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Comment on "Controlling the spectral shape of nonlinear Thomson scattering with proper laser chirping"

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Rykovanov, Geddes, Schroeder, Esarey and Leemans [Phys. Rev. Accel. Beams **19**, 030701 (2016); hereafter RGSEL] have recently reported on the analytic derivation for the laser pulse frequency modulation (chirping) which controls spectrum broadening for high laser pulse intensities. We demonstrate here that their results are the same as the exact solutions reported in Terzić, Deitrick, Hofler and Krafft [Phys. Rev. Lett. **112**, 074801 (2014); hereafter TDHK]. While the two papers deal with circularly and linearly polarized laser pulses, respectively, the difference in expressions for the two is just the usual factor of $1/2$ present from going from circular to linear polarization. In addition, we note the authors used an approximation to the number of subsidiary peaks in the unchirped spectrum when a better solution is given in TDHK.

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I. INTRODUCTION

In this comment we begin by noting that two important ideas used in Rykovanov, Geddes, Schroeder, Esarey and Leemans [1] (RGSEL) already appear in Terzić, Deitrick, Hofler and Krafft [2] (TDHK), their Ref. [47], for the linearly polarized case. In particular, the analytic requirement of constant lab-frame emission frequency and the use of a stationary phase argument prominent in TDHK are reused in RGSEL. The result is to derive essentially the same laser chirping prescription as appears in TDHK, but modified for circular polarization. It is the purpose of this comment to make clearer the appropriate connections between the results in TDHK and RGSEL.

II. DERIVATION OF THE EXACT LASER CHIRPING

First note how the key equation of RGSEL, their Eq. (22), can be found from the exact chirping prescription reported in TDHK. The equivalent equation for the linearly polarized laser pulse is derived in TDHK as an unnumbered equation preceding their Eq. (4):

$$\frac{d}{d\xi}[\xi f(\xi)] = \frac{1 + a^2(\xi)/2}{1 + a_0^2/2}. \tag{1}$$

From here deriving the exact laser chirping function is just solving a first order differential equation with a boundary condition. We use $f(0) = 1$ in TDHK. The solution for the exact modulation function in TDHK becomes, as reported in their Eq. (4):

$$f(\xi) = \frac{1}{1 + a_0^2/2}\left(1 + \frac{1}{2\xi}\int_0^\xi a^2(\xi')d\xi'\right). \tag{2}$$

The derivation of the corresponding exact modulation function for a circularly polarized pulse instead of the linearly polarized pulse from TDHK is straightforward. One should remove factors of $1/2$ from Eq. (1) for the circular polarization case to obtain

$$\frac{d}{d\xi}[\xi f(\xi)] = \frac{1 + a^2(\xi)}{1 + a_0^2}. \tag{3}$$

For the boundary condition $f(0) = 1$ (our preferred frequency normalization) one obtains

$$f(\xi) = \frac{1}{1 + a_0^2}\left(1 + \frac{1}{\xi}\int_0^\xi a^2(\xi')d\xi'\right). \tag{4}$$

For $f(\pm\infty) = 1$ (as chosen in RGSEL), after relating the modulation function from TDHK, f, to that of RGSEL, ϕ, as $\phi(\xi) = \xi f(\xi)$, we obtain their Eq. (21):

$$\phi(\xi) = \xi f(\xi) = \xi + \int_{-\infty}^\xi a^2(\xi')d\xi'. \tag{5}$$

Equation (19) of RGSEL, which led to their crucial Eq. (22), is derived using the stationary phase argument

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as in [3]. TDHK previously used the stationary phase method to give an alternate derivation of the exact frequency modulation condition.

Therefore, the chirping mechanism reported by RGSEL is exactly the same as the prescription of TDHK applied to circularly polarized pulses as opposed to linearly polarized pulses.

III. NUMBER OF SUBSIDIARY PEAKS IN THE UNCHIRPED SPECTRUM

Next, we note that RGSEL might have beneficially used the patently *exact* (to within the accuracy of the stationary phase approximation) expression for the number of subsidiary peaks in the unchirped spectrum, derived in TDHK and reported in their Eq. (1), instead of their approximation.

Equation (3) of RGSEL approximates the number of oscillations in the unchirped spectrum as

$$N_{\rm osc} = \omega_L \frac{a_0^2}{1+a_0^2} \frac{1}{\Delta\omega_L}, \tag{6}$$

where $\Delta\omega_L$ is the FWHM bandwidth of the laser, ω_L is the laser frequency and a_0 the laser pulse amplitude. The relationship to the Eq. (1) in TDHK is established if we recognize that the bandwidth of the laser pulse and the laser frequency are given as, respectively, $\Delta\omega_L = 2\pi/T$, $\omega_L = 2\pi c/\lambda$, resulting in

$$N_{\rm osc} = \frac{1}{1+a_0^2} \frac{cTa_0^2}{\lambda}, \tag{7}$$

which is to be compared to Eq. (1) of TDHK for the first harmonic $n_h = 1$:

$$N_\tau = \frac{cT}{\lambda} \int_0^\infty a^2(\bar{\xi}) d\bar{\xi}, \tag{8}$$

with $\bar{\xi} \equiv \xi/(\sqrt{2}\sigma)$. RGSEL use a half-sine laser envelope profile: $a(\eta) = a_0 \sin[\frac{\pi\eta}{\tau_L}]$ for $0 < \eta < \tau_L$. Substituting this expression for the laser profile into Eq. (8) yields

$$N_\tau = \frac{\pi}{4} \frac{cTa_0^2}{\lambda}. \tag{9}$$

We note is that the result of RGSEL depends on the amplitude a_0 in a different fashion than quadratic. Quadratic dependence of the number of subsidiary peaks on the amplitude a_0 was first empirically observed by Heinzl *et al.* [4] ($N_\tau \approx 0.24T[\text{fs}]a_0^2$), and later analytically explained and generalized to an arbitrary pulse shape and laser wavelength in THDK.

For the two examples reported in the left panel of Fig. 1 ($a_0 = 0.4$) and the right panel of Fig. (2) ($a_0 = 1$) of RGSEL, their estimates are significantly different from those of TDHK. The TDHK expression correctly predicts 8 subsidiary peaks in the left panel of Fig. 1 and 50 subsidiary peaks in the right panel of Fig. 2 of RGSEL. RGSEL estimates these to be 9 and 32, respectively.

In summary, we believe Rykovanov, Geddes, Schroeder, Esarey and Leemans have shown clearly and convincingly that our chirping prescription applies for circularly polarized lasers by accounting for the usual change in the field strength in going from linear to circular polarization. We have shown that our formula for the number of subsidiary peaks in the unchirped spectrum yields quantitative agreement when applied to cases presented in RGSEL.

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