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Empirical Identification of Factor Models

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Abstract:

In the conventional factor-augmented vector autoregression (FAVAR), the extracted factors cannot be used in structural analysis because the factors do not retain a clear economic interpretation. This paper proposes a new method to identify macroeconomic factors, which is associated with better economic interpretations. Using an empirical-based search algorithm we select variables that are individually caused by a single factor. These variables are then used to impose restrictions on the factor loading matrix and we obtain an economic interpretation for each factor. We apply our method to time series data in the United States and further conduct a monetary policy analysis. Our method yields stronger responses of price variables and muted responses of output variables than what the literature has found.

JEL Classification: C30, C32, C51, E58

Keywords: monetary policy; causal search; FAVAR; PC algorithm

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1 Introduction

For over thirty years, the vector autoregression (VAR) has served as a useful tool for central bankers to conduct monetary policy simulations. In a typical monetary VAR, a few variables are chosen to approximate the overall dynamics of the economy. More recently, Bernanke *et al.* (2005) proposed a "factor-augmented" vector autoregression (FAVAR) that replaces individual variables with unobserved macroeconomic factors extracted from a large dataset. The VARs are run using the extracted factors instead of variables and the impulse responses are obtained through the estimated responses of the factors.

The FAVAR has the advantage of generating responses for many variables without running out of degrees of freedom, but the factors themselves do not retain a clear economic interpretation. This is because there are infinite sets of "rotated" factors spanning the exact same space of macroeconomic variables, making it difficult to interpret (Stock and Watson, 2005). This problem limits FAVAR's use in structural economic analysis.

In this paper, we propose a new method that uses graph-theoretic causal search approach to give factors some meaningful economic interpretations. The implementation of our search algorithm takes several steps. First, we filter time series data using estimated principal components and remove non-contemporaneous causal relationships of variables. Second, we search for a set of "pure variables" that are individually caused by a single factor. Through repeated sampling, we calculate the percentage of inclusion, a measure that allows us to choose pure variables within a given category of macroeconomic variables (e.g., output, price). The variable with the highest percentage of inclusion within a given category is chosen as the pure variable for that category. These pure variables are then used to identify factors following Bai and Ng (2013).

In the second half of the paper, we apply our search algorithm to a large panel of macroeconomic data in the United States. A monetary FAVAR is estimated and impulse responses of the factors and selected variables to a monetary policy shock are computed. First, we confirm that the impulse responses of factors are in conformity with theoretical predictions. Next, we compare the responses of selected variables with those obtained using Bernanke *et al.* (2005)'s two-step principal component approach, which is commonly used in the literature.[3] Our FAVAR methods yield different impulse responses in selected variables from those of Bernanke *et al.* (2005), which we argue is more reasonable based on conventional wisdom. Our results are robust to alternative specifications, such as using a smaller number of factors for the filtering process, a shorter sample period, and fewer variables.

There have been many studies that try to give economic interpretations to estimated factors. One approach is to study the direct association between variables and corresponding factors through either factor loadings or pairwise correlation.[4] Another approach is to "rotate" the estimated principal components under certain conditions, so that they can be interpreted economically; see Lawley and Maxwell (1971) for a detailed discussion. Recently, Bai and Ng (2013) showed that when certain restrictions are imposed on either factors or factor loadings, it will make the estimated factors insensitive to further rotations and have economic interpretation as well. But it is not easy to justify these restrictions *a priori* without relying on the researcher's judgment. The search algorithm developed in this paper makes Bai and Ng (2013)'s proposed method empirically operational. Our paper is close in spirit to Swanson and Granger (1997) and

[3] For example, see Mumtaz and Surico (2008), Boivin and Giannoni (2010), Baumeister et al. (2013), Dave et al. (2013).
[4] For example, in the term structure literature it is common to relate latent factors with "level", "slope", and curvature" of the yield curve (Diebold et al., 2006; Dewachter et al.,2006).

Demiralp and Hoover (2003), who applied similar graph-theoretic causal search algorithm to identify the monetary policy shock in a small-scale VAR.[5]

Another contribution of our paper is to incorporate the economically interpretable factors to the otherwise standard factor-augmented regression analysis. Existing studies have shown that macroeconomic variables in the US can be represented by a few structural factors.[6] There are several methods in the literature on how to find the number of structural factors, but many of them avoid direct interpretations of factors.[7] In this paper, we provide a rigorous way of imposing identifying loading restrictions such that the identified macroeconomic factors have better economic interpretations.

The paper is organized as follows. Section 2 presents our method to identify factors with meaningful structural interpretations. Section 3 provides an empirical application using monetary FAVAR. Section 4 concludes.

2 Identifying Factors with Meaningful Structural Interpretation

2.1 Motivation

We start by laying out the basic econometric framework and provide motivation for our proposed method. The factor model for the i^{th} observable X_i is represented as

$$X_{it} = \lambda_i' F_t + \varepsilon_{it}, \qquad t = 1, \ldots T \tag{1}$$

[5] For other related works that apply graph-theoretic causal search algorithm in the VAR, see Bessler and Lee (2002), Demiralp and Hoover (2003), Demiralp et al. (2008), Demiralp et al. (2009), Demiralp et al. (2014), Hoover (2005), Moneta (2008), Moneta and Spirtes (2006), and Phiromswad (2014).
[6] For example, see Bai and Ng (2007), Forni et al. (2000, 2009), Forni and Gambetti (2010), Giannone et al. (2005, 2006), and Hallin and Liska (2007).
[7] One important exception is Belviso and Milani (2006) who identify factors by categorizing observed variables based on conventional wisdom (e.g., real activity, inflation, financial market) and extracting one latent factor from each category to be further used in monetary FAVAR analysis. For other related work that aims to identify different aspects of factors, see Boivin et al. (2009), Bork et al. (2010), and Reis and Watson (2010).

where F_t is a set of unobserved macroeconomic factors at period t, λ_i is a factor loadings that relate unobserved factors with the observed variable i, and ε_{it} is an idiosyncratic error term. In the factor-augmented vector autoregression (FAVAR) literature, it is further assumed that factors have the following autoregressive structure:

$$F_t = B_1 F_{t-1} + B_2 F_{t-2} + \ldots + B_P F_{t-P} + v_t \equiv B(L) F_{t-1} + v_t, \tag{2}$$

where $B(L)$ is a conformable lag polynomial of order P and v_t is a vector of reduced form shocks. Equation (2) can be regarded as a "reduced form" representation of the economy that takes the following form:

$$A_0 F_t = A_1 F_{t-1} + A_2 F_{t-2} + \ldots + A_p F_{t-P} + u_t \equiv A(L) F_{t-1} + u_t, \tag{3}$$

where F_{t-P} is the P^{th} lag of F_t and $A_0 \ldots A_P$ are matrices of structural parameters. u_t is a vector of structural shocks assumed to be uncorrelated with each other.

One challenge of estimating FAVAR is that factor and factor loadings are not separately identifiable, unless restrictions are applied. Bernanke *et al.* (2005) first apply the standard normalization restriction that sets the covariance matrix of the factors to be an identity matrix and then estimate a set of factors using the principal component analysis (PCA). These factors span the entire space of the variables in the model. However, there also exist infinite sets of "rotated" factors $\tilde{F}_t = H^{-1}\hat{F}_t$ that span the exact same space as the estimated set of factors $\hat{F}_t$ for a given nonsingular rotating matrix H. In other words, the estimated factors cannot retain clear economic interpretation. To overcome the problem of rotational indeterminacy, further restrictions must be applied on either the factors or the factor loadings.

Bai and Ng (2013) study conditions in which the estimated factors can avoid further rotational indeterminacy, i.e. H can be approximated to an identity matrix. In this paper, we adopt one type of restrictions in their paper, which the authors name as PC3. The factor loadings matrix takes the following form

$$\Lambda = \begin{pmatrix} I_K \\ \Lambda_2 \end{pmatrix}, \tag{4}$$

where Λ is the N-by-K matrix of factor loadings, I_K is the K-by-K identity matrix and Λ_2 is the (N - K)-by-K matrix.[8] To obtain the estimate of the factor loadings $\widehat{\Lambda}$, we first obtain the estimate $\widetilde{\Lambda}$ using PCA, and then calculate

$$\widehat{\Lambda} = \widetilde{\Lambda}\widetilde{\Lambda}_1^{\ -1},$$

where $\widetilde{\Lambda}_1$ represents the first K-by-K block of $\widetilde{\Lambda}$. In accordance with this, we further rotate the principal component estimate of the factor $\tilde{F}$ as

$$\hat{F} = \tilde{F}\widetilde{\Lambda}_1{}',$$

so that $X = \hat{F}\widehat{\Lambda}'$. Bai and Ng (2013) prove in their appendix that under certain regularity assumptions, the newly defined rotating matrix $H^{\dagger} \equiv H\widetilde{\Lambda}_1{}'$ that relates $\hat{F}$ and the unobserved macroeconomic factors F converges in probability to I_K.

Equation (4) requires each of the K factors be accompanied by at least one variable that is exclusively caused by this factor. We call these variables "*pure variables*". In practice, finding pure variables is not a straightforward task. Unlike in a model analysis where "output", "price", and "interest rate" all have a distinct economic interpretation, data series such as industrial production or consumer price index do not perfectly represent one concept. In addition, there may be many candidates within a given category. For example, choosing one representative "price" variable from many available candidates (e.g., Consumer price index: All items, Consumer price index: All items excluding food, Personal consumption expenditure price index) could be fairly difficult to justify. We propose a method that applies a search algorithm in

[8] The definition of factor loading here follows the notation of Bai and Ng (2013), which contains the time dimension through redefining the factors and observed data in matrix form instead of vector. For more details, see the original paper.

determining pure variables that is required in Bai and Ng (2013)'s PC3 method. Our method thus complements their method by making the factor identification empirically operational.

2.2 Overview of search algorithm

To identify pure variables, we use a graph-theoretic causal search algorithm.[9] First, we introduce some important terminologies. A detailed explanation of the algorithm can be found in Appendix A.

A *causal graph* represents a structural model with multiple equations. *Directed edges* illustrate direct causal relationships between two variables. For example, $X_A \rightarrow X_B$ shows that X_A causes X_B directly. This illustrates the situation of a structural equation in which X_B is the dependent variable and the regression coefficient of X_A is non-zero. A *directed path* from X_A to X_B exists if there are series of (at least one) directed edges from X_A to X_B such that all the arrows are pointing towards X_B. The same logic applies to the type of factor model that we considered, in which edges are directed from factors to variables but not from one variable to another. To illustrate this, suppose that a factor model as in Equation (1) has a specific causal structure as shown in Figure 1 Panel A.[10] The model has three unobserved factors ($K = 3$) and five observed variables ($N = 5$). The interpretation of the figure is that X_1 is caused by F_1, X_2 is caused by F_1 and F_2, X_3 is caused by F_2, and X_4 and X_5 are caused by F_3. F_1 is called a *common cause* of X_1 and X_2, F_2 is a common cause of X_2 and X_3, and F_3 is a common cause of X_4 and X_5. In this case, the factor loadings matrix is given as,

$$\Lambda = \begin{bmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & \alpha_3 & 0 \\ 0 & \alpha_4 & 0 \\ 0 & 0 & \alpha_5 \\ 0 & 0 & \alpha_6 \end{bmatrix},$$

[9] Detailed discussion of the graph-theoretic causal search methodologies can be found in Spirtes et al. (2000) and Pearl (2000).
[10] For simplicity, we assume that there is no dynamic structure as in Equation (3) and (4).

where the zero elements mean that there are no edges drawn between the corresponding factors and variables.[11]

The purpose of the causal search is to find the mapping between the causal graph and the probabilistic distribution of the data generating process. For our proposed method, the mapping is between the causal graph and conditional independence conditions. A set of conditional independence conditions is a collection of all conditional (i.e. conditioning on one or more variables) and unconditional (i.e. conditioning on an empty set) independencies among all variables. We use the mapping method based on Pearl (1988)'s *d-separation theorem*, which is standard in the literature.[12] Appendix A states the theorem in its entirety. The theorem is used to infer conditional independence conditions in a system of linear structural equations. It also implies the following

(1) Two variables are *dependent unconditionally* if there is a directed path either from X_A to X_B or from X_B to X_A.[13]

(2) When two variables share at least one common cause, they are dependent unconditionally.

(3) If both (1) and (2) above does not hold, X_A and X_B are *independent unconditionally*.

[11] In order to apply the identification scheme proposed by Bai and Ng (2013), we need to know which variables can be considered as "pure variables". If variables are reordered as X_1, X_3, X_4, X_2, and X_5, the factor loadings matrix would take the form consistent with the restrictions in equation (4). What is needed is the knowledge that X_1, X_3 and X_4 are pure variables. This is the purpose of our proposed search algorithm.

[12] Originally, Pearl (1988) studied only recursive structural equations. Spirtes (1995) and Koster (1996) independently illustrated that Pearl's d-separation theorem can also be extended to non-recursive structural equations.

[13] We assume the faithfulness assumption (Spirtes et al. 2000 p. 31) throughout the paper, which is a common assumption in the graph-theoretic causal search methodologies. A causal graph satisfies the faithfulness assumption if all conditional independence conditions entailed in the data generating process is consistent with those obtained from Pearl's d-separation theorem. However, it is important to note that there are examples where the faithfulness assumption is violated (see Sprites et al. 2000 p. 41; Pearl 2009 p. 62-63; Hoover 2001 p. 45-49, 151-153, 168-169).

According to (2), X_2 and X_3 in Figure 1 Panel A are dependent unconditionally because they share the same common cause. Likewise X_1 and X_2 are dependent unconditionally as F_1 is their common cause. However, X_1 and X_4 are independent unconditionally because neither (1) nor (2) applies.

With this in mind, a search procedure is constructed to search for all causal graphs that are consistent with the conditional independence conditions that are present in the data. It is possible that there is more than one causal graph representing the same conditional independence conditions (see *observational equivalence theorem* in Sprites *et al*., 2000 ch. 4). Figure 1 Panel B illustrates two causal graphs with the same conditional independence conditions among the observed variables (X_1 to X_5). Both graphs share the same conditional independence conditions in that, X_1, X_2, and X_3 are uncorrelated with X_4 and X_5. Some features (e.g., absence of an edge between two variables) are common to causal graphs. To find these features, we apply our identification method detailed next.

2.3 Implementation

Our proposed method utilizes the conditional correlation conditions estimated from data to identify which variable is exclusively generated by a single unobserved factor.[14] The method can be divided into three steps, i.e. (1) filtering variables, (2) finding the candidate for a pure variable (within a given category of variables), and (3) listing the pure variable through repeated sampling method. We summarize these steps in Table 1 and discuss them below.

2.3.1 Filtering Variables

The aim of filtering (steps 1a-c in the Table 1) is to eliminate the non-contemporaneous correlations that are caused by the dynamic structure of the model. Imagine two variables X_1 and

[14] We assume that the data generating process is linear which makes the tests of conditional correlation equivalent to the test of conditional independence.

X_2 that are *not* jointly caused by any single unobservable factor. These two variables should be contemporaneously uncorrelated with each other. However, these two variables could still become correlated over time through the dynamic structure in Equation (1) and (2). To see this more clearly, rewrite Equation (3) as follows:

$$F_t = A_0^{-1}A_1F_{t-1} + A_0^{-1}A_2F_{t-2} + \cdots + A_0^{-1}A_PF_{t-1} + A_0^{-1}u_t \qquad (5)$$

$$\equiv B_1F_{t-1} + B_2F_{t-2} + \cdots + B_PF_{t-P} + v_t \equiv B(L)F_{t-1} + v_{t.}$$

Substitute this into Equation (1) yields

$$X_{it} = \lambda_i'B_1F_{t-1} + \lambda_i'B_2F_{t-2} + \cdots + \lambda_i'B_PF_{t-P} + \lambda_i'v_t + \varepsilon_{it} \qquad (6)$$

$$\equiv D_1F_{t-1} + D_2F_{t-2} + \cdots + D_PF_{t-P} + \lambda_i'v_t + \varepsilon_{it},$$

where $D_p = \lambda_i{}'A_0^{-1}A_p$ for $p = 1$ to P. In Equation (6), the right hand side illustrates the dynamic structure of the system. Two variables X_1 and X_2 could be correlated with each other due to the past shocks that travel through time, even if they do not share the same unobserved factor contemporaneously (i.e. when $\lambda_i{}'A_0^{-1}$ is an identity matrix but A_1 to A_P are not). Therefore without the filtering process, the proposed causal search algorithm will not detect any information about the causal structure.[15]

In order to overcome this problem, we use factor-augmented regression (see Bai and Ng, 2006 and references therein). Since $F_{t\text{-}1}$ to $F_{t\text{-}P}$ are unobserved, we instead estimate $\hat{F}_{t-1}$ to $\hat{F}_{t-P}$ (by estimating $\hat{F}_t$ through PCA and lag it) which would span the spaces of $F_{t\text{-}1}$ to $F_{t\text{-}P}$. Then, we regress X_{it} on P lags of $\hat{F}_t$ and obtain the filtered variables $\tilde{X}_{it} \equiv X_{it} - \hat{X}_{it}$. The filtered variable can also be written as follows:

$$\tilde{X}_{it} = (\lambda_i{}'A_0^{-1})u_t + \varepsilon_{it} = \lambda_i'v_t + \varepsilon_{it} \qquad (8)$$

[15] If all variables are correlated over time but not through the same factors, and we do not apply the filtering, none of the pure variables can be identified since no edges will be removed in step 2b.

It can be seen that the filtered variables are not influenced by non-contemporaneous causal relationships in the system (i.e. from A_1 to A_P).[16] There are two elements that influence the correlations of the filtered variables. First is the factor loadings λ_i that encode how the unobserved factors cause the observed variables contemporaneously. The second is the A_0 matrix that encodes the contemporaneous causal relationship among the unobserved factors (F_t). By including multiple factors (e.g., selected based on some information criterion) with reasonable lag length (e.g., one year for monthly data) in the filtering process, we make sure that the non-contemporaneous correlations are eliminated.[17]

2.3.2 Finding the Candidate for a Pure Variable

Step 2a-d in Table 1 applies the search algorithm to find the candidate for a pure variable. The following proposition is important to understand how our proposed search algorithm identifies pure variables.

> **Proposition**: Let X_1 and X_2 be two distinct variables that are generated by Equation (1) to (4). If two filtered variables $\tilde{X}_1$ and $\tilde{X}_2$ are uncorrelated, then X_1 and X_2 do not share the same unobserved factor contemporaneously.

The proof of the proposition is presented in Appendix B. Here, we provide the intuition of this proposition. For simplicity, let A_0 be an identity matrix. Then, Equation (8) can be written as $\tilde{X}_{it} = \lambda'_i u_t + \varepsilon_{it}$. Notice the similarity between this equation and Equation (1). Let $\tilde{X}_1$ and $\tilde{X}_2$

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[16] Stock and Watson (2002) prove the consistency of the estimated parameters in a factor-augmented regression. We test conditional correlations by partitioned regression (a residual-based approach), which is equivalent to testing conditional correlations of the observed variables when lagged factors (estimated from PCA) are always included in the conditioning sets. Therefore, the result of Stock and Watson (2002) applies in our context as well.

[17] We note that the number of factors used to filter the variables does not have to exactly match the number of factors employed in identifying the factors. This is because the purpose the filtering process is to remove dynamic dependencies among variables rather than performing structural analysis. We will return to this point in the later empirical analysis.

be two distinct filtered variables associated with X_1 and X_2. If $\tilde{X}_1$ and $\tilde{X}_2$ are uncorrelated (contemporaneously) with each other, then X_1 and X_2 do not share the same unobserved factor (since λ_i' encodes the *exact* structural coefficients as well as zero restrictions used to generate X_{it}).

To see how we use steps 2a-d to search for pure variables, assume that Figure 1 Panel A is the data generating process. If we have access to the data generating process, we know that X_1 is a pure variable for F_1, X_3 is a pure variable for F_2, and X_4 and X_5 are pure variable for F_3. On the other hand, X_2 is not a pure variable as it is caused by more than one factor.

We begin in step 2a by allowing all variables to be connected with all other variables by undirected edges. This is equivalent to starting from the most general specification in which all variables are allowed to be caused by all other variables. With five observables, we begin the search with ten undirected edges shown in Figure 2 panel A (for N variables, there are a total of $N(N\text{-}1)/2$ edges).

Then, each edge is examined to determine whether the associated variables share any common factors or not. We examine all edges based on the elimination step of the PC algorithm as in Spirtes *et al*. (2000), which works efficiently for large datasets like ours.[18] The elimination step of the PC algorithm determines whether two variables are uncorrelated in any of the conditioning sets. When this happens (e.g., when two variables are uncorrelated unconditionally), it signifies that the two variables do not share any common cause (see the above proposition). We also remove an edge between the two variables. For testing conditional correlations, p-values are computed using Fisher's z-statistic.[19] We keep those edges that the method fails to remove under a pre-specified significance level (e.g., 0.05).

[18] The abbreviation "PC" stands for Peter and Clark, the first names of Spirtes and Glymour who invented this search algorithm.
[19] See Demiralp and Hoover (2003) footnote 8 for more discussion.

For the data generating process in Figure 1 Panel A, we observe that variables X_2 and X_3 are caused by the same unobservable factor F_2. This will necessarily make both $\tilde{X}_2$ and $\tilde{X}_3$ correlated with each other (for all sets of conditioning variables) through the common factor.[20] Consequently the edge between the two variables will be kept. Now, add another variable X_4 that is caused by a different factor F_3. In this case, variables $\tilde{X}_3$ and $\tilde{X}_4$ would be uncorrelated with each other and hence the edge between the two variables will be eliminated. For ten edges between five observables shown in Figure 2 panel A, seven edges can be removed. We illustrate this in Figure 2 panel B.

After the elimination step, variables are further grouped into "clusters" that are connected through edges. Figure 2 panel C shows that there are in total three clusters that can be formed (group 1: X_1 and X_2, group 2: X_2 and X_3, group 3: X_4 and X_5). Variables belonging to the same cluster are treated as sharing the same factors. We note that a variable that belongs to multiple clusters must be caused by more than one factor, thus it cannot be a pure variable. We drop these variables from the pool of candidates for pure variables. Figure 2 panel D shows the final result in which variable X_2 is excluded because it belongs to two clusters. Thus, we end up with X_1 as a pure variable for a factor, X_3 as another pure variable for *another* factor, and X_4 and X_5 as a set of pure variables for another factor. This is consistent with the data generating process in Figure 1 Panel A.

Our proposed method of finding pure variables closely follows the method developed by Silva *et al*. (2006). There are two main differences. First, Silva *et al.* (2006) assume an environment in which all observed variables share at least one similar unobserved factor. This implies that all variables are correlated with one another when conditioning on any set of

[20] This can also be illustrated based on the d-separation theorem of Pearl (1988). Two variables are correlated (or d-connected) since there is a directed path from the two variables through a common cause (which is the unobserved factor in this case).

variables, which does not help us in finding the pure variables based on correlation information alone. Instead, we assume that many variables can remain uncorrelated after conditioning on some set of variables. Second, Silva *et al*. (2006) only consider conditional correlation tests of the first order. We include higher-order conditional correlation tests as shown in Spirtes *et al*. (2000), which allow more than one variable to enter the conditioning set.[21] This makes the elimination process more robust.

2.3.3 Listing the Pure Variable Through Repeated Sampling Method

The purpose of the last step 3a-d is to identify pure variables with the highest degree of reliability. We apply the bootstrapping procedure proposed by Demiralp *et al*. (2008). Each bootstrap sample consists of randomly selected time series observations (with replacement) of the filtered variables $\tilde{X}_t$. Due to the randomness in our sample, the search algorithm could pick different variables as pure variables in a given bootstrap sample. The variables that are repeatedly chosen (i.e. with a high percentage of inclusion) are good candidates for pure variables, as a slight variation of the sample does not affect the conclusion from the search algorithm. But it is still possible that some of the pure variables might share a single common factor. When multiple candidates for pure variables exist, we choose the variable with the highest percentage of inclusion.

The last step is to use the selected pure variables as additional restrictions in equation (4) to identify the factor associated with the pure variable. Since this identification method avoids

[21] In principle, there are many ways we can do step 2b. Treating Equations (1) to (4) as the exact data generating process, we can conduct step 2b based on testing just the unconditional correlations among the filtered variables. However, we choose to adopt the PC algorithm of Spirtes et al. (2000), which consider testing conditional correlations as well. We believe this is more appropriate because i) it would encompass testing unconditional correlations alone, and ii) it allows for more parsimonious causal structure. For example, if the observed variables are also generated by other observed variables (i.e. not just the unobserved factors), then testing conditional correlations would be appropriate to eliminate an association among two variables. The PC algorithm is considered to be the most widely used graph-theoretic causal search algorithm (Demiralp and Hoover, 2003).

further rotational indeterminacy ($H \approx I_K$), the identified set of factors can be economically associated with the pure variable used in identification.

2.4 Cross-examining the Relevance of Pure Variables in Identifying Factors

It is important to note that our search algorithm does not guarantee that the variable with the highest percentage of inclusion is the variable exclusively caused by one macroeconomic factor. Consider a hypothetical example in which we add an *i.i.d.* random variable to the dataset which does not represent any meaningful macroeconomic factor. In a large sample, this variable should receive a percentage of inclusion of *hundred* percent based on our search algorithm, because it is not correlated with any of the variables in the dataset. To ensure that our search algorithm is selecting the relevant variable, we cross-examine the correlation coefficient between the selected pure variable and other (filtered) variables in the same macroeconomic category. If a pure variable does not correlate with any variables in the same category, it is also unlikely to share the same common factor with other variables. In this case, the variable will be discarded from our list of pure variables.[22]

One issue that may arise in empirical applications is that none of the variables within a given category is driven by a single factor but only jointly by multiple factors. In this case, all variables in a given category will have a *zero* percentage of inclusion and none of them can be used to identify factors. Therefore, it is important to include a large number of variables in each category so that the chance of finding a pure variable is maximized. In addition, it is useful to include disaggregated variables (e.g., IP: durable consumer goods within the output and income category) because disaggregated variables are less likely to be influenced by multiple factors

[22] For example, a pure variable that conceptually belong to the output category but is not correlated with the total industrial production index would not be a credible candidate for the output factor.

compared with aggregated variables. This would further increase the chance of finding pure variables.

3 Empirical Analysis

We apply our search algorithm to a monetary policy analysis using FAVAR. We compare our results with those in Bernanke *et al.* (2005, hereafter BBE).

3.1 Data

We use monthly data of the United States from January 1960 to December 2007. We select 124 variables that cover the major economic concepts and categorize them into eleven categories following BBE.[23] The choice of variables is fairly standard and many of them are used in macroeconomic factor analysis (Bernanke *et al.* 2005; Ludvigson and Ng 2009). Our dataset includes real output and income (18 variables), employment and hours (28 variables), consumption (5 variables), real inventory and orders (3 variables), housing starts and building permits (10 variables), money and credit quantity aggregates (11 variables), stock prices (8 variables), price indexes (19 variables), interest rates: level (9 variables), interest rates: spread (8 variables) and exchange rates (5 variables). Prior to applying the search algorithm, we remove outliers following the rule suggested by Stock and Watson (2005) so that variables with idiosyncratic behavior are not selected as pure variables.[24] The complete list of variables is shown in Appendix C.

Our first step is to filter these variables so that the non-contemporaneous correlations among variables are eliminated. We choose eleven principal components in the filtering process.

[23] The only change from BBE's categorization is the interest rate category. We distinguish levels and spread because these may capture different macroeconomic factors.
[24] We replace observations with absolute median deviations larger than six times the interquartile range with the median value of the preceding five observations.

This number is obtained by applying the Bai and Ng (2002)'s information criterion while choosing the maximum number of factors considered to twenty. The lag length is set to thirteen.[25]

3.2 Finding Pure Variables

The next step is to apply the search algorithm to the filtered variables. A total of 124 ×123/2 edges are initially drawn among the 124 variables. We keep those edges that the method fails to remove under the significance level of 0.05.[26] To ensure that our result is not sensitive to a particular sample, we apply the search algorithm onto 200 bootstrapped samples. For each sample, we record which edge is kept and which is eliminated.

Figure 3 visually summarizes how often the edges are kept within a given category for the 200 samples. To save space, we only present two categories that are of primary interest to macroeconomists, namely output/income and price indexes. Each symbol in the matrix shows how often the edge between two variables is kept. We separate our results into four categories using four symbols: edges that are kept in more than 75% of the 200 samples (filled circles), between 25%-75% of the samples (unfilled circles), between 0% -25% (dot), and 0% (blank). Many of the strong edges that are shown in filled circles tend to be drawn among variables that belong to a particular subcategory: for example, among the *producer* price index, three of the total of ten pairs of variables have edges not rejected in 75% of the samples (Panel B). This "block-diagonal" pattern is also observed in other categories that are not shown here.[27]

[25] This number is the same as the one in Bernanke et al. (2005). We experimented with different lag lengths, but the result is almost unaffected.
[26] This is the suggested value for the PC algorithm of Spirtes et al. (2000) based on simulation experiments with similar sample size. We also tried with other values of α (0.1, 0.025, and 0.01), but they did not affect the result in a significant way.
[27] This provides an informal support to our identification strategy in general, since the more the edges are concentrated within a group of variables, the higher the chance to detect a pure variable that is exclusively caused by a single factor.

Table 2 lists pure variables in three selected categories (output/income, price indexes, interest rates in levels), while the complete search result with 124 variables is delegated to appendix Table A.2. For example, within the output/income category, our search algorithm selects industrial production index for durable consumer goods (IP: Durable consumer goods) as the pure variable since it has the highest percentage of inclusion (28.0%) among the output/income variables.

Next we check the relevance of pure variables in identifying factors. Column (iii) of Table 2 shows that pure variables are highly correlated with conventional aggregate variables in each category. For example, the correlation coefficient of IP: Durable consumer goods (the identified pure variable) and total industrial production index (a conventional measure of aggregate production) is 0.65, indicating that IP: Durable consumer goods is relevant in identifying the output factor. We also check whether pure variables correlate with the remaining variables in their respective category and results are shown in Column (iv) of Table 2. We test the null hypothesis that the correlation coefficient is zero at the 5% significance level. If the hypothesis is rejected, we regard that as an evidence of correlation. We find that most of the variables are correlated with the pure variable in the same category, with only few exceptions.[28]

3.3 Monetary Policy Analysis

This subsection applies the search result to a monetary policy analysis using monetary FAVAR. To determine how many factors to identify, we rely on the criterion proposed by Hallin and Liska (2007). Based on their information criterion IC_1 and penalty function p_1, the criterion suggests using three structural factors. This number is reasonably close to what existing

[28] For example, within the price category three of the nineteen price indexes are not significantly correlated with the pure variable (PPI: Finished goods). However, this result was partially caused by how we grouped our price variables in the first place. If we treat PPI and CPI as two different categories, then PPI: Finished goods would be chosen as the pure variable in the first category, whereas CPI: All items less medical care would be chosen as the pure variable in the second category. Such distinction would increase the relevance of pure variable within the narrower subcategory.

studies on dynamic factor models have found to be appropriate for similar datasets.[29] We then select three pure variables from interest rate, price, and output categories, which are further used to identify factors. The joint behavior of these economic factors is of key interest to the monetary policymaker. To facilitate comparison with BBE, we follow their data transformation method. Further details of FAVAR are provided in Appendix D.

First, we impose identifying loading restrictions in Equation (4) in order to identify macroeconomic factors. The pure variables used in this process are the federal funds rate, PPI: Finished goods, and IP: Durable consumer goods. Each variable is selected from the categories of interest rates: level, price indexes, and output and income, respectively. The federal funds rate is used as the pure variable instead of the 3-month Treasury bill rate that our algorithm selects. It is a common practice in the literature to treat the orthogonal innovation to the federal funds rate as the monetary policy shock. In a separate analysis not shown here, we confirm that using the interest rate factor identified through the 3-month Treasury bill rate does not alter our results. For the other two variables, we simply choose the variable with the highest percentage of inclusion within the given category. In Appendix Table A.3, we show that the estimated factor is highly correlated with the variable used in identification.

Next, we examine how an orthogonal innovation to the federal funds rate affects the price and output factors identified using our method. Figure 4 shows the impulse responses of the factors to a 25 basis point increase in the federal funds rate. The dashed lines represent the 90 percent confidence interval calculated based on Kilian (1998) that applies bootstrapping on the VAR coefficients in the Equation (2). The interest rate factor shows a significantly positive response immediately after the policy shock. It returns to the original level within a year, and

[29] For example, Hallin and Liska (2007) suggest between one and four, depending on the sample period. Giannoni et al. (2005, 2006) use two factors, whereas Bai and Ng (2007) and Forni and Gambetti (2010) use four factors in their benchmark analysis.

stays below the original level thereafter. The last part is consistent with Forni and Gambetti (2010)'s argument that interest rate falls below the original level because the Federal Reserve would lower the rate in response to the lower output caused by the contractionary policy. The price factor shows a slightly positive (but not significant) response immediately after the shock, but then falls after one month and stays significantly below its original level for the next two years. Our identification method is successful in overcoming the "price puzzle" often seen in simple monetary VARs, i.e. a prolonged positive response of price to a contractionary monetary policy shock. The output factor falls significantly after the shock and recovers to its pre-shock level after twelve months.

Finally, we compare our results with those of BBE. As stated earlier in section 2, the main difference between the two approaches is that BBE focuses on the identification of space spanned by factors whereas ours aims to identify the individual factors that is associated with meaningful economic interpretation. Also when identifying the monetary policy shock, BBE remove the effects that the federal funds rate has on the extracted factors by pre-selecting a subset of variables that are "slow" in responding to the innovation in the federal funds rate, then run a regression to remove the effect of factors on the federal funds rate to assure that the innovation on the rate can be treated as an orthogonal shock. To facilitate comparison, we set the number of principal components extracted using BBE's method to two ($K = 2$), so that the total number of factors including the observed factor (=federal funds rate) becomes three, which is the same as our baseline specification.[30] Figure 5 shows the impulse responses of selected variables.

[30] We have also experimented with K = 3 for BBE (not shown), which coincides with what the authors call "preferred specification". Raising the number of principal component makes the price fall slightly faster, but it has no effect on the responses of output and interest rate variables.

The top panels show the responses of three interest rate measures. Here, the responses look relatively similar for all the selected measures. The middle panels show the responses of three price measures. In both models, prices eventually fall relative to the pre-shock level, but CPI: All items (mid-left panel) and CPI: All items less food (mid-center panel) did not fall almost the entire two years for the BBE whereas in our method both prices fall after six months. This difference in timing is important because clearly one of the main goals of a contractionary monetary policy is to stabilize the price level. The bottom panels of Figure 5 show the responses of three output measures. Although total industrial production index (IP, bottom-left panel) falls quickly in both models, the recovery path is very different between the two methods. In BBE it shows no sign of recovery even after two years whereas in our case IP starts to rise after one year. This is also more reasonable given that a typical recession does not last longer than two years.[31] In the case of BBE, capacity utilization rate (bottom-right panel) shows a mild fall that persists over a year, but in our model it falls sharply following the shock. It is more reasonable to have capacity utilization rate fall quickly because it is more consistent with the quick decline in the IP.

Overall, we conclude that our baseline FAVAR shows more economically plausible responses than the BBE, most notably the price responses.

3.4 Robustness Check

We check the robustness of the baseline result by first examining whether the choice of pure variables is affected by alternative model specifications or the sample period. Table 4 ranks the variables of three categories (output, price, interest rate) based on the percentage of inclusion. Column (i) shows the baseline specification. Column (ii) uses the variables filtered with three

[31] According to NBER, the average peak-to-trough duration during recessions that occured after 1960 was 11.6 months with the longest contraction observed from December 2007 to June 2009 (18 months).

principal components (“three factors analysis”) instead of eleven. We choose three so that the number of factors used in the filtering process exactly matches the number of factors used in the structural analysis, an approach that is implicitly adopted by many FAVAR studies including BBE. Column (iii) uses the post-1983 sample period instead of the full sample (“post 1983 analysis”) because many studies report a structural change in 1983.[32] We observe that the pure variables selected in the baseline specification also retain a high percentage of inclusion in alternative specifications. Within the given category, PPI: Finished goods is always ranked first in all three specifications, whereas IP: Durable consumer goods is ranked second in the three-factor analysis and ranked fifth in the post-1983 analysis.

Next, we examine whether our results in monetary FAVAR are sensitive to different choice of pure variables. We consider a case in which the researcher hand-selects pure variables instead of using algorithm-selected variables. Specifically, we assume that the researcher uses the total IP index and CPI: All items to represent output/income and price indexes, respectively. The first column of Figure 6 shows the impulse responses of the federal funds rate, CPI: All items, and the IP index to a 25 basis point innovation in the federal funds rate. While the federal funds rate and the IP index show similar responses to the baseline and alternative models in Figure 6, the CPI: All items falls much quicker in the alternative model with hand-selected variables. Adopting the hand-selected pure variables (instead of algorithm-selected variables) affects the speed and magnitude of the price response. The middle column of Figure 6 shows responses of the same set of variables when pure variables obtained from the shorter sample period (post-1983) were applied onto the full sample. Recall from Table 3 that the search with the subsample caused the pure variables for output/income to change. While the change in pure

[32] See Clarida et al. (1999); Estrella and Fuhrer (2003); Hoover and Jorda (2001); Demiralp et al. (2014) for more discussion.

variables itself may reflect structural changes, it has almost no effect on the monetary policy analysis conducted here, leading to a near-identical impulse responses in all three variables.

Finally, we consider a scenario in which the search is conducted for a limited number of variables (50 variables). This experiment is particularly important because with many variables in the search process, computing time rises very quickly.[33] We pre-select variables based on the FRED website's popularity index. The list consists of conventional aggregate variables and some of the disaggregated variables regarded as useful in forecasting macroeconomic variables by experts. The complete list of variables and the list of pure variables are shown in appendix Table A.1 and A.2. The right column of Figure 6 shows responses of the variables when the smaller dataset is used. The only variable visibly affected is the price, which falls quicker than the baseline. For industrial production index and federal funds rate the difference is less obvious, implying that working with small data is not too costly for the purpose of monetary policy analysis.

4 Conclusions

This paper proposes a new method that associates extracted factors with an economic interpretation. We show how to identify factors through a set of "pure variables", i.e. variables that are exclusively caused by a single underlying factor. In our method, each factor is economically related to a variable used in its identification. We conduct a monetary FAVAR analysis using the factors identified through pure variables. Our method yields different impulse responses in interest rates, price indexes, and output/income related variables from those of

[33] For the 124-variables case the total time to complete the search process is close to 100 hours, whereas with 50 variables the total time reduces to twelve hours.

Bernanke *et al.* (2005). We argue that the price and output responses are more reasonable in our approach.

There are several ways to apply our method to structural economic analyses. Using our method, factors can be treated as variables, which helps in identifying structural shocks other than the orthogonal shock to monetary policy by using conventional identification approaches commonly applied to small-scale VAR.[34] Another application is to use the estimated factors as observables in structural DSGE models. This could improve efficiency because factors are less subject to measurement errors.

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Appendix A: Graph-theoretic causal search methodologies

The objective of graph-theoretic causal search methodologies is to learn about causal structures from information regarding probabilistic distribution of the data generating process estimated from the data. This paper focuses mainly on a system of linear structural equations with independent errors. We also make a distinction that some variables in a system are observed while some are latent variables. A *causal graph* can be used to represent such a system. X is a *direct cause* of another variable Y (represented as $X \rightarrow Y$) in a causal graph G if and only if there is a non-zero coefficient associated with variable Y in the equation that X is the dependent variable (for example, an equation $X = \alpha Y + e$ with $\alpha \neq 0$ and e as a structural error term). Thus, a causal graph illustrates the underlying *zero* and *non-zero restrictions* in a system of linear structural equations. We also say that, in the above case, X is a *parent* while Y is a *child*.

A *directed path* from X to Y exists if and only if there is a series of directed edges (or a directed edge) pointing in the same direction from X to Y. In this case, we also say that X is an *ancestor* while Y is a *descendant*. By convention, every variable is its own ancestor. On the other hand, there is an *undirected path* from X to Y if and only if there is a series of directed edges (or a directed edge) from X to Y regardless of the direction implied by the arrow. A vertex is a "collider" in an undirected path if and only if there are two directed edges pointing into this vertex in the path (i.e. in a graph $X \rightarrow Y \leftarrow Z$, Y is a collider).[35] There are two types of collider;

[35] Based on an example given in Spirtes et al. (2000), engine's status (E) is a collider between the state of a car's battery (B, dead or not) and the state of a car's tank (T, with or no gas). Knowing that a car did not start, dead battery will tend to occur when the tank has some gas (and vice versa). Thus the two variables (B and T) become dependent conditioning on the common effect or E (i.e. the collider).

shielded and *unshielded* collider. For a shielded collider, the two directed edges are shielded by an edge. For an unshielded collider, the two directed edges aren't shielded.

Pearl (1988)'s *d-separation theorem* states the following

Pearl (1988)'s d-separation theorem (as presented in Spirtes *et al.* 2000 p. 44): Let **V** be a set of all variables of a causal graph **G** in which **X**, **Y**, and **Z** be distinct subsets of variables in **V**. **X** and **Y** are *d-separated* given **Z** (i.e. independent conditional on **Z)** if and only if, there is no undirected path U between **X** and **Y** such that i) every collider on U has a descendent in **Z**, and ii) no other variable on U is in **Z**; otherwise **X** and **Y** are *d-connected* given **Z** (i.e. dependent conditional on **Z)**.

For example, in a causal graph $X \rightarrow Y \rightarrow Z$, X and Z are dependent unconditionally but become independent conditional on Y since Y is other variable which is on the only undirected path from X to Y.

Appendix B: Proof of Proposition

Based on the d-separation theorem of Pearl, we provide the proof of the proposition in the text.

Restatement of Proposition: Let X_1 and X_2 be two distinct variables that are generated by Equation (1) to (4). If two filtered variables $\tilde{X}_1$ and $\tilde{X}_2$ are uncorrelated, then X_1 and X_2 do not share the same unobserved factor contemporaneously.

Proof of Proposition:

Part 1: If two filtered variables $\tilde{X}_1$ and $\tilde{X}_2$ are uncorrelated, then X_1 and X_2 are also uncorrelated contemporaneously. To verify this by contradiction, suppose that X_1 and X_2 are also correlated contemporaneously. Then, there must exist an undirected path which make X_1 and X_2 d-connected. Since $\tilde{X}_1$ and $\tilde{X}_2$ are also influenced by the same factor loadings and the A_0 matrix which encodes the contemporaneous causal relationship among the unobserved factors (F_t), then $\tilde{X}_1$ and $\tilde{X}_2$ must be correlated which is a contradiction.

Part 2: If X_1 and X_2 are uncorrelated contemporaneously, then X_1 and X_2 do not share the same unobserved factor contemporaneously. To verify this by contradiction, suppose that X_1 and X_2 share the same unobserved factor contemporaneously (i.e. sharing a common cause). By direct application of Pearl's d-separation theorem, X_1 and X_2 must be d-connected which is a contradiction.

Thus, if $\tilde{X}_1$ and $\tilde{X}_2$ are uncorrelated, then X_1 and X_2 do not share the same unobserved factor contemporaneously. Q.E.D.

Appendix C: Data

For the 124 variables case, series were taken from the Federal Reserve Economic Data (FRED) database, Federal Reserve Board (FRB), Bureau of Economic Analysis (BEA), U.S. Census (Census), Global Financial Database (GFD). The data source for individual series is shown in the last column of Table A.1. Prior to applying search algorithm, stationary transformations were applied based on the Augmented Dicky-Fuller unit root test with the optimal lag length chosen based on the Schwarz BIC. The type of transformation applied for individual series is shown in the third column of Table A.1. The number in the column represents the following transformation: 1 – no transformation; 2 – first difference; 4 – logarithm; 5 – first difference of logarithm; 6 – second difference of logarithm.

For the 50 variables case, all series were taken from the Federal Reserve Economic Data (FRED) database.[36] These variables have an asterisk next to the mnemonic in Table A.1. The number of variables is balanced among the following five broad categories so that the panel covers the major subset of the Federal Reserve's information set: output (8 variables), labor market (12 variables), expenditure (8 variables), nominal variables (11 variables) and financial market variables (11 variables). Within each concept, the individual variables were chosen based on the "popularity" of variable among the FRED users.

Appendix D: FAVAR estimation

To identify factors using our method we use three pure variables which are (a) IP: Durable consumer goods, (b) PPI: Finished goods, and (c) Effective federal funds rate. The first two variables are selected based on our search algorithm whereas the effective federal funds rate is chosen so that it represents the monetary policy measure that are widely accepted in the literature. When running the VAR with the factors, we recursively order them as (a)→(b)→(c) within the same period. Finally, the monetary policy shock in the first period is set so that it is equivalent to a 25 basis point increase in the policy rate.

[36] Downloaded on August 28th, 2012 from http://research.stlouisfed.org/fred2/

Table A.1: Description of Data: 124 Variables

Real output and income

1.	IPP	1960:01-2007:12	5	IP: Final products and nonindustrial supplies; s.a.	FRB
2.	IPFINAL	1960:01-2007:12	5	IP: Final Products (Market Group) ; s.a.	FRED
3.	IPCONGD*	1960:01-2007:12	5	IP: Consumer Goods; s.a.	FRED
4.	IPDCONGD*	1960:01-2007:12	5	IP: Durable Consumer Goods; s.a.	FRED
5.	IPNCONGD*	1960:01-2007:12	5	IP: Nondurable Consumer Goods ; s.a.	FRED
6.	IPBUSEQ*	1960:01-2007:12	5	IP: Business Equipment; s.a.	FRED
7.	IPNS	1960:01-2007:12	5	IP: Nonindustrial supplies; s.a.	FRB
8.	IPMAT*	1960:01-2007:12	5	IP: Materials; s.a.	FRED
9.	IPDMAT	1960:01-2007:12	5	IP: Durable Materials; s.a.	FRED
10.	IPNMAT	1960:01-2007:12	5	IP: Nondurable Materials; s.a.	FRED
11.	IPMAN	1960:01-2007:12	5	IP: Manufacturing (SIC); s.a.	FRED
12.	INDPRO*	1960:01-2007:12	5	IP Index; s.a.	FRED
13.	USASARTMISMEI	1960:01-2007:12	5	Total Retail Trade in United States	FRED
14.	MCUMFN	1960:01-2007:12	1	Capacity Utilization: Manufacturing (SIC); s.a.	FRED
15.	NAPM	1960:01-2007:12	1	ISM Manufacturing: PMI Composite Index; s.a.	FRED
16.	NAPMPI	1960:01-2007:12	1	ISM Manufacturing: Production Index	FRED
17.	DSPIC96*	1960:01-2007:12	5	Real Disposable Personal Income; s.a.a.r.	FRED
18.	W875RX1*	1960:01-2007:12	5	Real personal income excluding current transfer receipts ; s.a.a.r.	FRED

Employment and hours

19.	CE16OV*	1960:01-2007:12	5	Civilian Employment	FRED
20.	UNRATE*	1960:01-2007:12	1	Civilian Unempl. Rate	FRED
21.	UEMPMEAN*	1960:01-2007:12	2	Avg. (Mean) Dur. of Unempl.	FRED
22.	UEMPLT5*	1960:01-2007:12	2	Civilians Unempl. - Less Than 5 Weeks	FRED
23.	LNU03008756	1960:01-2007:12	2	# Unempl. for 5 to 14 Weeks	FRED
24.	LNU03008516	1960:01-2007:12	2	# Unempl. ^{3} 15 Weeks	FRED
25.	LNU03008876	1960:01-2007:12	1	# Unempl. for 15 to 26 Weeks	FRED
26.	LNU03008636	1960:01-2007:12	1	# Unempl. for 27 Weeks and over	FRED
27.	PAYEMS*	1960:01-2007:12	5	All Employees: Total nonfarm	FRED
28.	USPRIV*	1960:01-2007:12	5	All Employees: Total Private Industries	FRED
29.	USGOOD	1960:01-2007:12	5	All Employees: Goods-Producing Industries	FRED
30.	USMINE	1960:01-2007:12	5	All Employees: Mining and logging	FRED
31.	USCONS*	1960:01-2007:12	5	All Employees: Construction	FRED
32.	MANEMP	1960:01-2007:12	5	All Employees: Manufacturing	FRED
33.	DMANEMP	1960:01-2007:12	5	All Employees: Durable goods	FRED
34.	NDMANEMP	1960:01-2007:12	5	All Employees: Nondurable goods	FRED
35.	USTPU	1960:01-2007:12	5	All Employees: Trade, Transportation & Utilities	FRED
36.	USWTRADE	1960:01-2007:12	5	All Employees: Wholesale Trade	FRED
37.	USTRADE	1960:01-2007:12	5	All Employees: Retail Trade	FRED
38.	USFIRE	1960:01-2007:12	5	All Employees: Financial Activities	FRED

39.	SRVPRD	1960:01-2007:12	5	All Employees: Service-Providing Industries	FRED
40.	USGOVT	1960:01-2007:12	6	All Employees: Government	FRED
41.	AWHMAN*	1960:01-2007:12	1	Avg. Weekly Hours: Manufacturing	FRED
42.	AWOTMAN*	1960:01-2007:12	2	Avg. Weekly Overtime Hours: Manufacturing	FRED
43.	NAPMEI*	1960:01-2007:12	1	ISM Manufacturing: Employment Index	FRED
44.	CES2000000008*	1960:01-2007:12	5	Avg. Hourly Earnings: Construction	FRED
45.	CES3000000008*	1960:01-2007:12	5	Avg. Hourly Earnings: Manufacturing	FRED
46.	CES0600000008	1960:01-2007:12	5	Avg. Hourly Earnings: Goods-Producing	FRED
Consumption					
47.	PCE*	1960:01-2007:12	5	Real Personal Consumption Expenditures, Bil 05$, s.a.a.r. (deflate orig. vars by PCEPI)	FRED
48.	PCEDG*	1960:01-2007:12	5	Real PCE: Durable Goods, Bil 05$, s.a.a.r. (deflate orig. vars by PCEPI)	FRED
49.	PCEND*	1960:01-2007:12	5	Real PCE: Nondurable Goods, Bil 05$, s.a.a.r. (deflate orig. vars by PCEPI)	FRED
50.	PCES*	1960:01-2007:12	5	Real PCE: Services, Bil 05$, s.a.a.r. (deflate orig. vars by PCEPI)	FRED
51.	PCECARS	1960:01-2007:12	5	Real PCE: New Cars, Bil 05$, s.a.a.r. (deflate orig. vars by PCEPI)	BEA
Real inventory and orders					
52.	NAPMII*	1960:01-2007:12	1	ISM Manufacturing: Inventories Index (percent)	FRED
53.	NAPMNOI*	1960:01-2007:12	1	ISM Manufacturing: New Orders Index (percent)	FRED
54.	NAPMSDI*	1960:01-2007:12	1	ISM Manufacturing: Supplier Deliveries Index (percent)	FRED
Housing starts and building permits					
55.	HOUST*	1960:01-2007:12	5	Housing Starts: Total: Thous, s.a.	FRED
56.	HOUSTNE	1960:01-2007:12	4	Housing Starts in Northeast, Thous, s.a.	FRED
57.	HOUSTMW	1960:01-2007:12	4	Housing Starts in Midwest, Thous, s.a.	FRED
58.	HOUSTS	1960:01-2007:12	5	Housing Starts in South, Thous, s.a.	FRED
59.	HOUSTW	1960:01-2007:12	5	Housing Starts in West, Thous, s.a.	FRED
60.	PERMIT	1960:01-2007:12	5	Building Permits: Total, Thous, s.a.a.r.	FRED
61.	PermitNE	1960:01-2007:12	5	Building Permits in Northeast, Thous, s.a.a.r.	Census
62.	PERMITMW	1960:01-2007:12	4	Building Permits in Midwest, Thous, s.a.a.r.	Census
63.	PERMITS	1960:01-2007:12	5	Building Permits in South, Thous, s.a.a.r.	Census
64.	PERMITW	1960:01-2007:12	5	Building Permits in West, Thous, s.a.a.r.	Census
Money and credit quantity aggregates					
65.	M1SL*	1960:01-2007:12	5	M1 Money Stock	FRED
66.	M2SL*	1960:01-2007:12	5	M2 Money Stock	FRED
67.	RM2SL	1960:01-2007:12	5	Real M2 Money Stock (deflate orig. vars by PCEPI)	FRED
68.	CURRCIR	1960:01-2007:12	5	Currency in Circulation	FRED
69.	BOGAMBSL	1960:01-2007:12	5	Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements	FRED
70.	TRARR	1960:01-2007:12	5	Board of Governors Total Reserves, Adjusted for Changes in Reserve Requirements	FRED

71.	NONBORTAF*	1960:01-2007:12	5	Non-Borrowed Reserves of Depository Institutions Plus Term Auction Credit	FRED
72.	BUSLOANS*	1960:01-2007:12	5	Commercial and Industrial Loans at All Commercial Banks	FRED
73.	ΔBUSLOANS	1960:01-2007:12	2	Net Change in Commercial and Industrial Loans at All Commercial Banks (first diff. orig. vars)	FRED
74.	TOTALSL*	1960:01-2007:12	5	Total Consumer Credit Owned and Securitized, Outstanding	FRED
75.	CREDIT	1960:01-2007:12	2	Ratio, Nonrevolving Consumer Credit Owned and Securitized to Disposable Personal Income (%)	FRB
Stock prices					
76.	SP500*	1960:01-2007:12	5	S&P 500 Stock Price Index: Composite	FRED
77.	SP500IND	1960:01-2007:12	5	S&P 500 Stock Price Index: Industrials	GFD
78.	SP500KGOODS	1960:01-2007:12	5	S&P 500 Stock Price Index: Capital Goods	GFD
79.	SP500UTIL	1960:01-2007:12	5	S&P 500 Stock Price Index: Utilities	GFD
80.	SP500DYIELD	1960:01-2007:12	2	S&P 500 Monthly Dividend Yield (% per annum)	GFD
81.	SP500PERATIO	1960:01-2007:12	2	S&P 500 P/E Ratio	GFD
82.	DJIA*	1960:01-2007:12	5	Dow Jones Industrial Average	FRED
83.	DJUA*	1960:01-2007:12	5	Dow Jones Utility Average	FRED
Price indexes					
84.	NAPMPRI	1960:01-2007:12	1	ISM Manufacturing: Prices Index	FRED
85.	PPIFGS*	1960:01-2007:12	5	Producer Price Index: Finished Goods	FRED
86.	PPIFCG	1960:01-2007:12	5	Producer Price Index: Finished Consumer Goods	FRED
87.	PPIITM*	1960:01-2007:12	5	Producer Price Index: Intermediate Materials: Supplies & Components	FRED
88.	PPICRM*	1960:01-2007:12	5	Producer Price Index: Crude Materials for Further Processing	FRED
89.	CPIAUCSL*	1960:01-2007:12	6	Consumer Price Index for All Urban Consumers: All Items	FRED
90.	CPIAPPSL	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: Apparel	FRED
91.	CPITRNSL	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: Transportation	FRED
92.	CPIMEDSL	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: Medical Care	FRED
93.	CUSR0000SAC*	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: Commodities	FRED
94.	CUSR0000SAD	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: Durables	FRED
95.	CUSR0000SAS	1960:01-2007:12	6	Consumer Price Index for All Urban Consumers: Services	FRED
96.	CPIULFSL*	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: All Items Less Food	FRED
97.	CUSR0000SA0L2	1960:01-2007:12	5	Consumer Price Index for All Urban Consumers: All items less shelter	FRED
98.	CUSR0000SA0L5	1960:01-2007:12	6	Consumer Price Index for All Urban Consumers: All items less medical care	FRED
99.	PCEPI	1960:01-2007:12	6	Personal Consumption Expenditures: Chain-type Price Index	FRED
100.	PCEPID	1960:01-2007:12	5	Personal Consumption Expenditures: Chain-type price index: durable	BEA
101.	PCEPIND	1960:01-2007:12	5	Personal Consumption Expenditures: Chain-type price index: nondurable	BEA

102.	PCEPIS	1960:01-2007:12	6	Personal Consumption Expenditures: Chain-type price index: services	BEA
Interest rates: level					
103.	FEDFUNDS*	1960:01-2007:12	2	Effective Federal Funds Rate	FRED
104.	TB3MS*	1960:01-2007:12	2	3-Month Treasury Bill: Secondary Market Rate	FRED
105.	TB6MS*	1960:01-2007:12	2	6-Month Treasury Bill: Secondary Market Rate	FRED
106.	GS1*	1960:01-2007:12	2	1-Year Treasury Constant Maturity Rate	FRED
107.	GS5*	1960:01-2007:12	2	5-Year Treasury Constant Maturity Rate	FRED
108.	GS10*	1960:01-2007:12	2	10-Year Treasury Constant Maturity Rate	FRED
109.	AAA*	1960:01-2007:12	2	Moody's Seasoned Aaa Corporate Bond Yield	FRED
110.	BAA*	1960:01-2007:12	2	Moody's Seasoned Baa Corporate Bond Yield	FRED
111.	CPN3M	1960:01-2007:12	2	3-Month AA Nonfinancial Commercial Paper Rate	FRED
Interest rates: spread					
112.	TB3MS-FF	1960:01-2007:12	1	Spread: 3-Month TB - FF	FRED
113.	TB3MS-FF	1960:01-2007:12	1	Spread: 6-Month TB - FF	FRED
114.	GS1-FF	1960:01-2007:12	1	Spread: 1-Year Treasury Notes - FF	FRED
115.	GS5-FF	1960:01-2007:12	1	Spread: 5-Year Treasury Notes - FF	FRED
116.	GS10-FF	1960:01-2007:12	1	Spread: 10-Year Treasury Bond - FF	FRED
117.	AAA-FF	1960:01-2007:12	1	Spread: Moody's Aaa - FF	FRED
118.	BAA-FF	1960:01-2007:12	1	Spread: Moody's Baa - FF	FRED
119.	CPN3M-FF	1960:01-2007:12	1	Spread: 3-Month AA Nonfinancial CP - FF	FRED
Exchange rates					
120.	EXSZUS	1960:01-2007:12	5	Exchange Rate: Swiss Franc per U.S.$	FRED
121.	EXJPUS	1960:01-2007:12	5	Exchange Rate: Yen per U.S.$	FRED
122.	EXUSUK	1960:01-2007:12	5	Exchange Rate: Cents per Pound	FRED
123.	EXCAUS	1960:01-2007:12	5	Exchange Rate: Canadian $ per U.S.$	FRED
124.	TWEXMMTH	1960:01-2007:12	5	Trade Weighted U.S. Dollar Index: Major Currencies	FRED

Table A.2: Percentage of inclusion using alternative specifications

	(i)		(ii)	(iii)	(vi)	(v)
	Category / Variables		Baseline Analysis (in %)	Three Factors Analysis (in %)	Post 1983 Analysis (in %)	50 variables Analysis (in %)
1	Real output and income	IP: Final products and nonindustrial supplies; s.a.	2.50	5.00	**34.50**	
2		IP: Final Products (Market Group) ; s.a.	0.00	0.00	9.00	
3		IP: Consumer Goods; s.a.	0.00	0.00	8.00	1.00
4		IP: Durable Consumer Goods; s.a.	**20.00**	29.00	11.00	9.50
5		IP: Nondurable Consumer Goods ; s.a.	9.00	**30.00**	4.50	**43.50**
6		IP: Business Equipment; s.a.	11.00	19.50	9.50	26.50
7		IP: Nonindustrial supplies; s.a.	3.00	3.00	1.00	
8		IP: Materials; s.a.	2.00	0.50	2.00	2.50
9		IP: Durable Materials; s.a.	0.00	0.00	12.00	
10		IP: Nondurable Materials; s.a.	0.00	0.50	1.50	
11		IP: Manufacturing (SIC); s.a.	0.50	0.50	0.00	
12		IP Index; s.a.	15.00	9.00	3.00	0.00
13		Total Retail Trade in United States	9.00	20.50	12.00	
14		Capacity Utilization: Manufacturing (SIC); s.a.	0.00	0.00	0.00	
15		ISM Manufacturing: PMI Composite Index; s.a.	0.00	0.00	0.00	
16		ISM Manufacturing: Production Index	5.00	19.50	34.00	
17		Real Disposable Personal Income; s.a.a.r.	1.00	1.00	0.00	27.00
18		Real personal income excluding current transfer receipts ; s.a.a.r.	2.00	0.00	2.00	0.50
19	Employment, hours, and ave.hourly earnings	Civilian Employment	0.50	0.00	0.50	2.50
20		Civilian Unempl. Rate	0.00	0.00	2.50	0.50
21		Avg. (Mean) Dur. of Unempl.	0.00	0.00	0.00	13.50
22		Civilians Unempl. - Less Than 5 Weeks	0.50	1.00	0.00	2.00
23		# Unempl. for 5 to 14 Weeks	0.50	0.00	2.50	
24		# Unempl. ³ 15 Weeks	0.00	0.00	0.00	
25		# Unempl. for 15 to 26 Weeks	0.00	0.00	0.00	
26		# Unempl. for 27 Weeks and over	0.00	0.00	0.00	
27		All Employees: Total nonfarm	2.50	6.50	**56.50**	**54.00**
28		All Employees: Total Private Industries	2.50	5.50	3.50	0.00
29		All Employees: Goods-Producing Industries	0.00	0.50	20.00	
30		All Employees: Mining and logging	3.00	1.00	6.50	
31		All Employees: Construction	0.00	0.00	0.50	0.00
32		All Employees: Manufacturing	10.50	9.00	6.00	
33		All Employees: Durable goods	0.00	0.00	9.00	
34		All Employees: Nondurable goods	4.50	3.50	3.50	

35		All Employees: Trade, Transportation & Utilities	0.00	0.00	0.50	
36		All Employees: Wholesale Trade	0.00	0.50	0.50	
37		All Employees: Retail Trade	**22.50**	**10.00**	5.00	
38		All Employees: Financial Activities	1.00	0.00	0.50	
39		All Employees: Service-Providing Industries	0.50	0.00	1.00	
40		All Employees: Government	1.00	9.00	5.50	
41		Avg. Weekly Hours: Manufacturing	0.00	0.00	0.00	0.50
42		Avg. Weekly Overtime Hours: Manufacturing	0.00	0.00	4.00	0.00
43		ISM Manufacturing: Employment Index	0.00	0.00	2.50	1.50
44		Avg. Hourly Earnings: Construction	0.00	7.00	1.00	6.50
45		Avg. Hourly Earnings: Manufacturing	0.00	0.00	0.50	1.00
46		Avg. Hourly Earnings: Goods-Producing	17.50	7.00	28.50	
47	Consumption	Real Personal Consumption Expenditures, Bil 05$, s.a.a.r.	0.00	0.00	0.00	7.50
48		Real PCE: Durable Goods, Bil 05$, s.a.a.r.	0.50	0.00	0.00	19.00
49		Real PCE: Nondurable Goods, Bil 05$, s.a.a.r.	**12.00**	7.50	3.00	**31.00**
50		Real PCE: Services, Bil 05$, s.a.a.r.	0.00	0.00	2.50	11.00
51		Real PCE: New Cars, Bil 05$, s.a.a.r.	2.50	**10.00**	**15.00**	
52	Real inventories	ISM Manufacturing: Inventories Index (percent)	**6.50**	0.00	2.00	**0.50**
53	and orders	ISM Manufacturing: New Orders Index (percent)	0.00	0.00	**9.50**	0.00
54		ISM Manufacturing: Supplier Deliveries Index (percent)	0.00	0.00	0.00	**0.50**
55	Housing starts	Housing Starts: Total: Thous, s.a.	0.00	0.00	0.00	2.50
56		Housing Starts in Northeast, Thous, s.a.	0.00	0.00	0.00	
57		Housing Starts in Midwest, Thous, s.a.	0.00	0.00	0.00	
58		Housing Starts in South, Thous, s.a.	3.50	**18.00**	0.00	
59		Housing Starts in West, Thous, s.a.	0.50	0.50	9.00	
60		Building Permits: Total, Thous, s.a.a.r.	0.00	0.00	0.00	
61		Building Permits in Northeast, Thous, s.a.a.r.	**4.50**	5.00	1.50	
62		Building Permits in Midwest, Thous, s.a.a.r.	0.00	0.00	0.00	
63		Building Permits in South, Thous, s.a.a.r.	1.50	4.50	**18.50**	
64		Building Permits in West, Thous, s.a.a.r.	0.50	4.50	2.50	
65	Money and credit	M1 Money Stock	0.00	0.00	0.50	0.00
66	quantity aggregates	M2 Money Stock	0.00	0.00	0.50	0.50
67		Real M2 Money Stock	1.50	0.00	**4.00**	
68		Currency in Circulation	0.50	0.00	0.00	
69		Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements	0.00	0.00	2.50	

70		Board of Governors Total Reserves, Adjusted for Changes in Reserve Requirements	0.00	0.00	2.00	
71		Non-Borrowed Reserves of Depository Institutions Plus Term Auction Credit	**3.00**	4.00	0.50	**2.00**
72		Commercial and Industrial Loans at All Commercial Banks	2.50	2.00	3.00	0.50
73		Net Change in Commercial and Industrial Loans at All Commercial Banks	2.00	**5.50**	1.50	
74		Total Consumer Credit Owned and Securitized, Outstanding	0.00	0.00	1.00	1.50
75		Ratio, Nonrevolving Consumer Credit Owned and Securitized to Disposable Personal Income (%)	0.00	0.00	0.00	
76	Stock prices	S&P 500 Stock Price Index: Composite	0.00	0.50	4.50	2.50
77		S&P 500 Stock Price Index: Industrials	4.00	6.00	2.50	
78		S&P 500 Stock Price Index: Capital Goods	**71.00**	**64.00**	**50.50**	
79		S&P 500 Stock Price Index: Utilities	0.00	0.00	0.50	
80		S&P 500 Monthly Dividend Yield (% per annum)	0.00	0.00	0.00	
81		S&P 500 P/E Ratio	0.50	3.00	0.50	
82		Dow Jones Industrial Average	27.00	15.50	16.00	**17.00**
83		Dow Jones Utility Average	0.00	0.00	0.50	0.00
84	Price indexes	ISM Manufacturing: Prices Index	0.00	0.00	0.00	
85		Producer Price Index: Finished Goods	**5.00**	**3.00**	**63.00**	0.50
86		Producer Price Index: Finished Consumer Goods	0.50	0.50	6.50	
87		Producer Price Index: Intermediate Materials: Supplies & Components	0.00	0.00	0.00	0.00
88		Producer Price Index: Crude Materials for Further Processing	0.50	1.00	0.00	2.00
89		Consumer Price Index for All Urban Consumers: All Items	0.00	0.50	9.50	**9.00**
90		Consumer Price Index for All Urban Consumers: Apparel	1.00	1.50	5.00	
91		Consumer Price Index for All Urban Consumers: Transportation	0.00	0.00	13.50	
92		Consumer Price Index for All Urban Consumers: Medical Care	1.00	2.00	2.00	
93		Consumer Price Index for All Urban Consumers: Commodities	0.00	1.00	0.00	0.00
94		Consumer Price Index for All Urban Consumers: Durables	0.50	0.50	2.50	
95		Consumer Price Index for All Urban Consumers: Services	0.00	0.00	1.50	
96		Consumer Price Index for All Urban Consumers: All Items Less Food	0.00	0.00	1.50	7.00
97		Consumer Price Index for All Urban Consumers: All items less shelter	0.50	**3.00**	0.00	
98		Consumer Price Index for All Urban Consumers: All items less medical care	4.50	**3.00**	1.00	
99		Personal Consumption Expenditures: Chain-type Price Index	0.00	0.00	0.00	
100		Personal Consumption Expenditures: Chain-type price index: durable	0.00	0.00	2.50	
101		Personal Consumption Expenditures: Chain-type price index: nondurable	0.00	0.00	2.00	

102		Personal Consumption Expenditures: Chain-type price index: services	0.00	0.00	0.00	
103	Interest rates (level)	Effective Federal Funds Rate	0.00	1.00	12.50	7.00
104		3-Month Treasury Bill: Secondary Market Rate	**32.00**	**25.00**	15.50	**34.50**
105		6-Month Treasury Bill: Secondary Market Rate	13.50	11.50	3.00	33.50
106		1-Year Treasury Constant Maturity Rate	0.00	0.00	0.50	0.00
107		5-Year Treasury Constant Maturity Rate	7.50	10.00	3.50	10.00
108		10-Year Treasury Constant Maturity Rate	1.00	1.50	3.00	2.50
109		Moody's Seasoned Aaa Corporate Bond Yield	8.00	6.50	0.50	5.50
110		Moody's Seasoned Baa Corporate Bond Yield	11.50	8.00	**35.00**	18.00
111		3-Month AA Nonfinancial Commercial Paper Rate	3.00	0.00	3.50	
112	Interest rates (spread)	Spread: 3-Month TB - FF	0.00	3.50	3.00	
113		Spread: 6-Month TB - FF	17.50	**4.00**	0.00	
114		Spread: 1-Year Treasury Notes - FF	0.00	0.00	0.00	
115		Spread: 5-Year Treasury Notes - FF	**22.00**	0.00	**16.00**	
116		Spread: 10-Year Treasury Bond - FF	0.00	2.00	2.00	
117		Spread: Moody's Aaa - FF	1.50	0.00	1.00	
118		Spread: Moody's Baa - FF	11.00	1.50	0.00	
119		Spread: 3-Month AA Nonfinancial CP - FF	0.00	1.00	0.00	
120	Exchange rates	Exchange Rate: Swiss Franc per U.S.$	8.50	10.50	10.50	
121		Exchange Rate: Yen per U.S.$	**17.50**	20.00	11.50	
122		Exchange Rate: Cents per Pound	17.00	**28.00**	**18.50**	
123		Exchange Rate: Canadian $ per U.S.$	0.00	0.00	1.00	
124		Trade Weighted U.S. Dollar Index: Major Currencies	0.00	0.50	0.00	
	Average		3.84	4.03	5.72	8.36

Note: Percentage of inclusion is calculated from 200 bootstrapped samples in step 3 of the search algorithm. The variables with bold numbers are those with the highest percentage in each category

Table A.3: Pairwise correlations of the estimated factors and the variables used in identification

	Factor 1 ("output and income")	Factor 2 ("Prices")	Factor 3 ("Interest rates")
IP: Durable consumer goods	**0.57**	-0.11	-0.14
PPI: Finished goods	-0.12	**0.60**	0.51
Effective federal funds rate	-0.23	0.77	**0.92**

Note: The variables used in identifying factors in the baseline are as follows: Industrial production (IP): Durable consumer goods → Factor 1, PPI: Finished goods → Factor 2, Federal funds rate →Factor 3. IP Durable goods and PPI: Finished goods are variables that have the highest percentage of inclusion in the output and income category, and the price category, respectively.

Table 1: Search algorithm in steps

Step 1: filtering the variables (a) From the dataset $X_t=(X_{1t}, X_{2t}, \ldots, X_{Nt})'$, t=*1,…,*T, obtain K factors using the Principal Component Analysis (PCA) (b) Regress each X_{it} on P lags of $\hat{F}_t$. Then calculate the fitted value as follows: $\hat{X}_{it} = c_{i1}\hat{F}_{t-1} + c_{i2}\hat{F}_{t-2} + \ldots + c_{iP}\hat{F}_{t-P} \equiv C(L)\hat{F}_{t-1}$ (c) Compute the residuals $\tilde{X}_{it} = X_{it} - \hat{X}_{it}$. Call them filtered variables.
Step 2: finding the candidate for pure variable within a given sample (a) Let all observed variables be connected with undirected edges to one another in a causal graph. (b) For each pair of filtered variables that is uncorrelated conditioning on a set of filtered variables, remove the associated undirected edge in the graph. (c) Form all clusters of variables such that a cluster consists of variables that are all connected with one another by undirected edges in the graph. (d) Remove all observed variables that belong to more than one cluster.
Step 3: listing the pure variable through repeated sampling method (a) Generate multiple bootstrap replications $\tilde{X}^*_{it}$ for the filtered variables $\tilde{X}_{it}$. (b) For each bootstrap sample, apply the search algorithm in step 2. Record the variables that are chosen as pure variables. (c) After applying the method for the entire bootstrap sample, order the variables according to the percentage of inclusion in the list of pure variables. (d) Examine the pairwise correlation between the variables listed in 3c. If two variables are highly correlated, discard the lower-ranked variable. Continue the process until K variables are chosen as the candidates for pure variables.

Table 2: Selected pure variables

(i) Category	(ii) Pure variables	(iii) Selected variables and its correlation coeff. (in parenthesis) with the pure variable	(iv) # of variables (same category) correlated with the pure variables
Output and income	IP: Durable consumer goods	IP index (0.65), IP: Final products (Market group) (0.75), Capacity utilization: Manufacturing (0.19)	18 out of 18
Price indexes	PPI: Finished Goods	CPI: All items (0.47), CPI: All items less food (0.36), PPI: Crude materials (0.56)	16 out of 19
Interest rates: level	3-month Treasury bill rate	Effective federal funds rate (0.48), 10-year Treasury bond yield (0.45), Baa corporate bond yield (0.30)	9 out of 9

Note: In column (ii), a variable is selected as the pure variable if the percentage of inclusion is the highest among the variables in the same category of column (i). Column (iv) is based on a hypothesis test that the absolute value of the correlation coefficient between a given variable and the corresponding pure variable is zero. If the null hypothesis is rejected, then we regard them as correlated.

Table 3: Output, price, and interest rate variables, ranked by the percentage of inclusion

(a) Real Output and Income (total: 18 variables)

Ranking	(i) Baseline	(ii) Three Factors Analysis	(iii) Post 1983 Analysis
#1 (=pure variable)	**IP: Durable Consumer Goods**	IP: Nondur. Cons. Goods	IP: Final products and nonindustrial supplies
#2	IP Index	**IP: Durable Consumer Goods**	ISM Manufacturing: Production Index
#3	IP: Business Equipment	Total Retail Trade in US	IP: Durable Materials
#4	IP: Nondur. Cons. Goods	IP: Business Equipment	Total Retail Trade in US
#5	Total Retail Trade in US	ISM Manufacturing: Production Index	**IP: Durable Consumer Goods**

(b) Price indexes (total: 19 variables)

Ranking	(i) Baseline	(ii) Three Factors Analysis	(iii) Post 1983 Analysis
#1 (=pure variable)	**PPI: Finished Goods**	**PPI: Finished Goods**	**PPI: Finished Goods**
#2	CPI: All items less med. care	CPI: All items less shelter	CPI: Transportation
#3	CPI: Apparel	CPI: All items less med. care	CPI: All Items
#4	CPI: Medical Care	CPI: Medical Care	PPI: Finished Cons. Goods
#5	PPI: Finished Cons. Goods, PPI: Crude Materials, CPI: Durables, CPI: All items less shelter	CPI: Apparel	CPI: Apparel

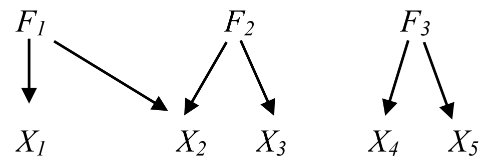
(c) Interest rates: level (total: 9 variables)

Ranking	(i) Baseline	(ii) Three Factors Analysis	(iii) Post 1983 Analysis
#1 (=pure variable)	**3-Month Treasury bill rate**	**3-Month Treasury bill rate**	Baa corporate bond yield
#2	6-Month Treasury bill rate	6-Month Treasury bill rate	**3-Month Treasury bill rate**
#3	Baa corporate bond yield	5-Year Treasury bond rate	Effective Federal Funds Rate
#4	Aaa corporate bond yield	Baa corporate bond yield	5-Year Treasury Rate
#5	5-Year Treasury bond rate	Aaa corporate bond yield	3-Month AA Nonfinancial Commercial Paper rate

Note: The three variables in each column are ranked based on the percentage of inclusion among the variables in the same category of (a) real output and income, (b) price indexes, and (c) interest rates: level. Post 1983 Analysis covers from January 1983 to December 2007. For the complete result, see Appendix Table A.2.

Figure 1: Illustration of A FAVAR model

Panel A: A typical FAVAR model



Panel B: Two FAVAR models with the same conditional independence conditions (of X_1 to X_5)

Model A Model B

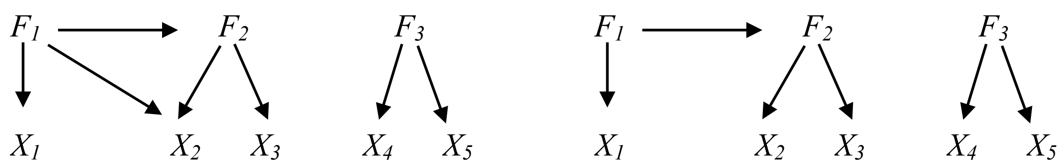
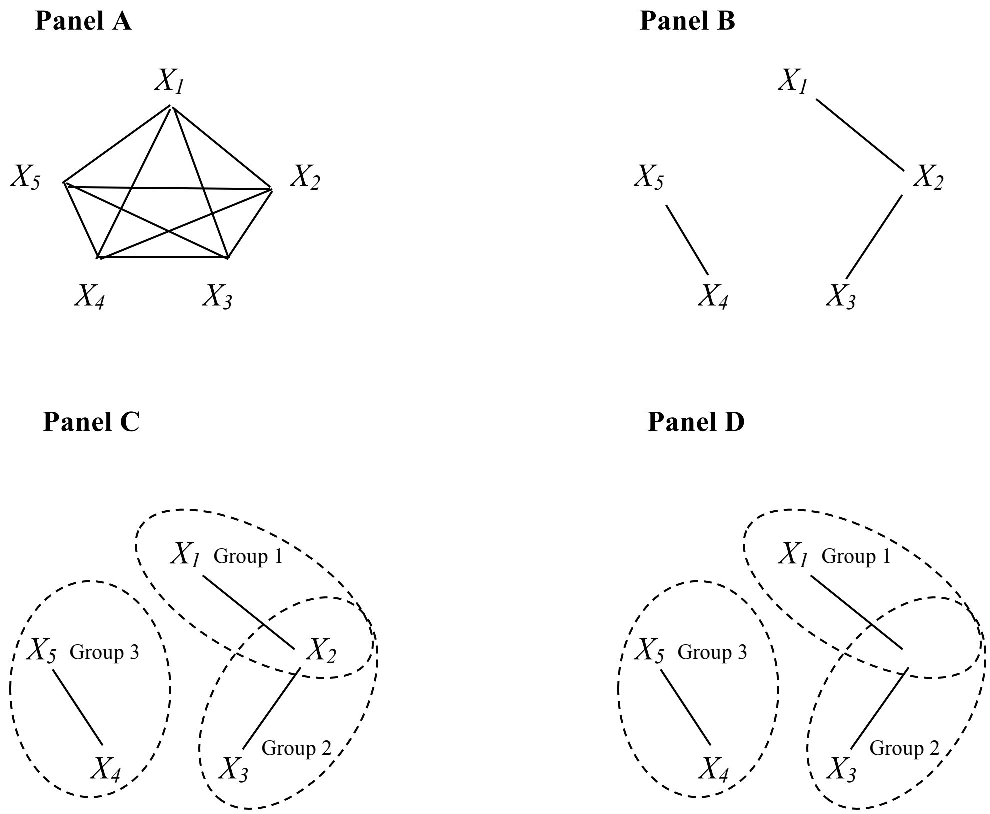


Figure 2: Illustration of steps 2a-d of the proposed search algorithm to find pure variables



Note: The above panels A-D correspond to steps 2a-d described in Table 1.

Figure 3: Edges kept among variables within selected categories

(a) Output/income

	Industrial Production (IP)												Other measures				Income	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1. IP: Final Products	-	○		·		·	·	·	·		·	○		·				
2. IP: Final Products: (market group)	○	-	●	·	○	·		·		·	·	○						
3. IP: Consumer Goods		●	-		●	○	·		·		·	·						
4. IP: Durable Cons. Goods (pure variable)	·	·		-		·		·	·		○							
5. IP: Nondurable Cons. Goods		○	●		-	·			·		·	·		·				
6. IP: Business Equipment	·	·	○	·	·	-			·		●	·		·				○
7. IP: Nonindustrial Supplies	·		·				-	·	·	○	·	○	·	○			·	
8. IP: Materials	·	·		·			·	-	·	●	○	●	·				·	·
9. IP: Durable Materials	·		·	·	·	·	·	·	-		●	·				·		
10. IP: Nondurable Materials		·					○	●		-	○	·	·	·		·	·	
11. IP: Manufacturing (SIC)	·	·	·	○	·	●	·	○	●	○	-	·						·
12. IP index	○	○	·		·	·	○	●	·	·	·	-		·				
13. Total Retail Trade in US							·	·		·			-	·				·
14. Capacity Utilization: Manufacturing	·				·	·	○			·		·	·	-				
15. ISM Manufacturing: PMI Composite Index															-	●		·
16. ISM Manufacturing: Production Index									·	·					●	-		
17. Real Disposable Income							·	·		·							-	●
18. Real Personal Income (excl. transfer)						○		·			·		·		·		●	-

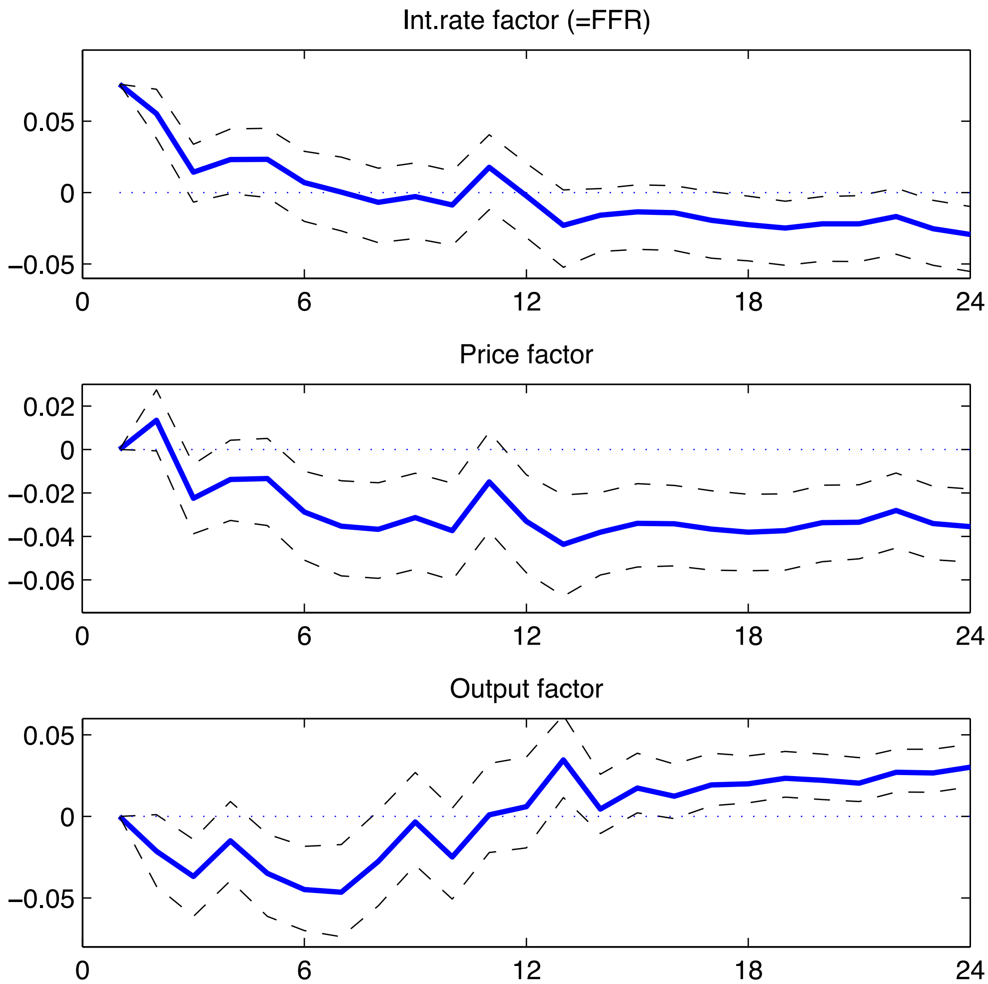
Figure 3: Edges kept among variables within selected categories (continued)

(b) Price Indexes

	Producer Price Index (PPI)					Consumer Price Index (CPI) / Personal Consumption Expenditure Price Index (PCEPI)													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1. ISM Manufacturing: Prices Index	–			●	·		·			·		·	·	·	·		·		
2. PPI: Finished Goods (pure variable)		–	●	·	·								·	·			·	·	
3. PPI: Finished Consumer Goods		●	–	●	·			·						·	·	·		○	
4. PPI: Intermediate Materials	●	·	●	–	○							·	·	○				·	
5. PPI: Crude Materials	·	·	·	○	–			·				·		·	·	·			·
6. CPI: All Items						–			·	●			○		●	·			
7. CPI: Apparel	·						–		·			·	○			·		·	
8. CPI: Transportation			·		·			–		·	·	·	·	·	○			●	·
9. CPI: Medical Care						·	·		–		·		·						·
10. CPI: Commodities	·					●		·		–			·	●				●	
11. CPI: Durables								·	·		–	·	·	·	·		●	·	
12. CPI:Services	·			·	·		·	·			·	–	○				·		●
13. CPI: All Items Less Food	·	·		·		○	○	·	·	·	·	○	–	●	○	·		·	·
14. CPI: All Items Less Shelter	·	·	·	○	·			·		●	·		●	–	●	●		·	
15. CPI: All Items Less Medical Care	·		·		·	●		○			·		○	●	–	·		·	
16. PCE Price Index (PCEPI)			·		·	·	·						·	●	·	–	●	·	●
17. PCEPI: Durable	·	·									●	·				●	–		
18. PCEPI: Nondurable		·	○	·			·	●		●	·		·	·	·	·		–	
19. PCEPI: Services					·			·	·			●	·			●			–

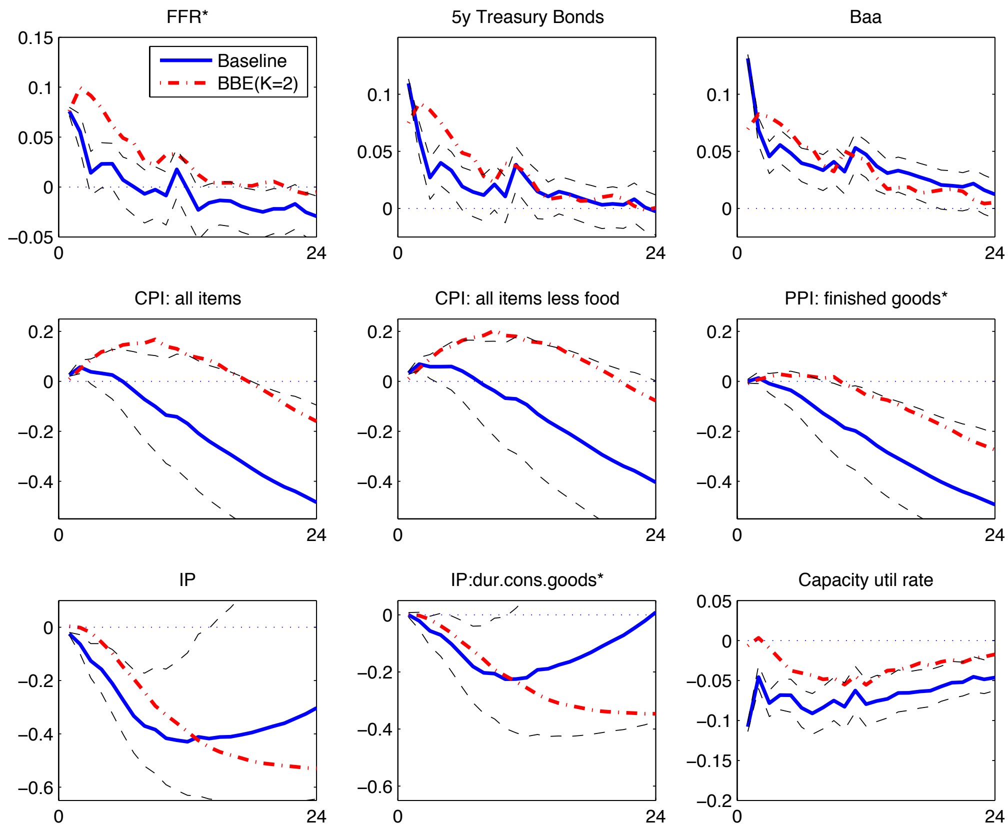
Note: The categorization of variables is based on Bernanke *et al.* (2005). The order of variables follows the order in Table A.1. A cell with filled circle means that the particular edge is kept in more than 75% of the total samples, an unfilled circle means that the edge is kept for 25% to 75% of the samples, a dot means that the edge is kept for more than 0% to less than 25% of the samples, and a blank means that there is no edge kept in the sample.

Figure 4: Impulse responses of factors to a shock in the monetary policy measure (baseline)



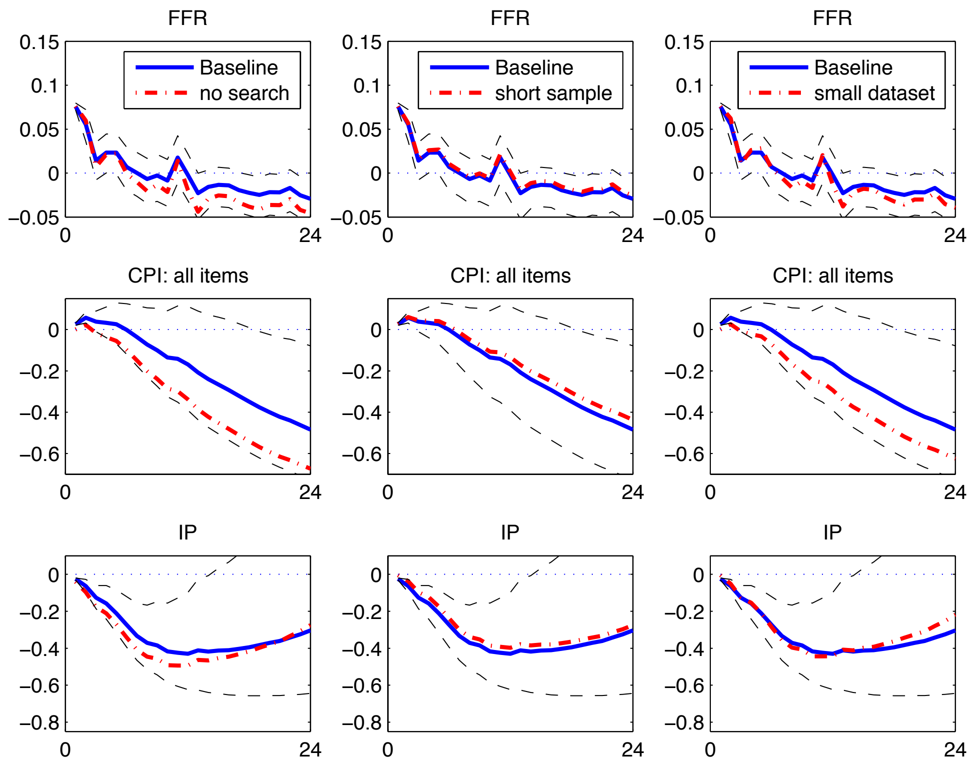
Note : The solid line represents the responses and the dashed line represents the 90 percent confidence interval calculated based on Kilian (1998). Sample period is from Jan. 1960 to Dec. 2007.

Figure 5: Comparison with BBE: Impulse response of selected variables



Note : The solid line represents the responses obtained through the authors' method ("Baseline") and the dashed line represents the 90 percent confidence interval calculated based on Kilian (1998). The dash-dotted line represents the responses obtained through Bernanke *et al.* (2005)'s method ("BBE"). The variables with an asterisk are the pure variables used in identifying factors for the baseline case. Sample period is from Jan. 1960 to Dec. 2007.

Figure 6: Impulse responses under alternative specifications



Note : The solid line represents the responses obtained through the authors' method ("Baseline") and the dashed line represents the 90 percent confidence interval calculated based on Kilian (1998). The pure variables used in identifying factors for the baseline case are (a) IP: Durable consumer goods, (b) PPI: Finished goods, and (c) Effective federal funds rate. The dash-dotted responses in the left panels are calculated by replacing the (a), (b) in the baseline with the total IP index and CPI: All items, respectively. The dash-dotted responses in the center panels are calculated by replacing (a) in the baseline with IP: final products and nonindustrial supplies obtained from the shorter sample period (1983-2007). The dash-dotted responses in the right panels are calculated by replacing (a), (b) in the baseline with IP: nondurable consumer goods and CPI: All items obtained from the smaller dataset (50 variables). Sample period is from Jan. 1960 to Dec. 2007.