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Nonlinear interaction of a detonation/vorticity wave

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The interaction of an oblique, overdriven detonation wave with a vorticity disturbance is investigated by a direct two-dimensional numerical simulation using a multidomain, finite-difference solution of the compressible Euler equations. The results are compared to those of linear theory, which predict that the effect of exothermicity on the interaction is relatively small except possibly near a critical angle where linear theory no longer holds. It is found that the steady-state computational results whenever obtained in this study agree with the results of linear theory. However, for cases with incident angle near the critical angle, moderate disturbance amplitudes, and/or sudden transient encounter with a disturbance, the effects of exothermicity are more pronounced than predicted by linear theory. Finally, it is found that linear theory correctly determines the critical angle.

I. INTRODUCTION

The passage of a weak shear disturbance through a reactive shock wave, or detonation, was examined by Jackson *et al.*[1] Supersonic engines based on oblique, overdriven, reacting shock waves have been proposed as a possible alternative to the SCRAMJET for high-speed propulsion.[2] It is still a matter of exploration as to whether or not such waves can be stabilized. Of particular interest in this investigation was the effect of heat release on the refraction and amplification of the vorticity disturbance and the simultaneous generation of acoustic and entropy signals behind the overdriven detonation. The detonation was assumed to be at an angle to the base flow, and the normal Mach number of the gas ahead of the detonation front was taken to be greater than the Mach number of a Chapman–Jouget wave. The vorticity disturbance was assumed to be a small amplitude, planar, shear wave with wave vector parallel to the base flow (i.e., transverse disturbances). There exists a critical angle, dependent upon the exothermicity of reaction and the overdrive, such that the relative velocity of the base flow behind the front is subsonic for $\theta < \theta_c$ and supersonic for $\theta > \theta_c$. In the former case, the amplitude of the generated acoustic disturbance is exponentially decaying behind the detonation; in the latter case, the amplitude is constant. The critical angle approaches zero as the Chapman–Jouget limit is approached. It was found that the vorticity was significantly amplified by the exothermicity and that the generated acoustic response is most affected by exothermicity near the critical angle. Furthermore, the manner in which the shape and structure of the detonation are altered by the disturbance was also investigated.

The analysis was accomplished by considering the detonation wave, which consists of a lead shock, an induction zone, and a fire zone, as a discontinuity on the length scale of the disturbance. The discontinuity separates an unburnt mixture of reactants from a burnt mixture of reaction products. After superimposing a vorticity disturbance in the form of a planar shear wave, the generalized Rankine–Hugoniot conditions were linearized about the base flow providing the conditions to determine the amplitudes and angles of the transmitted vorticity wave and the generated acoustic and entropy waves. The effect of the disturbance on the structure of the detonation was found by considering the limit of large activation energy E. The thickness of the induction zone (the region between the lead shock and the fire zone) is a measure of the thickness of the detonation. The lead shock and the fire zone were treated as discontinuities on a length scale comparable to the induction zone thickness. The equations governing the perturbations within the induction zone due to disturbances ahead of the lead shock were derived, and the fire zone position was determined as the point at which the the solutions become singular. It was found that disturbance amplitudes $O(E^{-1})$ have an $O(1)$ effect on the fire zone position.

The aim of the present investigation is to examine the predictions of linear theory and to determine by means of a direct two-dimensional numerical simulation the regions of validity for linear theory. For this investigation, the detonation is treated as a discontinuity, and the generalized Rankine–Hugoniot relations are used to provide appropriate jump conditions across the detonation. The effect of the disturbances on the internal structure of the detonation is not considered. Of particular interest is the behavior of the solution for angles near critical where linear theory, if it applies, predicts that exothermicity can have a significant effect on the generation of acoustic signals.

II. NUMERICAL SIMULATION

To study the interaction of a detonation wave with a vorticity disturbance, a numerical approach similar to the one utilized by Zang *et al.*[3,4] is considered. The major differences are that the present routine takes advantage of the periodic nature of the solution in the transverse direction and also uses a multidomain scheme in the normal direction.

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Rather than using coordinates in which the position of the detonation is stationary, it is assumed that at time $t = 0$ an infinite, planar detonation wave starts at $x = 0$ and propagates into the unburnt mixture. The position of the detonation $x_s(y,t)$ is calculated by using the improved shock-fitting approach of Kopriva *et al.*,[5] generalized to include exothermicity. The initial conditions are chosen such that in the absence of disturbances the detonation wave will propagate to the right with a Mach number (relative to the gas in front of the detonation wave) greater than the Chapman–Jouget Mach number. The base flow in the region ahead of the detonation has only a vertical component so that the net result of the velocity of the detonation front in the positive x direction and the vertical component produce a base flow at an angle θ in a coordinate system attached to the detonation front. Superimposed on the uniform base flow is a vorticity disturbance which propagates at the same angle θ to the x axis (see Fig. 1). The disturbance is assumed to have unit wavelength, $2\pi/|\mathbf{k}| = 1$, where $\mathbf{k}$ is the wave vector. The flow is therefore periodic in the y direction with period $2\pi/k_y = \sec\theta$. The initial flow behind the detonation front has a vertical component such that the tangential velocity across the front is continuous. It should be noted that the flow ahead of the front is prescribed and is used to impose the appropriate jump conditions across the discontinuity.

The physical domain in which the fluid motion is computed is given by

$$x_L \leqslant x \leqslant x_s(y,t), \quad 0 \leqslant y \leqslant \sec(\theta), \quad t \geqslant 0, \tag{1}$$

where x_s is the shock position and $x_s(y,0) = 0$. The left boundary x_L is some suitably chosen negative number (usually minus one). This domain takes advantage of the periodic nature of the solution in the y direction. In the x direction, a multidomain approach is used which allows for a greater number of grid points near the detonation front where greater accuracy is needed and fewer grid points away from the front where the solution is smoother. The interfaces between the domains are denoted by $x_i(y,t)$ where the left boundary is $x_0(y,t) = x_L$, the detonation front is $x_n(y,t) = x_s(y,t)$, and the planar interfaces between the domains are $x_i(y,t) = x_i(t)$, for $i = 1,\ldots,n-1$. Since the behavior of the solution near the critical angle is of interest, it is necessary to continue the calculations until the detonation wave penetrates a significant distance into the unburnt mixture. Because of this expanding domain, a greater number of grid points in the x direction is needed at the end of the calculation than is needed in the beginning. The resolution in each subdomain is monitored, and the number of grid points is increased when necessary. A linear interpolation is used to transfer the solution to the finer grid. The numerical routine also allows for ramping of the amplitude of the imposed disturbance, thereby decreasing in size the portion of the transient caused by the sudden encounter of the undisturbed detonation with the vorticity wave. The approach to steady state was smoother, allowing an accurate comparison to linear theory.

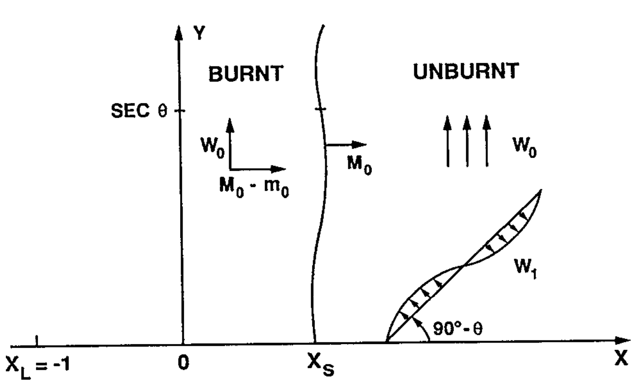

FIG. 1. Schematic of model problem in the physical domain. Velocity of unburnt mixture is $W_0 + W_1$.

The following change of variables is made to computational coordinates

$$X_i = \frac{x - x_{i-1}(y,t)}{x_i(y,t) - x_{i-1}(y,t)}, \quad i = 1,\ldots,n, \tag{2a}$$

$$Y = y\cos(\theta), \tag{2b}$$

$$T = t. \tag{2c}$$

The computational domains are therefore

$$0 \leqslant X_i \leqslant 1, \quad 0 \leqslant Y \leqslant 1, \quad T \geqslant 0. \tag{3}$$

The fluid motion is governed by the two-dimensional Euler equations. In terms of the computational coordinates these are

$$\mathbf{Q}_T + B\mathbf{Q}_{X_i} + C\mathbf{Q}_Y = 0, \quad i = 1,\ldots,n, \tag{4}$$

where

$$\mathbf{Q} = [P,u,v,S]^T, \tag{5a}$$

$$B = \begin{bmatrix} U_i & \gamma X_{i,x} & \gamma X_{i,y} & 0 \\ c^2 X_{i,x}/\gamma & U_i & 0 & 0 \\ c^2 X_{i,y}/\gamma & 0 & U_i & 0 \\ 0 & 0 & 0 & U_i \end{bmatrix}, \tag{5b}$$

$$C = \begin{bmatrix} V & \gamma Y_x & \gamma Y_y & 0 \\ c^2 Y_x/\gamma & V & 0 & 0 \\ c^2 Y_y/\gamma & 0 & V & 0 \\ 0 & 0 & 0 & V \end{bmatrix}, \tag{5c}$$

$$U_i = X_{i,t} + uX_{i,x} + vX_{i,y}, \tag{5d}$$

$$V = Y_t + uY_x + vY_y. \tag{5e}$$

Here, P is the natural logarithm of pressure, c is the local sound speed, and S is the entropy divided by specific heat at constant volume. The velocities u and v, in the x and y directions, respectively, are scaled by the sound speed ahead of the shock. The ratio of specific heats γ is taken to be 1.4 for all calculations.

As in Zang *et al.*,[3,4] the equations are discretized in each subdomain using the finite difference method of MacCormack,[6] which is a variant of the Lax–Wendroff method. Conditions at the right boundary, at the interface between the domains, and at the left boundary need to be imposed to determine the system completely. At the right boundary, the improved shock-fitting routine of Kopriva *et al.*[5] has been generalized to include exothermicity and time dependence of the flow ahead of the shock. Details are given in the Appendix. At the interface between domains, a routine similar to Kopriva[7] is used. Since the interfaces are perpendicular to the X axis, the derivatives in Y can be calculated by using the values of $\mathbf{Q}$ along the interface. The values of the derivatives in X, however, must be taken from the left or right of the interface or a combination of both. The method chosen here is

$$\mathbf{Q}_t^I + \tfrac{1}{2}(B + |B|^*)\mathbf{Q}_X^L + \tfrac{1}{2}(B - |B|^*)\mathbf{Q}_X^R + C\mathbf{Q}_Y^I = 0, \quad (6)$$

where $|B|^*$ is an approximation to $|B| = P|\Lambda|P^{-1}$, Λ is a diagonal matrix with the eigenvalues of B on the diagonal, and P is a matrix of the eigenvectors of B. The superscript I refers to values calculated on the interface and the superscripts R and L refer to the finite difference approximations to the X derivative in the subdomain to the right and to the left of the interface, respectively. The approximation used for these calculations is

$$|B|^* = \begin{bmatrix} \tfrac{1}{2}|U + c\phi| + \tfrac{1}{2}|U - c\phi| & 0 & \tfrac{1}{2}|U + c\phi| - \tfrac{1}{2}|U - c\phi| & 0 \\ 0 & |U| & 0 & 0 \\ \tfrac{1}{2}|U + c\phi| - \tfrac{1}{2}|U - c\phi| & 0 & \tfrac{1}{2}|U + c\phi| + \tfrac{1}{2}|U - c\phi| & 0 \\ 0 & 0 & 0 & |U| \end{bmatrix}, \quad (7)$$

where $\phi = (X_x^2 + X_y^2)^{1/2}$. The conditions imposed at the left boundary naturally depend on the inflow at the boundary being supersonic or subsonic. If the inflow is supersonic, then all four components of $\mathbf{Q}$ can be prescribed; however, if the inflow is subsonic, only three of the four components can be prescribed at the left boundary. For the parameters of this study, the inflow is always supersonic. It should be mentioned that the normal component of the flow behind the detonation is supersonic in the laboratory frame; however, the normal component of the flow behind the detonation is subsonic in a frame of reference moving with the detonation.

III. RESULTS AND COMPARISON WITH LINEAR THEORY

A. Linear theory

Consider a coordinate system attached to the detonation front such that the speed of the detonation is zero. The flow ahead of the front is assumed to be at an angle θ to the front, the normal component of the velocity has Mach number M_0, and the vertical component of the velocity is W_0 (see Fig. 1). The generalized Rankine–Hugoniot conditions for a detonation with heat release parameter α provides that the normal component of the flow behind the front has Mach number (relative to the upstream speed of sound)

$$m_0 = \frac{1 + \gamma M_0^2}{(\gamma + 1)M_0} - \frac{1}{(\gamma + 1)} \times \left(\frac{(M_0^2 - 1)^2}{M_0^2} - 2(\gamma + 1)\alpha\right)^{1/2} \quad (8)$$

and requires that the tangential components be continuous

$$w_0 = W_0. \quad (9)$$

Therefore, the angle of the base flow behind the detonation is given by

$$\tan\phi = (M_0/m_0)\tan\theta. \quad (10)$$

Superimposed on the upstream base flow is a sinusoidal disturbance in the velocity given by

$$\mathbf{U}_1 = \epsilon \mathbf{U}_0 \cos(\mathbf{k}'\cdot\mathbf{x}), \quad (11)$$

where $\mathbf{U}_0 = (M_0, W_0)$ and $\mathbf{U}_0\cdot\mathbf{k}' = 0$. When this constant pattern of vorticity is convected through the detonation front, the vorticity wave is refracted and amplified, and an acoustic and entropy wave are generated. Linear analysis provides analytic expressions for the amplitudes and angles of all three disturbances downstream. The result is that the following planar waves are superimposed on the base flow and the pressure downstream ($x < 0$):

$$\mathbf{u}_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \epsilon|\mathbf{U}_0|S\begin{bmatrix} \cos(\mathbf{k}\cdot\mathbf{x} + \delta_s) \\ 0 \end{bmatrix} + \epsilon|\mathbf{U}_0|P(x)\begin{bmatrix} \cos(\mathbf{k}_p\cdot\mathbf{x} + \delta_p) \\ \beta\sin(\mathbf{k}_p\cdot\mathbf{x} + \delta_p) \end{bmatrix}, \quad (12)$$

$$p_1 = -\epsilon\gamma|\mathbf{U}_0|M_0 \sec\phi P(x)\cos(\mathbf{k}_p\cdot\mathbf{x} + \delta_p), \quad (13)$$

where

$$S = \begin{cases} S^0, & \mu > 1, \\ S_0, & \mu < 1, \end{cases} \quad (14)$$

$$P = \begin{cases} P^0, & \mu > 1, \\ P_0\exp(\lambda x), & \mu < 1, \end{cases} \quad (15)$$

and μ is the local Mach number of the relative base flow behind the detonation front. The constants S^0, S_0, P^0, P_0, δ_s, δ_p, and λ are given in Jackson *et al.*[1] The vector $\mathbf{k}$ is perpendicular to the downstream base flow and the vector $\mathbf{k}_p$ is perpendicular to the direction of acoustic propagation which makes an angle ϕ' with the x axis (see Fig. 2). This angle is given by

$$\phi' = \begin{cases} \phi - \cot^{-1}(\beta), & \mu > 1, \\ -\tan^{-1}[(\mu/\beta_n)^2\cos^2\phi\tan\phi], & \mu < 1, \end{cases} \quad (16)$$

with $\beta = (|1 - \mu^2|)^{1/2}$ and $\beta_n^2 = 1 - \mu^2\cos^2\phi$. A similar disturbance is also superimposed on the temperature and density due to the generated entropy wave.

B. Nonlinear calculations

The comparison of the two-dimensional calculations to the linear theory consists of comparing the amplitudes of the

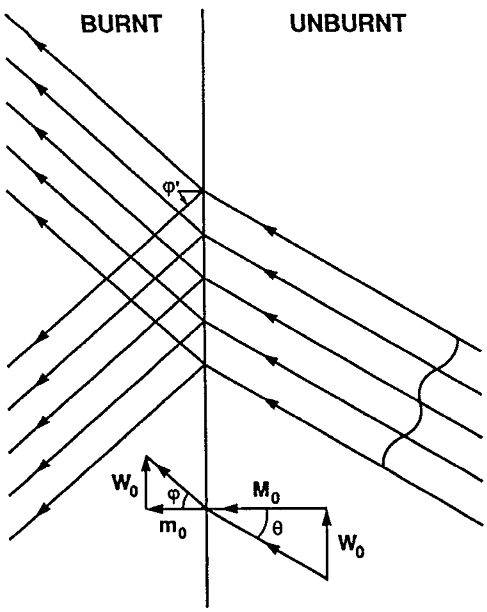

FIG. 2. Schematic of model problem for linear theory. Acoustic disturbances in the burnt gas propagate at angle ϕ' to the x axis. Vorticity and entropy disturbances are convected with the mean flow at angle ϕ.

vorticity and acoustic waves behind the detonations with the predictions. To achieve this comparison, a least-squares fit to the functional forms

$$u_y - v_x = S_1 \cos(k_y' y) + S_2 \sin(k_y' y), \tag{17}$$

$$P = P_0 + P_1 \cos(k_y' y) + P_2 \sin(k_y' y) \tag{18}$$

is performed for the computed vorticity and acoustic fields for each value of x. The amplitudes of the above vorticity and acoustic disturbances $\bar{S} = (S_1^2 + S_2^2)^{1/2}$ and $\bar{P} = (P_1^2 + P_2^2)^{1/2}$ are divided by $\epsilon|\mathrm{U}_0|k_y'$ and $\epsilon\gamma|\mathrm{U}_0|M_0 \sec\phi$, respectively, and graphed as a function of x for various times t. In the calculations presented here, the front is initially at position $x = 0$, and behind the front there is no disturbance (i.e., $\bar{P} = 0$ and $\bar{S} = 0$). As time progresses, the front moves downstream into a region with a nonzero vorticity disturbance. Therefore, the numerical solutions include transient behavior. The relaxation to steady state depends on both the acoustic and the vorticity responses as well as the angle of incidence. If θ is above the critical angle, then the predicted acoustic response is constant; therefore, steady state is indicated by a broad flat response behind the front. If θ is below the critical angle, an exponentially decaying acoustic response is predicted; therefore, steady state is indicated when the value of the acoustic response at the front remains essentially constant as the front propagates a sufficient distance. Since the vorticity response is constant in both cases, steady state is indicated when a broad flat vorticity response behind the shock has been achieved.

For many of the calculations involving either large disturbance amplitudes or angles of incidence near critical, the solution did not asymptote to a steady state even after the detonation front had penetrated a significant distance into the unburnt mixture. The nature of this transient behavior is important since real turbulence consists of sudden nonsteady phenomena, not steady plane waves. In general, the transient depends on the suddenness in which the front meets with the full disturbance, the amplitude of the disturbance, and the angle of incidence. To resolve the portion of the transient caused by the sudden encounter of the front with a nonzero disturbance, the calculations allow for a slow or fast ramping of the amplitude of the imposed disturbance; the amplitude as a function of time is given by

$$\epsilon(t) = \begin{cases} \epsilon_0 (M_0 t/R)^2 (3 - 2M_0 t/R), & t < R/M_0, \\ \epsilon_0, & t > R/M_0. \end{cases} \tag{19}$$

Therefore, when the position of the front is $x_s = R$, the flow ahead of the front will have its maximum disturbance amplitude.

The value of the exothermicity parameter α is chosen to be equal to two for all calculations. This value is chosen since linear theory predicts that the value of $\bar{S}$ will be significantly different from its value in the nonreacting case for almost all values of angle θ and the value of $\bar{P}$ will be significantly different for values of the angle near critical. The strength of the detonation is chosen so that the normal Mach number of the flow is 1.5 times the Chapman–Jouget number. The critical angle for these parameter values is $\theta_c = 24.89°$.

The pressure and vorticity responses to a 1% disturbance at an angle of $\theta = 40°$ for ramping parameter $R = 6.0$ and $R = 0.5$ are shown in Figs. 3(a) and 3(b), respectively. For these calculations, 32 grid points in the y direction are used. For slow ramping, the gas behind the front responds to each small increment in the amplitude of the disturbance, and a smooth approach to steady state is observed. For fast ramping, $R = 0.5$, the acoustic response shows significant overshoot of the predicted linear value. The vorticity response is somewhat smoother. The influence of heat release on the transient is considered by letting $\alpha = 0$, $R = 0.5$ [Fig. 3(c)] and prescribing the same normal Mach number; the overshoot and fluctuations of the acoustic response are considerably less than for the reacting shock, but the vorticity response shows essentially the same behavior as in the detonation.

As seen from Fig. 3(a), when the ramping is slow and the amplitude of the disturbance is small, the predictions of linear theory agree with the calculated acoustic response but there is a discrepancy in the vorticity response of about 2%. As mentioned in Zang *et al.*,[3,4] since the vorticity response involves computation of derivatives in both the x and y directions, the vorticity response calculations are possibly less accurate than the pressure response calculations. When only 16 grid points are used in the y direction, the discrepancy in the vorticity response increases to 4%–5% and the pressure response still agrees with the linear predictions. It should be noted that the calculated vorticity response is always less than the predicted values. Another important parameter which influences the transient behavior is the amplitude of the disturbance. For the 10% disturbance at $\theta = 40°$ [Fig. 3(d)], the acoustic response takes a longer time to asymptote to a steady state.

The comparison between linear theory and numerical simulations for other angles is seen in Fig. 4. The circles are

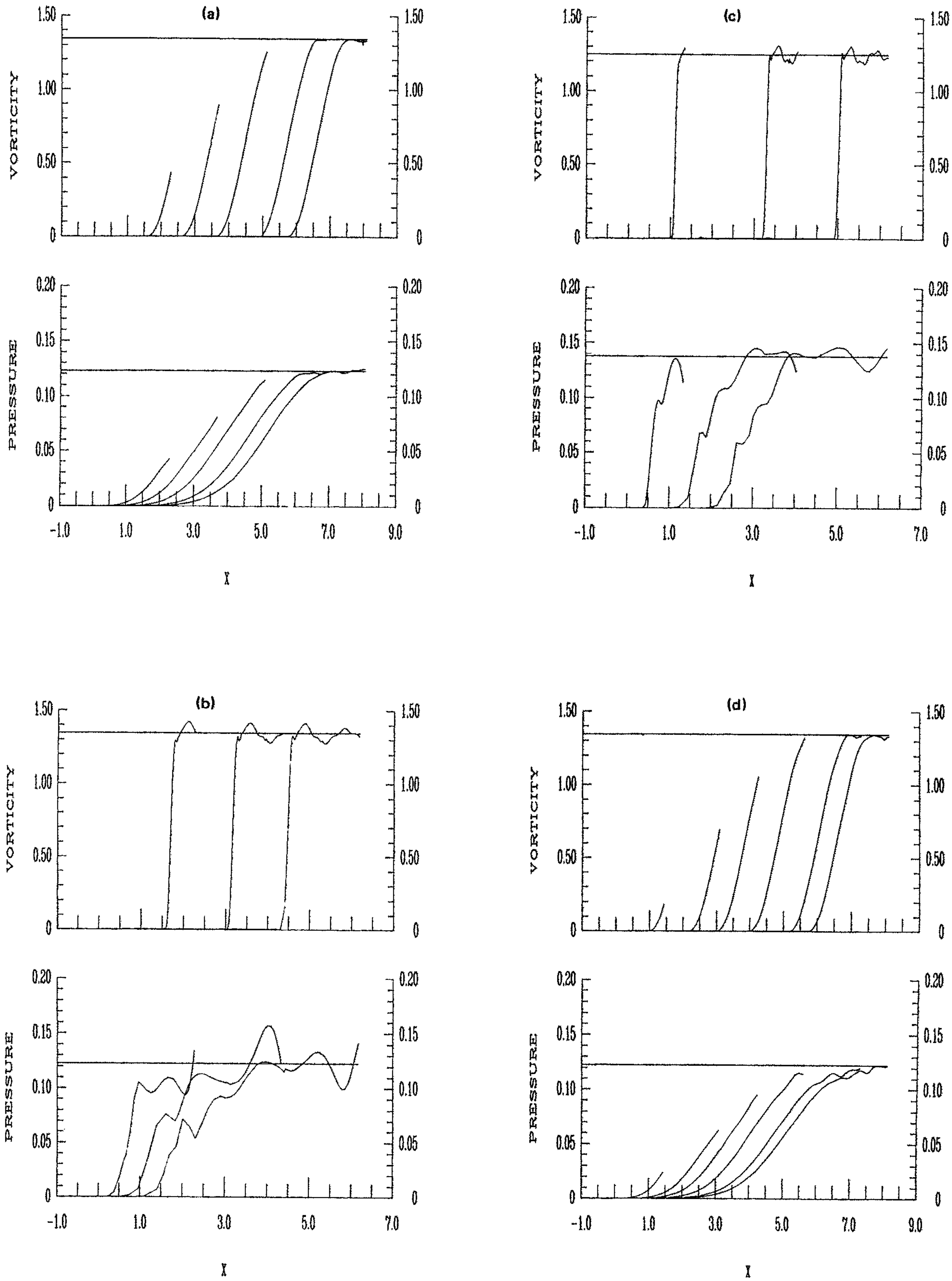

FIG. 3. (a) Pressure and vorticity response to a 1% slowly ramped disturbance: $R = 6$, $\alpha = 2$, and $\theta = 40°$. Solid line is linear theory. The curves correspond to $t = 0.380, 0.613, 0.847, 1.140, 1.341$. (b) Response to 1% disturbance with fast ramping: $R = 0.5$, $\alpha = 2$, and $\theta = 40°$. The curves correspond to $t = 0.377$, 0.719, 1.026. (c) Response to 1% disturbance with fast ramping: $R = 0.5$, $\alpha = 0$, and $\theta = 40°$. The curves correspond to $t = 0.219$, 0.669, 1.026. (d) Response to a 10% disturbance with slow ramping: $R = 6$, $\alpha = 2$, and $\theta = 40°$. The curves correspond to $t = 0.237$, 0.512, 0.704, 0.933, 1.208, 1.342.

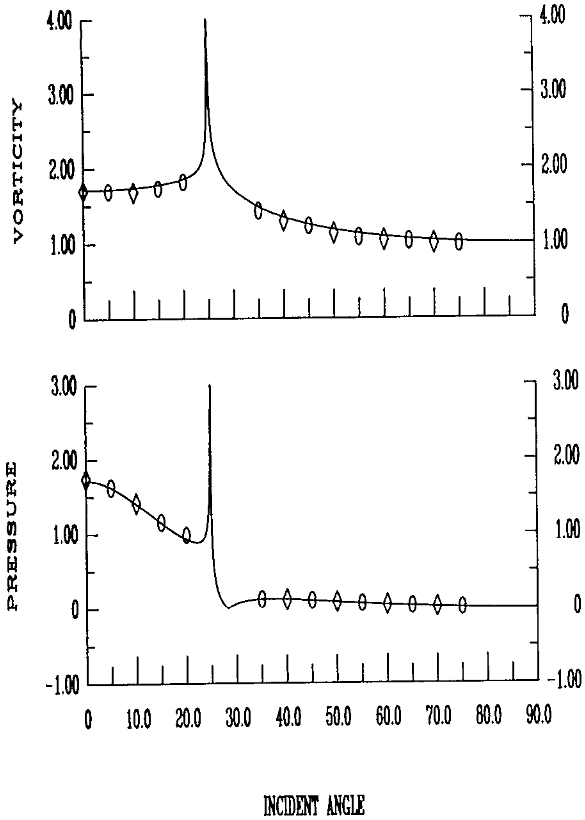

FIG. 4. Pressure and vorticity responses as a function of angle. Circles represent 1% disturbances and diamonds 10% disturbances. Solid line is linear theory for $\alpha = 2$.

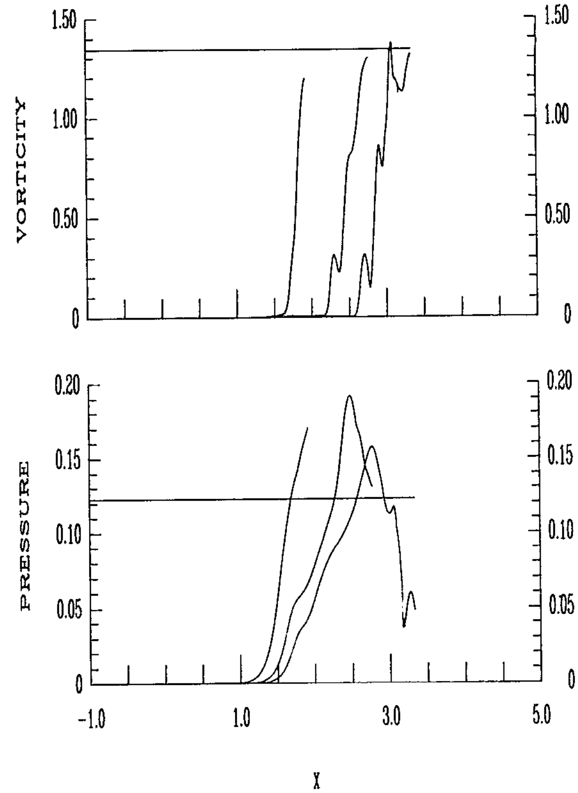

FIG. 5. Response to a 30% disturbance. $R = 3.0$, $\alpha = 2$, and $\theta = 40°$. The curves correspond to $t = 0.319, 0.459, 0.554$.

the calculated responses to a 1% disturbance and the diamonds are the calculated responses to a 10% disturbance. As previously found for the interaction of a nonreacting shock with a vorticity disturbance, the linear predictions and the calculated acoustic responses agree for disturbance amplitudes up to 10% when the angle of the incoming flow is not near the critical angle. The calculated vorticity responses are consistently about 2% below the predicted values when 32 grid points are used. Although many calculations were made for small amplitude disturbances (1% or less) and angles within 5° of critical, the results are not given in Fig. 4 since a steady-state value could not be reliably determined. The results of these calculations are discussed below.

For disturbance amplitudes larger than 10%, the solution did not reach a steady state. A typical run is shown in Fig. 5 for a 30% disturbance at $\theta = 40°$ with ramping parameter $R = 3$. Linear theory is a good predictor of the scale for the overall response, but the distortions produced in the detonation front prevent the solution from reaching steady state; in fact, the distortions prevented the calculations from proceeding further since the time step based on the CFL number became exceedingly small. Furthermore, since the front is no longer planar, the comparison between the least squares fit to the forms (17) and the linear case no longer holds. The same behavior was observed for 10% disturbances at angles within 5° of critical.

Linear theory predicts that exothermicity has its greatest effect on both the vorticity and pressure responses near the critical angle; therefore, it is important to resolve the behavior of the solution for angles near critical. As the angle of incidence approaches the critical angle, it is observed that the relaxation of the transients takes longer and longer. These transients are not related to the ramping of the disturbance amplitude as was shown for $\theta = 40°$. The response to a 1% disturbance with $\theta = 30°$ is shown in Fig. 6 (16 grid points in the y direction and ramping parameter $R = 6$). The transient nature of the solution provides for considerable overshoot of the predicted acoustic response with very little relaxation over the duration of the run. To continue the calculations past $x_s = 10$ would require adding additional grid points in the x direction to keep the same resolution making each time step considerably more expensive. In addition to the large pressure fluctuations, the vorticity response also displays some overshoot which is not observed for slowly ramped disturbances at angles more than 5° from critical. This is particularly surprising since calculations at other angles indicate that the vorticity response remains at 5% below the predicted value and it is quite smooth for such slowly ramped disturbances. The same parameter values were used but with fewer grid points in the x direction allowing integration to continue until $x_s = 24$. Although the resolution is less, a relaxation of the transients is observed; the acoustic response oscillates about a value accurately predicted by linear theory, and the vorticity response oscillates about a value that is approximately 5% below the value predicted by linear theory; there appears to be a slow decrease in the amplitude of these fluctuations. These results are consistent with steady states at larger angles of attack found using 16 grid points. A similar trend in the vorticity response for angles of attack slightly less than critical develops. The vorticity response displays a slight overshoot of the value predicted by linear theory which is not present for angles away from critical, and returns to a small oscillation about a value below the linearly predicted value. The overshoot of the predicted pressure and vorticity responses is observed for angles of attack between two and five degrees of the critical angle; however, relaxation of the transients is significantly slower as the angle of incidence nears critical. For $\theta = 24.9°$ (Fig. 7), it can be seen that the acoustic response does not reach the value predicted by linear theory but is still increasing when the calculation is stopped. The vorticity response is also still increasing, having reached only half of the value predicted by linear theory. This calculation was terminated due to the increasing expense of maintaining an accurate numerical resolution; however, the calculations were repeated using a very coarse grid, and it was found that even though the pressure and vorticity responses continue the slow increase, the pressure has achieved only 40% of the linear theory value and the vorticity only 60% of the linear theory value when $x_s = 50$. These results are only qualitative at best but seem to indicate that the lengthening of the transient response for angles within 2° of critical counteracts the transient overshoot of the pressure and vorticity responses found for other angles within 5° of critical.

IV. CONCLUSIONS

Nonlinear calculations of the response of an initially plane detonation wave to a vorticity disturbance show that the results of steady-state linear theory are useful in providing an overall scale of the response. In cases in which the angle of incidence is near critical, disturbance amplitudes are moderate, and/or there is a sudden encounter with a disturbance, the calculated responses display a transient overshoot of the linear prediction. It is found that exothermicity increases the overshoot in addition to increasing the value of the predicted linear response. Also, the significant departures of the predicted responses of the reacting shock from the nonreacting shock near the critical angles appear to be real, and the calculated responses show transient overshoot of the predicted values for angles between two and five degrees of critical. Closer to the critical angle, the lengthening of the relaxation time for the transient produces a competing effect. Also, it is found that critical angle of linear theory is an accurate predictor of the transition in the behavior of the acoustic response. Previously,[3,4] it was reported that the change from a constant pressure response to an exponentially decaying pressure response occurred at an angle significantly different than predicted by linear theory. For the situation presented here, it is determined that the long relaxation of the transients for angles slightly above critical makes the pressure response appear to be exponentially decaying; when in fact, if the calculations are of a sufficient duration, the pressure response eventually changes and eventually oscillates about a a constant value which is consistent with the value predicted by linear theory.

Since exothermicity is seen to increase both the vorticity

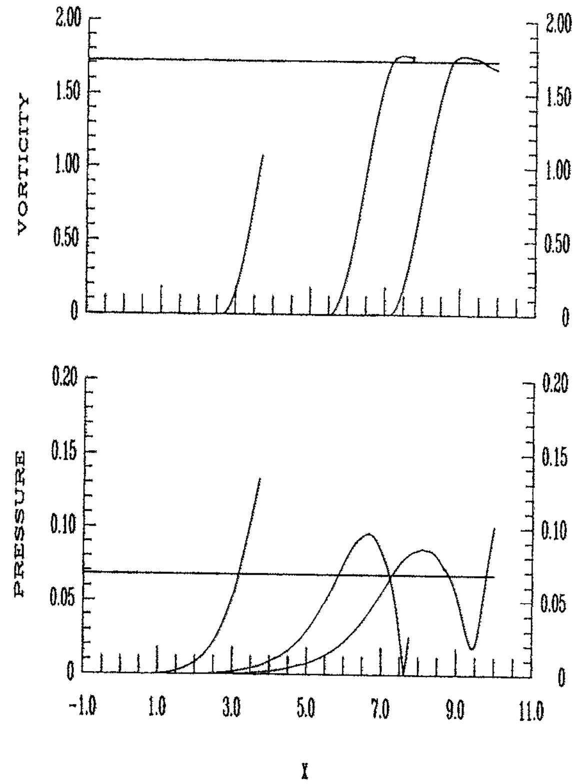

FIG. 6. Transient response to a 1% disturbance for $\theta = 30°$, $R = 6$ and $\alpha = 2$. The curves correspond to $t = 0.614$, 1.284, 1.657.

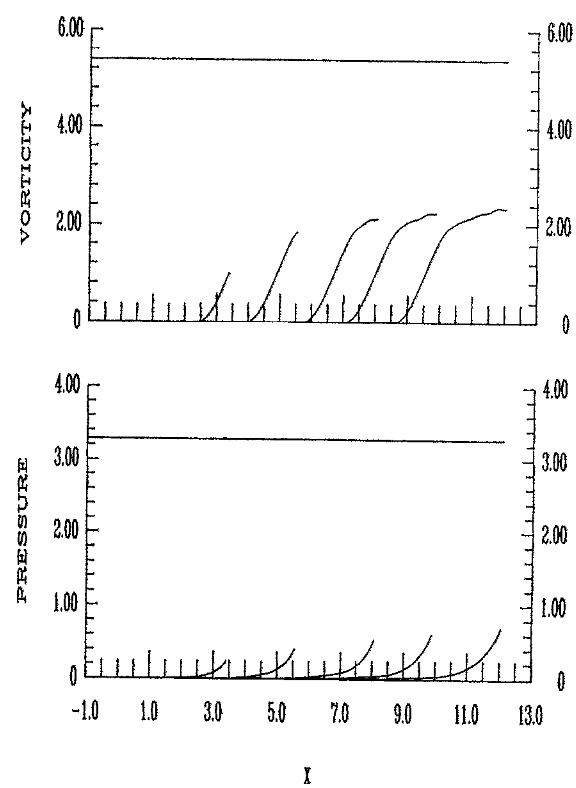

FIG. 7. Transient response to a 1% disturbance for $\theta = 24.9°$, $R = 6$ and $\alpha = 2$. The curves correspond to $t = 0.562$, 0.917, 1.335, 1.639, 1.999.

and acoustic responses for the detonation wave/vorticity wave interaction, it would be appropriate to study the interaction of a detonation wave with a fully developed vortex. Such a numerical study was presented in Meadows *et al.*[8] for the nonreacting shock using a shock capturing scheme which is more appropriate than the shock fitting scheme for the study of shock interaction with large disturbances. Of particular interest is the ability to capture secondary shocks which agree with experimentally observed features of the flow. This is not possible using shock-fitting methods.

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APPENDIX: GENERALIZED SHOCK FITTING

The improved shock-fitting scheme utilized by Kopriva *et al.*[5] is generalized to allow for exothermicity of the reacting shock and to allow for a time-dependent flow ahead of the front. The generalized Rankine–Hugoniot conditions for a stationary reacting shock can be simply written as

$$P_2 = P_1 + g(M), \tag{A1}$$

$$m = f(M), \tag{A2}$$

$$g(M) = \ln \frac{M\left[1 + \alpha + \frac{1}{2}(\gamma - 1)(M^2 - m^2)\right]}{m}, \tag{A3}$$

where the subscript 1 refers to known quantities ahead of the front and subscript 2 refers to the corresponding quantity behind the front; $f(M)$ is given by (8). Here, M is the normal Mach number of the flow ahead of the front, and m is the normal velocity behind the front divided by the sound speed ahead of the front. If all other velocities are nondimensionalized with respect to the speed of sound for the mean flow ahead of the front, then the prescribed flow has a local speed of sound c_1 at the front which differs from unity and depends upon both y and t. Denoting the front position by $\mathbf{r}_f(y,t)$, the speed of the front traveling in the positive x direction is $(d/dt)\mathbf{r}_f(y,t) = (U_f,0)$. The velocities of the gas in the frame of reference where the front moves $\mathbf{Q}_i = (u_i,v_i)$ are related to M and m by

$$c_1 M = U_f N_x - \mathbf{Q}_1 \cdot \mathbf{N}, \tag{A4}$$

$$c_1 m = U_f N_x - \mathbf{Q}_2 \cdot \mathbf{N}, \tag{A5}$$

where $\mathbf{N} = (N_x, N_y)$ is the normal to the shock front pointing in the direction of the domain subscripted by 1. The key to the shock fitting routine is the compatibility equation,

$$P_{2,t} + (\gamma/c)\mathbf{N}\cdot\mathbf{Q}_{2,t} = -\mathbf{Q}\cdot\nabla P - (\gamma/c)\mathbf{N}\cdot\mathbf{R} - c\mathbf{N}\cdot\nabla P - R_P, \tag{A6}$$

where

$$\mathbf{R} = (uu_x + vu_y, uv_x + vv_y), \quad R_P = \gamma u_x + \gamma v_y \tag{A7}$$

derived from the Euler equations. By differentiating the Rankine–Hugoniot relations and Eqs. (A4) and (A5), an equation for the acceleration of the detonation front can be found and is given by

$$\begin{aligned} U_{f,t}(y,t) = \{ & -c_1 c P_{1,t} + c_1 c C + (cG - \gamma F) \\ & \times (\mathbf{Q}_{1,t}\cdot\mathbf{N} + \mathbf{Q}_1\cdot\mathbf{N}_t) - (cG - \gamma F + \gamma c_1) U_f N_{x,t} \\ & + c_1 \gamma \mathbf{Q}_2\cdot\mathbf{N}_t + c_{1,t}[(cG - \gamma F)M + c_1 \gamma m]\} \\ & \times [(cG - \gamma F + \gamma c_1)N_x]^{-1}, \end{aligned} \tag{A8}$$

where $G = g'(M)$, $F = f'(M)$, and C is the right-hand side of Eq. (A6). During the calculations, the quantity C is evaluated using the solution of the Euler equations from the previous time step, and all other quantities are calculated using the appropriate jump conditions. The front position and velocity are updated for each time step using MacCormack's method.

[1] T. L. Jackson, A. K. Kapila, and M. Y. Hussaini, Phys. Fluids A **2**, 1260 (1990).
[2] D. T. Pratt, J. W. Huamphrey, and D. E. Glenn, AIAA Paper No. AIAA-87-1785, 1987.
[3] T. A. Zang, M. Y. Hussaini, and D. M. Bushnell, AIAA J. **22**, 13 (1984).
[4] T. A. Zang, M. Y. Hussaini, and D. M. Bushnell, Paper No. AIAA-82-0293, AAIA 20th Aerospace Sciences Meeting, Orlando, FL, January 1982.
[5] D. A. Kopriva, T. A. Zang, and M. Y. Hussaini, Florida State University Report No. FSU-SCRI-89-116 (to appear AIAA J.).
[6] R. W. MacCormack, *Lecture Notes in Physics* (Springer-Verlag, New York, 1971), Vol. 8, p. 151.
[7] D. A. Kopriva, in *Computational Acoustics: Algorithms and Applications*, edited by D. Lee and M. H. Schultz (Elsevier, New York, 1988), Vol. 2.
[8] K. R. Meadows, A. Kumar, and M. Y. Hussaini, AIAA J. **29**, 2 (1991).