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Measurement of magnetic fluctuations by $O \to X$ mode conversion

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The possibility of measuring magnetic fluctuations in a fusion plasma is considered by examining the O–X mode conversion. Under certain conditions and with good angular resolution, this mode conversion can be attributed to the presence of magnetic fluctuations even though the level of these fluctuations is much lower than that of density fluctuations. Some nonideal effects such as mode polarization mismatch at the plasma edge are also discussed.

I. INTRODUCTION

It is of considerable importance to develop diagnostics that can detect magnetic fluctuations in a fusion plasma, especially because of their expected interconnection with transport.[1] Such a diagnostic must overcome a universal feature present in tokamaks: the level of magnetic fluctuations, $\delta B/B_0$, is much lower than that of the electron density fluctuations, $\delta n/N_0$. Typically, $\delta n/N_0$ is determined by electron drift wave fluctuations:[2] $\delta n/N_0 \approx 3\rho_s/L_n$, where ρ_s is basically the electron gyroradius and L_n the density gradient length scale. On the other hand, magnetic fluctuations are driven by magnetohydrodynamic processes which have a low-frequency, long-wavelength coherent component and a high-frequency, short-wavelength incoherent part. Scattering experiments typically measure large m (≈ 100) fluctuations, where m is the poloidal mode number. These magnetic fluctuations have been measured in the edge region with Mirnov loops,[3] and, in very small devices, with interior magnetic probes.[4] High-frequency probe measurements[4] give $\delta B/B_0 \approx 10^{-4}$–$10^{-5}$ in the interior of Microtor for frequencies $\leqslant 30$ kHz, while $\delta B/B_0 \approx 10^{-5}$–$10^{-6}$ at the edge of Tokamak Fusion Test Reactor (TFTR)[3] in the frequency range 100–150 kHz. Thus, for the interior of TFTR one might expect a fluctuation ratio

$$\left(\frac{\langle \delta B_1^2\rangle/B_0^2}{\langle \delta n^2\rangle/N_0^2}\right)_{m\gg 1} \approx 10^{-4}\text{–}10^{-5}. \qquad (1)$$

Here we explore the possibilities of electromagnetic scattering of an ordinary (O) $\to$ extraordinary (X) mode as a possible diagnostic[5] in a high-temperature plasma. The incident O mode approaches a cutoff near the local electron plasma frequency while the X mode will propagate through this cutoff layer and be detected. In particular, for mode propagation perpendicular to the magnetic field **B**, the O mode is linearly polarized while the X mode is elliptically polarized. If the incident and scattered angles are chosen to be both perpendicular to the **B** field, then density fluctuations, being scalar in nature, cannot force this $O \to X$ mode conversion. However, the tensorial magnetic fluctuations can facilitate in this mode conversion.

These conclusions are strictly true only for exactly perpendicular propagation to **B**. Because the density fluctuation levels are so much higher than those of the magnetic fluctuations, Eq. (1), the effects of finite beamwidth and finite angular resolution should also be considered. These effects are examined here by following a bundle of modes using a toroidal ray tracing code and examining the refraction of this bundle. For an appropriate choice of scattering parameters, we find in Sec. II that refractive effects on the mode bundle are very small. This then allows us, in Sec. III, to use the scattering formalism of Sitenko[6] for a locally homogeneous plasma to derive the differential scattering cross section for an $O \to X$ mode conversion. Finally in Sec. IV, we show that the generation of X modes due to incident polarization mismatch with **B** at the plasma edge and the subsequent $X \to X$ scattering by density fluctuations will not mask the $O \to X$ conversion due to magnetic fluctuations.

II. RAY TRACING

To examine the refractive effects on a bundle of O modes incident nearly perpendicular to **B**, from either the low or high magnetic field side, we employ the TORCH ray tracing code.[7] The O modes (with incident frequency ω_i) will be reflected near the cutoff position r_c where the incident frequency becomes equal to the local electron plasma frequency, $\omega_i = \omega_{pe}(r_c)$, as shown in Fig. 1(a). On the other hand, in the neighborhood of r_c, a corresponding bundle of X modes will propagate out of the plasma perpendicular to **B** and with little refraction, as shown in Fig. 1(b). Moreover,[5] the O modes and X modes that propagate perpendicular to **B** retain their identity in that the polarization of these modes remains invariant relative to the local magnetic field in the eikonal approximation. This is also consistent with the observation[8] of tokamak emission at $2\omega_{ce}$ in which the perpendicularly polarized mode retained its orientation relative to **B** in the emitting layer as well as to **B** at the plasma edge.

One can thus conclude from Fig. 1 that under appropriate scattering orientations, the O–X mode conversion around r_c can be examined by treating the system as locally homogeneous.

III. SCATTERING THEORY

Consider the Sitenko[6] formalism for scattering from fluctuations in a locally homogeneous magnetized plasma.

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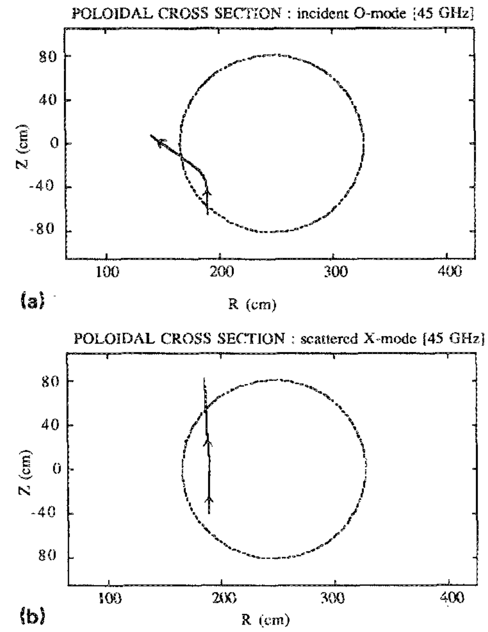

FIG. 1. (a) Result of ray tracing in the poloidal plane (R,Z) for a bundle of 45 GHz O modes (eight modes with a total angular width 4° about $\pi/2$) incident on the high field side in TFTR from below. The O modes approach a cutoff at r_c and are reflected (with some refraction) out of the plasma. (b) A bundle of 45 GHz X-mode ray trajectories in the poloidal plane emanating from r_c, perpendicular to **B**. Refractive effects in the poloidal plane are very small, and near the cutoff r_c there is no refraction in the X modes since the refractive index here is unity.

The incident plane electromagnetic wave satisfies the wave equation

$$\nabla\times\nabla\times\mathbf{E}_{\rm inc}+\frac{1}{c^2}\,\boldsymbol{\varepsilon}_i\cdot\frac{\partial^2\mathbf{E}_{\rm inc}}{\partial t^2}=0,$$

where $\boldsymbol{\varepsilon}_i$ is the plasma dielectric function. The scattered field $\mathbf{E}_s$ is given by

$$\nabla\times\nabla\times\mathbf{E}_s+\frac{1}{c^2}\,\boldsymbol{\varepsilon}_s\cdot\frac{\partial^2\mathbf{E}_s}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial\mathbf{J}}{\partial t},$$

where $\mathbf{J}$ is the current due to the interaction of the incident wave $(\omega_i,\mathbf{k}_i)$ with plasma density and magnetic fluctuations $\delta n_e(\omega,\mathbf{k})$ and $\delta\mathbf{B}(\omega,\mathbf{k})$. From the conservation of momentum and energy, the scattered wave number and frequency satisfy $\mathbf{k}_s=\mathbf{k}_i+\mathbf{k}$ and $\omega_s=\omega_i+\omega$. Thus

$$\left[\frac{c^2k_s^2}{\omega_s^2}\left(\frac{k_{s,\alpha}k_{s,\beta}}{k_s^2}-\delta_{\alpha\beta}\right)+\varepsilon_{\alpha\beta}(\omega_s,\mathbf{k}_s)\right]E_{s,\beta}(\omega_s,\mathbf{k}_s)$$
$$=E_{{\rm inc},\beta}(\omega_i,\mathbf{k}_i)\biggl(\frac{\omega_i}{\omega_s}[\delta_{\alpha\beta}-\varepsilon_{\alpha\beta}(\omega_i,\mathbf{k}_i)]\,\frac{\delta n_e(\omega,\mathbf{k})}{N_0}$$
$$+\frac{ie}{m_ec}\frac{\omega_i}{\omega_{\rm pe}^2}[\delta_{\alpha\gamma}-\varepsilon_{\alpha\gamma}(\omega_s,\mathbf{k}_s)]$$
$$\times\varepsilon_{\gamma\xi\eta}[\delta_{\xi\beta}-\varepsilon_{\xi\beta}(\omega_i,\mathbf{k}_i)]\delta B_\eta(\omega,\mathbf{k})\biggr),$$

where $\varepsilon_{\gamma\xi\eta}$ is the standard Levi–Citiva symbol, and N_0 the background density (with summation over repeated Greek subscripts). Hence the radiation field at the observation position $\mathbf{r}_s$ is

$$\mathbf{E}_s(\mathbf{r}_s,t)\approx\frac{1}{r_s}\sum_{k_s}{}'\frac{(\mathbf{J}\cdot\bar{\mathbf{e}}_s)\bar{\mathbf{e}}_s}{|\nabla\Lambda||K|^{1/2}}\exp[i\mathbf{k}_s\cdot\mathbf{r}_s-i\omega_st]+\cdots,$$

where Σ' denotes the sum over all wave numbers $\mathbf{k}_s$ that lie on the surface $\Lambda=0$ such that the group velocity (in the direction $\nabla\Lambda$) is parallel to the observer position $\mathbf{r}_s$. $\mathbf{e}_s$ is the normalized polarization of the scattered electric field and Λ is defined by

$$\Lambda\equiv\det\left[\frac{c^2k_s^2}{\omega_s^2}\left(\frac{k_{s,\alpha}k_{s,\beta}}{k_s^2}-\delta_{\alpha\beta}\right)+\varepsilon_{\alpha\beta}(\omega_s,\mathbf{k}_s)\right].$$

K is the Gaussian curvature of the surface $\Lambda=0$ at the points $\mathbf{k}_s$. From the average power at $\mathbf{r}_s$ one can readily determine the differential scattering cross section

$$d\sigma=G\biggl[|\xi|^2\frac{\langle\delta n^2\rangle_{\omega,\mathbf{k}}}{N_0^2}+\frac{\omega_{\rm ce}^2\omega_s^2}{\omega_{\rm pe}^4}a_\alpha a_\beta^*\frac{\langle\delta B_\alpha\delta B_\beta\rangle_{\omega,\mathbf{k}}}{B_0^2}$$
$$+\frac{2\omega_{\rm ce}\omega_s}{\omega_{\rm pe}^2}{\rm Im}\left(\xi a_\alpha^*\frac{\langle\delta n\delta B_\alpha\rangle_{\omega,\mathbf{k}}}{N_0B_0}\right)\biggr],\qquad(2)$$

where

$$\xi\equiv e_{s,\alpha}^*[\varepsilon_{\alpha\beta}(\omega_i,\mathbf{k}_i)-\delta_{\alpha\beta}]e_{i,\beta},$$
$$a_\alpha\equiv\varepsilon_{\alpha\beta\gamma}e_{s,\xi}^*[\varepsilon_{\xi\beta}(\omega_s,\mathbf{k}_s)-\delta_{\xi\beta}]\,[\varepsilon_{\gamma\rho}(\omega_i,\mathbf{k}_i)-\delta_{\gamma\rho}]e_{i,\rho},$$

$\mathbf{e}_i$ is the incident electric field polarization vector and $\omega_{\rm ce}$ is the electron gyrofrequency. G is a complicated geometric factor which only affects the overall magnitude of the cross section for a particular scattering process. It is considered in some detail by Hughes and Smith[9] following the analysis of Simonich and Yeh.[10] The third term in Eq. (2), the cross correlation term, can be shown to have a negligible effect on the differential cross section $d\sigma$ and so need not be considered further. Our major concern is to be able to make the dominant contribution to $d\sigma$ come from magnetic fluctuations, the second term in Eq. (2).

For perpendicular propagation to $\mathbf{B}$ ($\theta_i=\pi/2=\theta_s$, Fig. 2), one finds from Eq. (2) that an $O\rightarrow X$ mode conversion can only arise from the presence of magnetic fluctuations in the plasma. However, one must incorporate the effects of finite beamwidths and finite angular resolution, especially since the level of magnetic fluctuations is generally much lower than that of the density fluctuations. In Fig. 3 we plot the relative contributions of the second and first terms in $d\sigma$, Eq. (2), as a function of the relative intensities of magnetic/density fluctuations for various tokamaks for a scattering angle of $\Phi\approx0^\Omega$. Typically, we assume a total angular spread of 1^Ω about $\Phi=0^\Omega$ and consider an O mode

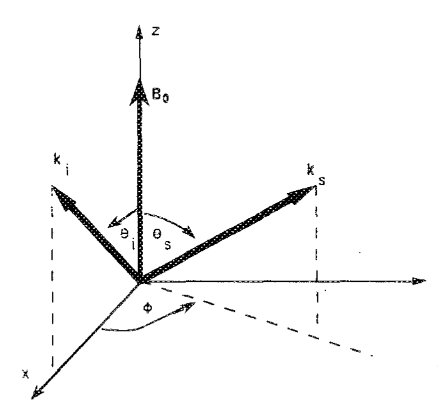

FIG. 2. Scattering geometry used in calculating the differential scattering cross section $d\sigma$ (here we choose $\theta_i=\theta_s=\pi/2$). The optimal scattering angle is forward scattering, $\Phi\approx0^\Omega$.

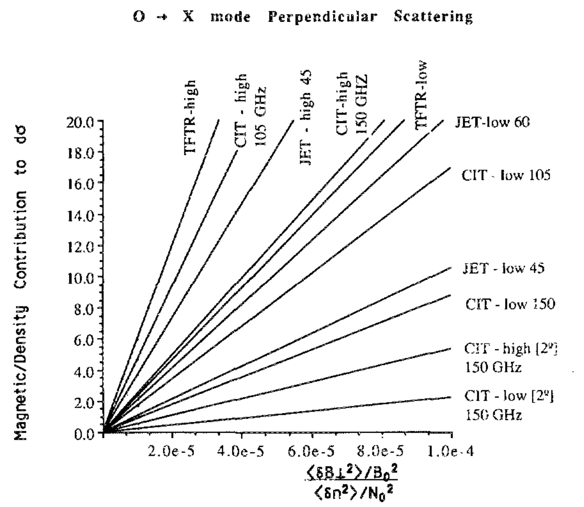

FIG. 3. Relative contribution of magnetic to density fluctuations to the differential scattering cross section $d\sigma$ as a function of their relative intensities for an angular width of 1° (unlabeled) or 2° (labeled) about $\pi/2$. Various tokamaks are considered (TFTR:$\omega_i = 45$ GHz, CIT: $\omega_i = 105$ and 150 GHz, JET:$\omega_i = 45$ and 60 GHz). The advantage of using the high-field side over the low-field side is somewhat illusionary. It is due to the larger gyrofrequency at r_c, but to sustain the same level of $\langle \delta \mathbf{B}_\perp \rangle / \mathbf{B}_0^2$ one requires a correspondingly higher level of magnetic fluctuations.

incident along the midplane on both the low-field and high-field sides. For typical drift wave fluctuation magnitudes[2] and a TFTR plasma with an incident O mode of frequency $\omega_i = 45$ GHz, an appropriate level of magnetic to density fluctuations is[1]

$$\frac{\langle \delta \mathbf{B}_\perp^2 \rangle / \mathbf{B}_0^2}{\langle \delta n^2 \rangle / N_0^2} \approx 6.0 \times 10^{-5}.$$

If the mode is incident on the low-field side, then the magnetic fluctuation contribution to the differential cross section for the $O \to X$ conversion is about 14 times greater than that arising from the density fluctuation contribution. However, if the O mode is injected on the high-field side (Fig. 1), then the ratio of magnetic to density contribution to the differential cross section rises to a factor of 36.

IV. DISCUSSION

An important nonideal effect that remains to be discussed is the $O \to X$ conversion that can occur due to mode polarization mismatch with **B** near the plasma edge.[11] This X mode can then be scattered by *density* fluctuations into another X mode, possibly masking the $O \to X$ conversion due to magnetic fluctuations. However, from a full wave calculation, Brambilla and Moresco[12] have shown that the power fraction converted to an X mode from the incident O mode due to polarization mismatch is $\approx 10^{-5}$ for low-shear tokamak plasmas. Hence, for small forward scattering angles ($\Phi \approx 0^\Omega$, see Fig. 2) one finds from Eq. (2) that the cross section ratio for $O \to X$ scattering by magnetic fluctuations to that for $O \to X$ due to edge mismatch with **B** and subsequent $X \to X$ due to density fluctuations (in the same frequency range as the magnetic fluctuations and an $\omega_{ce}/\omega_i \approx 3.96$)

$$\frac{d\sigma_{O \to X}^{\delta B}}{d\sigma_{O \cdot X \to X}^{\delta n}} \approx \frac{G_{\delta B}}{G_{\delta n}} \frac{\omega_{ce}^2}{\omega_i^2} \left(\frac{\langle \delta \mathbf{B}_\perp^2 \rangle / \mathbf{B}_0^2}{\langle \delta n^2 \rangle / N_0^2} \right) \times 10^5.$$

Moreover, the corresponding geometric factor for $X \to X$ scattering is $G_{\delta n} \approx \omega_i^2 / \omega_{ce}^2 \approx 0.064$, while for $O \to X$ scattering the geometric factor $G_{\delta B} \gg 1$, due to the cutoff layer. Thus, it appears that polarization mode mismatch at the edge will not mask the effect of magnetic fluctuation scattering even for magnetic to density fluctution ratios as low as 10^{-6}. Other nonideal effects, such as the horn limitations on the polarization of the emitted wave, will be considered in more detail elsewhere.

Hence it appears that millimeter $O \to X$ mode conversion can be used as a diagnostic for magnetic fluctuations.

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