return to the owner.

Optimum conservation of an exhaustible resource is based on the assumption that, for society, there exists a rate of time preference which effectively allocates resources over time in an optimal manner. That is, the optimal rate of production must be such that the sacrifices of postponing current satisfactions are offset by increased future satisfactions according to society's existing rate of time preference. This form of remuneration is approximated by

the market rate of interest. The next section will show how an existing rate of time preference leads to an optimal rate of production for an exhaustible resource under a competitive market system.

B. Optimal Rate of Production for an Exhaustible Resource

Society's marginal rate of time preference is the major determinant in finding the optimum rate of production for an exhaustible resource. This condition will be shown in an illustration of an exhaustible resource firm in a competitive market. The firm operates under the following assumptions:

A. The amount of resource available for production is completely known.

B. The owner invests, borrows, and lends at the same rate of interest or return.

C. The owner's objective is to maximize the present value of his asset, his exhaustible resource.

D. In order to accomplish (c) he may adjust only the rate of extraction, measured in units per period; in the long-run this will determine the capacity of the firm.

E. Conditions are constant through time. There are three aspects to this assumption:

1. The reserves are of uniform grade throughout their known total volume.

2. Supply prices of all inputs delivered at the
mine and selling prices of all products f.o.b. the firm are given, fixed, and unchanging through time. The manager is a price taker.

3. The owner knows the costs and prices in (2) with complete certainty.²

A diagram of this firm is shown in Figure 5. The difference between a conventional firm and an exhaustible resource firm is shown by the two different sets of cost curves, marginal and average cost curves as well as the total cost curve. Both sets show short-run diminishing returns; output is increased only by increasing inputs in a greater proportion. The curves indicate that, up a certain rate of output, costs will increase in lesser proportion than the revenue gained from it. Beyond a least-unit-cost rate, costs will rise in a greater proportion. With cost being a function of output we have an agine-shaped total cost curve and U-shaped average and marginal cost curves. The upward sloping portion of the marginal cost curve represents the limiting factors of production in the short-run.³

The conventional set of cost curves is indicated by dashed MC-AC and TC curves. This firm would produce at the rate where it would obtain the maximum profit per


³Scott, p. 31.
Fig. 5. Diagram of Production Rate for an Exhaustible Resource Firm in a Competitive Market. The dotted profit curve suggests a firm in equilibrium in a competitive market with no excess profits.
period, this is at point "B" where MR=MC. At this point the firm optimally allocates its resource because the marginal sacrifices for producing the good is equal to the marginal gain in satisfaction through consumption.

The second set of cost curves represent the current actual production costs of an exhaustible resource firm. The exhaustible resources firm appears to be making excess profits at output B; the optimum output is apparently A where profits are maximized under the condition that MR=MC. However, the exhaustible resource firm is subject to an additional cost, the foregone opportunity of earning income on the finite resource in a future period. This cost must be added to the production cost curves of the exhaustible resources firm in order to determine the optimum level of output. That is, the marginal cost of producing output consists of two parts: (1) the actual marginal cost of production, and (2) the opportunity cost of discounting future profits which will be unavailable when units of output are produced and consumed in the current period. The exhaustible resources cost curves in Figure 5 are lower than conventional curves by the difference of opportunity cost of present production.

Addition of the opportunity cost is necessary to determine the optimum output position of the exhaustible resource firm. The opportunity cost for a finite time horizon is defined to be dependent upon the social rate of time preference. That is, loss of satisfaction from
from foregone present consumption is compensated appropriately when the current equilibrium position includes the costs of foregone future consumption. In our example, net social satisfactions are balanced over time when production of the exhaustible exhaustible resource is OB. At this output, future profits of the firm offset the loss of current profits which would be earned if production were at OA, considering only current actual costs. Production at OB would be socially optimal because consumers are willing to forego consumption now in order to have it available for future use. Again, the mechanism for intergenerational equilibrium in the demand and supply of a natural resource is the social rate of time preference/rate of interest. Optimization for the exhaustible resource firm is symbolized by the following equation: \( MR = MC + MTP \), where TMP is marginal rate of time preference and MR is marginal revenue and MC is marginal cost. This equation is rearranged as:

\[
MR - MC = MTP,
\]

where MTP now becomes equal to marginal profit. Thus, for the firm to reach optimal present profits, denoted by point B, it must produce at a less rate than its actual cost curves indicate. It must give up a portion of present profits in the amount of \( MR-MC=MTP \), so as to produce at point B. Through maximizing profits in the present total profits, including future periods, will be maximized.
To maximize the present value of the asset the profits given up in each period must have an equal present value for all future periods till exhaustion. This is algebraically formulated as

\[(MR_0 - MC_0)e^{rt} = (MR_1 - MC_1)e^{-rt}\]

where \(e^{-rt}\) is the exponential growth rate of the market rate of interest, \(r\). This indicates society's marginal rate of time preference. The optimal rate of production for a finite exhaustible resource is

\[(MR_0 - MC_0) = (MR_t - MC_t)e^{-rt}\]

or

\[(MR_t - MC_t) = (MR_0 - MC_0)e^{rt}.

In summary, this section has shown how society's marginal rate of time preference influences the optimal rate of production for an exhaustible resource. We illustrated this by showing a firm producing in a perfect competitive environment. The diagram, Figure 5, showed that even though the firm was concerned with future time periods, as evidence of its lower cost curves (solid line), it still wished to produce at a rate which would maximize its current period's profits, \(MR=MC\). In order to do this the firm had to give up some of its present profits produced at point "A" according to its actual curves so as to produce at the rate indicated by the present society's needs, at \(MR=MC\). Thus for the firm the optimal rate of production is where \(MR=MC+MTP\), where MTP is marginal rate of time preference. This can be rearranged to \(MR-MC=MTP\). Thus the marginal profit
given up due to the opportunity cost of exhaustion is remunerated back to the firm through the present value of future profits. This is how a firm allocates a given resource optimally between present and future periods. In the next section we will construct a model which will emphasize the optimal rate of production for petroleum producers.

C. Optimal Production Model for Petroleum Production

This section will construct a model of optimal conservation for petroleum production. This model will show the determinants for optimal production which was illustrated in the previous section.

A diagram of the model is shown in Figure 6. The current rate of production, at point "C", is the rate which will maximize the present value of the resource and thus lead to an optimal conservation rate. The axis of the top panel of the graph measures net present value and current rate of production. Each of the several curves labeled $W_1$, $W_2$, etc., represents net present values obtainable from a given number of wells, the number increasing from $W_1$ to $W_2$, etc. Each curve has a maximum ($M_1$, $M_2$, etc.) net present value, but the third curve ($W_3$) has the highest. Thus the producer will drill at an output corresponding to $W_3$ and will produce at the current rate "C" in order to maximize the present value of his asset. The bottom panel indicates this optimum rate if current
Fig. 6. Determination of the Optimum Rate of Production and Number of Wells.
production occurs where the curve MWC, representing incremental well cost (assumed to be a constant amount) intersects the curve MPV, representing incremental maximum present value.\textsuperscript{4}

We will assume that the producer's minimum acceptable rate of return is equal to the market rate of interest, adjusted for uncertainty, which in turn is equal to society's marginal rate of time preference. This occurs when the postponement in present production is at the point where the last unit postponed yields a rate of return just equal to this adjusted market rate of interest. Given this constraint the producer will push investments in the number of wells where the last increment to investment yields a rate of return also equal to this market rate of interest adjusted for risk. Production at this rate maximizes society's net satisfactions through sacrificing present consumption for future consumption.

We can conclude that when production is at a rate where the rate of return is equal to society's rate of time preference optimal conservation occurs for an exhaustible resource. Where optimal conservation is defined as maximizing the present value of natural resources we can determine whether petroleum producers are doing this to crude-oil production by testing for maximum present value of

\textsuperscript{4}McDonald, p. 81.
production. The model to test for this will be constructed in the next section.

D. Optimal Conservation

In this section we will determine the variables necessary to be satisfied in order to obtain the behavior of optimal conservation. We will do this through analyzing it through the "investment theory of the firm." This will be done by studying a model of a firm wishing to optimize its revenue under the assumption of a limited time horizon. The limited time horizon is synonymous to a limited exhaustible resource. The model will be illustrated through an analysis of an investment in a tree which will be cut down and sold for lumber at some future date. The entrepreneur's objective is to find the optimum time period of investment into this tree which will maximize his net present value of investment. The optimum condition exists when the present value of revenue from the tree is equal to the amount invested into that tree, \( V-C=0 \).

The calculation of the present value from that tree occurs when the rate of growth of the potential revenue during the investment period is equal to the market rate of interest.\(^6\) In order to maximize this present value, the rate

\[ V = f(t)e^{-rt} \]

\(^6\)Let \( R=f(t) \) represent the revenue from the tree as a function of the investment period, \( t \). Since the cost of investment, \( C \), is a constant, maximization of \( V-C \) requires simply that the present value of revenue...
of growth of this revenue must be equal to a maximum average internal rate of return. That is the rate of discount which, if used to discount down to the present all the revenues from the investment, makes their present value equal to the total cost of investment. This maximization condition occurs when the average internal rate of return is equal to the marginal internal rate of return.\(^7\)

For optimal conservation to occur the minimum rate of return which must be equal to the marginal internal rate of return must also coincide with the maximum rate of return, \(R\), on the entrepreneur's own capital, \(K\). We find that \(R\) is maximized over the investment period of a single tree when the investment period \(t\) is chosen such that \(R\) equals the percentage rate of growth of the entrepreneur's

\[
\frac{f'(t)}{f(t)} = r
\]

\(^7\)The average internal rate of return is \(P_a\) in the equation \(f(t)e^{-P_a} = C\), which gives us

\[
P_a = \frac{1}{t} \log \left( \frac{f(b)}{C} \right)
\]

Differentiating this expression with respect to \(t\), and putting the result equal to zero we find that the maximum \(P_a\) is reached when

\[
P_a = \frac{f'(t)}{f(t)}
\]

or when the average internal rate of return equals the marginal internal rate of return.
net worth (value of the tree minus the principle of the debt and accumulated interest thereon). This is equivalent to saying that the rate of return on the entrepreneur's capital, \( R \), is maximized when it equals the marginal internal rate of return on that capital.

Thus when the maximized \( R \) equals the market rate of interest, and both are equal to the marginal internal rate of return in the condition of maximization, \( V-C=0 \), obtained. This conclusion is delineated in Figure 7. The diagram shows the proper investment period for a limited resource which will maximize the present value of that investment. The length of the investment period is measured on the x-axis, and the y-axis measures the logarithmic scale which measures the value of the investment and the

---

8 The rate of return on the entrepreneur's own capital is \( R \) in the formula \( R e^{kt}-f(t)-(C-K)e^{rt} \) which gives us

\[
R = \frac{1}{t} \log \left( \frac{f(t)-(C-K)e^{rt}}{K} \right)
\]

R is then maximized (i.e., \( R'(t)=0 \)) when

\[
R = \frac{f'(t)-r(C-K)e^{rt}}{f(t)-(C-K)e^{rt}}
\]

or

\[
(1) \ f'(t) = R[f(t)-(C-K)e^{rt}]+r(C-K)e^{rt}
\]

9 If \( r \) happens to equal the maximized \( R \) in equation (1), by substituting \( r \) for \( R \), i.e., \( r=R-f'(t)/f(t) \). When \( r+R \) is also equal to \( P_a \), where \( P_a \) is equal to the marginal internal rate of return, \( R \) will be maximized at the same value for \( t \) as that which \( P_a=f'(t)/f(t) \), i.e., at which \( P_a \) is maximized. Since \( P_a=r \), \( V-C \) is also maximized at this same \( t \), where the maximum is obtained with \( V-C=0 \).
Investment Period

Fig. 7. Optimal Investment Horizon of the Firm.
entrepreneur's net worth. The entrepreneur has an objective of simultaneously maximizing the value of his resource and at the same time maximizing the net worth of his capital. The upper curve shows the value of the resource as it grows over time (the function \( f(t) \)), and the lower curve represents the growth of the entrepreneur's net worth (i.e., the value of the resource minus the debt, as expressed by \( f(t) - (C-K)e^{rt} \)). The rate of interest, \( r \), is represented by the slope of the line \( V_d \) or any line parallel to this line. There is a whole family of such discount lines representing any given rate of interest; the one in the graph identifies the marginal rate of time preference of society. The \( V_d \) line which is tangent to the \( f(t) \) curve, at point \( L \), indicates the optimum investment period which will maximize the present value of the resource. The maximum present value is obtained by allowing the investment time horizon to be ON. This point determines the appropriate internal rate of return which will maximize the investment. C and K represent the total cost of the investment, and the entrepreneur's own capital respectively. The slope of the steepest line drawn from C that will touch the curve \( f(t) \) represents the maximum marginal internal rate of return that can be achieved. This is at point \( L \). Similarly, the slope of the steepest line that can be drawn from K tangent to the net worth curve \( (f(t) - (C-K)e^{rt}) \) indicates the maximum
rate of return, $R$, that can be achieved on the entrepreneur's capital. This is at point $Q$.\(^{10}\)

Thus the graph shows that optimal conservation defined as $V-C=0$ occurs when the slopes of the slopes of the $C$ and $K$ line equals the slope of the $V_d$ line or rate of interest. This situation holds that for optimization in production to occur the rate of return on entrepreneurs capital, $R$ (indicated by the market rate of interest), must be equal to the marginal internal rate of return. When this occurs the present value of the resource is maximized and this is what optimal conservation leads to.

In the next chapter we will test the rate of petroleum production in the United States according to this model. We will find the marginal internal rate of return and analyze its relationship to the minimal acceptable market rate of interest. This will tell us whether the production of petroleum, crude-oil, is behaving in an optimal conservationist manner based on the theory of exhaustible resources under the assumptions stated above.

\(^{10}\text{Lutz, p. 30.}\)
CHAPTER IV

EMPIRICAL ANALYSIS

A. Empirical Test

In this chapter we will conduct a test based on the model presented in Chapter III. The objective of this test is to determine whether or not the rate of petroleum production is consistent with optimal conservation. The test will cover a sample of new petroleum reserves produced in the United States from years 1959 through 1975. Using the discount cash flow rate of return method of analysis we will observe the relationship of a calculated marginal internal rate of return per period to an assumed market rate of interest adjusted for risk and uncertainty. The market rate of interest is an interpretation of society's marginal rate of time preference. If the calculated marginal internal rate of return is equal to the market rate of interest then the rate of production is optimal; if not, we have non-optimality.

The discount cash flow rate of return method of analysis is used because it coincides with our model of optimization in Chapter III. That is, optimization is recognized when the present value of net revenues minus the present value of exploration and development costs,
discounted by society's marginal rate of time preference, gives us a net present value of zero. That is,

$$\text{NPV} = V - C = 0$$

where

$$V = \frac{(P_1Q_0 - c_0)}{(1+i)^1} + \frac{(P_2Q_2 - c_2)}{(1+i)^2} + \ldots + \frac{(P_nQ_n - c_n)}{(1+i)^n}$$

The variables are:

- **NPV** = Net present value
- **V** = present value of net revenues
- **C** = development and exploration expenditures in present terms
- **P** = wellhead price
- **Q** = annual production
- **c** = annual operating cost
- **i** = discount rate, interpreted as society's marginal rate of time preference
- **1,2...n** = subscripts denoting the number of years equal to the life of the reservoir.

The data for this test is taken from a study of petroleum production costs conducted by John D. La Rue.\(^1\) He has compiled data pertaining to the production and exploration of domestic petroleum reserves for the years 1959 through 1975. We will use this data to find the marginal internal rate of return for each year and in this study.

\(^1\) Calculation of New Oil Costs, United States Years 1959 Through 1974. La Rue, Moore and Schafer, Petroleum Consultants (Dallas, Texas, 1975), pp. 1-95.
The reserves to be tested for optimality are found in Table 3, column 8. They are the total sum of new reserves through exploration and certain adjustments. Each year's total is the sum of petroleum obtained from the extension of old oil reserves, discovery of new reserves, and new reserves discovered in old fields. Also certain adjusted reserves are added which are obtained through the addition of existing reserves determined by the algebraic summation of positive and negative adjustments to reserves in all the fields of the United States. The objective of the test is to determine whether these reserves listed in Table 3, column 8, are the proper quantity to satisfy an optimal conservationist rate of production.

Fifteen percent is the maximum discount rate. La Rue feels this amount of return is necessary to maintain optimal exploration levels.\(^2\) For our study this figure is seen as the market rate of interest adjusted for risk and uncertainty.

The net cash flow is found by using historic yearly expenditures for petroleum exploration and development and the ultimate petroleum reserves added through drilling during the same year. These reserves are projected over their expected lives so that their future annual gross revenues from the sale of crude oil and its associated

\(^2\)Calculation of New Oil Costs, United States Years 1959 Through 1974. La Rue, Moore and Schafer, Petroleum Consultants (Dallas, Texas, 1975), p. 10.
### TABLE 2

**OIL RESERVE ADDITIONS THROUGH DRILLING**

**PLUS ALLOCATED HISTORICAL REVISIONS**

**UNITED STATES (a)**

(Millions of Barrels)

<table>
<thead>
<tr>
<th>Year</th>
<th>(1) Reserves at End of Year</th>
<th>(2) Revisions to Existing Reserves</th>
<th>(3) Reserves Added by Drilling During Year</th>
<th>(4) New Fields</th>
<th>(5) New Reservoirs in Old Fields</th>
<th>(6) Totals</th>
<th>(7) Allocated Ultimate Reserves</th>
<th>(8) Ultimate Reserve Added</th>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1946</td>
<td>20,874</td>
<td>1,255</td>
<td>1,159 (b)</td>
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<tr>
<td>1947</td>
<td>21,448</td>
<td>740</td>
<td>1,270 (b)</td>
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<tr>
<td>1948</td>
<td>23,280</td>
<td>1,959</td>
<td>1,440 269 (b)</td>
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<tr>
<td>1949</td>
<td>24,649</td>
<td>604</td>
<td>1,694 544</td>
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<td>663</td>
<td>1,334 408</td>
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<td>27,468</td>
<td>1,776</td>
<td>2,249 206</td>
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<td>30,300</td>
<td>465</td>
<td>1,543 207</td>
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<td>30,536</td>
<td>955</td>
<td>1,339 151</td>
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<td>1,779 166</td>
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<td>31,613</td>
<td>788</td>
<td>1,324 141</td>
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<td>1,041 92</td>
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<td>30,970</td>
<td>966</td>
<td>858 97</td>
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<td>30,991</td>
<td>899</td>
<td>1,419 127</td>
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<td>1965</td>
<td>31,352</td>
<td>1,783</td>
<td>793 237</td>
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<tr>
<td>1966</td>
<td>31,452</td>
<td>1,839</td>
<td>814 160</td>
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<td>31,377</td>
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<td>716 125</td>
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<td>1968</td>
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<td>777 166</td>
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<td>1969</td>
<td>29,632</td>
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<td>615 96</td>
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<tr>
<td>1970</td>
<td>29,401 (a)</td>
<td>2,089</td>
<td>631 263 (a)</td>
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<td>1971</td>
<td>28,463 (a)</td>
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<td>561 91</td>
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<td>1972</td>
<td>26,739 (a)</td>
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<td>459 123</td>
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<td>1973</td>
<td>25,700 (a)</td>
<td>1,552</td>
<td>390 116</td>
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<td>1974</td>
<td>24,650 (a)</td>
<td>1,311</td>
<td>369 226</td>
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<td><strong>TOTALS</strong></td>
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<td></td>
<td>33,965</td>
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<td>5,495</td>
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<td>45,275</td>
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</tbody>
</table>

Source: API. See Appendix A - I.

**NOTE:** Columns may not extend precisely because of computer rounding.

Definitions relating to reserve nomenclature may be found in Appendix A.

**EXPLANATORY NOTES:**

Column

(1) Proved oil reserves at end of year.
(2) Revisions to reserves included in Column (1).
(3) Extensions of old fields by drilling.
(4) New field discoveries.
(5) New reservoirs discovered in old fields.
(6) Column (3) + Column (4) + Column (5).
(7) (Total of Column (2) / Total of Column (6) ) x yearly value of Column (6).
(8) Reserves added during year, Column (6) + Column (7).

**FOOTNOTES:**

(a) Excludes Prudhoe Bay field in Alaska.
(b) Included in Column (5).
gas can be calculated. All costs, discounted exploration and development cost, were deducted to calculate net cash flows to the producer after federal income tax. The calculation of net cash flow for each year, 1959 through 1974, is shown in the appendix of this thesis.

The results of the analysis is shown in Table 4, column 1. These are the marginal internal rates of return, or the rate of discount used to discount to the present the calculated net cash flow after federal income taxes to a net present value of zero. According to the model in Chapter III, if the marginal internal rate of return is not equal to the market rate of interest the rate of production of an exhaustible resource does not coincide with optimal conservation. The results of our analysis show that the calculated marginal internal rate of return is less than the market rate of interest adjusted for risk. In the following section we will analyze the results to determine what they mean.

B. Meaning of Results

To fully understand the meaning of the results we must first understand the meaning of the market price per barrel of oil. These prices are stated next to the computed marginal internal rate of returns, in Column 2 of Table 4. The market price for a particular good indicates an equality between the quantity supplied and the quantity demanded for that good. The price represents equality between the
Table 4

Marginal Internal Rates of Return and Well Head Prices in the Production of New Domestic Petroleum Reserves for the Years 1959 Through 1974

<table>
<thead>
<tr>
<th>Year</th>
<th>Marginal Internal Rate of Return, %</th>
<th>Well Head Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959</td>
<td>15.3</td>
<td>2.90</td>
</tr>
<tr>
<td>1960</td>
<td>11.2</td>
<td>2.88</td>
</tr>
<tr>
<td>1961</td>
<td>11.7</td>
<td>2.89</td>
</tr>
<tr>
<td>1962</td>
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<td>2.90</td>
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</table>

Source:

Crude oil prices at the well head, United States average

Source: 1959-73 United States Bureau of Mines
1974 Federal Energy Association
1975 Federal Energy Association
marginal sacrifice of producing that good with the marginal gain in satisfaction from consuming it. Thus price acts as a mechanism which optimally allocates a good among alternative employments.

In dealing with an exhaustible resource price can act as a mechanism to optimally allocate resources among present and future time periods. The pricing mechanism responsible for this is the market rate of interest, which is an interpretation of society's marginal rate of time preference. At this price, the equilibrium rate of interest equates the marginal sacrifice of current satisfactions with the marginal gain to future satisfactions. Again the market acts as a mechanism to optimally allocate goods among alternative employments. Production at this point is the prerequisite for obtaining allocation according to intergenerational equity.

The empirical test appears to suggest a current condition which we depict in Figure 8. At a market price, \( P \), assuming the firm is a price-taker, the output level is \( OQ \) with a given supply curve \( S \). Adding the opportunity cost discussed above, the supply curve becomes \( S' \). It can then be observed that at output \( OQ \), the real cost of current production is greater than the selling price per unit of output. This is consistent with the results from Table 4. In order for \( OQ \) to be the optimal rate of production price must rise to \( P_e \). Table 4, Column 2, which includes the necessary rate of return on investment. At this level of production, price \( P_e \) is sufficient to compensate owners of resources
Fig. 8. Diagram of Results Obtained from Empirical Analysis of United States Petroleum Production. Hypothetical demand and supply curves.
when they are allocated from present to future consumption. Thus, if the firm were of the conventional type, output would be $OQ'$, given $P_e$. The difference between $OQ$ and $OQ'$ is consumption postponed for future use.

The empirical results suggest that production is occurring at $OQ$, but price is $P$, rather than $P_e$ (actually some price between, since the marginal internal rate of return is not zero for any year observed). Thus, producers are not being appropriately compensated for saving energy for future consumption. As a result of this we might also expect that investment in petroleum production would slow down, and firms would attempt more forward at an output level similar to $OQ"$. Column 8 of Table 3 tends to bear out this expectation. Specifically, reserves added have tended to decline throughout the period of analysis. Thus with a less than optimal rate of production according to society's preference of postponing consumption we found that producers are maximizing their private benefits but social benefits are not maximized. In general society is not being optimally compensated for foregoing present consumption in order to save it for future generations. This results in a present rate of petroleum production which is less than the optimal amount necessary to satisfy intergenerational equity.

A free competitive market is a capable mechanism for allocating a resource equitably between generations.
But in dealing with petroleum the market is subject to the externality of losing correlative property rights in petroleum. To compensate for this externality the petroleum market is constrained by state conservation regulations over the rate of production. The following chapter will analyze these regulations and determine whether they are responsible for the present inequitable distributions of petroleum.
CHAPTER V
CONCLUSION

In this chapter we will analyze the petroleum industry in an attempt to determine why we got the results of non-optimal conservation. The industry will be analyzed according to the influences placed on the rate of production by state conservation regulations. The petroleum industry is unique by the fact it operates under a regulated market system. The reason for this is that the production of petroleum is characterized by an externality. That is, due to poorly defined property rights, petroleum producers are subject to losing their petroleum to a neighboring producer. This externality is known as correlative property rights of petroleum. In order to protect the loss of these correlative property rights states have passed conservation regulations which control petroleum production. Thus these regulations will be analyzed in terms of how they affect the rate of production.

After analyzing these state regulations we will suggest an alternative to them known as "Unitization." We will conclude this chapter with an interpretation and summary of the results found in this thesis.
A. Reason for Conservation Regulation

Optimal conservation of petroleum occurs when oil producers drill the optimum number of wells and produce petroleum at the optimum rate of production. This situation occurs in a competitive free market system. But the petroleum industry is characterized by the condition we have termed correlative property rights. That is, due to the migratory nature of petroleum a producer is faced with a problem of losing his property in petroleum to a neighboring producer, thus an externality is created. If a free market situation were allowed to operate under such circumstances it could lead to excessive output and ruinous price competition. Thus states have passed conservation statutes to regulate the production of petroleum in order to control output and thus internalize the externality. In order to fully understand why these regulations are necessary, we will analyze the migratory nature of petroleum.

Petroleum is found in underground reservoirs as a fluid mixture of chemically distinct hydro-carbon compounds, ranging in number from few to many. The physical properties of a given mixture, in the reservoir, vary with temperature and pressure. Crude-oil is petroleum in the liquid state, where natural gas is petroleum in the gaseous state. It is the relationship between oil and gas which is the prevailing reason for the present conservation
regulations. Gas dissolved in oil plays a dual role: (1) it affects the fluidity and with it the mobility of the oil, making it flow more freely by lowering viscosity, by reducing specific gravity, and by lowering surface tension; (2) the dissolved gas which is under pressure in the undisturbed reservoir state expands upon release from this pressure (such as a well opening). This expanding provides power to move the oil toward the well base and bring it to the surface. ¹ The cheapest way to produce oil is to take full advantage of this gas pressure by controlling the rate of production through a proper well spacing outlay.

Each reservoir has its own specific gradient of natural drive. This depends on characteristics such as the nature of the crude, the amount of gas dissolved in it, and the porosity and permeability of the rock formation in the reservoir. It is these conditions which determine the optimal number of wells to be drilled in a reservoir. That is, they affect the size of the segment of a reservoir which a single well can drain in a given time. That is the optimal well spacing density. In general, wells ought to be located so as to take the fullest advantage of the natural drive available in the reservoir.

Because of this gas-oil relationship, petroleum is characterized by a migratory nature. If ownership of

a reservoir is divided among several landowners, petroleum will migrate away from one owner to another. These movements can be deliberately induced so as neighboring landowners can drain the petroleum away from his neighbor. And through slowing down his rate of production to coincide with society's time preference a producer is liable to lose his petroleum to his neighbor. Thus the whole issue of petroleum correlative property rights has to do with the fact that petroleum can migrate from one property owner to the next.

If an unregulated competitive market were allowed to operate under these conditions a chaotic situation would exist. Each producer would accelerate his production rate so as to attempt to steal his neighbor's petroleum. And at the same time he would invest in offset wells to stop his neighbor from stealing his petroleum. An offset well is one deliberately drilled for the specific purpose of counteracting or offsetting the capacity of a well located on an adjoining property to drain oil from underneath the property on which the offset well is located.\(^2\) In general, waste production and investment would occur in a free market setting. The price would drop due to excess supply and the cost of production would rise due to unnecessary investment in wells. Thus a detrimental situation for the petroleum industry would exist.

\(^2\)Ibid., p. 99.
Present state conservation regulations are set up to compensate for the externality of correlative property rights. Their objective is to control production to coincide with optimal conservation. Seeing that petroleum is characterized by a migratory nature the only way to control production is by controlling well spacing. Through influencing the optimal well spacing density regulations will attempt to control the production rate of petroleum to coincide with optimal conservation. The results we obtained from the empirical data, in Chapter IV, showed that the rate of production of crude-oil was in a non-optimal manner. The reason for this can be confined to whether the present regulatory statutes promote an optimal well spacing density or not. In the next section we will observe the regulatory system and observe how it influences well spacing.

B. The Regulatory System

Each petroleum producing state has its own specific forms of regulation. We will group these forms of regulations into three specific types.3 We will describe how each works and how they influence the rate of production through well spacing. After each description we shall

conclude whether well density proposed by each is optimal or not.

The first, and simplest, is the restriction of well density on the basis of some uniform state-wide density rule. Through controlling the number of wells to be tapped in a reservoir by selecting a minimum acreage allotment per well it is possible to control the rate of production to coincide with the optimal conservation rate. Under this method producers still have the freedom to capture his neighbor's petroleum; thus operators have incentives to drill wells as densely as regulation permits. They will produce as much as is demanded in the market. Regulation by spacing rules as a method for controlling production is practiced in the states of Illinois, Indiana, Ohio, Pennsylvania, Kentucky, and West Virginia.

This form of regulation cannot create an optimal well spacing density because any rule which specifies some minimum acceptable distance between wells, or some minimum acceptable acreage per well, is essentially arbitrary. If each reservoir is characterized by unique conditions such as depth, recoverable reserves per acre, drive, and gas-oil ratio, then a specific well density for one reservoir is not appropriate for another. Thus it is conceivably impossible to have an optimum well spacing situation where each reservoir is uniquely characterized by independent conditions. There is incentive for over-investment in
wells since the optimal density is not assured. Thus a non-optimal rate of production of petroleum exists.

The second type of production control is to limit each reservoir's output to its specific MER, maximum efficiency rate of production. The MER of a reservoir is the highest rate of producing petroleum that does not cause physical waste or reduce ultimate recovery.\(^4\) If the rate of production does not coincide with the MER, then the reservoir's ultimate recovery of crude oil is impaired because of the lack of pressure to push it to the surface. The permissible rate of production is determined separately for each affected reservoir in the light of its distinctive operating characteristics. Each well is assigned its share of the allowable reservoir output. Where operators know their MER of a new reservoir and their production allocation to wells in proportion to acreage drained, there is no incentive to drill more wells on a given acreage than can produce the proportionate share of the expected MER. Thus this form of controlling production will lead to wider spacing than the minimum required under the general spacing regulations. This, in addition to regulation of well spacing regulation, is practiced in Arkansas, Mississippi, Alaska, California, Utah, Colorado, Wyoming, Montana and Nebraska.

\(^4\)Zimmermann, p. 70.
This form of regulation also has a tendency for creating an non-optimal conservation situation. The MER of a reservoir is not necessarily its optimum rate of production. The principle behind a MER rate of production is to avoid waste in the ultimate amount of petroleum in the reservoir. It does not make any allowances for calculating society's rate of time preference. The MER is unresponsive to changes in the market situation in a changing economy. Even if the MER should approximate the optimum rate in the first years of production from a reservoir, it would not do so continuously throughout the reservoir's life in the face of changing economic conditions and prospects.\textsuperscript{5} Thus conservation is not optimized because of the lack of correlation between a reservoir's MER and society's rate of time preference.

The third form of regulation is controlling output through a method known as "market-demand prorationing." Prospective purchases for a specific state are periodically determined by a regulatory authority, and then this total is allocated among reservoirs and wells in proportion to their basic allowables. The basic allowable for a well is a statewide schedule of a well's allowable rate of production based on well depth and acreage drained. This form of regulation is practiced in Texas, Louisiana, New Mexico,

\textsuperscript{5}Mcdonald, p. 184.
Oklahoma, and Kansas. Their total is responsible for three-fourths of the nation's total crude-oil production. Since this form of regulation controls such a major proportion of crude-oil production it is the greatest factor which influences the results we obtained in our empirical test. This being the case, we will go into great detail on how this regulation system operates and how it influences spacing incentives.

Market-demand prorationing consists of procedures by which a state regulatory agency restricts statewide production in line with an estimated demand for the next ensuing period. This state total is next allocated back to reservoirs and individual wells. Each state has a system of statutes, rules, and regulations setting forth in considerable detail the framework of the system. Basically demand for a future ensuing period is obtained by public meetings conducted monthly or bi-monthly. At these meetings petroleum purchasers put forth "nominations" for crude-oil purchases. These "nominations" are estimates of how much purchasers will need in the ensuing period. These nominations are given under an assumption of an inelastic demand curve; thus price has no influence. The commission then forms a judgment on the probable quantity of oil to be demanded from that state for the forthcoming period. This is known as the "allowable" for the state.
This "allowable" is then prorated back to reservoirs and wells. The basic prorating tool is the "depth-factor," or "yardstick" schedule, adjusted by an acreage factor. Every reservoir is produced at some particular depth. According to this depth each well of the reservoir is assigned a "top allowable." This represents a one-hundred percent production rate figure; for example, a 5000-6000 foot well on a forty acre drilling unit has a top allowable of 102 barrels per day. The schedule of top allowables increase as depth and acreage increase.

The final rate of production is determined by adjusting the top allowable per well to the specific state allowable for that ensuing period. This is accomplished through a market-demand factor. This is the ratio of the total non-exempt share of allowed production to the total non-exempt basic allowable. That is, there are certain fields and wells which are exempt from market-demand restrictions, such as capacity water-flooded fields or discovery wells, which control their own capacity rates of production. The sum of this total allotment is subtracted from the total state allowable and the remainder is allocated to non-exempt fields and wells in proportion to their respective top allowable through the use of the market demand factor. This market demand factor, or ratio, is used as a scheduling tool to allocate production for each well. For example, for a well with a 102 barrel per day top allowable and a
market demand factor of .30, the actual allowable for the well is 31 barrels per day \((102 \times .30 = 31)\).\(^6\)

In general, well spacing density, under a market-demand prorationing system, is controlled through a depth-acreage schedule. The only way such a system will produce petroleum at an optimal rate is by the coincidence of having the depth-acreage schedule the same as a well density spacing promoted in a free competitive market situation. Well allowables vary directly with depth and acreage drained, and their pattern generally induces wider spacing at greater depths. We have a situation where given a depth and recoverable reserves per acre in a reservoir, induced well density is related not to a reservoir operating characteristic, but to a pattern of well allowables and expected market demand factors. Well density tends to optimal, not with respect to reservoir conditions but with respect to the regulatory restraints imposed. Thus there is still a situation of a non-optimal rate of production because well density tends to be non-optimal.

Now that prevailing regulatory programs have been analyzed we will see why their cumulative effect is responsible for the present non-optimal rate of new petroleum reserves. Through our empirical test we observed an industry profit maximization path is greatly constrained

\(^{6}\)Ibid., p. 8.
by regulation. Given this limit, producers can increase
profits only by drilling wells up to the limit allowed by
spacing regulations. We have showed that these conservation
regulations have a weakness of not being able to correlate
their regulated well density spacing to optimal well spacing
density. Assuming producers are rational they will
maximize their profits at the point where marginal revenue
equals marginal cost. However, under regulation the
parameter within which the producer must make this decision
is altered due to this constraint. What is optimal from
his point of view might not be optimal from society's view-
point. Given non-optimal constraints on well spacing a
producer may maximize his profits by producing at a rate of
production rational to him but the decision to produce at
this rate might be uneconomical and irrational from society's
standpoint. This is what the results in our test indicated
through the fact that the calculated marginal internal rate
of return was not equal to society's assumed marginal rate
of time preference. In short, due to non-optimal constraints
put on well spacing density through state conservation
regulation the production of petroleum is behaving in a
non-optimal conservationist manner.

The following section will suggest the conservation
program of "unitization" as an alternative to the present
non-optimal conservation regulations.
C. Unitization

Unitization is defined as the practice of unifying the ownership of an actual or prospective oil or gas pool by the issuance or assignment of units as undivided interest in the entire area with the provision for development and operation by an agent, trustee, or committee representing all holders of undivided interest therein. The policy of unitization connotes unit operation which means "the carrying on of development and production of a field, pool or area thereof as a geological unit under one control, with or without the exchange of transfer of undivided interest." In general, the unitization of a multi-owned reservoir is synonymous with a unit operation.

There are two forms of unitization agreements:

the first involves the merger of titles in leases and the second involves the pooling of interest in production.

In the former, there is an exchange of assignments among lessees so that each lessee becomes a co-tenant of each tract in proportion to his ascribed interest in the whole unitized area. The later type of agreement states each

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8Ibid.

lessee will retain full title to his lease while title in production is pooled and each lessee shares in total production and expenses in proportion to the percentage interest ascribed to his tract. In either case the property owner receives compensation on the basis of production attributed by the agreement to his land.

Since the essence of unitization is a provision for unifying separate property owners, decisions for the whole unit are made from a central management group. Usually this decision making body is comprised of a board of representatives from each producer and a central unit manager. This body's objective is to draw up a general agreement for operation of the unit. This agreement indicates the type of drive to the selected, the manner of creating or selecting the selected drive, the number and location of additional wells to be drilled, the number and location of wells to be plugged or converted to injection wells, the rate of oil or gas production, the handling of associated gas, and the disposal of salt water and other waste material. In general, the agreement sets up the mode of operation best suited for the reservoir as a whole.

The reason why unitization will control production to coincide with the optimal conservation rate is that it creates the incentive for optimal well spacing density. That is, where the maximum number of reservoir acres are

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10 Mcdonald, p. 208.
economically and effectively drained by one well within a reasonable period of time. This is possible because under the unitization set-up producers have the freedom to locate wells in conformance with the structural characteristics of their unit reservoir. The externality of losing their property rights has been neutralized; thus producers are free to take advantage of the natural drives of the reservoir. Thus there will be more efficient recovery with fewer wells. A unit manager may select that number and location of wells which would maximize the present value of the reservoir net of well cost at the time of decision. Without unitization the spacing pattern would depend on statewide spacing rules, MER restrictions, and depth-acreage allowable schedules. Such a pattern would coincide with the optimum density only by chance. Thus unitization allows a free unregulated market to determine the optimum spacing density which in turn determines the optimum rate of production in accordance with optimal conservation.

D. Conclusion

In this thesis we constructed on a model based on the pure theory of exhaustible resources to determine optimal conservation. Optimal conservation of a resource occurs when the resource's present value is maximized from the point of view of society as a whole. That is, the rate of production of an exhaustible resource must coincide with
society's marginal rate of time preference. Under the construction of our model we defined this point as when the present value of revenues from that resource minus the present value cost of exploration and development, discounted by society's marginal rate of time preference, gives a net present value of zero. Another way of interpreting this is when the marginal internal rate of return, for that period, is equal to the market rate of interest adjusted for risk and uncertainty.

We applied this model to domestic petroleum production for a sample period of years, 1959 through 1975. We determined that the rate of producing new petroleum reserves was non-optimal in accordance with our conservation model. The results showed that the present generation, from years 1960 through 1975, is subject to not enough petroleum production according to the optimal conservation theory. This generation is sacrificing too much in the form of foregoing present consumption so as future generations are able to consume this resource in the future. The objective of intergenerational equity is violated.

The reason for this is thought to prevail in the state conservation regulations over the petroleum industry. These regulations exist in order to preserve producers' correlative property rights in petroleum. Being that well spacing and production rates are interwoven and interdependent we look at the relationship of these conservation
regulations on the incentives for promoting optimal well spacing density. The three major forms of spacing regulations are statewide spacing rules, MER rate of production, and depth-acreage allowable schedules. We found that these regulations have a common weakness in lacking the administrative flexibility to adjust their fixed spacing rules to specific production characteristics of a reservoir. Thus producers operating under these non-optimal constraints have a tendency to produce at the rate where they maximize their own private benefits and not the benefits from society's standpoint. The regulations result in a greater than optimum well density and increase production costs for a given output level. The rate of return is thus lower than the free-market return.

We finally concluded by suggesting an alternative to state regulations, and that was a program of unitization. That is a program to unite all individual property owners of a reservoir, distributing benefits and costs proportionately, to act as one unit. This method allows a free unregulated market to determine the optimum well spacing density, thus being able to attain a rate of production consistent with the optimal rate of conservation.
SELECTED BIBLIOGRAPHY


APPENDIX

SAMPLES OF EACH YEAR, 1959 THROUGH 1974,
USED IN THE EMPIRICAL ANALYSIS
APPENDIX

Table 5 is a summary table of the cost of new oil found in the study conducted by John D. La Rue. We have the final figures of the net cash flow after income tax, shown in column 16, for each year 1959 through 1974. Since there is a net cash flow other than zero it means that the net present optimum is not satisfied at the maximum market rate of interest of fifteen percent. La Rue calculates a new present value of zero at fifteen percent by adjusting the present value of revenues by the price per barrel shown in column 18. Wherever there is a discrepancy between this calculated price and the wellhead price it means that the marginal internal rate of return is not equal to the market rate of interest adjusted for risk and uncertainty. The final marginal internal rate of returns calculated in Table 4, column 1, are the ones necessary to obtain a net present value of zero for each year 1959 through 1974. This appendix shows the present value of each category involved in calculating the final net cash flow to be used in calculating the final marginal internal rate of return.

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## APPENDIX

### SUMMARY TABLE, COST OF NEW OIL—UNITED STATES

**YEARS 1959 THROUGH 1974**

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<th>YEAR</th>
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**NOTE:** Figures are for total United States except Prudhoe Bay field in Alaska. All financial data expressed in constant dollars for year of initial projection. Columns may not add precisely because of computer rounding.

**Explanatory Notes:**

Column

1. Oil reserves added by drilling plus expected upward revisions. API; See Table 2, Column (8).
2. Gas reserves associated with oil.
3. Column (1) + Column (18) + Column (2) + Column (19).
4. 12.5% of Column (3). Minimum royalty.
5. Variable tax rate × (Column (3) - Column (4)). Tax rate is approximately 6%. IRS.
6. Direct operating costs including field labor and supplies, maintenance, general and administrative overhead. IRS; See Table 5.
7. Column (3) - Column (4) - Column (5) - Column (6).
8. Total capital investment attributable to oil reserves added in year including leasehold costs. IRS; See Table 7, Line (11).
9. Portion of total capital attributable to intangible drilling costs. IRS; See Table 7, Line (10).
10. Includes cost and percentage depletion based on law at time of first year projected.
11. Cumulative depreciation of tangible drilling costs and leasehold equipment. IRS; See Table 7, Line (9).
12. Column (7) - Column (9) - Column (10) - Column (11).
13. Investment tax credit is variable for each year depending on law at start of year. Zero some years.
14. Column (12) × 50% - Column (13).
15. Column (7) - Column (14) or alternatively Column (12) - Column (14) + Column (10) + Column (9) + Column (11).
16. Column (15) - Column (8).
17. Column (16) discounted at 15% per annum compounded annually. Must total zero.
18. Gross oil price at wellhead required for 15% discounted rate of return after federal income taxes.