2016

A New Interpretation of Carnap's Logical Pluralism

Teresa Kouri
Old Dominion University, tkouri@odu.edu

Original Publication Citation
Kouri, T. (2016). A new interpretation of Carnap's logical pluralism. *Topoi*, 1-10. doi:10.1007/s11245-016-9423-y

A New Interpretation of Carnap's Logical Pluralism

August 23, 2016

Abstract

Rudolf Carnap's logical pluralism is often held to be one in which corresponding connectives in different logics have different meanings. This paper presents an alternative view of Carnap's position, in which connectives can and do share their meaning in some (though not all) contexts. This re-interpretation depends crucially on extending Carnap's linguistic framework system to include meta-linguistic frameworks, those frameworks which we use to talk about linguistic frameworks. I provide an example that shows how this is possible, and give some textual evidence that Carnap would agree with this interpretation. Additionally, I show how this interpretation puts the Carnapian position much more in line with the position given in Stewart Shapiro (2014) than had been thought before.

1 Introduction

There is one slogan that many people who hold that Carnap was a logical pluralist agree upon: Carnap's pluralism is one in which a change in logic can occur only when there is a corresponding change in connective meaning. In this way, it is claimed that a Carnapian pluralism requires language change any time there is logical change. Logical change can only occur because of language change.

What I will show in this paper is that there is a different interpretation of Carnap (1950) and Carnap (1937), where he is better accounted for as a Shapiro (2014)-style pluralist. First, I will show that, for Carnap, the ordinary question of whether two connectives in different languages share a meaning is meaningless and has no answer. Second, I will suggest that the only way to make this question meaningful is to assess it with respect to a meta-linguistic framework, a framework which we use to talk about other linguistic frameworks. I give some evidence for thinking that Carnap would agree. Further, this will allow us to claim that, with respect to some meta-frameworks, connectives in different logics will share meanings, while with respect to others, they will not. This final step is what allows for logical change without language change. Finally, I show how this new interpretation puts the Carnapian position much more in line with that of Shapiro (2014). On this interpretation, obtained by extending what Carnap says about linguistic frameworks to meta-linguistic frameworks, when two linguistic frameworks are embedded in distinct meta-linguistic frameworks, the connective meanings in the two original frameworks may or may not be the same.

2 The Traditional Carnapian View

It will be useful to explain carefully why the typical slogan about Carnap's view takes him to be claiming that change in logic requires a change in connective meanings. Authors who often seem to be making use of such a slogan to describe Carnap's position include Cook (2010), Hellman (2006) and Restall (2002). Shapiro (2014) makes this claim more prominently. He states "the crucial Carnapian conclusion is that these [relevant and classical negations] are *different negations*" (Shapiro, 2014, p 107). Rarely does anyone explicitly assert that the slogan applies directly to Carnap's position, but much of what is said can be thought to imply it. In a way, this paper can be read as a cautionary note against taking the slogan seriously in interpreting Carnapian himself.

The slogan readily makes sense of passages like the following:

> Our attitude towards...requirements...[of a logic] is given a general formulation in the *Principle of Tolerance: It is not our business to set up prohibitions but to arrive at conventions.... In logic, there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own language. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical [as opposed to scientific] arguments. (Carnap, 1937, p 51/2)
>
> Let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. (Carnap, 1937, p xv)

These passages are typically taken to entail two things. The first is that we ought to be tolerant of different logics. As long as we can build a logic (which is given by its syntactic rules), and provide applications for it, that logic is legitimate. If we have two applications which call for different logics, then we end up with logical pluralism. The second passage is taken to imply that the meanings of the logical connectives are *given by* their inference rules. In this way, it is very easy to (mistakenly) make the inference that changing the logic requires changing the connective meanings. If connectives are defined by their inference rules, and changing the logic just amounts to changing the rules, then changing the logic seems to amount to changing the meanings of the connectives.

What I will claim in the next two sections is that this "sloganed" position conflates two notions: that of building a language from rules, and that of building a language from rules which the builder knows to be distinct (at a fundamental level) from the rules of other languages. As I will show, Carnap meant the former, as the question which corresponds to the latter cannot be answered ("Do these two languages have connectives which mean the same thing?"). A builder can build any language she wants, as expressed in the “i.e., his own language” phrase in the quote above. In this respect we have to be tolerant. However, a builder cannot claim that her language is different from any other without first making some other assumptions about a meta-framework in which she is making that claim.

3 Carnap’s Position

As we saw above, Carnap says several things which make it reasonable to assume that any change in the rules of a logic is necessarily a change in connective meanings. In order to show that this is not the case, we need details on three aspects of his view: we need to know what linguistic frameworks are, how they relate to connective meanings, and what a pseudo-question is.

Carnap claims that a linguistic framework is “a system of... ways of speaking, subject to... rules” (Carnap, 1950, p 242). We will assume here that the “ways of speaking” which Carnap mentions really are just the syntactic rules for a logical system.[1] Recall that the connectives are defined by “[letting] any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols” (Carnap, 1937, p xv). Thus, on this interpretation, the meanings of the connectives are determined by the rules and postulates of the linguistic framework.[2]

[1]These syntactic rules may have later been expanded to include model-theoretic rules. Early quotes from Carnap suggest a purely proof-theoretic view of logic. Though he held that proof theory was the best way to “do” logic early in his career, his position about logic being a purely proof-theoretic endeavor changed after he met and spoke with Tarski, who convinced him that model-theory was a legitimate enterprise (see, for example, Carnap (1947)).

[2]In particular, they will be given by the L-rules of the framework, which are the rules that govern transformations of logically true sentences into logically true sentences. We will not here concern ourselves with P-rules, which govern transformations of descriptively true sentences into descriptively true sentences. For more information, see (Carnap, 1937, p 133-5).

For Carnap, a theoretical question is one asked relative to a linguistic framework. It is internal to a linguistic framework, and asked assuming the rules of that framework. A non-theoretical question is one asked about reality itself, without a linguistic framework in mind. It is external to any given framework. There are two types of external questions: pragmatic questions and pseudo-questions. A pragmatic question is a question about which framework is best for a given purpose, and can be answered.[3] Pseudo-questions (non-pragmatic external questions) cannot be answered, on Carnap's view, and this is the sense in which they are illegitimate. For Carnap, a pseudo-question is "one disguised in the form of a theoretical question while in fact it is a non-theoretical [question]" (Carnap, 1950, p 245). Examples of pseudo-questions include questions of existence of abstract objects, which can only be answered relative to a given linguistic framework, and questions about the right logic, or which logical consequence relation is correct. Outside of a linguistic framework questions like this are only pseudo-questions.

Thus, linguistic frameworks are (for our purposes) syntactic logical systems, and their rules define the connectives. If we ask a question with respect to no linguistic framework, then it is external, and it is a pseudo-question unless it is pragmatic.

4 The External Question

We can now ask the following question: when do corresponding connectives in distinct linguistic frameworks have the same meaning? According to the slogan from section 2, the answer to this question on Carnap's behalf ought to be "never". A different, perhaps less drastic version of the slogan suggests that the answer

[3]Pragmatic questions are in principle answerable. There is some question as to whether they are actually external, though. See Steinberger (2015) for an interesting discussion about how pragmatically to select the appropriate linguistic framework.

ought to be something like "connectives are defined by their rules, and so if we change the rules, we necessarily change the meanings of the connectives." According to this version of the slogan, two linguistic frameworks share some connective meanings then they share some of the same logical rules. Each of these ways of answering the question ("never", and "only when the rules are the same") is flawed. This is because the question of when two linguistic frameworks have corresponding connectives with the same meaning is unanswerable; it is an external question which is not pragmatic. It is not pragmatic, since it is not about framework selection, but it is external, because it is not asked with respect to some given linguistic framework. It is a pseudo-question. Even if two linguistic frameworks have postulates and rules which "look exactly the same" (i.e. are typographically the same) we still cannot ask the question, since we still have no postulated meta-linguistic framework in which to compare the rules and postulates in question. The answer to the question, then, cannot be "never".

In addition to the fact that questions about the meaning of logical connectives are asked outside of a linguistic framework and are not pragmatic, there is at least one other reason why we ought to think they are pseudo-questions: they have a similar characteristic to the metaphysical questions which Carnap also dismisses as pseudo-questions. I will show that, first, there is no evidence acceptable by all parties that would settle the question one way or another, and second they are trivially analytic once asked in the appropriate way.

In Carnap (1950), he considers two philosophers debating the status of numbers. One thinks they are real entities, and the other does not. Of the debate, Carnap claims "I cannot think of any possible evidence that would be regarded as relevant by both philosophers, and therefore, if actually found, would decide the controversy or at least make one of the opposite theses more probable than the other" (Carnap, 1950, p 254). Because of this Carnap claims he feels "compelled to regard the external question as a pseudo-question, until both parties to the controversy offer a common interpretation of the question as a cognitive question; this would involve an indication of possible evidence regarded as relevant by both sides" (Carnap, 1950, p 255). Thus, if there is no evidence which would decide the issue, external questions are pseudo-questions. But notice that we can reconstruct the debate with meaning questions. Suppose we have a debate between a philosopher who holds that connective meanings are given by truth conditions, and another who holds that connective meanings are given by inference rules. Again, it seems unlikely that there would be evidence which both philosophers would agree is relevant. Natural language data seems to support neither (or both) as connectives in natural language do not behave like they do in formal language.[4] Additionally, with the exception of uncontroversial argument patterns which support both positions, no argument pattern will be accepted as relevant evidence by both philosophers. In general, in a piece of evidence supports one position, the other philosopher will find reason to dismiss it as irrelevant.

Further, Carnap writes that

> [The statement "there are numbers"] follows from [an] analytic statement and is therefore itself analytic. Moreover, it is rather trivial, because it does not say more than that the new system is non-empty... Therefore, nobody who meant the question "Are there numbers?" in the internal sense would either assert or even seriously consider a negative answer. This makes it plausible to assume that those philosophers who treat the question of the existence of numbers as a serious philosophical problem and offer lengthy arguments on either side, do not have in mind the internal question. And, indeed, if we were to ask them: "Do you mean the question as to whether the framework of numbers, if we were to accept it, would be found to be empty or not?", they would probably reply: "Not at all; we mean a question prior to the acceptance of the new framework"... Unfortunately, these philosophers have so far not given a formulation of their question in terms of the common scientific language. Therefore our judgment must be that they have not succeeded in giving to the external question and to the

[4] For good examples of the complexity of natural language connectives, see Horn (1989) on negation or Jennings (1994) on disjunction.

> possible answers any cognitive content. Unless and until they supply a clear cognitive interpretation, we are justified in our suspicion that their question is a pseudo-question. (Carnap, 1950, p 245)

Thus, questions which are trivially analytic once embedded into the appropriate framework (where trivially analytic just means that they follow "easily" from the empty set of premises and the rules and postulates of the framework in question, see Ebbs (2016) for more details) and which have not otherwise been formulated rigorously are pseudo-question if they are not meant to be trivial. However, meaning questions, in addition to number questions, are like this. Once we properly specify a meta-linguistic framework, which includes statements to the effect that for any two terms, they either share a meaning or do not, meaning questions become trivial. Further, we can only answer then once they are rigorously formulated, and once we have specific rules for what counts as sameness of meaning. But this is just to say that without such rules, they are external and illegitimate questions, in the same way the number questions are. Meaning questions, like metaphysical questions, cannot be resolved by evidence accepted by both parties as relevant and are trivially analytic when asked internally, and thus we have an extra reason to think of them as pseudo-questions.

What, though, if we postulated the existence of such a meta-linguistic framework, one which was capable of talking about the two linguistic frameworks in question? This would, I will show, give us an opportunity to answer the question of when two distinct frameworks have the same logical terminology. Additionally, I will show that the answer to whether two object-frameworks have corresponding connectives with the same meanings in a meta-framework will vary depending on our pragmatic goals, and hence the meta-framework we select.

First, when both frameworks in question are considered from the vantage point of a meta-linguistic framework, we could answer the question "when do two linguistic frameworks have logical terminology which means the same thing?".

The question is now being asked, not about what is *really* true, but about what is true with respect to the meta-framework. This makes it a theoretical question, and so it is no longer a non-pragmatic external question. So far so good. The new question is answerable, and not a pseudo-question.

Second, I hold that the same-meaning question will have different answers depending on our pragmatic goals, and hence meta-linguistic framework selection. There will be no single meta-framework which will do the trick for us here, there might be a whole spectrum of them. As with object-level frameworks, we pick one meta-framework which suits our pragmatic aims and operate with it. Given this choice, and presumably given that each meta-linguistic framework will come equipped with some rules and postulates for determining when two terms have the same meaning, we can infer the following. With respect to some meta-framework embeddings, corresponding connectives will share a meaning. With respect to others, though, they will not be the same.

As an example, let us consider two meta-linguistic frameworks and two object level frameworks. One object level framework is classical and the other intuitionistic.[5] The only thing that will concern us about the meta-linguistic frameworks are which rules they have for determining sameness of meaning. In this example, each is equipped with a different version of a double negation translation, and two terms are synonymous if they are inter-translatable. Here, I take it that these translations map synonyms to synonyms, as they do in natural language, and that synonyms share a meaning. Thus, translations preserve meaning.[6]

The first meta-framework has the typical Gödel-Gentzen translation from classical to intuitionistic logic, call it T_1, and is defined inductively as follows:

[5]Thanks to ?? for suggesting this example.

[6]There are important relations between synonymy and analyticity on Carnap's view. In effect, if Quine (1951) and subsequent authors are right, then Carnap cannot get his notion of analyticity, or his project, off the ground. As this paper is only meant to present an interpretation of Carnap's views, and not assess whether they are viable, I will not address this further here.

1 if ϕ is atomic, then $T_1(\phi) = \neg\neg\phi$

2 $T_1(\phi \wedge \psi) = T_1(\phi) \wedge T_1(\psi)$

3 $T_1(\phi \vee \psi) = \neg(\neg T_1(\phi) \wedge \neg T_1(\psi))$

4 $T_1(\phi \rightarrow \psi) = T_1(\phi) \rightarrow T_1(\psi)$

5 $T_1(\neg\phi) = \neg T_1(\phi)$

It is a known result that ϕ is provable classically if and only if $T_1(\phi)$ is provable constructively. In this sense, we have a translation between classical and intuitionistic logic. Now, consider a second translation, call it T_2, and assume that this is the translation available in the second meta-framework. T_2 is the same as T_1 for all clauses expect conjunction. The conjunction clause for T_2 is

2* $T_2(\phi \wedge \psi) = \neg\neg(T_2(\phi) \wedge T_2(\psi))$

T_2 also has the desired property that ϕ is provable classically if and only if $T_2(\phi)$ is provable constructively (the proof proceeds by simply replacing the inductive steps for the conjunction clause in the Gödel-Gentzen proof). Here, though, we can ask the following question: do intuitionists and classicists mean the same thing by "$\wedge$"? Well, if the translations provide us with a relation of synonymy, then because conjunction is translated as conjunction via T_1, while it is translated as a double negation of conjunction by T_2, we have sameness of meaning via T_1, but difference in meaning via T_2. In essence, it seems that if we are using T_1 the conjunctions mean the same thing, while if we are using T_2 they do not. Depending on our theoretical purposes, or on which meta-framework we are using, we will have access to different translations, and different logical terms will be synonymous.

On the face of it, if we think there is such a thing as *real* meaning, and that all translations must preserve *real* meaning, it might seem unlikely that we will be able to use different translations depending on our purposes and aims.

However, consider the difference between translating between intuitionism and classicism for the purposes of a logic class, where the Gödel-Gentzen translation might be sufficient and simplest, and translating between the two for the purposes of writing computer programs, where the Kolmogorov translation might be more successful (the Kolmogorov translation of ϕ is generated by affixing a double negation to every subformula of ϕ). Here, depending on our purposes, one translation will be better than another, and so we ensure that we select a meta-framework which can make sense of that translation, and thereby may generate differences in the "same meaning" relation.

Take another example, from (Field, 2009, p 346-7). He considers a translation between three logics: classical logic, intuitionist logic and some paraconsistent logic. In particular, he focuses on translating between the negations of the three logics. The classical negation is the typical boolean negation, the intuitionist negation is defined as $A \to \bot$, and the paraconsistent logic has two non-equivalent negation-like operators: one which obeys the De Morgan laws and double negation elimination, and another which is defined as $A \to \bot$. When considering a translation between classical and intuitionistic logic, Field claims it is best to translate the negations homophonically. When considering a translation between classical logic and the paraconsistent logic, Field claims it is best to translate the classical negation as the first of the negation-like operators (the one obeying double negation elimination and the De Morgan rules). One would think, then, that if translation preserved meaning, we ought to follow through and translate the intuitionist negation as the De Morgan negation in the paraconsistent logic as well. But there is a method for translating between the intuitionist logic and the paraconsistent logic which will preserve more of the behaviour of the intuitionist negation: we ought to translate the intuitionist negation as the negation-like operator which is defined as $A \to \bot$.

It seems like an available step at this point to say something like the following. Sometimes, translating the connectives by the transitive translation above will serve our theoretical purposes, as in the case of translation loosely between classical and intuitionist logic. Sometimes, however, it will not, and the best available translation will not map two things we thought might mean the same thing onto each other, as in the case of classical and intuitionistic logic together being translated into the paraconsistent logic. In those cases, translation will preserve something else, something which is not captured by similarity of "shape" (perhaps it preserves the inferences associated with each connective).

What our translation needs to do (i.e. what it needs to preserve) will depend in part on what our aims are, which is cashed out by what we choose our third language to be. We pick a third language to suit our pragmatic purposes, and then we see whether one of the available translations maps homophones to homophones. Sometimes it will be available, and sometimes it will not. In other words, sometimes the intuitionist negation will be synonymous with the De Morgan negation (when we are using the meaning-preserving transitive translation and taking classical logic into account), and sometimes the intuitionist negation will be synonymous with the $A \rightarrow \bot$ paraconsistent negation (when we are translating via the "closest in inference" translation). Depending on our theoretical purposes, different translations will be appropriate, and hence different connectives will be said to be synonymous.

In the remainder of this section, I will explain what two types of mistakes we might make in trying to address the original external question without being careful about which meta-linguistic framework we are using, and I will also address an immediate potential objection from Friedman (1988).

First, then, let us consider two examples of where this type of confusion might be a problem. In the first case, consider two linguistic frameworks, each defined by the Gentzen-style left and right logical rules for a sequent calculus, but one which has the full complement of structural rules, and one which does not have weakening. The connectives in both frameworks are defined by their logical rules on the Carnapian picture, since these are the "postulates and rules" to which the earlier quote was referring. In this case, one might this that the only way to embed these two object-frameworks into a meta-framework is to use one where the identity map between the two object frameworks counts as a good translation. However, in this case even though the logical rules look the same in both logics, it might not be best to translate them as rules which have the same meaning. Perhaps, for example, if we are trying to deduce what effect the structural rule of weakening had on the behaviour of the connectives. In this case, assuming that the connectives must mean the same thing might obscure the results of adding or removing the weakening rules. Additionally, the logics will be different (the first classical, the second relevant), and so if we think that the traditional view of Carnap's pluralism holds here, the connective meanings ought to be different. This is an example where rules which look the same may not be the same once considered from the perspective of a meta-linguistic framework, the framework which we use to compare two linguistic frameworks.[7]

In the second case, consider an adaptation of a case from (Shapiro, 2014, p 127-133). Here, we have two mathematicians, one classical and one constructive, discussing some form of analysis. The classicist normally defines her connectives via truth conditions, and the constructive mathematician normally uses proof conditions. One might think that the only way to embed these two object-

[7]There is some question as to whether Carnap would accept this type of suggestion. It seems that Carnap would have thought that structural rules are meaning determining as well. This type of holism seems to be part of what is expressed in the quote above, when he claims that the postulates and rules of inference determine the meaning of the "fundamental logical symbols". The point I am trying to make still stands, though. It would be a mistake to think that the connectives in question shared a meaning, since even on the traditional view, they were never candidates for having the same meaning in the first place. Thanks to a referee for suggesting this possibility.

frameworks into a meta-framework is to do so in such a way that none of the acceptable translations translate the connectives homophonically. However, during their exchange, and for the purpose at hand, they never discuss the connectives, nor do they consider any results which are not acceptable to both of them. In this situation then, it makes sense to talk about them as though they are "speaking the same language", that is, as though they are both using connectives which mean the same. This is an example where rules which do not look the same may in fact imbue corresponding connectives with the same once embedded into a meta-framework.[8]

Here, we see that if we are not careful about our meta-framework selection, we will be liable to make two types of mistakes. First, we may think that rules which look the same must mean the same thing. Second, we may think that rules which do not look the same cannot mean the same thing. Neither of these is good.

There is an immediate potential objection here.[9] I have given no method by which we individuate meta-frameworks. I have only made vague claims that somehow which translations the meta-frameworks contain individuate them. However, it is distinctly possible that there is only one meta-framework, perhaps one which contains all of the translations we might ever need. For example, primitive recursive arithmetic (PRA) might play this role. This is the position advocated for in Friedman (1988) (see also Friedman (2001)). This is a system which can encode all of the translations I mention above, but then there would only be one

[8]There is another option for interpretation here: it is possible to claim that the mathematicians are simply talking past each other. If we assume this, though, we would have to assume that any dispute they had would be a merely verbal dispute. However, it seems more charitable to assume that sometimes the mathematicians in question can have substantive disagreements, as in the case, for example, where they discuss the status of the intermediate value theorem, which the classicist is provable in the classical system, but not in the intuitionist system. See Shapiro (2014) for more details. Thanks to a referee for pushing me on this issue.

[9]Thanks to ?? for bringing it to my attention and to a referee for doing so as well and suggesting much of the literature discussed here.

meta-framework. Friedman claims

> Carnap takes the idea that primitive recursive arithmetic constitutes a privileged and relatively neutral "core" to mathematics and, moreover, that this neutral "core" can be used as a "metalogic" for investigating much richer and more controversial theories. (Friedman, 1988, p 87)

In other words, for Carnap, the only meta-language available is PRA, since in this language we can execute all of our logical and scientific investigation. What becomes of my claim that whether two object-level frameworks have connectives which share meanings is dependent on which meta-framework we embed them into? It seems, if there is only one meta-framework, that answer will never change. For any two frameworks, either their corresponding connectives will always share meanings, or they will always have distinct meanings, since we can only ever embed the object-level frameworks into a single meta-framework. There are two possible responses here.

First, we can side with Devidi and Solomon (1995), and point to textual evidence that Carnap would not hold that PRA was the only legitimate meta-framework. If Carnap thought that there was more than one available meta-framework, then there is no reason for us to assume there was only one. Devidi and Solomon cite section 45 of Carnap (1937) where he claims that

> Our attitude toward the question of indefinite terms conforms to the principle of tolerance; in constructing a language we can either exclude such terms (as we have done in Language I) or admit them (as in Language 1I). It is a matter decided by convention .... Now this holds equally for the terms of syntax. If we use a definite language in our formalization of a syntax... then only definite syntactical terms may be defined. Some important terms of the syntax of transformations are, however, indefinite (in general); as, for instance, 'derivable' 'demonstrable', and a fortiori 'analytic', 'contradictory',... and so on. If we wish to introduce these terms also, we must employ an indefinite syntax-language (such as Language II).

In this passage, Carnap suggests that just as the object language (here just "language") can include or exclude indefinite terms, so to can the meta-language (here, “syntax”). If this is right, then there are at least two legitimate meta-languages, and hence two legitimate meta-frameworks. Further, since the second meta-framework has more terms in the language than the first (“derivable, “contradictory”, etc.) there may be a different class of translations available in the second language. Hence, depending whether we embed our original object language into a meta-language with or without indefinite terms, we may find different results about our meaning question.[10]

Second, even if it does turn out to be the case that Friedman is correct, we have another avenue available to us. What Friedman presents is a single meta-language rich enough to address all of the mathematical and logical questions about the Carnapian picture. A single meta-language, and PRA in particular, cannot answer the meaning question we are trying to resolve for a simple reason: it does not contain a notion of meaning, or of same meaning. We need these notions to answer the questions with which we are concerned. Thus, though Friedman may be right that there is a unique meta-language for all mathematical pursuits on the Carnapian picture, in order to resolve meaning disputes, we need a meta-language with the appropriate terminology. Thus, PRA just won’t do for us in this situation. So again, we find ourselves with at least two available meta-languages: PRA, and one with a notion to express “same meaning”. Presumably, again, we will have access to different translations in both.

5 Carnap’s Agreement

There are passages where Carnap says things which support the interpretation of section 4. I will discuss several here.

[10]There is much literature on the topic of meta-language on Carnap’s view. See, for example, Tennant (2007) on a different response to Friedman’s PRA proposal. There, Tennant argues that “Friedman is demanding too much in thinking that the resources of such combinatorial analysis as Carnap requires should not exceed those of primitive recursive arithmetic.” (p 103).

First, let us reconsider the quotes of Carnap given above to support the traditional view.

> Our attitude towards...requirements...[of a logic] is given a general formulation in the *Principle of Tolerance: It is not our business to set up prohibitions but to arrive at conventions.... In logic, there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own language. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical [as opposed to scientific] arguments. (Carnap, 1937, p 51/2)
>
> Let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. (Carnap, 1937, p xv)

Here, it is important to note that we never explicitly have the claim that we can tell when two languages are distinct. Everyone might still be at liberty to build up their own language, but nothing here explicitly prevents that in order to compare any two languages, we must first embed them in a meta-language. In fact, given that whether two terms share a meaning is a theoretical question, we are *required* to address meaning questions from within a meta-linguistic framework, and so are required to embed any framework into a meta-framework to answer the question.

There is some sense, when interpreted as speaking loosely, that Carnap may be thought to be saying exactly the opposite here. On the face of it, what he seems to be saying here is something like "we recognize rules and postulates without needing any theoretical apparatus, and can tell when they are the same just by looking at them". I think this is reading of the Carnapian position is too loose. First, in order to interpret Carnap consistently, we ought to interpret him as not putting forward this loose interpretation as a serious component of his view, as it would require suggesting that some external questions are legitimate (the meaning ones). Second, Carnap insisted that we need a theoretical tool to engage in rigorous and meaningful discussion of logic. Reading Carnap as claiming the above would vitiate this. As I suggested above, meaning questions are external questions on his view, and can only be answered if we make them internal by asking them with respect to some framework. Thus, I claim, the loose interpretation just will not do here.

Taken as not speaking loosely, what Carnap can be thought to have said in these passages is that, though the rules do determine a connective's meaning, they only determine a meaning *within a particular framework*. Without considering the two frameworks from the perspective of a meta-framework, there is no way to know whether the meanings of the connectives in the two different frameworks are the same. There are two reasons why we are prevented from knowing this: first, because sometimes rules with the same shape will wind up meaning different things under the embedding into a meta-linguistic framework, and second because sometimes rules which intuitively look different will end up meaning the same under the embedding into a meta-linguistic framework (see the two possible "mistakes" in section 4). Since rules determine connective meaning, this implies the connectives will sometimes share a meaning and sometimes not. The passage above, then, can only be referring to rules determining a meaning for the connectives *within a given framework*. The passage makes no claim about considerations from outside of a framework (nor should it).

There are other passages where Carnap makes similar sounding claims. For example, in section 14, when he claims that "if we wish to determine what a sentence...means...we must find out what sentences are consequences of that sentence" (p 41). Similar claims can be found in section 16, 50 and 61.[11] Here, I take it, we must interpret him in the same way as we have above. We must interpret him as claim that if we wish to determine what a sentence means *in a particular linguistic framework*, then we must find out what sentences are consequences of

[11]Thanks to a referee for pointing me to these passages.

it *in that framework*. The same would go for the other passages that are similar in nature.

Second, it seems as though Carnap explicitly agrees that language comparison must be done in a meta-language (here, languages just are linguistic frameworks). Consider the following from (Carnap, 1937, section 62). There, he discusses translations from one language into another. These translations must occur within a meta-language:

> The interpretation of the expressions of a language S_1 is thus given by means of a *translation* into a language S_2, the statement of the translation being effected in a syntax-language S_3... (Carnap, 1937, p 228)

If all translations are effected in a meta-language, then there is reason to suspect that when using two distinct meta-languages, sometimes a term in S_1 will be translated into a particular term in S_2 and sometimes to a different term in S_2.

Third, Carnap holds that just what a translation must do dependens on our theoretical goals (seeCarnap (1937), p 228, where he discusses the special conditions which can be imposed on translations). If the above is correct, then it matters what S_3 is. However, if there were only one type of translation, then we might expect the relationship between S_1 and S_2 to always look the same no matter what we select as S_3. Given the examples from section 4, we can see that there will not always only be one type of translation available, nor will that translation always be required to do the same things. Carnap himself suggests that what a translation is required to do is variable; depending on our theoretical aims, a successful translation will be different. Sometimes it "must depend upon a reversible transformance, or it must be equipollent in respect of a particular language, and so on" (Carnap, 1937, p 228).[12] But it need not always be as such.

[12]Here, a transformance is a map between two languages such that "the consequence relation in [the first] is transformed into the consequence relation in [the second]." A reversible transformance is a transformance such that the reverse relation is also a transformance. Being equipollent in respect

It might not be reversible, and it might not be equipollent (bijective). Depending on what we are wanting our translation to do, we impose different adequacy conditions on it. So what a translation must do, and what it must preserve is a pragmatic choice.

Given that the original quotes taken to support the traditional view can be reinterpreted, and that Carnap explicitly says that two languages must be discussed in a meta-language and what translations have to preserve is variable, it looks like he would have supported the interpretation given in section 4.

Finally, it pays to keep in mind what Carnap says directly about intuitionism as a philosophical position. In section 43 of *Logical Syntax*, Carnap criticizes the intuitionists for ruling out languages which contain indefinite terms for all purposes. He says

> In any case, the material reasons so far brought forward for the rejection... of indefinite... terms are not sound. We are at liberty to admit or reject such definitions without giving any reason. But if we wish to justify either procedure, we must first exhibit its formal consequences. (p 165)

What he is saying here about intuitionism is that it is not *always* permissible to rule out indefinite terms.[13] However, if the formal consequences of a language which contains indefinite terms do not suit the purpose to which it is being applied, then I take it he would allow a ruling out of indefinite terms *for that purpose*. Thus, in some contexts, intuitionism is good, while in others it is not. Similarly, in Carnap (1939), he states

> If we compare the systems of classical mathematics and intuitionistic mathematics, we find that the first is much simpler and technically

of a language amounts to the requirement that any sentences which are mapped to each other are consequences of each other in the language we are respecting. The details here are not as critical to the view as the fact that these are different requirements.

[13]There are serious questions about whether the intuitionists Carnap took himself to be addressing would agree with these criticisms. See Koellner (Koellner) for more details.

> more efficient, while the second is more safe from surprising consequences. (p 50-1)

Here, we see Carnap suggesting that depending on our aims (either simplicity or safety when we do mathematics) one of classical or intuitionistic logic may fare better.

In these passages we see that Carnap both thought that intuitionism was good for something, and that the intuitionists were too restrictive in thinking it was good for everything. Take together with the passages above about translation and language comparison, we can thus infer that Carnap would agree with the proposal in section 4 and that classical logic and intuitionistic logic are two candidates for the L-rules of object-level linguistic frameworks.

6 Shapiro's Position

Finally, I would like to suggest that the section 4 interpretation puts Carnap in a position much closer to that of Stewart Shapiro than one might have thought. This is odd, given the way Shapiro interprets Carnap (see section 2). Carnap should be interpreted as a friend of the Shapiro position, rather than an opponent. I will look at two scenarios from Shapiro (2014).[14] Shapiro claims that

> For some purposes...it makes sense to say that the classical connectives and quantifiers have different meanings that their counterparts in intuitionistic, paraconsistent, quantum, etc. systems. In other situations, it makes sense to say that the meaning of the logical terminology is the same in the different systems. (Shapiro, 2014, p 127)

The purposes/situations in question here can be thought of as something like the S_3 discussed above. Shapiro's two examples come from comparing classical analysis and smooth infinitesimal analysis,[15] and asking whether the logical sys-

[14] See also ??

[15] Smooth infinitesimal analysis (SIA) is an intuitionistic analysis system, in which all functions are smooth. Importantly, it is such that 0 is not the only nilsquare (elements whose square is zero, i.e.

tems required for each have corresponding connectives which share a meaning. In the first scenario, we compare the systems when we are interested in differences between the logics themselves. In the second scenario, we compare the systems in terms of their mathematical consequences. For example, in the first case, two people might be comparing axioms of each system; they might discuss whether the existence of nilsquares is possible. In the second, they might be asking whether both systems prove some theorem; they might discuss whether both systems prove the intermediate value theorem.

Here is the rub: Shapiro claims that in the first case, "[it is] natural to speak of meaning shift" (p 128), while in the second case, "it is more natural to take the logical terminology in the different theories to have the same meaning" (p 130). In our language, in the first case, the logical connectives are different in each system, while in the second they are the same.

To put this in Carnapian terms, let classical analysis and smooth infinitesimal analysis be the first two linguistic frameworks, or S_1 and S_2. Then, when we are discussing the logics of the systems, as in the first scenario above, we will find ourselves in a meta-linguistic framework where the maps between S_1 and S_2 which are translations will not map $A*B$ in S_1 to $A'*B'$ (where $*$ is any connective and A' and B' are the translations of A and B) in S_2. Rather, they will map $A*B$ to some formula not equivalent to $A'*B'$ in S_2. This will be akin to the case when we considered T_2 above, where conjunction was mapped to a double negation of a conjunction, and so the meaning of conjunction was not the same across

elements x such that $x^2 = 0$). This is because every function is linear on the nilsquares. From this, it follows that 0 is not the only nilsquare even though there are no nilsquares distinct from 0. This would be inconsistent in classical logic (because of the validity of LEM), and so intuitionistic logic is required. More formally, in a classical system, the sentence $\neg\forall x(x = 0 \lor \neg(x = 0))$ is a contradiction. In an intuitionistic logic, since the law of excluded middle is not valid, the sentence cannot be true. Importantly for us, the SIA system has a very simple and straightforward proof of the fundamental theorem of the calculus (that the area under a curve corresponds to its derivative). Rather than, as usual, taking approximations of the rectangles under a curve as they approach a width of 0, we take a rectangle under the curve which has the width of a nilsquare. No approximations are necessary, and we do not need the concept of "approaching zero". See Bell (1998) for more details.

the two logics. In the second case, discussing the mathematical implications of each system, we ought to find ourselves in a meta-linguistic framework where we will be able to produce a trivial translation between CA and SIA. I suspect in the second case, the translation given by the identity map will be salient, where $A * B$ just gets mapped to $A' * B'$. In the first case, however, because our meta-language is so interested in the details of each system, such a translation will not be particular enough about what gets translated to what. This accords very well with the way Carnap seems to think about translation and cross-framework meanings as discussed in section 5.

This upshot of this that Shapiro might be a Carnapian logical pluralist (especially given what Carnap says about what a translation needs to preserve above), which would make the traditional slogan about Carnap's view mistaken. Additionally, by making this comparison, we can address the worry that there is something quite anachronistic about the interpretation of Carnap's project which I have just presented. There are several differences between the way that Carnap thought of logic and the way we do. Notably, the version of Carnap I am considering here only used syntactic rules, and his languages were not purely logical by our lights, as they included arithmetical terms. Both of these make it harder to assess what Carnap would say about our current state of affairs. By comparing him to a modern logical pluralist, I take it that we can at least learn something about what he might say, and can certainly learn something about what he would not say. If I am right, no version of Carnap would claim that "change in language necessitates a change in logic."

7 Conclusion

It is often claimed that Carnap must hold that there is language change whenever there is logical change. However, this assumes that we can ask questions about meanings outside of any linguistic framework. This assumes that we can answer the question “does logical change require language change *really*?”; it is illegitimate on Carnap’s view. What we should be asking is “does logical change require language change in framework X?”. By doing so, we embed the external question into a linguistic framework, and thus make it answerable. Additionally, when we embed the question into an additional linguistic framework, we see that sometimes there is meaning change when there is logical change, and sometimes there is not. In a sense, this means that Carnap must be Carnapian about the meta-theory.

References

Arthur, P. (1963). *The Philosophy of Rudolf Carnap*, Volume XI. Open Court.

Bell, J. L. (1998). *A Primer of Infinitesimal Analysis*. Cambridge University Press.

Carnap, R. (1937). *The Logical Syntax of Language*. New York: Harcourt, Brace and Company.

Carnap, R. (1939). *Foundations of logic and mathematics*. International encyclopedia of unified science. The University of Chicago press.

Carnap, R. (1947). *Meaning and Necessity - A Study in Semantics and Modal Logic*. University of Chicago.

Carnap, R. (1950). Empiricism, Semantics and Ontology. *Revue Internationale de Philosophie 4*.

Carnap, R. and W. Quine (1995). *Dear Carnap, Dear Van: The Quine-Carnap Correspondance and Related Work*. Berkeley, CA: University of California Press.

Cook, R. T. (2010). Let a Thousand Flowers Bloom: A Tour of Logical Pluralism. *Philosophy Compass 5/6*, 492–504.

Devidi, D. and G. Solomon (1995). Tolerance and Metalanguages in Carnap's Logical SYntax of Language. *Synthese 103*, 123–139.

Ebbs, G. (2016). Carnap on Ontology. In *Carnap, Quine and Putnam on Methods of Inquiry*. Cambridge Univeristy Press.

Field, H. (2009). Pluralism in Logic. *The Review of Symbolic Logic 2*(2), 342–359.

Friedman, M. (1988). Logical Truth and Analyticity in Carnap's "Logical Syntax of Language". *Essays in the History and Philosophy of Mathematics*, 82–94.

Friedman, M. (2001). Tolerance and Analyticity in Carnap's Philosophy. In J. Floyd and S. Shieh (Eds.), *Future Pasts: The Analytic Tradition in Twentieth Century Philosophy*, Number August 2012, pp. 223–256.

Friedman, M. and R. Creath (2007). *The Cambridge Companion to Carnap*. Cambridge Companions to Philosophy. Cambridge University Press.

Hellman, G. (2006). Mathematical pluralism: The case of smooth infinitesimal analysis. *Journal of Philosophical Logic 35*(6), 621–651.

Horn, L. R. (1989). *A Natural History of Negation*. Center for the Study of Language and Information - Lecture Notes Series. Center for the study of language and information.

Jennings, R. (1994). *The Genealogy of Disjunction*. Oxford University Press, USA.

Koellner, P. Truth in Mathematics : The Question of. pp. 1–44.

Quine, W. V. O. (1951). Two Dogmas of Empiricism. *Philosophical Review 60*(1), 20–43.

Restall, G. (2002). Carnap's Tolerance, Language Change and Logical Pluralism. *Journal of Philosophy 99*, 426–443.

Shapiro, S. (2014). *Varieties of Logic*. OUP.

Steinberger, F. (2015). How Tolerant Can You Be? Carnap on Rationality. *Philosophy and Phenomenological Research* (2009), 1–24.

Tennant, N. (2007, September). Carnap, Godel, and the Analyticity of Arithmetic. *Philosophia Mathematica 16*(1), 100–112.

Wagner, P. (2009). *Carnap's Logical Syntax of Language*. History of analytic philosophy. Palgrave Macmillan.