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Dennis M. Bushness

Jerry N. Hefner

Robert L. Ash
*Old Dominion University*, rash@odu.edu

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# Effect of compliant wall motion on turbulent boundary layers

Dennis M. Bushnell and Jerry N. Hefner

*Langley Research Center, National Aeronautics and Space Administration, Hampton, Virginia 23665*

Robert L. Ash

*Old Dominion University, Norfolk, Virginia 23508*

A critical analysis of available compliant wall data which indicated drag reduction under turbulent boundary layers is presented. Detailed structural dynamic calculations suggest that the surfaces responded in a resonant, rather than a compliant, manner. Alternate explanations are given for drag reductions observed in two classes of experiments: (1) flexible pipe flows and (2) water—backed membranes in air. Analysis indicates that the wall motion for the remaining data is typified by short wavelengths in agreement with the requirements of a possible compliant wall drag reduction mechanism recently suggested by Langley.

## I. INTRODUCTION

The current energy shortage places additional emphasis upon research to increase the efficiency of transportation, including aircraft.[1] Since, for long-haul aircraft, the viscous drag is approximately 40–50 percent of the total drag, any sizable decrease in turbulent skin friction translates directly into an appreciable fuel saving. Reference 2 provides a discussion of various techniques for reducing viscous drag, as well as drag-due-to-lift. A more detailed review of various $C_f$ (skin friction coefficient) reduction approaches[3] concluded that five techniques (laminar flow control, slot injection, particle injection, polymers, and compliant skins) were worthy of further detailed research.

The laminar flow control technology is quite mature, and most of the remaining important problems are in the practical areas of maintainability and reliability. Further research is, however, still required for optimization of suction rate and distribution. Skin friction reduction due to injection of low momentum air near the wall (slot or porous injection) can definitely cause large local $C_f$ reductions,[4] but recent studies[5] have shown that, as one might expect, the penalties for collecting and ducting the slot air outweigh the expected drag reduction benefits on aircraft; therefore, this method is presently suitable only for local regions where excess air, perhaps from a laminar flow control system, might be available. Particle injection can also provide a drag reduction,[6] but applying this technique to aircraft (particularly considering the various anti-pollution regulations) is not currently feasible. Polymer injection produced large $C_f$ reductions,[7–10] but the effect is limited to liquids.

The remaining technique, compliant walls, is the subject of the present critical review. Blick[11] discussed the field, with primary emphasis upon the University of Oklahoma research. The introduction sections of Refs. 12, 13, and 14 provide fairly up-to-date synopses.

The purpose of the present paper is to investigate the entire subject for both the transitional and the turbulent flow cases and discuss possible correlations between computed wall motion and observed drag reduction. The paper also presents alternate explanations for "drag reductions" observed in several cases reported in the literature.

Considering the demonstrated success with laminar flow control on the X-21 aircraft[15] one might reasonably ask why anything further (another drag reduction technique) is needed for aircraft. There are two responses. First, laminar flow control has thus far been applied on wings, and not fuselages. On the present wide body transports, approximately one-half of the surface area is on the fuselage. Since the fuselage boundary layer is relatively thick and generally has small pressure gradients, it may be much more suitable for applying compliant walls rather than for laminar flow control (although there has been no concentrated effort to apply laminar flow control to the fuselage). The second and more serious comment concerns the high altitude operating conditions necessary for laminar flow control. The low unit Reynolds numbers required for a successful laminar flow control system (primarily dictated by roughness and radiated noise from the fuselage) are only available at altitudes above 35 000 ft and, ideally, over 40 000 ft. However, most operational flights (and 60% of aircraft fuel) are for stage lengths less than 1500 miles and therefore considerable fuel is burned at altitudes below the levels required for laminar flow control. The application of laminar flow control is optimal for high altitude, long range aircraft, but something else (perhaps compliant walls) is needed for the considerable traffic in the shorter stage length, lower altitude cases. With these considerations, and because there does not seem to be any other potentially viable viscous drag reduction scheme, compliant wall research should obviously be pursued until the potential for drag reduction under turbulent boundary layers is either conclusively proved or disproved.

The current state of compliant wall technology can best be described as confusing and inadequate. Although the mechanism for altering the drag must be connected with the flow induced wall motion, to the present authors' knowledge, except for some recent attempts at

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Langley, only the study by Grosskreutz[16] measured wall motion for either successful, or unsuccessful, compliant wall experiments. Extensive wall motion measurements must be made, particularly in cases where drag reduction is obtained, before any theoretical approach to the problem (or derived design methodology) can be reasonably validated. The problem is obviously one of fluid-structural dynamic interaction, and yet the structural side has been somewhat ignored by the experimentors. Another confusing issue is the various options available for eventual application to aircraft, i.e., (1) delaying/stretching out transition vs reducing $C_f$ in fully turbulent flow and (2) use of truly compliant (flow perturbation following) walls vs resonant (excited eigenmode) walls. Also, theoretical research indicated that compliant walls with low damping were required for flow stabilization whereas practical applications and experiments necessarily utilize resonant walls with appreciable damping. Studies are available for practical application of the transition-retarding option.[17]

An additional use for compliant walls is reduction of turbulent boundary layer noise. Both analytical[18] and experimental[19,20] work are available, but this option has not yet received extensive study. Reference 12 discusses the possible application of compliant walls for drag reduction in other modes of transportation besides aircraft.

Compliant wall research evidently originated from consideration of the drag of dolphins.[21-25] However, the possible presence of such walls on dolphins is problematical. In an article in *Scientific American*[21] Gray surmized, based upon the observed 22 mph speed of dolphins and their (then) known physiological propulsion efficiency, that the dolphin must have some means of producing extensive regions of laminar flow (although it was not mentioned, lower drag can obviously also be due to considerably reduced transitional/turbulent drag levels as well as to more extensive regions of laminar flow). Kramer,[22] in 1961, published a suggestion as to how extensive regions of laminar flow might be produced on the dolphin. He based his suggestion, which he termed "distributed flow damping" or compliant walls, upon a detailed examination of porpoise skin. He found that the skin is quite pliable and contains a considerable quantity of absorbed water. His initial tests (starting in 1956) of man-made coatings simulating porpoise skin did indeed yield lower drag in sea water. In a later paper[22] Kramer estimated that the porpoise could obtain up to a 40% drag reduction just due to the favorable pressure gradient influence on transition Reynolds number (body shaping); he further concludes that an additional drag reduction is necessary to explain the porpoise speed and again advances the compliant skin concept as a method for stabilizing the boundary layer fluctuations by supplying distributed damping. However, tests conducted by Lang *et al.*[24,25] indicate that, while the porpoise can indeed reach high speeds (on the order of 22 knots), the animal does so only for relatively short time periods, and this may be accomplished by running the muscles into "oxygen deficit." Lang's studies of porpoises observed during relatively controlled "open water" tests yielded drag coefficients in approximate agreement with fully turbulent predictions during high speed coasting (where the drag coefficient may be determined more accurately than, but may not be a true representation of, the propulsive case).

Therefore, whether the porpoise uses a compliant skin still seems to be somewhat a matter of contention, but an important result of this line of inquiry is the drag reduction obtained by Kramer on periodic "porpoise-like" skins where evidently, the wall motion induced in the surface by the boundary layer disturbance field caused a lower drag. It is not entirely clear from Kramer's results whether the lower drag is a result of (a) delaying transition, (b) reducing the intensity of bursts in the transitional region, or (c) reducing the turbulent skin friction. The present authors' conjecture that at least the latter two effects [(b) and (c)] may have occurred. As a footnote to the porpoise skin discussion there is available a hydrophilic coating for use on boats. The surface of this coating evidently becomes somewhat similar to a compliant wall when water-saturated. Some details concerning the coating are available in Ref. 11. The coating was tested both in the laboratory[26] and on ships.[27] Some drag reduction is observed, but the mechanism involved has not been studied and the reduction is evidently not large.

The present critical review first briefly considers the structural dynamics and flow stability areas. The major portion of the paper concerns the theory and experiments for compliant walls under turbulent boundary layers.

## II. STRUCTURAL DYNAMIC CONSIDERATIONS FOR COMPLIANT WALLS

The compliant wall problem is one of fluid-structural dynamic interaction. Considerable structural dynamics and fluid-structural interaction research is available,[28,29] but many compliant wall experimenters have not used this technology to design their walls. Although this may be stating it somewhat strongly, the usual approach seems to have been, "this feels soft, let's try it."

The problem is not straightforward. The walls are exposed to a relatively wide-band spectral loading and the formulation must include the influence of wall-motion-induced aerodynamic forces. The prime variables are multitudinous and include: (a) flow parameters such as speed, Reynolds number, boundary layer thickness, and state (laminar, transitional, or turbulent), pressure gradient and flow medium (air or water, etc.); (b) structural configuration (vertical and/or horizontal layers, two or three dimensions, with or without pretensioned members) and the possibility of nonlinear effects such as caused by membranes backed with small air gaps; and (c) material property parameters such as density, modulus, damping, degree of anisotropy, and the possible variation of these properties with temperature, vibration frequency, and aging. Given a set of flow parameters and a stable of available materials, the type of structure is dictated by the surface response desired.

For the case of a simple two-dimensional convected surface wave, the fundamental wall motion parameters are wavelength, wave amplitude, and wave speed or frequency. However, since the structure is continuous, the surfaces will, in general, react over a whole range of amplitudes, wave speeds, and wave lengths. In addition, since the forcing field is three-dimensional, the induced motion could also be three-dimensional, depending upon the degree of "compliance." It is also possible that favorable alteration of the turbulence field demands the presence of unsymmetric surface waves having particular wall curvature or pressure gradient distributions.

Some attempts have been made[12,13,30–33] to utilize structural dynamics technology, of varying sophistication, to design compliant walls according to various assumed aerodynamic (or disturbance damping/alteration) criteria. One such criterion ("roughness" effect) is quite obvious from difficulties encountered in the experiments. The amplitude of the motion must be kept low enough to avoid causing a "roughness" effect which would increase, rather than decrease, the drag. The occurrence of static divergence conditions (large standing waves) have commonly been observed in compliant wall experiments[34–36] at high flow speeds and generally lead to drag increases. For application to the turbulent case one could use the common roughness height rule-of-thumb of $k^+ \leq 6$ to determine an allowable wave amplitude, where $k^+$ equals the wave or roughness height in usual law of the wall coordinates. [A superscript (+) is used throughout this paper to denote law of the wall units where one unit is given by $\nu/u_\tau$, kinematic viscosity divided by frictional velocity.] For most turbulent flows this limits the maximum wave height to less than 0.5 mm. However, except for the static divergence case, the Langley experience is that it is quite difficult, for air flows, to excite practical walls to amplitudes even the order of 0.02 mm (based upon both surface amplitude measurements and structural dynamic calculations).

There is another design criteria which is probably correct, the waves must move or at least have fairly high frequency. Most experiments with fixed wavy walls[37] (and the zero wave speed results of Ref. 38) used wave amplitudes which exceed the roughness criteria just mentioned and thus measured increases in drag. Reference 38 indicates that a fixed wavy wall has a pressure drag due to the phase shift between the surface contour and the wave-induced pressure field. Unpublished experiments at Langley with fixed waves smaller than those of Ref. 38 indicated, within an accuracy of $\simeq \pm 3$ percent, no net change in drag for low-speed turbulent flow. These latter data seem to suggest that drag reduction requires moving or relatively high frequency waves. Aerodynamic design criteria which might be used for compliant walls is discussed more fully in the theoretical sections of this paper.

One basic structural dynamics consideration that should be emphasized is the difference between the "compliant" walls dictated by stability theory[39,40] and the usual flexible walls employed in experiments. The successful compliant wall, according to stability theory, has extremely small damping and modulus of elasticity and can therefore respond, with little phase lag, to the wall forcing function produced by the flow disturbance field. However, such surfaces are practically impossible to construct. Therefore, the experimental data are taken with structures which are not optimal in the stability theory sense, but are actually "resonant surfaces" in that the flow-disturbance forcing function excites wall vibration modes.

Compliant wall experiments must, practically speaking, be conducted using resonant walls. The design of structures to provide the desired wall motion is complex and requires design techniques based on aerodynamic wall motion criteria obtained either from wall motion measurements during successful experiments or from rather sophisticated theory.

## III. INFLUENCE OF WALL MOTION ON STABILITY/ TRANSITION

Although the major purpose of the present paper concerns the influence of wall motion upon the drag in turbulent boundary layers, in this section a brief review of the laminar/transitional results (which are primarily theoretical) is provided for later theoretical application in the turbulent case. In particular, the turbulent wall burst formation and pre-burst flow may involve a relatively brief region (brief in time and space) where stability theory applies in the near wall region. Influencing this portion of the flow may be sufficient to break the feed-back loop which allows the turbulence to be self-sustaining (see also Ref. 40, p. 126).

### A. "Conventional compliant" walls

Pelt[41] reviewed the various approaches to the stability theory of compliant walls through approximately 1964, including the results of Benjamin,[42] Landahl,[39] Betchov,[43] Boggs and Tokita,[44] Hains and Price,[45] Nonweiler,[46] and Linebarger.[47] Theoretical research since the Pelt review comprises further work by Benjamin,[48] (including an excellent summary paper[40]), Landahl and Kaplan,[49] Kaplan,[50] Takematsu,[51] Burden,[52] and Amfilokhiyev *et al.*[53] From the number of authors cited, this has obviously been a popular problem, with most of the work accomplished in the early sixtys (primarily due to the stimulus of Kramer's results[22,23]). However, attempts to experimentally validate the theoretical calculations generally used either resonant surfaces, which do not correspond to the theoretical assumptions employed, or active walls.[54] In fact, Landahl[39] concludes that the Kramer surfaces were so far removed from the conditions necessary for stabilization according to the theory that the Kramer drag reduction must be due to a favorable modulation of the later stages of transition or turbulent portions of the body.

Burden[52] suggests the following compliant wall characteristics as necessary for stabilization of oscillations in the early stages of transitional flow (see also Kaplan[50]): (a) wall density the order of the fluid density (obviously out of the question for air); (b) shear modulus the order of the fluid dynamic pressure; (c) small wall dissipation; and (d) impervious wall. He further states, in agreement with earlier authors, that too much flexibility, or too much dissipation will allow the growth of instabilities not otherwise found in the zero-pressure-gradient rigid wall case. The definitive review of research in this area is that of Benjamin.[40] It should be noted that all of these approaches used steady state stability theory in the sense that the wall motion was not allowed to "back react" upon the assumed mean profile. For moderate to large wall displacements the surface motion induces a traveling pressure signal[38] which can modulate the mean profile and this invalidates a key assumption in the conventional stability theory approach to compliant walls. The relaxation of this assumption is discussed in a subsequent section of this paper.

Although Wehrmann[54] had to use an "active" or driven wall to simulate the compliant surface assumed in the theories, he was able to show, experimentally, that, depending upon the phase between the driven wall wave and the Tollmien-Schlichting wave generated by a vibrating ribbon, large reductions in velocity fluctuation intensity could be obtained. The largest reduction occurred near the wall.

In summary, there is a large body of theoretical research (which is probably correct, judging by the available checks on rigid wall stability theory and the results of Ref. 54) which suggests that certain types of compliant walls can (a) increase the lower critical Reynolds number, and (b) reduce the spatial and temporal amplification rates of unstable waves. The problem, particularly for the air case, is that the walls required to produce these effects evidently cannot be constructed with current technology. Consequently, we have a possible drag reduction effect of highly compliant walls which, while intriguing, is probably not practical, especially for air flows.

### B. Stability with resonant walls

The Kramer surfaces[22] are obviously more resonant than compliant due to their large damping. In the present review, we treat the Kramer data[22,55–57] as primarily indicating an alteration of the transitional and turbulent burst frequency/structure rather than a delay of the onset of transition, because of the amount and slope of the drag reduction obtained with the surface at large Reynolds numbers (up to $15\times10^6$).[22] Therefore, the detailed discussion of the Kramer data is deferred until the section on turbulent experiments.

There is essentially no applicable theory for the influence of resonant (excited structural eigenvalue, fairly wide-band motion, random phase) walls on transition except perhaps for piecewise application of the fairly large, and recent, effort on stability in modulated or periodic boundary layer flows. These latter efforts (theory and available experiments) are discussed in the next section since they were developed primarily for active or driven oscillations (wall or stream); the present discussion focuses on passive walls.

An experimental study was conducted at MIT[58] to simulate a compliant wall by a series of streamwise-connected Helmholtz resonators. "The vertical motions of plugs of air adjacent to the flow was intended to model the oscillations of a flexible membrane." These experimental results indicate earlier (increased instability), rather than retarded transition primarily because of the fairly large size and discrete nature of the surface perturbations. Attempts to duplicate Kramer's work by delaying transition in ground facilities during the early sixtys were also unsuccessful.[59,60]

The major investigations on the influence of resonant surfaces on stability were conducted by Babenko. This work included porpoise skin studies[61] and experimental stability investigations using a wide variety of surfaces.[62–64] These studies concentrated quite heavily upon the characterization of the structural dynamic parameters for the surfaces tested. The experimental results indicate that for compliant surfaces the wave number corresponding to initial loss of stability was larger than for rigid surfaces. Measurements of neutral stability curves provided Reynolds numbers for the loss of stability (lower critical Reynolds number) for wide variations in surface design and material parameters. Variables include surface material, damping, density, modulus, and tension (for those surface using membranes).

The data indicate that significant changes in critical Reynolds number occurred for resonant walls, with measured values both above and below the conventional Blasius value, depending upon the surface parameters. On the basis of the Babenko results, it seems that not only compliant walls but resonant walls as well, can favorably alter the stability boundaries of laminar flows. Considerably more experimental and theoretical research is required in this area, but the results may have eventual practical application, particularly for water flows, where one can build resonant walls over a wide range of wall motion parameters (e.g., frequency, amplitude, wave speed, spectra). This research further suggests that Kramer's assertions that he delayed transition with his fairly high damping surfaces may be correct. As a final note to this section there are also the data of Karplus,[65] who measured a decrease in Reynolds number for transition, and a lengthening of the transition region itself; the results are of a preliminary nature.

### C. Stability of periodic boundary layers

As already stated, the modulation produced by a resonant wall can be sufficient to significantly alter the effective mean velocity profile under investigation. There are two effects; one is the obvious influence of alternately increasing and decreasing, through profile modification, the disturbance amplification rate, so that the integrated amplitude ratio may be greater (or less) depending upon the amplitude, frequency, and wave speed of the imposed oscillation. The second influence involves the timewise dynamics of the modulated boundary layer in that the application of ordinary stability theory in a quasi-steady sense is not sufficient, and one must include time derivatives of the mean profile parameters in the theory. It is the present authors' contention that this timewise modulation of the mean flow and the subsequent influence upon stability may be the key to further theoretical understanding of the experimental results obtained using resonant walls for both laminar and turbulent flows.

Several types of modulations are possible. The free stream (or wall) velocity can be varied primarily in the in-plane or stream direction.[66] This imposes a cyclic variation which diffuses in a viscous manner through the profile. On the other hand, one can impose a pressure gradient modulation upon the flow, in which case the entire profile is modified. This pressure modulation can be imposed either upon the flow as a whole (i.e., vary the imposed pressure drop in a Poiseuille flow with time) or can be generated locally at the wall by the use of traveling surface waves. This latter method probably occurs for the resonant or active wall case.[13]

As noted earlier, the rigid wavy wall does not seem to produce a favorable effect, one needs moving surface waves, or at least stationary, nodal time-dependent surface motion. The stability theory calculation for a rigid wavy wall (with parallel flow assumption[67]), indicates a destabilization effect.

There are two recent, and quite excellent, reviews in this area of stability for modulated viscous flows,[68,69] the latter being an outgrowth of work in Refs. 70–72. These reviews are highly recommended by the present authors as entry points for this relatively new, and quite exciting, technology. It appears to be important to account for the true time-dependent nature of the complete problem; a quasi-steady stability analysis of the modulated flow[73] is not sufficient in many cases. For instance, if one examines a flow containing an inflection point during some portion of the cycle, this instantaneous profile will generally have a reduced lower critical Reynolds number compared with the unmodulated profile. The initial reaction is to brand the modulation as destabilizing, since, somewhere in the cycle, the lower critical Reynolds number is reduced. Such a conclusion could be erroneous. A more satisfactory approach (but incorrect for high frequency) is to use the quasi-steady analysis to generate local amplification rates and then integrate these over complete cycles of the modulation.[69] For stabilization, the trick seems to involve using modulations which are not so large that they trigger enormous amplification during part of a cycle. Therefore, one needs small-to-moderate amplitude modulations but not so small as to have little or no measurable effect upon the flow, a problem with resonant surfaces in air flows where an extremely low wall modulus is required to produce any reasonable surface amplitude. Another requirement for stabilization, although this varies with the particular type of flow,[68] is a relatively high frequency modulation. Theoretical papers of particular interest include Refs. 74 and 75. Some of the limited experimental backup data (using active or driven disturbances) are given in Refs. 76–79.

In summary, the results of both theoretical and experimental studies in time-modulated flows indicate that it may be possible to stabilize boundary layer flows using low-to-moderate amplitude/relatively high frequency oscillations. In the compliant wall case these oscillations, in the form of pressure modulations due to traveling surface waves, could result from the use of resonant walls. Further research is obviously needed to determine whether or not these indications are correct.

## IV. THEORETICAL $C_f$ REDUCTION MECHANISMS FOR TURBULENT FLOWS WITH MOVING WALLS

Based upon Refs. 80 and 81, it is probable that the turbulent drag reduction mechanisms to be discussed in this section are also applicable to the later (turbulent burst) stage of transition, and therefore, one or more of these mechanisms could cause a stretch-out and reduction of drag for the transition region as well as a turbulent skin friction reduction. This postulation is further bolstered by the common sensitivity of transitional and turbulent flow regions to changes in boundary conditions (other than wall motion) such as pressure gradient, wall suction/blowing, and longitudinal curvature.

The influence of wall modulation upon shear flows is part of the general problem of the behavior of turbulent flows subjected to periodic disturbances. This question is investigated in some detail in Refs. 82 and 83, but the pertubation amplitudes are relatively small, and the studies do not consider the possibility of altering the basic turbulent structure via this approach. Experimental attempts to introduce substantial periodic disturbances into turbulent wall layers (other than by wall motion) are documented in Refs. 84 and 85. Reference 84 describes the influence of large (up to 34%) narrow-band fluctuations in free stream velocity at Strouhal numbers less than 1 (based on boundary layer thickness and free stream velocity). This is a low frequency in terms of the modulations required by unsteady stability theory to increase stability (as described in the previous section). No appreciable alteration of the turbulence structure was observed. In Ref. 85 a turbulent boundary layer was subjected to an imposed sound field over a wide frequency range. The experimental trends observed are in agreement with the unsteady stability theory results in that high frequency sound caused a reduction in surface heating and lower frequency sound caused an increase. However, the effects are not large, only of the order of 4%.

This experiment, along with the compliant wall burst modulation theory and interpretations of compliant wall data (both described later in this paper) as well as the unsteady stability theory, indicate that a reduction in the turbulence transport rate requires a high frequency modulation (of moderate amplitude). The present authors suggest that future experiments in the area of turbulence control for wall flows concentrate on the high frequency range to prove or disprove this evidentiary material.

### A. Sublayer models

#### *1. Analysis by J. E. Ffowcs Williams*

Ffowcs Williams[86] uses the assumption of two-dimensional disturbances and linearized (sublayer) equations to derive an expression for shear stress near a compliant membrane. From examination of this expression he concludes that moving waves in a surface with low wave speed can produce a region of "negative Reynolds stress" which could "starve the turbulent eddies" and "may lead to a reduction in the turbulent level." Therefore, for a shear stress decrease, this approach seems to indicate the use of surfaces designed to generate low speed waves (less than about eighty percent of the free stream velocity, the convection speed of the gross wall pressure fluctuations). The approach of Blick[87] is quite similar.

*2. The sublayer approach of Semenov*

Semenov[88] provides an alternate sublayer type analysis, using equations similar to those of Sternberg.[89] He predicts that the Reynolds stress could decrease, or increase, depending upon the characteristics of the surface motion, and is careful to point out the importance of the surface wave phase angle. Comparisons between the predictions of Ref. 88 and the data of Ref. 90 (shown in Fig. 3 of Ref. 88) indicate quite reasonable agreement for the change in Reynolds stress as a function of frequency due to a resonant wall. Both hot-wire data[90] and theory give a reduction due to the wall motion at low frequency (region of "energy containing eddies") and an increase for higher frequencies. To the present authors' knowledge the theory has not yet been extensively applied, evidently due to the lack of the necessary empirical input for cases other than Ref. 90. The approach probably could be used in parametric studies to predict the type of surface motion necessary for drag reduction (and thus provide design data for experimental surfaces).

*3. A mixing length approach*

Amfilokhiyev[91] assumes that the elasticity of a surface (wall motion) reduces the Prandtl wall slope ($K \simeq 0.4$ for rigid walls) and indicates skin friction values as a function of the assumed decrease in $K$. This approach does not agree with the data of Ref. 90, where $K$ was still $\simeq 0.4$ in the drag reduction case. In Ref. 90 the sublayer thickness was increased, thereby indicating, in terms of mixing length turbulence closure, an increase in the Van Driest wall damping constant, $A^+$.

*4. Analysis of Zimmermann[14]*

He considers the surface motion as a small disturbance to the basic turbulent flow, which is assumed known in detail. Further assumptions include low damping in the surface and small thickness, an approach to the surface dynamics which is similar to that used in the conventional stability theory of compliant walls. The Navier–Stokes equations are then linearized using the small disturbance assumption and solved by a Green's function approach. He uses measured pressure-velocity correlation data from rigid walls to estimate the change in shear stress due to the flexible surface and predicts only a small reduction for air, but a measurable ($\simeq 5\%$) reduction for water where the coupling is greater between the fluid and flexible wall.

*5. Reynolds stress at surface*

Hueristic arguments concerning the possibility of producing an altered Reynolds stress condition simply due to the horizontal and vertical velocity fluctuations, $u'$ and $v'$ associated with the surface motion itself are available in Refs. 92 and 93.

**B. Drag reduction mechanisms based upon turbulent burst modulation**

The "sublayer" theoretical approaches just described for the problem of the wall motion effect upon turbulence were based upon traditional modeling of sublayer flow and use of Reynolds averaging. Analysis of the more recent research on the detailed structure of wall turbulence, particularly the flow visualization and conditional sampling data, yields an alternate theoretical approach,[13] based upon the possible modulation of the pre-burst flow. The implications from the detailed data on rigid walls are first summarized briefly and then the possible modification of the turbulence production process using high frequency wall motion is discussed.

*1. Coherent structures and a possible feedback mechanism for turbulent wall flows*

There is considerable evidence that a "quasi-ordered" or "coherent" series of fluid dynamic events are responsible for the production of turbulence in wall flows. These events occur randomly in time and space and originate above the surface in the near wall ($y^+ < 100$) region. Several excellent reviews are available which analyze and summarize this relatively recent information,[94–96] especially Ref. 97. Reference 81 attempts to model turbulent flows using this information. Stated briefly, a low speed streak occurs very near the wall; this streak undergoes retardation with increases in time and space with a "burst" or eruption of the low speed fluid occurring for the more severe retardations. This burst and subsequent sweep provides the bulk of the Reynolds stress and turbulence production. The flow between events and the pre-burst retardation region is relatively quiescent (low $\overline{u'v'}$). This latter statement is important for theoretical calculations of these events, because in order to reproduce the retardation one should not use the fully developed or time-averaged Reynolds stress level in the flow, but instead a rather small fraction of that level. There is still considerable controversy as to the relationship between the retardation and the burst or ejection. Many authors suggest that the (retardation-influenced) pre-burst profile undergoes an instability growth/amplification in a fashion similar to the burst formation in transitional flows. Landahl (see Sec. 7 of Ref. 80) suggests that the burst may be due to space-time focusing of instability waves, leading to the observed catastrophic growth. Both experiment and theory indicate the existence of a highly inflected retarded profile (inflection point at $y^+ \simeq 25$, Fig. 31 of Ref. 97) as the required pre-burst condition. (In Landahl's theory the inflection is a necessary, but not sufficient, condition.) This near wall turbulence production, through discrete bursts, bears remarkable resemblance to the earlier theories.[98,99]

A logical question is, what sets up the requisite pre-burst profile, why does the profile become retarded? It has long been known (Ref. 100 and others) that the larger scale, more intense portion of the wall pressure spectrum has a convection speed the order of 0.7 to $0.8U_\infty$ and appears to originate at $y^+ > 100$. There is evidence[97,101,102] that the retardation is caused by this portion of the wall pressure field, which in turn is probably due to the growth and interaction of old bursts produced upstream. In fact, Burton[94] measured a strong correlation between the occurrence of a burst and the imposition on the wall flow of a large moving adverse pressure gradient signal with a magnitude of the order of three times the rms wall pressure intensity. This adverse pressure gradient was followed by a favorable gradient, perhaps at least partially responsible for the sweep portion of the event cycle. It should be noted that recent evidence appears to show more high frequency energy in the wall pressure fluctuation spectra than was originally indicated, perhaps associated with the burst process (see Fig. 9 of Ref. 97 and Ref. 103). However, the burst initiation seems to involve the lower frequency portion of the spectrum. It should be noted that not all outer-flow imposed, low frequency wall pressure fluctuations produce bursts. Evidently, only the large amplitude, long lasting pulses can cause sufficient retardation to produce the necessary pre-burst conditions. Preliminary quasi-steady retarded near wall calculations, using low levels of Reynolds stress and the burst-producing pressure pulses measured by Burton,[94] indicated that the pressure gradients involved are sufficient to cause severe retardation of the near wall flow.[13] The recognition of the low Reynolds stress condition of the pre-burst flow is critically important to the success of such calculations. Further, much more detailed and complete calculations of the pre-burst flow (with and without resonant wall motion) are currently underway by Orszag in collaboration with Langley Research Center, National Aeronautics and Space Administration.

Therefore, there exists a possible feedback mechanism in which older bursts, which have grown and migrated up to the law of the wall and outer portions of the boundary layer, interact and produce a pressure field which contains pulses of sufficient duration and amplitude to induce new bursts in the near wall flow. This near wall region contains a high level of background turbulence but fairly low levels of $\overline{u'v'}$ during the non-burst periods.

It is interesting that an effort to artificially induce bursts using a stationary pressure pulse[104] did not seem to alter the characteristic burst frequency of a turbulent wall flow.

### *2. Burst modulation due to wall motion*

One approach to a theory of drag reduction due to wall motion is to postulate that certain types of surface motion can alter or interrupt some portion of the feedback process just described. The portion of the process most available for alteration by wall motion is the pre-burst, near wall retardation region. Other drag reducing procedures, such as the injection of polymers, may alter this or some other part of the feedback loop.

Reference 13 suggests that the moving surface waves associated with a resonant wall can produce a modulation of the pre-burst retarded flow. The details of this modulation are a function of the wavelength, wave speed (or frequency), and amplitude of the surface motion; however, for short, steep traveling waves the wall motion can produce a modulated pressure gradient, (modified for viscous effects and wave speed as per the Kendall[38] data) alternately positive and negative. This modulation is superimposed upon the adverse gradient signal which normally triggers the bursts. The suggestion[13] is that this modulation, if it has high frequency and moderate amplitude (see the section on stability theory for periodic flows), can delay the burst formation. Considering the expected stabilizing influence of the following favorable gradient portion of the outer flow generated signal, only a small delay may be sufficient to completely obviate the burst. Longer waves (lengths in excess of 500 wall units) would presumably trigger as many extra bursts (due to the long adverse gradient portion of the wave) as the number of bursts eliminated by the stabilizing influence of the favorable gradient part of the wave train. An alternate approach to a similar mechanism is to analyze the pre-burst flow using the breakdown mechanism of Landahl.[105,106] As a minimum, the wall frequency required is probably the order of 50 times the fundamental burst frequency ($\lambda^+ \sim 100$, $C/U_\infty \sim 0.5$).

The burst alteration mechanism just described is only a suggestion; further, more detailed calculations and experiments are necessary to either prove, or disprove, this approach. However, it is known that small surface waves can alter much larger scales of turbulence in the atmosphere over a sea surface[107,108] and the high frequency surface motion dictated by the mechanism is in general agreement with wall motions computed for successful drag reduction experiments (next section of report). Detailed calculations using this burst modulation model are underway.

## V. CRITICAL ANALYSIS OF RESONANT WALL EXPERIMENTS UNDER TURBULENT BOUNDARY LAYERS

In this section we attempt to examine the available data with the following two questions in mind: (1) Is there another possible, (and defensible) explanation for the drag reduction observed, and (2) for the successful cases which survive the filter of question 1, what is the computed wall motion, is it high or low frequency (Strouhal number much greater than, or less than, one)? We also mention, for completeness, some of the experiments on drag over resonant water waves in air and the unsuccessful compliant wall experiments, which are discussed first.

### A. "Unsuccessful" (small reduction, no change or $C_f$ increase) experiments

It should be noted beforehand that this class of experiments has not received the detailed study by the present authors which it perhaps deserves. We have spent most of our efforts trying to understand the successful experiments, in an attempt to infer the type of wall motion responsible for the observed drag reduction (if no other cause for the reduction other than wall motion input could be found). On the basis of the theoretical and experimental results thus far, it is reasonable to conjecture that a favorable influence (reduction) on $C_f$, if indeed such a thing can occur, is probably only possible over a relatively narrow range of wall motion conditions. Without an attempt to control, measure or widely vary the wall motion, hitting the correct combination in an experiment is a pure happenstance. Nevertheless, there are a few obvious comments which can be made concerning these unsuccessful studies.

### 1. *Ritter and Porteous*[109]

This water experiment used the original Kramer surface material (periodic, "stubbed" rubber—discussed in considerable detail in Sec. VD). A pertinent comment from Ref. 110 on these data is that the free stream turbulence level was probably too high to constitute a valid test of the laminarizing properties of the surface. The speed was limited due to the formation of local blisters in the coating at high dynamic pressure. Skin friction was estimated from measured values of momentum thickness, a procedure which is not always sufficiently accurate.[12] Problems with the fairing at the leading edge of the coating caused premature transition.

What is fascinating about these data is (1) the use of a periodic surface (spatially repeated substructure, although the modulus of the rubber may have been somewhat high considering the reduced speed range of the tests), and (2) the data in Fig. 5 of Ref. 109 which indicate that drag levels at various distances back along the body have a premature turbulent-like variation with Reynolds number, *and this distribution is below the developed turbulent level.* The amount of this drag reduction is greater farther forward on the body, but due to the later transition on the rigid body (better surface finish) the compliant wall drag levels are generally above the rigid case (and therefore, the authors of Ref. 109 did not consider the data as indicating a drag reduction). The present authors strongly suggest that the fact that these compliant data, obtained using a periodic surface, have a turbulent-like variation with Reynolds number (but at a lower $C_f$ level) may be highly significant. Based on the discussion in the paper thus far, a single stub spacing would not be expected to work well over a wide Reynolds number range, and also the well-known peak in $P_w'/q_\infty$ (ratio of root mean square wall pressure to free stream dynamic pressure) associated with the end of transition may help explain the apparent variation of performance with body distance. This experiment should probably be redone, with more complete instrumentation, to higher speeds, and using a series of periodic surfaces designed over a range of wavelength, wave amplitude, and wave speed (or frequency).

### 2. *Ritter/Messum*[111]

This is a continuation of the research of Ref. 109, but using small flat plates rather than cylindrical bodies. Again, the stream turbulence levels were quite high, and Kramer's stubbed coatings were used. The data, taken only for turbulent boundary layer flow on six different skins, exhibit considerable scattering, but two types of skins gave a drag reduction. One skin showed a consistent 7–14% drag reduction, again exhibiting a variation with Reynolds number parallel to, but below, the rigid turbulent case. A second skin gave a drag reduction (7–15%) except at high speed where standing waves (roughness) may have occurred.

### 3. *Laufer and Maestrello*[112]

From Ref. 113, one of the problems in this experiment was the possible reflection of flow-induced surface waves from the downstream end of the channel. Periodic surfaces were used (in some of the tests). A further difficulty in this experiment is that air (rather than water) was used as the test medium. Therefore, the velocity of the flow must be considerably higher to provide a level of wall pressure fluctuation (forcing function) necessary to excite the fairly stiff surfaces used. However, when the flow speed is increased the frequency requirements which the surface waves must meet also increase and therefore, a very careful surface structural design effort is needed. The Langley experience with the design of periodic surfaces for air flows shows that one can get reasonable amplitudes with long wave length (low frequency), but as one tries to increase the frequency of the surface response the amplitude of the motion is considerably reduced, even using very low modulus materials. The periodic surface design problem for air flows is not straightforward. The general conclusion from Ref. 112 is that the surfaces used were too rigid, except for one case, and in that case the duct cross-section changed, due to the pressure loading on the flexible surface, thus invalidating the results.

### 4. *Smith*[114]

This is a pipe flow experiment using a nonperiodic (gel-coated) surface and diesel fuel as the test fluid. Only drag increases were measured, and these were ascribed to the roughness of the gel coating. For the higher Reynolds number data, long wave instabilities may have occurred, giving increased roughness. There are several other pipe experiments besides Ref. 114, but these gave substantial drag reductions, and therefore, the discussion of the main portion of the pipe data is postponed until Sec. VB.

### 5. *Dinkelacker*[115, 116]

This is one of the most carefully conducted of the compliant surface tests. Water was the test medium and the facility was a pipe inlet flow. In particular, Dinkelacker attempted to determine, for his experiments (a) the repeatability of the rigid tube data, (b) influence of small steps in the tube walls (found to be significant), and (c) possible occurrence/effect of organ pipe acoustic modes, along with studies of other possible influences upon his experimental results. In his own words,[115] "The main outcome of this investigation was the establishment of a detailed picture of the difficulties which may occur in experiments with flexible walls." Some results of the study seemed to indicate a reduction in drag.

### 6. *Taneda/Honji*[117]

This study measured drag on towed flat plates in water, using non-periodic surfaces. The drag of the flexible surface covered plates was generally greater than the rigid ones (due perhaps to early transition). Also, the turbulent levels for the flexible data were generally equal to, or greater than, the rigid levels.

### 7. *Mattout*[118]

A fairly detailed study using both resonant and active (driven) walls in water. For the driven wall case a 25% drag reduction and a thrust[122] is obtained. For the flexible wall, only a 7% reduction was observed. For the driven wall case, the waves were quite long (4.2 to 50 cm) compared with the boundary layer thickness which was on the order of 2 cm. The passive walls were membrane covered foams, which are not really periodic surfaces for the relatively small pore size used. Mattout invested considerable effort in determing the modulus of his passive surfaces and used both Polyvinyl chloride and Mylar membranes. Prime variables included membrane tension and flow velocity. Figure 39 of Ref. 119 provides a thumbnail review of the compliant wall results up to 1970.

### 8. *Grosskreutz*[16, 93, 123]

This was an attempt to build nonisotropic effects into the wall and is one of the very few studies with wall motion data. The basic concept was to preload the surface so that the surface motion always produced a positive correlation of Reynolds stress, i.e., $u$ and $v$ at the surface were either both positive or both negative. Therefore, the surfaces (which were periodic due to the discrete pre-loading used) directly controlled the Reynolds stress at the wall. The tests were conducted in water, and the surfaces used are somewhat similar to Kramer's (see Fig. 2 of Ref. 93) except that the periodicity length is greater than in the Kramer case. Grosskreutz measured a reduction in momentum thickness on one of the surfaces for the low velocity (low unit Reynolds number) cases. The present authors speculate that there is a possibility that the favorable low unit Reynolds number results may have been due to thicker sublayers where the periodic spacing used was closer to the wavelengths required for burst modulation.

### 9. *Hansen/Hunston*[34]

These experiments were run on a rotating disk in water using layered (nonperiodic) surfaces. There is insufficient conditional sampling data for the three-dimensional boundary layers on rotating disks to determine the type of wavelengths and three-dimensional waveforms which might be required for burst modulation. The results of the experiment[34] were null in that no drag reduction occurred. At high speeds, where static divergence waves formed, the drag increased. One should probably stay with zero pressure gradient, two-dimensional, turbulent flows for testing of wall motion effects and prove/disprove the case for drag reduction there, before considering three-dimensional and pressure gradient effects.

### 10. *Kawamata et al.*[124]

This study used a floating element model on the wall of a water tunnel. Material properties such as damping coefficient and stiffness are given for several of the surfaces used, which were fairly thick rubber or Neoprene membranes backed by either air or olive oil. Due to the nonperiodicity of the surfaces, the wavelengths were probably quite large. The authors claim large drag reductions, but there are more drag increase data than drag reduction. The most worrisome point is that sizable reductions are observed in the fully laminar case, perhaps indicating possible problems with accuracy using the floating element balance (flow/pressure gradients in the gap, etc.) or other experimental problems. These experiments should be carefully analyzed and perhaps repeated.

### 11. *McAlister and Wynn*[125]

This was an attempt to confirm the results of Blick *et al.* at the University of Oklahoma (Refs. 36, 90, 126–132, reviewed in Ref. 11). In Sec. VC the present authors suggest, based upon recent Langley analysis and experimental results, a possible alternative explanation for the drag reductions observed during the Oklahoma liquid-backed membrane tests. This alternate explanation may account for the difficulty which McAlister and Wynn and also Lissaman and Harris[35] experienced in reproducing the liquid-backed Oklahoma results. McAlister and Wynn used an airfoil type model mounted on flexures for direct force measurements and tested in air. There may be an absolute accuracy problem in these experiments associated with the combination of pressure and skin friction drag sensed by the load cells and the percentage of overall surface area covered with compliant surface. The flexible surfaces used were layered and non-periodic and were probably too stiff to respond well to the relatively low dynamic pressure of the air flow. No drag reduction was observed.

In summary, even though labeled "unsuccessful," several of these studies did observe some drag reduction ($\simeq$ 5–15 percent) particularly for conditions condusive to small wavelength, steep waves (low unit Reynolds number, water flow, periodic surfaces with small cell size).

## B. Flexible pipe experiments

Successful drag reduction experiments with turbulent flows in flexible pipes have been reported by Pelt,[41] Teslo and his co-workers,[133,134] and Klinzing *et al.*[135] All of these investigators have reported significant reductions in the pressure drop for a flexible pipe when compared with a rigid pipe. However, these data should be questioned because of the extreme sensitivity of pressure drop to changes in diameter. A theoretical model is discussed which indicates that significant drag reductions can be attributed directly to changes in tube shape rather than a compliant effect (see also Ref. 136 for possible effects of geometry change).

Hydrostatic displacement of the tubes employed in Pelt's experiments has been studied in some detail.[137] Although that analysis does not apply directly here, an important result was that, for the materials and geometry employed in Pelt's experiments, the radial tube displacement, $w(x)$, could be well approximated by

$$w(x) = (R_0/\alpha)\left[R_0 P(x) - \nu_T N_x(x)\right]$$

for axial ($x$) locations away from the ends. In this equation, $R_0$ is the tube radius, $P$ is the local fluid pressure, $\nu_T$ is Poisson's ratio, $N_x$ is the axial load, and $\alpha$ is a constant given by

$$\alpha = E\delta_T\left(1 + \frac{\delta_T^2}{12 R_0^2(1-\nu_T^2)}\right) ,$$

where $E$ is Young's modulus for the tube and $\delta_T$ is the tube thickness. The tube was modeled as a shell, the influence of end conditions on the tube deformation was restricted to very small regions in the vicinity of each end (less than two diameters). These approximations are also valid for the flexible tube employed by Teslo *et al.*[133,134] By assuming quasi-parallel flow, the tube deformation can be coupled with the flow-induced pressure drop in a straightforward manner.

Assuming that the rigid pipe friction factor, $\bar{\lambda}$ can be employed locally for the flexible pipe (quasi-parallel flow) and that $\gamma$ varies with Reynolds number, $R_e$ according to

$$\bar{\lambda} = C\, R_e^{-\beta} = 0.3164\, R_e^{-1/4}$$

which is related to the local pressure gradient by

$$\frac{dP}{dx} = \frac{\bar{\lambda}}{D}\,\frac{1}{2}\,\rho V^2 ,$$

where $D$ is the local pipe diameter, $\rho$ is the fluid density, and $V$ is the local mean velocity, the local pressure gradient can be developed as a function of diameter. If $\gamma_0$ is the rigid pipe, fully developed pressure gradient, $dP/dx_r$, given by

$$-\gamma_0 = \frac{dP}{dx_r} = -\frac{\bar{\lambda}_0}{D_0}\,\frac{1}{2}\,\rho V_0^2 ,$$

it can be incorporated in the last equation along with the conservation of mass requirement

$$VD^2 = V_0 D_0^2 ,$$

to write

$$dP/dx = -\gamma_0\left(\frac{D_0}{D}\right)^{5-\beta} .$$

Furthermore, the local diameter is given by

$$D(x) = D_0 + 2w = D_0\left(1 + \frac{R_0 P(x) - \nu N_x}{\alpha}\right)$$

and the axial load, $N_x$ is related to the applied tension, $T$ and local fluid pressure, $P(x)$ by

$$N_x = T + (R_0/2)\,P(x) ,$$

when the pressure at the end of the tube ($x = L$) is zero. Combining the last three equations

$$\frac{dP}{dx} = \frac{-\gamma_0}{\left\{\left[1 - \frac{\nu}{\alpha}T + \frac{R_0}{\alpha}\left(1 - \frac{\nu}{2}\right)P\right]^{5-\beta}\right\}^{-1}}$$

which can be integrated between $x = 0$ and $L$ to yield

$$\left[1 - \frac{\nu}{\alpha}T + \frac{R_0}{\alpha}\left(1 - \frac{\nu}{2}\right)P_0\right]^{6-\beta} = \left(1 - \frac{\nu}{\alpha}T\right)^{6-\beta} + \frac{1}{\alpha}R_0(6-\beta)\left(1 - \frac{\nu}{2}\right)\gamma_0 L$$

or

$$P_0 = \frac{\alpha}{R_0\; 1 - \frac{\nu}{2}}\left\{\left[\left(1 - \frac{\nu}{\alpha}T\right)^{6-\beta} + \frac{1}{\alpha}\left(6-\beta\right)\left(1 - \frac{\nu}{2}\right)\gamma_0 R_0 L\right]^{1/(6-\beta)} - \left(1 - \frac{\nu}{\alpha}T\right)\right\} ,$$

where $P_0$ is the pressure at $x = 0$, which in this case corresponds to the estimated pressure drop.

Using the flexible pipe data in Table I, the pressure

TABLE I. Flexible pipe data.

| Pipe material | Thick rubber[a] | Thin rubber[a] | Tygon[a] | Texin[a] | Polyethlene[b] |
|---|---|---|---|---|---|
| $L$, m | 2.79 | 2.79 | 2.79 | 2.79 | 2.80 |
| $R_0$, mm | 4.8 | 4.8 | 4.8 | 4.0 | 25.0 |
| $\delta_T$, mm | 5.6 | 3.6 | 1.6 | 0.8 | 0.16 |
| $E$, N/cm$^2$ | 140 | 70 | 250 | 1000 | 15 000 |
| $\nu_T$ | 0.49 | 0.49 | 0.45 | 0.45 | 0.45 |
| $T$, N/cm[c] | 5.2 | 2.4 | 3.8 | 3.3 | NA |
| measured pressure drop, N/cm$^2$ | 18.1 | 12.8 | 18.6 | 22.0 | 0.110 |
| $P_0(T=0)$, N/cm$^2$ | 20.3 | 12.2 | 13.4 | 24.8 | 0.128 |
| pressure drop for a rigid pipe, N/cm$^2$ | 24.7 | 18.0 | 18.0 | 31.1 | 0.128 |

[a]Data from Pelt, Ref. 21.
[b]Data from Teslo and Zhoga, Ref. 37.
[c]Tension estimation based on reported elongation.

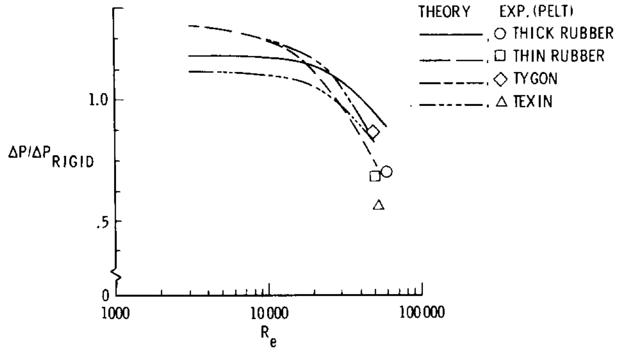


FIG. 1. Variation of flexible pipe pressure drop with Reynolds number.

drop has been calculated from this last equation for various Reynolds numbers and is shown in Fig. 1. Since the effect of gravity (which was important in Pelt's[41] vertical flow experiments) has been ignored, the calculated pressure drops cannot agree with Pelt's measurements, particularly at low Reynolds numbers where hydrostatic effects dominate. However, at Reynolds numbers of 50 000 the calculated pressure drop agrees with the measured data to within about 10% in all cases except for tygon tubing where the theoretical prediction was about 30% lower than the measured value.

The close agreement between theory and experiment may be fortuitous because end effects and viscoelastic effects have not been considered. End conditions would generally increase the pressure drop whereas viscoelastic creep would likely result in diametral increases and subsequent reductions in pressure drop. The latter effect may be responsible for the 25% drag reduction reported in Teslo *et al.*[133,134] Teslo and Filipchuk[134] note that a 3% increase in diameter occurred in their experiments, which is not predicted by the current theory. A 3% increase in diameter will cause a 15% reduction in pressure drop. Conversely, a 25% reduction can be caused by a 5% change in diameter which may be within the uncertainty of their measurements. Although the present investigation cannot show positively that the data of Teslo *et al.* are a result of pipe deformations, it appears very likely that creep may have occurred during those tests.

### C. Re-evaluation of compliant wall experiments in air with water substrates

A large portion of the successful compliant wall air data (which indicate drag reductions to 50%) were obtained at the University of Oklahoma during the years 1966 through 1969.[126–132] A re-evaluation of the Oklahoma experiments with water substrates (most of the data) was conducted at the Langley Research Center and is presented in Ref. 138.

The Oklahoma experiments, conducted on compliant surfaces in air at subsonic speeds with foam and/or fluid substrates, measured, directly, the skin friction drag on floating panel models which were mounted flush with the tunnel floor. The models, shown schematically in Fig. 2, were mounted on a long, vertical, single-column beam. Weights were attached to the upstream portion of the models to counterbalance bending moments caused by the surface skin friction. This arrangement permitted very low drag forces to be measured on the compliant model panels (e.g., forces as small as 0.5% of the hard plate drag could be detected).

The models for Ref. 127 had a compliant surface 63.8 cm long by 18.1 cm wide whereas the model surface for Ref. 128 was 38.1 cm long 24.1 cm wide. Various fluid substrates with different viscosities (i.e., air, water, and solutions of water and Polyox) were explored in the investigation of Ref. 127 which was conducted at a free stream velocity of 11.6 m/sec. Skin friction drag reductions up to 50% were reported for these tests. Reference 128 explored the effect of free stream velocity (i.e., $U_\infty \simeq 5.2-67.1$ m/sec) on the compliant wall drag reduction for porous polyurethane foam substrates (some of which were saturated with water), and drag reductions up to 38% were achieved. Both investigations used thin polyvinyl chloride sheets, either 0.0064 or 0.0089 cm thick, for the compliant wall skins.

The following re-evaluation of the Oklahoma water-backed membrane tests stems from thus-far unsuccessful attempts at the Langley Research Center to experimentally verify the University of Oklahoma experiments. As noted previously, Ref. 35 and 125 were also unable to reproduce the liquid-backed Oklahoma experiments. No drag reductions were obtained in Langley experiments conducted in a small subsonic wind tunnel on floating panel compliant wall models with liquid substrates. The size of the models tested, the fluid substrates, and the flow conditions were similar to those reported in Refs. 127 and 128, and are summarized in Table II. The major difference between the Langley experiments and those at the University of Oklahoma was the method of obtaining the direct skin friction drag measurements. Whereas the University of Oklahoma experiments used the single-column, cantilever beam type of arrangement, the Langley experiments supported the floating panels by thin wires attached to each of the four corners of the test panels outside the air stream (see Fig. 3). The drag force was then obtained by determining the amount of weight necessary to null the test plate to the original windoff position.

In the Langley attempts to duplicate the University of Oklahoma experiments, small standing waves were measured on the compliant surface as the free stream velocity was increased from 15 to 30 m/sec. These waves

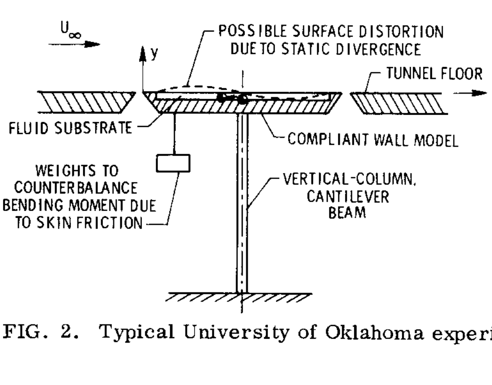


FIG. 2. Typical University of Oklahoma experiment.

TABLE II. Comparison of Langley Research Center and University of Oklahoma experiments in air with water substrates.

| | Langley research center | University of Oklahoma Looney/Blick | University of Oklahoma Chu/Blick |
|---|---|---|---|
| Stream velocity | 9–30 m/sec | 11.6 m/sec | 5 – 67 m/sec |
| Model surface | | | |
| Length | 66 cm | 63.8 cm | 38.1 cm |
| Width | 20.3 cm | 18.1 cm | 24.1 cm |
| Depth | 1.27 cm | 0.79 cm | 1.27 cm |
| Membrane | | | |
| Material | PVC | PVC | PVC |
| Thickness | 0.0064 cm | 0.0064 and 0.0089 cm | 0.0064 cm |
| Longitudinal tension | 3.5 N/m | 4 – 111.7 N/m | 55.3 N/m |
| Lateral tension | 3.5 N/m | 1.9 – 55.5 N/m | 53.6 N/m |

resembled sine waves with half of the wave protuding over the upstream portion of the model and the other half of the wave being recessed over the downstream end of the model. Reference 128 and private communications with Dr. Edward F. Blick of the University of Oklahoma acknowledged the existence of small standing waves in the Oklahoma tests at certain free stream velocities. It is concluded that this information, coupled with the results of the Langley drag reduction experiments, suggest that standing waves in the University of Oklahoma experiments could have caused a shift in the model center of gravity and this shift may have created a bending moment on their single column balance that was interpreted as a reduction in the skin friction drag.

Reference 138 analytically determined the amplitude of a simple sine wave necessary to produce a center of gravity shift and resulting bending moment large enough to account for an apparent 40 percent reduction in skin friction drag on a single-beam balance. The assumed wave shape is shown in Fig. 2. Based on a hard plate average skin friction coefficient of 0.00389 for the tests of Ref. 127, the apparent drag reduction was approximately 1.9 g. For the balance moment arm of 85.7 cm in the University of Oklahoma tests, this apparent drag reduction could have generated a bending moment of 1.63 g-m. The maximum amplitude of a sine wave necessary to shift the model center of gravity upstream and to create this bending moment was only 0.013 cm (assuming that the compliant wall skin was always in contact with the water substrate). As Ref. 138 points out, surface motion this small would obviously be very difficult to detect with the unaided eye and could probably have been overlooked.

To check the analytically determined magnitude of the possible surface deflections, Ref. 138 presents measured surface deflections for the Langley experiments of Fig. 3 on a compliant wall model with 0.0064 cm thick polyvinyl chloride skin stretched over a water-filled cavity. The tests were representative of those reported in Ref. 127. The model had a compliant surface 66.0 cm long by 20.3 cm wide and was tested at a free stream velocity of 16.2 m/sec. Uniform tension ($\simeq$ 3.5 N/m) was applied laterally and longitudinally to the compliant wall membrane with a vacuum-tensioning device. During the Langley drag reduction tests the tension was actually varied over a wide range, and no drag reduction was observed. The value of 3.5 N/m for tension applies to the data runs for surface deformation measurements. An optical system developed by Weinstein at Langley, was used to measure the compliant wall surface deflections. The system used two photo detectors, 1.37 cm apart, driven on a track over the full length of the model surface. Each photodetector measured the instantaneous surface angle over a spot 0.13 cm in diameter to within 0.002 of a degree. However, for the data of Ref. 139, only one detector was used to obtain a time-average surface angle. The original wind-off and then the wind-on surface angles were measured, and the differences between the two were integrated to obtain the flow-induced change in surface position.

The surface position measurements of Ref. 138 on the polyvinyl chloride compliant skin model with water-filled substrate are presented in Fig. 4 for three longitudinal pressure gradients. Pressure gradients of these magnitudes were found in Refs. 127 and 128. References 127 modified the upper tunnel wall in an attempt to eliminate or reduce the gradients whereas Ref. 128 corrected the drag measurements by calculating the bending moment induced by action of the measured variation in the surface pressures on the edges of the balance. In the study of Ref. 138, the gradients were obtained by moving the tunnel side walls ± 0.318 cm from the mean zero pressure

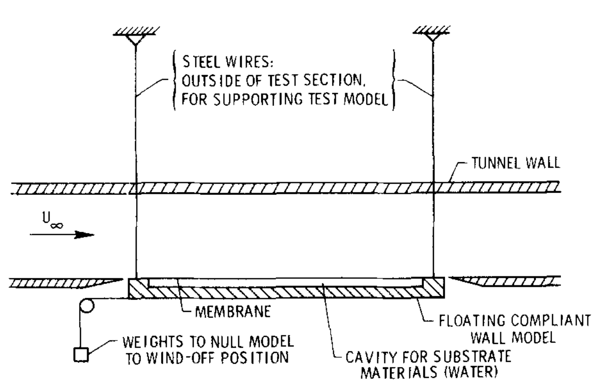


FIG. 3. Air flow over water-backed membranes. Langley attempted redo of typical University of Oklahoma experiments.

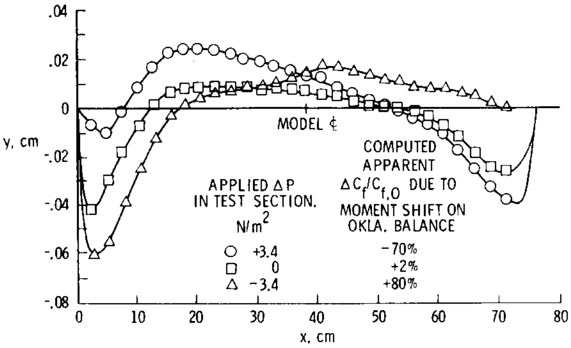


FIG. 4. Surface position measurements for compliant surface with water substrate in Langley experiments. $U_\infty = 16$ m/sec, static divergence mode.

gradient position. These changes produced a 1.1% variation in the steam velocity with a 2.2% variation in static pressure. The corresponding changes in the static pressure ($\Delta p$) over the 76.2 cm long model were approximately $\pm 3.4$ N/m$^2$.

Reference 138 showed the large effect of only small pressure gradients on the formation of standing waves on the compliant wall surface (see Fig. 4). For the nearly zero pressure gradient the surface protruded outward by approximately 0.009 cm. The positive gradient created a bulge over the upstream portion of the model whereas the negative gradient caused the bulge to shift to the downstream portion of the surface. The model fairing plate around the compliant surface caused the surface to dip over the first 7.6–16.5 cm for all three gradients; the surface dipped over the last 23 cm for the zero and slightly positive gradients.

The water volume under each of the three waves in Fig. 4 was integrated in the study of Ref. 138 to determine the bending moment caused by the transfer of the water mass. The slightly positive gradient shifted the center of gravity upstream of the model centerline and generated a 3 g-m bending moment. The near zero and negative pressure gradients shifted the model center of gravity downstream of the model centerline and generated 0.1 g-m and 3.3 g-m bending moments, respectively. As Ref. 138 showed, these bending moments in each situation would be sufficient to significantly alter the drag reductions reported in Ref. 127 and 128 and hence compromise the validity of the data (e.g., the 3 g-m bending moment for the positive gradient could have indicated an apparent 70% drag reduction). A further problem with tests of this type is that, from the Langley experiments, the balance reading is quite sensitive to even small air leaks in the enclosure surrounding the tunnel and balance system. An excellent seal is mandatory for accurate data.

*1. Structural dynamics analysis of water-backed membranes*

Having suggested that drag reduction did not occur for air flow over water-backed membranes, it is of interest to determine whether their characteristic surface response is in agreement with the arguments thus far which indicate that high frequency surfaces are necessary for drag reduction. The influence of water-backing on membrane surface motion has been studied by Blick[87] and Ash and Balasubramanian.[32] Those analyses are similar, but the somewhat more detailed calculations of Ash and Balasubramanian[32] are extended in the present work. Details of their calculations are given in Ref. 32 and only the salient features of the model are discussed here.

Membrane motion has been assumed driven by three pressure force contributions: a direct turbulent wall pressure field and two reaction pressure fields induced by wall motion. The turbulent wall pressure was assumed to obey Taylor's "frozen pattern" hypothesis so that individual wave number pressure contributions could be analyzed one at a time. The turbulent wall pressure spectrum was modeled using Bull's[139] data, but modified for very low frequences (Strouhal numbers based on displacement thickness and free stream velocity, $\omega\delta^*/U_\infty$, below 0.05) by assuming the spectrum level fell off with the fourth power of the wavenumber ($\omega/U_\infty$). A potential flow model was employed to estimate the induced pressure field resulting from membrane motions over the water backing, and a subsonic acoustic wave equation could be solved to estimate the induced pressure on the turbulent air boundary layer side of the membrane when wave velocities were different from the flow velocity.

Membrane deflection was developed using a normal mode expansion approach. Subsequently, surface motion calculations were obtained for individual turbulent wall pressure frequency contributions, which included the two induced pressure effects, and a spectrum of expected surface motion values was generated from the compilation of those frequencies. Calculation results are shown in Fig. 5 for a typical case corresponding to the Langley data. It is obvious from these results that the surface motion is at quite a low frequency, having very little motion at frequencies larger than the Strouhal number one level. This indicates that surface wavelengths are larger than the boundary layer thickness, and therefore more than an order of magnitude larger than values necessary for drag reduction according to the burst modulation mechanism described herein. Therefore, the null result (no drag reduction) on these surfaces is, in a sense,

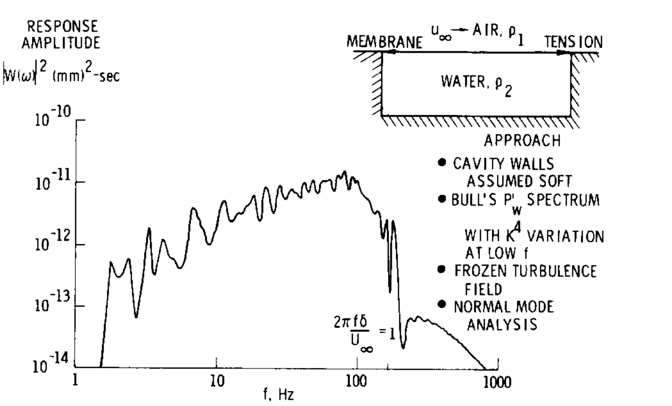


FIG. 5. Predicted surface motion of water-backed membranes. Typical Langley experiments, $U_\infty = 16$ m/sec.

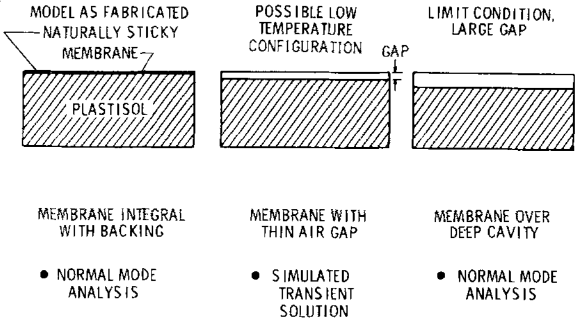


FIG. 6. Possible geometric configurations. Langley membrane/Plastisol model.

somewhat encouraging as far as verification of this theory is concerned.

### D. High frequency/periodic surfaces

The turbulence modification model (modulation of preburst flow) discussed previously requires short wavelength, high frequency surface motions for effective compliant wall interaction. Since that motion does not normally occur on structural panels, it is important to determine whether such motion could have been present during previously reported highly successful experiments. Three sets of such experiments cannot at this point be dismissed by alternate explanations: (1) Kramer's experiments[55–57] in water, (2) Walter[90] experiments in air, and (3) the experiment of Fischer *et al.*[12] at low temperature (on the order of 0 °C) in air. Aside from being in air, the experiments of Walters[90] and Fischer *et al.*[12] have marked similarities and will be discussed first.

In these experiments in air, both investigators employed thin rectangular membrane surfaces stretched over a soft elastic substrate. Walters[90] employed a thin polyvinyl chloride skin stretched over a porous polyurethane foam, while Fischer *et al.*[12] used a Mylar skin stretched over a sticky, gelatinous layer of polyvinyl chloride plastisol. It should be noted that even though these substrates were structurally soft, both were very rigid from the standpoint of the available turbulent pressure forces. Walters[90] found that no drag reduction occurred when the membrane skin was attached to the substrate by means of a spray adhesive, but drag reduction did occur when the membrane simply rested against the substrate. Fischer *et al.*[12] apparently achieved significant drag reduction only when the wind tunnel temperature was near 0 °C. Recalling that the substrate employed by Fischer was naturally sticky, and noting further that a 20 °C temperature drop would result in significant thermal contraction of the substrate, it is likely that the substrate pulled away from the Mylar skin during the successful test. Both experiments strongly suggest that the substrate does not act effectively as a continuous spring during successful tests. Rather, those data indicate that the substrate may act intermittently to alter the motion of a conventional membrane. Specifically, if the membrane is separated from the substrate by a thin air gap, as shown in Fig. 6, it is likely that the substrate functions as a "wavelength chopper." That is, by interfering with the downward displacement of long wavelength fundamental membrane vibration modes, the substrate can act to drive the membrane into short wavelength higher harmonic vibration modes. That possibility has been examined by Ash *et al.*[13] and Ash and Balasubramanian[32] and their results can be extended here.

The equations governing the motion of a membrane over a thin air gap are, in their simplest form, nonlinear.[13] However, they can be analyzed using conventional finite-difference techniques. Because of the nonlinearity, it was very desirable to determine the dynamic surface motion under conditions of a simulated, nonfrozen, turbulent wall pressure loading. Details of the turbulence simulation are discussed briefly by Ash and Balasubramanian[32] and simulated surface motion histories are also shown.

Again referring to Fig. 6, the membrane substrate system employed by Fischer *et al.*[12] could change from (1) an integral system of a membrane attached to a continuous elastic spring, to (2) a membrane separated from its substrate by a thin gap, and finally (3) to a membrane over a cavity which is deep enough to eliminate contact between the membrane and substrate. Those three configurations have been subjected to the same simulated turbulent wall pressure to show the influence of those structural changes on the response amplitude spectra and are shown in Fig. 7. It can be seen that the integral system produces a smoother, lower amplitude response, whereas the narrow air gap case yields a higher frequency motion.

Recently, unpublished experiments have been performed at Langley Research Center in which attempts were made to control the thin air gap behind a membrane in a low speed air flow. The major finding of those tests was that a specified gap cannot be maintained beneath a membrane with any reasonable tolerance over a large surface. (Small localized "bubbles" did however occur and produced drag reductions of 10–15%.) That work is continuing, but alternate methods for producing short wavelength, high frequency surface motion are being examined. The most direct method for producing short wavelength motion is to employ a ribbed periodic substructure beneath a thin skin. The rib spacing can be used to control the surface wavelength and frequency.

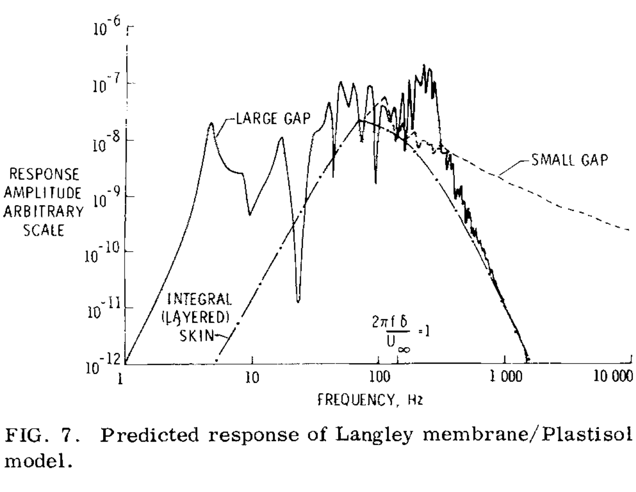


FIG. 7. Predicted response of Langley membrane/Plastisol model.

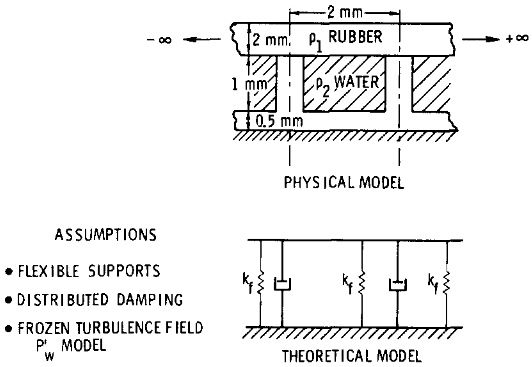


FIG. 8. Predicted surface motion—Kramer's periodic structure.

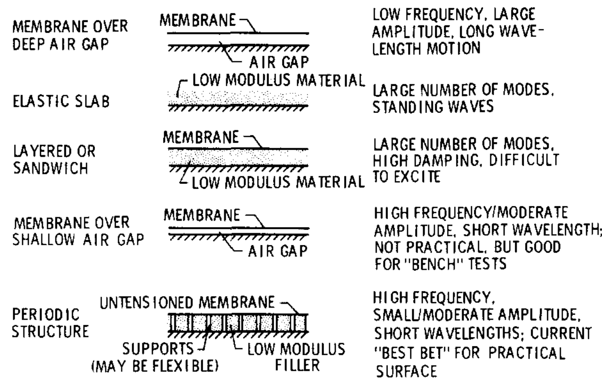


FIG. 10. Resonant compliant wall structural concepts.

However, analysis has shown that in the low speed, turbulent, air boundary layers currently available for testing, reasonable amplitudes of surface motion cannot be achieved. That would not be the case in water and since Kramer's[55] original model designs employed the types of periodic structures just suggested, a reexamination of his experiments is appropriate.

A cross section of one of Kramer's[55] successful compliant surfaces is shown in Fig. 8, where the structural elements are periodic, i.e., repeated with the same dimensions. If one assumes that Kramer achieved his drag reduction beneath a fully turbulent boundary layer, the reported average skin friction coefficients can be used to estimate a turbulent wavelength, $\lambda^+$, for the compliant surface. The variation of $\lambda^+$ was found to be between 200 and 500 wall units over the velocity range (7 to 18 m/sec) of his successful tests.

It has been rather commonplace to discount Kramer's results, for many reasons. First, because the tests were performed by towing test models behind a motor boat in Long Beach Harbor, it has been reasoned that the tests may not have been as closely controlled as most laboratory facility experiments. Kramer acknowledges that his experiments were affected by the season (experiments in summer were less encouraging than at other times of the year) and attempts to duplicate his results in towing tanks[59] and water tunnels[109] were unsuccessful or inconclusive.

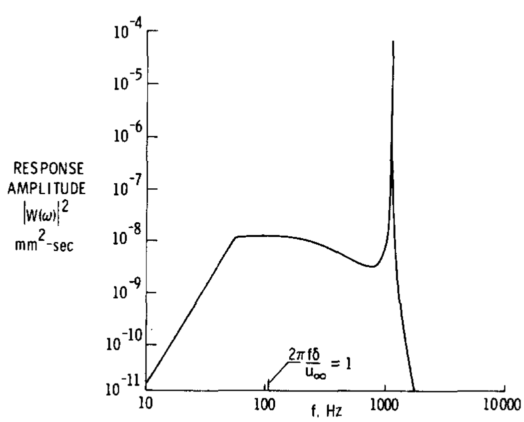


FIG. 9. Steady-state response of Kramer stubbed periodic surface. $U_\infty = 18$ m/sec, water flow.

The structural dynamics response of Kramer's model has been examined recently at Langley Research Center assuming interaction with a fully turbulent boundary layer. For a two-dimensional model of the configuration shown in Fig. 8, an approximately equivalent spring-dashpot (also shown) has been used to study the system, employing the method of space harmonics.[29] A frozen turbulence pattern was assumed with the same spectrum employed in the water-backed membrane analysis. The spectrum of surface motion is displayed in Fig. 9 for the test conditions just mentioned and it is observed that significant high frequency motion can be developed by that surface adding further support to a high frequency requirement and, by inference, the turbulence modification explanation.

To conclude this section, there is sufficient evidence in support of the turbulent burst modulation theory for compliant wall drag reduction to merit further investition of high frequency surfaces. Furthermore, surface motion measurements during successful tests are crucially needed, as are reliable numerical predictions of expected surface motion for a particular design (which also must be verified by experimental data). To date, the surface design concepts shown in Fig. 10 have been examined on the basis of high frequency surface motion potential. Of these concepts only the membrane over the thin air gap and the periodic structures are considered promising at this time for significant modification of low speed turbulent air boundary layers.

## VI. AIR-SEA INTERFACE AS A PROBLEM OF WALL MOTION INFLUENCE UPON TURBULENCE

As final note on the influence of wall motion on wall turbulence structure we briefly discuss the air-sea interface problem. As already mentioned, Refs. 107 and 108 indicate that the suppression of small (capillary) waves on the ocean surface can alter much larger scales of turbulence in the atmosphere above the surface. In the absence of capillary waves, the air boundary layer appears to lose coherence with the ocean waves. The usual influence of the large ocean waves on skin friction is to increase drag[140] (evidently due to a roughness effect for traveling waves,[41] and also zero wave speed case[37]). Although there is still controversy, the wind generated waves on this resonant surface (air-sea interface) can evidently become large enough to separate the air flow between waves.[142] Nevertheless, the present authors suggest that the information on the air-sea interface problem can at least be used to indicate regions in the three space of wavelength, wave speed, and wave amplitude (normalized in law of the wall coordinates) which one should stay away from (drag-increasing) in the design of resonant walls.

## VII. SUMMARY

In this paper we have reviewed the following subjects pertinent to the use of walls with moving waves for drag reduction:

(a) The need, in aeronautics, for drag reduction research using moving walls under turbulent boundary layers.

(b) Simple structural dynamic considerations which indicate that practical walls must be of the resonant type and also periodic for control of wavelength.

(c) The influence of wall motion upon boundary layer stability for highly compliant and also resonant walls and the stability of periodic flows. Recent Soviet experimental research indicates that resonant walls can also influence transition.

(d) Theoretical models of the drag reduction mechanism for moving walls under turbulent boundary layers. The current best strategy, in the opinion of the present authors, is to further investigate an approach based upon modulation of the pre-burst flow. This modulation may result in delay of burst formation and the subsequent elimination of a reasonable percentage of the total number of bursts. This mechanism requires considerable further research but does provide a crude guideline for future experiments (small wavelength, steep wave high frequency surfaces).

(e) A critical analysis of the available compliant wall data (resonant walls under turbulent boundary layers) indicates that the best performance has been obtained using high frequency response surfaces. Alternate explanations (other than alteration of the turbulence structure) are given for the drag reductions observed in pipe flows and also for liquid-backed membranes in air.

As a suggestion, further experiments should be conducted in water using both periodic and active (driven) surfaces, but over a much wider range of wavelength than used in past experiments, particularly at the low end (smaller wavelengths). Due to the low dynamic pressures associated with air tests for velocities much less than transonic, subsequent air tests should initially concentrate on driven surfaces, especially with small wavelengths. All experiments should measure surface motion and, if a drag reduction is observed, the influence of the wall motion upon the bursting process should be determined. Also, redundant drag measurements should be mandatory.

The purpose of the active wall research is to provide controlled experiments to determine what type of wall motion (wave speed, length, amplitude, etc.) is required to produce a drag reduction. These criteria can then be used to design passive surfaces which will work (or we find out that such surfaces cannot be designed using today's technology and therefore a substantial supporting materials/structures research effort is needed).

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