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NUMERICAL SOLUTIONS FOR WEAKLY SINGULAR HAMMERSTEIN EQUATIONS AND THEIR SUPERCONVERGENCE

HIDEAKI KANEKO, RICHARD D. NOREN AND YUESHENG XU

ABSTRACT. In the recent paper [7], it was shown that the solutions of weakly singular Hammerstein equations satisfy certain regularity properties. Using this result, the optimal convergence rate of a standard piecewise polynomial collocation method and that of the recently proposed collocation-type method of Kumar and Sloan [10] are obtained. Superconvergence of both of these methods are also presented. In the final section, we discuss briefly a standard product-integration method for weakly singular Hammerstein equations and indicate its superconvergence property.

1. Introduction. We consider the Hammerstein equation with weakly singular kernel

\[ \varphi(s) - \int_a^b g_\alpha(|s-t|)k(s,t)\psi(t,\varphi(t))\,dt = f(s), \quad a \leq s \leq b, \]

where

\[ g_\alpha(s) = \begin{cases} 
  s^{\alpha-1} & \text{for } 0 < \alpha < 1 \\
  \log s & \text{for } \alpha = 1.
\end{cases} \]

Throughout this paper, we assume that

(i) \( k \in C([a, b] \times [a, b]) \)

(ii) \( \psi \in C([a, b] \times (-\infty, \infty)) \) and satisfies the Lipschitz condition \( |\psi(t, y_1) - \psi(t, y_2)| \leq A|y_1 - y_2| \).

In the recent paper [7], it was shown that under assumptions (i), (ii) and

(iii) \( AG < 1 \), where \( G \equiv \sup_{a \leq s \leq b} \int_a^b |g_\alpha(|s-t|)k(s,t)|\,dt \),

there is a unique solution to equation (1.1).

Generalizing the argument of C. Schneider [14], regularity properties of the solution \( \varphi \) were also obtained in [7]. For our present purposes, these results can be summarized as follows: