2006

Response to "Comment on Variational Approach to the Volume Viscosity of Fluids" [Phys. Fluids 18, 109101 (2006)]

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Original Publication Citation

Zuckerwar, A. J., & Ash, R. L. (2006). Response to "Comment on Variational approach to the volume viscosity of fluids" [phys. Fluids 18, 109101 (2006)]. *Physics of Fluids, 18*(10), 109102. doi:10.1063/1.2361310

Response to "Comment on 'Variational approach to the volume viscosity of fluids'" [Phys. Fluids 18, 109101 (2006)]

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(Received 30 August 2006; accepted 7 September 2006; published online 18 October 2006)

We respond to the Comment of Markus Scholle and therewith revise our material entropy constraint to account for the production of entropy. © *2006 American Institute of Physics*.
[DOI: 10.1063/1.2361310]

The following is our Response to the Comment by M. Scholle on the subject article.[1]

Inconsistency No. 1: We agree with a reviewer of Scholle's comment and reiterate his reply:

"For a steady flow, α is independent of time at a given position but not necessarily constant with position as the convective part of the material derivative is $\mathbf{v}\cdot\nabla\alpha=-A$. I do not see that integrating this in time implies linear growth in α."

In other words, in steady flow explicit time derivatives $\partial/\partial t$ are zero, in which case integration over time is not meaningful.

Inconsistency No. 2: We agree that the assumption of negligibly small entropy production, as expressed in our Eq. (28), which we call the "acoustic approximation," restricts the analysis to a small subclass of flows. We take this opportunity to generalize the analysis for the case where entropy production is not negligibly small. We again assume a single dissipative process. Unless otherwise indicated, equation numbers (28) and above refer to those of our original article; equation numbers in the present Response begin again with (1).

To include entropy production, we replace Eq. (28) with[2,3]

$$\frac{DS}{Dt}-\frac{L}{T}A^2=0. \tag{1}$$

This replacement will not affect the result of the variational procedure [Eq. (32)], because A is neither an independent variable nor a coordinate of the variation. Upon substitution of Eq. (8) (in the original paper) into (1) the entropy production becomes

$$\frac{DS}{Dt}=\frac{1}{LT}\left(\frac{D\xi}{Dt}\right)^2. \tag{2}$$

However, the entropy production term appearing in the dynamic equation of state [differentiating Eq. (41) with respect to time],

$$\frac{D\rho}{Dt}=\left(\frac{\partial\rho}{\partial P}\right)_{\xi S}\frac{DP}{Dt}+\left(\frac{\partial\rho}{\partial\xi}\right)_{PS}\frac{D\xi}{Dt}+\left(\frac{\partial\rho}{\partial S}\right)_{P\xi}\frac{DS}{Dt}, \tag{3}$$

as well as in the equation of motion [Eq. (39)],

$$\frac{D\mathbf{v}}{Dt}=-\nabla\Omega-\frac{1}{\rho}\nabla P+\alpha\nabla\frac{D\xi}{Dt}+\beta\nabla\frac{DS}{Dt}, \tag{4}$$

cannot be ignored under these conditions. Upon substituting Eq. (2) into (3) and (4) we find

$$\frac{D\rho}{Dt}=\left(\frac{\partial\rho}{\partial P}\right)_{\xi S}\frac{DP}{Dt}+\left(\frac{\partial\rho}{\partial\xi}\right)_{PS}\frac{D\xi}{Dt}+\left(\frac{\partial\rho}{\partial S}\right)_{P\xi}\frac{1}{LT}\left(\frac{D\xi}{Dt}\right)^2, \tag{5}$$

$$\frac{D\mathbf{v}}{Dt}=-\nabla\Omega-\frac{1}{\rho}\nabla P+\alpha\nabla\frac{D\xi}{Dt}+\frac{\beta}{LT}\nabla\left(\frac{D\xi}{Dt}\right)^2. \tag{6}$$

We can eliminate the progress variable ξ from the equation of motion [Eq. (6)] by letting

$$\alpha=-\frac{C}{\rho}\left(\frac{\partial\rho}{\partial\xi}\right)_{PS}, \tag{7}$$

and

$$\beta=-\frac{C}{\rho}\left(\frac{\partial\rho}{\partial S}\right)_{P\xi}, \tag{8}$$

where C is a constant. Compatibility with the equations of acoustical propagation requires that

$$C=\frac{\tau_{VS}}{\kappa_S^\infty}=\frac{\tau_{PS}}{\kappa_S^0}=\tau_{PS}\rho_0a_0^2. \tag{9}$$

Upon using

$$\rho\kappa_S^\infty=\left(\frac{\partial\rho}{\partial P}\right)_{\xi S}\quad\text{and}\quad\frac{D\rho}{Dt}=-\rho\nabla\cdot\mathbf{v}$$

and comparing with the last two terms of (5), we obtain

$$\alpha\frac{D\xi}{Dt}+\frac{\beta}{LT}\left(\frac{D\xi}{Dt}\right)^2=-\frac{C}{\rho}\left[\frac{D\rho}{Dt}-\left(\frac{\partial\rho}{\partial P}\right)_{\xi S}\frac{DP}{Dt}\right]$$
$$=\tau_{PS}\rho_0a_0^2\nabla\cdot\mathbf{v}+\tau_{VS}\frac{DP}{Dt}. \tag{10}$$

When we substitute (10) into (6), we recover our original

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volume viscosity terms in the equation of motion:

$$\rho \frac{D\mathbf{v}}{Dt} = - \rho \nabla \Omega - \nabla P + \nabla \left[\tau_{PS} \rho_0 a_0^2 \nabla \cdot \mathbf{v} + \tau_{VS} \frac{DP}{Dt} \right], \tag{11}$$

and consequently the same modified Navier-Stokes equations (45) or (46) as before. We conclude that the production of entropy by internal processes, no matter how large, has no effect upon the macroscopic motion of a fluid. This is not a surprising result, for the entropy production in viscous shear flow likewise is not manifest in the Navier-Stokes equation.

We thank Professor Scholle for his Comment.

[1]A. J. Zuckerwar and R. L. Ash, "Variational approach to the volume viscosity of fluids," Phys. Fluids **18**, 047101 (2006).

[2]H. J. Bauer, "Phenomenological theory of the relaxation phenomena in gases," in *Physical Acoustics IIA*, edited by W. P. Mason (Academic, New York, 1965).

[3]S. R. deGroot and P. Mazur, "Viscous flow and relaxation phenomena," in *Non-Equilibrium Thermodynamics* (North Holland, Amsterdam, 1962).