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Takeshi Yagihashi
Old Dominion University

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Abstract

This paper discusses whether the parameter invariance problem as in Lucas (1976) applies to the standard new Keynesian DSGE model when the credit channel is left out from its structure. We simulate a financial crisis in which the credit market friction is positive and we shift the monetary policy to stimulate the economy. We evaluate the cost of omitting the credit channel by examining the changes of the estimated model parameters and by using policy outcomes. We find that although some parameters incur nontrivial changes after the policy shift, overall these parameter changes have little impact on the conduct of monetary policy.

Keywords: DSGE model, Lucas Critique, Bayesian estimation, Financial Accelerator model

*Economics Department, 2021 Constant Hall, Norfolk, VA 23529; tyagihas@odu.edu; Tel.+1(757)683-3512; Fax.+1(757)683-5639.

1 Introduction

Lucas (1976) once said that any model should be "structural" so that the hypothetical policy shift does not trigger any spurious shift in model parameters, otherwise the model does not serve well in policy discussions. Soon after, the analytical tool known as the Dynamic Stochastic General Equilibrium (DSGE) model was developed and has become the workhorse for analyzing the aggregate economy. Unlike previous generations of models, its behavioral equations characterizing model variables are derived from an optimizing principle, and further pinned down by structural parameters that describe preferences and technology of agents. Today, many researchers have come to embrace DSGE models as a practical solution to the Lucas' above critique and many central banks have developed their own versions of the DSGE model that closely resemble their national economy.

The financial crisis in 2008 has brought fresh criticism to DSGE models for the lack of "credit channel". The credit channel here refers to monetary policy transmission process through the supply of loanable funds. Prior to the financial crisis, it was customary for central banks to introduce the credit channel in an informal manner, in which arbitrary dynamic equations served as "proxies for short-run effects such as credit constraints, house price effects, confidence and accelerator effects" (Harrison *et al.*, 2005).[1] After the financial crisis, many central banks have started to incorporate the credit channel into its core structure. Such refinement was largely regarded as a positive move because these models can explicitly analyze the workings of the credit channel providing accuracy to policy discussions. But there are also skepticism on whether such modifications would necessarily change our view on how monetary policy should be conducted.[2] The cost of using a misspecified DSGE model is that it could mislead policymakers to adopt a suboptimal policy to stabilize the economy. However if the magnitude of the cost is trivial in an economic sense, policymakers might still prefer to use the simpler DSGE model e.g. Calvo (1983) and Yun (1996), especially when they do not have complete confidence in

[1] See also Bayoumi *et al.* (2004), Coenen *et al.* (2007), and Erceg *et al.* (2006).

[2] For more discussion on this topic, see for example Faust (2012), Caballero (2010).

how the credit channel works in practice and / or how it should be modelled.

The objective of this paper is to assess the potential cost of using a DSGE model that does not feature the credit channel at its core (="approximating model"). We assume the policymaker is ignorant of the credit channel in the true model economy (="data-generating model"). The policymaker uses an approximating model that does not feature the credit channel to estimate model parameters. We then examine how the parameter estimates obtained from the approximating model are individually affected by a *policy shift* and how these parameter changes affect the overall policy outcome if the policymaker were to design its policy based on the estimated parameters. We evaluate the direct cost in two ways: (a) magnitude of the changes in the estimated parameters, and (b) the resulting destabilization of inflation and output caused by parameter changes. We also evaluate how large the "opportunity " cost of using a misspecified model is, relative to the correctly-specified credit channel model.

For our benchmark data-generating model we choose the Financial Accelerator model of Bernanke, Gertler and Gilchrist (1999). The main reason for this model choice is its wide use among policymakers and academics, making our work relevant to a broader set of models and applications.[3] The model also has a transparent structure because the credit channel mechanism is captured by a single structural parameter that measures the degree of the credit market friction. When this parameter is set to zero, the credit channel is effectively shut off and the financial accelerator model becomes isomorphic to the simple new Keynesian model that we use as our approximating model. Thus any parameter instability detected in the approximating model can be fully attributed to the credit channel misspecification.

The main findings are summarized as follows. First, two of the estimated parameters, i.e. labor supply elasticity and price stickiness, exhibit economically non-trivial

[3] In 2008, Federal Reserve governor Mishkin stated that the financial accelerator mechanism describes well the macroeconomic risk that the monetary policymaker faces (Mishkin, 2008), and since then many central banks such as the European Central Bank, the Bundesbank, the Riksbank have formally incorporated the credit channel as in Bernanke *et al.* (1999) into their DSGE models. For works in the academic literature, see for example Goodfriend and McCallum (2007) and Iacoviello (2005).

changes before and after the policy shift, but the remaining parameters remain relatively unchanged. Second, we find that parameter changes as a whole have a relatively small effect on policy outcomes evaluated using either the approximating model or the data-generating model. Our result indicates that the misspecification itself does not alter the policymaker's belief about their own policy outcomes and that the misspecification does not mislead the policymaker to pick a worse policy evaluated based on the (unknown) credit channel model economy. However, the policymaker could perform much better if it were to use the correctly specified credit channel model because the opportunity cost of not using the right model is found to be much larger than the direct cost obtained solely based on parameter changes. Our findings are robust under different degrees of credit market frictions and with alternative data-generating models.

This paper adds to the growing literature that studies the link between parameter invariance problem and model misspecifications. While works such as Chang, Kim, and Schorfheide (2010), Cogley and Yagihashi (2010), Leeper and Sims (1994), Lubik and Surico (2010), and Rudebusch (2005) all have studied this problem from different angles, no one has examined the cost of parameter invariance problem in the context of the credit channel. This paper intends to fill in the gap.

The next section explains the models in detail. We present in the third section the main results with regard to parameter changes and discuss its policy implications. The fourth section conducts the robustness checks. The last section concludes.

2 The Model

The data-generating model closely follows the Financial Accelerator model (FA-DGM) of Bernanke *et al.* (1999). We first describe the basic structure of the FA model and highlight the differences with the simpler new Keynesian approximating model (NK-AM). We then explain how the monetary policy shift is modeled. After calibrating the structural parameters, we compute the equilibrium.

2.1 Financial Intermediary and Entrepreneur

In the FA-DGM, there is imperfection in the capital market that generates a risk premium between the return on capital (R^k) and the risk-free interest rate (R). A risk-neutral financial intermediary collects funds from the representative household, then lends out to individual entrepreneurs. Entrepreneurs use these funds to buy capital (K) from capital producers and rent them out to intermediate goods producers. The demand for loanable funds is expressed as

$$E_t R_{t+1}^k = E_t \left[\frac{MPK_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right], \tag{1}$$

where $E_t R_{t+1}^k$ is the ex ante return on capital, MPK is the marginal product of capital, Q is the capital price, and δ is the depreciation rate of capital. Equation (1) states that the demand for loanable funds is implicitly determined by the sum of rental rate of capital and the direct gain of holding the capital.

Entrepreneurs are subject to the risk of insolvency through the idiosyncratic productivity disturbance. This disturbance is randomly distributed across entrepreneurs and a low realization makes entrepreneurs unable to repay their loan. The representative financial intermediary cannot tell which entrepreneur becomes insolvent until the loan payment stops, in which case, the financial intermediary collects all that is left to the entrepreneurs' capital by paying the agency cost AC associated with liquidation of asset. The optimal contract problem can be formulated as the entrepreneurs' profit maximization problem subject to a financial intermediary's participation condition. Solving this problem results in an expression for the supply of loanable funds

$$E_t R_{t+1}^k = E_t \left[\left(\frac{Q_t K_{t+1}}{N_t} \right)^v R_{t+1} \right], \tag{2}$$

where v is the credit market friction that is calculated using several model parameters, such as the unit agency cost. N is the net worth of the entrepreneur, which accumulates endogenously following the process

$$N_{t+1} = \gamma \left[R_t^k Q_{t-1} K_t - \left(R_t + \frac{AC_t}{Q_{t-1} K_t - N_t} \right) (Q_{t-1} K_t - N_t) \right] + W_t^e, \tag{3}$$

where W^e is entrepreneurs' income and γ is entrepreneurs' constant probability of surviving to the next period.[4] Equation (2) states that the more leveraged entrepreneurs are in financing their investment, the larger the financial premium R^k/R has to be, so that the financial intermediary can cover the increased cost of insolvency.

In the context of our experiment, the credit channel misspecification is captured in Equation (1) and (2). In the NK-AM, the credit market friction is nonexistent and the return on capital equals the risk-free rate at all times. Thus the demand for loanable funds can be written as

$$R_{t+1} = E_t \left[\frac{MPK_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right],$$

whereas the supply of loanable funds in the NK-AM is replaced by the consumption Euler equation derived from the representative household's optimal saving decision.

2.2 Monetary Policymaker

In the DSGE literature, the preferred method of modeling monetary policy is the Taylor rule (Taylor, 1993) that has reaction coefficients on pre-selected endogenous variables. We extend the rule by adding target inflation as an additional policy parameter. In recent years, policymakers and academics have discussed about the possible shift of the target inflation as a way to stimulate the economy.[5] Our modification allows this type of policy shift to be analyzed within the otherwise conventional model framework.[6]

In our model, monetary policy is determined in two steps. The policymaker first chooses a particular level of target inflation, then chooses the Taylor rule coefficients in order to minimize the (discounted) sum of expected variances for output and inflation. More specifically, the reaction coefficients ρ_Π, ρ_Y in the Taylor rule are chosen as follows

$$R^n_{t+1} = (\Pi_t)^{\rho_\pi} (Y_t)^{\rho_Y} \exp(e_{m,t}),$$

[4] Entrepreneurial income is introduced to assure that entrepreneurs always enter the economy with positive wealth.

[5] See for example Bakhshi *et al.* (2007), Blanchard *et al.* (2010), Evans (2011), Sahuc (2006), and Williams (2009).

[6] We note that this modification is not intended to replicate the "unconventional" monetary policy by the Fed that took place in the recent years. To keep Taylor rule functional, we assume that the zero-lower-bound does not bind.

$$\min_{\rho_\Pi,\rho_Y \geq 0} WL_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t' \Gamma_t, \tag{4}$$

where R^n_{t+1} is the nominal interest rate that is chosen at time t, $\Gamma_t = [\widehat{\Pi_t}, \widehat{Y_t}]'$ is the vector of inflation and output, and $e_{m,t} \sim N(0, \sigma_m^2)$ is a monetary policy shock. WL_0 is the loss measure which approximates the policymaker's objective of stabilizing the economy both today as well as in the future.[7] All the structural equations in the model serve as additional constraints in this optimization problem. Further details on how to compute ρ_Π, ρ_Y are provided in Appendix A.

One consequence of allowing policymakers to target inflation is that its expansionary monetary policy can affect the economy through aggregate supply as well as through demand. This is because target inflation appears in the aggregate supply equation known as the (new Keynesian) Phillips curve. Following the new Keynesian literature, we assume that firms face a fixed probability ρ of not being able to adjust their price in a given time period. The firm's optimal relative price p^* can be expressed as

$$p_t^* \equiv \frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} MC_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1} Y_{t+j}}, \tag{5}$$

where ϵ is the elasticity of demand, MC is the marginal cost, Y is the aggregate output, $\Delta_{j,t+j}$ is the j-period stochastic discount factor. Linearizing equation (5) around the steady state yields the following two equations in which model variables are expressed in terms of deviation from the steady state

$$\begin{aligned} \widehat{\Pi}_t &= \beta\overline{\Pi} E_t \widehat{\Pi}_{t+1} + \kappa \widehat{MC}_t + \varsigma \left(\sigma \widehat{C}_t - \widehat{Y}_t\right) \\ &\quad + \varsigma \left[(\epsilon - 1) E_t \widehat{\Pi}_{t+1} + E_t \widehat{\Phi}_{t+1}\right] + e_{cp,t}, \end{aligned} \tag{6}$$

$$\widehat{\Phi}_t = \left(1 - \rho\beta\overline{\Pi}^{\epsilon-1}\right)\left(\widehat{Y}_t - \sigma \widehat{C}_t\right) + \rho\beta\overline{\Pi}^{\epsilon-1}\left((\epsilon-1) E_t \widehat{\Pi}_{t+1} + E_t \widehat{\Phi}_{t+1}\right), \tag{7}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation, C is consumption, $e_{cp,t} \sim N(0, \sigma_{cp}^2)$ is the cost push shock, and Φ is an auxiliary variable that captures the behavior of firms' stochastic discount factor.[8] In Equation (6), $\kappa = \left(1 - \rho\overline{\Pi}^{\epsilon-1}\right)\left(1 - \rho\beta\overline{\Pi}^{\epsilon}\right)/\rho\overline{\Pi}^{\epsilon-1}$ represents

[7] We adopt this rather "ad-hoc" description of the loss function to keep calculation of policy coefficients tractable.

[8] For more details on derivation, see Ascari and Ropele (2007).

the "slope" of the Phillips curve, which is a decreasing function of target inflation $\overline{\Pi}$. Intuitively, when the target inflation is set higher, firms put less weight on the current marginal cost in choosing their prices because they expect the general price level to rise quickly over time. This makes the Phillips curve "flatter" in aggregate.[9] Under a flatter Phillips curve, an expansionary monetary policy shock of the same size will yield a stronger output response.

Table 1 provides a numerical example on how the target inflation affects the coefficients ρ_{Π}, ρ_Y in the NK-AM using the calibrated parameter values (explain in the next section). We observe that higher target inflation always leads to smaller coefficients for output and inflation. Also the loss measure in Equation (4) increases as the target inflation is raised. If the policymaker's objective is to minimize the welfare loss in the long-run, the optimal target inflation would always be set to zero. Thus our loss measure can be interpreted as the long-run cost of deviating from the optimal zero inflation in exchange of stimulating the economy in the short run.

2.3 Other Parts of the Model

Structural equations that describe the behavior of households, firms, and fiscal government are commonly shared among the FA-DGM and NK-AM. Here we show these equations, closely following the original specifications in Bernanke *et al.* (1999).

The representative household chooses consumption C_t, labor supply L_t, and bond holding D_{t+1} to maximize its lifetime utility subject to a budget constraint. Solving the utility maximization problem yields the standard consumption Euler and the intratemporal efficiency condition with regard to the labor-leisure choice

$$C_t^{-\sigma} = \beta R_{t+1} E_t C_{t+1}^{-\sigma},$$

$$\frac{\zeta L_t^{1/\tau}}{C_t^{-\sigma}} = W_t,$$

[9] When we apply our calibrated values to the model parameters, a 3% point increase in the the annual target inflation from 2% to 5% decreases κ by 26% (0.31 to 0.23).

where β is the subjective discount factor, σ is the risk aversion, τ is the labor supply elasticity, and ζ is the weight of the disutility of labor in the household utility. W_t is the real wage that is determined in a competitive factor market.

There are three types of firms and each of them involves in a different production stage. The competitive capital producers purchase raw output I to produce capital goods which are sold to entrepreneurs. In the production process, capital producers incur capital adjustment costs specified as

$$X_t = \frac{\chi}{2}\left(\frac{I_t - \delta K_t}{K_t}\right)^2 K_t,$$

where χ captures the cost of adjustment. Solving the profit maximization problem yields

$$Q_t = \left[1 - \chi\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1}.$$

The intermediate goods producers, indexed as z in the unit interval, face monopolistic competition in the market. Each producer hires labor and capital from an economy-wide competitive market and transforms into output via the constant return to scale production function.

$$Y_t(z) = A_t K_t^{\alpha}(z) L_t^{1-\alpha}(z),$$

where α is the capital share of income. The aggregate technology shock A is modeled as,

$$\begin{aligned} A_t &= (A_{t-1})^{\rho_A} \exp\left(e_{A,t}\right), \\ e_{A,t} &\sim N(0, \sigma_A^2). \end{aligned}$$

Their pricing behavior is captured in the behavioral equation (5) as we saw in the previous section.

The competitive final good producers aggregate the intermediate goods produced by firm z into a composite final good. Their production function can be written as

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}.$$

Finally, the government expenditure shock is modeled as

$$\begin{aligned} G_t &= (G_{t-1})^{\rho_G} \exp\left(e_{G,t}\right), \\ e_{G,t} &\sim N(0, \sigma_G^2). \end{aligned}$$

2.4 Calibration

The choice of private sector parameters largely follows the literature. Table 2 shows the calibrated values for the parameters that exist in both the DGM and the AM. The labor supply elasticity τ is set to one. The price stickiness parameter ρ implies average price nonadjustment of 6.7 months.

Table 3 shows the calibrated values for the parameters that are left out of estimation.[10] Some of them are commonly used in the DGM / AM while others are only used in the DGM. The discount factor β implies an annual real interest rate of 4.3%. The elasticity of demand ϵ implies a markup of 10% in the steady state. The government share of output $\overline{G}/\overline{Y}$ is set to match the historical average. We compute the credit market friction parameter ν following Bernanke *et al.* (1999) so that it reflects the most recent data during 1988Q1 - 2009Q2.[11] We use a value of 0.073, which is slightly higher than that in Bernanke *et al.* (1999).

Table 4 column (1) shows the policy parameters $\overline{\Pi}, \rho_{\Pi}, \rho_Y$ for the low inflation policy. Prior to the financial crisis, the annual target inflation is set to 2%, which is roughly the annualized quarter-to-quarter PCE inflation during the past two decades. Taylor rule coefficients are computed using the estimated parameter values in the pre-crisis period.[12] The Taylor rule parameters are $\rho_{\Pi} = 4.05$ and $\rho_Y = 0.37$. The values are in line with the empirical literature that finds a large reaction to inflation and a relatively muted reaction to output gap in the pre-crisis period.[13]

Table 4 column (2) shows policy parameters for the reflationary policy. The shift from the low inflation policy is modeled as a one-time, permanent, unanticipated shift in

[10] We do not calibrate the value for ζ, because it does not play any role in explaining the dynamics of the economy.

[11] First, time series of risk premium, default rate, and leverage ratio are constructed as in Yagihashi (2011), then the averages of each series are calculated, and finally the structural parameters are chosen so that the model-implied risk premium, default rate and leverage ratio match with those in the data.

[12] As we will see later in Table 7, estimates in the pre-crisis period is identical to the calibrated values in Table 2. This is because the credit channel is shut off ($\nu = 0$) and there will be no bias arising from model misspecification.

[13] For example, Fernandez-Villaverde and Rubio-Ramirez (2007) report that ρ_{Π} is in the range of 3.5 to 4.5 during 1980-2000 in the United States.

the policy parameters, which means that a learning process is suppressed and immediate convergence to a new rational expectations equilibrium is assumed. We choose the target inflation to be 5%. This number is from Fernandez-Villaverde and Rubio-Ramirez (2007) that have estimated the *implied* target inflation during the S&L crisis in the late 1980's. The Taylor rule coefficients are calculated using this new target inflation as well as the parameter estimates obtained *after* the crisis. Some of the parameter estimates have changed from the original DGM values due to the credit channel misspecification. The new policy coefficients are $\rho_\Pi = 2.24$ and $\rho_Y = 0$, notably smaller than those under the low inflation policy. Combining the results in Table 1 and Table 4 allows us to "decompose" the overall policy shift into the target inflation effect and the estimated parameter effects: higher target inflation alone results in the change in the Taylor rule coefficients from $[\rho_\Pi, \rho_Y] = [4.05, 0.37]$ to $[\rho_\Pi, \rho_Y] = [2.03, 0]$, whereas the estimated parameters effect is considerably smaller $[\rho_\Pi, \rho_Y] = [2.24, 0]$. This indicates that much of the overall change can be attributed to the change in the target inflation.

2.5 Solving for the Equilibrium

The rational expectations equilibrium is obtained in three steps. First, we solve for the model's deterministic steady state. Second, the model equations are log-linearized around the steady state and stacked into a system of linear expectational difference equations. Lastly, the system is solved to find the approximate equilibrium law of motion.

Table 5 presents the key steady state values in both models. In the FA-DGM the model implied annual capital output ratio is 1.6, whereas in the NK-AM the ratio is 2.1. The larger capital output ratio in the latter reflects the zero risk premium assumption, which lowers the marginal product of capital and raises the capital output ratio in the steady state. The difference in the capital output ratio further causes the expenditure shares of output in the steady state to differ.

To reconfirm how the FA-DGM works under positive steady state inflation, we subject our model economy to the expansionary monetary policy shock and compare model outcomes in three scenarios. In the first scenario, we combine zero credit market friction with a low inflation policy (= 2%). In the second scenario, we combine positive credit market friction (= 0.073) with a low inflation policy. The last scenario combines positive credit market friction with a high inflation policy (= 5%). To facilitate comparison, we keep the Taylor rule coefficients the same for all cases. Figure 1 shows the impulse responses for selected variables.

There are visible differences between the first scenario and the second scenario. When the credit market friction is present, an expansionary monetary policy shock lowers the cost of both external and internal financing, stimulating investment demand and raising the price of capital. The increase in the price of capital further raises the net worth. Because capital and net worth adjust sluggishly over time, the risk premium is kept low in the future periods. The low risk premium implies lower cost of external financing, which further pushes up the investment demand. The model generates a positive feedback loop known as the financial accelerator effect.

We also observe notable differences between the second scenario and the third scenario. In this case, the gaps in the impulse responses are purely generated through policy with higher target inflation. A higher target inflation generates additional stimulus effect through a flatter Phillips curve. This effect is directly seen in the output response in which the size of expansion is found to be large. Note that the model also generates a muted response of inflation under the reflationary policy, which is expected from a flatter Phillips curve but the quantitative effect is relatively small.

3 Results

3.1 Estimation Strategy

In order to assess the parameter invariance problem, we use the simulation method developed in Cogley and Yagihashi (2010). It assumes that the policymaker does not know the FA-DGM and instead uses the NK-AM to interpret data emanating from the DGM. We simulate three sets of data from the DGM: the "old" subsample with a zero credit market friction, the "new" subsample with a positive credit market friction and the same policy parameters as in the "old" sample, and the "final" subsample with a positive credit market friction and a set of new policy parameters. The timing in the model is summarized as follows:

1. Pre-crisis period: no credit market friction in the DGM, low inflation policy, "old" subsample is generated in the DGM, policymaker obtains the "old" estimates using the AM and computes the Taylor rule coefficients based on these estimates.

2. Financial crisis happens: positive credit market friction in the DGM, low inflation policy, "new" subsample is generated in the DGM, policymaker obtains the "new" estimates using the AM. Taylor rule coefficients are still kept the same as in step 1.

3. Policy shift: the policymaker adopts the high inflation policy and computes the new Taylor rule coefficients based on the "new" estimates in step 2.

4. Post-policy shift: positive credit market friction in the DGM, high inflation policy, "final" subsample is generated in the DGM, the policymaker obtains the "final" estimates using the AM.

The policymaker obtains parameter estimates by "fitting" the AM to each of the dataset generated through the DGM. More formally, the estimation can be stated as solving the following minimization problem

$$\arg\min KLIC = \int \log\left(\frac{p_{DGM}(\mathbf{Y}|\boldsymbol{\theta}_{DGM})}{p_{AM}(\mathbf{Y}|\boldsymbol{\theta}_{AM})}\right) p_{DGM}(\mathbf{Y}|\boldsymbol{\theta}_{DGM})d\mathbf{Y}, \tag{8}$$

where $KLIC$ stands for the distance metric known as the Kullback-Leibler Information Criterion.[14] $p_i(\mathbf{Y}|\boldsymbol{\theta}_i)$ represents the likelihood function for both models $i = (DGM, AM)$, and the vector $\mathbf{Y}$ represents endogenous variables common to both models, and $\boldsymbol{\theta}_i$ represents a vector of model-specific parameters that are partitioned into policy parameters ($\boldsymbol{\theta}_i^{pol}$) and private sector parameters ($\boldsymbol{\theta}_i^{priv}$). The Bayesian consistency theorem assures that the estimates converge in probability to what is called the "pseudo-true value" estimates of the private sector parameters ($\widehat{\boldsymbol{\theta}}_{AM}^{priv}$) while treating policy parameters as known

[14] For more details on the consistency theorem, see the appendix in Gelman *et al.* (2000)

constants.[15] Due to the misspecification in the AM, there will always be an asymptotic bias i.e. $\widehat{\boldsymbol{\theta}}_{AM}^{priv} \neq \boldsymbol{\theta}_{DGM}^{priv}$. These results are then used to assess to what extent the policy shift is responsible for the changes in the estimated parameters.

To focus solely on the parameter invariance problem due to misspecification in the credit channel, sufficiently long subsamples (500,000 periods) are generated.[16] The large sample size assures that the asymptotic standard errors are tiny, allowing us to focus on how point estimates change across experiments.[17] We choose output, inflation, nominal interest rate, and consumption as "observables" that are available to the policymaker. The number of observables is matched to the number of structural shocks in the AM to avoid stochastic singularity.

The structural parameters are estimated by maximizing the posterior kernel density that involves the Bayesian prior. The role of prior is to facilitate computation by introducing curvatures into the likelihood function.[18] Table 6 summarizes the priors for individual parameters. The prior distributions are chosen so that they respect the domain of original parameters. The standard deviations of the priors are intentionally set loosely so that the estimates can quickly converge to the "pseudo-true" values.

3.2 Parameter Estimates: Before and After the Financial Crisis versus Before and After the Policy Shift

The first experiment we conduct is on how the parameter estimates change before and after the policy shift. Before we get to the policy shift, we first examine how the parameter estimates change before and after the *financial crisis* while the policy rule remains unchanged. The pseudo-true values ($\widehat{\boldsymbol{\theta}}_{AM}^{priv}$) and the associated standard errors for each subsamples are shown in columns (1) and (2) of Table 7. Posterior standard errors are

[15] We assume that the policymaker has perfect knowledge of their own policy. Thus the policy parameters are dropped from its estimation.

[16] In practice we generated 550,000 sample periods, then discarded the first 50,000 in order to ensure that initial conditions have worn off.

[17] In reality, policymakers also face issues of estimation uncertainty and the identification problem. See An and Schorfheide (2007) and Canova and Sala (2009) for discussions.

[18] See An and Schorfheide (2007) for more discussion on the use of the Bayesian method.

driven down to near zero in both cases due to the large sample size.[19] The "old" estimates are identical to the values used in the DGM. This is because, as implied by Equation (2), the zero credit market friction makes the credit channel irrelevant to the model outcome in the DGM. Thus the NK-AM "perfectly" approximates the unknown FA-DGM. In column (2), the "new" estimates are obtained under a positive credit market friction. Many parameter estimates now change: most notable ones are the labor supply elasticity τ (from 1 to 1.63), the government expenditure shock σ_G (from 0.01 to 0.014), and the risk aversion σ (from 1 to 1.17). The change in the risk aversion is non-trivial because it implies that household perceived welfare cost of the business cycle increases by 17% (Lucas, 1987). While these three parameters are loosely related to the credit channel, we also observe changes in the price stickiness parameter ρ (from 0.55 to 0.52) that are more difficult to justify. This change in the price stickiness implies that the slope parameter κ in Equation (6) increases by 22%, partially offsetting the flatter Phillips curve through reflation. When the policymaker observes this, it might wrongly conclude that the current monetary policy is not expansionary enough.

Next, we examine whether the parameters remain invariant before and after the policy shift. Using data generated from the reflationary policy, the policymaker obtains the "final" estimates reported in column (3) of Table 7. Again due to the misspecification, these estimates have changed from the "new" estimates in column (2). In particular, two parameters stand out: one is the labor elasticity parameter τ that increases from 1.63 to 2.78 and the other is the price stickiness parameter ρ that decreases from 0.52 to 0.49. We regard these changes as non-trivial, because it matches the magnitude of the crisis scenario in which the credit market friction increases from 0 to 0.073. The further fall in the price stickiness could be of particular concern to the policymaker, because such a change further steepens the slope of the Phillips curve κ. Under the "final" estimate the value for κ is 0.36. This is even larger than the pre-crisis value of 0.31.

[19] An exception is the labor supply elasticity, which is caused by the inherent identification problem common in the baseline NK-AM. Cogley and Yagihashi (2010) conduct a separate simulation exercise that shows this weak identification does not affect the results on our parameter invariance problem.

Empirically, the price stickiness is found to be negatively correlated with inflation (e.g. Fernandez-Villaverde and Rubio-Ramirez, 2007). While our result is consistent with this empirical finding, we emphasize that the change in ρ in our case was purely caused by the misspecification in the credit channel. Cogley and Yagihashi (2010) examine the misspecification in the Phillips curve and they find that the policy shift *does not* affect price stickiness. This demonstrates that the parameter invariance problem is context-specific.

3.3 Parameter Invariance and the Central Bank's "Belief"

The second experiment examines how the changes in parameters collectively affect policy outcomes evaluated using the *approximating model*. Based on our model setting, the policymaker's "belief" with regard to its own policy is formed through observing the single loss measure in Equation (4), which captures the stabilization effect on inflation and output. The calculation of the loss measure uses the estimated structural parameters as input. Because the credit channel misspecification changes the parameter estimates before and after the policy shift, the "perceived" loss is likely to change as well.

For the purpose of clarification, we now rename the baseline reflationary policy as the "NK-optimal" policy, which is the first-best policy according to the *NK-AM*. As a competing policy we introduce the "NK-*suboptimal"* policy in which the policymaker raises its target inflation rate but ignores the "new" parameter estimates when calculating the Taylor rule coefficients. Because of informational constraint imposed on the NK-suboptimal policy, the loss under this policy is expected to be larger than the loss in the NK-optimal policy. A large difference in the loss measure casts doubt on the use of the approximating model.

Table 8 shows the result. The loss measure increases by 1.68% and 2.37% after the policy shift under the NK-optimal policy (column 1) and the NK-suboptimal policy (column 2), respectively. To put these numbers into perspective, a reflation from 2% to 3% is associated with an additional loss of 2.5% as shown in Table 1. Thus the loss associated with the credit channel misspecification is slightly smaller than the cost of a 1% higher inflation.

The small difference in losses can be explained by the similarity of the Taylor rule coefficients across policies. As the top two rows of Table 8 show, both policies have zero output reaction coefficients and only a small difference in the inflation reaction coefficients. The Taylor rule coefficients are not affected by much because the *overall* parameter changes after the financial crisis are not large. As a result, the change in the loss measure under the NK-optimal policy becomes trivial.

This experiment demonstrates that the NK-AM is quite robust to the credit channel misspecification when used together with the policy rule considered in this paper. As a result, the effect on the policymaker's own assessment about the policy outcome remains relatively small.

3.4 Optimal Policy in the FA-DGM

The last experiment asks whether the policymaker can use the AM as a guide to reduce the loss measure evaluated in the *data-generating model*. Chang, Kim and Schorfheide (2010) provide an example in which model misspecifications induce the fiscal policymaker to adopt a policy associated with an adverse welfare outcome. We are interested in whether the same could occur in the case of the credit channel misspecification.

We introduce two policies that help us in assessing the performance of the baseline NK-optimal policy. The first policy treats the FA-DGM as unknown as before, but we give the policymaker an opportunity to reoptimize its policy based on the parameter estimates obtained *after* the policy shift. We call this policy the "NK-reoptimized policy". If this reoptimized policy yields a larger loss than the loss in the NK-optimal policy, we interpret this as the policymaker being misguided by its own model. In this case, the use of the NK-AM should be avoided.

The second policy treats the FA-DGM *as known* to the policymaker. The Taylor rule coefficients are thus obtained by minimizing the loss criterion using the FA-DGM. We call this policy the "FA-optimal" policy. By construction, this policy will yield the lowest loss possible evaluated using the FA-DGM. This is used to measure the opportunity cost of using the misspecified model when the credit channel model is used to generate the data.

Table 9 shows how the losses compare under the three policies. We note that the NK-reoptimized policy has a smaller loss than the NK-optimized policy, even though the difference is tiny (-0.07%). This means that the NK-AM is almost neutral in choosing a policy evaluated in the unknown FA-DGM. Compared to the FA-optimal policy, both the NK-optimal and the NK-reoptimized policies perform significantly worse. Their loss measures are more than 10% higher than the loss under the FA-optimal policy.

These differences in losses can be again attributed to the differences in the Taylor rule coefficients. Coefficients for the FA-optimal policy are much larger than those generated using the NK-optimal policy. This is because the FA-optimal policy tries to counter the fluctuation caused by the financial accelerator mechanism.

One might wonder whether it is possible to improve upon the NK-optimal policy to get closer to the FA-optimal policy. Figure 2 draws a contour map of the loss measure for possible combinations of ρ_Π and ρ_Y. The numbers associated with each contour are normalized so that the FA-optimal rule produces a relative loss of one.[20] The NK-optimal policy is depicted as a dot in the south-west corner. We observe that once the dot moves towards the north-east direction, the loss quickly starts to fall, meaning that the potential gain from using a better approximating model is large. However, the strongly skewed shape of the contour map also shows that the difficulty of achieving a better policy outcome in the post-crisis economy. For example, if the policymaker were to simply raise ρ_Π while keeping ρ_Y fixed, that would deviate the policy away from the FA-optimal policy. On the other hand, raising ρ_Y while keeping ρ_Π fixed would soon hit the indeterminacy region, which is clearly suboptimal.

This experiment demonstrates that the NK-AM does not necessarily induce the policymaker to pick a bad policy when the credit channel misspecification is present. However the model also does very little in further reducing the loss measure. To significantly improve the policy outcome, the policymaker needs to be equipped with a model that features the financial accelerator mechanism.

[20] The contour lines also represent gross deviations from the FA-optimal policy.

4 Robustness Checks

4.1 The Role of Credit Market Friction

In the benchmark FA-DGM, the set of credit channel parameters are specifically calibrated using historical data in the United States during 1988Q1-2009Q2. One might be interested in how the NK-AM performs when different credit market frictions are assumed. The size of the credit market friction enhances the response of output and inflation through the risk premium. This implies that a higher friction may cause more changes in the estimated parameters.

For this experiment, we examine two scenarios, "modest" period and "disaster" period, in addition to the "baseline" period that we used in the previous analysis. The modest period features a lower credit market friction whereas the crisis period features a higher credit market friction than the baseline. They imply different steady-state risk premiums, default rates, and leverage ratios as summarized in Table 10. The "modest" case is close to Bernanke *et al* (1999)'s baseline parameterization which does not include the period of 2007-09 financial crisis.

Table 11 shows the estimation result. To save space, we only report the changes in the estimated parameters before and after the policy shift.[21] For the "modest" period case in column (2), many of the parameters stay invariant relative to the baseline case. In the "disaster" period (column (3)), the parameter changes are similar to the baseline. One notable difference is price stickiness, which falls twice as much in the disaster period compared to the baseline, making the Phillips curve much steeper from the pre-crisis level. In most cases, the *direction* of change is monotonically related to the credit market friction, with the exceptions of the labor supply elasticity and the size of technology shock. Overall we find similar results as our baseline that the changes in parameters as a whole is rather limited.

[21] For the baseline case, this is equivalent to the gap between column (2) and (3) of Table 7.

4.2 An Alternative Credit Channel Model

Next, we consider a different DGM that features a different type of credit channel. Recently there is a heightened interest among policymakers in explicitly modeling the banking sector, which is absent in the FA model. Works by Curdia and Woodford (2010), Gertler and Kiyotaki (2010), and Gertler and Karadi (2011) have gained much popularity within the central banking community. The main benefit of their models is that it can be used to analyze the economic effect of injecting liquidity into the banking sector.

We choose the model of Gertler and Karadi (2011) as our benchmark DGM (hereafter the GK model). This model introduces an incentive misalignment problem between the depositors and the banks: the bank owners have incentives to terminate their businesses and run away with the deposited funds. In order to keep them in the business, sufficiently high returns have to be provided in the form of spread $R_k - R$. When such incentive constraint is binding, there will be an endogenously determined leverage ratio that constrains how much the bank capital QK can be expanded relative to the bank net worth N. This function of the GK model is qualitatively similar to the financial accelerator mechanism in the earlier model.

The GK model has seven equations in addition to those of the NK model. Two equations are of particular importance. The first equation represents the value of capital, or more specifically, the time t opportunity cost of terminating the business evaluated in terms of bank's capital,

$$\upsilon_t = E_t \left[(1-\theta)\, \beta\Lambda_{t,t+1} \left(R_{k,t+1} - R_{t+1} \right) + \beta\Lambda_{t,t+1}\theta \left(\frac{Q_{t+1}K_{t+2}}{Q_t K_{t+1}} \right) \upsilon_{t+1} \right], \tag{9}$$

where θ represents the exogenous survival rate of the bankers. The second equation represents the value of net worth, or the opportunity cost of terminating the business in terms of bank's net worth,

$$\eta_t = E_t \left[(1-\theta) + \beta\Lambda_{t,t+1}\theta \left(\frac{N_{t+1}}{N_t} \right) \eta_{t+1} \right]. \tag{10}$$

Banks optimally choose their leverage QK/N so that the total opportunity cost of terminating the business $\upsilon_t Q_t K_{t+1} + \eta_t N_t$ is equal to the amount of funds that they could potentially divert. The remaining equations in the GK models are shown in Appendix B.[22]

Although the credit channel is modeled differently in the GK model and the FA model, our simulation results are strikingly similar to those of the FA-DGM on how the parameter estimates change across policy shifts. We apply the same reflationary policy shift that was applied in the previous experiment. The new results are summarized in Table 12. The column "new" shows the pseudo-true values before the policy shift. We note that many parameter estimates are quite different from the parameters used in the DGM. When compared to the FA-DGM case (Table 7), the risk aversion parameter σ and the capital adjustment cost parameter χ show much larger asymptotic biases this time. It confirms the obvious fact that the two models are inherently different in structure.

The next column "final" shows the pseudo-true values after the policy shift. Again, the asymptotic biases are estimated to be large. However, when we look at the direction of the parameter change, eight out of ten changes are in the *same* direction as in the FA-DGM case. Specifically, the price stickiness falls by a significant margin, which confirms our previous finding.

This experiment demonstrates that even though different DGM generates different estimates in the AM, the changes in parameters are intimately related to the nature of the type of model misspecification. Thus the main result of our paper may be extended to a wide variety of credit channel models, including those that involves the explicit banking sector.

5 Conclusion

This paper examines the parameter invariance problem of the workhorse DSGE model when a misspecification occurs in the credit channel. The financial accelerator model is chosen to represent the complicated reality and the simpler new-Keynesian model is used as the estimating model by the policymaker. A reflationary monetary policy that involves

[22] We also apply the same parameter values as in Gertler and Karadi (2011).

an increase in the target inflation is simulated using estimated structural parameters. We find that after the policy shift, both labor supply elasticity and price stickiness fail to remain invariant due to model misspecification. While the additional exercises demonstrates the relative robustness of the simpler new Keynesian model, it also shows that there is much room left in improving the policy outcome through correctly specifying the credit channel in the estimation model. Finally, the paper examines how the new Keynesian model performs under a different degree of credit market friction and under an alternative data-generating credit channel model. We find that much of the baseline results hold in these cases as well.

In the empirical context, our findings provide an alternative explanation to what the literature regard as the parameter instability problem associated with DSGE models. For example, Smets and Wouters (2005), Fernandez-Villaverde and Rubio-Ramirez (2007), and Canova (2009) report that the estimated structural parameters in DSGE models look quite different for different subsamples that are split by a historically well-known policy shift. The result of this paper suggests that the parameter instability could possibly be caused by the lack of the credit channel in the estimation model.

In the experiments, we kept the model relatively small in favor of transparency. A possible extension to our paper is to repeat the exercise with a larger model *a la* Smets and Wouters (2003) with more structural model equations. This would introduce additional cross-equations restrictions in the model that generates new tensions between the estimating model and the generated data. It would be interesting to see whether the labor supply elasticity and price stickiness parameter in the new setting will be affected in an non-trivial manner, as we see in this paper.

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Appendix A: Computing the Optimal Policy

Our hypothetical central banker chooses policy parameters ρ_π, ρ_y so that it minimizes the expected (discounted) quadratic loss subject to the law of motion of the NK model economy. The objective function is

$$\begin{aligned} & \min_{\rho_\pi,\rho_y} E_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t' \mathbf{W} \Gamma_t \\ s.t. \mathbf{Y}_t &= \Xi s_t \\ s_{t+1} &= \Psi_1 s_t + \Psi_2 e_{t+1} \qquad e_t \backsim iidN(0,\Sigma_\epsilon) \end{aligned}$$

where $\Gamma = [\widehat{\Pi}, \widehat{Y}]'$ is the vector of inflation and output that enters the loss function and $\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 0 & w_y \end{bmatrix}$ is the weight matrix that defines the relative importance of the stabilization component to the policymaker.[23] On the constraint side, $\mathbf{Y}$ is the vector of all variables, s is the vector of predetermined endogenous variables and e is the vector of exogenous shocks. Ξ, Ψ_1, Ψ_2 are time-invariant coefficient matrices that include policy coefficients ρ_π, ρ_y. To solve this minimization problem, we first define the period loss function as

$$\Gamma_t' \mathbf{W} \Gamma_t = s_t' \mathbf{R} s_t.$$

Next, we conjecture a quadratic value function that takes the form of

$$s_t' \mathbf{P} s_t + \iota = s_t' \mathbf{R} s_t + \beta E_t \left[s_{t+1}' \mathbf{P} s_{t+1} + \iota \right],$$

then we use the reduced form to replace s_{t+1}, we obtain

$$\begin{aligned} s_{t+1}' \mathbf{P} s_{t+1} &= [\Psi_1 s_t + \Psi_2 e_{t+1}]' \mathbf{P} [\Psi_1 s_t + \Psi_2 e_{t+1}] \\ &= s_t' \Psi_1' \mathbf{P} \Psi_1 s_t + s_t' \Psi_1' \mathbf{P} \Psi_2 \varepsilon_{t+1} + \varepsilon_{t+1}' \Psi_2{}' \mathbf{P} \Psi_1 s_t + \varepsilon_{t+1}' \Psi_2' \mathbf{P} \Psi_2 \varepsilon_{t+1}, \end{aligned}$$

thus we have

$$\begin{aligned} E_t \left[s_{t+1}' \mathbf{P} s_{t+1} \right] &= s_t' \Psi_1' \mathbf{P} \Psi_1 s_t + E_t \left[\varepsilon_{t+1}' \Psi_2' \mathbf{P} \Psi_1 \varepsilon_{t+1} \right] \\ &= s_t' \Psi_1' \mathbf{P} \Psi_1 s_t + tr \left(\mathbf{P} \Psi_2 \Omega \Psi_2{}' \right), \end{aligned}$$

[23] In the calculation, we set $w_y = 1$.

where $\Omega \equiv E_t\left[\varepsilon_{t+1}\varepsilon'_{t+1}\right]$. It follows that the value function can be expressed as

$$\begin{aligned} s'_t\mathbf{P}s_t + \iota &= s'_t\mathbf{R}s_t + \beta s'_t\Psi'_1\mathbf{P}\Psi_1 s_t + \beta\left[tr\left(\mathbf{P}\Psi_2\Omega\Psi_2{}'\right) + \iota\right] \\ &= s'_t\left[\mathbf{R} + \beta\Psi'_1\mathbf{P}\Psi_1\right]s_t + \beta\left[tr\left(\mathbf{P}\Psi_2\Omega\Psi_2{}'\right) + \iota\right]. \end{aligned}$$

Equating powers of s_t yields

$$\mathbf{P} = \mathbf{R} + \beta\Psi'_1\mathbf{P}\Psi_1.$$

This Ricatti Equation has a unique solution $\mathbf{P}$. We can further solve for ι by equating the constant terms of the value function

$$\begin{aligned} \iota &= \beta\left[tr\left(\mathbf{P}\Psi_2\Omega\Psi_2{}'\right) + \iota\right] \\ \iota &= \frac{\beta}{1-\beta}tr\left(\mathbf{P}\Psi_2\Omega\Psi_2{}'\right). \end{aligned}$$

For any given initial value of $s = s_0$, there exists an optimal monetary policy that solves the following minimization problem

$$\min_{\rho_\pi,\rho_y} s'_t\mathbf{P}s_t + \iota,$$

where we use the steady state values as our initial values.

Appendix B: Model of Gertler and Karadi (2011)

The new model adds seven equations to the NK-AM. These are as follows [24]

$$\upsilon_t = E_t\left[(1-\theta)\beta\Lambda_{t,t+1}\left(R_{k,t+1} - R_{t+1}\right) + \beta\Lambda_{t,t+1}\theta\left(\frac{Q_{t+1}K_{t+2}}{Q_tK_{t+1}}\right)\upsilon_{t,t+1}\right] \tag{B-1}$$

$$\eta_t = E_t\left[(1-\theta) + \beta\Lambda_{t,t+1}\theta\left(\frac{N_{t+1}}{N_t}\right)\eta_{t+1}\right] \tag{B-2}$$

$$\phi_t \equiv \frac{Q_tK_{t+1}}{N_t} = \frac{\eta_t}{\lambda - \upsilon_t} \tag{B-3}$$

$$\frac{N_{t+1}}{N_t} = \left(R_{k,t+1} - R_{t+1}\right)\phi_t + R_{t+1} \tag{B-4}$$

$$N_t = N_{e,t} + N_{n,t} \tag{B-5}$$

[24] For more explanation on how these equations are derived, see Gertler and Karadi (2011) and the online appendix.

$$N_{e,t} = \theta \left[(R_{k,t} - R_t) \phi_{t-1} + R_t \right] N_{t-1} \tag{B-6}$$

$$N_{n,t} = (1 - \theta) \varphi Q_t K_t. \tag{B-7}$$

Equation (B-1) and (B-2) represent the value of capital and net worth, as explained in the main text. Equation (B-3) defines the optimal leverage ratio ϕ for the bank. λ is the fraction of funds that the banker can divert from the deposit. Equation (B-4) is the law of motion for the aggregate net worth. Equation (B-5) defines the aggregate net worth, which consists of the net worth of the existing banks N_e and the net worth of the newly entering banks N_n. Equation (B-6) is the law of motion for the net worth N_e. Equation (B-7) describes the start-up funds for the new banks. φ is the fraction of capital that is transferred.

Table 1: Target Inflation and Monetary Policy

(Annual) Target inflation $\Pi^{ss} \equiv \overline{\Pi}^4 - 1$	2%	3%	5%	7%
Slope of NKPC κ	0.31	0.28	0.23	0.18
Taylor rule coefficient: inflation ρ_Π	4.05	3.19	2.03	1.88
Taylor rule coefficient: output gap ρ_Y	0.37	0.19	0	0
Loss Measure WL	7.6673	7.8592	8.2762	8.7843
Δ% from Π^{ss} =2%	-	2.5%	7.9%	14.6%

Note: the unit of loss measure is 1E-4 = 0.0001. NK-AM is used for calculation

Table 2: Estimated Parameters

Parameter name	Values
Risk aversion σ	1
Labor supply elasticity τ	1
Capital adjustment cost χ	0.25
Price stickiness ρ	0.55
Persistence of tech. shock ρ_A	0.9
Persistence of gov. shock ρ_G	0.9
Size of tech. shock σ_A	0.01
Size of gov. shock σ_G	0.01
Size of cost push shock σ_{cp}	0.01
Size of mon. policy shock σ_m	0.01

Table 3: Other Structural Parameters

Parameter name	Values
Discount factor $\beta = 1/\overline{R}$	0.99
Capital share α	0.33
Elasticity of demand ϵ	11
Government spending share of output $\overline{G}/\overline{Y}$	0.21
Entrepreneur's share of output $\overline{C^e}/\overline{Y}$	0.01
Depreciation rate δ	0.025
Credit market friction υ	0.073
Survival rate of entrepreneur γ	0.973

Table 4: Policy Parameters

	(1) Low inflation policy	(2) Reflationary policy
(Annual) Target inflation $\Pi^{SS} \equiv \overline{\Pi}^4 - 1$	2%	5%
Taylor rule coefficient: inflation ρ_Π	4.05	2.24
Taylor rule coefficient: output gap ρ_Y	0.37	0
cf. Credit market friction	0	0.073

Table 5: Steady-state Values

	(1) FA model	(2) NK model
Credit market friction ν	0.073	0
Consumption spending share of output $\overline{C}/\overline{Y}$	0.64	0.59
Investment spending share of output $\overline{I}/\overline{Y}$	0.16	0.21
(Annual) Capital output ratio $\overline{K}/4\overline{Y}$	1.6	2.1
(Annual) Ex ante real return on capital $(\overline{R}^k)^4$	1.088	1.043
(Annual) Risk premium $(\overline{R}^k)^4 - \overline{R}^4$	4.5%	0%
(Annual) Default rate $F(\omega^*)$	6.3%	0%
Leverage ratio $\overline{QK}/\overline{N}$	1.8	1

Table 6: Priors

	Distribution	Mode	95% credible set
Risk aversion σ	Gamma	1	[0.523, 2.087]
Labor supply elasticity τ	Gamma	1	[0.523, 2.087]
Capital adjustment cost χ	Gamma	0.25	[0.133, 0.438]
Price stickiness ρ	Beta	0.55	[0.197, 0.851]
Persistence of tech. shock ρ_A	Beta	0.9	[0.635, 0.967]
Persistence of gov. shock ρ_G	Beta	0.9	[0.635, 0.967]
Size of tech. shock σ_A	Inv-Gamma	0.01	[0.006, 0.084]
Size of gov. shock σ_G	Inv-Gamma	0.01	[0.006, 0.084]
Size of cost push shock σ_{cp}	Inv-Gamma	0.01	[0.006, 0.084]
Size of mon. policy shock σ_m	Inv-Gamma	0.01	[0.006, 0.084]

Note: the 95% credible set is calculated as the highest posterior density interval from the random draws.

Table 7: Parameter Estimates

	DGM values	Estimates (1) "old"	(2) "new"	(3) "final"
Risk aversion σ	1	1.00 (0.001)	1.17 (0.002)	1.14 (0.001)
Labor supply elasticity τ	1	1.00 (0.066)	1.63 (0.061)	2.78 (0.109)
Capital adjustment cost χ	0.25	0.25 (0.000)	0.24 (0.000)	0.24 (0.000)
Price stickiness ρ	0.55	0.55 (0.005)	0.52 (0.003)	0.49 (0.002)
Persistence of tech. shock ρ_A	0.9	0.90 (0.000)	0.93 (0.000)	0.93 (0.000)
Persistence of gov. shock ρ_G	0.9	0.90 (0.001)	0.95 (0.0001)	0.95 (0.001)
Size of tech. shock σ_A	0.01	0.010 (0.000)	0.010 (0.0000)	0.010 (0.000)
Size of gov. shock σ_G	0.01	0.010 (0.000)	0.014 (0.000)	0.015 (0.000)
Size of cost push shock σ_{cp}	0.01	0.010 (0.001)	0.010 (0.000)	0.010 (0.000)
Size of mon. policy shock σ_m	0.01	0.010 (0.000)	0.010 (0.000)	0.010 (0.000)
cf. credit market friction in DGM ν	-	0	0.073	0.073

Note: Posterior standard errors are shown in parentheses.

Table 8: Losses in the Approximating Model

	(1) NK-optimal	(2) NK-suboptimal
Taylor rule coefficient: inflation ρ_Π	2.24	2.03
Taylor rule coefficient: output gap ρ_Y	0	0
(a) WL under "new" estimates	11.0674	11.0730
(b) WL under "final" estimates	11.2539	11.3355
Δ% from (a) to (b)	+1.68%	+2.37%

Note: target inflation is set to 5%. The unit of the loss measure is 1E-4 = 0.0001.

Table 9: Losses in the Data-generating Model

	(1) FA-optimal	(2) NK-optimal	(3) NK-reoptimized
Taylor rule coefficient: inflation ρ_Π	8.61	2.24	2.20
Taylor rule coefficient: output gap ρ_Y	3.33	0	0
Loss measure WL	8.5121	9.3921	9.3859
Δ% from NK-optimal case	-9.37%	-	-0.07%
Δ% from FA-optimal case	-	+10.34%	+10.27%

Note: target inflation is set to 5%. The unit of the loss measure is 1E-4 = 0.0001.

Table 10: Steady-state Values, with Different Credit Market Friction

	(1) Baseline	(2) Modest period	(3) Disaster period
Credit market friction parameter ν	0.073	0.056	0.127
Consumption spending share of output $\overline{C}/\overline{Y}$	0.64	0.63	0.66
Investment spending share of output $\overline{I}/\overline{Y}$	0.16	0.17	0.14
(Annual) Capital output ratio $\overline{K}/4\overline{Y}$	1.6	1.7	1.4
(Annual) Ex ante real return on capital $(\overline{R}^k)^4$	1.088	1.077	1.116
(Annual) Risk premium $(\overline{R}^k)^4 - \overline{R}^4$	4.5%	3.5%	7.4%
(Annual) Default rate $F(\omega^*)$	6.3%	5.8%	7.7%
Leverage ratio $\overline{QK}/\overline{N}$	1.8	1.9	1.6

Note: the case with baseline parameterization in Table 2, 3 is named as the “baseline”.

Table 11: Changes in Estimates, with Different Credit Market Friction

	(1) Base-line	(2) Modest period	(3) Disaster period
Risk aversion σ	-0.024	-0.015	-0.028
Labor supply elasticity τ	$+1.150$	-0.006	$+0.472$
Capital adjustment cost χ	-0.004	-0.003	-0.005
Price stickiness ρ	-0.027	$+0.001$	-0.054
Persistence of tech. shock ρ_A	$+0.004$	±0.000	$+0.004$
Persistence of gov. shock ρ_G	-0.001	±0.000	-0.005
Size of tech. shock σ_A	-0.0002	-0.0001	-0.0001
Size of gov. shock σ_G	$+0.0015$	$+0.0013$	$+0.0015$
Size of cost push shock σ_{cp}	$+0.0001$	±0.0000	$+0.0001$
Size of mon. policy shock σ_m	±0.0000	±0.0000	±0.0000
cf. credit market friction in DGM ν	0.073	0.056	0.127

Note: each number represents the change from the "new" estimate to the "final" estimate.

Table 12: Parameter Estimates, with Different Credit Channel Model

	DGM values	GK Estimates "new"	GK Estimates "final"	Changes GK	Changes cf. BGG
Risk aversion σ	1	2.50 (0.008)	2.21 (0.011)	-0.289	-0.024
Labor supply elasticity τ	1	0.16 (0.001)	0.22 (0.002)	$+0.062$	$+1.150$
Capital adjustment cost χ	0.25	1.18 (0.009)	1.09 (0.016)	-0.088	-0.004
Price stickiness ρ	0.55	0.69 (0.001)	0.58 (0.001)	-0.101	-0.027
Persistence of tech. shock ρ_A	0.9	0.92 (0.000)	0.93 (0.000)	$+0.008$	$+0.004$
Persistence of gov. shock ρ_G	0.9	0.71 (0.001)	0.79 (0.001)	$+0.077$	-0.001
Size of tech. shock σ_A	0.01	0.008 (0.000)	0.007 (0.000)	-0.0008	-0.0002
Size of gov. shock σ_G	0.01	0.137 (0.001)	0.120 (0.001)	-0.0168	$+0.0015$
Size of cost push shock σ_{cp}	0.01	0.011 (0.000)	0.011 (0.000)	±0.0000	$+0.0001$
Size of mon. policy shock σ_m	0.01	0.010 (0.000)	0.010 (0.000)	±0.0000	±0.0000

Note: posterior standard errors are shown in parentheses.

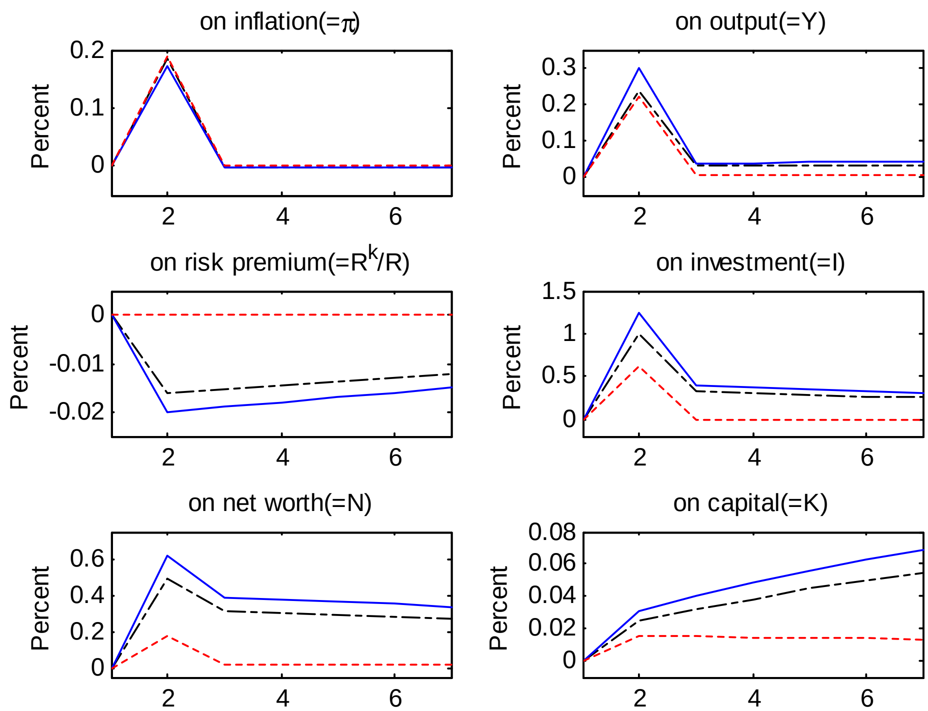

Figure 1: Impulse Responses to an 1% Expansionary Monetary Policy Shock: zero credit market friction with low inflation (Dotted line), positive credit market friction with low inflation (Dash-dotted line) and positive credit market friction with high inflation (Solid line)

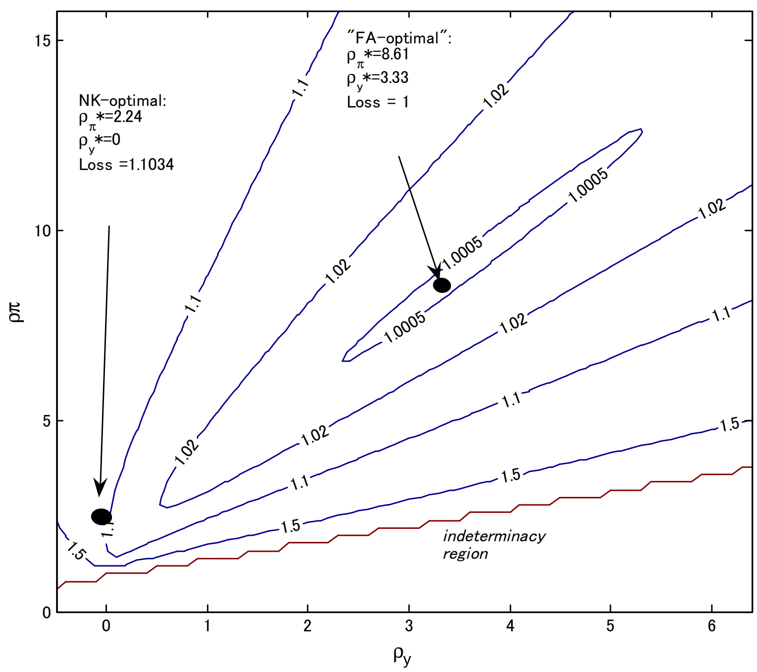

Figure 2: Isoloss Contour Map Showing Relative Loss Under Different Policies: the numbers in the figure are normalized so that the FA-optimal rule produces a relative loss of one and the contour lines represent gross deviations from the FA-optimal policy.