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Black Male Students and The Algebra Project: Mathematics Identity as Participation

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In this article, the authors examine the mathematics identity development of six Black male students over the course of a 4-year The Algebra Project Cohort Model (APCM) initiative. Mathematics identity here is defined as participation through interactions and positioning of self and others. Data collection included nearly 450 minutes of video recordings of small-group, mathematics problem solving in which student actions, coded as acts of participation, were tallied. These tallied actions were conceptualized descriptively in terms of mathematics identity using the lenses of agency, accountability, and work practices. The analyses suggest that the APCM students' confidence in self and peers increased over the 4 years, they consistently chose to engage in mathematics, and their reliance on knowledgeable others lessened. Opportunities for future research and implications for policy makers and other stakeholders are discussed.

KEYWORDS: Black male students, mathematics identity, mathematics teaching and learning, The Algebra Project

Many urban high school mathematics classrooms have disproportionate numbers of students who are often described in policy reports and media as “at risk” (Durbin, 2012). This imbalance is especially true for Black male students who are often labeled as learning deficient, targeted for disciplinary action, and positioned for future incarceration (Booker & Mitchell, 2011; Gregory, Skiba, & Noguera, 2010). Many Black male mathematics learners have been historically, and continue to be, underserved by schools and society at large, especially those attending urban schools and qualifying for reduced-price meals (Anyon, 2006; Haberman, 1991/2010). Nevertheless, research has shown that when Black male students become aware of and have opportunities to learn mathematics in culturally receptive climates they take on productive mathematics identities (Berry, Ellis,

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& Hughes, 2014). Conversely, Black male students positioned in restrictive school climates with limited learning opportunities often experience negative outcomes (Gibson, Wilson, Haight, Kayama, & Marshall, 2014). Furthermore, when Black male students take on productive mathematics identities they are better equipped to self-advocate for more positive learning opportunities and improved outcomes for themselves, their communities, and society at large (Hope, Skoog, & Jagers, 2015).

To understand how Black male students might take on productive mathematics identities, we explored the mathematics identity development of six Black male students who chose to participate in The Algebra Project Cohort Model (APCM) initiative during their 4 years of high school. The overarching research question and accompanying sub-questions that guided the exploration were:

How did the mathematics identity of six Black male students participating in The Algebra Project Cohort Model initiative develop over their 4 years of high school?

- i. What types of *agency* were students observed exercising and how did their agency evolve?
- ii. How did students' observed work practices (i.e., small-group problem solving) influence their mathematics identity development?
- iii. To whom were students observed being accountable to and how did their accountability evolve?

Review of Literature

There has been substantial scholarship over the past 20 years that explores identity from many perspectives. Cultural and social psychologists, anthropologists, sociologists, and social scientists in general have reframed how we think about identity. In the mathematics education literature, this reframing has been driven by concepts derived out of a variety of theories such as critical theory, critical race theory, feminist theory, sociocultural theory, poststructural theory, and so forth (see, e.g., Berry, 2008, Gutstein, 2007; McGee & Martin, 2011b; Stinson, 2013). Nonetheless, for the study reported here, we take a narrower view of identity. We define *mathematics identity* simply as participation. Specifically, we explore how six Black male students' mathematics identities developed over 4 years of high school using nearly 450 minutes of video recordings of small-group, mathematics problem solving. To contextualize our study, we discuss two connected areas of research: (a) "reform" in mathematics education, and (b) Black male students and mathematics identity.

Reform in Mathematics Education

To position Black children in reform efforts, we drew on Berry, Pinter, and McClain's (2013) critical review of K–12 mathematics education reform efforts from the mid 1950s to the early 2000s. Their review of reform mathematics focused on what was taught, how it was taught, who taught it, and, most importantly, who got access to it. They concluded that the needs of Black children in mathematics education reform efforts have not been attended to over the decades. Segregation has been re-enacted through testing and tracking in many schools, and the brilliance of Black children has been largely ignored by the majority of mathematics educators and researchers.

Recently, Martin (2015) argued that mathematics education reform for several decades has yielded few benefits for *the collective Black*¹ as he critiqued the National Council of Teachers of Mathematics' (NCTM) latest policy document *Principles to Actions: Ensuring Mathematics Success for All* (NCTM, 2014) at the 2015 NCTM Research Conference held in Boston, Massachusetts. His critique included categorizing the long-standing rhetoric about equity and “mathematics for all” as political, questioning the audience for whom the document was written, and calling for mathematics educators to consider revolutionary reform designed for the collective Black.²

For the most part, extant reform efforts have neither targeted nor yielded substantive improvements for the collective Black, in general, and Black male students, in particular. In this article, we discuss aspects of mathematics education reform in spite of this oversight because that is what exists (for now) and these efforts are pertinent for situating our project.

Over the last several decades, national organizations such as the NCTM (e.g., 1991, 2000, 2014) and the National Research Council (2001) have called for significant cultural changes in mathematics classrooms. The NCTM, for example, called for classrooms that are co-created by teachers and students, “where students of varied backgrounds and abilities work with expert teachers, learning important mathematical ideas with understanding, in environments that are equitable, challenging, supportive, and technologically equipped for the twenty-first century” (NCTM 2000, p. 4). The latest national call for change is embedded in the Common Core State Standards for Mathematical Practice (CCSS, 2010). Specific recommendations for mathematics education reform efforts have also emerged from mathematicians and mathematics educators. Mathematicians have suggested that Black students, in particular, need opportunities to engage in doing mathematics in ways that

¹ This term was used by Martin (2015), defined as African American, Latin@, Indigenous, and poor; he attributes the term and definition to Eduardo Bonilla-Silva.

² It appeared that the predominantly White audience received his remarks with loud silence.

are both social and cultural (Fullilove & Treisman, 1990; Maton, Hrabowski, & Greif, 1998). Similarly, mathematics educators have advocated for pedagogical approaches that are student-centered and collaborative versus the traditional didactic approaches that have persisted for decades (Franke, Kazemi, & Battey, 2007; Hiebert et al., 1997; Lampert, 1990/2004). The consensus among many: students participating with peers in ways that fosters sense making while using their cultural experiences and ways of knowing within and beyond school settings supports mathematics learning (e.g., Moses & Cobb, 2001; Walker, 2006). Critical mathematics educators have also stipulated that these student-centered, collaborative approaches are more effective for Black students when they are carried out by teachers who are culturally aware versus those who believe that learning and teaching are race neutral (Martin, 2012; Matthews, Jones, & Parker, 2013; McGee & Martin, 2011a; Stinson, Jett, & Williams, 2013).

In addition to student-centered, collaborative pedagogical approaches, mathematics educators have advocated for using high-level, cognitively demanding tasks (e.g., Stein, Smith, Henningsen, & Silver, 2000). These educators claim that the cognitive level of the task affords different types of teaching and learning opportunities. High-level tasks that require students to engage mathematically, to seek connections to other mathematical ideas, and to prove their approaches, require teachers to facilitate learning differently than low-demand tasks that only require students to recall memorized facts that teachers, in turn, validate. The types of pedagogies needed for facilitating high-level tasks are typically more student-centered, such as examining student work and listening to their explanations to inform instructional decisions, and requiring students to use mathematical processes and practices in learning (CCSS, 2010; Doerr, 2006; Henningsen & Stein, 1997; NCTM, 1991, 2000). However, classrooms where students engage collaboratively in cognitively demanding tasks are not available to all students, in particular Black students (Ladson-Billings, 2006).

In fact, too many Black students attend poor performing schools. According to Balfanz and Legters (2004), in 2002 almost half (46%) of Black students attended high schools with weak *promoting power*³ where graduation was not the norm; most of these schools were in urban areas with high poverty. Few reform efforts have been meaningfully enacted in schools, in general, and urban high-poverty schools, in particular, for many reasons that are beyond the scope of this article (for a complete discussion see Marrus, 2015). Mathematics education in urban, high-poverty schools typically manifests as perpetual remediation, discipline, and other authoritative actions (Bracey, 2013; Ladson-Billings, 2006; Love & Kruger, 2005; Patterson, 2014). Bracey (2013) captured the essence of mathematics education re-

³ Promoting power is an indicator of high school dropout rates, calculated as a percentage comparison of seniors to freshmen 4 years earlier; 60% fewer seniors than freshmen represent weak promoting power (Balfanz & Legters, 2004).

form within the current climate of accountability: “The net result is a lack of opportunity to engage Black children beyond prescriptive remediation to pass annual yearly performance ... mandates” (p. 173). In summary, effective mathematics education reform efforts for Black students must provide opportunities for and access to: (a) pedagogies that are student centered and collaborative; (b) teachers who are culturally aware and well prepared; and (c) high-level mathematics courses with cognitively demanding tasks.

Black Male Students and Mathematics Identity

Supporting positive and productive mathematics identity development for Black male students requires they have access to teachers who: (a) explicitly and publicly hold high expectations for them to learn rigorous mathematics; (b) create receptive, engaging, and supportive learning environments; and (c) are culturally aware and responsive while exercising decentralized teaching authority (Ladson-Billings, 1994, 1997; Stinson, Jett, & Williams, 2013). From this perspective, we review literature about Black male students’ mathematics identity development.

Martin (2009, 2013) argued that discussions about Black students’ mathematics identities cannot be independent of discussions about race and racism in the United States. The historical rhetoric in the United States around mathematics teaching and learning often positions Black students implicitly and explicitly as mathematically deficient compared to White students who are positioned as the norm. This positioning, unfortunately, is often supported by mathematics education research and educational policies (Martin, 2013). Therefore, for Martin (2009), mathematics identity

refers to the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person’s self-understanding and how others see him or her in the context of doing mathematics. (pp. 136–137)

Central elements of mathematics identity that emerge from this definition include perceptions by others and beliefs about self in relation to mathematics learning and doing.

Perceptions by others influence the ways we think about ourselves and the actions we take. One perception about Black male students held by others is the stereotypical image of the non-academic, street “thug.” This stereotypical image not only influences but also can threaten Black (male) students’ mathematics identity development (Steele, 1997, 2006; Steele, Spencer, & Aronson, 2002). Steele (2006) referred to this phenomenon as *stereotype threat* and defined it as “the threat of being viewed through the lens of a negative stereotype, or the fear of doing something

that would inadvertently confirm that stereotype” (p. 253). Researchers have reported ways that Black male students have navigated the peril of stereotype threat in mathematics learning contexts (e.g., Berry, 2008; McGee, 2013a; Stinson, 2008). For instance, McGee (2013a) analyzed interviews with 11 successful Black male high school juniors and seniors. McGee described the students as having defiant reactions to the stereotype; some ignored the threat and persevered and others worked harder to attain academic achievement. In either case, students developed productive mathematics identities using internal coping mechanisms when faced with negative perceptions by others.

Watson (2012) uncovered another type of perception that is more covert by nature. She studied mathematics teachers whom she described as *norming suburban* when asked to describe their students. The act of norming suburban uses middle-class, White cultural perceptions as the standard by which all other groups “are compared, judged, and subordinated” (p. 987). Neither innocent nor objective comparisons emerge when norming suburban because it requires one “to posit, either implicitly or explicitly, that teaching in suburban schools is better, and base this belief on the perceived inferiority of urban students,” all the while not using “race language” (p. 988). Watson outlined a three step suburban norming process: (a) assume groups are monolithic with respect to behaviors, values, and beliefs; (b) decide if these cultures are negative or positive; and then (c) establish hierarchies among groups. Norming suburban appears to be a form of stereotype threat that does not attend directly to characteristics such as race and class. Students, particularly those in the lowest hierarchical group, however, are likely to notice teachers who adopt norming suburban practices and discourses (Berry, 2008).

Stinson (2006) reviewed historical and theoretical perspectives surrounding Black male students schooling experiences and presented three discourse clusters often used by others when discussing Black male students: the discourse of deficiency, the discourse of rejection, and the discourse of achievement. The discourse of deficiency is the perception that Black children are products of genetics, families, communities, and sociocultural spaces that are historically lesser than and not sufficient. This discourse leads to perceptions by others that Black male students, in particular, are incapable, lacking, and otherwise deficient with respect to mathematics learning and achievement. School officials and policy makers who adopted deficiency perceptions for Black students often select intervention options that are typically segregating and anti-intellectual, such as labeling, tracking, isolating remediation, and authoritative pedagogies. The discourse of rejection is the perception that Black male students reject either a productive intellectual identity or the collective Black identity; the intervention here is often nurturing support programs, such as African-centric rites of passage programs. The discourse of achievement is the perception that Black students are able to achieve intellectually and mathematically. Leonard and Martin (2013) took up the discourse of achievement to compile their

edited volume *The Brilliance of Black Children in Mathematics: Beyond the Numbers and Toward New Discourse*, which approaches mathematics learning and identity development of Black children from the perception of brilliance, omitting the discourses of deficiency and rejection altogether.

Beliefs about self as articulated by successful Black male students starkly contrast the mathematical identity descriptions presented about them received via educational research, policy reports, and media outlets (Ladson-Billings, 2006; Martin, 2012); and there are too few of these stories told within extant literature (Martin, 2013; McGee, 2013a). Berry (2008) reports one such student's self-account who described mathematics as "an easy subject for him to learn because he likes it and he loves the challenge of problem solving" (p. 464). This mathematically talented and engaged Black male student's account was shared during middle school; he had been identified as academically gifted in the fourth grade. The account reported by Berry described the student's relationship with his father that included mathematical challenges with games and puzzles done at home. In sixth grade, however, he encountered a teacher with whom he did not connect. This teacher appeared set on removing him from her class and presumably the gifted program. The student with parental advocacy persevered and passed the teacher's class earning a B. Accounts of Black students' mathematics learning experiences that include social and cultural influences using students' "voices" (e.g., Berry, 2008; Jett, 2010; McGee, 2013b; Stinson, 2013) or strongly influenced by students' voices (e.g., Grant, 2014; McGee & Martin, 2011a, 2011b) are adding new positive perceptions and characterizations for how Black students see themselves in relation to doing and learning mathematics.

Conceptual Framework: Mathematics Identity as Participation

Mainstream education scholars have explored the notion of identity development to better understand how people think about themselves or how others perceive them in relation to learning (e.g., Cobb & Hodge, 2002; Gee, 2000; Gilpin, 2006; Greeno, 1997). In these cases, mathematics identity is conceptualized as mathematics participation: the ways that students interact with others and position themselves and others in relation to mathematics engagement. These mainstream conceptualizations, however, most often do not consider the socio-cultural and -political contexts of learners and of learning. With this limitation in mind, Varelas, Martin, and Kane (2013) used a socio-cultural and -political critical lens to develop the content learning and identity construction framework for researching learning in mathematics and science classrooms. This framework considers content learning and identity construction as requisite. They described identities as "lenses through which we position ourselves and our actions and through which others position us" (p. 324). Positioning influences learning opportunities in which students may en-

gage, and changes in positioning result in different learning opportunities. Opportunities are essential for an exploration of identity as participation.

Students' self-perceptions are central to the actions (or inactions) they pursue within social systems, such as mathematics classrooms (Gresalfi, 2009; Nasir & Hand, 2006; Nasir, McLaughlin, & Jones, 2009; Varelas et al., 2013). Within the lens of mathematics identity, self-perceptions, the perceptions of others, and the situated contexts converge and influence identity related processes (Esmonde, 2009; Esmonde, Brodie, Dookie, & Takeuchi, 2009; Nzuki, 2010). Esmonde and colleagues defined three identity related processes, referred to as work practices for cooperative groups: collaborative, individual, and helping. In this study, we explore student mathematics identity development within the context of small-group, mathematics problem solving, and through observation we sought to interpret their mathematics identity development in terms of mathematics agency, accountability, and work⁴ practices (with a focus on collaborative and individual practices only).

Mathematics Identity and Participation

This study characterizes participation as *observable* mathematics engagement and uses participation as the overarching construct for students' mathematics identity. This two-tiered construction of students' mathematics identity has foundations in educational psychology and mathematics education literature: (a) exercised agency (Bandura, 2006; Gresalfi, Taylor, Hand, & Greeno, 2009; Gutstein, 2007; Hand, 2010) and (b) student accountability (Ares, 2006; Cobb, Gresalfi, & Hodge, 2009; Cobb & Hodge, 2002; Yackel & Cobb, 1996). These constructs, agency and accountability, manifest as observable student actions (i.e., agency) or inactions in mathematics learning contexts, and students chose participation or non-participation based on afforded opportunities that are influenced by feelings of accountability.

Mathematics identity and agency. Our perspective of agency is grounded in Bandura's (2005) agentic perspective of social cognitive theory: "To be an agent is to influence intentionally one's functioning and life circumstance. In this view, people are self-organizing, proactive, self-regulating, and self-reflective" (p. 9). In other words, people make intentional choices in their self-interests, which, from our perspective, are manifestations of identity as (observable) participation, or non-participation, which is also agency exercised. Gresalfi and colleagues (2009) explain the possession and exercise of agency:

It is important here to dispel the notion that people "have" or "lack" agency. In virtually any situation, even the most constrained, people are able to exercise agency; at the basic level, by complying or resisting. The ways that agency can be exercised, and the

⁴ The *work* in this study is mathematics problem solving within a small group of three to four students.

consequences for doing so, are what change in a particular context. Said differently, an individual can *always* exercise agency, it is the nature of that exercise that differs from context to context. (p. 53; emphasis in original)

Here, Gresalfi and colleagues are suggesting that close attention be paid to issues of power and authority within the mathematics classroom when considering distribution of agency. Many critical researchers have acknowledged that mathematics classrooms and mathematical tasks are not neutral or without power dynamics, equitable access, and opportunity for engaging (e.g., Esmonde & Langer-Osuna, 2013; McGee & Martin, 2011a; Tyner-Mullings, 2012; Varelas et al., 2013). While power dynamics and equity are not prominent in this study, we recognize that these dynamics influence agency and accountability and the importance of being mindful of such within both research and practice. Otherwise, there is no commitment to social justice and the status quo continues.

Mathematics identity and accountability. Accountability is a prominent element of construction of competency as participation (Cobb et al., 2009). Cobb and colleagues articulated competency in terms of curricula with respect to agency distribution (i.e., *accountable for what*) and in terms of the culture for discourse in terms of accountability (i.e., *accountable to whom*). Similar to the other participation components described thus far, this component is observable and interpretations can be made to categorize what was observed. The second portion of this participation component, accountable to whom, includes five levels: (a) teacher only or class only; (b) teacher and peer; (c) small group only; (d) teacher and small group; and (e) teacher, small group, and class. For this study, as students were situated in small groups for problem solving and the proctor followed a non-helping protocol (discussed later), our focus for whom students were accountable included: (a) expert: directs discourse to knowledgeable other, in this case, proctor or a peer positioned by the student as expert or more knowledgeable; (b) peers: expressed concern for peer in relation to mathematics at hand; or (c) self: positioning self as expert/knowledgeable or expressed disinterest in peer or others' perspectives.

In summary, mathematics identity as participation was framed using agency and accountability. Observable incidents of participation were used as the overarching construct that situated actions of agency and accountability related to mathematics problem solving. We connected our study to recommended reforms for improving mathematics teaching and learning and to extant understandings about Black male students and their mathematics identity development.

Methods

Interpretive qualitative analyses were employed for the purpose of understanding student mathematics identity development as related to mathematics participation (Schwandt, 1994). Descriptive statistics were also used to aid in pattern

discovery and recognition. The social and cultural contexts selected for exploration were the small-group, problem-solving assessments observed throughout the 4 years of the APCM initiative. Video recordings of these assessments were the primary data source.

The Algebra Project

The underlying genesis of The Algebra Project was influenced by concerns of mathematical equity and access (Moses & Cobb, 2001). Therefore, the primary goal for the APCM initiative was to transform urban and rural students' perceptions of themselves from adopting mathematical identities given to them by others (e.g., at risk students) to mathematics learners and leaders who possess mathematics literacy. In other words, "young people finding their voice instead of being spoken for is a crucial part of the process" (Moses & Cobb, 2001, p. 19).

The APCM initiative was designed for accelerating mathematics understanding for mathematics students who are likely to be underserved by schools and society at large. It was comprised of three parts: a cohort structure, curriculum and pedagogy, and community outreach. Worth noting explicitly, The Algebra Project consistently seeks to work with students from the lower quartile,⁵ but "interventions" neither advocate for nor include remedial approaches, and students are not positioned as deficient. Instead, The Algebra Project curriculum begins with students sharing an experience from which mathematical understandings are developed and abstracted, an experiential learning approach (Kolb, 1984). The APCM initiative is built on 15 years of experience in middle and high school pilot programs that included instructional materials development funded by the National Science Foundation (Moses, Dubinsky, Henderson, & West, 2008). A robust discussion of The Algebra Project curriculum⁶ would likely be interesting, but is beyond the scope of this article. The APCM initiative endeavors to create opportunities for students to actively engage in mathematics that develops mathematical identities while building mathematical literacy.

Participants and Context

The first author, in Years 1 and 2 of the project, visited participants' classroom several days per month to work with the APCM teacher and the local university mathematician, the principal investigator for the local project. In Years 3 and 4,

⁵ How one measures and determines hierarchies that order students and relegates some to the lower quartile is of no consequence because The Algebra Project seeks to work with all students perceived as underserved or otherwise labeled through deficiency discourses.

⁶ The Algebra Project curriculum is available for inspection and comment through a Public Curriculum Portal accessible at <http://www.algebra.org/curriculum/>.

she visited the classroom once or twice per year, and attended two of the four summer institutes.⁷ During site visits, she supported the teacher and sometimes took an active role during instruction with the students and she collected data to support the research project. Through these activities, she got to know the students and they got to know her. In Year 1, she conceptualized the need for and developed the small-group assessment protocol (discussed later).

The study reported here was part of a larger The Algebra Project research project that spanned five urban and rural sites across the United States. The goals for the larger APCM research project included: (a) students graduating from high school in 4 years; (b) students, upon graduation, enrolling in credit-earning mathematics courses for those choosing post-secondary options; and (c) students developing and participating in productive peer cultures for learning mathematics (Moses et al., 2008).

The research reported here focuses on one of the sites from the large project; a small, urban community located in the midwestern United States. The APCM student cohort was comprised of 19 students in their first year of high school, the only high school in the community. The students, with parental or guardian consent, agreed to take two, 50-minute classes of mathematics each day with the same teacher for all 4 years of high school. Most of the students and their parents (or guardians) knew the APCM teacher as a member of their community prior to entering high school. The APCM teacher is White, but she raised her bi-racial (Black) son, who was academically successful and a star on the football team, in the community. Her son was about two years older than the APCM cohort students. However, all of the children in the community who engaged in sports did so within the community leagues, and the majority of the male cohort students were also on the high school football team. The first author, on many occasions, observed students gravitating to the APCM teacher in times of need. Several of the Black male students referred to her using familial terms, such as “school mom” or “second mother.” The APCM teacher was observed reciprocating the students’ affections. For instance, she maintained a snack cabinet to feed hungry students; admonished poor decision making, in or out of school, while encouraging and expecting better in the future; and returned many unsolicited hugs. After her son graduated, the APCM teacher continued to participate in the community with students and to attend extra-curricular events.

The state department of education designated 16 of the 19 cohort students as “Not Proficient” as freshmen based on a score received on the eighth-grade, state-mandated mathematics achievement test. The school and society at large, from our

⁷ Summer institutes were held each summer to provide students with opportunities to engage in mathematics and to develop leadership and other positive dispositions. The institute locations alternated between a large urban university and a moderately sized rural university, where the students lived on the campus of the hosting university for 2 weeks each year.

perspective, underserved these students; our project endeavored to serve them differently with respect to developing mathematics literacy.

Six Black male students from the cohort were purposefully selected for this study because they represented a sample cohort. The first consideration for selection, and most obvious, was to represent the cohort demographically; the cohort was predominantly comprised of students who identified as Black (90%) and male (71%).⁸ A conscious decision to select only Black male students was made for several reasons: (a) the vast majority of cohort students identified as Black and male during the 4 years of high school; (b) in Year 4, there was only one student who identified as not Black and that student was new to the cohort; and (c) a personal interest of the first author to study Black students and their mathematics identity development. The other important consideration for selecting the sample was to restrict selection to 4-year participants and to those participants with sufficient data. Thus, the small-group assessment videos (discussed later) were viewed to identify an initial list of eligible students, and two factors were used to cull that list: (a) the student was present for at least one small-group video segment for each of the 4 years;⁹ and (b) an equal number of students positioned by peers as leaders. Using these as criteria, six Black male students were selected. It is worth noting that no students who identified as female appeared in more than two years of the small-group assessment videos. Reasons for absences were twofold: either the student was absent on a particular group assessment day or there were technical challenges while videotaping. It was not uncommon for more students to be absent on assessment days, especially during the earlier years. We always announced research related data collection activities and allowed students to opt out without penalty, per the Institutional Review Board agreement. Moreover, the research team was responsible for videotaping, and especially in the earlier years, unintended errors occurred such as failure to turn the camera on or uncharged batteries.

The six Black male students selected included: (a) three students who were regularly positioned by peers as class leaders; (b) two students who were more outspoken during class, one was positioned as a leader by peers and the other was not; and (c) two students who tended to be less vocal during class, one was positioned as a leader by peers and the other was not. Of the six students, only one was not an athlete, but all students engaged in extracurricular activities at school. A descriptive summary of the six students was compiled from the first author's experiential knowledge and relationships with the students, class observations, and informal conversations with the APCM teacher (see Table 1).

⁸ The demographic percentage calculations represent averages calculated using cohort enrollment data over the 4 years of the APCM initiative.

⁹ There was one exception, Ray was not in a Year 1 video, but he appeared in two Year 2 videos, and one was used for his Year 1 assessment.

The local university mathematician, mentioned earlier, was task developer and proctor for all of the small-group assessments. As the principal investigator and participant researcher, he participated in curriculum development with other mathematicians from The Algebra Project, and supported the teacher as a local consultant for The Algebra Project curriculum during the 4 years she taught the APCM students. In that capacity, he became well known by the students because during their freshman year he regularly visited (2 to 4 days per week) and participated in mathematics instruction in collaboration with the teacher.

Table 1
Student Descriptions

Name ^a	Leader ^b	Achievement ^c	Summary of In-Class Persona
Hal	No	Moderate	A gregarious personality, a collaborative, confident, and enthusiastic mathematics engager; he identified passing the state test for mathematics as impactful
Neo	Yes	High	A hard worker, soft spoken student leader, logical defender of mathematical ideas, ready mathematics participant and collaborator; teacher calls him dependable
Ray	No	Low	A hard and persistent worker who puts forth great effort, encourages peer participation and focus; vocal in class, may have undiagnosed learning disability
Reg	Yes	High	Confident, loyal, and success oriented, supportive of peers, focused and driven, and non-judgmental; recognized student leader
Rex	No	Low	A hard worker, willing to work with others, ready participant, may have undiagnosed learning disability
Ted	Yes	High	Class leader who led by example, willing to work with anyone, ready participant; passing the state test for mathematics was impactful

^a All names are pseudonyms.

^b Leader as positioned by peers.

^c Estimate of achievement as measured by state test scores and school grades.

Assessment Protocol

The small-group assessment protocol had three components: (a) pose the problem, answering only questions related to task clarification; (b) provide no hints or validation during problem solving; and (c) encourage students to rely on peers for support. The mathematician, in most every instance, faithfully executed this

protocol over the 4 years, as evidenced by the video recordings. The purpose of the group assessments was to gain insights about: (a) The Algebra Project curriculum effectiveness in relation to students' mathematics understanding,¹⁰ and (b) the students' sociocultural development for mathematics learning.

We assert that the small-group, problem-solving assessment protocol and the established relationship between the mathematician and the students afforded a narrow and fertile context for addressing the research questions. Over the years, the proctor and students built an amicable and trusting relationship, based on observations by the first author. Gillen (2014), a long-time veteran teacher of The Algebra Project and social justice advocate, found from his extensive experience that an environment where students have sufficient opportunity to engage mathematically within a receptive climate affords freedom for them to engage through a myriad of roles. All of the assessments were proctored using the same protocol and students were free to choose to work individually or collaboratively with no negative consequences. Because of these factors and the nature of an Algebra Project classroom as described by Gillen, we posit that the small-group assessment context minimized inherent power dynamics that exist in typical learning environments. The environment afforded students opportunities to engage with limited or no barriers, and therefore afforded an unobstructed view of these students' mathematics identity as it emerged and evolved.

Data Collection

Data collected for this study included six different tasks, captured using 15 video recordings of small-group assessments; the average length of the recordings was about 30 minutes (in total, approximately 450 minutes). The assessment proctor also created analytic field notes and collected document artifacts for each of the problems. The video recordings were captured using a stationary camera or non-professional videographer. The camera was placed or held near the small groups in order to record the participants' discourses, interactions, and body language. The local university mathematician proctored all of the assessments and generated analytic field notes about the group members and their work; these notes were used to add clarity to the video recordings. For example, if a student had a misconception about the mathematics, the proctor's analytic notes may have described the misconception; or if a student created a drawing and they were pointing out a particular

¹⁰ All of the problem-solving tasks, after the first problem, were designed using a context different from that used for instruction. This design component is significant because The Algebra Project curriculum situates mathematics learning within student shared experiences. The types of tasks developed for the small-group assessments were not unique and could be characterized as worthwhile tasks (Stein et al., 2000).

aspect of it while talking, the proctor may have kept a copy of the drawing with his notes.

The APCM curriculum addresses mathematics topics from the high school content standards outlined by the Common Core State Standards for Mathematical Practice (CCSS, 2010). The group assessment tasks used for this investigation targeted mathematics content from several CCSS high school content standards, including algebra, functions, and geometry. For example, one of the Year 1 tasks focused on algebraic reasoning, a subset of the CCSS algebra standard. Reg, Rex, and Ted worked on that task; Reg and Ted were in the same group for their Year 1 assessment, they have the same Group #, while Rex worked on the same problem, but in a different group (see Table 2).

Table 2
Mathematics Topics and Student Groupings by Year

	Reg	Rex	Ted	Hal	Neo	Ray
Year 1 Math Topics	Algebraic reasoning	Algebraic reasoning	Algebraic reasoning	Linear functions (continuous)	Linear functions (continuous)	Linear functions (piecewise)
Group #	1	2	1	3	3	4^a
Year 2 Math Topics	Linear functions (piecewise)	Linear functions (piecewise)	Linear functions (piecewise)	Geometric construction (with paper)	Geometric construction (with paper)	Geometric construction (with paper)
Group #	5	6	4	7	8	7
Year 3 Math Topics	Area of composite shapes	Area of composite shapes	Area of composite shapes	Area of composite shapes	Area of composite shapes	Area of composite shapes
Group #	9	10	10	11	11	9
Year 4 Math Topics	Direct variation (velocity comparison)	Direct variation (velocity comparison)	Direct variation (velocity comparison)	Direct variation (velocity comparison)	Direct variation (velocity comparison)	Direct variation (velocity comparison)
Group #	12	13	14	15	15	15

^a Ray's Year 1 task actually occurred during Year 2. Task selection for each year was done in a way to align the mathematics topics of his peers.

Video Recording Data Analysis

The analysis of video data was done using qualitative software (NVivo version 10) that allowed multiple researchers to analyze the same data, which simplified comparative analysis of coding, and diminished the amount of transcription required. The protocol used for video analysis was as follows: (a) segment each video recording into time segments of about 1 to 2 minutes; (b) watch each segment, and assign descriptive themes (i.e., nodes) that captured observed phenomena, paying specific attention to each target student; and (c) for recordings with two

or more target students in the same small group, recordings were watched multiple times, at least once for each target student (counting re-watched video recordings, nearly 700 minutes of recordings were analyzed).

The group assessment video recordings were coded and analyzed for recurrent themes, allowing inferences to be made and supported (or disputed) via the data—warranted claims were made (Wolcott, 2001). The analysis process was multi-level. The first-level analyses assigned thematic categories (i.e., nodes) to recording segments. The second-level used descriptive statistics and graphical representations of coded node frequencies to search for patterns. Heavily coded nodes (i.e., those with relatively high frequency counts) and patterns were used to draw inferences related to the research questions. Then the video data corpus was searched to find representative examples, evidence that confirmed or disputed inferences—the third-level analyses. The evidentiary data served to warrant the inferences from which claims were made (Erickson, 1986). A second researcher, a doctoral student whose worldview and biases differ from the first author's, was recruited to work collaboratively with the first author to improve the validity of findings while increasing the efficiency of the video analyses—a somewhat forth-level of analyses. Using multiple researchers to analyze the video data increased the trustworthiness of the analyses, which strengthens the validity of the findings (Lather, 1986).

The first author used the video analysis protocol to code the first 2 years of video recordings after creating an initial codebook that defined thematic nodes based on the grounding literature. The nodes in the codebook were organized hierarchically beneath the primary constructs: *agency*, *accountable for what*, and *accountable to whom*. Analyzing the video segments led to defining emergent nodes during the analysis process to capture unanticipated phenomena observed that had not been included in the initial codebook; an approach described by Schwandt (1994) as an emic perspective. Additionally, *mathematical work practices* as an organizing category was added late in the analysis process and after revisiting the literature. The emergence of this category is described later in the Results section, as it was not part of the original analysis plan. Late during the analyses, we reorganized the a priori categories because of patterns in the data, which Lather (1986) described as face validity, which also strengthens qualitative research findings.

The videos were watched several times by one or two researchers to establish and maintain an acceptable inter-rater reliability standard (Landis & Koch, 1977) with the Kappa coefficient $> .70$ and percent coding agreement $> 85\%$ throughout the coding process. To that end, the two researchers' coded one video recording from Year 4; comparison analyses were run that showed coding did not meet the pre-established standard. The researchers met to review the coding and collaboratively re-coded and refined the codebook definitions; refining the codebook during the analysis process ensured shared understandings for node definitions and consistency of coding between researchers. Collaborative coding continued until the

two researchers reached coding reliability agreement of > 85%. The two researchers then independently coded videos from Years 2 and 3; comparison analyses were run and the inter-rater reliability standard was met. Overall, the first author analyzed video recordings for Years 1 through 3, and the second researcher analyzed video recordings for Year 4.

Results

The purpose of this study was to determine how mathematics identity developed for six Black male students who choose to participate in the APCM initiative; students agreed to take two periods of mathematics taught by the same teacher for all 4 years of high school.¹¹ The six Black male students participated in the APCM for all 4 years, passed the state's graduation achievement test, and graduated high school in 4 years. According to exit interview data, these six students' paths after graduation included college (2-year or 4-year), work, or military service. Their exit interviews also revealed that each student claimed mathematical readiness to pursue their planned path for the future. We turn our attention to the results from analyzing the video data.

Mathematics Agency

Three themes were evident from analyzing the 4 years of small-group, problem-solving assessments: confidence, collaboration, and personal effort. The most heavily coded thematic nodes aggregated over the 4 years from the categories *agency* and *accountable for what*, the primary constructs for the analysis, are shown in Table 3. Interestingly, we noticed that several of the heavily coded thematic nodes might have been categorized as discourse practices. Manouchehri and St. John (2006) characterized discourse for mathematics learning as being comprised of both reflection and action for the purposes of gaining understanding of their peers' perspectives and influencing them. The heavily coded nodes align with these characterizations and purposes: the discourses were reflective and action oriented for the purpose of gaining understanding of peer's conceptions or garnering peer support.

¹¹ Scheduling conflicts for credit attainment for graduating precluded students from taking the double periods of mathematics during their final year of high school.

Table 3
The Predominant Codes Aggregated Over Year 1 to Year 4

Thematic Nodes	Hal	Neo	Ray	Reg	Rex	Ted	Totals
Engaged in peer collaboration ^a	22	23	5	8	10	25	93
Engaged in individual process ^a	5	9	26	19	12	15	86
Collaborative sense making	17	16	2	10	3	14	62
Explaining ideas	13	12	7	10	7	12	61
Listening for understanding	9	13	6	3	7	22	60
Sharing ideas with peers	14	13	2	5	6	16	56
Consulting expert source	13	7	7	12	6	10	55
Listening to peer	11	13	8	3	7	10	52
Asking clarifying questions	7	7	6	4	7	13	44
Acknowledging contributions by others	7	6	2	5	1	14	35

^aWork practices for mathematical engagement

One example of this type of discourse practice occurred as Hal worked to understand how to fold a circular piece of paper in a way to construct parallel lines. As Hal grappled with the problem, he engaged in a mostly nonverbal way with a peer (Student 1, not a study participant) in his group:

- Hal:** [Explains to the proctor why the lines he has constructed are not parallel. As the two other group members continue to work on folding their papers. He glances at Student 1's folding a couple of times as he continues to contemplate his work.]
- Student 1:** [After making several folds and examining his paper closely, he rotates the paper twice to examine the lines] "I don't know if it's right" [student giggles.]
- Hal:** [Reaches over and picks up the paper his peer had been folding for closer examination.] It's not bad, Dog.
- Student 1:** [Nods in acknowledgement of Hal's praise.]

(Video recording, Year 2)

In this interaction, there is little dialogue, but after taking Student 1's paper, Hal goes on to explain why he believes Student 1's constructed lines are parallel. This interaction clearly depicts a discourse practice in which Hal was reflective—by comparing his approach to Student 1's—and led to action—explaining why Student 1's lines were parallel. The purpose of the discourse was an example of Hal attempting to understand Student 1's mathematics.

A second example depicts Hal's effort to garner peer support for an idea, using the reflection/action discourse practice. This example occurred in Year 4 in a small group comprised of Hal, Neo, and Ray. In this episode, Hal and Ray are engaged in making sense of the problem involving two moving cars, A and B, travel-

ing at different rates, and their locations are described in terms of the other car and a referent red line at specific time intervals:

- Hal:** There is no doubt, that it [car A] was one meter past the red line and car A was two meters past the red line, and if they're both exactly at four meters at three [sec], that means that it [car B] caught up, and passed, not passed it, but it just equaled up with it. So that it's [car B] clearly faster. They're going the same speed.
- Neo:** Car B is at first, like already ahead of car A; and then car A tied it up. And car A is going faster.
- Hal:** No, it says [referring to the problem], that it's [car A] two meters past it [car B] already, so it's [car A] in front of car B, it's one meter past the red line.
- Neo:** Oh, Ok. I thought...
- Hal:** So, it's kind of lined up like this [begins drawing and talking softly about his sketch.]

(Video recording, Year 4)

In this episode Hal again reflects on input received from a peer as he listens to Neo's explanation, but then disputes Neo's position using evidence from the text of the problem, which convinces Neo of Hal's position. This reflective process leads to Hal taking action: depicting his thinking via a sketch. Hal's purpose in the discourse appears to be garnering peer support: Hal is seeking Neo's support before investing in creating a pictorial representation. This action is representative of agency as articulated by Bandura (2005); Hal's exercised agency was self-regulated and negotiated within the group's social system.

The two most heavily coded thematic nodes for participation represent two distinct student work practices for mathematics engagement, individual and collaborative (Esmonde, 2009). The total coding for these nodes *engaged in peer collaboration* (93) and *engaged in individual process* (86) are much greater than the total for the next most heavily coded node, *collaborative sense making* (62; see Table 3). These two most heavily coded nodes were interpreted as students *choosing* to engage mathematically, as they did not opt to engage in off task behaviors or otherwise not participate. Close examination of this analysis led to two things: (a) reorganizing the nodes hierarchically around these two heavily coded nodes; and (b) examining how the students' participation acts split across the two nodes.

The qualitative analyses were done using software (NVivo version 10), which allowed for easily restructuring of nodes at any point within the analyses without disturbing prior analyses. The node restructuring led to additional analyses using this emergent perspective—looking at the students' participation through the lens of their observed work practices. Immediately obvious, we found that some students mostly worked collaboratively (i.e., Hal, Neo, and Ted), while others opted to work independently (i.e., Ray and Reg), and one student (i.e., Rex) worked almost equally across the two work practices (see Table 3).

When we reorganized the heavily coded thematic nodes by student work practices for engagement, individual and collaborative, some nodes were combined beneath an existing node or a new node was defined. In the end, the most heavily coded nodes were found primarily under the collaborative work practice. Node descriptions are provided to make clear the meanings used for the coding process: categorizing what was observed (individual or collaborative) and students' verbal utterances, actions, and gestures (see Table 4).

Table 4
Definitions for Heavily Coded Thematic Nodes Organized by Categories

Heavily Coded Thematic Nodes	Definitions of Nodes
<u>Category: Individual work practice</u>	
Asking clarifying questions	Independently initiates questioning, clarifying and probing questions
Consulting expert source	Seeks help and/or support from someone perceived as expert
<u>Category: Collaborative work practice</u>	
Acknowledging contributions by others	Making public recognition of a peer's mathematical contribution
Collaborative sense making	Expressing efforts to understand while engaging with others
Listening to peers	Making public the effect of a peer's verbal expression
Passive peer interactions	Nonverbal response to another's action
Sharing ideas with peers	Making public, verbally or by action, ideas, explanations, and artifacts

Mathematical Identity Development Over 4 Years

The reorganized codebook provided the foundation for our response to the primary research question: How did the mathematics identity of six Black male students participating in the APCM initiative develop over their 4 years of high school? We found the aggregate coding frequencies (i.e., totals for the six students by years) for the heavily coded nodes; doing so afforded an overall perspective of the students' mathematics identity development across the 4 years. Summaries depicting the evolution of the students' identity are shown using summary line charts, using the following lenses: (a) students exercising individual problem-solving practices, (b) students exercising collaborative problem-solving practices; and (c) for whom students were observed being accountable.

Individual problem-solving practices. With respect to individual work practices, there were only two heavily coded nodes *asking clarifying questions* and *consulting expert source* that emerged from the analyses (see Figure 1). These two participation behaviors were observed most during Years 2 and 3. Interestingly, stu-

dents consulting a perceived expert source was highest in Year 2, and then declined for each year after. However, the students' questioning increased from Year 1 and peaked in Year 3. We posit that this summary suggests that these students transitioned from reliance on a knowledgeable other to a greater reliance on self and collaborations among peers.

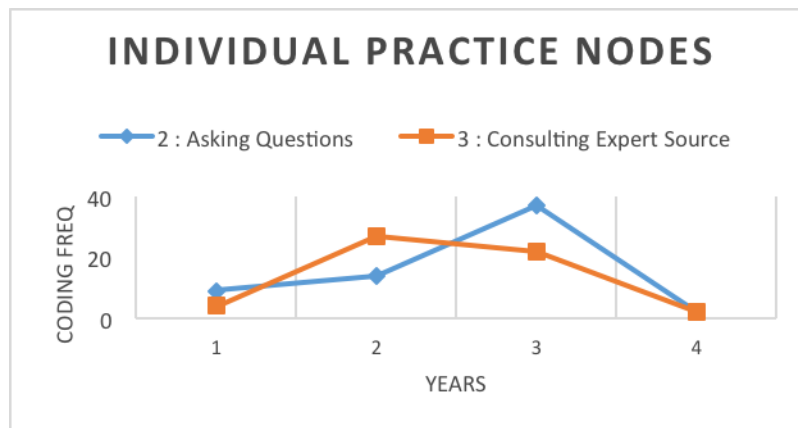


Figure 1: Summary of heavily coded nodes categorized by individual practice for all students by years.

Two examples from the *asking clarifying questions* node from Years 2 and 3 are presented to illustrate this finding. In Year 2, Rex was observed asking more questions than he did in all of the other years combined. In the first example, students are given prices for purchasing Jelly bracelets from an online vendor. They are given a variety of information, such as the price for a specific number of bracelets and volume shipping costs. The information, however, is not presented simplistically in a way that suggests a linear relationship. Rex asks several questions, such as: “Isn’t it [Jelly bracelet cost] going up?”; “What did you get?”; and “So, how much is it for one bracelet?” (Video recording, Year 2) These questions were asked of the group and are often focused on getting to the answer or seeking confirmation. Rex’s group members offered little in response to his questions, which led him to pose questions to the proctor about the given information. The proctor acknowledges that there is sufficient information to solve the problem, to which Rex replied, “Well, I didn’t find it.” (Video recording, Year 2); a response that suggests that Rex is done, unable to solve the problem and has no other options. Rex’s questioning was focused narrowly on getting support for finding the right answer to the problem.

The second example illustrates a different perspective that shows the evolution of questioning among the students. In Year 3, Reg asked the most questions. The problem presented was about finding the linear measure, width, on each side of

a rug centered in a room, given the room dimensions and the area of the rug. In addition to asking fewer questions seeking support or confirmation, Reg poses questions to the proctor: “Has any group solved this problem, yet?”; “Have we learned what we need to know in order to solve this question?”; and “Are we over thinking this?” (Video recording, Year 3) Reg’s questions appear to be seeking understanding about his preparedness to solve the problem. These questions are not searching for hints or support, but rather validation that he possesses all that is needed for success. What underlies these questions is an inherent trust that exists between the student and proctor given the student’s willingness to ask such questions and then to accept the response without question, even when he was faced with uncertainty about his solution. Reg persevered in problem solving after this exchange.

Collaborative problem-solving practices. There were four heavily coded nodes for student collaborative work practices that emerged from the analyses (see Figure 2), three of which are aligned with discourse practices (e.g., Manouchehri & St. John, 2006): *collaborative sense making*, *listening to peers*, and *sharing ideas*.



Figure 2: Summary of heavily coded nodes categorized by *collaborative practice* for all students by years.

The shape of the lines that show a summary of coding for these three nodes are similar in shape, the lines are relatively flat between Years 1 and 2, peak in Year 3, and then fall in Year 4. These trends suggest that student identity development related to discourse practices followed an increasing trajectory and peaked in Year 3; however, we hesitate to consider Year 4 because of previously stated reasons and the lack of observed participation in Year 4.

Ted was observed as the most collaborative participant as measured by observed behaviors in this study. Therefore, we selected examples from video segments featuring Ted to show a progression over the years as an illustrative case for

all students (see Table 5). The Year 1 discourse practice is, for the most part, not collaborative, even though students are talking to one another. In Year 2, Ted gives up on the problem until he is lured back by a question posed by the proctor to Ray. In Year 3, Ted listens to peers and without invitation he alerts them of an error in their mathematics strategy for solving the problem. While he does not know the correct solution at the time, he was sufficiently engaged to recognize the potential pitfall and share his perspective with peers to redirect their trajectory.

Each year, Ted's level of mathematical engagement and discourse with peers seemed to gain in complexity in the sense that Year 1 was more collective than collaborative—kids voicing ideas but not using them to improve mathematical understanding—and by Year 3 discourses were unsolicited peer supportive for mathematics learning. While Year 2 was clearly between the two extremes, greater perseverance emerged and participation continued after claiming, “done.” Thus, over the years, the discourse frequency and complexity increased.

Table 5

**Ted's Progression of Discourse Practice:
An Illustrative Example of One Aspect of Mathematics Identity Development**

Year #: Problem/Context	Description of Discourse
Year 1: A trip is represented using a linear model	Ted initiates conversation by sharing his idea; Reg responds, makes no comment about Ted's idea, and shares his approach. Ted listens to Reg explain his idea more than once with little insight to further his solution; Reg disengages, “Can I work on my own?”
Year 2: Find the total cost to purchase and ship Jelly bracelets, given complex pricing information	Ted declares himself done; the proctor poses a question to Ray, “Why are the price differences the same and then different?” Ted did not appear to be listening, but in response asks, “Which one?” Ted reengages with the problem.
Year 3: Find the width around a rug centered in a room, given room dimensions and area of the rug	Ted watches as Rex and the other group member discuss an idea; Ted recognizes an error in their logic. Ted takes Rex's paper and by drawing on his paper, shows the group members the width they seek.

The growing discourse practice may also explain the continual rise in students *acknowledging contributions by others*, the fourth heavily coded node, which should not be overlooked (see Figure 2). By far, Ted was the most observed engaging in this behavior, with the greatest number of instances observed in Year 4, making this node not as representative of the group, but Ted, the individual.

Accountability. The aggregated coding of nodes from the thematic category *accountable to whom* is shown in Table 6. These nodes were coded less heavily in comparison to those coded for the participation nodes because interpreting whom one is accountable is not always transparent to an observer. Nonetheless, there were

patterns; notice that the students were observed as most accountable to *peers* when looking at aggregate totals across the years. However, the aggregate totals for observed accountability to *expert* and *self* are almost evenly split. Interestingly, Hal, Neo, Rex, and Ted were most accountable to peers, which is supported through analysis results that Hal, Neo, and Ted were observed being the most collaborative among the students. According to the APCM teacher, Reg mentored Ray and Ray modeled himself after Reg (Informal communication, Year 4), and interestingly, they were the only two among the sample with the least amount of coding for accountable to peers and with the most coding for accountable to expert and self.

The descriptions used for this study for *accountability to whom* from the researchers' perspective follow. Accountability to expert most often suggests that the learner is not sufficiently empowered and lacks autonomy for mathematics or mathematical understanding. There were many instances of this across the years where students requested validation from the proctor and their peers by asking questions such as: "So it took her 10 seconds to get to the end of the field?" (Hal, Video recording, Year 1); "What's up with this, [Proctor]?" (Reg, Video recording, Year 2); "With your math knowledge is that correct?" (Hal, Video recording, Year 4)

Table 6

Aggregated Coding of Nodes from One Category Showing Analyses of Small-Group, Problem-Solving Assessment Videos Over 4 Years

Accountable to Whom	Hal	Neo	Ray	Reg	Rex	Ted	Total
Expert: seeks other to model or guide knowledge construction and validate	8	6	7	15	4	4	44
Peers: constructs understanding and validity collaboratively	17	22	2	5	7	18	71
Self: confident and autonomous constructs and validates independently	5	8	8	11	4	7	43

Accountability to self, means a level of learner confidence and belief with sufficient mathematical autonomy that he readily shares ideas with others so they may value, critique, or dispute them. From this stance, the proctor may be considered a peer at times; the proctor was positioned to share facts and not give hints or validation. The proctor was observed not exercising his authority within the groups' power hierarchy. Accountability to peers is between accountability to expert or self if one viewed the three along a continuum.

From a student perspective within a social system of a small group, being accountable to peers suggests sufficient confidence for overcoming the risk of being

wrong. The difference between peers and self is that for those who are accountable to peers, they are sufficiently free to rely upon others for support and adjust their ideas based on collaborations. On the other hand, those accountable to self may be more autonomous yet less open to “hear” critiques from others, which may be cognitively limiting, based on the situation, of course.

From the summary line chart shown in Figure 3 we can see that observed instances of reliance on peers peaked in Year 3 and was maintained in Year 4. Observed instances of reliance on self grew steadily from Year 2 through Year 4 and over the same period observed instances of reliance on expert decreased steadily. These observations taken with the accountable to whom perspectives, we conclude that students made a shift from relying on knowledgeable others to relying on themselves and/or peers and they were sufficiently confident to risk being wrong, yet free enough to be influenced by collaborations.



Figure 3: Summary of heavily coded nodes categorized by *accountable to whom* for all students by years.

Summary of Findings: Mathematics Identity Development

The first point to make is that the APCM initiative created a receptive climate and we posit the freedom and nurturing was fertile ground for the six Black male students’ mathematics identity development during high school. The students expressed, their teacher described, and the researchers observed students’ mathematics confidence. One student described his mathematics confidence as, “I can do anything that I put my mind to” (Hal, Interview, Year 4). This simple statement is emblematic of the way these students saw themselves mathematically (i.e., their mathematics identity), and it suggests that their confidence was connected to personal effort. What is not captured by this particular statement is the value the stu-

dents placed on their peers and collaborations for learning mathematics, an aspect that was evident across the observations.

Another key aspect of these students' mathematical identity was their intentional choice to engage mathematically. The students opted to engage in individual and/or collaborative practices; however, even for those who favored individual practices they were observed in discourse through asking questions, which often led to verbal discourses with others. Overall, we observed students engaged in rich discourse practices that were reflective and action oriented for the purpose of gaining understanding or garnering peer support. The evolution of these six Black male students included transitioning from reliance on knowledgeable others to reliance on self and peer collaborations or mathematical participation, which reified the observed increase in the number of discourses and their complexity within their small-group, problem-solving assessments. Therefore, it was not surprising to realize that confidence appears to be the foundation for our students' mathematics identity development when viewed through the lens of mathematical agency, a relationship established by Bandura (2002).

Discussion and Conclusion

Research on reform-based mathematics calls for classrooms environments where mathematical autonomy and freedom abound and are available for all learners (Carpenter, Fennema, Franke, Levi, & Empson, 2000; Carpenter & Romberg, 2004; Hiebert et al., 1997; West & Staub, 2003). Such autonomy and freedom, however, cannot be taken for granted, especially from those students (urban and rural) who are underserved (Gillen, 2014). We agree that pedagogical content knowledge and mathematical knowledge for teaching (e.g., Ball & Bass, 2003; Ball, Hill, & Bass, 2005; Hiebert et al., 1997; Hufferd-Ackles, Fuson, & Sherin, 2004) are necessary conditions in creating effective learning environments; however, these knowledges are not sufficient conditions for creating equitable learning environments for all children (e.g., Martin & Herrera, 2007; NCTM 2000, 2014). Although NCTM (2000) established the Equity Principle long ago, equity continues to elude many mathematics classrooms, especially those with large numbers of racial and ethnic minority students (Berry, 2008; Martin, 2008).

Further research is needed to understand the full impact of practices and policies on student mathematics identity development, and to articulate specific reforms needed to free our children and our classrooms from those practices and policies that inhibit mathematics literacy, leadership, and freedom (Hope et al., 2015). We must serve underserved students differently so that they are afforded opportunities to choose mathematics literacy. As the research reported here demonstrates, historically underserved students develop different mathematics identities when provided access to classroom environments that are not reliant on traditional remediation ap-

proaches. We know that mathematics efficacy and confidence are essential dispositions for exercising mathematics agency (Bandura, 2002), which supports our finding that the students' confidence played a role in the ways they engaged in the small-group problem solving. We also posit that the receptive climate and freedom in the groups contributed to the ways their participation manifested in relation to their mathematics identities. These findings were not unanticipated. In another study about APCM students, Grant (2014) examined students' verbal and written reflections about the ways they interact with peers while learning mathematics. She found that APCM students from two cohorts, one urban and the other rural, described productive classroom culture for mathematics learning as students getting along with peers, working hard, and supporting one another.

In the end, the study reported illustrates the development of mathematics identity as participation of six Black male students; the findings add to the literature extolling the virtues of Black learners. The documented participatory freedom exercised by the students may be useful for those looking for new approaches in transforming the culture of participation and agency in mathematics classrooms. The study, however, is limited in the sense that it looked closely at only six Black male students in one mathematics teacher's classroom, over a 4-year time period. Moreover, the structure of the APCM initiative—students studying with one teacher for all 4 years—worked for the students and teacher reported here. It is important to note, however, that there were instances with other APCM cohorts where that was not the case. Some teacher–student relationships were not synergistic, and did not promote effective mathematics learning.

Nevertheless, one implication of this study is that the APCM initiative provides guidance for those interested in creating equitable and receptive environments for underserved students generally, and for Black male students specifically. Policy makers and other stakeholders have claimed interests aligned with equity as evidenced by names such as “No Child Left Behind” and “Race to the Top.” These and other initiatives funded through public and private organizations are all innocently labeled as accountability measures. These labels, however, are misleading and have been far reaching with many unintended negative consequences for U.S. schools and mathematics classrooms. These mandates manifest in education systems as hierarchies where teachers and students are at the bottom, with little or no choice or autonomy (Gillen, 2014). Gillen and others (e.g., Leonard & Martin, 2013) argue, and we concur, that students need different learning environments and opportunities if they are to develop the types of positive mathematics identities described here.

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