2017

Logical Pluralism from a Pragmatic Perspective

Teresa Kouri Kissel
Old Dominion University, tkouri@odu.edu

Original Publication Citation
Kouri Kissel, T. (2017). Logical pluralism from a pragmatic perspective. *Australasian Journal of Philosophy, 96*(3), 578-591.
doi:10.1080/00048402.2017.1399151

Logical Pluralism from a Pragmatic Perspective

Teresa Kouri Kissel

Abstract: This paper presents a new view of logical pluralism. This pluralism takes into account the way the logical connectives shift depending on the context in which they occur. Using the Question-Under-Discussion Framework as formulated by Craige Roberts, I identify the contextual factor which is responsible for this shift. I then provide an account of the meanings of the logical connectives which can accommodate this factor. Finally, I suggest that this new pluralism has a certain Carnapian flavor. Questions about the meanings of the connectives or the best logic outside of a specified context are not legitimate questions.

Keywords: Logical pluralism; Questions Under Discussion; Connectives; Carnap; Polysemy

1. Introduction

There are several options for logical pluralism on the table. This paper presents a view of logical pluralism which can account for the fact that there are some contexts in which distinct logics have logical terms which are synonymous, and some contexts in which distinct logics have logical terms which are not synonymous. None of the logical pluralisms on the table can accommodate this. I will discuss the factor which affects the meanings of the logical connectives, and then sketch an account of such meanings which works in light of this factor. Ultimately, I will suggest that we cannot ask about the meanings of the logical connectives outside of a context, as doing so is asking an external question, in the Carnapian sense, and is illegitimate.

2. The Intuitions

There are two particular conversational thought experiments I will focus on throughout this paper. In both conversations, two participants are discussing two analysis systems: classical analysis (CA) and smooth infinitesimal analysis (SIA). Briefly, SIA is an intuitionistic analysis system, in which all functions are smooth. Importantly, it is such that 0 is not the only nilsquare (a nilsquare is an element whose square is zero, i.e. elements x such that $x^2 = 0$). This is because every function is linear on the nilsquares. From this, it is provable that 0 is not the only nilsquare even though 'there are nilsquares distinct from 0' leads to a contradiction. This would be inconsistent in classical logic (because of the validity of the law of excluded middle), and so intuitionistic logic is required.[1]

The conversations we are interested in are as follows.

Conversation 1: Here, the participants are discussing the two analysis systems in a classroom setting.

> Rudolf: In my system, I can prove the fundamental theorem of the calculus. Does your system prove it?
>
> Geoffrey: I can prove a version of the fundamental theorem, too. I use nilsquares, numbers whose square is 0. They are not distinct from 0, but it is also the case that 0 is not the only one!
>
> Rudolf: What? Such nilsquares cannot exist. There is only one nilsquare and it is 0.
>
> Geoffrey: Well, using your negation, sure.
>
> Rudolf: Oh, I see. Your negation must behave differently from mine.

[1]See Bell [1998] for more details.

Conversation 2: Again, the participants are discussing the two analysis systems in a classroom setting.

> JC: In my system the fundamental theorem of the calculus holds. Does it hold in yours?
>
> Greg: In mine too. I use non-trivial nilsquares to prove it.
>
> JC: There are no non-trivial nilsquares in my system. Interesting. We both prove the same thing differently.

Intuitively, in both conversations, two logics are in play: the logic of CA, classical logic, and the logic of SIA, intuitionistic logic. In Conversation 1, between Rudolf and Geoffrey, the logical connectives in each logic in question seem to mean something different (the participants even suggest as much, stating that the negations must be different). In Conversation 2, they mean the same thing. At the very least, JC and Greg seem to be talking about the same fundamental theorem. If the theorems are the same, then the logical terminology in the formal statements of the fundamental theorems must have the same meaning.

In the next sections, I will develop a method for accounting for both conversations, and show how we can account for the change in connective meaning by a contextual factor. I will then sketch an account of meaning which accords well with this shift, and suggests that this fits well with the way Carnap might have thought about things. Finally, I will address several objections.

3. The New System

I will use Roberts's Question Under Discussion (QUD) framework to analyze these conversations. I will show that the factor which affects whether two connectives mean the same thing is whether the assumption that 'if two words sound alike, are spelled the same way, and are generally used in the same sentences in the same way then they mean the same thing' can be preserved in the common ground. I call this the 'correlation as identity proposition', or CIP. CIP is a default for most conversations, and like any default, it can be given up. Conversations where CIP is part of the common ground are conversations where the logical connectives mean the same thing. Conversations where *CIP* cannot be part of the common ground are conversations where the logical connectives can mean something different. It is important to note that the conversations I will discuss have three important features. The conversations I consider are all 'deductive conversations'. These are conversations about deductive reasoning or where the participants are doing or discussing deductive reasoning. Lectures in a typical mathematics class count, as does a mathematician working out a proof by herself on a blackboard. Anything that can be construed as the exchange of mathematical or deductive information between a speaker and an audience, even including those cases in which the speaker is the sole audience member, is a deductive conversation.

Additionally, I will assume that the conversational contexts are sufficiently regimented to carry the appropriate logics with them. This will help give us a transcription between the natural language conversation and a formal language counterpart for each deductive conversation. In most cases, this will be simple enough, at least for the object level assertions within a conversation.[2] For example, in conversation 1, the statements 'such nilsquares cannot exist' and 'there is only one nilsquare and it is 0' will be easily transcribable. The

[2]It is not clear that the object-level transcription will always be so easy. Field [2009: 346-7] provides an example where translating negation into formal language is quite complicated. I ignore this complication here.

meta-theoretic claims, though, will pose a further problem: it is not at all clear how to transcribe claims like ‘Your negation must behave differently from mine’ and ‘Using your negation, sure’.

The biggest problem in attempting to transcribe the meta-language claims is that the connectives are both used and mentioned in the meta-language. I will assume that the conversational participants in each conversation are using the same meta-language connectives. In the conversations, the meta-theoretic statements merely serve as signposts about what to expect the meanings of the connectives to be in such a context. This allows us to focus explicitly on the meanings of the connectives and logical terminology in the mathematical systems in question, while using the semantic claims as guides for how the conversational participants might charitably interpret and accommodate each other’s utterances.

Finally, I will assume that something like a principle of charity and accommodation is in play. Each conversational participant is meant to be as charitable as possible to each other participant. For example, if it looks like a participant has uttered something irrational or unjustifiable, the remaining participants are obliged to do everything they can to rationalize the utterance. Participants must accommodate as much as possible to ensure they do not have to believe their fellow participants are irrational. For more details, see Grice [1969] and Lewis [1979].

The requirement of charity is very much in line with the Hilbertian perspective of Shapiro [2004]. The Hilbertian perspective, roughly, is the position that most mathematical theories, so far as they are coherent, are legitimate. Here, Shapiro and I share a similar point of view: so long as a deductive utterance can be accommodated, it can be seen to be coming from the perspective of a legitimate logical theory. One major difference between the Shapiro position and the position advocated here is that the QUD framework can be extended beyond mathematical contexts. One can think of this new view either as an extension of the Shapiro point of view to contexts which involve more than just mathematics, or as a combination of the views taken to be held by Carnap [1937] and Beall & Restall [2006]. Shapiro holds the view that the meanings of the logical connectives can shift from one mathematical context to another, and moreover that ‘means the same thing as’ is vague, so that sometimes the classical and intuitionist connectives mean the same thing while sometimes they do not. Carnap and Beall & Restall, on the other hand, each seem to hold the opposite ends of this view. On the face of it, Carnap holds that connectives in different logics can never share a meaning, while Beall & Restall hold that they always do.

3.1 QUD framework

The framework I will consider is a direct adaptation of the system in Roberts [2012] provided by for analyzing natural language conversations. The formal details are available in the Appendix. The system has a typical Common Ground (CG) (which contains the propositions the participants are agreeing to treat as true for the purposes of the conversation), and an additional resource for tracking the knowledge and goals of the interlocutors: a list of questions under discussion, QUD. Questions are added to QUD when all interlocutors agree that their conversation goal is to answer a question, and removed from QUD when a satisfactory answer is provided, or the question is deemed to be unanswerable.

We need to make two small changes to this framework to make it suit our purpose. First, Roberts [2012] discusses the system with respect to propositions. However, I will use the term ‘proposition’ only roughly, for something like a formalized sentence, in order to avoid having to make substantial (controversial) claims about just what propositions are. This pluralism can be made to fit many views about the nature of propositions. Second, it is ordinarily assumed that CG cannot entail an answer to a question for that question to be legitimate, and added to QUD. We will not assume that here. If we couldn't add questions whose answer were entailed by CG, then when our common ground includes the axioms for a theory, we would not be able to ask any question about what that theory entails. Rather, we will assume that if CG entails an answer to such a question from an available set of assumptions, then that question cannot be added to QUD. The term 'available' here has been adapted from Stalnaker [2014: 24]. There, he uses it to pick out those propositions which are currently being entertained by the conversational participants, and 'easy' to infer, where what is easily inferable is something like 'not taking too many steps' or 'being well within the abilities of the conversational participants to deduce'. What propositions are available will be, in most cases, a context sensitive matter.

The conversations as described above fall into one of two categories: conversations where $CIP \in CG$ and conversations where the interlocutors are forced to remove CIP from CG.[3] In the first case, the logics in play in the conversation will have connectives which mean the same thing. In the second case, the logics in play in the conversation may have connectives which do not share meanings.

This will roughly line up with the framework in Shapiro [2014]. I suspect that his 'logical' contexts will usually correspond to contexts in which we must remove CIP from CG, and that his 'mathematical' contexts will usually correspond to the contexts in which we can leave CIP in place. This may not always happen. For example, though CA proves the intermediate

[3]Whether $CIP \in CG$ does not affect the status of the proposition expressed by CIP as true or false, but only whether the conversational participants are treating it as true for the purposes of the conversation. This means that the theory I'm giving here is a pragmatic theory. In section 4.2, I will suggest that there is no further theory to provide. Thanks to a referee for pushing me on this issue.

value theorem, SIA does not. On my view, it would be possible for a conversation between two people about the intermediate value theorem to be one in which we must remove CIP from CG, which would result in a conversation where the corresponding connectives might not share a meaning, and where the intermediate value theorems in each system were not the same. This would not be a natural occurrence on Shapiro's view.

3.2 Conversation 1

In conversation 1, Rudolf and Geoffrey are discussing their two analysis systems. One possible question in this conversation is 'in each system, do there exist types of things which are not distinct from 0, but such that 0 is not the only one?'. Another is something like 'what is consistent in your system given the meaning of your negation?'.

Intuitively, if we assumed that both participants would interpret themselves as using the same connectives, we would have a problem with charity and accommodation. Geoffrey has uttered something which can be taken to imply the negation of a classical truth. His utterance can be taken to imply that there exists some non-singleton, non-empty, set of things which are not distinct from 0. Since the double negation cancels in classical logic, this is equivalent to saying that these things both are and are not distinct from 0 for the classical analyst, a clear contradiction. Adding the existence of these types of nilsquares to a classical system will result in a contradiction. Rudolf cannot accept the existence of these objects and maintain a consistent theory. If the conversation continues after the existence of such nilsquares is posited (and not just by a simple 'no' from Rudolf), then we must assume that Rudolf is assuming that Geoffrey is using a different negation. So, the most charitable interpretation of this conversation is one in which the negation connectives have different meanings.

The story in Roberts's system works out the same way. What we wind up showing is that the presupposition that the connective meanings were shared is what makes the original CG defective. The conversation proceeds as normal (in fact, the first two utterances in both conversations are incredibly similar) until Rudolf challenges Geoffrey with another question: a question expressing skepticism about the possibility of nilsquares, perhaps something like 'how is it that your nilsquares exist without contradiction?'. This is a clarification question, a meta-linguistic question which signals a request for more information (see Ginzburg [2012] for a description of clarification questions). It indicates, in effect, that something has gone wrong in the conversation which Rudolf does not understand.

This should not be unexpected, either. Geoffrey has already hedged about exactly what it is he proved. In suggesting 'a version of the fundamental theorem', he hedges about whether or not it is actually the same theorem. It seems Geoffrey knows in advance that he is going to run into problems with Rudolf's understanding of his claims about nilsquares. He could have simply responded 'yes, I prove that theorem, too', but rather chose to supplement with additional information. This strongly suggests that Geoffrey anticipated Rudolf's clarification question.

In order to answer the clarification question, Geoffrey suggests that the negations in question must be different, which Rudolf accepts with his final contribution to the conversation. Here, we must make a change to CG. Since we assumed that CIP was in CG from the beginning, then this is what we have to change. The default is that these words have the same meaning, until we are put into a position where the only way to accommodate the differences is to assume that they do not. The way Rudolf and Geoffrey have to accommodate each other is by adding to the common ground that the connectives in question actually mean something different, even though they sound/look the same.

This shows us that the original common ground was defective. When the original question ‘does your system prove the fundamental theorem?’ is posed, Rudolf is assuming that both he and Geoffrey are talking about the same fundamental theorem, and are therefore both looking for an answer to a single question. This cannot be the case. They must both prove a very closely related thing, though not exactly the same thing. They are discussing near translational equivalents. This is an effect of the common ground originally containing CIP and the forced removal of it. [4]

In the appendix, I provide a formalization of this conversation in the Roberts framework.

3.3 Conversation 2

In conversation 2, JC and Greg, are seeing what is common to their two theories. Here, one of the questions is something like ‘what tools does each system use to prove the FTC?’. JC and Greg are behaving as though there is one fundamental theorem which they both prove. If we think that the statements of the fundamental theorem mean the same thing, then because those statements contain logical operators, those operators must mean the same thing as well. This means that we have to treat the connectives as meaning the same thing. However, it still requires two distinct logical systems: one which licenses non-trivial nilsquares and one which does not.

Importantly, nothing flags the common ground as defective. We are given no reason to change the assumption that $CIP \in CG$, and so there is no need to do so.[5]

[4]One possibility not addressed here is that ‘in my system’ is something more like ‘in my book’. In this way, we might liken the debate between Geoffrey and Rudolf to a debate between two historians wondering about the nature of Cleopatra (this example and issue are due to a referee). Certainly, if two historians were disagreeing about Cleopatra, we would not want to interpret them as discussing two distinct historical figures. We might think the same should be true for the logical case. However, in the logical cases, we cannot point to historical records or relics, but only to the behaviors of the conversational participants and the rules and/or truth conditions they profess to associate with each connective. Thus, it makes more sense to let how many connectives are (pragmatically) in play be guided by the behavior of the participants.

[5]One thing to notice here is that if this conversation continued, it is distinctly possible that Greg’s next utterance would be something like ‘You must mean something different by “proof” than I do.’ In this case, it

In the appendix I provide the formalization of this conversation in the Roberts framework, where the results are as expected.

3.3 The Factor Affecting Connective Meanings

So far, so good. What is important for this project is the following: whether $CIP \in CG$ can directly affect the meanings of the logical connectives in question.

When $CIP \in CG$ the logical connectives mean the same thing across different logics, while if the interlocutors are forced to remove CIP from CG, the assumption that the connectives must mean the same thing is given up. The factor responsible for the intuitions in section 2 is the presence or absence of CIP from CG at the time we are assessing whether the connectives share a meaning.[6]

4 A New Theory of Connective Meaning

We need an account of the meaning of negation (and the rest of the connectives) which can be

would be possible that we would have to remove CIP from the common ground, but that it would not affect the meanings of the logical connectives, but rather the meaning of 'proof'. This is interesting, since it shows two things. First, it makes explicit the fact that there are two logics in play in the conversation. Second, it shows that CIP does not need to be restricted to just the logical connectives, but can affect much of what is going on in the conversation. One particularly interesting aspect of this lack of restriction is that rather than different connectives being in operation, it might be different standards of implication (thanks to a referee for this suggestion). Terrés Villalonga [forthcoming] and Caret [forthcoming] each propose a view where the standards of implication might shift in a way that has nothing to do with connective meanings. I have not addressed whether the standards of implication are the same across both conversations. It seems likely that each conversation has two standards of implication in play, but whether both standards in conversation 1 are the same as both standards in conversation two is another question. I suspect that we will find that we cannot compare the standards across the conversations without embedding them into the appropriate context. A full examination of this topic is beyond the scope of this paper.

[6] I need to give a story about what happens in 'mixed' conversations. That is, what happens in conversation where CIP cannot be part of the common ground at a certain point, but can be part of the common ground before and after that point. We ought to maintain that if CIP is removed from the common ground and then 'added back' that the meanings of the connectives or 'proves' might change throughout the conversation. This means that any time a new goal becomes salient, the status of CIP might change. But this is to be expected. We can account for the purpose of a conversation in terms of the conversational goals. Thus, since we have been maintaining that the right logic and meanings of the connectives are relative to a purpose and a context, so we ought to accept that when out purpose changes, sometimes the meanings will change.

treated in such a loose way. I sketch a possibility in this section to show that there is logical space for such a theory of meaning, and draw some conclusions about it.

4.1 Connective Meanings as Polysemous

There is something it seems everyone is after when they claim to have provided a new formal definition of negation. This is true not only for the classical logician, the intuitionist and the relevance logician, but also the linear logician, the dialetheic logician, etc. I claim that they are all, roughly, after the same thing, which we might call 'pre-theoretic negation', and the concepts they wind up arriving at are related to each other in a polysemous way.[7] We might also call this a 'fluid concept', in the sense of Lynch [1998]. There, he states that 'the search for the essence or common property expressed by a [fluid] concept is futile' [Lynch 1998: 62].

We have some idea of what 'pre-theoretic negation' might be, but giving any more than broad-stroke descriptions of it is impossible. This puts 'pre-theoretic negation' the weird position of being neither a natural nor formal language negation; it is not robust enough to be used effectively in conversation nor to stand in logical relationships to other connectives.

It is natural to think that pre-theoretic negation might be defined by a single model-theoretic or proof theoretic clause. Perhaps a model-theoretic clause, like minimal truth conditions, in the sense of Restall [2002], or the claims developed in Hjortland [2013] would do the trick. It should be clear, though, that not all negation clauses will fit this mold. What of those which can only be defined relative to a proof theory, for example? If we start with a proof-theoretic

[7]Two words are polysemous when they are ambiguous but related. So, while 'bat' is ambiguous between the mammal and the baseball equipment, it is not polysemous. On the other hand, 'wood' is ambiguous between the thing trees are made of and the thing a lot of trees make up, they are related to each other, and so are polysemous. I will make no claims here about whether all polysemous words are always related by a 'pre-theoretic' notion.

clause, we have a similar problem: our pre-theoretic meaning of negation will not be able to accommodate negation in logics which are exclusively model-theoretic. However, there is something common to the clauses given by the model-theorist and the proof-theorist: they are both *precisifications of* the pre-theoretic meaning. This claim is further supported by the fact that a person using one of them may very well be understood by a person using another. In the terms we used earlier, they are logical terms which usually abide by the antecedent of CIP. This is the sense in which they are polysemous.

The interesting feature of the QUD system is that we can use the conversational goals and questions under discussion to resolve much of the polysemy. Questions under discussion are excellent tools for helping to identify the topic of any given conversation. Once we identify just what it is we are talking about, we can identify which precisification of negation is in play. For example, if we are talking about or doing classical mathematics, then we can safely say that we are using classical negation. However, if we are talking about or doing non-trivial naive set theory, we can safely say we are using a paraconsistent negation. Further, this allows us to make sense of some additional intuitions. In the two conversations above, it was easy for the conversations to continue either by removing CIP from CG, or assuming that the negations in question were the same. However, imagine a similar conversation, but this time between a classical logician and a dialetheist. It seems in this conversation, when the classicist discovers that her interlocutor is using a negation with which both $A$and $\neg A$ can be true, she is allowed to claim that she does not know whether that negation means the same thing as hers. In this case, it seems possible that the ways each participant has spelled out the pre-theoretic meaning of the connective are just too far apart to make sense of together.

Polysemy presents us with a nice option in logical space. Section 3 showed that connective meanings can vary from context to context. However, without some general idea of what makes some formal negation connective a negation rather than disjunction, and some idea of what types of things can shift from context to context, the results of section 3 would be impeded. The story about polysemy and pre-theoretic negation shows that both requirements can be fulfilled, and so the results in section 3 are left intact.

4.2 Real Meaning as an External Question

Here, then, we have a system which governs the pragmatic story of when two connectives share a meaning. It does not address the question of whether any two sentences involving negation *really* mean the same thing. 'Pre-theoretic negation' is not a robust enough notion of 'same meaning' to do any significant work. No logic will flow from the pre-theoretic meaning; it is simply too thin to stand in any logical relation to the pre-theoretic meaning of the other connectives. In the Carnapian sense, I claim that *real* meaning questions, asked outside of an appropriate context, are external and illegitimate.

For Carnap, a non-theoretical question is one asked about reality itself; it is external to any given framework (see Carnap [1950]). There are two types of external questions: pragmatic questions and pseudo-questions. A pragmatic question is a question about which framework is best for a given purpose, and can be answered.[8] Pseudo-questions (non-pragmatic external questions) cannot be answered, on Carnap's view, and this is the sense in which they are illegitimate. Examples of pseudo-questions include questions of existence of abstract objects, which can only be answered relative to a given linguistic framework, and questions about the right logic. Outside of a linguistic framework questions like this are only pseudo-questions.

So it goes, I claim, for questions of the *real* meanings of the logical connectives. They are

[8]See Steinberger [2016] for an interesting discussion about how pragmatically to select the appropriate linguistic framework.

like questions of the existence of abstract objects. We do not have enough information outside of an appropriate context to know precisely what they mean. We simply cannot ask that question and expect an answer.

We can ask instead about when two connectives, as they are spelled out *in a context*, share a meaning robust enough to be logical. Two sentences involving negation cannot be said to share a logical meaning simpliciter, because then we would have to ask whether they share a logical meaning without some conversational goal guiding our inquiry. But without such a goal, or a determination of the status of CIP, we do not know anything but the pre-theoretic meanings of the connectives.

More generally, what I have proposed is that logical connectives simply have no robust meaning outside of their meaning in a context. This means that the question of the meaning of the logical connectives, when asked without a purpose or application of the logic in mind, are unanswerable. Here, we have extended the Carnapian picture: not only is it illegitimate to ask for the right logic outside of a pragmatic structure, it is also illegitimate to ask for the meanings of the logical connectives for any particular logic outside of a context. Without a context, without a goal, we can have no logical meaning. Once we fill out these details, we are at liberty to 'build up [our] own logic' [Carnap 1937: 52], to preach Carnapian tolerance, and moreover, we can build that logic so that it shares connectives with another, or so that it does not. The Carnapian slogan becomes 'in logic [and connective meanings,] there are no morals' [Carnap 1937: 52].

5 Objections and Replies

Objection: What is the difference between this view and a view that claims that the meanings of the logical connectives are ambiguous? Why posit polysemy at all?

Reply: Though the polysemy view about the meanings of the connectives is indeed a type of ambiguity view about those meanings, there is an additional benefit to positing polysemy rather than mere ambiguity. That benefit is that this view can better make sense of disagreements, and with that, it can make sense of what Plunkett and Sundell [2013] and Thomasson [2016] call 'metalinguistic negotiation'. With mere ambiguity, there is no clear answer to what people disagree about when they argue over the meanings of the logical connectives. With polysemy, there is. Those in disagreements over the meanings of the logical connectives can be said to be disagreeing over the pre-theoretic meaning of each connective. The position I present does hold that they are wrong to do so outside of a properly constructed context, but they may nonetheless be engaged in such a disagreement. This benefit also allows us to make sense of meta-linguistic negotiations, which Plunkett and Sundell [2013: 3] define as 'disagreement[s] about the proper deployment of linguistic representations'. Metalinguistic negotiations are exactly the kind of thing that Geoffrey and Rudolf are engaged in when they decide they must each be using different versions of the connectives; they disagree about how to use the connective, and then come to a negotiated conclusion about what it means when each of them uses it. If the connectives were merely ambiguous, we would not see this type of behavior, since there would be nothing in common between the connectives which we could pick out as the thing the participants were negotiating the use of.

Further, this use of polysemy rather than ambiguity has an important impact on an argument often provided against pluralism. Opponents of pluralism often suggest that logical disagreement is not possible when logical pluralism is on the table, since people using different logics are talking past each other, and having a merely verbal dispute (Quine [1986] might be thought of as a making an argument of this type).[9] This would be bad for the pluralist, as it seems genuine logical disagreement is possible. The adoption of a view of connective meanings on which they are polysemous and related by sharing a pre-theoretic meaning solves this problem. We find that genuine logical disagreement is indeed possible. In the case where CIP is removed from CG and there is more than one meaning for negation in use in the conversation, this disagreement would be a pragmatic and external disagreement about which connective it is best to use. It becomes a metalinguistic negotiation about which extension of the pre-theoretic meaning of negation is best for the purpose at hand. On the other hand, when CIP remains in CG, and the interlocutors are using the same connectives, they can still have genuine logical disagreement. This case is easier, though, since there is little chance the interlocutors accidentally talk past each other as they are using connectives with the same meaning. This means, quite interestingly, that a difference in meaning does not necessarily lead to a verbal dispute. Substantive disagreement is possible even when conversational participants are using terms with different meanings. For a further development of this point, see Wyatt and Payette [forthcoming].

Objection: Formal language connectives mean just what they are formally defined to mean, and there is no context sensitivity there. Without such formal language context sensitivity you have not really shown that whether two formal language connectives mean the same thing is a context sensitive matter, as you claim.[10]

Reply: In a sense, this objection is correct. If the formal connectives are explicitly defined by the conversational participants, and those definitions are part of CG, then there will be no context sensitivity at the formal level. We will be left merely with natural language context

[9]Thanks to a referee for raising this concern, and suggesting that different meanings do not necessarily lead to merely verbal disputes.

[10]Thanks to a referee for suggesting this line of attack.

sensitivity, and the question of whether the natural language connectives have the meanings ascribed to the formal language connectives in CG.

However, the more interesting cases, and the two cases described in conversation 1 and 2, do not have definitions of the formal language connectives in CG.[11] Because of this, the formal connective meanings themselves are up for grabs. In conversation 1, the only constraints on such meanings is that there must be two sets for each logic, and that Rudolf and Geoffrey are using distinct meanings. This gives us several options: perhaps we have truth conditions and inferential roles, and Rudolf is using truth conditional meanings and Geoffrey inferential roles. Or perhaps Rudolf is using natural deduction rules to define his connectives, and Geoffrey is using proof-conditions. Nothing about the conversation so far settles this question, and so what exactly the formal connectives mean is up for debate. Because of this openness we can say that the formal language meanings of the connectives are contextual in the same way 'red' might be. In some contexts the meaning of 'red' is underspecified. Sometimes two people might disagree over whether something is red, and there may be two distinct meanings floating about. One might mean by 'red' 'things with wavelengths between 640 and 660' and the other might mean 'things with wavelengths between 600 and 700'. The context may not settle the answer, but might tell us whether one or two specific meanings are in play. It is the same with the formal language connective meanings: if no explicit meanings are added to CG, then the specific meanings are up for grabs and can be more or less settled by the context. This makes the formal language connective meanings, in addition to the natural language connective meanings, contextually dependent, as required.

[11]This way of thinking about the situation does require us to separate the formal connective meanings from the logical rules. On this picture, the rules in a logic for a particular connective do not necessarily fully specify the meaning of that connective.

Objection: Your proposal about connective meanings is vague and needs more details.

Reply: This is unavoidable. The very point of the proposal is that the meanings of the connectives are a hazy matter. We find that any time we settle on one meaning more specific than 'somehow closely related to the accepted clauses' or 'sharing a family resemblance', we rule out some things we would want to count as negation, and allow some things we would not. For further work on this ruling in and ruling out problem see Kouri [2016], Priest [1987], and Hjortland [2013]. This makes sense of our intuitions that we can only find robust meanings of connectives within a specified context, and that the connectives are fluid, in the sense of Lynch [1998]. Adding more precision to the proposal would prevent this fluidity and context sensitivity.

6 Conclusion

I have shown how one can construct a logical pluralism which relies on the linguistic system presented in Roberts [2012] which makes sense of conversations 1 and 2, and which can be easily extended beyond mathematical contexts. On this view, sometimes the logical connectives in such a conversation have the same meaning, and sometimes they have pairwise distinct meanings. Questions about the meanings of the logical connectives cannot be asked outside of a context of use. Such questions are illegitimate; they are Carnapian pseudo-questions.

Acknowledgements: I owe a debt of gratitude to Stewart Shapiro, who read many early drafts. I am grateful for comments on previous drafts from Ethan Baruer, Roy Cook, Geoffrey Hellman, Hannes Leitgeb, Chris Pincock, Craige Roberts, Marcus Rossberg, Gil Sagi, Giorgio Sbardollini, Kevin Scharp, and Neil Tennant, as well as several referees.

Additionally, attentive audiences at The Ohio State Philosophy-Linguistics Workshop (2015), The University of Connecticut Logic Group (2015), Pluralism Week at the Veritas Research Center at Yonsei University (2016), the Buenos Aires Workshop on the Philosophy of Logic (2017), the Logic Seminar at the University of Melbourne (2017), and the Minnesota Center for Philosophy of Science Foundations Interest Group (2016) provided very useful feedback.

References

Beall, J.C. and Greg Restall 2006. *Logical Pluralism*. Oxford: Oxford University Press.

Bell, John L. 1998. *A Primer of Infinitesimal Analysis*. Cambridge: Cambridge University Press.

Caret, Colin R. forthcoming. The collapse of logical pluralism has been greatly exaggerated. *Erkenntnis*.

Carnap, Rudolf 1937. *The Logical Syntax of Language*. New York: Harcourt, Brace and Company.

Carnap, Rudolf 1950. Empiricism, Semantics and Ontology. *Revue Internationale de Philosophie* 4/11: 20-40.

Field, Hartry 2009. Pluralism in Logic. *The Review of Symbolic Logic* 2/2: 342–359.

Ginzburg, Jonathon 2012. Grounding and CRification, in *The Interactive Stance*, Oxford: Oxford University Press:146–217.

Grice, H. P. 1969. Utterer's meaning and intention. *The Philosophical Review* 78/2: 147–177.

Hjortland, Ole Thomassen 2013. Logical Pluralism, Meaning-Variance and Verbal Disputes. *Australasian Journal of Philosophy* 91/2: 355-373.

Kouri, Teresa 2016. Restall's Proof-Theoretic Pluralism and Relevance Logic. *Erkenntnis* 81/6: 1243-1252.

Lewis, David 1979. Scorekeeping in a Language Game. *Journal of Philosophical Logic* 8/1: 339–359.

Lynch, Michael 1998. *Truth in Context: An Essay on Pluralism and Objectivity*. Cambridge, Massachussets: MIT Press.

Plunkett, David and Timothy Sundell 2013. Disagreement and the semantics of normative and evaluative terms. *Philosophers' Imprint* 13/23: 1-37.

Priest, Graham 1987. *Doubt Truth to be a Liar*. New York: Oxford University Press.

Quine, W. V. 1986. *Philosophy of Logic*. Cambridge, Massachusetts: Harvard University Press.

Restall, Greg 2002. Carnap's Tolerance, Language Change and Logical Pluralism. *Journal of Philosophy* 99/8: 426–443.

Roberts, Craige 2012. Information Structure in Discourse: Towards an Integrated Formal Theory of Pragmatics. *Semantics and Pragmatics* 5/6: 1–69.

Shapiro, Stewart 2014. *Varieties of Logic*. Oxford: Oxford University Press.

Stalnaker, Robert 2014. *Context*. Oxford: Oxford University Press.

Steinberger, Florian 2016. How Tolerant Can You Be? Carnap on Rationality. *Philosophy and Phenomenological Research* 92/3: 645-668.

Terrés Villalonga, Pilar forthcoming. From Natural to Formal Language: A Case for Logical Pluralism. *Topoi*.

Thomasson, Amie L. 2016. Metaphysical disputes and metalinguistic negotiation. *Analytic Philosophy*, 4/1: 1-28.

Wyatt, Nicole and Gillman Payette forthcoming. Logical Pluralism and Logical Form. *Logique et Analyse*.

Institutional Affiliation:

Department of Philosophy and Religious Studies

Old Dominion University

Norfolk, Virginia, USA

Appendix

The Roberts [2012] framework is given formally by the following.

1. I: the set of interlocutors at time t
2. G: a function from pairs of individuals in I and times t to sets of goals in effect at t such that for each $i \in I$ and each t, there is a set $G(i,t)$ which is i's set of goals at t
3. G_{comm}: the set of common goals at t; i.e. $\{g | \forall i \in I, g \in G(i,t)\}$
4. M: the set of moves made by interlocutors up to t with the following distinguished subsets: A, the set of assertions, Q, the set of questions, R, the set of requests, and Acc, the set of accepted moves
5. $\leq$: a total order on M that reflects the chronological order of moves
6. CG: the common ground; i.e. the set of shared presupposed propositions at t
7. QUD: the set of questions under discussion at t; i.e. a subset of $Q \cap Acc$ such that for all $q \in QUD$, CG does not entail an answer to q and the goal of answering q is a common goal

The conversational score is updated as follows:

1. Assertion: if an assertion is accepted by all interlocutors, then the proposition asserted is added to CG and Acc

2. Question: if a question is accepted by all interlocutors, then the set of propositions associated with the question is added to QUD. A question is removed from QUD if and only if either its answer is entailed by CG or it is determined to be unanswerable
3. Request: if a request is accepted by an interlocutor, i, then the goal associated with the request is added to $G(i,t)$ and the proposition that $i$intends to comply with the request is added to CG

Formalization of Conversation 1

I assume that t_0 is the time before the first utterance, t_1 the time of the first utterance and t_5 the time of the last. I is {Rudolf, Geoffrey}. At t_0, CG includes the axioms of classical analysis, and the CIP. One might have thought that the axioms of SIA, and the fact that SIA uses intuitionistic logic, would also be included in CG at t_0. However, it seems clear, especially given Geoffrey's hedge and supplementing with additional information, that though Geoffrey knows the axioms, Rudolf does not, and it is common knowledge that Rudolf does not. Thus, they are not candidates to be included in CG. G_{comm}, M, A, Q, QUD, and Acc are empty.

At t_1, Rudolf promotes the proposition that his system proves the fundamental theorem to a simple, clear and straightforward implication in CG, A and Acc. At the same time, he adds to Q the question of whether the other system proves the theorem. It also gets added to QUD, since in this context, even though CG implies it, it is not yet an available implication. Rudolf's goals at t_1 include finding out whether Geoffrey's system proves the theorem.

At t_2, Geoffrey answers the question by claiming that the other system proves a version of the fundamental theorem. Additionally, Geoffrey attempts to add new information to the CG: that his system uses nilsquares to prove this theorem. He also proposes to add to CG and

A that nilsquares are not distinct from 0, but it is also the case that 0 is not the only one.

At t_3, Rudolf tries to block these two additions from entering Acc and CG by claiming that the utterance is incoherent. 'Non-trivial nilsquares do not exist' is added to A, but not Acc. This is the moment where he asks Geoffrey a clarification question about nilsquares, and challenges Geoffrey's assumption that one can posit the existence of non-trivial nilsquares without thereby engendering a contradiction.

At t_4, Geoffrey implies that because of his distinct negation meaning, the existence of such nilsquares is not contradictory. He promotes to an available implication in CG that his logic, connectives and mathematical system, are different from Rudolf's. He also adds this assertion to A. Here we see that Geoffrey and Rudolf must accommodate each other by changing a fundamental assumption in the common ground, namely CIP. Even though the connectives are homophonic in both systems, they cannot be treated as meaning the same thing in this case. This is a radical change, but is necessary for the conversation to continue charitably. CIP is removed from CG.

At t_5, Rudolf accepts this. Propositions about the difference in connective meanings, logics, and the existence of nilsquares are added to CG. Geoffrey's assertions that non-trivial nilsquares exist in his system and that the negation connective does not entail that these are contradictory are added to Acc and CG, and the proposition that SIA proves a fundamental theorem is promoted to an available implication in CG. All is well. We can make sense of this conversation if we assume that the initial common ground was defective, in that it contained CIP. By the end, Rudolf and Geoffrey have rectified this, and have discovered that they both prove a theorem which looks like the fundamental theorem, but because of the addition to the common ground that words which satisfy the antecedent of the CIP do not have to mean the same thing, 'the' theorem might not be the same theorem in both systems.

Formalization of Conversation 2

I assume that t_0 is the time before the first utterance, t_1 the time of the first utterance and t_3 the time of the last. I is {JC, Greg}. At t_0, CG includes some basic information about analysis and the proposition that both participants use different analysis systems and different logics, and G_{comm}, M, A, Q, QUD, and Acc are empty.

At t_1, JC adds the proposition that his system proves the fundamental theorem to A and it is also added to Acc, and promoted in CG to an available implication. At the same time, he adds to Q the question of whether the other system proves the same theorem. It also gets added to QUD. JC's goals at $t_1$include finding out whether Greg's system proves the theorem.

At t_2, Greg answers the question affirmatively. The question posed by JC is removed from QUD and the proposition that Greg's system proved the fundamental theorem is promoted in CG to a simple, clear and straightforward theorem, and added to A and Acc. JC's goal of finding out whether this new system proved the theorem has been satisfied. Additionally, Greg adds new information to the CG: that his system uses nilsquares to prove this theorem. That is, he adds the propositions that his system licenses non-trivial nilsquares, and that non-trivial nilsquares are used in the proof of the fundamental theorem. He adds no new questions.

At t_3, JC adds to the common ground that his system has no non-trivial nilsquares. This proposition is promoted in CG, and added to A and Acc. Again, nothing said suggests the common ground was defective, and so we can maintain that the logical connectives mean the same thing in both logics. At the end of the day, $CIP \in CG$.