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Generalized Compton Amplitudes in Quantum Chromodynamics

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GENERALIZED COMPTON AMPLITUDES IN QUANTUM CHROMODYNAMICS

by

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ABSTRACT

GENERALIZED COMPTON AMPLITUDES IN QUANTUM CHROMODYNAMICS

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In this dissertation we describe results of our studies of generalized Compton amplitudes. We have calculated the one-loop corrections to the amplitude in the coordinate representation in terms of nonlocal string light-ray operators. We have also developed a consistent approach to the problem of constructing the gauge invariant Compton amplitude and obtained an expression for the explicitly gauge invariant amplitude which includes all the generalized target-mass corrections.

Acknowledgments

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Chapter 1

Introduction

Quantum chromodynamics has been one of the most challenging frontiers of the modern physics for the last three decades.

Before the constituent quark model was introduced ([1], [2]) there were only several phenomenological models used to describe the strong interaction of hadrons. The quark model was able to give a successful classification of the whole spectrum of hadrons specifying their quantum numbers. The main postulate of the model states that both baryons (strongly interacting fermions) and mesons (strongly interacting bosons) consist of more fundamental particles called quarks. Quarks interact with each other by means of gluon exchange.

An important impact on the development of the theory of strong interactions was data on deep inelastic scattering (DIS) which led to the creation of the parton model ([3, 4, 5]). This data shows that inside the nucleons there is a cloud of quasi-free point-like particles or partons. Their quantum numbers coincided with the quantum numbers of the quarks.

*The style specifications used in this thesis follow those of Physical Review D.

The next big step was the introduction of a new quantum number, the color ([6, 7, 8]). Color was necessary to describe baryons and mesons as bound states of quarks.

Compton scattering provides a unique tool for studying internal structure of hadrons. A general Compton amplitude probes hadronic structure by two electromagnetic currents. In quantum chromodynamics, the photons couple to the quarks of the hadron. The photon-quark vertex is pointlike only in the simplest approximation: gluon interactions produce radiative corrections which may be uncalculable within the perturbative QCD framework, if the relevant Feynman integrals are dominated by regions of soft momenta. Still, it is possible to preserve the simple almost pointlike structure of the photon-quark coupling by incorporating the asymptotic freedom property of QCD and choosing a specific kinematics in which the behavior of the relevant amplitude is dominated by integration regions where momenta are large. In the coordinate representation, this corresponds to a situation when the separation between the two photon vertices of the Compton amplitude is light-like. In other words, the leading asymptotic behavior in the momentum space is governed by the leading light-cone singularity of the product of two electromagnetic currents $J^{\mu}(0)J^{\nu}(z)$. The light-cone operator product expansion was originally applied to the forward virtual Compton amplitude [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], whose imaginary part gives the cross section of deep inelastic scattering, the classic process for measuring the parton distribution functions. Another example of a light-cone dominated Compton-like amplitude is the form factor $\gamma^{*}\gamma \to \pi^{0}$ [22, 26, 29], which describes the process providing the cleanest information about the pion distribution amplitude [28, 30]. Recently, a lot of attention was focussed on studies of deeply virtual Compton scattering [31, 32, 41, 42, 46, 47] which can provide information about so-called skewed [33, 32] and double [43, 44] parton distributions. The leading large-Q^2 behavior of deeply virtual Compton scattering (DVCS) is also governed by the light-cone singularities of the coordinate representation version of the Compton amplitude [32, 41, 42]. As shown in [45], the light-cone singularities of the Compton amplitude play an important role in the description of the wide-angle real Compton scattering (WACS). It should be emphasized that in the coordinate representation, the Compton amplitudes for all four processes mentioned above are identical, despite the fact that in the momentum representation each of these processes has its own rich structure, visibly different from that of the other ones. Hence, the study of the generalized Compton amplitude in the coordinate representation provides the basis for a unified approach to several basic hard processes.

The advantages of the coordinate representation become especially clear when one needs to calculate corrections to the leading-order (or handbag) approximation. In particular, the calculations of the one-loop radiative corrections to DIS [9], $\gamma^*\gamma \to \pi^0$ form factor [26, 29, 30], and DVCS [41, 46] represent calculations of three different processes. In this dissertation, we show that one can perform the calculation of just the generic Compton amplitude in the coordinate representation. All the nontrivial procedures: renormalization (subtraction of the UV divergences), factorization (separation of contributions due to short and long distances) are performed at this stage. The next step is to perform parametrization of the long-distance part (matrix element of a light-cone operator) in terms of the relevant parton functions. Mathematically, this corresponds to taking the relevant Fourier transform. The final result strongly depends on the kinematics of the process under study: all the differences between the expressions for specific processes appear at this stage. This feature of our approach gives an opportunity to perform a nontrivial "quality control" for the results of these complicated calculations. In particular, we demonstrate that our expression for the one-loop Compton amplitude reproduces the classic DIS results by Bardeen et al. [9].

The coordinate representation of the Compton amplitude can be used as a starting point for a calculation of the target-mass corrections. These corrections are purely kinematical, and for this reason, they can be calculated exactly. For DIS, the nucleon mass corrections $(m_p^2/Q^2)^N$ can be calculated within the Nachtmann-Georgi-Politzer formalism [17, 25]. In DVCS, in addition to the nucleon mass, one should deal with the invariant momentum transfer t, which also leads to exactly calculable kinematic $(t/Q^2)^N$ corrections. A self-consistent treatment of the $O(t)$ terms is very important. In particular, the asymptotic expressions for the DVCS amplitudes [31, 32] were derived under assumption that the $O(t/Q^2)$ terms can be neglected. A straightforward use of these expressions in the $t \neq 0$ case leads to inconsistencies: the DVCS amplitude is not EM gauge invariant [48], i.e., the EM current conservation is apparently violated. In the second part of this dissertation, using the coordinate representation and double distribution formalism [34, 43, 44], we obtain the expression for the tree-level DVCS amplitude which includes all the generalized target-mass $(m_p^2/Q^2)^N$ and $(t/Q^2)^N$ corrections. We also show that this expression is electromagnetic (EM) gauge invariant. Hence, this expression can serve as a starting point for self-consistent phenomenological applications for the DVCS process, such as calculations of the DVCS cross section and, especially, the single-spin asymmetry [31, 48] which is proportional to t and for this reason cannot be reliably obtained from the asymptotic expressions of refs. [31, 32] valid only in the $t \to 0$ limit.

This dissertation is organized as follows:

In the first part of the thesis we consider Compton scattering amplitude in the coordinate representation. First we write the lowest order (tree level) amplitude in the coordinate representation and show how the transition to the more traditional momentum representation is done. Then we calculate one-loop correction. We give a detailed description of the calculation technique of Feynman integrals in the coordinate representation. We use the dimensional regularization and the minimal subtraction scheme. We write the amplitude in terms of non-local string light-ray operators.

Next we perform factorization and extract the Wilson coefficient function. Calculated in the coordinate representation the latter has a great advantage of being universal for many different processes. Therefore the one-loop corrections found in the coordinate representation can be used for analysis of DIS, DVCS, WACS etc. In the last two chapters of the part I of the thesis we present two examples to illustrate this, a forward process (deep inelastic scattering) and a non-forward process (Compton scattering). For the latter we explicitly calculate both real and imaginary parts using the methods of the theory of generalized functions. Some interesting formulas derived in the process of the calculations are listed in the appendices.

The goal of the second part of this thesis is to obtain an explicitly gauge invariant expression for the virtual Compton scattering amplitude. We introduce a modified symmetric parametrization of a non-forward matrix element in the coordinate representation. We derive a condition which provides gauge invariance of the modified parametrization. This condition leads to a system of differential equations which are solved.

Then we perform Fourier transformation and derive an expression for the Compton amplitude in the momentum representation. We show that obtained amplitude is indeed explicitly gauge invariant.

Chapter 2

Notations

A few words about used notations are in order.

Through out the paper we use

$$\bar{u} \equiv 1 - u. \tag{1}$$

Notation

$$(pq) \tag{2}$$

means the scalar product of two Lorentz vectors p and q. The same notation is used for both $d-$dimensional and four-dimensional vectors.

The "hat"

$$\hat{k} = k^{\beta}\gamma^{\beta} \tag{3}$$

means contraction of Lorentz vector k and a Dirac gamma matrix.

A generalized "plus" function (functional, to be precise) $\left[\frac{g(x)}{x-a}\right]_+$ is defined as

$$\int dx \, \left[\frac{g\,(x)}{x-a}\right]_+ F(x) = \int dx \; g(x)\frac{F(x) - F(a)}{x-a}. \tag{4}$$

where a probe functions $F(x)$, as usual, have a compact support, and point of singularity, $x = a$ is inside the limits of integration.

We use $T^{\mu\nu}(z)$ and $T^{\mu\nu}(q,p)$ for the Compton amplitude in the coordinate and the momentum representation respectively.

Notation $M_n^{\mu\nu}(z)$ is reserved for the contribution of the diagram pictured on the figure n.

Chapter 3

Tree Level (Born) Diagrams

3.1 Tree Level Diagrams in Coordinate Representation

The perturbative expansion for the Compton amplitude starts at zero order in the strong coupling constant, with the purely QED diagrams in which the photons interact with the quarks.

The expressions for two (s- and u-channel) tree level (or Born) diagrams in the coordinate space (see Fig.1) are simply given by the quark propagator switched between the initial and final quark fields, with the quarks being treated as massless fermions.

As all the loop calculations are to be performed in a d-dimensional space-time to exploit the benefits of dimensional regularization, here we also write the expression for the massless fermion propagator in a d-dimensional space-time:

$$\hat{S}(z_1 - z_2) = \frac{i\,\Gamma\left(\frac{d}{2}\right)(\hat{z}_1 - \hat{z}_2)}{2\pi^{d/2}(-(z_1 - z_2)^2)^{d/2}}. \quad (5)$$

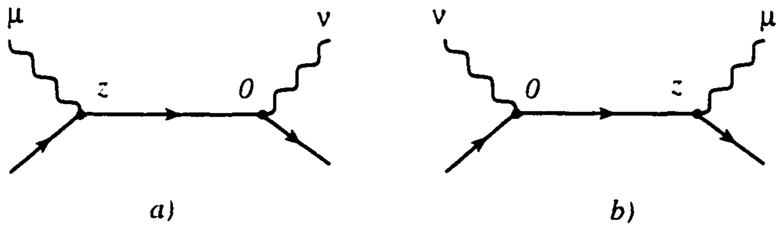

FIG. 1: Tree level (Born) diagrams: $a)$ s-channel and $b)$ u-channel

This leads to

$$\frac{-i\,\Gamma\left(\frac{d}{2}\right)\bar{\psi}(0)\gamma^{\nu}\hat{z}\gamma^{\mu}\psi(z)}{2\pi^{d/2}(-z^2)^{d/2}} \quad (6)$$

and

$$\frac{i\,\Gamma\left(\frac{d}{2}\right)\bar{\psi}(z)\gamma^{\mu}\hat{z}\gamma^{\nu}\psi(0)}{2\pi^{d/2}(-z^2)^{d/2}} \quad (7)$$

for each of the two tree level diagrams. The summation over the quark color indices is implied.

Using the following gamma-matrix formula ([51])

$$\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu} = \left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - g^{\mu\nu}g^{\alpha\beta}\right)\gamma^{\beta} + i\varepsilon^{\mu\alpha\nu\beta}\gamma^{\beta}\gamma_5 \quad (8)$$

we express the original bilocal quark operators in terms of more convenient ones, which have only one Lorentz index (basically, it is a decomposition on 16 gamma matrices basis, see [53]), e.g., $\bar{\psi}(0)\gamma_{\beta}\psi(z)$ and $\bar{\psi}(0)\gamma_{\beta}\gamma_5\psi(z)$ in case of the s-channel diagram.

The next complication arises from the fact that these operators contain higher-twist contributions: their Taylor expansion would contain local operators which are not automatically symmetric with respect to interchange of their indices.

To isolate the leading twist part we are interested in, we use the formulas given in Ref.[27]:

$$\left[\bar{\psi}(\alpha z)\gamma^{\mu}\psi(\beta z)\right] \stackrel{\text{l.t.}}{=} \frac{\partial}{\partial z^{\mu}} \int_0^1 dt\ \bar{\psi}(\alpha t z)\hat{z}\psi(\beta t z) \tag{9}$$

$$\left[\bar{\psi}(\alpha z)\gamma^{\mu}\gamma_5\psi(\beta z)\right] \stackrel{\text{l.t.}}{=} \frac{\partial}{\partial z^{\mu}} \int_0^1 dt\ \bar{\psi}(\alpha t z)\hat{z}\gamma_5\psi(\beta t z), \tag{10}$$

where the notation $\stackrel{\text{l.t.}}{=}$ indicates that the leading-twist parts of the left hand side and the right hand side coincide.

Here for the first time we encounter the non-local string operator:

$$\int_0^1 dt\ \bar{\psi}(\alpha t z)\hat{z}\psi(0). \tag{11}$$

The term "string" is used because the argument of $\bar{\psi}$ takes all the values on the "string" from 0 to z.

As a result, the contribution of the tree level diagrams to the Compton scattering amplitude can be expressed in the form

$$\begin{aligned} T_0^{\mu\nu}(z) &= \\ &\frac{-i\,\Gamma(\frac{d}{2})}{2\pi^{d/2}(-z^2)^{d/2}}\left(i\varepsilon^{\nu\mu\alpha\beta}z^{\alpha}\partial^{\beta}\int_0^1 \left(\bar{\psi}(0)\hat{z}\gamma_5\psi(tz) + \bar{\psi}(tz)\hat{z}\gamma_5\psi(0)\right) dt \right.\\ &\left. + \left\{ z^{\nu}\partial^{\mu} + z^{\mu}\partial^{\nu} - g^{\mu\nu}(z\partial)\right\}\int_0^1 \left(\bar{\psi}(0)\hat{z}\psi(tz) - \bar{\psi}(tz)\hat{z}\psi(0)\right) dt \right), \end{aligned} \tag{12}$$

in which the $\mu \leftrightarrow \nu$ symmetric and antisymmetric parts are now explicitly separated and all the nonperturbative information is accumulated in the matrix elements of operators of the two types:

$$\begin{aligned} \mathcal{O}_V(0,z) &\equiv \frac{i}{2}\left[\bar{\psi}(0)\hat{z}\psi(z) - \bar{\psi}(z)\hat{z}\psi(0)\right] \\ \mathcal{O}_A(0,z) &\equiv \frac{1}{2}\left[\bar{\psi}(0)\hat{z}\gamma_5\psi(z) + \bar{\psi}(z)\hat{z}\gamma_5\psi(0)\right]. \end{aligned} \tag{13}$$

Now, only twist-2 operators (symmetric and traceless) appear in the Taylor expansion of these bilocal operators on the light cone $z^2 = 0$.

3.2 Tree Level Diagrams in Momentum Representation

The next step in our analysis is to specify the form of the hadronic matrix element dictated by the relevant hard process and parametrize it in terms of the relevant nonperturbative (parton) functions. Let us list here the most important examples:

- Studying the $\gamma^*\gamma \to \pi^\circ$ transition form factor, one arrives at the matrix element $\langle \pi(p) \,|\, \mathcal{O}_A \,|\, 0 \rangle$ parametrized by the pion distribution amplitude, $\phi_\pi(y)$.

- The deep inelastic scattering (DIS) cross section can be calculated through the imaginary part of the forward virtual Compton amplitude. In this case, one would deal with the matrix elements $\langle N(p) \,|\, \mathcal{O}_{V,A} \,|\, N(p) \rangle$ parametrized by spin-averaged $f(x)$ or polarized $g(x)$ parton densities, respectively.

- In the case of deeply virtual Compton scattering (DVCS) one would get nonforward matrix elements $\langle N(p')\,|\,\mathcal{O}_{V,A}\,|\,N(p)\rangle$ which can be parametrized in two alternative ways: through double $F(x,y;t)$ or skewed $\mathcal{F}_\zeta(X)$ parton distributions.

The double distributions (DDs) have hybrid properties: they look like parton densities with respect to their first argument x and like distribution amplitudes with respect to the second argument y. Therefore the DVCS example contains two other ones as limiting cases and it is instructive to analyze the DVCS amplitude to illustrate how the transition from the coordinate to the more traditional momentum representation and parton description can be obtained.

In the DVCS case, there are initial and final nucleons (with momenta p and p' respectively), an initial highly virtual photon (with momentum $q, q^2 << 0$) and a final real photon (with momentum $q', q'^2 = 0$).

The nonforward matrix element of the $\mathcal{O}_V$ operator can for instance be parametrized in terms of double distributions $F(x,y;r^2)$ (see [34] and [44] for further details):

$$\begin{aligned}\langle p'|\ \bar{\psi}(0)\hat{z}\psi(z) - \bar{\psi}(z)\hat{z}\psi(0)\ |p\rangle\ &= \\ = \ \bar{u}(p')\hat{z}u(p)\int_0^1 dx\int_0^{\bar{x}} dy\ F(x,y)\left(e^{ixpz-i\bar{y}rz} - e^{-ixpz-iyrz}\right), \qquad (14)\end{aligned}$$

where $r = p - p'$ is the momentum transfer.

Here we neglected $O(z^2)$ terms and terms vanishing in the $r \to 0$ limit. The latter include the hadron helicity flip term proportional to $\bar{u}(p')(\hat{z}\hat{r} - \hat{r}\hat{z})u(p)$ and the Polyakov-Weiss term [49] proportional to $(rz)\bar{u}(p')u(p)$ and containing a single integration over y. For our illustration purposes we do not need these terms here. This topic is covered in greater details in the second part of this thesis.

Consider the $\mu \leftrightarrow \nu$ symmetric part of the DVCS amplitude. Taking $d = 4$ and using the parametrization written above for the two tree-level diagrams we obtain

$$\begin{aligned} T_0^{\mu\nu}(z) &= \\ &\frac{i}{2\pi^2(-z^2)^2}\left\{g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta} - g^{\mu\nu}g^{\alpha\beta}\right\} z^\alpha \partial^\beta \; \bar{u}(p')\hat{z}u(p) \\ &\int_0^1 dt \int_0^1 dx \int_0^{\bar{x}} dy \; F(x,y;r^2)\left(e^{ixpz - i\bar{y}rz} - e^{-ixpz - iyrz}\right). \end{aligned} \tag{15}$$

Now, in order to get the amplitude in momentum representation (i.e.calculate the Fourier transform) we multiply this expression by e^{-iqz} and perform the integration $\int d^4z$.

Introducing the shorthand notations

$$\begin{aligned} l_1 &= -xp - yr \\ l_2 &= xp - \bar{y}r \\ k_1 &= -q + tl_1 \\ k_2 &= -q + tl_2, \end{aligned} \tag{16}$$

taking the derivative ∂^β and performing integration over z we get

$$\begin{aligned} T_0^{\mu\nu}(q,p) &= \\ &-i\left\{g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta} - g^{\mu\nu}g^{\alpha\beta}\right\}\int_0^1 dt \int_0^1 dx \int_0^{\bar{x}} dy \; F(x,y) \\ &\Bigg[\; \bar{u}\gamma^\beta u \left(\frac{k_1^\alpha}{k_1^2} - \frac{k_2^\alpha}{k_2^2}\right) \; + \; t\,\bar{u}\gamma^\xi u \; \times \\ &\quad \left(l_1^\beta\left(\frac{g^{\alpha\xi}}{k_1^2} - \frac{2k_1^\alpha k_1^\xi}{k_1^4}\right) - l_2^\beta\left(\frac{g^{\alpha\xi}}{k_2^2} - \frac{2k_2^\alpha k_2^\xi}{k_2^4}\right)\right)\Bigg]. \end{aligned} \tag{17}$$

One can clearly see one of the advantages of the coordinate representation notations: its simplicity compared to the momentum representation.

Note that the integrand is a rational function of t. Therefore the artificial extra integration we introduced to single out the leading twist can be done explicitly.

Here we restrict ourselves to the even simpler case in which the nucleon mass and the invariant momentum transfer, r^2, are neglected:

$$p'^2 = p^2 = r^2 = 0. \tag{18}$$

The skewedness ζ in this case coincides with the Bjorken variable

$$\zeta = x_{Bj} = \frac{-q^2}{2(pq)}. \tag{19}$$

As discussed in [34], in the Bjorken limit, the hard subprocess amplitude in (17) depends only on the special combination $X = x + \zeta y$ of the variables x and y :

$$\begin{aligned} T_0^{\mu\nu}(q,p) &= \\ &\bar{u}\left\{q^\mu\gamma^\nu + q^\nu\gamma^\mu - g^{\mu\nu}\hat{q} + \frac{\zeta}{pq}2p^\mu p^\nu\hat{q}\right\}u\,\frac{i\zeta}{2(pq)}\int_0^1 dt\int_0^1 dx\int_0^{\bar{x}} dy\; F(x,y) \\ &\left[\frac{1}{\zeta + t\,(X-\zeta)} + \frac{1}{tX-\zeta} - \frac{t\,(X-\zeta)}{(\zeta + t\,(X-\zeta))^2} - \frac{tX}{(tX-\zeta)^2}\right], \end{aligned} \tag{20}$$

where we dropped all terms vanishing in this limit.

Subsequent integration over t results in

$$\begin{aligned} T_0^{\mu\nu}(q,p) &= \\ &\frac{i}{2(pq)}\,\bar{u}\left\{q^\mu\gamma^\nu + q^\nu\gamma^\mu - g^{\mu\nu}\hat{q} + \frac{\zeta}{(pq)}2p^\mu p^\nu\hat{q}\right\}u\;\times \end{aligned}$$

$$\times \int_0^1 dx \int_0^{\bar{x}} dy\ F(x,y) \left[\frac{1}{X-\zeta} + \frac{1}{X}\right]. \tag{21}$$

To derive this formula we used the approximation $\bar{u}\gamma^\mu u \sim p^\mu$. In this approximation we can rewrite the above amplitude expressed through the momentum of the outgoing photon, q' as follows

$$\begin{aligned} T_0^{\mu\nu}(q,p) &= \\ &\frac{i\bar{u}\hat{q}u}{2(pq)^2}\left\{q'^\mu p^\nu + q'^\nu p^\mu - g^{\mu\nu}(pq')\right\} \times \\ &\times \int_0^1 dx \int_0^{\bar{x}} dy\ F(x,y) \left[\frac{1}{X-\zeta} + \frac{1}{X}\right]. \end{aligned} \tag{22}$$

Furthermore, we can integrate $F(x,y)$ over y which converts it into the skewed parton distribution $F_\zeta(X)$.

Chapter 4

One-Loop Corrections

There are four distinctive types of the diagrams which contribute at one-loop to the Compton amplitude in the coordinate representation. They are

- handbag or box diagram
- vertex correction diagram with the vertex located at the origin
- vertex correction diagram with the vertex located at the point z
- self energy diagram.

Besides, each diagram comes in two channels: $s-$channel and $u-$channel (crossed diagram). So that we have 8 diagrams in total.

4.1 Handbag (Box) Diagrams

We start the study of the one-loop QCD corrections to the tree level diagrams with the so called handbag or box diagrams. Fig.2 makes these names self-explanatory.

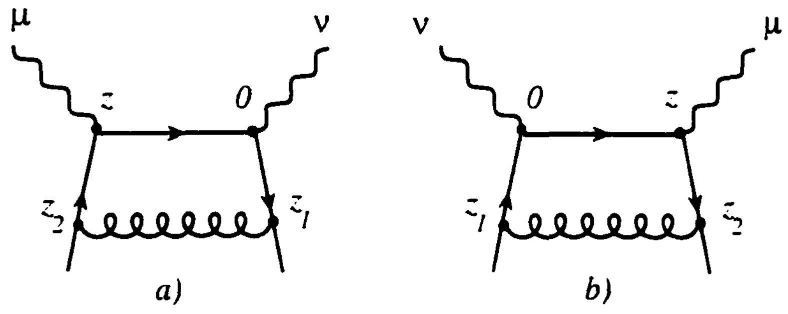

FIG. 2: Handbag (box) diagrams: $a)$ s-channel and $b)$ u-channel.

These two diagrams have a very similar structure. To be specific, we will concentrate on the calculation of the s-channel box diagram.

In the Feynman gauge, its contribution in the coordinate representation is given by

$$\begin{aligned} M_{2a}^{\mu\nu}(z) &= \int d^d z_1 \int d^d z_2 \, D(z_1 - z_2) \times \\ &\times \bar{\psi}(z_1)\gamma^\alpha \hat{S}(z_1)\gamma^\nu \hat{S}(-z)\gamma^\mu \hat{S}(z - z_2)\gamma^\alpha \psi(z_2), \end{aligned} \tag{23}$$

where $D(z_1 - z_2)$ is a massless boson propagator in a d-dimensions:

$$D(z_1 - z_2) = \frac{\Gamma\left(\frac{d}{2} - 1\right)}{4\pi^{d/2}(-(z_1 - z_2)^2)^{d/2-1}}. \tag{24}$$

With the chosen coordinate labeling, it is more convenient to start with the integral over z_1. Below, we give a rather detailed discussion of how we calculate this integral to illustrate the basic techniques of dealing with the Feynman integrals in the coordinate representation. In fact, for massless particles, the Feynman integrals in the momentum and coordinate representation are remarkably similar.

Just like in the momentum representation, the first step is to combine the denominator factors of two z_1-dependent propagators by means of the d–dimensional Feynman formula:

$$\frac{\Gamma(a)\Gamma(b)}{(-(z_1 - z_2)^2)^a\,(-z_1^2)^b} = \int_0^1 du \frac{\Gamma(a+b)u^{a-1}\bar{u}^{b-1}}{(-(z_1 - uz_2)^2 - u\bar{u}z_2^2)^{a+b}}. \tag{25}$$

The next standard step is to carry out the shift of the integration variable $z_1 \to z_1 + uz_2$.

However, even after this change, the external quark field $\bar{\psi}(z_1 + uz_2)$ still depends on the integration variable z_1. This is a new feature specific for the Feynman integrals in the coordinate representation. At this point, it is worth emphasizing that for the given operator product the coefficient function is process independent and hence does not depend on the target. Therefore, any target can be chosen. For our purposes it is the best to choose the massless on-shell quarks as the target. In the coordinate representation that implies the following condition on the quark field operator:

$$\frac{\partial^2}{\partial z^2}\psi(z) = 0. \tag{26}$$

In the same manner as the momentum-space on-shell condition $p^2 = 0$ substantially simplifies momentum loop integrations, the equation (26) plays an important role in the calculations in the coordinate representation performed in the present dissertation.

To make use of the on-shell condition (26) we expand the quark field operator into the Taylor series around the point uz_2 :

$$\bar{\psi}(z_1 + uz_2) = \bar{\psi}(uz_2) + z_1^\alpha \bar{\psi}_{,\alpha}(uz_2) + \frac{1}{2} z_1^\alpha z_1^\beta \bar{\psi}_{,\alpha\beta}(uz_2) + \dots . \quad (27)$$

An important result. which is easy to verify. is that integration over z_1 causes the contraction of the indices of the derivatives. Therefore all but the first two terms of the Taylor expansion would contain ∂^2 and vanish due to the on-shellness condition:

$$\bar{\psi}_{,\alpha\alpha}(z) \equiv \partial^2 \bar{\psi}(z) = 0. \quad (28)$$

The two nonvanishing terms can be integrated easily and the result is

$$\int d^d z_1 \frac{i\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)\bar{\psi}(z_1)z_1^\xi}{8\pi^d(-z_1^2)^{d/2}(-(z_1-z_2)^2)^{d/2-1}} =$$
$$\frac{(-1)}{8\pi^{d/2}} \int_0^1 du \left(\frac{\bar{\psi}(uz_2) z_2^\xi \Gamma\left(\frac{d}{2}-1\right)}{(-z_2^2)^{d/2-1}} - \frac{\bar{u}\bar{\psi}_{,\xi}(uz_2)\Gamma\left(\frac{d}{2}-2\right)}{2(-z_2^2)^{d/2-2}} \right). \quad (29)$$

This expression should be integrated over z_2 along with

$$\frac{-(\Gamma\left(\frac{d}{2}\right))^2}{4\pi^d(-z^2)^{d/2}} \Bigg\{ \gamma^\xi \Big[(d-4)\gamma^\nu \hat{z}\gamma^\mu (\hat{z}-\hat{z}_2) + 2(\hat{z}-\hat{z}_2)\gamma^\nu \hat{z}\gamma^\mu$$
$$-2\gamma^\mu\gamma^\nu \hat{z}(\hat{z}-\hat{z}_2) + 2\hat{z}\gamma^\nu\gamma^\mu(\hat{z}-\hat{z}_2) \Big]$$
$$+4\Big[g^{\xi\nu}\hat{z}\gamma^\mu(\hat{z}-\hat{z}_2) - (z-z_2)^\xi \gamma^\nu \hat{z}\gamma^\mu +$$
$$g^{\xi\mu}\gamma^\nu \hat{z}(\hat{z}-\hat{z}_2) - z^\xi \gamma^\nu\gamma^\mu(\hat{z}-\hat{z}_2) \Big] \Bigg\} \frac{\psi(z_2)}{(-(z-z_2)^2)^{d/2}}. \quad (30)$$

Note that due to the symmetry of the box diagram, the second term in Eq.(29) does not contribute at the leading twist level.

The result for the Diagram $2a$ (s-channel box) is

$$
\begin{aligned}
M^{\mu\nu}_{2a}(z) &= g^2 \frac{(-i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}} \\
&\int_0^1 du \int_0^1 dv\; v\,\bar{\psi}(uvz) \left\{ \frac{2(d-2)}{d-4} \left[z^\mu\gamma^\nu + z^\nu\gamma^\mu - g^{\mu\nu}\hat{z} \right] + 2(d-2) \times \right. \\
&\left. \left[\frac{2z^\mu z^\nu}{z^2} - g^{\mu\nu}\right]\hat{z} + (d-6)\left(1-\frac{1}{d-4}\right)\left[\gamma^\nu\hat{z}\gamma^\mu - \gamma^\mu\hat{z}\gamma^\nu\right] \right\} \psi(vz). \quad (31)
\end{aligned}
$$

Again, we can rewrite this expression in terms of the light-ray string operators using the formula (9). However, because of the subtleties of defining the γ_5 matrix in d dimensions, we prefer to leave the $\mu \leftrightarrow \nu$ antisymmetric term "as is", keeping it in the form of the commutator of gamma matrices until we will return (after necessary renormalization subtractions) into the physical 4-dimensional space. This transformation gives

$$
\begin{aligned}
M^{\mu\nu}_{2a}(z) &= g^2 \frac{(-i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}} \int_0^1 du \int_0^1 dv \times \\
&\left\{ \frac{2(d-2)\,\bar{v}}{d-4} \left[z^\mu\partial^\nu + z^\nu\partial^\mu - g^{\mu\nu} z\cdot\partial \right] \int_0^1 dt\; \bar{\psi}(uvtz)\hat{z}\psi(vtz) \right. \\
&\quad + 2(d-2)\,v \left[\frac{2z^\mu z^\nu}{z^2} - g^{\mu\nu}\right] \bar{\psi}(uvz)\hat{z}\psi(vz) \\
&\left. + v(d-6)\left(1-\frac{1}{d-4}\right) \bar{\psi}(uvz)\left[\gamma^\nu\hat{z}\gamma^\mu - \gamma^\mu\hat{z}\gamma^\nu\right]\psi(vz) \right\} \quad (32)
\end{aligned}
$$

for the Diagram $2a$.

The result for the Diagram $2b$ (u-channel box) differs only in the overall sign and the interchange of both the vector indices ($\mu \leftrightarrow \nu$) and arguments of quark spinors ψ, $\bar{\psi}$:

$$M_{2b}^{\mu\nu}(z) = g^2 \frac{i\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}}$$
$$\int_0^1 du \int_0^1 dv\; v\bar{\psi}(vz) \left\{ \frac{2(d-2)}{d-4}\left[z^\mu\gamma^\nu + z^\nu\gamma^\mu - g^{\mu\nu}\hat{z}\right] + 2(d-2) \times \right.$$
$$\left. \left[\frac{2z^\mu z^\nu}{z^2} - g^{\mu\nu}\right]\hat{z} + (d-6)\left(1 - \frac{1}{d-4}\right)\left[\gamma^\mu\hat{z}\gamma^\nu - \gamma^\nu\hat{z}\gamma^\mu\right] \right\} \psi(uvz). \quad (33)$$

In terms of the light-ray string operators expression for the Diagram $2b$ reads

$$M_{2b}^{\mu\nu}(z) = g^2 \frac{i\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}} \int_0^1 du \int_0^1 dv$$
$$\left\{ \frac{2(d-2)\,\bar{v}}{d-4}\left[z^\mu\partial^\nu + z^\nu\partial^\mu - g^{\mu\nu} z\cdot\partial \right] \int_0^1 dt\; \bar{\psi}(uvtz)\hat{z}\psi(vtz) \right.$$
$$+\, 2(d-2)\,v\left[\frac{2z^\mu z^\nu}{z^2} - g^{\mu\nu}\right]\bar{\psi}(vz)\hat{z}\psi(uvz)$$
$$\left. +\, v(d-6)\left(1 - \frac{1}{d-4}\right)\bar{\psi}(vz)\left[\gamma^\mu\hat{z}\gamma^\nu - \gamma^\nu\hat{z}\gamma^\mu\right]\psi(uvz) \right\}. \quad (34)$$

We note here that while two structures,

$$\left[z^\mu\partial^\nu + z^\nu\partial^\mu - g^{\mu\nu} z\cdot\partial\right] \quad (35)$$

and

$$\left[\gamma^\mu\hat{z}\gamma^\nu - \gamma^\nu\hat{z}\gamma^\mu\right] \quad (36)$$

exactly reproduce the tensor structures of the tree level diagrams, the loop integration also created a completely new tensor structure:

$$\left[\frac{2z^\mu z^\nu}{z^2} - g^{\mu\nu}\right]. \tag{37}$$

4.2 Vertex Correction Diagrams

In the coordinate representation, QCD gluon-exchange corrections to the QED vertices produce two distinct types of Feynman integrals, depending on whether the vertex is at the origin or at the space-time point z. Let us first calculate loop corrections for the vertex at the origin (see Figure 3).

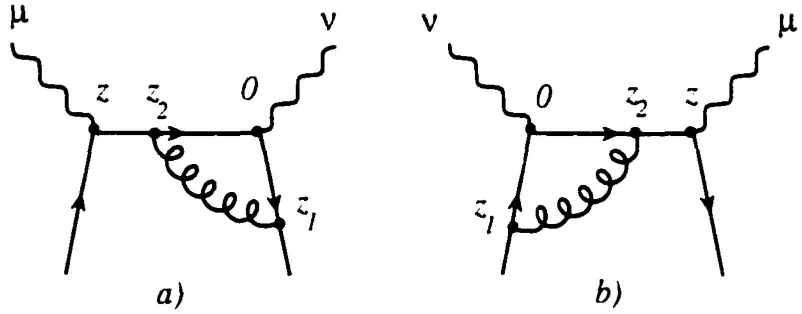

FIG. 3: Gluon corrections to the vertex located at the origin.

The Feynman integral for the diagram Fig.3a is given by

$$\begin{aligned} M_{3a}^{\mu\nu}(z) &= \int d^d z_1 \int d^d z_2 \, D(z_1 - z_2) \times \\ &\times \bar{\psi}(z_1)\gamma^\alpha \hat{S}(z_1)\gamma^\nu \hat{S}(-z_2)\gamma^\alpha \hat{S}(z_2 - z)\gamma^\mu \psi(z). \end{aligned} \tag{38}$$

In this case, it is easier to start with the integration over z_1 which is applied to exactly the same integral as in (29). Now, as the diagram is clearly asymmetric, both terms in (29) would contribute into the leading twist. Performing the second integration along with the remaining part gives the following result for the Diagram 3a (s−channel):

$$
\begin{aligned}
&M_{3a}^{\mu\nu}(z) = \\
&g^2\frac{(-i)\,\Gamma\left(\frac{d}{2}\right)}{16\pi^d(-z^2)^{d-2}}\int_0^1 du\left[\Gamma\left(\frac{d}{2}-2\right)\frac{\left(u^{1-d/2}-1\right)}{d/2-1}\left[\bar{\psi}(uz)-\bar{\psi}(0)\right]\gamma^\nu\hat{z}\gamma^\mu\psi(z)\right. \\
&\quad+\Gamma\left(\frac{d}{2}-1\right)\frac{d-3}{2-d}\left(\frac{\bar{\psi}(0)}{\frac{d}{2}-2}+\bar{\psi}(uz)-\left[\bar{\psi}(uz)-\bar{\psi}(0)\right]u^{1-d/2}\right)\gamma^\nu\hat{z}\gamma^\mu\psi(z) \\
&\quad\left.+\Gamma\left(\frac{d}{2}-1\right)\bar{\psi}(uz)z^\nu\gamma^\mu\psi(z)\right]+ \\
&g^2\frac{(-i)\,\Gamma\left(\frac{d}{2}\right)}{16\pi^d(-z^2)^{d-3}}\int_0^1 du\;\bar{\psi}_{,\nu}(uz)\gamma^\mu\psi(z)\times \\
&\quad\left[\Gamma\left(\frac{d}{2}-2\right)\left(\frac{(d/2-2)u+u^{2-d/2}}{d/2-1}-1\right)+\Gamma\left(\frac{d}{2}-1\right)\frac{v^{2-d/2}-v}{d-2}\right]. \qquad (39)
\end{aligned}
$$

At this stage of our analysis, it becomes convenient to introduce the parameter ε, $\varepsilon = d - 4/2$, which describes the difference between the dimension d of the fictitious space-time in which we perform our calculations and 4, the dimension of the physical $d = 4$ space-time. As always integrals, divergent at $d = 4$, would yield the poles in ε, in dimensional regularization. With this the above expression can be rewritten in a little bit simpler form:

$$
\begin{aligned}
M_{3a}^{\mu\nu}(z) &= g^2\frac{(-i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{16\pi^d(-z^2)^{d-2}}\times \\
&\int_0^1 du\left[\frac{1}{\varepsilon}\frac{\left(u^{1-d/2}-1\right)}{d/2-1}[\bar{\psi}(uz)-\bar{\psi}(0)]\gamma^\nu\hat{z}\gamma^\mu\psi(z)+\bar{\psi}(uz)z^\nu\gamma^\mu\psi(z)+\right.
\end{aligned}
$$

$$\frac{d-3}{2-d}\left(\frac{\bar\psi(0)}{\varepsilon}+\bar\psi(uz)-[\bar\psi(uz)-\bar\psi(0)]u^{1-d/2}\right)\gamma^\nu\hat z\gamma^\mu\psi(z)\Bigg]+\frac{g^2}{16\pi^d}\times$$

$$\frac{i\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{(-z^2)^{d-3}\left(\frac{d}{2}-1\right)}\int_0^1 du\,\bar\psi_{,\nu}(uz)\gamma^\mu\psi(z)\left[\frac{1-u^{-\varepsilon}}{\varepsilon}+1-\frac{u^{-\varepsilon}}{2}-\frac{u}{2}\right]. \quad (40)$$

There are two types of poles in the last expression. The second (trivial) pole $\frac{\bar\psi(0)}{\varepsilon}$ corresponds to the UV divergence of the diagram and is removed by the vertex renormalization. The other type corresponds to the IR divergence and is caused by the fact that we used massless propagators in the loop integral.

The Feynman integral for the u-channel vertex correction diagram (see Fig.3a) is given by:

$$M_{3b}^{\mu\nu}(z) = g^2\frac{(i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{16\pi^d(-z^2)^{d-2}}\times$$

$$\int_0^1 dv\left[\frac{1}{\varepsilon}\frac{\left(v^{1-d/2}-1\right)}{d/2-1}\bar\psi(z)\gamma^\mu\hat z\gamma^\nu[\psi(vz)-\psi(0)]+\bar\psi(z)\gamma^\mu z^\nu\psi(vz)+\right.$$

$$\left.\frac{d-3}{2-d}\bar\psi(z)\gamma^\mu\hat z\gamma^\nu\left(\frac{\psi(0)}{\varepsilon}+\psi(vz)-[\psi(vz)-\psi(0)]v^{1-d/2}\right)\right]+\frac{g^2}{16\pi^d}\times$$

$$\frac{(-i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{(-z^2)^{d-3}\left(\frac{d}{2}-1\right)}\int_0^1 dv\,\bar\psi(z)\gamma^\mu\psi_{,\nu}(vz)\left[\frac{1-v^{-\varepsilon}}{\varepsilon}+1-\frac{v^{-\varepsilon}}{2}-\frac{v}{2}\right]. \quad (41)$$

The other vertex correction diagram has vertex at the point z "wrapped" by the loop. This leads to the different integrals.

The Feynman integral for the diagram Fig.4a is given by

$$M_{4a}^{\mu\nu}(z) = \int d^dz_1\int d^dz_2\; D(z_1-z_2)\times$$

$$\times\bar\psi(0)\gamma^\nu\hat S(-z_2)\gamma^\alpha\hat S(z_2-z)\gamma^\mu\hat S(z-z_1)\gamma^\alpha\psi(z_1). \quad (42)$$

Again we first calculate the integral over z_1

$$\frac{i\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{8\pi^d}\int d^dz_1\frac{(z-z_1)^\xi\;\psi(z_1)}{(-(z-z_1)^2)^{d/2}\;(-(z_1-z_2)^2)^{d/2-1}} =$$

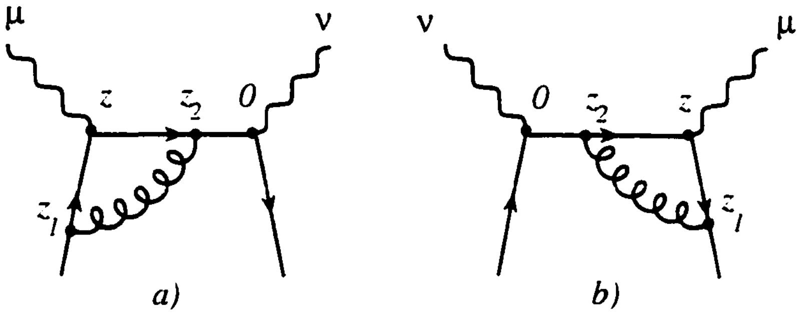

FIG. 4: Gluon corrections to the vertex located at the point z.

$$= \frac{(-1)}{8\pi^{d/2}} \int_0^1 du \left(\frac{(z-z_2)^\xi\, \psi(uz_2+\bar{u}z)\, \Gamma\left(\frac{d}{2}-1\right)}{(-(z-z_2)^2)^{d/2-1}} + \frac{\bar{u}\, \psi_{,\xi}(uz_2+\bar{u}z)\, \Gamma\left(\frac{d}{2}-2\right)}{2\,(-(z-z_2)^2)^{d/2-2}} \right). \tag{43}$$

This expression should be integrated over z_2 along with

$$\frac{-g^2\left(\Gamma\left(\frac{d}{2}\right)\right)^2\, \bar{\psi}(0)\gamma^\nu \hat{z}_2}{4\pi^d(-z_2^2)^{d/2}(-(z-z_2)^2)^{d/2}} \Big[(6-d)\,(\hat{z}-\hat{z}_2)\,\gamma^\mu\gamma^\xi - 4(z-z_2)^\mu\gamma^\xi + 4(z-z_2)^\xi\gamma^\mu - 4\,(\hat{z}-\hat{z}_2)\,g^{\mu\xi} \Big]. \tag{44}$$

The integration over z_2 results in

$$M_{4a}^{\mu\nu}(z) = g^2\frac{(-i)\,\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{16\pi^d(-z^2)^{d-2}}\times$$
$$\int_0^1 dv \left[\frac{1}{\varepsilon}\frac{\left(\bar{v}^{1-d/2}-1\right)}{d/2-1}\bar{\psi}(0)\gamma^\nu\hat{z}\gamma^\mu[\psi(vz)-\psi(z)] + \bar{\psi}(0)\gamma^\nu z^\mu\psi(vz) + \frac{d-3}{2-d}\bar{\psi}(0)\gamma^\nu\hat{z}\gamma^\mu\left(\frac{\psi(z)}{\varepsilon} + \psi(vz) - [\psi(vz)-\psi(z)]\bar{v}^{1-d/2}\right)\right] + \frac{g^2}{16\pi^d}\times$$

$$\frac{(-i)\,\Gamma\big(\frac{d}{2}\big)\Gamma\big(\frac{d}{2}-1\big)}{(-z^2)^{d-3}(\frac{d}{2}-1)}\int_0^1 dv\,\bar{\psi}(0)\gamma^\nu\psi_{,\mu}(vz)\left[\frac{1-v^{-\varepsilon}}{\varepsilon}+1-\frac{v^{-\varepsilon}}{2}-\frac{v}{2}\right]. \quad (45)$$

The corresponding $u-$channel diagram gives:

$$\begin{aligned} M_{4b}^{\mu\nu}(z) &= g^2\frac{i\,\Gamma\big(\frac{d}{2}\big)\Gamma\big(\frac{d}{2}-1\big)}{16\pi^d(-z^2)^{d-2}}\times \\ &\int_0^1 du\left[\frac{1}{\varepsilon}\frac{\left(\bar{u}^{1-d/2}-1\right)}{d/2-1}[\bar{\psi}(uz)-\bar{\psi}(z)]\gamma^\mu\hat{z}\gamma^\nu\psi(0)+\bar{\psi}(uz)z^\mu\gamma^\nu\psi(0)+\right. \\ &\left.\frac{d-3}{2-d}\left(\frac{\bar{\psi}(z)}{\varepsilon}+\bar{\psi}(uz)-[\bar{\psi}(uz)-\bar{\psi}(z)]\bar{u}^{1-d/2}\right)\gamma^\mu\hat{z}\gamma^\nu\psi(0)\right]+\frac{g^2}{16\pi^d}\times \\ &+\frac{i\Gamma\big(\frac{d}{2}\big)\Gamma\big(\frac{d}{2}-1\big)}{(-z^2)^{d-3}(\frac{d}{2}-1)}\int_0^1 du\,\bar{\psi}_{,\mu}(uz)\gamma^\nu\psi(0)\left[\frac{1-u^{-\varepsilon}}{\varepsilon}+1-\frac{u^{-\varepsilon}}{2}-\frac{u}{2}\right]. \quad (46)\end{aligned}$$

4.3 Self Energy Diagram

The last and the simplest one is the so called self energy diagram depicted on the figure Fig.5.

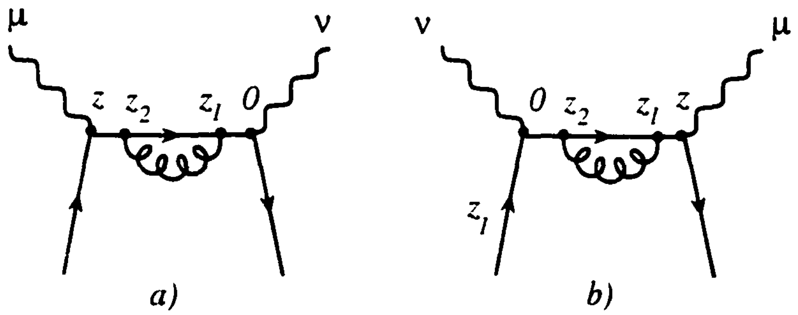

FIG. 5: Self energy diagrams: $a)$ $s-$channel and $b)$ $u-$channel.

The Feynman integral for the diagram Fig.5a is given by

$$
\begin{aligned}
M_{5a}^{\mu\nu}(z) &= \int d^d z_1 \int d^d z_2\, D(z_1 - z_2) \times \\
&\times \bar{\psi}(0)\gamma^\nu \hat{S}(-z_2)\gamma^\alpha \hat{S}(z_2 - z_1)\gamma^\alpha \hat{S}(z_1 - z)\gamma^\mu \psi(z) \qquad (47)
\end{aligned}
$$

In the self energy diagrams the gluon-fermion loop is interwined with no vertices. Because of that, the result of integration exactly reproduce the tensor structure of the tree level diagram.

For the s-channel self energy diagram we get

$$
M_{5a}^{\mu\nu}(z) = g^2 \frac{(-i)\ \Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}}\ \bar{\psi}(0)\gamma^\nu \hat{z}\gamma^\mu \psi(z)\ \frac{1}{\varepsilon}, \qquad (48)
$$

and the corresponding crossed (u-channel) self energy diagram gives

$$
M_{5b}^{\mu\nu}(z) = g^2 \frac{(i)\ \Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{32\pi^d(-z^2)^{d-2}}\ \bar{\psi}(z)\gamma^\mu \hat{z}\gamma^\nu \psi(0)\ \frac{1}{\varepsilon}. \qquad (49)
$$

For this diagram the pole $\frac{1}{\varepsilon}$ corresponds to pure UV divergence of the loop integral.

Chapter 5

Coefficient Function

5.1 Compton Amplitude and Operator Product Expansion

The Compton amplitude for non-forward scattering of a virtual photon off a hadron is given by

$$T_{\mu\nu}(p, p', q) = i \int d^4 z \, e^{-i(qz)} \langle p', S' | T(J_\mu(z) J_\nu(0)) | p, S \rangle. \tag{50}$$

where q is the momentum of the virtual photon, $-q^2 = Q^2 >> 0$.

The factorization theorem (see [35],[36],[37],[38],[39],[40],[34],[42],[41]) states that in quantum chromodynamics the later can be factorized in two parts: the hard one, depending on large momentum Q, and the soft one, depending on small momenta p, p' (for the sake of simplicity we omit vector indices here):

$$T(p,p',q) = \sum_i \tilde{C}_i \left(Q, \mu^2, g^2\right) \tilde{A}_i \left(p, p', \mu^2, g^2\right). \tag{51}$$

Here μ is a scale dividing large and small distances (also called a factorization scale) and g^2 is a (renormalized) coupling constant. Due to the asymptotic freedom of quantum chromodynamics, the hard part can be found by means of the perturbation theory and the soft part contains non-perturbative effects.

In terms of coordinate representation this corresponds to light-cone generalization of Wilson's operator product expansion (OPE) ([10],[11],[21]-[25]).

The operator product expansion expresses the singularities of the operator products as a series of nonsingular operators with the coefficients being singular complex numbers (for the introduction see [57]):

$$\hat{O}(x)\,\hat{O}(y) = \sum_i C_i(x-y)\hat{A}_i(x,y), \tag{52}$$

where $C_i(x-y)$ is a set of singular C-number functions (coefficient functions) and $\hat{A}_i(x,y)$ denotes a family of non-singular (in the limit $x \to y$) non-local operators. In our case they would be the string light-ray operators. These non-local operators can in turn be expanded into a Taylor series of local operators. For the string light-ray operators operator product expansion (52) takes the form of convolution:

$$\begin{aligned} \sum_i C_i(x-y) \otimes \hat{A}_i(x,y) &= \\ \int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u)\, C_i(x-y,u,v)\hat{A}_i(x,y,u,v). \end{aligned} \tag{53}$$

To carry out the factorization procedure and separate the contributions corresponding to coefficient functions from operators, we follow the method described in [9] (see also reference [26]).

Let us outline it briefly. We can write our OPE symbolically as

$$T(g^2) = C(g^2)A(g^2). \tag{54}$$

In the one-loop approximation the functions involved are given by

$$\begin{aligned} T(g^2) &= 1 + \frac{g^2}{16\pi^2}T^{(1)} + \dots \\ C(g^2) &= 1 + \frac{g^2}{16\pi^2}C^{(1)} + \dots \\ A(g^2) &= 1 + \frac{g^2}{16\pi^2}A^{(1)} + \dots . \end{aligned} \tag{55}$$

We insert these three equations into Eq.(54) and keep only terms of order g^2 to obtain

$$1 + \frac{g^2}{16\pi^2}T^{(1)} = 1 + \frac{g^2}{16\pi^2}C^{(1)} + \frac{g^2}{16\pi^2}A^{(1)}. \tag{56}$$

The coefficient function in one-loop approximation can be found as

$$C(g^2) = 1 + \frac{g^2}{16\pi^2}T^{(1)} - \frac{g^2}{16\pi^2}A^{(1)}. \tag{57}$$

hence, to find the coefficient function, we need to perform the following steps

1. calculate the one-loop corrections to the virtual Compton amplitude

2. find non-local operators, $A(g^2)$

3. subtract the latter from the former.

While one-loop corrections to the virtual Compton amplitude were calculated in chapter 4, the non-local string operators can be found as the product of the tree-level operators contracted with the evolution kernel $B_{qq}(u, v)$ (see

(59) below) and a pure pole term, $\frac{1}{\varepsilon}$. This simplicity is one of the advantages of using the dimensional regularization.

The light-ray quark-quark evolution kernel $B_{qq}(u,v)$ can be found as the coefficient of the pure pole terms:

$$\frac{1}{\varepsilon}\int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u)\ \left[B_{qq}(u,v)\ \bar{\psi}(uz)\hat{O}^{\mu\nu}\psi(\bar{v}z)\right]_{d=4}. \qquad (58)$$

The sum of all diagrams leads to the result first obtained in [20] and in [27]. In the symmetric notation employed here the evolution kernel was first obtained in [34].

$$B_{qq}(u,v) = g^2\left(1 + \left[\frac{\bar{u}}{u}\right]_+ \delta(v) + \left[\frac{\bar{v}}{v}\right]_+ \delta(u) - \frac{1}{2}\delta(u)\delta(v)\right). \qquad (59)$$

As the same ε is used to regularize both UV and IR behaviour the last term of (59) removes the total UV divergence.

5.2 Coefficient Function in Coordinate Representation

We use a minimal subtraction scheme, in which renormalization is done by removing pure poles, $\frac{1}{\varepsilon}$. When the limit $d \to 4$ is taken the expression of the type

$$\frac{1}{\varepsilon}(z^2)^\varepsilon - \frac{1}{\varepsilon} \qquad (60)$$

leads to the logarithm function in the result. Parameter μ is inserted to make the argument of the logarithm dimensionless: $\ln(z^2\mu^2)$. We also use shorthand notation for the combination $\delta = \gamma_E + \ln\pi$, where γ_E is Euler-Mascheroni constant. This combination is common for the dimensional regularization.

The above factorization procedure for both s-channel and u-channel handbag diagrams gives

$$\frac{i\, g^2}{16\pi^4(-z^2)^2}\int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u) \times$$
$$\Big\{ 2\left[g^{\mu\nu} - \frac{2z^\mu z^\nu}{z^2}\right] \left(\bar{\psi}(uz)\hat{z}\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\hat{z}\psi(vz)\right)$$
$$+ \left[\ln(z^2\mu^2) + \delta - 1\right]\ \bar{\psi}(uz)\left[z^\mu\gamma^\nu + z^\nu\gamma^\mu - g^{\mu\nu}\hat{z}\right]\psi(\bar{v}z)$$
$$- \left[\ln(z^2\mu^2) + \delta - 1\right]\ \bar{\psi}(\bar{u}z)\left[z^\mu\gamma^\nu + z^\nu\gamma^\mu - g^{\mu\nu}\hat{z}\right]\psi(vz)$$
$$+\frac{1}{2}\left[\ln(z^2\mu^2) + \delta + 3\right]\ \bar{\psi}(uz)\left[\gamma^\nu\hat{z}\gamma^\mu - \gamma^\mu\hat{z}\gamma^\nu\right]\psi(\bar{v}z)$$
$$-\frac{1}{2}\left[\ln(z^2\mu^2) + \delta + 3\right]\ \bar{\psi}(\bar{u}z)\left[\gamma^\mu\hat{z}\gamma^\nu - \gamma^\nu\hat{z}\gamma^\mu\right]\psi(vz)\Big\} \qquad (61)$$

The sum of both vertex correction and self energy diagrams (including crossed diagrams) results in

$$\frac{i\, g^2}{16\pi^4(-z^2)^2}\int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u) \times$$
$$\Big\{ \delta(v)\left[\frac{\ln u + \bar{u}\left(\frac{1}{2} + \delta + \ln(z^2\mu^2)\right)}{u}\right]_+ \left(\bar{\psi}(uz)\gamma^\nu\hat{z}\gamma^\mu\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\gamma^\mu\hat{z}\gamma^\nu\psi(vz)\right)$$
$$+\ \delta(u)\left[\frac{\ln v + \bar{v}\left(\frac{1}{2} + \delta + \ln(z^2\mu^2)\right)}{v}\right]_+ \left(\bar{\psi}(uz)\gamma^\nu\hat{z}\gamma^\mu\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\gamma^\mu\hat{z}\gamma^\nu\psi(vz)\right)$$
$$+\ \delta(u)\ \delta(v)\ \frac{1}{2}\left[4 - \delta - \ln(z^2\mu^2)\right] \left(\bar{\psi}(uz)\gamma^\nu\hat{z}\gamma^\mu\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\gamma^\mu\hat{z}\gamma^\nu\psi(vz)\right)$$
$$-\ \delta(v)\left(\bar{\psi}(uz)z^\nu\gamma^\mu\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)z^\mu\gamma^\nu\psi(vz)\right)$$
$$-\ \delta(u)\left(\bar{\psi}(uz)z^\mu\gamma^\nu\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)z^\nu\gamma^\mu\psi(vz)\right)\Big\}$$
$$+\ \frac{i\, g^2}{16\pi^4(-z^2)}\int_0^1 du\left(\ln u + \frac{\bar{u}}{2}\right) \times$$
$$\Big\{\ \bar{\psi}_{,\nu}(uz)\gamma^\mu\psi(z) - \bar{\psi}(z)\gamma^\mu\psi_{,\nu}(uz) + \bar{\psi}_{,\mu}(\bar{u}z)\gamma^\nu\psi(0) - \bar{\psi}(0)\gamma^\nu\psi_{,\mu}(\bar{u}z)\ \Big\} \qquad (62)$$

In order to single out the leading twist contribution and extract the coefficient functions associated with the different tensor structures we rewrite expressions (61) and (62) in terms of light-ray strings operators. That can be done with the help of the following formulas:

$$\left[\bar{\psi}(\alpha z)\gamma^{\mu}\psi(\beta z)\right] \stackrel{\text{l.t.}}{=} \frac{\partial}{\partial z^{\mu}} \int_0^1 dt\ \bar{\psi}(\alpha t z)\hat{z}\psi(\beta t z) \tag{63}$$

$$\left[\bar{\psi}(\alpha z)\gamma^{\mu}\gamma_5\psi(\beta z)\right] \stackrel{\text{l.t.}}{=} \frac{\partial}{\partial z^{\mu}} \int_0^1 dt\ \bar{\psi}(\alpha t z)\hat{z}\gamma_5\psi(\beta t z) \tag{64}$$

$$\left[\bar{\psi}(z)\gamma^{\mu}\psi_{,\nu}(\alpha z)\right] \stackrel{\text{l.t.}}{=} \frac{1}{\bar{\alpha}}\left(\mathcal{D}_{\nu} - \partial_{\nu}\right)\partial^{\mu} \int_0^1 dt\ \bar{\psi}(tz)\hat{z}\psi(\alpha t z) \tag{65}$$

$$\left[\bar{\psi}(0)\gamma^{\mu}\psi_{,\nu}(\alpha z)\right] \stackrel{\text{l.t.}}{=} \frac{1}{\alpha}\partial_{\nu}\partial^{\mu} \int_0^1 dt\ \bar{\psi}(0)\hat{z}\psi(\alpha t z), \tag{66}$$

where again the notation $\stackrel{\text{l.t.}}{=}$ indicates that the leading-twist parts of the left hand side and the right hand side are the same.

Here $\partial_{\nu} \equiv \frac{\partial}{\partial z^{\nu}}$ and $\mathcal{D}_{\nu}$ is the derivative of the string operator with respect to the total translation. It is defined as

$$\mathcal{D}_{\nu}\ \bar{\psi}(\alpha z)\hat{O}\psi(\beta z) = \frac{\partial}{\partial x^{\nu}}\left[\bar{\psi}(\alpha z + x)\hat{O}\psi(\beta z + x)\right]_{x=0}. \tag{67}$$

Using Eqs.(63) - (66) the sum of all four factorized diagrams and their crossed counterparts can be rewritten in terms of non-local light-ray string operators as follows

$$\frac{i\, g^2}{16\pi^4(-z^2)^2}\int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u)\ \int_0^1 dt$$
$$\Big\{ (C(u,v)+2)\ i\varepsilon^{\mu\nu\alpha\beta} z^\alpha \partial^\beta\ \left(\bar{\psi}(utz)\hat{z}\gamma_5\psi(\bar{v}tz) + \bar{\psi}(\bar{u}tz)\hat{z}\gamma_5\psi(vtz)\right)$$
$$+\ (C(u,v)-2)\left[z^\mu\partial^\nu + z^\nu\partial^\mu - g^{\mu\nu} z\cdot\partial \right]\times$$
$$\times\left(\bar{\psi}(utz)\hat{z}\psi(\bar{v}tz) - \bar{\psi}(\bar{u}tz)\hat{z}\psi(vtz)\right)$$
$$-\ \frac{\delta(v)+\delta(u)}{2}\left[z^\mu\partial^\nu + z^\nu\partial^\mu \right]\left(\bar{\psi}(utz)\hat{z}\psi(\bar{v}tz) - \bar{\psi}(\bar{u}tz)\hat{z}\psi(vtz)\right)\Big\}\ +$$
$$\frac{i\, g^2}{16\pi^4(-z^2)^2}\int_0^1 du \int_0^1 dv\ \theta(\bar{v} > u)\, 2\left[g^{\mu\nu} - \frac{2z^\mu z^\nu}{z^2}\right]\times$$
$$\times\left(\bar{\psi}(uz)\hat{z}\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\hat{z}\psi(vz)\right)\ +$$
$$\frac{i\, g^2}{16\pi^4(-z^2)}\int_0^1 du \int_0^1 dt\left(\frac{\ln u}{\bar{u}} + \frac{1}{2}\right)$$
$$\Big\{ \left[\mathcal{D}^\nu\partial^\mu - \partial^\nu\partial^\mu \right]\left(\bar{\psi}(utz)\hat{z}\psi(tz) - \bar{\psi}(tz)\hat{z}\psi(utz)\right)$$
$$+\ \left[\partial^\nu\partial^\mu\right]\left(\bar{\psi}(\bar{u}tz)\hat{z}\psi(0) - \bar{\psi}(0)\hat{z}\psi(\bar{u}tz)\right)\Big\}, \tag{68}$$

where

$$C(u,v) = \ln(z^2{\mu_0}^2)\left\{ 1 + \delta(v)\left[\frac{\bar{u}}{u}\right]_+ + \delta(u)\left[\frac{\bar{v}}{v}\right]_+ - \frac{\delta(u)\,\delta(v)}{2}\right\}$$
$$+\delta(v)\left[\frac{2\ln u + \bar{u}}{2u}\right]_+ + \delta(u)\left[\frac{2\ln v + \bar{v}}{2v}\right]_+ + \delta(u)\,\delta(v) + 1 \tag{69}$$

and $\ln(z^2\mu^2) + \delta \equiv \ln(z^2{\mu_0}^2)$.

In the expression (68) the first structure gives the coefficient function corresponding to the antisymmetric non-local operator $\mathcal{O}_A(0,z)$ (see Eq. (13)) and the rest are the coefficient functions corresponding to different tensor structures going along with the symmetric non-local operator $\mathcal{O}_V(0,z)$.

Chapter 6

Examples

6.1 Deep Inelastic Scattering

Calculations of the coefficient function in the coordinate representation has a great advantage of being process independent. Once computed in the coordinate space the coefficient function can be applied to different processes. One just needs to introduce a proper parametrization of the corresponding matrix elements (see [34] for details) and perform transformation to the momentum space.

To illustrate this procedure and compare our results with those obtained in well known the paper by Bardeen *et al.* [9], we consider the relatively simple case of the deep inelastic scattering (DIS) (for the good introduction to the topic see [55] and [56]).

The cross section of the DIS process can be calculated as an imaginary part of the forward process matrix element i.e. the matrix element between the states with the same momentum (see [50]).

In this case the derivative with respect to the total translation, $\mathcal{D}_\nu$ vanishes because it is proportional to the transferred momentum (the difference of the initial and final momenta). This allows us to recombine terms in (68) and, as a result, we are left with only two string operators:

$$\bar{\psi}(utz)\hat{z}\psi(\bar{v}tz) \quad - \quad \bar{\psi}(\bar{u}tz)\hat{z}\psi(vtz) \tag{70}$$

$$\bar{\psi}(utz)\hat{z}\gamma_5\psi(\bar{v}tz) \quad + \quad \bar{\psi}(\bar{u}tz)\hat{z}\gamma_5\psi(vtz). \tag{71}$$

Therefore only two functions are needed to parametrize their matrix elements. Here we restrict ourselves to the first one.

For zero transferred momentum, the parametrization of the matrix element depends only on (pz) and simply reads

$$\begin{aligned} \langle p| \; \bar{\psi}(uz)\hat{z}\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\hat{z}\psi(vz) \; |p\rangle \; &= \\ 2(pz) \int_0^1 dx \; f(x) \left(e^{ix(pz)(1-u-v)} - e^{-ix(pz)(1-u-v)}\right). \end{aligned} \tag{72}$$

We proceed as follows

- (i) substitute the above parametrization of the forward matrix element into our general result (68)
- (ii) calculate all derivatives
- (iii) perform Fourier transformation $\int d^4Z\, e^{-iqZ}$ (where q is the momentum of the incoming photon, $q^2 = -Q^2$)
- (iv) Expand the result as the series of powers of $x\omega_B$, with

$$\omega_B \equiv \frac{1}{x_B} = -\frac{2p\cdot q}{q^2} \tag{73}$$

is the inverse Bjorken variable

- (v) introduce the moments f_n of the distribution function $f(x)$ defined as

$$f_n = \int_0^1 dx x^{n-1} f(x); \tag{74}$$

perform integration over u, v, t and x.

The intermediate calculations are quite long and tedious so that we list here only the final expressions for different diagrams.

For the box diagram the above procedure would result in

$$\frac{i\,g^2}{4\pi^2} \sum_{n=2,4...} f_n {\omega_B}^n \left\{ \frac{p^\mu p^\nu}{(pq)^2} q^2 \frac{1}{n(n+1)} \left(\frac{1}{n-1} - \frac{n}{2} \right) + \frac{q^\mu q^\nu}{q^2} \frac{2}{n+1} \right.$$
$$\left. - g^{\mu\nu} \frac{n-1}{n(n+1)} - d^{\mu\nu} \frac{1}{n(n+1)} \left(\ln \frac{Q^2}{4\pi {\mu_E}^2} - \sum_{k=1}^{n} \frac{1}{k} + n + \frac{1}{n} \right) \right\}, \tag{75}$$

with

$$d^{\mu\nu} = \frac{q^\mu p^\nu + q^\nu p^\mu}{(pq)} - \frac{p^\mu p^\nu}{((pq))^2} q^2 - g^{\mu\nu}. \tag{76}$$

And the other three diagrams would lead to

$$\frac{i\,g^2}{4\pi^2} \sum_{n=2,4...} f_n {\omega_B}^n \left\{ \frac{p^\mu p^\nu}{((pq))^2} q^2 \frac{1}{(n-1)} \left(\frac{1}{2} - \frac{1}{n} \right) - g^{\mu\nu} \frac{1}{n} \right.$$
$$+ d^{\mu\nu} \left(2 \sum_{k=1}^{n} \frac{1}{k^2} - 2 \sum_{k=1}^{n} \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j} - \frac{5}{2n} + \frac{9}{2} - \frac{3}{2} \sum_{k=1}^{n} \frac{1}{k} \right.$$
$$\left. \left. - \left(\frac{3}{2} - 2 \sum_{k=1}^{n} \frac{1}{k} \right) \ln \frac{Q^2}{4\pi {\mu_E}^2} \right) \right\} \tag{77}$$

Structures with $p^\mu p^\nu$ cancel each other and the sum is expressed in terms of two gauge invariant tensor structures: $d^{\mu\nu}$ (defined above) and

$$e^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}. \tag{78}$$

And the final result is given by

$$\frac{i\,g^2}{4\pi^2} \sum_{n=2,4...} f_n \omega_B{}^n \left\{ e^{\mu\nu} C^e{}_n + d^{\mu\nu} C^d{}_n \right\}, \tag{79}$$

with moments of two coefficient functions

$$\begin{aligned} C^e{}_n &= -\frac{2}{n+1} \\ C^d{}_n &= 2\sum_{k=1}^{n} \frac{1}{k^2} - 2\sum_{k=1}^{n} \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j} + \left(\frac{1}{n(n+1)} - \frac{3}{2} \right) \sum_{k=1}^{n} \frac{1}{k} + \frac{9}{2} - \frac{3}{2n} \\ &\quad - \frac{1}{n^2} - \frac{2}{n+1} - \left(\frac{1}{n(n+1)} + \frac{3}{2} - 2\sum_{k=1}^{n} \frac{1}{k} \right) \ln \frac{Q^2}{4\pi {\mu_E}^2}. \end{aligned} \tag{80}$$

The notation $\ln\frac{Q^2}{4\pi {\mu_E}^2}$ is a shorthand for $\gamma_E + \ln\frac{Q^2}{4\pi\mu^2}$.

This result exactly reproduces the one obtained in the paper by Bardeen *et al.* [9].

See appendix B for some of less obvious formulas which have been derived for the above calculations.

6.2 Nonforward Process

As another example, let us consider a non-forward process. For the sake of simplicity here we restrict ourselves to only one tensor structure, $g^{\mu\nu}$.

To extract the $g^{\mu\nu}$-term it is enough to use just the following simplified parametrization for the matrix element.

$$\begin{aligned} \langle p'|\ \bar{\psi}(uz)\hat{z}\psi(\bar{v}z) - \bar{\psi}(\bar{u}z)\hat{z}\psi(vz)\ |p\rangle &= \\ (p+p')\cdot z \int_0^1 dx \int_0^{\bar{x}} dy F(x,y) \left(e^{i l_1 z} - e^{i l_2 z}\right), \end{aligned} \qquad (81)$$

with

$$\begin{aligned} l_1 &= xp\,(1-u-v) - r\,(\bar{u} - y(1-u-v)) \\ l_2 &= -xp\,(1-u-v) - r\,(u + y(1-u-v)). \end{aligned}$$

and

$$F(x,y) \equiv F_a(x,y) + F_{\bar{a}}(x,y). \qquad (82)$$

The contribution of the two box diagrams to the $g^{\mu\nu}$−term is then

$$\begin{aligned} &\left[g^{\mu\nu} + \frac{2z^\mu z^\nu}{(-z^2)}\right] \frac{(p+p')\cdot z}{(-z^2)^2} \left(e^{i l_1 z} - e^{i l_2 z}\right) - \\ &\quad - g^{\mu\nu} \frac{(p+p')\cdot z \ \ln z^2 {\mu_1}^2}{(-z^2)^2} \left(e^{i l_1 z} - e^{i l_2 z}\right). \end{aligned} \qquad (83)$$

The integration over t is exactly canceled by the effect of $(z\cdot\partial)$ operator: it can be replaced with $(t\cdot\frac{\partial}{\partial t})$ and integrated by parts. Note that this is not true for the general case, $z^\alpha\partial^\beta$.

Now we should perform the Fourier transformation $\int d^4z\ e^{-i\ qz}$ of the last expression.

The relevant integrals are

$$\int d^4z\ e^{ikz}\frac{z^\alpha \ln(-z^2\mu^2+i\varepsilon)}{(-z^2+i\varepsilon)^2}=\frac{2\pi^2 k^\alpha}{-k^2-i\varepsilon}\left(1-\ln\left(\frac{-k^2}{4\mu^2}-i\varepsilon\right)-2\gamma_E\right), \quad (84)$$

$$\int d^4z\ e^{ikz}\frac{z^\alpha}{(-z^2+i\varepsilon)^2}=\frac{2\pi^2 k^\alpha}{-k^2-i\varepsilon}, \quad (85)$$

$$\int d^4z\ e^{ikz}\frac{z^\alpha z^\mu z^\nu}{(-z^2+i\varepsilon)^3}=$$
$$-\frac{\pi^2}{2}\left(\frac{k^\alpha g^{\mu\nu}+k^\mu g^{\alpha\nu}+k^\nu g^{\alpha\mu}}{-k^2-i\varepsilon}+\frac{2k^\alpha k^\mu k^\nu}{(-k^2-i\varepsilon)^2}\right). \quad (86)$$

We introduce the same notations we previously used for tree-level non-forward case (see chapter 3). The skewedness ζ (coincides with the Bjorken variable):

$$\zeta = x_{Bj}\equiv\frac{-q^2}{2(pq)}, \quad (87)$$

and a special combination X of the variables x and y :

$$X\equiv x+\zeta y, \quad (88)$$

Integration over u and v results for the two box diagrams in

$$g^{\mu\nu}\frac{i\,g^2}{8\pi^2}\frac{(p+p')\cdot q}{2(pq)}\int_0^1 dx\int_0^{\bar{x}} dy\ F(x,y)\left\{S_1(X,\zeta)\ln\frac{2(pq)}{4\pi\mu_E{}^2}+S_2(X,\zeta)\right\}, \quad (89)$$

where

$$S_1(X,\zeta)=\frac{\ln X-\ln\zeta}{X-\zeta}+\frac{\ln(X-\zeta+i\varepsilon)-\ln(-\zeta+i\varepsilon)}{X} \quad (90)$$

$$S_2(X,\zeta) = \frac{\ln^2(X-i\varepsilon)-\ln^2(\zeta-i\varepsilon)}{2(X-\zeta)} + \\ + \frac{\ln^2(\zeta-X-i\varepsilon)-\ln^2(\zeta-i\varepsilon)}{2X}. \quad (91)$$

The calculation for the remaining three diagrams is quite similar. The only difference is that the non-trivial kernel gives many different integrals over u and v some of which can not be integrated in terms of elementary functions (All these integrals are listed in the appendix C). These special contributions (dilogarithm function) however cancel each other and the result reads

$$g^{\mu\nu}\,\frac{i\,g^2}{8\pi^2}\frac{(p+p')\cdot q}{2(pq)}\int_0^1 dx\int_0^{\bar{x}} dy\;F(x,y)\times \\ \times\left\{\ln\frac{2(pq)}{4\pi\mu_E{}^2}\;S_3(X,\zeta)+S_4(X,\zeta)\right\}. \quad (92)$$

with

$$S_3(X,\zeta) = \frac{3}{2}\left(\frac{1}{X-i\varepsilon}+\frac{1}{X-\zeta+i\varepsilon}\right)+ \\ +\left(\frac{1}{X}-\frac{1}{X-\zeta+i\varepsilon}\right)\ln\frac{-X+i\varepsilon}{X-\zeta+i\varepsilon} \quad (93)$$

$$S_4(X,\zeta) = \frac{3}{2}\left(\frac{\ln(X-i\varepsilon)-3}{X-i\varepsilon}+\frac{\ln(\zeta-X-i\varepsilon)-3}{X-\zeta+i\varepsilon}\right)+ \\ \frac{\zeta}{2X}\frac{\ln^2(\zeta-X-i\varepsilon)-\ln^2(\zeta-i\varepsilon)}{X-\zeta+i\varepsilon}+\frac{\zeta}{2(X-\zeta)}\frac{\ln^2(\zeta-i\varepsilon)-\ln^2(X-i\varepsilon)}{X-i\varepsilon} \\ -\frac{3}{2}\left(\frac{1}{X-\zeta+i\varepsilon}-\frac{1}{X}\right)\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon}-\frac{3}{2}\left(\frac{1}{X-i\varepsilon}-\frac{1}{X-\zeta}\right)\ln\frac{X}{\zeta}. \quad (94)$$

Combining (89) and (92) the final result can be written as

$$g^{\mu\nu}\,\frac{i\,g^2}{8\pi^2}\frac{(p+p')\cdot q}{2(pq)}\int_0^1 dx\int_0^{\bar{x}} dy\;F(x,y)\times$$
$$\times\left\{\ln\frac{2(pq)}{4\pi{\mu_E}^2}\;\mathcal{S}_A(X,\zeta)+\mathcal{S}_B(X,\zeta)\right\}, \qquad (95)$$

where

$$\begin{aligned}
\mathcal{S}_A(X,\zeta) &= S_1(X,\zeta)+S_3(X,\zeta) = \\
&\frac{3}{2}\left(\frac{1}{X-i\varepsilon}+\frac{1}{X-\zeta+i\varepsilon}\right)+\frac{1}{X-i\varepsilon}\ln\frac{X-i\varepsilon}{\zeta-i\varepsilon} \\
&+\frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon}
\end{aligned} \qquad (96)$$

$$\begin{aligned}
\mathcal{S}_B(X,\zeta) &= S_2(X,\zeta)+S_4(X,\zeta) = \\
&\frac{3}{2(X-\zeta)}\ln\frac{X}{\zeta}+\frac{1}{2}\frac{\ln^2(\zeta-X-i\varepsilon)-\ln^2(\zeta-i\varepsilon)}{X-\zeta+i\varepsilon} \\
&+\frac{3}{2X}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon}+\frac{1}{2}\frac{\ln^2(X-i\varepsilon)-\ln^2(\zeta-i\varepsilon)}{X-i\varepsilon} \\
&+3\left(\frac{1}{X-i\varepsilon}+\frac{1}{X-\zeta+i\varepsilon}\right)\frac{\ln(\zeta-i\varepsilon)-3}{2}.
\end{aligned} \qquad (97)$$

Note that both $\mathcal{S}_A(X,\zeta)$ and $\mathcal{S}_B(X,\zeta)$ have poles at $X=\zeta$ and $X=0$. Therefore, even for the real double distribution function F, the amplitude of the process would be complex. Because its real and imaginary parts have different physical meaning it would be instructive to calculate them. To do it, we treat the amplitude as a functional of the distribution function F and our coefficient functions $\mathcal{S}_A(X,\zeta)$ and $\mathcal{S}_B(X,\zeta)$ as generalized functions. See Appendix C for details of this calculation. The real and imaginary parts of the coefficient functions (and of the amplitude as long as F is real) are given by

$$\begin{aligned}
\mathbf{Re}\, \mathcal{S}_A(X,\zeta) &= \frac{3}{2}\left(\frac{1}{X} + P\frac{1}{X-\zeta}\right) + \frac{1}{X}\ln\frac{X}{\zeta} + P\frac{\ln|X-\zeta| - \ln\zeta}{X-\zeta} - \\
&\quad -\frac{\pi^2}{2}\delta(X-\zeta) \\
\frac{1}{\pi}\mathbf{Im}\, \mathcal{S}_A(X,\zeta) &= \delta(X-\zeta)\ln\frac{\zeta}{1-\zeta} - \left[\frac{\theta(X-\zeta)}{X-\zeta}\right]_+ - \frac{3}{2}\delta(X-\zeta)
\end{aligned} \tag{98}$$

$$\begin{aligned}
\mathbf{Re}\;\; \mathcal{S}_B(X,\zeta) = {} & \frac{3}{2}\left(\frac{1}{X} + P\frac{1}{X-\zeta}\right)(\ln\zeta - 3) + \frac{3}{2X}\ln\frac{|X-\zeta|}{\zeta} \\
& + \frac{3}{2\,(X-\zeta)}\ln\frac{X}{\zeta} + P\,\frac{\ln^2|X-\zeta| - \ln^2\zeta}{2(X-\zeta)} \\
& - \frac{\pi^2}{2}\left[\frac{\theta(X-\zeta)}{X-\zeta}\right]_+ - \frac{\pi^2}{2}\ln(1-\zeta)\,\delta(X-\zeta) \\
& + \frac{\ln^2 X - \ln^2\zeta}{2X} \\
\frac{1}{\pi}\mathbf{Im}\;\; \mathcal{S}_B(X,\zeta) = {} & \frac{3}{2}\left(\delta(X-\zeta)\,(\ln\zeta - 3) - \frac{1}{X}\theta(X-\zeta)\right) \\
& - \left[\frac{\theta(X-\zeta)\ln|X-\zeta|}{X-\zeta}\right]_+ + \frac{\pi^2}{6}\delta(X-\zeta) \\
& + \frac{1}{2}\delta(X-\zeta)\left(\ln^2\zeta - \ln^2(1-\zeta)\right).
\end{aligned} \tag{99}$$

Part II

Gauge Invariant Compton Amplitude

Chapter 7

Symmetric Parameterization

7.1 Gauge Invariance of Nonforward Compton Amplitude

There have been numerous studies of different parametrizations of off-diagonal matrix elements (see [31],[32],[34],[33]). Our objective is to obtain full explicitly gauge invariant amplitude for Deeply Virtual Compton Scattering (DVCS). We start working in the process independent coordinate representation and modify the existing parameterization for the non-forward matrix element in such a way that it explicitly reveals gauge invariance of the corresponding process. Then we perform transformation to the momentum space and calculate the DVCS amplitude.

If we take the result (22) for the DVCS amplitude derived in the $p^2 = p'^2 = r^2 = 0$ limit:

$$T_0^{\mu\nu}(q,p) =$$

$$\frac{i\,\bar{u}\hat{q}u}{2(pq)^2}\left\{q'^{\mu}p^{\nu}+q'^{\nu}p^{\mu}-g^{\mu\nu}(pq')\right\}\times$$
$$\times\int_0^1 dx\int_0^{\bar{x}} dy\; F(x,y)\left[\frac{1}{X-\zeta}+\frac{1}{X}\right]+$$
$$\varepsilon^{\mu\nu\alpha\beta}\frac{p^{\alpha}q'^{\beta}}{(pq')}\,\bar{u}\gamma_5\hat{q}u\int_0^1 dx\int_0^{\bar{x}} dy\; G(x,y) \tag{100}$$

and try to extend it to general nonforward kinematics case when $r^2 \neq 0$, we can see that the amplitude satisfies the EM gauge invariance condition with respect to final photon, $q'^{\nu}T^{\mu\nu}=0$. Though $q'^{\mu}T^{\mu\nu}$ also vanishes, the EM gauge invariance requires $q^{\mu}T^{\mu\nu}=0$ which is satisfied only if $r^{\mu}T^{\mu\nu}=0$. However, for symmetric part of $T^{\mu\nu}$ we have

$$r^{\mu}T^{\mu\nu}_{sym}\sim(A^{\nu}r^2+B(\zeta p^{\nu}-r^{\nu})). \tag{101}$$

Note that $\zeta p^{\nu}-r^{\nu}\sim r^{\nu}_{\perp}+O(r^2)$. Hence, the amplitude is gauge invariant only with $O(r^2)$ accuracy, i.e. the correct gauge invariant amplitude should contain extra $O(r^2)$ terms, possibly, structures different from those present in (100).

These terms originate from two sources. First, one should write down all the terms which appear in Eq.(17), including $O(m_p^2)$ and $O(r^2)$ ones. Second, one should use a more accurate parametrization for the nonforward matrix element

$$\langle p'|\;\bar{\psi}(0)\hat{z}\psi(z)-\bar{\psi}(z)\hat{z}\psi(0)\;|p\rangle\;=$$
$$\bar{u}(p')\hat{z}u(p)\int_0^1 dx\int_0^{\bar{x}} dy\,F(x,y)\left(e^{ix(pz)-i\bar{y}(rz)}-e^{-ix(pz)-iy(rz)}\right), \tag{102}$$

including with it subleading $O(z^2)$ and $O(z^4)$ terms in the right hand side. These terms also produce $O(r^2/Q^2)$ corrections.

The necessity of having $O(z^2)$ terms in the right hand side of (102) can be seen from the fact that the left hand side of this equation satisfies D'Alembert equation:

$$\Box_z \langle p'| \bar{\psi}(0)\hat{z}\psi(z) - \bar{\psi}(z)\hat{z}\psi(0) |p\rangle = 0 \tag{103}$$

due to equation of motion $D\psi = 0$. However, applying $\Box_z$ operator to the right hand side one gets the terms containing $(xp + yr)^2$ and $(xp - \bar{y}r)^2$ which vanish only if $p^2 = r^2 = (pr) = 0$.

Note that $k_1 = xp + yr$ and $k_2 = xp - \bar{y}r$ play the role of the parton momenta. Therefore, the restriction $k_1^2 = 0$, $k_2^2 = 0$ just states that quarks (partons) are on-shell. As it is well-known, the on-shellness of external particles is a crucial element in proof of EM gauge invariance of the scattering amplitude. If the external lines are off-shell, the amplitude in general is not gauge invariant. This implies that if we want to construct a gauge invariant expression for DVCS (or any other nonforward) amplitude, we should require that parametrization of nonforward matrix element satisfies D'Alembert equation. As we will see below, this can be reached by adding $O(z^2)$, $O(z^4)$ etc terms in the right hand side of relevant parametrization. Fortunately, it is not necessary to compute the whole series of $O(z^n)$ corrections. Since, the Born amplitude has $\sim \frac{1}{z^2}$ structure, only $O(z^2)$ part of operator (matrix element) contribution should be included: the next powers of z^2 would cancel the quark propagator singularity (z^2 in the denominator) resulting in the $\delta^4(q - xp - yr)$ terms which produce zero contribution.

7.2 Modified Parametrization

The starting point for our analysis is the following symmetric parameterization of the non-forward matrix element of the flavor-singlet operator (see [34] and

[44]) *

$$\Big\langle p-r|\ \bar\psi(\frac{Z}{2})\hat Z\psi(-\frac{Z}{2}) - \bar\psi(-\frac{Z}{2})\hat Z\psi(\frac{Z}{2})\ |p\Big\rangle =$$
$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha\, e^{-ix(PZ)-i\frac{\alpha}{2}(rZ)} \times$$
$$\Big\{\bar u(p-r)\hat Z u(p) f(x,\alpha) + \bar u(p-r)u(p)2(PZ)h(x,\alpha)\Big\}, \qquad (104)$$

with P being the average nucleon momentum, $2P = p - r + p$.

We modify it by adding extra terms with the next powers of Z^2 to each distribution: †

$$\Big\langle p-r|\ \bar\psi(\frac{Z}{2})\hat Z\psi(-\frac{Z}{2}) - \bar\psi(-\frac{Z}{2})\hat Z\psi(\frac{Z}{2})\ |p\Big\rangle =$$
$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha\, e^{-ix(PZ)-i\frac{\alpha}{2}(rZ)} \Bigg\{ \bar u(p-r)\hat Z u(p)\left[f(x,\alpha) + \frac{Z^2}{4}f_2 + \frac{Z^4}{32}f_4\right]$$
$$+ \bar u(p-r)u(p)\left[2(PZ)h(x,\alpha) - i\frac{Z^2}{2}h_2 - i\frac{Z^4}{16}h_4\right]\Bigg\}. \qquad (105)$$

New functions (f_2, f_4, h_2 and h_4) keep many properties of the original symmetric distributions.

In particular, the support area for both original and modified distribution functions is the same. It is shown on the Fig.6.

*As argued by M. Polyakov and C. Weiss [49], it makes sense to write the (PZ)-independent terms as a separate integral over a single variable α rather than to include them into a singular part of distribution function. Due to the fact that this integrals is an additive term all analysis below remains valid.

†The numerical coefficients $\frac{1}{32}$, $\frac{i}{16}$, *etc.* have been chosen for the sake of simplicity of the amplitude in the momentum representation.

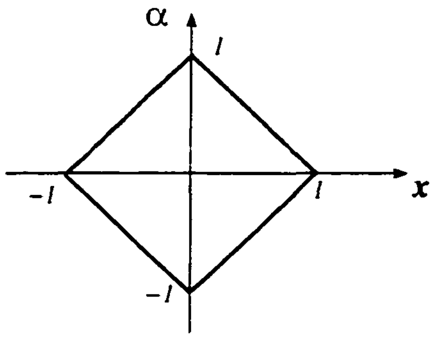

FIG. 6: Support area for distribution functions.

The double distribution functions f and h (and f_2, h_2 etc) obey the following boundary conditions

$$
\begin{aligned}
f(x, 1-|x|) = f(x, -1+|x|) &= 0 \\
h(x, 1-|x|) = h(x, -1+|x|) &= 0.
\end{aligned}
\tag{106}
$$

Due to hermiticity and time-reversal invariance of non-forward matrix elements, the double distributions are even functions of α :

$$
\begin{aligned}
f(x,\alpha) &= f(x,-\alpha) \\
h(x,\alpha) &= h(x,-\alpha),
\end{aligned}
\tag{107}
$$

and (for the flavor-singlet case) are odd functions of x :

$$
\begin{aligned}
f(x,\alpha) &= -f(-x,\alpha) \\
h(x,\alpha) &= -h(-x,\alpha).
\end{aligned}
\tag{108}
$$

Other properties of symmetric double distributions were discussed in [44].

The operator

$$\hat{O}\left(-\frac{Z}{2},\frac{Z}{2}\right) = \bar{\psi}(\frac{Z}{2})\hat{Z}\psi(-\frac{Z}{2}) \tag{109}$$

can be expanded in series in powers of Z^2 :

$$\hat{O} = \hat{O}^{\text{twist}-2} + Z^2\hat{O}^{\text{twist}-4} + \dots \tag{110}$$

Each term in this expansion possesses certain *twist* (twist of the operator is its dimension minus its spin)(see [10] - [18]). For the Compton amplitude in the momentum representation this corresponds to the expansion in the inverse powers of the large parameter Q^2 (for the case of the DVCS $q^2 = -Q^2$ is the momentum of the virtual photon).

If the functions parametrizing contributions due to different twists to Compton amplitude are dynamically independent, contributions for each particular twist should be gauge invariant. We will consider the leading-twist contribution, i.e. assume that matrix elements of higher twist operators vanish.

So that when one uses parametrization (104) or similar to calculate Compton scattering amplitude, the result of this calculation is usually rather long, and the task of demonstrating its gauge invariance is quite complicated.

The central idea beyond the modified parametrization (105) is that the new functions $f_2(x,\alpha)$, $h_2(x,\alpha)$, $f_2(x,\alpha)$ and $h_4(x,\alpha)$ are not independent but rather to be specified in such a way that the right hand side of Eq.(105) have only terms of the certain twist. As a result the Compton amplitude will be explicitly gauge invariant.

The condition

$$\Box_z \left\langle p - r \right| \bar{\psi}(\frac{Z}{2})\hat{Z}\psi(-\frac{Z}{2}) \left| p \right\rangle^{\text{twist}-2} = 0 \tag{111}$$

cuts out terms of the leading twist. Imposing this condition on the right hand side of parametrization (105) we will determine the extra functions (f_2, f_4, h_2 and h_4).

At first glance it may seem that it would be enough to restrict ourselves to the Z^2 order (f_2, h_2) in the modified parametrization (105). It indeed would be true if all we have to deal with were matrix element of $\bar{\psi}(\frac{Z}{2})\hat{Z}\psi(-\frac{Z}{2})$ operator. However (due to its tensor structure) the expression for the Compton scattering process also contains derivatives of this operator. Hence the next order (Z^4) terms should be included. We will return to this issue in more detail in chapter (9).

In the lowest order in Z^2, the condition (111) leads to the system of partial differential equations for functions f_2 and h_2:

$$\begin{cases} \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}\right) h_2(x,\alpha) = Mx\ f(x,\alpha) - \left(Px + r\frac{\alpha}{2}\right)^2 \frac{\partial h(x,\alpha)}{\partial x} \\ \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha} - 1\right) f_2(x,\alpha) = -\left(Px + r\frac{\alpha}{2}\right)^2 f(x,\alpha). \end{cases} \tag{112}$$

In the next order in Z^2, the condition (111) gives a similar pair of partial differential equations for functions f_4 and h_4:

$$\begin{cases} \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha} - 1\right) h_4(x,\alpha) = Mx\ f_2(x,\alpha) - \left(Px + r\frac{\alpha}{2}\right)^2 h_2(x,\alpha) \\ \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha} - 2\right) f_4(x,\alpha) = -\left(Px + r\frac{\alpha}{2}\right)^2 f_2(x,\alpha). \end{cases} \tag{113}$$

These equations can be solved. The solutions for the first pair f_2 and h_2 are

$$\begin{cases} h_2(x,\alpha) = \int_1^\infty dt \left[\left(Px + r\frac{\alpha}{2}\right)^2 t \frac{\partial h}{\partial x}(tx, t\alpha) - Mxf(tx, t\alpha) \right] \\ f_2(x,\alpha) = \left(Px + r\frac{\alpha}{2}\right)^2 \int_1^\infty dt\ f(tx, t\alpha). \end{cases} \tag{114}$$

For the second system of equations the solutions for functions f_4 and h_4 are

$$\begin{cases} h_4(x,\alpha) = \int_1^\infty dt \left[\left(Px + r\frac{\alpha}{2}\right)^2 h_2(tx, t\alpha) - \frac{Mx}{t} f_2(tx, t\alpha) \right] \\ f_4(x,\alpha) = \left(Px + r\frac{\alpha}{2}\right)^2 \int_1^\infty \frac{dt}{t}\ f_2(tx, t\alpha). \end{cases} \tag{115}$$

Here we note the fact that while the functions $f_2(x,\alpha)$ and $f_4(x,\alpha)$ depend only on one original distribution $f(x,\alpha)$, the functions $h_2(x,\alpha)$ and $h_4(x,\alpha)$, which correspond to the scalar structure $\bar{u}u$ in our parametrization, are derived from both $f(x,\alpha)$ and $h(x,\alpha)$.

Chapter 8

Kinematics of the Process

For the purpose of building a gauge invariant amplitude we found that it is more convenient to express the amplitude in terms of the average photon (Q) and nucleon (P) momenta rather than in terms of the initial and final momenta.

The main reason for that is when the amplitude is expressed in terms of these momenta the gauge invariance conditions for the incoming and outgoing photons would be similar and we can take full advantage of this symmetry.

We adopt the notations according to figure 7.

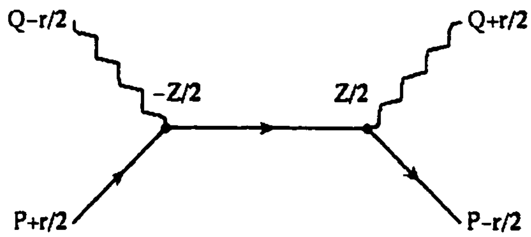

FIG. 7: Kinematics of the process.

In general, for n independent vectors the amplitude is a function of $(n^2 + n)/2$ independent scalar variables. In our case, there are four momenta but only three of them are independent due to the energy-momentum conservation. Thus the formula gives 6 independent invariants.

Moreover, the on-shell conditions for the outgoing photon momentum and both initial and final nucleon momenta effectively reduce the number of dynamic variables to 3.

Any three non-vanishing variables can be used. For our purposes we choose r^2, (QP) and (Qr) to be the dynamic variables.

From on-shell conditions for the initial and final nucleons (respectively)

$$\left(P + \frac{r}{2}\right)^2 = m^2 \tag{116}$$

$$\left(P - \frac{r}{2}\right)^2 = m^2 \tag{117}$$

and from the outgoing photon on-shell condition

$$\left(Q + \frac{r}{2}\right)^2 = 0 \tag{118}$$

we express the remaining three variables

$$(Pr) = 0 \tag{119}$$

$$Q^2 = -(Qr) - \frac{r^2}{4} \tag{120}$$

$$P^2 = m^2 - \frac{r^2}{4}. \tag{121}$$

The Dirac equation gives two more conditions on the spinor "sandwiches" of momenta P and r :

$$\bar{u}\hat{P}u = m\bar{u}u \tag{122}$$

and

$$\bar{u}\hat{r}u = 0. \tag{123}$$

Chapter 9

Fourier Transformation

Once the parametrization of the matrix element is set, the next step of our analysis is to calculate the Compton amplitude in the more traditional momentum representation by performing an appropriate Fourier transformation. The latter has direct physical meaning since particle physics experiments measure either energy or momentum of the interacting particles.

If p and $p' = p - r$ are the initial and final nucleon momenta and q is the incoming photon momentum (see figure 7) we can re-express the Fourier transformation (see [54],[50]) in terms of our variables as follows

$$\int d^4Z e^{-i(qZ)} \left\langle p - r | \ \bar{\psi}(0)\hat{Z}\psi(Z) - \bar{\psi}(Z)\hat{Z}\psi(0) \ |p \right\rangle =$$
$$\int d^4Z e^{-i(qZ)} e^{-i(r\frac{Z}{2})} \left\langle p - r | \ \bar{\psi}(\frac{Z}{2})\hat{Z}\psi(-\frac{Z}{2}) - \bar{\psi}(-\frac{Z}{2})\hat{Z}\psi(\frac{Z}{2}) \ |p \right\rangle =$$
$$\int d^4Z e^{-i(QZ)} \left\langle P - \frac{r}{2} | \ \bar{\psi}(\frac{Z}{2})\hat{Z}\psi(-\frac{Z}{2}) - \bar{\psi}(-\frac{Z}{2})\hat{Z}\psi(\frac{Z}{2}) \ |P + \frac{r}{2} \right\rangle . \quad (124)$$

For the sake of simplicity, we are going to restrict ourselves to the symmetric part of the amplitude, $T^{\mu\nu}_{sym}$.

As it was shown in chapter (3) of this thesis, the tree-level diagrams for Compton amplitude in the coordinate representation give rise to the following expression:

$$T^{\mu\nu}(Z) = i\,\Gamma\left(\frac{d}{2}\right)\frac{\bar{\psi}(\frac{Z}{2})\gamma^{\mu}\hat{z}\gamma^{\nu}\psi(-\frac{Z}{2})}{2\pi^{d/2}(-Z^2)^{d/2}} - i\,\Gamma\left(\frac{d}{2}\right)\frac{\bar{\psi}(-\frac{Z}{2})\gamma^{\nu}\hat{z}\gamma^{\mu}\psi(\frac{Z}{2})}{2\pi^{d/2}(-Z^2)^{d/2}}. \quad (125)$$

By means of formulas (9) and (10), the symmetric part of the last expression can be written in terms of the non-local string light-ray operators.

The Fourier transformation to the momentum space reads (we put d to be equal to 4)

$$T^{\mu\nu}_{sym}(Q,P,r) = \frac{i}{2\pi^2}\int d^4Z\frac{e^{-i(QZ)}}{(-Z^2)^2}\left\{ z^{\nu}\partial^{\mu} + z^{\mu}\partial^{\nu} - g^{\mu\nu}z^{\xi}\partial_{\xi}\right\}\int_0^1 d\lambda \left\langle P-\frac{r}{2}\right|\bar{\psi}(\lambda\frac{Z}{2})\hat{Z}\psi(-\lambda\frac{Z}{2}) - \bar{\psi}(-\lambda\frac{Z}{2})\hat{Z}\psi(\lambda\frac{Z}{2})\left|P+\frac{r}{2}\right\rangle. \quad (126)$$

To proceed, we substitute our modified parametrization of the hadronic matrix element (105) into the last equation and calculate the derivative over Z^{α}:

$$\begin{aligned}T^{\mu\nu}_{sym}(Q,P,r) &= \frac{i}{2\pi^2}\int d^4Z\frac{e^{-iQZ}}{(-Z^2)^2}\left\{ g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\sigma\rho}\right\}Z^{\rho}\\ &\int_{-1}^{1}dx\int_{-1+|x|}^{1-|x|}d\alpha\int_0^1\frac{d\lambda}{\lambda}e^{-i\lambda Z(Px+r\alpha/2)}\\ &\left[\bar{u}\gamma^{\sigma}u\left(f+\frac{\lambda^2Z^2}{4}f_2\right) + \bar{u}\hat{Z}u\,Z^{\sigma}\left(\frac{\lambda^2}{2}f_2 + \frac{\lambda^4Z^2}{8}f_4\right) + \right.\\ &\quad\bar{u}u\left(2P^{\sigma}h - iZ^{\sigma}\lambda h_2 - iZ^{\sigma}\frac{\lambda^3Z^2}{4}h_4\right) + \\ &\left. i\,\bar{u}\hat{Z}u\,b^{\sigma}\left(f+\frac{\lambda^2Z^2}{4}f_2\right) + i\,\bar{u}u\,b^{\sigma}\left(2(PZ)\,h - i\frac{\lambda Z^2}{2}h_2\right)\right]. \quad (127)\end{aligned}$$

We dropped all the terms with the powers greater than Z^4 in the numerator because we have just Z^4 in the denominator. Now it becomes clear why we had to include both Z^2 and Z^4 terms in the parametrization: the presence of the derivative in the equation (126).

Fourier transformation involves six different integrals over Z :

$$\int d^4Z \frac{Z^\beta e^{i(kZ)}}{(-Z^2)^2} = -\frac{2\pi^2 k^\beta}{k^2} \tag{128}$$

$$\int d^4Z \frac{Z^\beta e^{i(kZ)}}{(-Z^2)} = \frac{8\pi^2 k^\beta}{k^4} \tag{129}$$

$$\int d^4Z \frac{Z^\alpha Z^\beta e^{i(kZ)}}{(-Z^2)^2} = 2i\pi^2 \left[\frac{g^{\alpha\beta}}{k^2} - 2\frac{k^\alpha k^\beta}{k^4} \right] \tag{130}$$

$$\int d^4Z \frac{Z^\alpha Z^\beta e^{i(kZ)}}{(-Z^2)} = 8i\pi^2 \left[4\frac{k^\alpha k^\beta}{k^6} - \frac{g^{\alpha\beta}}{k^4} \right] \tag{131}$$

$$\int d^4Z \frac{Z^\alpha Z^\beta Z^\gamma e^{i(kZ)}}{(-Z^2)^2} = 4\pi^2 \left[4\frac{k^\alpha k^\beta k^\gamma}{k^6} - \frac{g^{\alpha\beta}k^\gamma + g^{\alpha\gamma}k^\beta + g^{\beta\gamma}k^\alpha}{k^4} \right] \tag{132}$$

$$\int d^4Z \frac{Z^\alpha Z^\beta Z^\gamma e^{i(kZ)}}{(-Z^2)} = 32\pi^2 \left[\frac{g^{\alpha\beta}k^\gamma + g^{\alpha\gamma}k^\beta + g^{\beta\gamma}k^\alpha}{k^6} - \frac{k^\alpha k^\beta k^\gamma}{k^8} \right]. \tag{133}$$

Basically, we just need to substitute the above integrals into our equation (127) for the Compton amplitude. However, at this point the expressions become very long and complicated. To proceed further, we take advantage of modern system of symbolic computation.

For this purpose, we used the symbolic processing language REDUCE which was found to be very useful for both time efficient and error-proof calculations. There is also a special package for calculations with the Dirac gamma matrices in REDUCE. The long output of REDUCE program was ported to the simple Perl (Practical Extraction and Report Language) program which transformed it directly into the LATEX format.

Chapter 10

Gauge Invariance

10.1 Representation in Terms of Internal Momenta k and b

Our goal is to obtain the gauge invariant Compton scattering amplitude expressed in terms of external momenta P, r and Q. All the data measured in a Compton scattering experiment are some combinations of these momenta and their quadratic forms.

However we found it to be quite useful to perform an intermediate step - express the amplitude in terms of two "internal" momenta, k and b.

These momenta are defined as follows:

$$\begin{aligned} b^{\beta} &= -\lambda\left(xP^{\beta} + \frac{\alpha}{2}r^{\beta}\right) \\ k^{\beta} &= b^{\beta} - Q^{\beta}. \end{aligned} \tag{134}$$

Written in terms of these vectors, the expression for the amplitude is relatively short. This allows us to simplify the analysis of its gauge invariance.

This demonstrate another advantage of using a program for the symbolic calculations: instead of redoing all the algebraic manipulations we just need to change one REDUCE operator. Moreover, calculation of the contraction of the amplitude and the average photon momenta Q^μ can be done in the same simple fashion.

The expression for the amplitude is

$$\frac{i}{2\pi^2}\int_0^1 d\lambda \int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha \left(\bar{u}u\mathcal{T}_u^{\mu\nu} + \mathcal{T}_\gamma^{\mu\nu} \right) \tag{135}$$

The scalar part, $\mathcal{T}_u^{\mu\nu}$, is a coefficient of $\bar{u}u$ term and depends on both $f(x,\alpha)$ and $h(x,\alpha)$ functions. It reads

$$\begin{aligned}
\mathcal{T}_u^{\mu\nu} = \\
&-4k^\mu k^\nu \frac{\lambda}{k^8}\left(2f_2k^2m\lambda^2x + 6f_4m\lambda^4x + h_2k^4 + 2h_4k^2\lambda^2\right)\\
&-(b^\mu k^\nu + k^\mu b^\nu)\frac{2}{k^6}\left(fk^2m\lambda x + 2f_2m\lambda^3x - 2hk^2(Pk) + h_2k^2\lambda\right)\\
&-2(b^\mu P^\nu + P^\mu b^\nu)\frac{h}{k^2}\\
&-2(k^\mu P^\nu + P^\mu k^\nu)\frac{h}{k^2}\\
&+g^{\mu\nu}\frac{2}{k^6}\Big(fk^2m\lambda x\left((bk) - k^2\right) + f_2m\lambda^3x\left(2(bk) - k^2\right) + 2f_4m\lambda^5x\\
&\quad +hk^2\left(k^2(Pb) + k^2(Pk) - 2(bk)(Pk)\right) + h_2k^2(bk)\lambda + h_4k^2\lambda^3\Big).
\end{aligned} \tag{136}$$

The other structure, $\mathcal{T}_\gamma^{\mu\nu}$, contains terms with gamma matrices $(\bar{u}\gamma u)$ and depends only on the distribution $f(x,\alpha)$:

$$\begin{aligned}
\mathcal{T}_\gamma^{\mu\nu} = \\
&-8k^\mu k^\nu \frac{\bar{u}\hat{q}u\lambda^2}{k^8}(f_2k^2 + 3f_4\lambda^2) - 2(b^\mu k^\nu + k^\mu b^\nu)\frac{\bar{u}\hat{q}u}{k^6}(fk^2 + 2f_2\lambda^2)
\end{aligned}$$

$$+g^{\mu\nu}\frac{\bar{u}\hat{q}u}{k^6}\left(fk^2\left(2(bk)-k^2\right)+f_2\lambda^2\left(4(bk)-k^2\right)+4f_4\lambda^4\right)$$
$$-\bar{u}(b^\mu\gamma^\nu+b^\nu\gamma^\mu)u\,\frac{1}{k^4}(fk^2+f_2\lambda^2)$$
$$-\bar{u}(k^\mu\gamma^\nu+k^\nu\gamma^\mu)u\,\frac{1}{k^6}\left(fk^4+3f_2k^2\lambda^2+4f_4\lambda^4\right). \tag{137}$$

We note the fact that while the second amplitude $\mathcal{T}_\gamma^{\mu\nu}$, is indeed expressed only in terms of vectors k, b, their squares and their scalar product, the scalar one, $\mathcal{T}_u^{\mu\nu}$, depends also on the average nucleon momentum P. This is caused, of course, by the fact that this momentum was explicitly introduced in the scalar part of our parametrization (105).

10.2 Demonstration of Gauge Invariance

In our symmetric notations the standard condition of gauge invariance of the amplitude (see for example [50])

$$\begin{cases} q_\mu T^{\mu\nu}=0 \\ q'_\nu T^{\mu\nu}=0 \end{cases} \tag{138}$$

becomes

$$\begin{cases} \left(Q-\frac{r}{2}\right)^\mu T^{\mu\nu}=0 \\ \left(Q+\frac{r}{2}\right)^\nu T^{\mu\nu}=0. \end{cases} \tag{139}$$

For the symmetric part of the amplitude which we consider here it splits into two independent conditions on Q and r :

$$\begin{cases} Q^\mu T^{\mu\nu}_{sym}=0 \\ r^\nu T^{\mu\nu}_{sym}=0. \end{cases} \tag{140}$$

Below we consider $Q^\mu T^{\mu\nu}_{sym}$ condition in greater details.

The scalar part of this condition (a coefficient of $\bar{u}u$ term in $Q^\mu T^{\mu\nu}_{sym}$) is given by

$$\begin{aligned}
&Q^\nu \frac{2}{k^8}\Big[fk^4 m\lambda x\left(b^2-k^2\right) + f_2 k^2 m\lambda^3 x\left(2b^2+4(bk)-5k^2\right) \\
&\quad +2f_4 m\lambda^5 x\left(6(bk)-5k^2\right) \\
&\quad +hk^4\left(k^2(bP)-2b^2(Pk)+k^2(Pk)+k^2(pQ)\right) \\
&\quad +h_2 k^4\lambda\left(b^2+2(bk)-2k^2\right)+h_4 k^2\lambda^3\left(4(bk)-3k^2\right)\Big] + \\
&b^\nu \frac{2}{k^8}\Big[fk^4 m\lambda x\left(k^2-b^2\right)+2f_2 k^2 m\lambda^3 x\left(3k^2-b^2-2(bk)\right) \\
&\quad +12f_4 m\lambda^5 x\left(k^2-(bk)\right)+2hk^4\left(b^2(Pk)-k^2(bP)\right) \\
&\quad +h_2 k^4\lambda\left(3k^2-b^2-2(bk)\right)+4h_4 k^2\lambda^3\left(k^2-(bk)\lambda^3\right)\Big] + \\
&2p^\nu \frac{h}{k^2}\Big[k^2-b^2\Big]. \qquad (141)
\end{aligned}$$

The other part contains terms with gamma matrices. It reads

$$\begin{aligned}
&2Q^\nu \frac{\bar{u}\hat{Q}u}{k^8}\Big[fb^2k^4 + f_2 k^2\lambda^2\left(2b^2+4(bk)-3k^2\right) \\
&\qquad +12(bk)f_4\lambda^4 - 8f_4 k^2\lambda^4\Big] \\
&-2b^\nu \frac{\bar{u}\hat{Q}u}{k^8}\Big[fb^2k^4 + 2f_2 k^2\lambda^2\left(b^2+2(bk)-2k^2\right) \\
&\qquad +12(bk)f_4\lambda^4 - 10f_4 k^2\lambda^4\Big] \\
&-\frac{\bar{u}\gamma^\nu u}{k^6}\Big[fk^4\left(b^2-k^2\right)+f_2 k^2\lambda^2\left(b^2+2(bk)-3k^2\right) \\
&\qquad +4(bk)f_4\lambda^4 - 4f_4 k^2\lambda^4\Big]. \qquad (142)
\end{aligned}$$

To verify that all calculations were done correctly and the result is indeed gauge invariant we need to show that both expressions vanish when the distribution functions $f_2(x,\alpha)$ and $f_2(x,\alpha)$ satisfy (114) and (115), respectively.

As an example we consider the expression with $\gamma-$matrices. It contains three tensor structures which are linearly independent and hence every coefficient should be equal to zero.

Let us consider as an example the second one, containing b^ν :

$$2\frac{b^\nu}{k^8}\Big(b^2 f k^4 + 2b^2 f_2 k^2\lambda^2 + 4(bk) f_2 k^2\lambda^2 - \\ -4f_2 k^4\lambda^2 + 12(bk) f_4\lambda^4 - 10 f_4 k^2\lambda^4 \Big). \tag{143}$$

At this point, we derive the following formula

$$\left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial\alpha}\right)\frac{1}{(k^2)^n} = -2n\frac{(bk)}{(k^2)^{n+1}}. \tag{144}$$

which holds true due to the relation between vectors k and b. It will be crucial for our proof as it enables us to perform calculations in terms of composite vectors k and b without touching underlying true vectors P and r. This simplifies the resulting expressions significantly.

In chapter 7 we established the relations between the functions $f(x,\alpha)$, $f_2(x,\alpha)$ and $f_4(x,\alpha)$ and expressed the two latter through the integrals of the base function $f(x,\alpha)$. The most natural way to show that (143) is zero would be to substitute the expressions (114) and (115) into it.

However, it is usually easier to deal with differential expressions than with the integral ones. So, instead of expressing $f_2(x,\alpha)$ through $f(x,\alpha)$ we go backwards and express $f(x,\alpha)$ as a function of $f_2(x,\alpha)$ in the first term of (143) using (112):

$$2b^\nu\frac{fk^4b^2}{k^8} = 2b^\nu\frac{\lambda^2}{k^4}\left(1 - x\frac{\partial}{\partial x} - \alpha\frac{\partial}{\partial\alpha}\right) f_2(x,\alpha). \tag{145}$$

To get rid of partial derivatives, we recall that this expression is actually wrapped by integrals over x, α and λ. Integrating it by parts, we get

$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha\, 2b^{\nu} \frac{\lambda^2}{k^4} \left(1 - x\frac{\partial}{\partial x} - \alpha\frac{\partial}{\partial \alpha}\right) f_2(x,\alpha) =$$
$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha \left[2b^{\nu} f_2 \lambda^2 \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}\right) \frac{1}{k^4} \right.$$
$$\left. + 2f_2(x,\alpha)\frac{\lambda^2}{k^4}\left(1 + 1 + 1 + x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}\right) b^{\nu}\right]. \qquad (146)$$

There are no surface terms due to the boundary conditions [106].

Exploiting formula (144) the last expression can be rewritten as

$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha\; 8\lambda^2 b^{\nu} f_2(x,\alpha) \left(\frac{1}{k^4} - \frac{(bk)}{k^6}\right). \qquad (147)$$

Now we repeat the above procedure with f_2 : combine (147) and the terms with f_2 from (143) and express them through f_4 by means of (114)

$$2b^{\nu}\frac{f_2\lambda^2}{k^6}\left(4k^2 - 4(bk) + 4(bk) + 2b^2 - 4k^2\right) =$$
$$4b^{\nu}\frac{f_2\lambda^2 b^2}{k^6} = 4b^{\nu}\frac{\lambda^2 b^2}{k^6}\left(2 - x\frac{\partial}{\partial x} - \alpha\frac{\partial}{\partial \alpha}\right) f_4(x,\alpha). \qquad (148)$$

Once again we integrate it by parts over x and α :

$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha\, 4b^{\nu} \frac{\lambda^4}{k^6} \left(2 - x\frac{\partial}{\partial x} - \alpha\frac{\partial}{\partial \alpha}\right) f_4(x,\alpha) =$$
$$\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha \left[4b^{\nu} f_4 \lambda^4 \left(x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}\right) \frac{1}{k^6} \right.$$
$$\left. + 4f_4(x,\alpha)\frac{\lambda^4}{k^6}\left(4 + x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}\right) b^{\nu}\right]. \qquad (149)$$

Using formula (144) to calculate the partial derivatives, we note that the result exactly cancels the remaining (f_4) terms in (143):

$$4b^\nu \frac{f_4\lambda^4}{k^8}\left(5k^2 - 6(bk)\right) + 2b^\nu \frac{f_4\lambda^4}{k^8}\left(12(bk) - 10k^2\right) = 0. \tag{150}$$

Exactly the same calculations can be (and were) performed for every tensor structure in the projection (142). We found that it is indeed zero when the relations between the distribution functions are taken into account.

Similar calculations for the scalar structure of the projection (141) are a bit more complicated due to the presence of both f and h functions in it. Moreover, there exists an explicit P term in the scalar part of the parametrization (105), for this reason the second projection cannot be expressed in terms of just two vectors b and k. This fact is reflected in the equation for $h_2(x, \alpha)$: it contains not only a uniform differential operator $x\frac{\partial}{\partial x} + \alpha\frac{\partial}{\partial \alpha}$ but also a single partial derivative over x. Hence formula (144) is not sufficient for performing the integration by parts,re-expressing $h(x, \alpha)$ through $h_2(x, \alpha)$ and so on. To this end we derived another formula

$$\frac{\partial}{\partial x}\frac{1}{(k^2)^n} = 2n\lambda\frac{(Pk)}{(k^2)^{n+1}}, \tag{151}$$

which helps to demonstrate gauge invariance of the scalar structure.

Chapter 11

Resulting Compton Amplitude

The resulting expression for the explicitly gauge invariant Compton Scattering amplitude is

$$\frac{i}{2\pi^2}\int_0^1 d\lambda \int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} d\alpha \, \left(\bar{u}u\mathcal{T}_u^{\mu\nu} + \mathcal{T}_\gamma^{\mu\nu} \right) \tag{152}$$

The scalar part of the amplitude, $\mathcal{T}_u^{\mu\nu}(Q,P,r)$, is a coefficient of $\bar{u}u$ term and depends on both $f(x,\alpha)$ and $h(x,\alpha)$ distributions. It reads

$$\begin{aligned}
&\mathcal{T}_u^{\mu\nu}(Q,P,r) = \\
&\frac{g^{\mu\nu}}{4k^6}\Big[2fk^2m\lambda x(4m^2\lambda^2x^2 - 4k^2 + 4(QP)\lambda x + 2(Qr)\lambda\alpha + r^2\lambda^2(\alpha^2 - x^2)) \\
&\quad +4f_2m\lambda^3x(4m^2\lambda^2x^2 - 2k^2 + 4(QP)\lambda x + 2(Qr)\lambda\alpha + r^2\lambda^2(\alpha^2 - x^2)) \\
&\quad +16f_4m\lambda^5x + hk^2(-16k^2m^2\lambda x - 8k^2(QP) + 4k^2r^2\lambda x + 16m^4\lambda^3x^3 \\
&\quad +32m^2(QP)\lambda^2x^2 + 8m^2(Qr)\lambda^2x\alpha - 4m^2r^2\lambda^3x^3 + 4m^2r^2\lambda^3x(\alpha^2 - x^2) \\
&\quad +16(QP)^2\lambda x + 8(QP)(Qr)\lambda\alpha - 4(QP)r^2\lambda^2x^2 + 4(QP)r^2\lambda^2(\alpha^2 - x^2) \\
&\quad -2(Qr)r^2\lambda^2x\alpha - r^4\lambda^3x(\alpha^2 - x^2)) + 2h_2k^2(4m^2\lambda^3x^2 + 4(QP)\lambda^2x \\
&\qquad\qquad +2(Qr)\lambda^2\alpha + r^2\lambda^3(\alpha^2 - x^2)) + 8h_4k^2\lambda^3 \Big]
\end{aligned}$$

$$
\begin{aligned}
&-4Q^\mu Q^\nu \frac{\lambda}{k^8}\Big[2f_2k^2m\lambda^2x+6f_4m\lambda^4x+h_2k^4+2h_4k^2\lambda^2\Big]\\
&-2P^\mu P^\nu \frac{\lambda x}{k^8}\Big[2m\lambda^2x^2(fk^4+4f_2k^2\lambda^2+6f_4\lambda^4)+hk^4(4m^2\lambda^2x^2\\
&\qquad -4k^2+4(QP)\lambda x-r^2\lambda^2x^2)+4h_2k^4\lambda^2x+4h_4k^2\lambda^4x\Big]\\
&-r^\mu r^\nu \frac{\lambda^2\alpha^2}{2k^8}\Big[2fk^4m\lambda x+8f_2k^2m\lambda^3x+12f_4m\lambda^5x\\
&\qquad +hk^4(4m^2\lambda x+4(QP)-r^2\lambda x)+4h_2k^4\lambda+4h_4k^2\lambda^3\Big]\\
&-\Big(P^\mu Q^\nu+Q^\mu P^\nu\Big)\frac{1}{k^8}\Big[2m\lambda^2x^2(fk^4+6f_2k^2\lambda^2+12f_4\lambda^4)\\
&\qquad +hk^4(4m^2\lambda^2x^2-2k^2+4(QP)\lambda x-r^2\lambda^2x^2)\\
&\qquad +x(6h_2k^4\lambda^2+8h_4k^2\lambda^4)\Big]\\
&-\Big(P^\mu r^\nu+r^\mu P^\nu\Big)\frac{\lambda\alpha}{k^8}\Big[2m\lambda^2x^2(fk^4+4f_2k^2\lambda^2+6f_4\lambda^4)\\
&\qquad +hk^4(4m^2\lambda^2x^2-2k^2+4(QP)\lambda x+r^2\lambda^2x^2)\\
&\qquad +4x(h_2k^4\lambda^2+h_4k^2\lambda^4)\Big]\\
&-\Big(Q^\mu r^\nu+r^\mu Q^\nu\Big)\frac{\lambda\alpha}{2k^8}\Big[2fk^4m\lambda x+12f_2k^2m\lambda^3x+24f_4m\lambda^5x\\
&\qquad +hk^4(4m^2\lambda x+4(QP)-r^2\lambda x)+6h_2k^4\lambda+8h_4k^2\lambda^3\Big]. \qquad (153)
\end{aligned}
$$

The other structure, $\mathcal{T}_\gamma^{\mu\nu}(Q,P,r)$, contains terms with gamma matrices, $(\bar{u}\gamma u)$. This part of the amplitude depends only on the distribution $f(x,\alpha)$:

$$
\begin{aligned}
&\mathcal{T}_\gamma^{\mu\nu}(Q,P,r) \;=\\
&g^{\mu\nu}\frac{\bar{u}\hat{Q}u}{2k^6}\Big[fk^2(4m^2\lambda^2x^2-2k^2+4(QP)\lambda x+2(Qr)\lambda\alpha\\
&\qquad +r^2\lambda^2(\alpha^2-x^2))+2f_2\lambda^2(4m^2\lambda^2x^2-k^2+4(QP)\lambda x\\
&\qquad +2(Qr)\lambda\alpha+r^2\lambda^2(\alpha^2-x^2))+8f_4\lambda^4\Big]\\
&-8Q^\mu Q^\nu\frac{\bar{u}\hat{Q}u\,\lambda^2}{k^8}\Big[f_2k^2+3f_4\lambda^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
&-4P^\mu P^\nu \frac{\bar{u}\hat{Q}u\,\lambda^2 x^2}{k^8}\Big[fk^4+4f_2k^2\lambda^2+6f_4\lambda^4\Big]\\
&-r^\mu r^\nu \frac{\bar{u}\hat{Q}u\,\lambda^2\alpha^2}{k^8}\Big[fk^4+4f_2k^2\lambda^2+6f_4\lambda^4\Big]\\
&-2\Big(P^\mu Q^\nu+Q^\mu P^\nu\Big)\frac{\bar{u}\hat{Q}u\,\lambda x}{k^8}\Big[fk^4+6f_2k^2\lambda^2+12f_4\lambda^4\Big]\\
&-2\Big(P^\mu r^\nu+r^\mu P^\nu\Big)\frac{\bar{u}\hat{Q}u\,\lambda^2 x\alpha}{k^8}\Big[fk^4+f_2k^2\lambda^2+6f_4\lambda^4\Big]\\
&-\Big(Q^\mu r^\nu+r^\mu Q^\nu\Big)\frac{\bar{u}\hat{Q}u\,\lambda\alpha}{k^8}\Big[fk^4+6f_2k^2\lambda^2+12f_4\lambda^4\Big]\\
&+2\bar{u}\Big(P^\mu\gamma^\nu+P^\nu\gamma^\mu\Big)u\,\frac{\lambda x}{k^6}\Big[fk^4+2f_2k^2\lambda^2+2f_4\lambda^4\Big]\\
&+\bar{u}\Big(Q^\mu\gamma^\nu+Q^\nu\gamma^\mu\Big)u\,\frac{1}{k^6}\Big[fk^4+3f_2k^2\lambda^2+4f_4\lambda^4\Big]\\
&+\bar{u}\Big(r^\mu\gamma^\nu+r^\nu\gamma^\mu\Big)u\,\frac{\lambda\alpha}{k^6}\Big[fk^4+2f_2k^2\lambda^2+2f_4\lambda^4\Big]. \qquad (154)
\end{aligned}
$$

This result allows for any given parametrization (which is set by a model on distribution functions $f(x,\alpha)$ and $h(x,\alpha)$) of the nonforward matrix element to calculate auxiliary distributions f_2, f_4, h_2 and h_4 and then construct the explicitly gauge invariant amplitude.

Chapter 12

Discussions and Conclusion

The main results obtained in this thesis are

1. Developed a method of calculating the one-loop corrections for the generalized Compton scattering amplitude terms of nonlocal string light-ray operators in the coordinate representation.

2. Calculated a generic one-loop Compton amplitude in the coordinate representation which can be used for many different processes:

   - deep inelastic scattering
   - $\gamma^*\gamma \to \pi^0$ form factor
   - deeply virtual Compton scattering
   - wide angle real Compton scattering.

3. Calculated coefficient function in one-loop approximation.

4. As an example of application of the developed technique reproduced a result for DIS obtained by Bardeen *et al.* [9].

5. As another example calculated one-loop corrections for the case of non-forward process.

6. Developed a technique of separating the real and imaginary parts of the Compton amplitude by means of the methods of the theory of generalized functions.

7. Developed a consistent approach to the problem of constructing the gauge invariant Compton amplitude.

8. Obtained an expression for the explicitly gauge invariant Compton amplitude which includes all the generalized target-mass $(m_p^2/Q^2)^N$ and $(t/Q^2)^N$ corrections.

In conclusion we note that the next logical step in the analysis of the generalized Compton amplitude is to calculate an explicitly gauge invariant amplitude in the one-loop approximation.

Appendix A

Integral Formulas

This appendix lists the integral formulas which were derived for calculation of the one-loop corrections in chapter 4.

In this appendix the following notation is used in the formulas below.

$$(z \cdot \partial)\, \psi(az) = z^{\mu} \psi_{,\mu}(az). \tag{155}$$

where $\psi_{,\mu}(az)$ (which is quite common in general relativity) means the derivative $\frac{\partial \psi_{,x}}{\partial x^{\mu}}$ calculated at $x = az$.

$$\int_0^1 du \int_0^1 dv\ \bar{u}\ v^{2-d/2}(z \cdot \partial)\psi(uvz) = \\ = \int_0^1 dv \frac{v^{1-d/2} - 1}{d/2 - 1} \left[\psi(vz) - \psi(0)\right] \tag{156}$$

$$\int_0^1 du \int_0^1 dv\ \bar{v}\ \bar{u}^{2-d/2}(z \cdot \partial)\psi((1 - \bar{u}v)z) = \\ = \int_0^1 du \frac{\bar{u}^{1-d/2} - 1}{1 - d/2} \left[\psi(uz) - \psi(z)\right] \tag{157}$$

$$\int_0^1 du \int_0^1 dv\ v^{1-d/2}\psi(uvz) = \\ = \int_0^1 dv \frac{v^{1-d/2}-1}{d/2-1}\left[\psi(vz)-\psi(0)\right] - \frac{\psi(0)}{d/2-2} \tag{158}$$

$$\int_0^1 du \int_0^1 dv\ \bar{u}^{1-d/2}\psi((1-\bar{u}v)z) = \\ = \int_0^1 du \frac{\bar{u}^{1-d/2}-1}{d/2-1}\left[\psi(uz)-\psi(z)\right] - \frac{\psi(z)}{d/2-2} \tag{159}$$

$$\int_0^1 du \int_0^1 dv\ u\ v^{2-d/2}(z\cdot\partial)\psi(uvz) = \\ = \int_0^1 dv \left[\psi(vz)-\psi(0)\right] \frac{2+(d-4)v^{1-d/2}}{d-2}. \tag{160}$$

Note that the last term is $O(\varepsilon)$. Also $\psi(0)$ can be dropped out (compare to the next one):

$$\int_0^1 du \int_0^1 dv\ v\ \bar{u}^{2-d/2}(z\cdot\partial)\psi((1-\bar{u}v)z) = \\ = \int_0^1 du\ \psi(uz)\frac{2+(d-4)\bar{u}^{1-d/2}}{2-d} \tag{161}$$

Appendix B

DIS Example Formulas

Some of the integrals used in calculations of Section 6.1

$$\int_0^1 dx \left[\frac{\ln\bar{x}}{\bar{x}}\right]_+ x^n = \sum_{k=1}^{n}\frac{1}{k}\sum_{s=1}^{k}\frac{1}{s} \tag{162}$$

$$\int_0^1 dx \left[\frac{x}{\bar{x}}\right]_+ x^n = -\sum_{k=1}^{n}\frac{1}{k+1} \tag{163}$$

$$\int_0^1 dx \ln\bar{x}\; x^n = -\frac{1}{n+1}\sum_{k=1}^{n+1}\frac{1}{k}\,. \tag{164}$$

Some of the series used in calculations of Section 6.1

$$\frac{\ln(1-x)}{1-x} = -\sum_{n=0}^{\infty} x^n \sum_{k=1}^{n} \frac{1}{k} \tag{165}$$

$$\frac{\ln(1+x)}{1+x} = -\sum_{n=0}^{\infty} (-1)^n x^n \sum_{k=1}^{n} \frac{1}{k} \tag{166}$$

$$\frac{\ln(1-x)}{(1-x)^2} = -\sum_{n=0}^{\infty} (n+1) x^n \sum_{k=1}^{n} \frac{1}{k+1} \tag{167}$$

$$\frac{\ln(1+x)}{(1+x)^2} = -\sum_{n=0}^{\infty} (-1)^n (n+1) x^n \sum_{k=1}^{n} \frac{1}{k+1} . \tag{168}$$

Appendix C

Nonforward Example Formulas

In this appendix we present some of the integrals which were used in the calculations of the nonforward amplitude in chapter 6.2.

Small complex number $i\varepsilon$ is provided to specify the contour of integration and values of complex logarithm in the expressions which diverge at $X = \zeta$. Note that in the region of interest both X and ζ are positive.

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{v}}{v}\right]_+ \frac{\delta(u)}{X(1-u-v)+\zeta u} = \frac{1}{X} \tag{169}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{v}}{v}\right]_+ \frac{\delta(u)}{X(1-u-v)-\zeta\bar{u}+i\varepsilon} = \frac{1}{X-\zeta+i\varepsilon}+ \\ + \left(\frac{1}{X} - \frac{1}{X-\zeta+i\varepsilon}\right)\Big(\ln(\zeta - i\varepsilon) - \ln(\zeta - X - i\varepsilon)\Big) \tag{170}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{u}}{u}\right]_+ \frac{\delta(v)}{X(1-u-v)+\zeta u} = \\ \frac{1}{X} + \left(\frac{1}{X} - \frac{1}{X-\zeta}\right)\Big(\ln X - \ln\zeta\Big) \tag{171}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{u}}{u}\right]_+ \frac{\delta(v)}{X(1-u-v) - \zeta\bar{u} + i\varepsilon} = \frac{1}{X - \zeta + i\varepsilon}. \tag{172}$$

The dilogarithm function ($n = 2$ Spence's integral) is defined in accordance with reference [58]:

$$\text{dilog}(x) = -\int_x^1 \frac{\ln t}{1-t}\, dt. \tag{173}$$

The following functional relationships of dilogarithm are relevant to our calculations:

$$\text{dilog}(x) + \text{dilog}(1-x) = \frac{\pi^2}{6} - \ln x \ln(1-x), \qquad 0 \leq x \leq 1 \tag{174}$$

and

$$\text{dilog}(1-x) + \text{dilog}(1+x) = \frac{1}{2}\text{dilog}(1-x^2), \qquad 0 < x \leq 1\,. \tag{175}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\ln v}{v}\right]_+ \frac{\delta(u)}{X(1-u-v) + \zeta u} = -\frac{1}{X}\, \text{dilog}(0) \tag{176}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\ln v}{v}\right]_+ \frac{\delta(u)}{X(1-u-v) - \zeta\bar{u} + i\varepsilon} =$$
$$\frac{1}{X - \zeta + i\varepsilon}\left(\ln\frac{X}{X - \zeta + i\varepsilon}\ln\frac{\zeta}{\zeta - X - i\varepsilon} - \frac{\pi^2}{6} + \text{dilog}\frac{X}{X - \zeta}\right) \tag{177}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\ln u}{u}\right]_+ \frac{\delta(v)}{X(1-u-v) + \zeta u - i\varepsilon} =$$
$$\frac{1}{X}\left(\ln\frac{X - \zeta + i\varepsilon}{X}\ln\frac{\zeta}{X} - \frac{\pi^2}{6} + \text{dilog}\frac{X - \zeta}{X}\right) \tag{178}$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\ln u}{u}\right]_+ \frac{\delta(v)}{X(1-u-v)-\zeta\bar{u}+i\varepsilon} = -\frac{1}{X-\zeta+i\varepsilon}\,\mathrm{dilog}(0) \quad (179)$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{v}}{v}\right]_+ \delta(u) \frac{\ln(X(1-u-v)+\zeta u - i\varepsilon)}{X(1-u-v)+\zeta u - i\varepsilon} = \frac{1}{X}\Big(\ln(X) - \mathrm{dilog}(0) \Big) \quad (180)$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{v}}{v}\right]_+ \delta(u) \frac{\ln(-X(1-u-v)+\zeta\bar{u} - i\varepsilon)}{X(1-u-v)-\zeta\bar{u} + i\varepsilon} = \frac{1}{X-\zeta+i\varepsilon}\Big[\ln(-X+\zeta-i\varepsilon) - \mathrm{dilog}\frac{\zeta}{\zeta-X} + +\frac{\zeta}{2X}\left(\ln^2(\zeta - X - i\varepsilon) - \ln^2(\zeta - i\varepsilon)\right)\Big] \quad (181)$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{u}}{u}\right]_+ \delta(v) \frac{\ln(X(1-u-v)+\zeta u - i\varepsilon)}{X(1-u-v)+\zeta u - i\varepsilon} = \frac{1}{X}\Big[\ln(X) - \mathrm{dilog}\frac{\zeta}{X} - -\frac{\zeta}{2(X-\zeta)}\left(\ln^2(X) - \ln^2(\zeta)\right)\Big] \quad (182)$$

$$\int_0^1 du \int_0^{\bar{u}} dv \left[\frac{\bar{u}}{u}\right]_+ \delta(v) \frac{\ln(-X(1-u-v)+\zeta\bar{u} - i\varepsilon)}{X(1-u-v)-\zeta\bar{u} + i\varepsilon} = \frac{1}{X-\zeta+i\varepsilon}\Big(\ln(\zeta - X - i\varepsilon) - \mathrm{dilog}(0) \Big)\,. \quad (183)$$

Appendix D

Real and Imaginary Parts

To separate the real and imaginary parts of (95) we adopt a technique described in [52].

Let us consider the following generalized function

$$\frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon}=\frac{\ln(X-\zeta+i\varepsilon)}{X-\zeta+i\varepsilon}-\frac{\ln(-\zeta+i\varepsilon)}{X-\zeta+i\varepsilon} \tag{184}$$

which corresponds to the functional

$$\int_0^1 dX\ F(X)\ \frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon}, \tag{185}$$

where probe functions $F(X)$ are real, sufficiently smooth and satisfy the following boundary conditions:

$$\begin{aligned} F(0) &= 0 \\ F(1) &= 0. \end{aligned} \tag{186}$$

To separate its real and imaginary parts let us first shift variable of integration

$$\begin{aligned} y &= X - \zeta \\ F(y+\zeta) &= \varphi(y). \end{aligned} \tag{187}$$

The imaginary part of the second term in the r.h.s of (184) can be easily found by means of the well-known formula:

$$\begin{aligned} -\frac{\ln(-\zeta + i\varepsilon)}{y + i\varepsilon} &= -(\ln\zeta + i\pi)\left(P\frac{1}{y} - i\pi\delta(y)\right) = \\ &- \ln\zeta\, P\frac{1}{y} - \pi^2\delta(y) - i\pi P\frac{1}{y} + i\pi\delta(y)\ln\zeta\,. \end{aligned} \tag{188}$$

In order to deal with the first term of the right hand side of (184) we note that the following relationship holds true

$$\frac{\ln(y+i\varepsilon)}{y+i\varepsilon} = \frac{1}{2}\frac{d}{dy}\ln^2(y+i\varepsilon) = \frac{1}{2}\left[\ln^2(y+i\varepsilon)\right]'. \tag{189}$$

Let us now calculate the derivative of the generalized function $\ln^2(y+i\varepsilon)$. By definition:

$$\begin{aligned} &\frac{1}{2}\int_{-\zeta}^{1-\zeta} dy\,\varphi(y)\left[\ln^2(y+i\varepsilon)\right]' = -\frac{1}{2}\int_{-\zeta}^{1-\zeta} dy\,\varphi'(y)\ln^2(y+i\varepsilon) = \\ &= \frac{\pi^2\varphi(0)}{2} - \frac{1}{2}\int_{-\zeta}^{1-\zeta} dy\,\varphi'(y)\ln^2|y| - i\pi\int_{-\zeta}^{0} dy\,\varphi'(y)\ln|y| = \\ &= \int_{-\zeta}^{1-\zeta} dy\,\varphi(y)\left(\frac{\ln|y|}{y} + \frac{\pi^2}{2}\delta(y)\right) - i\pi\int_{-\zeta}^{0} dy\,\varphi'(y)\ln|y|\,. \end{aligned} \tag{190}$$

Let us now combine the last term with the last term of (188):

$$i\pi\varphi(0)\ln\zeta - i\pi\int_{-\zeta}^{0} dy\,\varphi'(y)\ln|y| =$$

$$i\pi\varphi(0)\ln\zeta - i\pi\lim_{\varepsilon\to 0}\int_{-\zeta}^{-\varepsilon} dy\,\varphi'(y)\ln|y| =$$
$$\lim_{\varepsilon\to 0}\left[i\pi\ln\zeta\varphi(0) + i\pi\int_{-\zeta}^{-\varepsilon} dy\,\frac{\varphi(y)}{y} - i\pi\varphi(-\varepsilon)\ln\varepsilon\right]. \tag{191}$$

Now we may replace $\varphi(-\varepsilon)\ln\varepsilon$ by $\varphi(0)\ln\varepsilon$, since the limit of the difference is zero. Further,

$$\varphi(0)\ln\zeta - \varphi(0)\ln\varepsilon = -\int_{-\zeta}^{-\varepsilon} dy\,\frac{\varphi(0)}{y}. \tag{192}$$

The result for (191) may then be written as

$$\lim_{\varepsilon\to 0} i\pi\int_{-\zeta}^{-\varepsilon} dy\,\frac{[\varphi(y)-\varphi(0)]}{y} = i\pi\int_{-\zeta}^{1-\zeta} dy\,\varphi(y)\left[\frac{\theta(-y)}{y}\right]_{+}. \tag{193}$$

And for the real and imaginary parts of (184) we get

$$\begin{aligned}
\mathbf{Re}\,\frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon} &= P\frac{1}{X-\zeta}\ln\left|\frac{X-\zeta}{\zeta}\right| - \frac{\pi^2}{2}\delta(X-\zeta)\\
\mathbf{Im}\,\frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon} &= \pi\left[\frac{\theta(\zeta-X)}{X-\zeta}\right]_{+} - \pi P\frac{1}{X-\zeta}.
\end{aligned} \tag{194}$$

The latter can be also written as

$$\mathbf{Im}\,\frac{1}{X-\zeta+i\varepsilon}\ln\frac{X-\zeta+i\varepsilon}{-\zeta+i\varepsilon} = \pi\delta(X-\zeta)\ln\frac{\zeta}{1-\zeta} - \pi\left[\frac{\theta(X-\zeta)}{X-\zeta}\right]_{+}. \tag{195}$$

We note that since we shifted back to the original variable X, symbol $\left[\cdots\right]_{+}$ in formulas (194) and (195) means the subtraction of $F(\zeta)$, and not $F(0)$.

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