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NUMERICAL SOLUTIONS FOR WEAKLY SINGULAR HAMMERSTEIN EQUATIONS AND THEIR SUPERCONVERGENCE

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ABSTRACT. In the recent paper [**7**], it was shown that the solutions of weakly singular Hammerstein equations satisfy certain regularity properties. Using this result, the optimal convergence rate of a standard piecewise polynomial collocation method and that of the recently proposed collocation-type method of Kumar and Sloan [**10**] are obtained. Superconvergence of both of these methods are also presented. In the final section, we discuss briefly a standard product-integration method for weakly singular Hammerstein equations and indicate its superconvergence property.

1. Introduction. We consider the Hammerstein equation with weakly singular kernel

$$\varphi(s) - \int_a^b g_\alpha(|s-t|)k(s,t)\psi(t,\varphi(t))\,dt = f(s), \quad a \le s \le b, \tag{1.1}$$

where

$$g_\alpha(s) = \begin{cases} s^{\alpha-1} & \text{for } 0 < \alpha < 1 \\ \log s & \text{for } \alpha = 1. \end{cases}$$

Throughout this paper, we assume that

(i) $k \in C([a,b]\times[a,b])$

(ii) $\psi \in C([a,b]\times(-\infty,\infty))$ and satisfies the Lipschitz condition $|\psi(t,y_1)-\psi(t,y_2)| \le A|y_1-y_2|$.

In the recent paper [**7**], it was shown that under assumptions (i), (ii) and

(iii) $AG < 1$, where $G \equiv \sup_{a\le s\le b}\int_a^b |g_\alpha(|s-t|)k(s,t)|\,dt$,

there is a unique solution to equation (1.1).

Generalizing the argument of C. Schneider [**14**], regularity properties of the solution φ were also obtained in [**7**]. For our present purposes, these results can be summarized as follows: