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Ante Rem Structuralism and the No-Naming Constraint

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Tim Räz (2015) presents what he takes to be a new objection to Stewart Shapiro's *ante rem* structuralism (ARS). Räz claims that ARS conflicts with mathematical practice. I will explain why this is similar to an old problem, posed originally by John Burgess (1999) and Jukka Keränen (2001), and show that Shapiro can use the solution to the original problem in Räz's case. Additionally, I will suggest that Räz's proposed treatment of the situation does not provide an argument for the *in re* over the *ante rem* approach.

1 ARS and the Old Problem

Shapiro's account of *ante rem* structuralism (ARS) is that mathematical objects are places in structures. Each place is characterized by its relationships to other places in the same structure. For example, the natural number three is the fourth place in the natural number structure, characterized by the fact that it is the successor of the third place in the same structure, the successor of the successor of the second place, etc.

One of the motivational constraints on any account of mathematical objects, for

Shapiro, is that it not require changes to mathematical practice. It must be consistent with what mathematicians actually do. This constraint is called the faithfulness constraint (see Shapiro (1997, p 35)). ARS's characterization of mathematical objects seems to accord well with mathematical practice, since mathematicians often speak of abstracting away from particular instantiations, and so it seems ARS initially satisfies this constraint.

One of the problems often suggested raised by opponents of ARS is that it is not able to distinguish between certain mathematical objects we know to be distinct, and which mathematicians treat as distinct. Burgess (1999) and Keränen (2001) suggest, for example, that since there is no "structural" difference between i and $-i$ in the field of complex numbers, they cannot be distinguished. There are two ways one might use the word "distinguish" here. The first is metaphysical: we can metaphysically distinguish one object from another if the one has a property the other lacks. The second is semantic: we can semantically distinguish one object from another by referring to one (or, at least, talking about one) without talking about the other. The problem posed by Burgess (1999) and Keränen (2001) is concerned with metaphysical distinguishability. (Later, when we turn to Räz's problem, we will be concerned with semantic distinguishability.) The complex numbers i and $-i$ seem to share all of their "structural" properties: they are both square roots of -1, for example. The roots are metaphysically indistinguishable. Worse still, it seems that every point on the Euclidean plane is metaphysically indistinguishable from every other; no point on the plane has any structural property that another does not. Another example is a graph with n vertices and no edges. The vertices have exactly the same relationships to each other, namely none; they are metaphysically indistinguishable.

This is an instance where we must be careful about what a structural relationship amounts to. Just what types of relationships these are is unsettled. Originally, critics suggested that they must be formulae in one free variable (see Keränen (2001)). More recently, other proposals have been made, the weakest being that these relationships are exactly the non-reflexive relationships in any structure (see Leitgeb and Ladyman (2008) and Ladyman (2005)). There is also a question as to whether identity is a structural relation. If these relationships can include, for example, non-reflexive relationships, then we will be able to distinguish the two square roots of -1; they are additive inverses of each other, but neither is an additive inverse of itself. Shapiro himself holds that identity, and non-identity, are structural relationships (see Shapiro (2008)).[1] However, aside from including identity as a structural relationship, there still seems to be no way to distinguish metaphysically between the vertices in a graph with no edges. There simply are no non-reflexive relationships there (other than non-identity).

The Keränen-Burgess problem is not an isolated issue. Any structure that admits of non-trivial automorphisms will have places that share all their structural properties. This is because automorphisms preserve structural relations, no matter how we define them. If this is the case, and we think that Leibniz's principle of the identity of indiscernibles holds and that identity is not a primitive structural relation, then Shapiro will be forced to conclude that there is only one imaginary root, and only one point on the Euclidean plane. This would be absurd and would violate Shapiro's faithfulness constraint.

Shapiro's solution to this criticism involves two parts. First, one must both deny that the principle of the identity of indiscernibles holds for abstract objects, and allow identity and non-identity to count as structural relations. The principle of the identity of indiscernibles (normally attributed to Leibniz) is the principle that if two objects share all of the same properties (i.e. they are indiscernible), then they are identical.

[1] I will use the term "structural relationship" throughout, but not settle on a definition here. My criticisms of Räz hold independently of what counts as a structural relationship, especially since he takes the Burgess (1999) and Keränen (2001) issue to be solved (p. 118).

As it is questionable whether it even holds for all physical objects (e.g. photons, see Saunders (2003)), this is less controversial than it might at first appear.

Secondly, one needs to put in place a mechanism for talking about these indiscernible objects. Mathematicians do it, and so in order to abide by the faithfulness constraint, our philosophy of mathematics must be capable of explaining how they successfully discuss these objects. In other words, we need a mechanism to distinguish, semantically, the metaphysically indistinguishable objects. Shapiro (2008) uses parameters to explain this phenomenon. In essence, when we are speaking of indiscernible objects, it suffices to introduce a parameter into the discourse which "picks out" one of the indiscernible objects, and opt to use the parameter. Since all the objects in question have the same properties, it only matters that we are speaking of one of them, not which one of them.

This move is clearly appealing to the rule for existential elimination in first order logic. As long as the parameter has not already been used, when we have $\exists x F(x)$, we say "let b be any one of them", i.e. suppose that $F(b)$ and suppose nothing else about b; then .... Similarly the *ante rem* structuralist can say

> in this structure [the complex numbers], there is at least one square root of $-1 : \exists x(x^2 = -1)$. So they let i be one such square root, and go on from there. (Shapiro, 2008, p 300)

According to Shapiro, this explains how mathematicians are able to talk about indiscernible objects.

Consider a second example: trying to make Meno's solution to doubling the square fit the ARS framework.[2] The proof usually proceeds by "abstractly" drawing a line between diagonally opposite corners of any square which we wish to double, and constructing a square with the new line as an edge. It is simple and straightforward.

[2]Thanks to Chris Pincock for suggesting this example.

However, it is potentially hazardous for the *ante rem* structuralist. It seems that the only structural relations of the square are represented by the edges. Since each corner has the same number of edges attached to it, and each corner has exactly one other corner it is not directly attached to, each corner of the given square is metaphysically indistinguishable from any other. How does the structuralist even pick a corner to begin with? First, she must add identity and non-identity to the list of structural relations of the given square. Then, she can state that there are four corners by stating $\exists x_1, x_2, x_3, x_4(\text{CornerOfSquare}(x_1, x_2, x_3, x_4) \wedge x_1 \neq x_2 \wedge ... \wedge x_3 \neq x_4)$. Next, she can use the parametrization technique to "pick out" one of these four corners. She says "let A be such an x_i". With this in hand, she can say "let C be such an x_i such that $C \neq A$ and there is no direct edge from A to C". With A and C in hand, she can proceed with Meno's construction, since these are opposite corners. Moreover, it does not matter which corners the parameters "A" and "C" pick out, just that they are diagonal opposites, since this is all that is required for the proof. Thus, the *ante rem* structuralist can proceed with the proof.

By adding identity and non-identity as structural relations, and removing the law of the identity of indiscernibles, the original problem of Burgess (1999) and Keränen (2001) involving metaphysically indistinguishable objects is not a problem for ARS at all, and moreover Shapiro has provided a solution to the potential problem of semantically indistinguishable objects as well, by using parameters as tools to talk about indistinguishable objects.[3]

[3]A second solution, from Kouri (2010), has similar results, but uses different tools. Rather than introducing parameters, we might also make use of a choice function. It is possible that mathematicians are implicitly using a choice function in their reasoning when they refer to indiscernible objects. In effect, this solution just makes explicit the feeling of "picking one arbitrarily" that Shapiro seeks to capture with parameters. This second solution likely comes to the same thing as Shapiro's.

2 Räz's Claims

Räz claims to have a new, similar, problem for ARS. In our terms, his is a problem of semantic indistinguishability. This arises, he holds, because of a tension between the faithfulness constraint and what he calls the no-naming constraint:

> Certain mathematical structures with symmetries have the property that we cannot name or refer to the objects, or places, in these structures because they are *too homogeneous*. (Räz, 2015, p 118, Räz's emphasis)

Räz is correct in claiming that this constraint applies to ARS, as long as we assume that genuine names uniquely pick out objects. Shapiro agrees that he is subject to such a constraint (though he would not see it as "constraining"):

> There simply is no naming *any* point in Euclidean space, nor any place in a finite cardinal structure and in some graphs, no matter how much we idealize on our abilities to pick things out. The objects are too homogeneous for there to be a mechanism, even in principle, for singling out one such place, as required for reference, as that relation is usually understood. (cited by Räz, p 118, but from Shapiro (2008, p 291))

So, then, how does the tension between the no-naming constraint and the faithfulness constraint arise? Räz's example of this can be found on pages 119-124. I will consider a simpler adaptation here. We will consider maps on a group of four objects which swap two objects and leave the other two fixed. These maps are examples of permutations. The cycle type of such a map is $\{1^2, 2^1, 3^0, 4^0\}$, because there are two cycles of length 1 (leaving two elements unchanged) and one cycle of length 2 (swapping two elements). There are no cycles of length 3 or 4. It is easy enough to count how many cycles of a given cycle type there are. In this instance, we can simply list them by which two elements they swap. Suppose our group of four objects consisting of objects named a, b, c, d. Then the six cycles would be "a-swapped-with-b", "a-swapped-with-c", "a-swapped-with-d", "b-swapped-with-c", "b-swapped-with-d" and "c-swapped-with-d". There are six in total.

Here is where Räz claims the problem arises: mathematical practice allows us to count how many permutations there are of each cycle type (as in the example above), while ARS does not. ARS, he claims, cannot count the number of cycles of the type $\{1^2, 2^1, 3^0, 4^0\}$ since ARS cannot *name* the places in a structure with exactly four elements and no other relations. Consider for example the first two cycles "counted" above: 'a-swapped-with-b" and "a-swapped-with-c". According to Räz, ARS has no method of semantically distinguishing these two cycles. In fact, the *ante rem* structuralist cannot distinguish between any two of the six cycles above. Räz claims "all that can possibly matter for the *ante rem* structuralist is that two (nonidentical) places are swapped, while two further places, not identical to the former two, are left alone. There is one such situation, not two, or six" (Räz, 2015, p 122). Thus, the tension between faithfulness and no-naming arises: mathematical practice can count six such cycles, ARS can only count one.

Räz claims that because mathematicians can count the number of permutations "different permutations...that belong to the same cycle type are always distinguishable" (Räz, 2015, p 122). This is what Räz calls the mathematicians' "indirect answer" (p 122) to whether or not the places in the cycle-type structure of size four can be named. He claims that they can be named because they can be counted. For Räz, the fact that "we can recover all the cycles belonging to a cycle type" amounts to distinguishing such cycles. Thus, he claims, mathematicians can distinguish something which the *ante rem* structuralist cannot.

There is a problem here. Räz seems to insist that a mathematician's ability to count the number of permutations of a given cycle type implies that they are distinguishable. This is not the case. It is known, for example, that there are exactly two roots of -1, and that there are $2^{\aleph_0}$ points on the Euclidean plane, even though these are not distinguishable. The only thing counting allows us to do is to determine how many of something there are, not distinguish them. So, just because a mathematician can count the number of cycles of a given cycle type does not mean the mathematician can distinguish them. Thus, the "indirect answer" that Räz points to on the part of the mathematician is that the places can be counted; this indirect answer implies nothing about whether the places can be distinguished. Moreover, if they cannot be distinguished, then they cannot be named. The indirect answer to the question of whether the different permutations of the same cycle type can be named that Räz points to on behalf of the mathematician is that they can be counted, and nothing more.

The claim Räz makes then must amount to the claim that, since ARS cannot name the permutations of each cycle type in the cycle type structure, it cannot count the permutations of any given cycle type. This means that the no-naming constraint forces a proponent of ARS to give up the faithfulness constraint. Though mathematicians can count places in this structure, ARS cannot account for this behaviour. According to Räz, "there is no way for the *ante rem* structuralist to recover, or count, different permutations [that instantiate each cycle type]" (Räz, 2015, p 122). If Räz is right, then the *ante rem* structuralist must choose between the no-naming constraint and the faithfulness constraint. Since no such choice is possible, contends Räz, ARS fails as a viable philosophy of mathematics.

Unfortunately for Räz, this criticism will not hold water. I will show, in the next section, that it is similar to the old Keränen-Burgess problem, and as such can be solved by a version of the parametrization technique mentioned above. Lastly, I will suggest that Räz's proposed solution requires something like a parametrization technique as well, and so is no better off than ARS.

3 Räz and Keränen & Burgess

Räz claims that the problem of structures being "too homogeneous" and hence subject to the no-naming constraint is distinct from the problem suggested by Burgess (1999) and expanded by Keränen (2001). He takes this to be the case since his criticism of ARS does not rely on the use of the principle of identity of indiscernibles. It turns out that Räz's problem is similar enough to this old problem to be solved by the same method. In effect, any structure admitting of non-trivial automorphisms will be "too homogeneous" (in Räz's words) or "non-rigid" (in Keränen's words).

The difference, which must be what Räz is alluding to when he claims his problem is new, is that Keränen suggests that this means that the *ante rem* structuralist must identify the places, and Räz suggests that this means that the *ante rem* structuralist cannot remain faithful to mathematical practice, because it cannot talk about the right number of things. It is a difference, then, between metaphysical distinguishability and semantic distinguishability. Though Räz is correct in stating that Keränen's explanation of the problem uses the principle of identity of indiscernibles while his does not, it does not mean the problems are totally distinct. In the next section, I will show that, in fact, the parametrization solution to the original problem can be extended to solve Räz's problem.

4 The old solution works

The first step to adopting any solution to an indistinguishability problem is to accept that any map of a given cycle type will be metaphysically indiscernible from any other map of that same cycle type. This is simply a fact. What Räz claims is that it is not possible for an *ante rem* structuralist to count the number of permutations for any given cycle type, and that since mathematicians can do this, it is a violation of faithfulness. However, ARS can count the permutations by making use of parameters and second order logic. Räz even says as much:

> We could skolemize [sic] the axiom of the cardinal-four structure
> $\exists x_1, x_2, x_3, x_4 (x_1 \neq x_2 \wedge ... \wedge x_3 \neq x_4 \wedge \forall y (y = x_1 \vee y = x_2 \vee y = x_3 \vee y = x_4))$
> by eliminating the outermost existential quantifiers by introducing new parameters a, b, c, d for each quantifier...Drawing on second-order logic, it is even possible to deduce formally that there are exactly six different permutations [of the type $\{1^2, 2^1, 3^0, 4^0\}$] on the cardinal-four structure, *i.e.*, on the structure with exactly four objects. (Räz, 2015, p 123)

However, Räz claims that "using parameters does not solve the problem" (Räz, 2015, p 123). What he claims we need to be able to do is distinguish between the six permutations we counted by the second order logic method. Räz holds that to do this "we would need a one-one correspondence between parameters and places" (p 123), which is not possible, since this " would essentially amount to naming the places by using parameters" (p 124). This is partly right: we cannot require parameters to uniquely pick the same object each time they are used, as this would violate the no-naming constraint. What Räz claims here, though, is not quite right. We can, for example, embed $\mathbb{R}^2$ into Euclidean space without thereby naming all of the points in Euclidean space. Just because there is a one-to-one map does not mean that the elements in the domain serve as names for those in the range. The claim, then, must be that though parameters are acceptable for talking about places in structures, they are not acceptable for talking about functions on those places. Importantly, Räz is mistaken in one more respect: ARS can semantically distinguish, i.e. talk about, the six permutations without having to assume there is only one of them. There are two ways to do so: by using something like "parameters of parameters" or by using parameters to build embeddings into larger structures.

The first method to do this is to treat the permutations (which are maps) as parameters of parameters. We can write a formula stating this which would also involve existential quantification over the four places, as well as the six permutations. However, once we have this, we can use the parametrization technique twice, amounting to something like two rounds of existential elimination. In the first instance, we settle on parameters for places, and in the second for functions.

More specifically, this technique would proceed as follows.[4] We start with something Räz admits the *ante rem* structuralist has access to: the one-step parametrization above, i.e. Räz's a, b, c and d from page 123. Then, we use these parameters in a second order existential formula. Let $\Phi(f, x_1, x_2, x_3, x_4)$ abbreviate the following

[4]Thanks to Neil Tennant for providing the details of this formalism.

fourfold conjunction:

$$\forall x((\bigvee_{1\leq j\leq 4} x = x_j) \leftrightarrow \exists y\, y = f(x)) \quad [\mathrm{dom}(f) = \{x_1, x_2, x_3, x_4\}]$$

$$\wedge\, \forall z((\bigvee_{1\leq j\leq 4} z = x_j) \leftrightarrow \exists x\, z = f(x)) \quad [\mathrm{rng}(f) = \{x_1, x_2, x_3, x_4\}]$$

$$\wedge\, \forall x \forall x'(f(x) = f(x') \rightarrow x = x') \qquad [f \text{ is one-one}]$$

$$\wedge\, \exists x \exists x'(x \neq x' \wedge f(x) = x' \wedge f(x') = x \wedge \forall z \forall y((z \neq x \wedge z \neq x' \wedge y = f(z)) \rightarrow y = z))$$

[f swaps two distinct objects and is the identity function wherever else it is defined]

Now consider the claim

> For any four distinct objects x_1, x_2, x_3, x_4, there exist exactly six distinct permutations $f_1, f_2, f_3, f_4, f_5, f_6$ that swap exactly two of those objects, leaving the other two undisturbed.

This is regimented as follows:

$$\forall x_1 \forall x_2 \forall x_3 \forall x_4 \Big((x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_1 \neq x_4 \wedge x_2 \neq x_3 \wedge x_2 \neq x_4 \wedge x_3 \neq x_4) \rightarrow$$

$$\exists f_1 \exists f_2 \exists f_3 \exists f_4 \exists f_5 \exists f_6 \Big[\bigwedge_{1\leq i<j\leq 6} \neg \forall x((\bigvee_{1\leq k\leq 4} x = x_k) \rightarrow f_i(x) = f_j(x))$$

$$\bigwedge_{1\leq i\leq 6} \Phi(f_i, x_1, x_2, x_3, x_4)$$

$$\bigwedge \forall g[\Phi(g, x_1, x_2, x_3, x_4)$$

$$\rightarrow \bigvee_{1\leq i\leq 6} \forall x((\bigvee_{1\leq k\leq 4} x = x_k) \rightarrow f_i(x) = g(x))]$$

$$\Big]\Big)$$

We then parametrize, using a, b, c and d, as in the first level parametrization above to:

$$(a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d) \rightarrow$$

$$\exists f_1 \exists f_2 \exists f_3 \exists f_4 \exists f_5 \exists f_6 \Big[\bigwedge_{1\leq i<j\leq 6} \neg \forall x((\bigvee_{x_k=a,b,c,d} x = x_k) \rightarrow f_i(x) = f_j(x))$$

$$\bigwedge_{1\leq i\leq 6} \Phi(f_i, a, b, c, d)$$

$$\bigwedge \forall g[\Phi(g, a, b, c, d)$$

$$\rightarrow \bigvee_{1\leq i\leq 6} \forall x((\bigvee_{x_k=a,b,c,d} x = x_k) \rightarrow f_i(x) = g(x))]$$

$$\Big]$$

We can then use a second set of parameters, H, K, L, M, N, P, to get:

$$(a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d) \rightarrow$$

$$\bigwedge_{H,K,L,M,N,P=f_i\neq f_j=H,K,L,M,N,P} \neg \forall x((\bigvee_{x_k=a,b,c,d} x = x_k) \rightarrow f_i(x) = f_j(x))$$

$$\bigwedge_{f_i=H,K,L,M,N,P} \Phi(f_i, a, b, c, d)$$

$$\bigwedge \forall g[\Phi(g, a, b, c, d)$$

$$\rightarrow \bigvee_{f_i=H,K,L,M,N,P} \forall x((\bigvee_{x_k=a,b,c,d} x = x_k) \rightarrow f_i(x) = g(x))]$$

Now, since we know that the parameters a, b, c, and d represent distinct objects, we can also deduce from this that the parameters H, K, L, M, N, P represent distinct objects. I take it this is the type of procedure Räz is alluding to when he suggests "it is even possible to deduce formally that there are exactly six different permutations on the cardinal-four structure." This is what allows us to semantically distinguish the functions; we can talk about exactly one of them – without assuming that there is only one of them – by using a parameter to talk about the one. So the problem is solved, and we can semantically distinguish the permutations in question. However, it might be the case that Räz is objecting to precisely the move that we can have parameters of parameters. If that is the case, then there is a second option.

The second option is to use parameters to talk about the four indistinguishable places in a structure, and then embed that structure into a larger one to talk about the permutations. Räz suggests something along these lines towards the end of the paper. He states

> We can think of finite cardinal structures in terms of cycle types, but also in terms of conjugacy classes or partitions of natural numbers. One advantage of these different representations is that we can use our knowledge of one of the representations for all the others. (Räz, 2015, p 124)

However, what he seems to miss is that this strategy is open to the *ante rem* structuralist as well as the mathematician. ARS has the capacity to embed simpler structures into more robust structures and, in Räz's words, use our knowledge of that structure for all the others. Thus, even if it turns out that the *ante rem* structuralist cannot count the number of permutations in any given cycle type, she is welcome to embed the cycle type structure into a more robust one, say one where the places in the structure are just the first four natural numbers, and use that to figure out that there are a certain number of permutations of any given type. We see this happen often enough. For example, one way to count the square roots of -1 is to embed the complex number structure into the real plane, $\mathbb{R}^2$. We know what is true of the simpler structure in the more robust structure and the original language must have been true all along, and so this embedding allows us to count the roots of -1 in the original complex number structure. Since we have parametrized the original structure, we know that there are distinct places in the original structure which correspond to the places in the larger structure, and so we know there is something in the smaller structure corresponding to what we have counted in the larger structure.

Räz claimed that ARS violated faithfulness because mathematicians could count the number of permutations of any given cycle type. Here, though, we see that the *ante rem* structuralist can count the number of permutations of a given cycle type. ARS has the tools required to do exactly what Räz suggests they cannot. It can give the same indirect answer that mathematicians are in a position to give as to whether the permutations of a given cycle type are distinguishable: they can be counted. I have provided two techniques which show how this is possible.

5 Räz's *In Re* Solution

Räz proposes an alternative solution to the conflict he claims is problematic for ARS. He suggests that the solution is to treat mathematical objects as *in re*. In this section, I will discuss his solution, and show that Räz fares no better addressing this nearby problem than an *ante rem* structuralist.

Considering a mathematical object as *in re* is opposed to considering it as *ante rem*. For Räz, to treat a mathematical object as *in re* is to treat it as its instantiation, rather than to treat it as something prior to any instantiation (which is how an *ante rem* system treats mathematical objects). Ultimately he thinks that the solution to conflict between faithfulness and no-naming is to develop a system that treats mathematical objects in two ways: as *in re* and as *ante rem*. This means mathematical objects can be treated both as prior to their instantiations, and as the instantiations themselves. Shapiro's *ante rem* system, on the other hand, treats mathematical objects as being strictly prior to their instantiations (claims Räz).[5]

Räz says a number of things about what it means to treat mathematical objects as *in re*. However, he seems to vacillate between treating these instantiations as merely parameterizations of *ante rem* structures, or as properly named objects with more properties than their *ante rem* counterparts. On page 124, he gives as an example of instantiating an abstract structure "let π be a permutation of type x...". This is very similar to Shapiro's use of parameters to make sense of mathematicians' apparent reference to indiscernible objects. The first step in such an analysis is to say something like "let i be one such square root...". If all Räz means by *in re* is that we can use parameters as Shapiro describes, then there is no difference between Räz's position and ARS. Thus, the first option is no different from Shapiro's ARS-plus-parameters position as its stands, so we will assume that he means the second: that instantiations of structures are properly named instances of them. On Räz's *in re* system, then, mathematical objects are just the places in instantiations of structures. Considering the mathematical objects *in re*, Räz claims, we can count the permutations in the

[5]There is a significant problem in Räz's reading of Shapiro's position here. Räz seems to think that ARS cannot also view structures as instantiated. However, this is not the case. Though mathematical objects are strictly speaking prior to their instantiations, there is nothing that prevents the ante rem structuralist from adopting the places-as-offices perspective (rather than places-as-objects) from Shapiro (1997). Under this perspective, we consider the places in structures as occupied by objects, rather than as the objects themselves. This would allow an ante rem structuralist to consider a structure as an in re object. Räz seems to ignore this possibility. I will not address this discrepancy further here, for even ignoring this issue, Räz's position fails to avoid the problem he presents.

cycle-type structure by counting the permutations in an instantiation of the cycle-type structure where the places are named.

How, then, does Räz propose to solve the counting problem? Well, if we take any instantiation of the cycle-type structure mentioned above, we will be able to count the number of permutations of any particular type. The names of the places in the instantiation (Räz uses the instantiation where the places are named $1, 2, 3$ and 4) are non-structural properties. However, using these non-structural properties we can count the permutations of any given type. This counting is very simple, as once the places are named we can easily distinguish previously indiscernible objects. For example, the permutation that "swaps" 1 and 2, but sends 3 and 4 to themselves is easily distinguished from the permutation that "swaps" 3 and 4, but sends 1 and 2 to themselves. Thus, we have at least two permutations of the cycle type $\{1^2, 2^1, 3^0, 4^0\}$, and we can easily count all six in this way.

Here is the problem: what we count is the number of permutations of an instantiation, not of the original structure. In order to count the number of permutations of a structure via its instances, we would have to know which named place in the instantiation corresponded to which place in the original structure. This violates the no-naming constraint. ARS addresses this issue via parameterizations, so there is no difficulty for Shapiro. However, this violation means that the problem is still there for the *in re* approach *unless we have parameters to begin with.* We have not counted the places in the original structure, but merely an instantiation of it. This leaves the problem unsolved.

This also leaves Räz without one potential advantage over ARS. We might have thought that it was the use of parameters which was problematic, and not just parameters representing objects other than places in structures. However, Räz's solution fares no better here. He will need to make use of something similar to maintain a correspondence between structures and their instantiations.

6 Conclusion

Räz claims he has presented a new problem for ARS. I have shown that it is not an entirely new problem, but rather is very similar to the problem presented by Burgess (1999) and Keränen (2001). As such, the solution to this problem from Shapiro (2008) works here, as well as a new solution via structural embeddings. Finally, I suggested that Räz's proposed solution, even if it was needed, is no solution at all, and would require parameters all the same.[6]

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