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Mohamed Mostafa Yousef Bassyouny Elshabasy
Old Dominion University

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Mitigating Crack Propagation in a Highly Maneuverable Flight Vehicle Using Life Extending Control Logic

by

Mohamed Mostafa Yousef Bassyoueny Elshabasy
B.S. June 1995, Alexandria University, Egypt
M.S. June 2001, Alexandria University, Egypt

A Dissertation Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirement for the Degree of DOCTOR OF PHILOSOPHY AEROSPACE ENGINEERING OLD DOMINION UNIVERSITY May 2009

Approved by:

_______________________________
Brett A. Newman (Director)

_______________________________
Osama A. Kandil (Member)

_______________________________
Duc T. Nguyen (Member)

_______________________________
Keejoo Lee (Member)
ABSTRACT

Mitigating Crack Propagation in a Highly Maneuverable Flight Vehicle Using Life Extending Control Logic

Mohamed Mostafa Yousef Bassyouny Elshabasy
Old Dominion University, May 2009
Director: Dr. Brett A. Newman

In this research, life extending control logic is proposed to reduce the cost of treating the aging problem of military aircraft structures and to avoid catastrophic failures and fatal accidents due to undetected cracks in the airframe components. The life extending control logic is based on load tailoring to facilitate a desired stress sequence that prolongs the structural life of the cracked airframe components by exploiting certain nonlinear crack retardation phenomena. The load is tailored to include infrequent injections of a single-cycle overload or a single-cycle overload and underload. These irregular loadings have an anti-intuitive but beneficial effect, which has been experimentally validated, on the extension of the operational structural life of the aircraft. A rigid six-degree-of-freedom dynamic model of a highly maneuverable air vehicle coupled with an elastic dynamic wing model is used to generate the stress history at the lower skin of the wing. A three-dimensional equivalent plate finite element model is used to calculate the stress in the cracked skin. The plate is chosen to be of uniform chord-wise and span-wise thickness where the mechanical properties are assigned using an ad-hoc approach to mimic the full scale wing model. An in-extensional 3-node triangular element is used as the gridding finite element while the aerodynamic load is calculated using the vortex-lattice method where each lattice is laid upon two triangular finite elements with common hypotenuse. The aerodynamic loads, along with the base-excitation which is due to the motion of the rigid aircraft model, are the driving forces acting on the wing finite element
model. An aerodynamic control surface is modulated based on the proposed life extending control logic within an existing flight control system without requiring major modification. One of the main goals of life extending control logic is to enhance the aircraft's service life, without incurring significant loss of vehicle dynamic performance. The value of the control-surface deflection angle is modulated so that the created overstress is sufficiently below the yield stress of the panel material. The results show that extension in crack length was reduced by 40% to 75% with an absence of damage mitigation logic. Moreover, the desired structural integrity is satisfied without affecting the air vehicle dynamic performance.

Members of Advisory Committee: Dr. Osama A. Kandil Dr. Duc T. Nguyen Dr. Keejoo Lee
This dissertation is dedicated to my late parents, Kawther Salem and Mostafa Elshabasy (May ALLAH shower them with his mercy)
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NOMENCLATURE

English and Greek Symbols

\( G \)  Strain energy release rate
\( K \)  Stress intensity factor
\( K_c \)  Critical stress intensity factor
\( \rho \)  Radius of curvature at the point of interest
\( \sigma \)  Applied stress
\( 2\gamma \)  Surface energy connected with traction-free surface
\( E \)  Modulus of elasticity
\( a \)  Crack length
\( G_c \)  Critical strain energy, which is required for creating new unit area
\( \sigma_{max}, S_{max} \)  Maximum stress applied in cyclic loading
\( \sigma_{min}, S_{min} \)  Minimum stress applied in cyclic loading
\( \sigma_a \)  Stress amplitude
\( \sigma_r \)  Stress range
\( \sigma_m \)  Mean stress
\( R \)  Stress ratio, i.e., ratio between minimum and maximum stress
\( K_t \)  Stress concentration factor
\( N_i \)  Number of loading or straining cycles required to develop a microcrack
\( N_p \)  Number of cycles required to propagate a crack to some critical dimension
\( n_i \)  Number of applied load cycles at constant stress level \( S_i \)
\( N_i \)  Fatigue life at constant stress level \( S_i \), obtained from the S-N curve
\( \alpha \)  Material parameter that differ from one material to another
\( \beta \)  Material dependent parameter that must be experimentally determined

\( a \)  Half length of effective fully elastic crack
\( S_y \)  Yield strength
\( S_u \)  Ultimate strength
\( S_{flow} \)  Flow stress
\( a_c \)  Critical crack length
\( K_c \) Fracture toughness calculated at the critical crack length
\( S_{ul} \) Underload stress value
\( P \) Roll rate
\( Q \) Pitch rate
\( R \) Yaw rate
\( \gamma \) Heading angle
\( \theta \) Pitch angle
\( \phi \) Roll angle
\( \theta_{th} \) Throttle percentage
\( \delta_h \) Horizontal stabilizer deflection
\( \delta_f \) Flaperon deflection
\( \delta_r \) Rudder deflection
\( \delta_{sb} \) Speed brake deflection
\( \delta_{lef} \) Leading edge flap deflection
\( H_e \) Engine angular momentum
\( I_x \) Moment of inertia about \( X \) body axis
\( I_y \) Moment of inertia about \( Y \) body axis
\( I_z \) Moment of inertia about \( Z \) body axis
\( I_{xz} \) Product of inertia with respect to \( X \) and \( Z \) body axes
\( I_{yz} \) Product of inertia with respect to \( Y \) and \( Z \) body axes
\( I_{xy} \) Product of inertia with respect to \( X \) and \( Y \) body axes
\( V_T \) Aircraft total velocity
\( M \) Mach number
\( C_{X_T} \) Total \( X \) body axis force coefficient
\( C_{X_{lef}} \) Leading edge flap \( X \) body axis force coefficient
\( C_{X_{sb}} \) Speed brake \( X \) body axis force coefficient
\( C_{X_q} \) Pitch rate \( X \) body axis force coefficient
\( C_{X_{hs}} \) Horizontal stabilizer \( X \) body axis force coefficient
\( C_{Y_T} \) Total \( Y \) body axis force coefficient
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<tr>
<td>$C_{Y_{sb}}$</td>
<td>Speed brake $Y$ body axis force coefficient</td>
</tr>
<tr>
<td>$C_{Y_{r}}$</td>
<td>Roll rate $Y$ body axis force coefficient</td>
</tr>
<tr>
<td>$C_{Y_{sf}}$</td>
<td>Flaperon $Y$ body axis force coefficient</td>
</tr>
<tr>
<td>$C_{Y_{r}}$</td>
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<tr>
<td>$C_{Z_{r}}$</td>
<td>Total $Z$ body axis force coefficient</td>
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<tr>
<td>$C_{Z_{lf}}$</td>
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</tr>
<tr>
<td>$C_{Z_{sb}}$</td>
<td>Speed brake $Z$ body axis force coefficient</td>
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<tr>
<td>$C_{Z_{q}}$</td>
<td>Pitch rate $Z$ body axis force coefficient</td>
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<td>$C_{R_{r}}$</td>
<td>Total rolling moment coefficient</td>
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<tr>
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<tr>
<td>$C_{l_{r}}$</td>
<td>Yaw rate rolling moment coefficient</td>
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<tr>
<td>$C_{l_{r}}$</td>
<td>Rudder rolling moment coefficient</td>
</tr>
<tr>
<td>$C_{s_{p}}$</td>
<td>Sideslip angle rolling moment coefficient</td>
</tr>
<tr>
<td>$C_{m_{r}}$</td>
<td>Total pitching moment coefficient</td>
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<tr>
<td>$C_{m_{r}}$</td>
<td>Total yawing moment coefficient</td>
</tr>
<tr>
<td>$C_{m_{sf}}$</td>
<td>Leading edge flap yawing moment coefficient</td>
</tr>
<tr>
<td>$C_{n_{q}}$</td>
<td>Yaw rate yawing moment coefficient</td>
</tr>
<tr>
<td>$C_{n_{r}}$</td>
<td>Roll rate yawing moment coefficient</td>
</tr>
</tbody>
</table>
\( C_{np} \)  
Sideslip angle yawing moment coefficient

\( C_{ns} \)  
Rudder yawing moment coefficient

\( C_{nsf} \)  
Flaperon yawing moment coefficient

\( \bar{c} \)  
Wing mean aerodynamic chord

\( b \)  
Wing span

\( \eta_{\delta_s} \)  
Horizontal stabilizer effectiveness factor

\( x_{cm} \)  
Position of the air vehicle center of mass

\( x_{cm_r} \)  
Reference position of air vehicle center of mass

\( P_1 \)  
Engine power command

\( P_2 \)  
Engine intermediate power command

\( P_3 \)  
Engine current power command

\( \tau_f \)  
Thrust time constant

\( T_{mil} \)  
Military thrust

\( T_{idle} \)  
Idle thrust

\( T_{max} \)  
Maximum thrust

\( u \)  
Velocity component in \( X \) body axis

\( v \)  
Velocity component in \( Y \) body axis

\( w_{A/C} \)  
Velocity component in \( Z \) body axis

\( \bar{X} \)  
Aerodynamic force in \( X \) body axis

\( \bar{Y} \)  
Aerodynamic force in \( Y \) body axis

\( \bar{Z} \)  
Aerodynamic force in \( Z \) body axis

\( \bar{L} \)  
Aerodynamic moment around \( X \) body axis

\( \bar{M} \)  
Aerodynamic moment around \( Y \) body axis

\( \bar{N} \)  
Aerodynamic moment around \( Z \) body axis

\( \bar{q} \)  
Dynamic pressure

\( p \)  
Static pressure

\( u_x, u_y, u_z \)  
Displacement component of finite element node in \( x, y, \) and \( z \) axes

\( w \)  
Lateral displacement of the middle surface
\( \psi_x \) Rotation of the surface normal about the negative x axis
\( \psi_y \) Rotation of the surface normal about the positive y axis
\( \xi_1, \xi_2, \xi_3 \) Area coordinates
\( L_b, M_i \) Interpolation functions
\( \{w_i\} \) Element out of plane nodal displacements
\( \{w_r\} \) Element rotational nodal displacements
\( A \) Area of finite triangular element
\( \{\kappa\} \) Curvature vector
\( \{\varepsilon\} \) Element strain vector
\( \{\gamma\} \) Element shear strain vector
\( [C_e] \) Element strain interpolation matrix
\( [C_{nr}], [C_{rv}] \) Element shear strain interpolation matrix
\( \sigma_x \) Normal stress in the local x axis
\( \sigma_y \) Normal stress in the local y axis
\( \sigma_z \) Normal stress in the local z axis
\( \tau_{xy} \) Shear stress
\( \nu \) Poisson’s ratio
\( F_a \) Surface traction due aerodynamic loads
\( [\Phi] \) Matrix of eigen vectors
\( \{q^n\} \) Vector of modal coordinates
\( [\bar{M}] \) Modal mass matrix
\( [\bar{K}] \) Modal stiffness matrix
\( \bar{\Omega} \) Vorticity vector
\( \Gamma \) Strength of vector tube
\( \Phi \) Velocity potential
\( x_m, y_m, z_m \) Cartesian coordinates of the panel control point
\( \vec{V}_{m,n} \) Velocity induced at the \( m^{th} \) control point by the \( n^{th} \) vortex
\( \hat{C}_{m,n} \) Influence coefficient
\( \delta_m \) Slope of the mean camber at the control point \( m \)
\( \phi_w \) Dihedral angle
\( \alpha_{sh} \) Shear relaxation or correction factor
\( F_{tn} \) Traction at panel \( n \)
\( C_n \) Panel mean chord length
\( \delta_n \) Slope of mean camber at the control point of panel \( n \)
\( C_L \) Theoretical total lift coefficient
\( A_n \) Area of the panel \( n \)
\( \sigma_1, \sigma_2 \) Principal normal stresses
CHAPTER 1

INTRODUCTION

1.1 Problem Motivation and Description

The complexity of modern commercial and military aircraft, together with their various subsystems, have dramatically increased the costs for both initial purchase and in-service support. These increased costs, together with reduced national budgets and profit margins, coupled with rising fuel and operational costs, have forced aircraft to remain in service for many years beyond what was originally anticipated. These trends are the origin of an aging aircraft predicament. The reality of aging aircraft, and the consequences of aging, was first established on 13 March 1958 when the United States Air Force (USAF) lost two B-47 aircraft because of accelerated, long-term fatigue which led to cracks in the wing structure. The in-flight structural failure of an Aloha Airlines 737-200 on 28 April 1988 (see Figure 1.1) brought immediate attention to the aging aircraft issue. This failure was due to long-term corrosion fatigue that lay hidden from routine maintenance inspections. This accident in particular resulted in several large industry and government programs addressing aging aircraft. Several works describe some of these programs. Embedded within these aging aircraft programs are research and development initiatives to address many problems, the most significant of which deals with aircraft structural integrity resulting from aging problems due to fatigue cracking, stress corrosion cracking, corrosion-fatigue interactions, wear, fatigue mitigation, corrosion prevention, failure analysis, and life prediction technologies.

The journal model for this dissertation is the Journal of Guidance, Control, and Dynamics.
Maintenance of fatigue damage in aircraft components requires routine monitoring of crack size, stop drilling treatment, replacement of parts, tear down and build up of complex structures, and many other labor intense processes. Commercial aviation support, which includes repair, parts, and maintenance for fatigue-related damage, reached $47.5 billion in 1999. To compound the problem, commercial air carriers are facing aged airframe fleets. Table 1.1 shows the average fleet age for selected air carriers as reported by the Federal Aviation Administration (FAA) to the Bureau of Transportation Statistics (BTS). Note that average fleet age ranges from 12 to 50 years of service. The situation is no better in the military aviation facet. Surveys have found that over 51% of all the aircraft operated by USAF are older than 15 years, with 44% having greater than 25 years in service. Figure 1.2 shows the average age of active-service U.S. Navy aircraft rose from 11 years to beyond 16 years between 1980 and 2000. Over the period from 1949 through 1999, the average age of active-service U.S.
Air Force aircraft continually rose and now exceeds 20 years as shown in Figure 1.3. Of even greater concern is that some aircraft, namely the B-52, KC-135, and T-37, are expected to remain in service until 2015, when they will have been used for over 50 years. When an aircraft structure exhibits a rapid increase in the number of fractures in critical areas, a decision to undertake major modifications, perform structural replacement, or retire the aircraft, all being very costly, is required. Although it appears impossible not to reach this point for any high service aircraft, certain cost savings upstream of this point can be achieved by improved monitoring of fatigue damage, better fatigue growth prediction, and adjusted maintenance and repair schedules. Additionally, to slow the accumulation rate of fatigue damage, operational changes such as gust avoidance systems and active or passive load alleviation systems can be considered.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Model</th>
<th>Average Fleet Age (Years)</th>
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<tbody>
<tr>
<td>Airbus</td>
<td>A300-600</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>A310-2CF</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>A319-1</td>
<td>12</td>
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<tr>
<td></td>
<td>A320-1/2</td>
<td>16</td>
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<tr>
<td></td>
<td>A300X4</td>
<td>26</td>
</tr>
<tr>
<td>Boeing</td>
<td>B-727-1</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>B-727-2</td>
<td>31</td>
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<tr>
<td></td>
<td>B-737-C</td>
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<tr>
<td></td>
<td>B-747-1</td>
<td>37</td>
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<td></td>
<td>B-757</td>
<td>17</td>
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<tr>
<td></td>
<td>B-767-2</td>
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<td></td>
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<td>B747-2/3</td>
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<tr>
<td>Douglas</td>
<td>DC-10-1</td>
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<td>DC-10-4</td>
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<td></td>
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<td>MD-90</td>
<td>13</td>
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<tr>
<td>Lockheed</td>
<td>L-1011</td>
<td>32</td>
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<tr>
<td>Aircraft</td>
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<tr>
<td>L-188A</td>
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</tr>
<tr>
<td>L-382E</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>L1011-5</td>
<td>28</td>
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</tr>
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</table>

One area having potential for reducing maintenance, overhaul, and retirement expenses is the consideration of advanced breakthrough concepts and technologies that dramatically slow the rate of fatigue damage throughout the service life. For example, new fracture mechanics methodologies provide significant improvement in understanding nonlinear crack-growth behavior. Recent experimental and theoretical development has
focused on characterizing and modeling nonlinear crack-growth behavior including acceleration, retardation, and complete arrest of crack propagation due to overload applications. In addition, recent investigations show the existence of non-intuitive optimal overload stress and interval parameters that minimize crack-growth. Initial investigations to exploit this nonlinear behavior during flight by active feedback control are described in References 17-19. A second set of investigations noted in References 20-23 also have been considered. In the second set of investigations, the flight control system performs load tailoring functions, including both alleviation and/or amplification, of excitations in order to maintain the optimal overload stress conditions. A system of this type could be thought of as a generalization to typical gust and maneuver load alleviation systems widely used in commercial and military aircraft today. Each of these studies employed and/or emphasized various modeling assumptions, control strategies and theories, short term or long-term structural enhancements, and impacts on flight dynamic performance and stability. Each study indicates a significant potential for enhanced structural integrity, motivating further investigations undertaken here.

This dissertation focuses on mitigating crack extension in a highly maneuverable flight vehicle using life extending control logic. The feasibility and potential of life extension control (LEC) logic for reducing fatigue within aerospace vehicle structural components is explored. Reduced fatigue damage shall be addressed by exploiting nonlinear crack retardation behavior through load tailoring with a flight control system. A full envelope model of a highly maneuverable rigid aircraft with separate flexible wing model and control system coupled to a dynamic crack-growth model is used in the multidisciplinary and multiscale investigation. A complete mission from just after takeoff
to just prior to landing is simulated to provide a realistic structural loading environment. The control system consists of a baseline component providing stability augmentation and autopilot functions, and a separate component for load tailoring to increase structural life. Objectives of this research are to

1. Model crack propagation dynamics imbedded within a flight system,
2. Explore feasibility of the LEC concept,
3. Quantify potential enhancement to structural integrity from the LEC concept,
4. Identify practical implementation for the LEC concept, and
5. Assess flight stability and performance trades with the LEC concept.

This work is highly related to Reference 23 and is an extension thereof. However, unique aspects will set this work apart from that in Reference 23. First, online monitoring of the structural deterioration is considered on a mission-by-mission basis, as opposed to continuous time monitoring. Further, the control logic is simpler for implementation purposes. Moreover, this research utilizes improved and more accurate models for crack propagation, wing structural dynamics, and flight mechanics. The LEC concept exhibits high risk and hence may not be initially suitable for commercial applications. For this reason, the research focuses on a military airframe application. An aircraft that is widely used and which will eventually face aging concerns is the USAF F-16. For those reasons, the F-16 is selected as the research application. Another primary reason for this selection is that several models for the F-16 structural wing, flight dynamics, and core flight control system are readily available. This research will integrate various engineering models addressing fracture mechanics, structural dynamics, flight dynamics, and flight
control to investigate the feasibility and benefit of LEC. Thus, literature review in each of these disciplines is addressed next.

1.2 Fatigue and Fracture Mechanics Literature

In any given structural component, up to three types of discontinuities, each with its own size or number distribution, will exist, as shown in Figure 1.4. The first type consists of intrinsic material discontinuities. These discontinuities are the result of the material production process (alloying, heat treating, forming, and so on) and include porosity, microcracks, inclusions, and surface pits. The maximum intrinsic discontinuity size for this type effectively defines the initial discontinuity size for continuing damage analysis. The second group of discontinuities is introduced during fabrication (machining, assembly, finishing, and so on). These defects include machining marks, scratches, or any type of damage that could produce a crack like discontinuity. Establishing the maximum size of a fabrication discontinuity that could escape detection and thus exist in a structural component at the time that the mechanical part enters service is of interest. The third group, which is formed by and grows with in-service usage, is of primary interest to long-term structural integrity. Damage tolerance requirements are imposed during the operational phase by requiring that cracks (both fabrication and service induced) do not grow beyond a critical size within a specified period of time (either the design service life or the required test interval).

Repetitive or cyclic loads applied to these structural components will lead to propagation of the inherent flaws. Even at stress levels well below yield strength, these microscopic flaws can accumulate and spread with continuity of load repetitions until they develop into significant damage that can lead to catastrophic failure. This damage
mechanism is called fracture mechanics and one of the motivators of that mechanism is called fatigue. Strength failures of load bearing structures can be either of the yielding or fracture types. For fracture dominant failures, the size scale of the defects that are of major significance is essentially macroscopic, since general plasticity is not involved but is present only in local stress-strain fields associated with the defects. The evolution of structural design to include fracture mechanics has proceeded through a series of stages, depicted in Figure 1.5. The fifth stage represents the kind of activity that has been initiated within the past thirty years. In this stage explicit recognition is given to the perspective that cracks exist in every engineering structure, whether arising from initial defects in the material, from fabrication flaws, or from service conditions.

![Figure 1.4 Discontinuity Number-Size Distributions in Structural Components](image)

Traditionally, material fatigue performance is characterized by the Wöhler curve, commonly denoted as a S-N curve. This curve is a graph of the magnitude of a cyclical stress ($S$ or $\sigma$) against the cycles to failure ($N$). S-N curves are derived from tests on samples of the material to be characterized where a regular sinusoidal stress is applied by
a testing machine that also counts the number of cycles to failure. This process is sometimes known as coupon testing. Each coupon test generates a point on the plot. Common load terminologies are indicated in Figure 1.6. Fatigue failure under constant amplitude loading generally forms the basis of fundamental studies. Constant amplitude stress-life, strain-life, and crack-growth rate properties have contributed much to the understanding of fatigue behavior, materials selection, and life predictions. Damage accumulation theories have been developed to analytically characterize this type of experimental data to predict the remaining amount of structural life. Damage models by Miner, Manson, and others are typical but have several limitations when attempting to describe in-service airframe structural geometries and loading environments. Cycle integration based damage accumulation approaches appear to be superior.

Figure 1.5 Evolution of Structural Design
Aircraft structural components often consist of complicated geometries containing numerous sources of stress risers, such as, holes, notches, fillets, taper, curvature, corners, edge discontinuities, rivets, welds, fasteners, and many others. The stress field close to these regions is significant and can influence fatigue life. For example, Figure 1.7 shows a stress-load cycle curve for both notched and plain 7075-T6 aluminum specimens. The structural life of the notched specimen is extensively reduced relative to the unnotched specimen. Furthermore, the loading environment during flight is highly variable and includes both deterministic and stochastic traits associated with load mean, cyclic amplitude, overload strength, and load sequence and frequency. These loadings also originate from various sources including once-per-flight events, maneuvering, and atmospheric turbulence. The loading is not easily modeled by constant amplitude sinusoidal signals. The service loading time history of actual aircraft structural components can be quite complex, as shown in Figure 1.8.

![Figure 1.6 Typical Cyclic Loading Parameters](image)

Early theories of fracture phenomena were conceived by Griffith and were based on energy balance. Irwin refined the theory, introduced the idea of strain energy release rate \((G)\), and connected this theory to stress intensity factor \((K)\). Irwin contributed another major advance by showing that the energy approach is equivalent to the stress intensity
approach, according to which fracture occurs when a critical stress distribution ahead of the crack tip is reached. Demonstration of the equivalence of $G$ and $K$ provided the basis for development of the discipline of linear elastic fracture mechanics (LEFM), which allows for the invariance of stress in a region near the crack tip. Thus, tests on suitably shaped and loaded specimens to determine the critical stress intensity factor ($K_c$) make it possible to determine what flaws are tolerable in deployed structures under given conditions. Furthermore, materials can be compared as to their utility in situations where fracture is likely. Another branch of fracture mechanics, elastic-plastic fracture mechanics (EPFM), can be traced back to Wells' work on crack opening displacement (COD), which was published in 1961.\textsuperscript{42} EPFM is still very much an evolving discipline.

Concerning further fracture mechanics developments, a new parameter for fracture was provided by Rice, the so called J-integral.\textsuperscript{43} This parameter is independent of the integration path around the crack tip and is also used as a crack-growth criterion. From the LEFM point of view, the J-integral is equivalent to the strain energy release rate.

![Figure 1.7 S-N Curves for Different Specimens of 7075-T6 Aluminum Alloy](image-url)

Figure 1.7 S-N Curves for Different Specimens of 7075-T6 Aluminum Alloy
Figure 1.8 Standard Fighter Load Spectrum

Figure 1.9 shows an axially loaded specimen with far-field stress $\sigma_0$ containing an internal crack of length $2a$. LEFM shows that stress is maximum at the crack tip and decreases to the nominal applied stress with increasing distance away from the crack. With this insight, it is clear why cracks tend to originate from rivet holes and other stress riser like corners, keyways, grooves, welding defects, localized corrosion, etc.

Considering a hypothetical, infinite thin sheet of homogeneous and isotropic material (see Figure 1.10), which is subjected to remote tension in the $y$ direction, the normal stress near the crack tip along the $y$ axis is represented by an infinite series as

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} + \cdots \right]$$

(1.1)

where $K$ is a positive multiplying "stress intensity factor" that depends on the boundary loading conditions and crack size. Parameters $r$ and $\theta$ locate the point of interest with respect to the crack tip as shown in Figure 1.10. According to Equation (1.1), the stress value varies inversely with the square root of the radial distance from the crack tip and is
theoretically infinite at the crack tip, where \( r = 0 \). Based on the boundary conditions of the infinite sheet with thin crack, the stress intensity factor is

\[
K = \sqrt{\pi a \sigma_y}
\]  
(1.2)

where \( a \) is the half crack length and \( \sigma_y \) is the remote applied stress. Reference 44 lists many refinements to this theory. In Equation (1.2), the parameter \( (K / \sigma_y)^2 \) is often considered as a measure of fatigue resistance since it is proportional to crack length.

Figure 1.9 Stress Profile Along x-x

Another measure of fatigue resistance, which is based on consideration of small plastic regions near the crack tip, is the crack opening displacement \( (\delta) \), which is illustrated in Figure 1.11.\(^{45,46}\) The crack opening displacement is the height of the effective crack at the elastic-plastic boundary of the actual crack. An expression for \( \delta \) is

\[
\delta = \frac{4}{\pi} \frac{K^2}{E \sigma_y}
\]  
(1.3)
where $E$ denotes the material modulus of elasticity. The forms of Equations (1.2) and (1.3) describe only static fracture mechanics relationships.

To capture the fundamental behavior of crack-growth, considerable research has addressed dynamic relationships. In particular crack-growth rate laws such as

$$
\frac{da}{dN} = f(a, K, R, \cdots)
$$

have been explored where $R$ is the ratio between the minimum and maximum applied stresses, $a$ is the crack length at cycle $N$, and $K$ is the stress intensity factor. With such relationships and applicable loading characteristics, analytical or numerical integration can be performed to project crack length vs. service life behavior. As the theoretical-based crack-growth laws possess various inaccuracies, the most widely accepted technique for growth law development is a semi-empirical approach built around the factor $\sqrt{a\sigma}$. Paris and Erdogen\textsuperscript{47,48} recommend a growth law of the form

$$
\frac{da}{dN} = C_0 (K_{\text{max}})^n
$$
where $C_0$ and $n$ are empirical constants and $K_{\text{max}}$ is the maximum stress intensity factor for constant amplitude cyclic loading. A modified form that fits a wide range of materials, geometries, and loadings was developed as

$$\frac{da}{dN} = C_0 (\Delta K)^n$$

(1.6)

where $\Delta K = K_{\text{max}} - K_{\text{min}}$ and $K_{\text{max}}$ and $K_{\text{min}}$ correspond to the maximum and minimum stress intensity factors for a variable amplitude loading.46

Analytical and/or numerical prediction of crack propagation is usually based on test data which is applicable to the case under consideration. The important parameters include the type of material, structural geometry, environmental conditions, load sequence, and other factors. Such data is typically available in a nonlinear plot of crack-growth rate $\frac{da}{dN}$ versus the stress intensity factor $K$, i.e., $\frac{da}{dN} = f(\Delta K, K_{\text{max}})$. When the loading conditions are known, life prediction can be made by an integration procedure using

$$\int_{N_{ad}}^{N_{ac}} dN = \int_{a_d}^{a_c} \frac{da}{f(\Delta K, K_{\text{max}})}$$

(1.7a)

in which $a_d$ is the minimum detected crack size and $a_c$ is the critical crack length. For the case of constant amplitude loading at $S_{\text{min}}/S_{\text{max}} = 0$, the integration is derivable in closed-form when a power relationship, $f(\Delta K, K_{\text{max}}) = C \Delta K^n$, is assumed.

$$\int_{N_{ad}}^{N_{ac}} dN = \int_{a_d}^{a_c} \frac{da}{C \Delta K^n} = \frac{1}{C S_a} \int_{a_d}^{a_c} \frac{da}{(\sqrt{\pi a})^n}$$

(1.7b)
In Equation (1.7b), \( N_d \) is the number of cycles at \( a_d \) (the first detected crack length) and \( N_c \) is the number of cycles at \( a_c \) (the critical crack length).

Until recently, crack-growth retardation and/or acceleration effects due to overload and underload were not accounted for in crack-growth rate expressions, such as in Equation (1.6). To model this behavior, such relationships must incorporate stress state memory functionality.\(^{49,50}\) A significant breakthrough in this area is the development of the crack closure concept. The crack closure concept is the basis for most loading history dependency models within growth-rate laws. This concept implies that cracks under fatigue loadings can be fully or partly closed while the material is still under tension. According to Elber,\(^{51}\) only after the load completely opens the crack at stress intensity factor \( K_{op} \) will the crack tip be stressed. Larger \( K_{op} \) values result in a reduced, effective stress range \( \Delta K_{eff} = K_{max} - K_{op} \), and this \( \Delta K_{eff} \) instead of \( \Delta K \) would be the crack propagation rate controlling parameter. In other words, Equation (1.6) is replaced by

\[
\frac{da}{dN} = A(\Delta K_{eff})^m
\]

\[
= A \left( \frac{K_{max} - K_{op} (1-R)}{1-R} \right)^m, \text{ where } R = \frac{K_{min}}{K_{max}}
\]

\[
= A \left( \frac{K_{max} - K_{op} - K_{max} R + K_{op} R}{1-R} \right)^m
\]

\[
= A \left( \frac{K_{max} - K_{min} - (K_{op} - K_{op} R)}{1-R} \right)^m
\]

\[
= A \left( \frac{\Delta K - \Delta K_{th}}{1-R} \right)^m
\]
where $\Delta K_{th}$ denotes the threshold stress intensity factor range and $A$ and $m$ are growth rate constants.

![Diagram of crack opening displacement](image)

(a) Crack Tip  
(b) Replacement of Actual Crack and Plastic Zone by Effective Elastic Crack ($a' = a + r_y$)

(c) COD: Effective Crack Height at Elastic-Plastic Boundary

Figure 1.11 Crack Opening Displacement

Crack-growth immediately following application of an overload and during subsequent constant amplitude cycling will be retarded, as shown in Figure 1.12. The overload introduces a large plastic zone near the crack tip as shown in Figure 1.13. The material in this zone is stretched to a permanent deformation, but after unloading it still has to fit within the surrounding elastic material. The elastic material resumes its original size upon load release, but the material in the plastic zone does not. Therefore, the surrounding elastic material will exert compressive stresses on the plastically deformed material at the crack tip. The resulting residual stress system is also depicted in Figure 1.13. The residual compressive stresses tend to close the crack tip over some distance.
Subsequent cycling can cause crack-growth only if the residual stresses are overcome to a degree that the crack tip is opened again. As soon as the crack has grown through the area of residual stresses, the original exponential crack propagation behavior will resume again.

Figure 1.12 Overload Induced Retardation, 2024-T3 Al-Alloy

Figure 1.13 Overload Induced Residual Compressive Stresses at Crack Tip
Multiple periodic overloads have been found to cause additional retardation and a relative high single overload may totally arrest crack-growth at subsequent low amplitude cycling. Applying negative loads in a constant amplitude test cause practically no interaction effects, but they are detrimental in an indirect way. As reported by Newman concerning tests conducted by Yisheng and Schijve, overload causes an immediate crack-growth delay, and the application of an underload, immediately after the overload, reduces some of the crack-growth delay caused by the overload (see Figure 1.14). Exploitation of the retardation effect through state space growth law predictions is central to this research.

Estimates of fatigue crack propagation under variable amplitude loading can be implemented with two different philosophies:

- Conservative integration of constant amplitude data, or
- Integration of variable amplitude data using a semi-empirical retardation model.

Conservative integration, which is analytic in nature, simply means that interaction effects are neglected. This procedure will lead to an under estimate of structural life since interaction effects give rise to retardation in most cases except excessively large overloads. The cycle-by-cycle discrete integration using variable amplitude loads can be carried out numerically and typically makes use of a semi-empirical retardation model. This integration procedure is more accurate than the conservative procedure at the expense of complication in accounting for interaction effects. Any theory predicting sequence effects should include the evaluation of residual stresses and crack closure. Two noteworthy attempts have been made to implement this strategy for the prediction of fatigue life by Morrow and Impellizzeri.
Based on Elber's hypothesis, several analytical models have been developed to account for load interaction mechanisms. In these models, the retardation phenomenon is only considered within the plastic zone formed in the vicinity of the crack tip. The most famous models in the literature are those developed by Wheeler and Willenborg. According to their models, retardation takes place as long as the crack with length $a$ and with its accompanying secondary plastic zone remains within the primary plastic zone created by the preceding overload (see Figure 1.15). The main difference between the two models is that Willenborg accounted for the retardation effect by reducing both the maximum and minimum stress intensity factors acting on the crack tip, while Wheeler takes into account the retardation effect caused by direct reduction of the crack propagation rate $da/dN$ using a retardation function. Generally, the load interaction models can be divided into four categories:

- $da/dN$ models, such as the Wheeler model, which use retardation functions to directly reduce the crack propagation rate,
• $\Delta K$ models, which use the retardation function to reduce the value of the stress intensity range,\textsuperscript{58}

• $R_{\text{eff}}$ models, such as the Willenborg\textsuperscript{60} model, which use an effective stress ratio $R_{\text{eff}}$, calculated by reducing the maximum and minimum stress intensity factors acting on the crack tip (see Figure 1.16), and

• $K_{\text{op}}$ models, which use estimates of the opening stress intensity factor $K_{\text{op}}$ to directly account for Elber-type crack closure.\textsuperscript{61}

![Figure 1.15 Yield Zone Retardation Model Used by Wheeler and Willenborg](image)

Figure 1.15 Yield Zone Retardation Model Used by Wheeler and Willenborg

![Figure 1.16 Stress Intensity Factors in $R_{\text{eff}}$ Model](image)

Figure 1.16 Stress Intensity Factors in $R_{\text{eff}}$ Model
Another class of crack-growth models is based on the crack opening stress and is interpreted as a state space system. Crack opening stress, $S^o$, is defined as the far-field stress required to overcome the asperity-induced contact stresses along the crack and it can be determined experimentally by conducting a compression test. When the material yields under compression, the applied stress is defined as $-S^o$. Newman\textsuperscript{62} found that $S^o$ is a function of the max stress, the stress ratio $S_{min}/S_{max}$, and the thickness of the tested specimen. The majority of dynamic crack-growth models have focused on a constant amplitude stress cycle,\textsuperscript{63} where the crack opening stress $S^o$ is assumed to be constant. On the other hand, Anderson\textsuperscript{44} observed that $S^o$ depends on the history of cyclic stress, especially if the stress amplitude is variable. Therefore, for variable amplitude loading, the history of stress cycles has been taken into account by researchers like Newman\textsuperscript{64,65} and Ray.\textsuperscript{66}

The proposed models by Patankar\textsuperscript{50} and Ray,\textsuperscript{67,68} referred to as state space models, are based on the crack closure concept. The state variables in this kind of model include the crack length and the crack opening stress. The state space model is capable of capturing the effects of stress overload and underload (including compressive stresses) on crack retardation and acceleration. Furthermore, the state space model recursively computes the crack opening stress via a simple functional relationship that does not require a stacked array of peaks and valleys of stress history for its execution. Consequently, the savings in both computation time and memory requirement are very significant. As such, the state space model is suitable for real-time damage monitoring, prediction, and control for in-service structures. As the implementation of the state space model mainly depends on updating the crack length, the selection of crack length as a state variable is an obvious
choice. However, selection of crack opening stress as another state variable is not so obvious. Jacoby demonstrated that retardation in the crack propagation might extend beyond the overload plastic zone. Therefore, selection of $S^o$ as a state variable is preferable to the selection of the plastic zone radius as another state variable. After studying various data sets of crack-growth, Shijve said "The problem of predicting crack-growth rates in-service cannot be solved without a thorough knowledge of load-time history in-service...and knowledge of load sequence is essential." The application of an overload should generate a positive pulse to excite an appropriate state space equation. Once this overload pulse reaches its peak, decay of $S^o$ should be very slow. Hence, upon application of a large positive overload, the peak of $S^o$ may be significantly larger than its steady state value. Upon application of another overload with a value less than the previous one and when $S^o$ is still larger than its steady state value, the smaller second overload should not induce any significant effect. Accordingly, the selection of $S^o$ as a state variable is quite effective for the crack-growth model.

Development of the flight load profile presumes that the sequence of loads is essentially deterministic, but this is not true because gusts and maneuvers generally occur in a random fashion. The portion of the profile most deterministic, however, is the ground-to-air-to-ground cycle and its occurrence once-per-flight in programmed growth tests is significant. Once-per-flight loads may include cabin pressurization and the reversal of load on the wings. Landing impact, ground rolls during landing and take off, taxiing, turning, and braking constitute the source of ground-induced loadings. The loadings experienced during operation depend to a large degree upon the type of aircraft (e.g., Fighter, Bomber, Transport, etc.) and upon the particular mission being flown.
Corresponding maneuver loads result when the airframe is accelerated around one or more of its axes by the deflection of the control surfaces. Naturally induced loadings due to atmospheric turbulence result when the airframe penetrates air masses moving transversely. Localized buffeting turbulence is initiated mainly by the shape of the aircraft and can occur in regions and cavities such as control surfaces and bomb bays. Wake turbulence is particularly significant during air-to-air refueling. Sonic loads occur in the vicinity of power plants and as pseudo noise in turbulent and separated airflow. Figure 1.17 includes the order of magnitude data for load periods and number of loads occurring in the service life of several types of aircraft.

When dealing with variable amplitude loads in conjunction with integral based damage accumulation predictions, cycle-counting plays an extremely vital role. This simplification procedure produces a lower fidelity but still representative load sequence from full spectrum recorded data. The recorded data is usually load, stress, or strain versus time. Any cycle-counting algorithm should extract and preserve from the recorded data all information relevant to the fatigue behavior of the component and neglect or underestimate non-relevant information. The three most important features are the variations in stress amplitude, mean stress, and the sequence of their fluctuations. Many cycle-counting techniques are available and may be classified into four groups, namely, the Level Crossing, Range/Mean, Rainflow, and Probability Density Function (PDF). The Rainflow counting method is discussed in detail, and a modification of this method is used in this research.
Figure 1.17 Typical Cycle Periods and Total Cycles for Aircraft Fatigue Loads

Matsuiski and Endo\textsuperscript{75} introduced a cycle-counting method which separates high and low amplitude cycles and records them in a physically meaningful way by reducing complex loading data into a series of threshold nominal stresses. The procedure has universally become the accepted method of both counting cycles and simulating the reconstructed spectra. There exist two different descriptions of rainflow counting, which look different but are known to give the same result.\textsuperscript{74} One of these two methods, depicted in Figure 1.18, is visually clear and explains why this method was given its name and the second one is detailed in Reference 75. In Figure 1.18, the illustrated short stress-time history with its time axis pointing down may be treated as if it was a section through a “pagoda” with rain flowing down every roof starting at the highest point and dripping off at each extremity. A flow stops if it:

a. Reaches the end of the signal,

b. Joins water dripping down from a higher extremity, or

c. Comes opposite a peak of the same or larger size as that at which it dripped from.
Horizontally, the length of each terminated flow is recorded as a half-cycle stress range. The first half-cycle starts at 1 and stops opposite 5 (criterion c), giving it amplitude of 1-4. The half-cycle starting at 2 stops opposite 4 (criterion c), resulting in a 2-3 amplitude. The half-cycle starting at 3 terminates when joining the “water” dripping down from 2 (criterion b), giving it a 3-2 amplitude and forming a pair with the previous half-cycle. Continuing the process results in three single half-cycles and three equal pairs. With the data, the stress history can be simplified into a 1-4-7-10 stress block and 3 half cycles 2-3-(4), 5-6-(7), and 8-9-(10).

![Stress Diagram](image)

**Figure 1.18 Rainflow Counting with Pagoda Description**

### 1.3 Structural Dynamics and Aerodynamics Literature

Airframe structural analysis and design is a challenging subject that has evolved over many decades. References 76-78 describe some of the common methodologies and techniques employed from a pure structural mechanics perspective. Because the airframe interacts with the motion, flow, and thermal environments, aerothermoelastic
perspectives for airframe analysis and design are also common.\textsuperscript{79-81} The general built-up airframe structure is complex. These airframes employ many integrated components (skin, panel, spar, rib, stringer, bulkhead, rivet, fastener, weld, etc.) that possess difficult geometries (curve, sweep, taper, twist, slender, thin, notch, cutout, etc.) and advanced materials (metal alloys, composites, low inertia but high strength and high temperature, treated/finished, etc.). Such complexities ultimately lead to adoption of computational structural mechanics (CSM) procedures for airframe analysis and design, the most widely accepted and popular technique being the finite element method (FEM).\textsuperscript{82-84} The FEM is a discretization procedure for the underlying continuous structural model that employs a network of small elements with assumed internal behaviors described by shape functions that are made compatible with adjoining elements. FEM is widely used in the aerospace industry because of the method’s generality, versatility, and reliability.

Application of the FEM to these kinds of complex structures face major obstacles regarding prohibitive preparation time for model data, large computation requirements, and associated costs. Modeling fidelity at this level is not practical, and fortunately not required, for this research. Often equivalent continuum models are used to approximate and simulate these complex airframe structures. This idea is reasonable as long as the complex structure behaves physically in a close manner to the proposed continuum model. References 85-90 describe several equivalent beam and equivalent plate theories that have been investigated and explored. For aerospace wing structures, a number of studies have focused on using equivalent cantilevered beam models to represent simple wings composed of laminated or anisotropic materials, and they have yielded accurate results for the specific global behavior that was studied.\textsuperscript{85, 89} However, using an equivalent
cantilevered plate may be more promising because a wing, especially ones that have a low aspect ratio (such as the F-16 wing), is likely to behave more as a cantilevered plate than a cantilevered beam. In Reference 90, Reissner and Stein state “For analysis of thin solid wings of small aspect ratios that might be utilized in high-speed airplanes and missiles, the beam theory is no longer adequate. Wings of this type are more nearly plate than beams and should be analyzed by plate theory.” In the absence of large axial forces, fixed-free and fixed-sliding plates are approximated as in-extensional members.91-93 Tessler and Hughes94 presented a robust and accurate, three-node, nine degrees-of-freedom triangular plate element based on Reissner-Mindlin plate theory. They concluded that the performance of this element is excellent for both moderately thick and thin plate regimes and they call the element MIN3. Utilization of equivalent plate theory with the MIN3 element is selected for this research.

Different approaches are developed in the literature to determine the aerodynamic loads acting on the wing model. These methods are either analytical or numerical.95-98 Most of the analytical methods are based on the interpretation of the fluid flow phenomena that are present in a region of flow near an airframe structure. These phenomena include the thin layer of highly viscous flow near the surface (referred to as the boundary layer), vortex flows (small tornado-like flows which are generated near wing tips and sharp leading edges), and shock wave interactions. Complexity of the flow due to viscosity, rotationality, compressibility, and abrupt changes in the airframe geometries, hampers progress in the analytical approach of aerodynamic analysis. Moreover, the rapid progress in the computational capabilities motivated researchers to implement and investigate various numerical aerodynamic approaches. Numerical
schemes are versatile and include flexibility to account for the nonlinear flow and provide more realistic representation of aerodynamic loading.

There exits at least two ways to numerically model the aerodynamic loads. One way is to model the entire flow by one of the computational fluid dynamics (CFD) schemes based on finite difference or finite volume methods to directly solve the fluid flow equations. The area of CFD involves computational techniques that use the finite numerical approximations to solve governing partial differential equations. This approach is the most accurate thus far and it is considered the ultimate approach. Unfortunately the method is very expensive in computational time because it needs to grid the entire flow. Another way to numerically model the aerodynamic loads is to use potential flow models based on singularity elements. Singularity element models are based on a general method for calculating the incompressible potential flow about arbitrary body shapes. These methods use singularities distributed over the wing and body and calculate this distribution as the solution of an integral equation. Most of these methods specify a source distribution of variable strength (e.g., source-panel models), a dipole distribution of variable strength (e.g., doublet-lattice and doublet-panel methods), or a vortex distribution of variable circulation (e.g., vortex-lattice and vortex-panel methods). From the above mentioned methods, doublet-lattice and vortex-lattice methods have been extensively used over the years in the calculation of the aerodynamic loads in potential flows. A general unsteady vortex-lattice method (VLM) was developed by Konstadinopoulos, et al. The method is an extension of the vortex-lattice technique and is not limited by aspect ratio, camber, or angle of attack, as long as vortex breakdown does not occur above the surface of the wing and separation occurs only along sharp
edges. The procedure treats steady flows more efficiently than the specialized steady flow vortex-lattice approach.

In this dissertation, the aerodynamic loads are calculated using the linear vortex-lattice method, which is thought to be well-understood in terms of capabilities and limitations. For example, the design and shape optimization of various commercial and military aircraft are based upon calculating the aerodynamic loads using the vortex-lattice approximation. The linearity of the vortex-lattice method requires the angle of attack to remain small, between 0° and 5°. The recorded values of the angle of attack during the investigated mission in this dissertation maintain this requirement of the vortex-lattice method.

1.4 Flight Dynamics and Control Literature

Flight dynamics analysis and synthesis is an inherently multidisciplinary topic. In many settings, an ideally rigid vehicle assumption is appropriate. The rigidity of the aircraft means that all points in the aircraft structure maintain fixed relative positions in space at all times. Aerodynamic stability and control derivatives, inertial distributions, propulsion behavior, and unconstrained multiaxis dynamics all contribute to the mathematical model. Linear analysis of longitudinal and lateral-directional modes is a common activity in conceptual and preliminary studies. Eventually higher fidelity nonlinear models which can be simulated in a digital computer are required. In this setting, equations of motion of the rigid vehicle are one directionally decoupled into rotational and translational equations where the coordinate origin is chosen to be at the center of the mass. The rotational motion of the aircraft will then be equivalent to yawing, pitching, and rolling motions about the center of the mass as if it were a fixed
point in space. The remaining components of the motion will be three components of translation of the mass center. The state model derived is called a six degrees-of-freedom (6 DOF) model. Nonlinear simulation with the 6 DOF model is used to investigate topics such as flight envelope determination, stability and performance assessments, handling and ride quality analysis, control law development, etc. A model of this type, to conduct a realistic flight mission, is needed in this research.

Complete rigidity is not achievable in practice where minimal use of structural mass is desired. For example, it is common to observe flexing of the wings of large commercial aircraft during flight. Even a compact fighter configuration with a sturdy fuselage and short stubby wings (such as the F-16) will exhibit flexibility effects under large loads. The true natural mathematical description of a flexible aircraft is in terms of partial differential equations, and a great deal of complications is needed to arrive at the appropriate governing relationships. A flexible aircraft dynamics model becomes quite complicated as the degrees-of-freedom associated with flexible modes, such as body bending and wing flexure, are considered. An approximate and simpler approach is presented in Reference 109. Further, Reference 110 provides a thorough survey of flexible aircraft modeling and simulation.

A flight control system (FCS) is the heart of any modern aircraft, providing inner-loop stabilized control for high-performance and stealthy airframes, as well as assisting in outer-loop trajectory management. The early generation of FCS was mechanically based, as depicted in Figure 1.19, where direct mechanical linkages were used between the pilot's cockpit controls (pitch/roll stick and rudder pedals) and the control surfaces such as tail-plane, ailerons, and rudder. This arrangement is inherently of high integrity and
low risk. Incorporating automatic control loops in the mechanical implementation was possible but difficult to implement and operationally limited. Recently, the main emphasis of FCS schemes is on digital computing with the use of inertial motion and air stream sensor units. Direct mechanical linkages have been removed and replaced with electrical signaling. Although this arrangement provides a significant reduction in complexity, to achieve the same level of integrity, multiple signal sources and several lanes of computing are necessary to provide redundancy. Hence, any failed equipment can be isolated and safe operation is ensured. Subsequent generations of FCS have been developed on many advanced aircraft with a shift toward a digital fly-by-wire control methodology, schematically shown in Figure 1.20.29

With the advent of digital computer technologies, nearly every modern aircraft concept under consideration today incorporates a FCS as an essential component for success.111 Among the category of modern, highly maneuverable aircraft, the F-16 is a primary example of a relaxed stability airframe requiring artificial stability supplements from control. The pitch stability of this vehicle is heavily dependent upon the FCS to the extent that the vehicle cannot be manually stabilized and flown without the digital fly-by-wire system. The control system changes fundamental response behaviors to task-tailored response types that are appropriate for various flight phases, such as takeoff and landing, high-altitude cruise, low-altitude terrain contour following, air refueling, etc. The control system is every bit as important as the aerodynamic shape and structural layout in achieving overall vehicle performance.

Most of the control design literature specifically associated with flight control, such as References 29 and 30, are directed toward applications where the aircraft dynamic
model is approximated reasonably well by a rigid-body model. Emphasis is typically given to stability augmentation systems and command augmentation systems such as pitch and yaw dampers, pitch rate command systems, roll rate command systems, and autopilot hold systems. The general arrangement of these types of FCS can be represented as the block schematic shown in Figure 1.21. Both conventional-based and contemporary-based design methods can be employed to construct systems displayed in Figure 1.21. Conventional-based methods include the ubiquitous Nyquist, Bode, Nichols, and Evans techniques, and variations thereof such as quantitative feedback theory (QFT), sequential loop closure, generalized gain/root loci, and singular value loop shaping. Some of the more popular contemporary-based techniques include linear quadratic regulator / linear quadratic Gaussian / loop transfer recovery (LQR/LQG/LTR), infinity norm control ($H_\infty$), eigen-space assignment, and model following. Design by parameterization is another available technique. A baseline control system of this type to fly the realistic mission will be developed in this investigation.

Considerably less emphasis has been given to flight control of flexible aircraft with structural considerations. When the vehicle becomes so flexible that structural dynamics contribute significant percentages to the total acceleration, and when significant coupling exists between rigid-body and structural motions, highly specialized flight control systems are required to provide acceptable dynamic characteristics. These types of control system are commonly referred to as ride control system, structural mode control systems, and more generally aeroelastic flight control systems. Design of such aeroelastic flight control systems which include possibly separate but interacting subsystems for traditional stability augmentation and for structural dynamics suppression is a complex
multivariable problem requiring an integrated synthesis perspective. Some significant research and applications are listed in References 118-122. Other recent studies have also been conducted on control of highly flexible vehicles.\textsuperscript{123-126}

Figure 1.19 Typical Mechanical Flight Control System

Figure 1.20 Typical Digital Fly-By-Wire Flight Control System
Figure 1.21 General Structure of the Flight Control System

One particular class of flight control systems closely related to the goal of extending structural life is commonly referred to as maneuver and gust load alleviation systems. These systems, considered passive in nature, utilize multiple surfaces to reduce structural loads during maneuvering or gust encounters. In terms of flight control, there appears to be very little past work on direct control of crack-growth and fatigue damage reduction, until recently. References 30 and 127 briefly discuss this type of control system and objective, but indicate very little research has been conducted on this topic. Reduction of crack-growth and fatigue damage in overall airframe structures is achieved indirectly, to some extent, by structural mode control systems. Reference 135 briefly describes the level of fatigue damage reduction that might be expected with such systems. The LEC logic proposed by Ray is based on a trade-off between the air vehicle dynamic performance and the desired structural durability. The logic was implemented by a designed robust controller based on the specifications of flight performance and allowable fatigue crack damage at critical points of aircraft structures that serve as
indicators of the effective service life. Another LEC logic was proposed by Yu, which was fundamentally based on changing the control parameters of the FCS to achieve the desired maneuver level when the logic was activated. The logic is activated when the number of cycles of the nominal high stresses between two successive overloads reaches an appropriate interval. The recorded stress history is also used to calculate the optimal or suboptimal overload stress, which, in turn, is used to determine the appropriate maneuver level. The desired maneuver was mainly pitching motion as the simulations show that it dominantly affects the bending stresses in the wing spar.

1.5 Dissertation Outline

This dissertation is composed of 7 chapters, including the current Chapter 1. Chapter 2 covers the dynamic crack propagation model. Emphasis is also given to validating the model with the experimental literature. In Chapter 3, a high fidelity model of the highly maneuverable F-16 aircraft is presented. The discussed model is of the rigid type containing six degrees-of-freedom and the response of the aircraft as a result of step inputs of different control surfaces is analyzed. Chapter 4 consists of 2 main sections. In the first section, a finite element formulation of the equivalent cantilevered plate that mimics the full scale wing is presented. The aerodynamic loads are calculated using the vortex-lattice method which is discussed in detail in the second section of Chapter 4. The flight control system for the aircraft model and the design of flown missions is discussed in Chapter 5. Chapter 6 contains the discussion of the proposed LEC, which is followed by comparisons of the results in the presence of the LEC with those in case of logic absence. The dissertation is concluded in Chapter 7. This chapter also contains proposals for the future investigation.
Figure 1.22 indicates the overall schematic of the LEC system. System components include the crack propagation model, the aircraft flight model, the flexible wing model, the wing aerodynamics model, the stability and control augmentation system, the autopilot, and the life extending control. The dissertation chapters develop these individual models and then integrate them to address the topic of mitigating crack extension in a highly maneuverable flight vehicle using life extending control logic.
CHAPTER 2

DYNAMIC CRACK PROPAGATION MODEL

2.1 Crack Propagation Model Selection

The analytical prediction of crack propagation is ultimately based on test data that is applicable to the case under consideration. Therefore, several requirements listed below are used to choose between various analytical crack propagation models available in the literature.

- Is the model consistent with data obtained from experiments conducted on the specimen under the same load conditions?
- Is the model versatile enough to account for different load interaction effects (underload, overload, and underload-overload combinations)?
- Does the model satisfy real-time damage monitoring requirements?

After investigating several dynamic crack propagation models from the literature based on the above criteria, combining Newman\textsuperscript{62} and Ray\textsuperscript{67} models was focused to give promising results.\textsuperscript{136,137} The selected nonlinear dynamic model used for fatigue crack growth predictions has desirable characteristics that fulfill the previously mentioned requirements, especially for considering underload effects in combination with injected overloads in a uniformly loaded structural component.

2.2 Analytical Crack Propagation Model

The analytical model presented in this chapter contains three major sections: parameter specification, state initialization, and dynamic propagation, all of which are discussed next. This model is programmed in software on a digital computer for
preliminary numerical investigations and is to be integrated with the LEC logic presented in Chapter 6.

2.2.1 Parameter Specification

The specimen considered in this model represents the ideal presentation of aircraft panels. This specimen under stress loading is illustrated in Figure 2.1 and the specimen material is assumed to be an isotropic and homogenous metallic alloy. Parameters related to the specimen geometry which include half-width $W$, thickness $t$, notch half-width $B$, and notch height $h_n$ are all defined in the first section of the analytical model. Mechanical properties of this alloy, such as yield stress $S_y$, ultimate stress $S_u$, and modulus of elasticity $E$, should also be defined in this parameter definition section. The FASTRAN II manual$^{138}$ is used for defining the maximum, minimum, and incremental constraint parameters that include $\alpha_{\text{max}}$, $\alpha_{\text{min}}$, $\Delta \alpha_{\text{max}}$, and $\Delta \alpha_{\text{min}}$. These model parameters are used for evaluating the constraint factor $\alpha$ as will be shown later. The parameter specification section of the analytical model is a minor section. In addition to the parameter definition section, the analytical model is composed of two major sections, the state initialization section and the dynamic propagation section, which are discussed next.

2.2.2 State Initialization

In this section, the dynamic model state is initialized by certain static relationships, which are functions of the model parameters. Two intermediate variables $S_{\text{flow}}$ and $\eta$ are defined in Equation (2.1). The equation of the variable $\eta$ is obtained semi-empirically from the experimental data of centered crack specimens of the metallic alloy material.$^{68}$
Unless otherwise specified, e.g., based on experimental data of a specific configuration, the constraint factor $a$ at cycle $i$ is calculated based on the parameter $\theta$, as will be shown in the dynamic propagation section. Parameter $\theta$ is computed as follows.

$$\theta = \frac{a_{\text{max}} - a_{\text{min}}}{\ln(\Delta a_{\text{max}}) - \ln(\Delta a_{\text{min}})}$$

To initialize the dynamic propagation model, one needs the initial stress peak, $S_0^{\text{max}}$, and initial stress valley, $S_0^{\text{min}}$, of the stress profile, and the initial crack length $a_0$. If the initial crack opening stress ($S_0^o$) is not given, then it can be estimated from the nonlinear function $\sigma$ given below.

$$S_0^o = S_0^{\text{max}}$$

$$S_0^{\text{max}} = \sigma(S_0^{\text{max}}, S_0^{\text{min}}, a_0, F(a_0, W))$$
Function $\sigma$ is defined as

$$\sigma_0 = \left( A_0^0 + A_0^4 R_0 + A_0^2 R_0^2 + A_0^3 R_0^3 \right) S_{\max}^0$$

where

$$R_0 = S_{\text{mod}}^{0\text{max}}$$

$$S_{\text{mod}}^{0\text{max}} = S_{0\text{min}}^{\text{mod}}$$, otherwise

$$S_{\text{mod}}^{0\text{max}} = \frac{\alpha_0 S_{0\text{min}}^{\text{mod}} + S_{\text{min}}^{\text{mod}}}{\alpha_0 + 1}$$

where $i$ represents the current cycle number,

$$A_0^0 = \left[ 0.825 - 0.34 \alpha_0 + 0.05 \alpha_0^2 \right] \left[ \cos \left( \frac{\pi S_{0\text{max}}^{\text{mod}}}{2 S_{\text{flow}}^0} F(\alpha_0, W) \right) \right]^{\frac{1}{\alpha_0}},$$

where $F(\alpha, W) = \sqrt{\sec \left( \frac{\pi \alpha}{2 W} \right)}$.

$$A_0^1 = \left[ 0.415 - 0.071 \alpha_0 \right] \left[ \frac{S_{\text{max}}^{\text{mod}}}{S_{\text{flow}}^0} \right],$$

if $R_0 > 0$

$$A_0^3 = 2 A_0^0 + A_0^1 - 1$$

$$A_0^2 = 1 - (A_0^0 + A_0^1 + A_0^3), \text{ else } A_0^0 = A_0^3 = 0$$

Once the initial crack opening stress is known, the dynamic propagation section of the crack model can be activated.

### 2.2.3 Dynamic Propagation

To execute the nonlinear dynamic section of the model, the peaks and valleys of the remote stress profile, $\{S_{0\text{max}}^i, S_{0\text{min}}^i\}$ for $i=1,2,3,\ldots$, will serve as input to the crack model. In turn, these stresses determine the effective stress intensity range, $\Delta K_i^{\text{eff}}$. The major part of the dynamic propagation section is devoted to calculating $\Delta K_i^{\text{eff}}$ as

$$\Delta K_i^{\text{eff}} = F_i \sqrt{\pi a_{i-1}} \left[ S_{i\text{max}}^{\text{mod}} - \max \left( S_{i\text{min}}^{\text{mod}}, S_{i-1}^0 \right) \right] U \left( S_{i\text{max}}^{\text{mod}} - S_{i-1}^0 \right)$$

where $F_i = F(a_{i-1}, W)$
Note that the effective stress intensity range is driven by the difference of the maximum stress and the larger of either the minimum stress or the crack opening stress. In Equation (2.4), \( U\left(S_{i\text{max}} - S_{i-1}^{\circ}\right) \) denotes the Heaviside step function, which is responsible for arresting the crack propagation when the maximum stress is less than the crack opening stress of the previous cycle.

Another part of the dynamic propagation section is the determination of crack opening stress taking into consideration the retardations and accelerations due to various load interaction effects. The crack opening stress is calculated as follows.

\[
S_i^O = \left( \frac{1}{1-\eta} \right) S_{i-1}^O + \left( \frac{\eta}{\eta+1} \right) S_i^{oss} + \left( \frac{1}{\eta+1} \right) \left( S_i^{oss} - S_{i-1}^O \right) U \left( S_i^{oss} - S_{i-1}^{\circ} \right) + \left( \frac{1}{\eta+1} \right) \left( S_i^{oss} - S_i^{oss\text{ old}} \right) U \left( S_{i-1}^{\min} - S_i^{\min} \right) \left[ 1 - U \left( S_i^{oss} - S_{i-1}^{\circ} \right) \right] \tag{2.5}
\]

where \( S_i^{oss} = \sigma(S_{i\text{max}}, S_{i\min}, \alpha_i, F(a_i, W)) \) and \( S_i^{oss\text{ old}} = \sigma(S_{i-1\text{max}}, S_{i-1\min}, \alpha_{i-1}, F(a_{i-1}, W)) \). The term \( S_i^{oss} \) is the steady state opening stress. Note the inputs \( S_i^{oss\text{ old}} \) and \( S_i^{oss} \) to Equation (2.5) are different from the instantaneous crack opening stress \( S_i^{\circ} \) found in Equation (2.4) under variable amplitude loading. The unit step function \( U \left( S_i^{oss} - S_{i-1}^{\circ} \right) \) in the third term of Equation (2.5) allows a fast rise and a slow decay of the crack opening stress value. The fourth term accounts for the effects of reverse plastic flow that occurs during the unloading conditions. Depletion of the normal plastic zone occurs when the minimum stress \( S_{i\min} \) decreases below its value in the previous cycle, which is incorporated via the unit step function \( U \left( S_{i-1}^{\min} - S_i^{\min} \right) \).

The final part of the dynamic propagation model is used to compute the crack propagation rate from \( \Delta K_i^{\text{eff}} \). This effective stress intensity range is used to determine the
crack growth rate, see Equation (1.8), which is discretely integrated for crack length prediction.

\[ \Delta a_i = h(\Delta K_i^{\text{eff}}) \]
\[ a_{i+1} = a_i + \Delta a_i \quad (2.6) \]

In Equation (2.6), \( h(\Delta K_i^{\text{eff}}) = A(\Delta K_i^{\text{eff}})^m \), where \( A \) and \( m \) are material dependent constant parameters.

The complete recursive relations for \( i \geq 1 \) in the dynamic portion of the analytical crack propagation model are given in Equation (2.7). These relations, along with the parameter specification and state initialization relations, are implemented in digital software.

\[ F_i = F\left(a_i, W\right) \]

if \( S_{i+1}^{\text{max}} > S_i^{\text{min}} \) then

\[ \Delta K_i^{\text{eff}} = F_i \sqrt{\pi} a_{i-1} \left[ S_{i+1}^{\text{max}} - \max\left(S_i^{\text{min}}, S_{i+1}^{\text{max}} \right) \right] U\left(S_i^{\text{max}} - S_i^{\text{min}} \right) \]

else

\[ \Delta K_i^{\text{eff}} \]
end if

\[ \Delta a_i = h(\Delta K_i^{\text{eff}}) \]

if \( \Delta a_i > 0 \) then

\[ \alpha_i = \alpha_{\text{max}} + \left( \ln(\Delta a_i) - \ln(\Delta a_{\text{min}}) \right) g \]
\[ \alpha_i = \min(\alpha_i, \alpha_{\text{max}}) \]
\[ \alpha_i = \max(\alpha_i, \alpha_{\text{min}}) \]

else \( \alpha_i = \alpha_{\text{max}} \)
end if

\[ S_i^{\text{max, odd}} = \sigma\left(S_i^{\text{max}}, S_i^{\text{min}}, a_i, F\left(a_i, W\right)\right) \]
\[ S_i^{\text{max}} = \sigma\left(S_i^{\text{max}}, S_i^{\text{min}}, a_i, F\left(a_i, W\right)\right) \]

if \( S_i^{\text{max}} \geq S_i^{\text{min}} \) then

\[ \Delta S_i^{\text{o}} = \left(S_i^{\text{max}} - S_i^{\text{min}}\right) \]

end if

\[ S_i^{\text{max}} = \left(S_i^{\text{max}} > S_i^{\text{min}}\right) \]
\[
\text{else}
\]
\[
\text{if} \left( S_{i}^{\text{ass}} \geq S_{i-1}^o \right) \text{then}
\]
\[
\Delta S_i^o = \left( \frac{\eta}{1+\eta} \right) \left( S_i^{\text{ass}} - S_{i-1}^o \right) + \left( \frac{1}{1+\eta} \right) \left( S_i^{\text{ass}} - S_i^{\text{ass-\text{old}}} \right)
\]
\[
\text{else}
\]
\[
\Delta S_i^o = \left( \frac{\eta}{1+\eta} \right) \left( S_i^{\text{ass}} - S_{i-1}^o \right)
\]
\[
\text{endif}
\]
\[
\text{endif}
\]
\[
a_i = a_{i-1} + \Delta a_i
\]
\[
S_i^o = S_{i-1}^o + \Delta S_i^o
\]
\[
S_i^o = \max \left( S_i^o, 0 \right)
\]

### 2.3 Case Study

The dynamic crack growth model will be used for study of nonlinear behavior due to overload and underload injection on a nominal stress loading. The structural component specimen, shown in Figure 2.1, is made from 7075-T6 aluminum alloy. Table 2.1 contains the values of the different parameters of the structural component, which is subjected to three arrangements of load history indicated in Figures 2.2a-2.2c. Each load history has a baseline constant amplitude cyclic behavior. Superimposed on top of this baseline is a periodic application of an overload, overload followed by underload, and overload preceded by underload, respectively. The number of cycles is counted starting from an initial crack length \( a_{\text{initial}} \) of 12.3 mm till a prescribed critical value of crack size \( a_{\text{final}} \) equaling 25 mm is reached. Loading parameter \( N1 \) represents the number of cycles between each successive irregular injection and \( N2 = 1 \). Note that the chosen value of \( a_{\text{initial}} \) is larger than the initial discontinuity lengths assumed by the American Society for Nondestructive Testing (ASNT) and listed in Table 2.2.\(^{139}\) Note also that the value of \( a_{\text{final}} \)
is less than the critical crack length $a_c$, beyond which immediate repair is necessary, which can be calculated from Equation (2.8).\(^{140}\)

$$a_c = \frac{1}{\pi} \left( \frac{K_c}{F S_{\text{max}}} \right)$$

(2.8)

Parameter $K_c$ is the fracture toughness calculated at $a_c$ and $F$ is the geometric factor, which is a function of crack length and crack width.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (mm)</td>
<td>1.016</td>
</tr>
<tr>
<td>$W$ (mm)</td>
<td>76.2</td>
</tr>
<tr>
<td>$S_u$ (MPa)</td>
<td>570</td>
</tr>
<tr>
<td>$S_y$ (MPa)</td>
<td>505</td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>72,000</td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
<td>1.8</td>
</tr>
<tr>
<td>$a_{\text{min}}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\Delta a_{\text{max}}$ (mm)</td>
<td>$5.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Delta a_{\text{min}}$ (mm)</td>
<td>$5.0 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

2.2a Uniform Load History with Overload Injection

2.2b Uniform Load History with Overload Followed by Underload
The crack model is initially excited by constant amplitude uniform loading with a single irregular load injected after 17,000 cycles with characteristics similar to the irregularities shown in Figure 2.2 yielding three separate cases. The uniform load is applied after the irregular load till the crack length reaches $a_{\text{final}}$. The uniform baseline cyclic stress profile oscillates between $S_{\text{max1}}$ and $S_{\text{min}}$ with no overload or underload. Loading conditions include $S_{\text{max1}} = 68.9 \text{ MPa}$, $S_{\text{max2}} = 137.8 \text{ MPa}$, $S_{\text{min}} = 0.345 \text{ MPa}$, and $S_{ul} = -40 \text{ MPa}$. The crack propagation results, plotted in Figure 2.3, show that application of overload allows the specimen to survive beyond its nominal life. For the indicated
loadings and specimen, the life can be extended by about 8,000 cycles due to the overload relative to the uniform load case.

The effect of overload is further explained by plotting the crack opening stress versus the number of cycles, as seen in Figure 2.4. In this figure, the steady state crack opening stress value is \( S^o = 55.85 \text{MPa} \). This value is maintained up to 17,000 cycles and also continues for the uniform loading case. For the irregular loading cases, the crack opening stress rapidly increases after the overload injection up to the value \( S^o = 81.3 \text{MPa} \). As the nominal stress loading continues beyond 17,000 cycles, the crack opening stress level slowly decreases back to the steady value.

In Figure 2.3a, one can notice that the crack is completely arrested directly after the overload was applied and this behavior lasts up to the cycle 21,000 point. Notice that during this period, the crack opening stress values (see Figure 2.4) are greater than the maximum stress of the base loading (\( S_{\text{max}} \)) that is equal to 68.9 MPa. Then the crack begins to propagate as the nominal stress cycles continue beyond 21,000 cycles, where the crack opening stress values are less than \( S_{\text{max}} \). The larger the stress difference beyond 21,000 cycles is, the steeper the crack propagation rate becomes. This full crack propagation arrest, directly after the overload, is due to the large plastic zone formed ahead of the crack tip and encloses the smaller plastic zone formed by the base stress loading. The moment the overload is applied, a large zone of material will be subjected to a local stress beyond the yield point. Once the overload is removed, the large plastic zone formed should fit in a smaller volume. Thus, the crack head will be compressed as long as the secondary “smaller” plastic zone is enclosed in the primary “larger” plastic zone. This process leads to complete arrest of the crack propagation for hundreds of cycles until
the complete depletion of the primary plastic zone occurs ahead of the crack tip. Recall that

2.3a Overall Behavior Under the Four Load Histories

2.3b Terminal Behavior for the Irregular Load Histories

Figure 2.3 Crack Length vs. Number of Cycles to Threshold Crack Length
the secondary plastic is formed ahead of the crack tip due to uniform loading history. The retardation effect will vanish completely once the secondary plastic enclave crosses the boundary of the primary plastic zone.

2.4a Overall Behavior

2.4b Detail Behavior

Figure 2.4 Crack Opening Stress vs. Number of Cycles
The net effect of a single cycle overload is an abrupt increase in the crack propagation rate, which is followed by a slower crack propagation rate that lasts until the primary plastic zone is completely depleted, as seen in Figure 2.5. This behavior holds even when the overload was combined with an underload. If the underload is injected directly before the overload, it will not have a significant effect on the retardation phenomena. On the contrary, when the application of the underload occurs directly after the overload, it causes a small reduction in the life extension benefit by a few hundreds of cycles, as seen in Figure 2.3b.

2.4 Model Validation

To validate the dynamic crack propagation model, consider one of the cases shown in Figure 2.2. Consider the case of injecting overload stress in a uniform load history where $S_{\text{max}1} = 68.9 \text{ MPa}$, $N_l = 2,500 \text{ cycles}$, $S_{\text{min}} = 0.345 \text{ MPa}$, $1 \leq S_{\text{max}2} / S_{\text{max}1} \leq 6$, and $N_2 = 1$. The load is applied such that the crack can propagate from $a_{\text{initial}} = 12.3 \text{ mm}$ till a prescribed critical crack size $a_{\text{final}} = 25 \text{ mm}$. For every fixed stress ratio $(S_{\text{max}2}/S_{\text{max}1})$, the propagated crack length is plotted against the number of cycles, as shown in Figure 2.6. For all cases, the repeated sequence input results in typical, exponential, crack growth behavior. During the initial increase in the overload stress ($68.9 \leq S_{\text{max}2} \leq 143.9 \text{ MPa}$), a corresponding decrease in crack growth rate can be noticed. For the final range ($143.9 \leq S_{\text{max}2} \leq 323.9 \text{ MPa}$), the crack growth rate picks up. An approximate value of $S_{\text{max}2} = 143.9 \text{ MPa}$ corresponds to minimal overall crack growth. Thus, minimum crack growth does not correspond to the minimum overload stress. In other words, there is an optimal overload ratio at which the crack propagation rate will be the minimum. An
important item to note is that after each overload application, a crack retardation segment appears, but cannot be observed in Figure 2.6 due to the axis scaling.

Plotting the number of cycles to reach the critical crack size vs. the applied overload ratios, as shown in Figure 2.7, will give more insightful explanation for the previous comments. In Figure 2.7, the optimal overload ratio is fairly close to 2.1, which is approximately the optimal value found experimentally by Dawicke and Newman for the same specimen and the same loading parameters, as seen in Figure 2.8. The closeness of Figures 2.7 and 2.8 validate the analytical dynamic crack growth model for further use.

2.5a Overall Behavior
2.5b Detail Behavior

Figure 2.5 Crack Propagation Rate vs. Number of Cycles

2.5 Optimal Overload Ratio Parameterization

The main parameters that can affect the optimal overload ratios are the underload values are applied directly after the overload injection ($S_{ul}$) and the number of cycles between each of the two successive overload applications ($NI$). To assess the effect of $S_{ul}$ and the $NI$ on the optimal overload values, the load cases depicted in Figures 2.3a and 2.3b, which roughly approximate the irregular in-flight loadings, are applied to the case study described in Section 2.3. In Reference 137, a thorough investigation about the effect of the mentioned parameters was conducted. For example, $NI$ was varied from 1,000 cycles to 8,000 cycles while the underload ratio was fixed at a certain value. Additionally the ratio $S_{ul}/S_{max}$ was varied from 0.005 (approximately no underload) to -
3.193 (significant underload) at a fixed value of $Nl$. The relationship between the normalized overload ($S_{\text{max2}}/S_{\text{max1}}$) and the number of cycles for the crack to grow between $a_{\text{initial}}$ and $a_{\text{final}}$ at different values of $Nl$ or $S_{ul}$ are plotted in multiple figures. Figure 2.9 shows that, at fixed underload value of -40 MPa, changing the $Nl$ value from between 1,000 cycles and 8,000 cycles will lead to a dramatic change in the optimal overload ratio. Figure 2.10 shows the effect of changing the underload value on the optimal overload ratio, while $Nl$ was fixed at 1,000 cycles. One can notice that increasing the underload value between the limits defined earlier will lead to a change in the optimal overload ratio.

![Figure 2.6 Crack Length vs. Number of Cycles ($S_{\text{max1}} = 68.9$ MPa and Different Overload Ratios)](image-url)
Figure 2.7 Number of Cycles to Threshold vs. Overload Stress Ratio ($S_{\text{min}} = 0.345 \text{ MPa}$, $S_{\text{max}1} = 68.9 \text{ MPa}$, $N_1 = 2,500$ cycles)

Figure 2.8 Experimental Results from Dawicke
From Figure 2.9, one can also notice that by increasing the number of cycles between each two successive loadings, $N_l$, for fixed underload ratio, the number of cycles for the crack to grow from $a_{\text{initial}}$ to $a_{\text{final}}$ will decrease. The reason for the decrease in life is that the retardation effect is only a temporary phenomenon immediately following an overload injection and will decrease or completely vanish with the increase of period between each two consequent overload injections. Moreover, the injection of underload directly after overload will reduce the retardation effect as explained before. At the fixed value of $N_l = 7,000$ cycles, the effect of applying underload directly after the overload on the number of cycles for the crack to grow from the predefined value $a_{\text{initial}}$ to the threshold value $a_{\text{final}}$ is shown in Figure 2.11.

Figure 2.9 Number of Cycles to Threshold vs. Overload Ratio ($N_l = 1,000 - 8,000$ cycles, Overload Followed by $-40$ MPa Underload)
Figure 2.10 Number of Cycles to Threshold vs. Overload Ratio ($N_I = 1,000$ cycles, Different Underload Ratios)

Figure 2.11 Number of Cycles to Threshold vs. Underload Ratio ($N_I = 7,000$ and Different Overload Ratios)
Comparing Figure 2.11 with experimental data in Figure 2.12, one can observe the analytical predictions behave in the same way as the test results, except that Dawicke\textsuperscript{15} was testing the specimen until it resulted in complete failure. Note also that the abrupt change in the specimen lives (Figure 2.11) at the underload value of 0 MPa, is common and matches the results of the experiments conducted by Dawicke\textsuperscript{15} at the same underload value. In Figure 2.11, it is also noticed that there is no change in the specimen’s life in the case of application of underload only, i.e., $S_{\text{max2}}/S_{\text{max1}} = 1$. This behavior satisfies the second criterion of the model validity requirements and this characteristic also matches with the fact that applying negative underload to a uniform load history causes practically no interaction effect.\textsuperscript{141}

Figure 2.12 Cycles to Failure vs. Underload Ratio
CHAPTER 3

FLIGHT DYNAMICS MODEL

3.1 Aircraft Model Selection

Ultimately, the loading experienced by the imbedded crack in the structural component arises from the motion of the aircraft through the atmospheric environment. Therefore, several requirements are needed in the flight dynamics model to facilitate realistic LEC investigations. First, a full envelope model is needed to simulate a complete flight mission to expose the crack to various loadings during climb, cruise, maneuvering, and descent phases, both in calm and turbulent conditions. Second, the model should account for nonlinear motion terms to accurately represent load amplitudes correctly. Third, the model needs to exhibit a suite of control inputs to implement the LEC logic. Finally, because the LEC concept is high risk, the model should likely be in the military sector. A well-known model, readily available in the literature, that satisfies these requirements, is the approximate F-16 dynamics in Reference 26. Figure 3.1 shows the configuration, structural cutaway, and control surfaces for this highly maneuverable compact, multi-role fighter aircraft.

3.2 Analytical Flight Dynamics Model

The analytical model presented in this chapter is an ideal rigid (except for an internal spinning rotor) and constant mass F-16 that includes the motion model, the aerodynamic model, and the engine model. The model used in this dissertation, according to the aerodynamic data range and the implementation of the leading edge flap, is a high fidelity nonlinear model. In Reference 26, the aerodynamic force and moment coefficients can be found at angles of attack $\alpha$ from $-20^\circ$ to $+90^\circ$ and sideslip angles $\beta$ of $-30^\circ$ to
Moreover, the leading edge flap is implemented as a control surface that allows the F-16 to fly at large angles of attack by reducing the tendency to stall. The flight dynamics model is programmed in software on a digital computer for preliminary numerical analysis and is to be integrated with the LEC logic given in Chapter 6.

3.2.1 Motion Model

The reference frames, shown in Figure 3.2, are used to drive the dynamic model’s equations of motion. All used reference frames are orthogonal and right-handed. In Figure 3.2, four subscripted letters are used to distinguish between the different frames. Subscript “B” stands for the body frame which is attached to and moves with the aircraft, “S” refers to the stability axes reference frame that is obtained from the body-fixed reference frame by a left-handed rotation through angle of attack $\alpha$. Note $X_B$ points along the nose, $Y_B$ points along the right wing, and $Z_B$ points “down”. The wind axes reference frame is denoted by “W” and is obtained from the stability axes reference frame.
by a rotation around the $Z_s$ axis through sideslip angle $\beta$. The lift, drag, and side forces are defined naturally in the wind frame. The letter "E" stands for the earth as the inertial frame. In the earth-fixed reference frame, the $Z_E$ axis points to the center of the earth, the $X_E$ axis points in some arbitrary direction, e.g., north, and the $Y_E$ axis is perpendicular to the $X_E$ axis in the flat earth plane. This frame is useful for describing the position and orientation of the aircraft, relative to the earth and/or the origin.

A number of assumptions have to be made before proceeding with the derivation of the motion equations. The aircraft is a rigid-body, which means that any two points on or within the airframe remain fixed with respect to each other. This assumption is quite valid for fighter aircraft. The earth is flat and non-rotating and regarded as an inertial reference. This assumption is valid when dealing with control design for aircraft, but not when analyzing inertial guidance systems. The mass is constant during the time interval over which the motion is considered. In other words, the fuel consumption is neglected during this time interval. The mass distribution of the aircraft is symmetric relative to the $X_BZ_s$ plane. This assumption implies that the products of inertia $I_{xy}$ and $I_{yz}$ are equal to zero, and is valid for most aircraft.

Under the above assumptions the motion of the aircraft has 6 degrees-of-freedom (rotation and translation in 3 dimensions). The corresponding equations of motion of the rigid aircraft can be decoupled into rotational and translational equations if the coordinate origin is chosen to be at the center of mass (cm). The rotational motion of the aircraft will then be equivalent to yawing, pitching, and rolling motions about the center the mass as if it was a fixed point in space. The remaining components of the motion will be three components of translation of the cm.
The state variables selected to obtain the ordinary differential equations (ODE) of the rigid aircraft model will be twelve variables related to the potential and kinetic energy. Three components are needed to specify the potential energy in the gravitational field $P_N$, $P_E$, $h$, where $P_N$ is the position in the north direction, $P_E$ is the position in the east direction, and $h$ is the altitude in the vertical direction. Three components of velocity $u$, $v$, $w_{A/C}$ are required for specifying the transnational kinetic energy, where $u$, $v$, and $w_{A/C}$ are the translational velocity components in the body frame. Three components of angular velocity $P$, $Q$, $R$ are used to specify the rotational kinetic energy, where $P$, $Q$, and $R$ are the rotational velocity components in the body frame. Finally, three additional state equations will be required for the vehicle attitude: $\psi$, $\theta$, $\phi$, the Euler angle yaw, pitch, and roll variables. In general the basic rigid model will contain 12 state variables that are collected in the state vector $\bar{X}$, where

$$\bar{X} = [P_N\ P_E\ h\ u\ v\ w_{A/C}\ P\ Q\ R\ \psi\ \theta\ \phi]^T$$  (3.1)
Classical mechanics are employed to derive the governing motion equations, such as outlined in Reference 28. The nonlinear dynamic equations of the rigid model are a set of first order ordinary differential equations and can be classified as follows.

**Force Equations:**

\[
\begin{align*}
\dot{u} &= Rv - Qw_{_{AIC}} - g \sin \theta + \frac{1}{m}(\ddot{X} + F_T) \\
\dot{v} &= Pu_{_{AIC}} - Ru + g \sin \phi \cos \theta + \frac{1}{m}(\ddot{Y}) \\
\dot{w}_{_{AIC}} &= Qu - P v + g \cos \phi \cos \theta + \frac{1}{m}(\ddot{Z})
\end{align*}
\] (3.2) (3.3) (3.4)

**Moment Equations:**

\[
\begin{align*}
\dot{P} &= (c_1 R + c_2 P)Q + c_3 \bar{L} + c_4 (\bar{N} + H_e Q) \\
\dot{Q} &= c_5 PR - c_6 (P^2 - R^2) + c_7 (\bar{M} + F_T z_T - H_e R) \\
\dot{R} &= (c_8 P - c_2 R)Q + c_9 \bar{L} + c_{10} (\bar{N} + H_e Q)
\end{align*}
\] (3.5) (3.6) (3.7)

**Kinematic Equations:**

\[
\begin{align*}
\dot{\phi} &= P + \tan \theta (Q \sin \phi + R \cos \phi) \\
\dot{\theta} &= Q \cos \phi - R \sin \phi \\
\dot{\psi} &= \frac{Q \sin \phi + R \cos \phi}{\cos \theta}
\end{align*}
\] (3.8) (3.9) (3.10)

**Navigational Equations:**

\[
\begin{align*}
\dot{P}_N &= u \cos \psi \cos \theta + \nu (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) + \\
&\quad w_{_{AIC}} (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\
\dot{P}_E &= u \sin \psi \cos \theta + \nu (\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) - \\
&\quad w_{_{AIC}} (\sin \psi \sin \theta \cos \phi + \cos \psi \sin \phi) \\
\dot{h} &= u \sin \theta - \nu \cos \theta \sin \phi - w_{_{AIC}} \cos \theta \cos \phi
\end{align*}
\] (3.11) (3.12) (3.13)
In Equations (3.5) to (3.7), the \( c_i \) constants are defined in terms of the moments of inertias, \( I_x, I_y, \) and \( I_z \) and the product of inertia \( I_{xz} \) as follows.

\[
\begin{align*}
\Gamma c_1 &= (I_y - I_z) I_z - I_{xz}^2 \\
\Gamma c_2 &= (I_x - I_y + I_z) I_{xz} - I_{xz}^2 \\
\Gamma c_3 &= I_z \\
\Gamma c_4 &= \frac{I_x - I_z}{I_y} \\
\Gamma c_5 &= I_x (I_x - I_y) + I_{xz}^2 \\
\Gamma c_6 &= \frac{I_{xz}}{I_y}
\end{align*}
\]

\( \Gamma = I_x I_z - I_{xz}^2 \) (3.14)

Also, the parameter \( H_e \) that exists in the moment equations represents the engine angular momentum which, in general, is variable. In the current model, the value of \( H_e \) is 216.9 kg m\(^2\)/s (160 slug ft\(^2\)/s) corresponding to full throttle opening. In Equations (3.2) to (3.4) the vehicle mass is denoted by \( m \) and is assumed constant. Parameter \( g \) represents the gravitational acceleration which is \( g = 9.8 \text{ m/s}^2 \) (\( g = 32.2 \text{ ft/s}^2 \)). Thrust produced by the engine, \( F_T \), is assumed to act parallel to aircraft’s \( X_g \) axis, which causes the thrust to only appear in Equations (3.2) and (3.6). In Equation (3.6), the constant \( z_T \), which represents the offset distance of the thrust vector from the cm, is assumed to be zero. Finally, the aerodynamic forces are denoted by \( X, Y, \) and \( Z \), while aerodynamic moments are \( L, M, \) and \( N \).

The set of first order ODEs can be represented as a vector equation by

\[
\dot{\mathbf{X}} = f(\mathbf{X}, \mathbf{U})
\]

(3.15)

In this equation the symbol \( \mathbf{U} \) denotes the control input vector, specific elements of which are given in subsequent aerodynamic and engine model subsections. Note Equation (3.15) implies the aerodynamic loads are expressible in term of the state and
control input vectors. A 4\textsuperscript{th} order Runge-Kutta algorithm is used to integrate the motion state equations.

3.2.2 Aerodynamic Model

The aerodynamic forces acting on the aircraft \(\overline{X}, \overline{Y},\) and \(\overline{Z}\), are obtained from the following equations.

\[
\begin{align*}
\overline{X} &= \overline{q} S C_{X_T} (\alpha, \beta, P, Q, R, \delta, \ldots) \\
\overline{Y} &= \overline{q} S C_{Y_T} (\alpha, \beta, P, Q, R, \delta, \ldots) \\
\overline{Z} &= \overline{q} S C_{Z_T} (\alpha, \beta, P, Q, R, \delta, \ldots)
\end{align*}
\] (3.16)

The total aerodynamic force coefficients \(C_{X_T}, C_{Y_T},\) and \(C_{Z_T}\) are usually obtained from wind tunnel data and flight tests, and are functions of the aerodynamic attitude angles, angular velocity rates, and control surface deflections \(\delta\). The aerodynamic moments \(\overline{L}, \overline{M}, \overline{N}\) can be expressed in a similar way as the aerodynamic forces,

\[
\begin{align*}
\overline{L} &= \overline{q} S b C_{l_T} (\alpha, \beta, P, Q, R, \delta, \ldots) \\
\overline{M} &= \overline{q} S C_{m_T} (\alpha, \beta, P, Q, R, \delta, \ldots) \\
\overline{N} &= \overline{q} S b C_{n_T} (\alpha, \beta, P, Q, R, \delta, \ldots)
\end{align*}
\] (3.17)

where dynamic pressure is \(\overline{q} = \frac{1}{2} \rho V_T^2\), \(S\) is the reference wing area, \(\overline{c}\) is the wing mean aerodynamic chord, \(b\) is the wing span, \(\rho\) is atmospheric density, and \(V_T\) is the total velocity. The total aerodynamic moment coefficients are denoted \(C_{l_T}, C_{m_T}, C_{n_T}\). The definitions of the five aerodynamic control vector elements and their mechanical limits are listed in Table 3.1.
Table 3.1 Control Input Definitions

<table>
<thead>
<tr>
<th>Input</th>
<th>Definition</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ₉₁</td>
<td>Horizontal Stabilizer Deflection, deg</td>
<td>-25° to +25°</td>
</tr>
<tr>
<td>δ₉₂</td>
<td>Trailing Edge Flaperon Deflection, deg</td>
<td>-21.5° to +21.5°</td>
</tr>
<tr>
<td>δₛₓ</td>
<td>Rudder Deflection, deg</td>
<td>-30° to +30°</td>
</tr>
<tr>
<td>δₛₚ</td>
<td>Speed Brake Deflection, deg</td>
<td>0° to +60°</td>
</tr>
<tr>
<td>δₛₑ</td>
<td>Leading Edge Flap Deflection, deg</td>
<td>-2° to +25°</td>
</tr>
</tbody>
</table>

The total aerodynamic coefficients $C_{X_{r}}, C_{Y_{r}}, C_{Z_{r}}, C_{L_{r}}, C_{m_{r}},$ and $C_{n_{r}}$ are computed based on the high fidelity aerodynamic data tables in Reference 26. These coefficients are usually expressed as a baseline component plus correction terms that are denoted by the symbol $\delta$. The baseline component is primarily a function of angle of attack, $\alpha$, sideslip angle, $\beta$, and Mach number, $M$. Mach dependence can be removed from the baseline component and treated as a correction term in the case of data of subsonic speeds. As the available aerodynamic data tables were conducted at subsonic flow conditions, the effect of Mach number is neglected. In this model, the aerodynamic data shows strong dependency on the horizontal stabilizer deflection $\delta_{h}$, so $\delta_{h}$ is included as an independent variable for the baseline component. Normally, total coefficient equations have been used to sum the various aerodynamic contributions to a given force or moment coefficient as follows.

$X_B$ Axis Force Coefficient $C_{X_{r}}$:

$$C_{X_{r}} = C_{X}(\alpha, \beta, \delta_{h}) + \delta C_{X_{lsf}}\left(1 - \frac{\delta_{lsf}}{25}\right) + \delta C_{X_{sb}}\left(\frac{\delta_{sb}}{60}\right) + \frac{Q_{c}}{2V_{T}}\left[C_{X_{lsf}}(\alpha) + \delta C_{X_{lsf}}(\alpha)\left(1 - \frac{\delta_{lsf}}{25}\right)\right]$$

(3.18)

where

$$\delta C_{X_{lsf}} = C_{X_{lsf}}(\alpha, \beta) - C_{X}(\alpha, \beta, \delta_{h} = 0^\circ)$$

$Y_B$ Axis Force Coefficient $C_{Y_{r}}$:
\[C_{Y_T} = C_Y(\alpha, \beta) + \delta C_{Y_{nf}} \left(1 - \frac{\delta_{\text{ef}}}{25}\right) + \left[\delta C_{Y_{\gamma_{ef}}} + \delta C_{Y_{\gamma_{20^\circ}}} \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right]\left(\frac{\delta_f}{20}\right)
+ \delta C_{Y_{\delta_{20^\circ}}} \left(\frac{\delta_f}{30}\right) + \frac{Rb}{2V_T} \left[ C_Y(\alpha) + \delta C_{Y_{\gamma_{nf}}} (\alpha) \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right]
+ \frac{Pb}{2V_T} \left[ C_Y(\alpha) + \delta C_{Y_{\gamma_{nf}}} (\alpha) \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right] \tag{3.19}\]

where

\[\delta C_{Y_{nf}} = C_Y(\alpha, \beta) - C_Y(\alpha, \beta)\]
\[\delta C_{Y_{\gamma_{ef}}} = C_{Y_{\gamma_{ef}}}(\alpha, \beta) - C_Y(\alpha, \beta)\]
\[\delta C_{Y_{\gamma_{20^\circ}}} = C_{Y_{\gamma_{20^\circ}}}(\alpha, \beta) - C_Y(\alpha, \beta)\]
\[\delta C_{Y_{\delta_{20^\circ}}} = C_{Y_{\delta_{20^\circ}}}(\alpha, \beta) - C_Y(\alpha, \beta)\]

**Z_B Axis Force Coefficient** \(C_{Z_T}\):

\[C_{Z_T} = C_Z(\alpha, \beta, \delta_h) + \frac{Qc}{2V_T} \left[ C_Z(\alpha) + \delta C_{Z_{nf}} (\alpha) \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right]\left(\frac{\delta_{\theta}}{60}\right) \tag{3.20}\]

where

\[\delta C_{Z_{nf}} = C_{Z_{nf}} (\alpha, \beta) - C_Z (\alpha, \beta, \delta_h = 0^\circ)\]

**Roll Moment Coefficient** \(C_{l_T}\):

\[C_{l_T} = C_l(\alpha, \beta, \delta_h) + \delta C_{l_{nf}} \left(1 - \frac{\delta_{\text{ef}}}{25}\right) + \left[\delta C_{l_{\gamma_{ef}}} + \delta C_{l_{\gamma_{20^\circ}}} \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right]\left(\frac{\delta_f}{20}\right)
+ \delta C_{l_{\delta_{20^\circ}}} \left(\frac{\delta_f}{30}\right) + \frac{Rb}{2V_T} \left[ C_l(\alpha) + \delta C_{l_{\gamma_{nf}}} (\alpha) \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right]
+ \frac{Pb}{2V_T} \left[ C_l(\alpha) + \delta C_{l_{\gamma_{nf}}} (\alpha) \left(1 - \frac{\delta_{\text{ef}}}{25}\right)\right] + \delta C_{l_j} (\alpha) \beta \tag{3.21}\]

where
\[ \delta C_{b_{Zf}} = C_{b_{Zf}}(\alpha, \beta) - C_{b}(\alpha, \beta, \delta_h = 0^\circ) \]
\[ \delta C_{t_{Zf+20^\circ}} = C_{t_{Zf+20^\circ}}(\alpha, \beta) - C_{t}(\alpha, \beta, \delta_h = 0^\circ) \]
\[ \delta C_{t_{Zf+20^\circ LF}} = C_{t_{Zf+20^\circ LF}}(\alpha, \beta) - C_{t_{Zf}}(\alpha, \beta) - \left[ C_{t_{Zf+20^\circ}}(\alpha, \beta) - C_{t}(\alpha, \beta, \delta_h = 0^\circ) \right] \]
\[ \delta C_{t_{Zf+30^\circ}} = C_{t_{Zf+30^\circ}}(\alpha, \beta) - C_{t}(\alpha, \beta, \delta_h = 0^\circ) \]

Pitch Moment Coefficient \( C_{m_f} \):

\[
C_{m_f} = C_m(\alpha, \beta, \delta_h)\eta_{\delta_h}(\delta_h) + C_{2T}\left(x_{cm_r} - x_{cm}\right) + \delta C_{m_{g_{ef}}} \left(1 - \frac{\delta_{lef}}{25}\right) \\
+ \frac{QC}{2V_T}\left[C_{m_{q}}(\alpha) + \delta C_{m_{q_{ef}}} \left(1 - \frac{\delta_{lef}}{25}\right)\right] + \delta C_{m}(\alpha) + \delta C_{m_{n_{ef}}} (\alpha, \delta_h)
\]

where

\[ \delta C_{m_{g_{ef}}} = C_{m_{g_{ef}}}(\alpha, \beta) - C_m(\alpha, \beta, \delta_h = 0^\circ) \]

Yaw Moment Coefficient \( C_{n_r} \):

\[
C_{n_r} = C_n(\alpha, \beta, \delta_h) + \delta C_{n_{ef}} \left(1 - \frac{\delta_{lef}}{25}\right) + \delta C_{n_{r}}(\alpha) + \delta C_{n_{r_{ef}}} (\alpha, \delta_h) \\
+ \delta C_{n_{r_{ef}}} \left(\frac{\delta_{r}}{30}\right) + \frac{Rb}{2V_T}\left[C_{n_r}(\alpha) + \delta C_{n_{r_{ef}}} (\alpha) \left(1 - \frac{\delta_{lef}}{25}\right)\right] \\
+ \frac{Pb}{2V_T}\left[C_{n_r}(\alpha) + \delta C_{n_{r_{ef}}} (\alpha) \left(1 - \frac{\delta_{lef}}{25}\right)\right] + \delta C_{n_r}(\alpha, \beta) + C_Y \left[x_{cm_r} - x_{cm}\right] \frac{\bar{c}}{b}
\]

where

\[ \delta C_{n_{g_{ef}}} = C_{n_{g_{ef}}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_h = 0^\circ) \]
\[ \delta C_{n_{r}} = C_{n_r}(\alpha, \beta) - C_n(\alpha, \beta, \delta_h = 0^\circ) \]
\[ \delta C_{n_{r_{ef}}} = C_{n_{r_{ef}}}(\alpha, \beta) - C_{n_{r_{ef}}}(\alpha, \beta) - \delta C_{n_{r_{ef}}} \]
\[ \delta C_{n_{r_{ef}}} = C_{n_{r_{ef}}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_h = 0^\circ) \]

The horizontal stabilizer effectiveness factor \( \eta_{\delta_h}(\delta_h) \) appearing in Equation (3.22) is provided in tabular form as a function of the horizontal stabilizer deflection. The strength of this term decreases near the maximum deflection angle of the horizontal stabilizer.
Note the pitch and yaw moment coefficient equations allow for a variable mass center point denoted by $x_{cm}$, relative to the nominal point $x_{cm_0} = 0.35 \bar{c}$. Parameter $x_{cm}$ is chosen to be equal to the reference position in the current model. Finally, the angle of attack $\alpha$ and the sideslip angle $\beta$ are defined in terms of the body axes velocity components as

$$\alpha = \tan^{-1}\left(\frac{w_{A/C}}{u}\right), \quad \beta = \sin^{-1}\left(\frac{v}{V_T}\right)$$

(3.24)

where $V_T = \sqrt{u^2 + v^2 + w_{A/C}^2}$

3.2.3 Engine Model

The vehicle is powered by an afterburning turbofan jet engine. The thrust response to the throttle inputs can be computed using the mathematical model schematically shown in Figure 3.3. The engine model, shown in Figure 3.3, is a variable time constant first order system representing spool-up and spool-down lags in the turbine engine.\(^{26}\) The throttle command gearing is shown in Figure 3.4. Figure 3.4 describes how to obtain engine power command $P_I$ as a function of the throttle opening $\theta_{th}$. Note that the lower limit of the throttle position $\theta_{th}$ is set to 5% in order for the engine not to be turned off. The maximum limit of $\theta_{th}$ is 100% at full throttle opening. The variable $P_2$ denotes intermediate power command to the engine, and $P_3$ denotes current engine power, which is a state variable representing the time delay in the engine response. The power command terms $P_I, P_2, P_3$ are represented as percentages of the maximum power ($0 \leq P_i \leq 100$). The engine gyroscopic effects were simulated by representing the engine angular momentum at a fixed value of $H_e = 216.9 \text{ kg m}^2/\text{s}$ (see Equations (3.5) to (3.7)).

The engine response was modeled with a first order lag which varied as shown in Figure 3.5, depending on the value of $P_2 - P_3$. The decay rate of the turbine engine is
represented by \( \frac{1}{T} \), where \( \tau \) is the thrust time constant. \( T_{mil} \) denotes the military thrust representing maximum thrust generated at the normal operating conditions. The idle thrust representing thrust generated at the idle condition is denoted by \( T_{idle} \). \( T_{max} \) denotes maximum thrust representing thrust generated with full afterburner condition engaged. These thrust values are presented as a function of altitude and Mach number over the range of \( 0 \leq h \leq 15,240 \) m and \( 0 \leq M \leq 1.0 \). According to the thrust logic shown in Figure 3.3, when \( P_3 \) is over 50% of maximum power, the engine dynamic model uses \( T_{mil} \) and \( T_{max} \) to compute the engine thrust. If \( P_3 \) does not exceed 50%, \( T_{idle} \) and \( T_{mil} \) are used to compute the engine thrust.

![Figure 3.3 Logic Diagram for Engine Dynamic Model](image-url)
According to the thrust logic shown in Figure 3.3, the engine state space equation is

\[ \dot{P}_3 = \frac{1}{\tau_r} (P_2 - P_3) \]  
(3.25)

where

\[ P_2 = \begin{cases} 
P_1 & \text{if } P_1 \geq 50 \text{ and } P_3 \geq 50 \\
60 & \text{if } P_1 \geq 50 \text{ and } P_3 < 50 \\
40 & \text{if } P_1 < 50 \text{ and } P_3 \geq 50 \\
P_1 & \text{if } P_1 < 50 \text{ and } P_3 < 50 
\end{cases} \]  
(3.26)

and

\[ \frac{1}{\tau_r} = \begin{cases} 
5 & \text{if } P_1 \geq 50 \text{ and } P_3 \geq 50 \\
f(P_2 - P_3) & \text{if } P_1 \geq 50 \text{ and } P_3 < 50 \\
5 & \text{if } P_1 < 50 \text{ and } P_3 \geq 50 \\
f(P_2 - P_3) & \text{if } P_1 < 50 \text{ and } P_3 < 50 
\end{cases} \]  
(3.27)

and

\[ f(P_2 - P_3) = \begin{cases} 
1.0 & \text{if } (P_2 - P_3) \leq 25 \\
0.1 & \text{if } (P_2 - P_3) \geq 50 \\
1.9 - 0.036 (P_2 - P_3) & \text{if } 25 < (P_2 - P_3) < 50 
\end{cases} \]  
(3.28)

A 4\textsuperscript{th} order Runga-Kutta algorithm is used for integrating the engine state space equation, for which one can get the current engine power \( P_3 \). Once \( P_3 \) is computed, the thrust value can be calculated as

\[ F_T = \begin{cases} 
T_{idle} + (T_{mil} - T_{idle}) \left( \frac{P_3}{50} \right) & \text{if } P_3 < 50 \\
T_{idle} + (T_{max} - T_{mil}) \left( \frac{P_3 - 50}{50} \right) & \text{if } P_3 \geq 50 
\end{cases} \]  
(3.29)

3.3 Trimmed Flight

In normal flight it is usual for the pilot to adjust the controls of an aircraft such that on releasing the controls, the aircraft continues to fly at the chosen condition. The aircraft is
then said to be trimmed. The trim state defines the initial conditions about which the dynamics of interest may be studied. The objective of trimming is to bring the forces and moments acting on the aircraft into a state of equilibrium. Therefore, the condition for an aircraft to remain in steady trimmed flight requires that the forces and moments acting on the aircraft sum to zero and that it is stable. In order to calculate the trim condition of the F-16 model, it is convenient to assume steady rectilinear symmetric (wings level) flight. In symmetric wing level flight, the angle of attack is constant with no sideslip angle ($\beta = 0^\circ$). The velocity components $u$ and $w_{A/C}$ are non-zero constant values, while the component $v$ is set equal to zero. All the angular rates $P$, $Q$, and $R$ should be zero. Both the roll angle $\phi$ and yaw angle $\psi$ are specified to be zero. In a wing level flight condition, pitch angle $\theta$ is equal to the angle of attack $\alpha$ as the flight path angle $\gamma$ is zero. For the control inputs, the speed brake deflection $\delta_{sb}$ is set to zero and the throttle position $\theta_{th}$, the horizontal stabilizer deflection $\delta_h$, and the leading edge flap deflection $\delta_{lef}$ are non-zero constants. The flaperon deflection $\delta_f$ and the rudder deflection $\delta_r$ are specified to be zero.

![Figure 3.4 Power Command Variation with Throttle Position](image)
Figure 3.5 Variation of Inverse Thrust Time Constant with Incremental Power Command

Applying the previous conditions to the nonlinear equations of motion mentioned before, one ends up with the nonlinear algebraic Equations (3.30) to (3.32) with the added kinematic constraint Equation (3.33) and the leading edge flap schedule according to Equation (3.34). These equations can be solved simultaneously using a nonlinear equation solver such as Zero-Finders in the MATLAB software tool. The solution results are the non-zero values of $\theta, \alpha, \theta_h, \delta_h,$ and $\delta_{\text{le}}$.

\[
g \sin \theta - \frac{1}{m} \left( \vec{X}(\alpha, \delta_h, \delta_{\text{lef}}, V_T) + F_T(\theta_h) \right) = 0.0 \tag{3.30}
\]

\[
g \cos \theta + \frac{1}{m} \left( \vec{Z}(\alpha, \delta_h, \delta_{\text{lef}}, V_T) \right) = 0.0 \tag{3.31}
\]

\[
\vec{M}(\alpha, \delta_h, \delta_{\text{lef}}, V_T) + F_T(\theta_h) z_T = 0.0 \tag{3.32}
\]

\[
\Gamma - \theta + \alpha = 0.0 \tag{3.33}
\]

\[
\delta_{\text{le}} - 1.38 \alpha + 9.05 \frac{\vec{g}}{p}(h, V_T) - 1.45 = 0.0 \tag{3.34}
\]
The algorithms underneath the Zero-Finders function are based on iteratively adjusting the independent variables of the nonlinear algebraic equations until some solution criterion is met. The solution will be approximate but can be made arbitrary close to the exact solution by tightening up the criterion. Also, the solution may not be unique. For example, steady state level flight at an arbitrary engine power level can generally correspond to two different airspeeds and angles of attack. For the trimming condition at an altitude of 3,000 ft and \( V_T = 500 \text{ ft/s} \), the calculated state and control inputs are

\[
\begin{bmatrix}
\theta & \alpha & \theta_h & \delta_{\text{ref}} \\
2.5259^\circ & 2.5259^\circ & 14.9578\% & -0.5706^\circ & 3.6387^\circ
\end{bmatrix}
\] (3.35)

Using \( \theta_{\text{int}} \), the engine power intermediate command \( P_2 \) and the current engine power \( P_3 \) can be computed and correspond to \( [P_2 P_3] = [9.7054\% \ 9.7054\%] \).

Capabilities of aircraft design are parameterized with altitudes and Mach numbers. Various limits, which include aerodynamics, propulsion, structural, and sea level, constrain the aircraft flight capabilities. The maximum and minimum tolerable Mach numbers are calculated at all possible flight altitudes between sea level and the ceiling point. The pairs of Mach numbers and altitudes are plotted to form an envelope that represents the utmost flight capability of the aircraft. The boundary is called the flight envelope or performance envelope where out of which the aircraft can not practically fly because of violating the above-mentioned limits. Figure 3.6 shows the flight envelope of the model under investigation.

### 3.4 Transient Flight

To validate the trimming flight solution and the numerical computations, the vehicle motion response due to applying the initial states and the initial control inputs representing the trimmed flight solution in Equation (3.35) are illustrated in Figures 3.7-
3.9. The $4^{th}$ order Runga-Kutta numerical integration method is used to integrate the differential equations of the F-16 dynamic model, Equations (3.2)-(3.13). To start the numerical integration, the values of the states and inputs of the trimming conditions are used as the initial values of the states and the inputs, respectively.

![Flight Envelope, $\delta_{\text{ef}} = 0$](image)

**Figure 3.6** Flight Envelope, $\delta_{\text{ef}} = 0$

From Figure 3.7, one can notice that there are no changes in the velocity components, $u$, $v$, and $w_{A/C}$. In Figure 3.8, there are no changes in the pitch and yaw rates and the change in the roll rate is nearly zero. Note that the roll rate change is visible only because of the scale of the ordinate. The propagation of the leading edge flap deflection is shown in Figure 3.9. The deflection of the leading edge flap is dependent on the angle of attack $\alpha$, the static pressure $p(h)$, and the dynamic pressure $\bar{q}(h,V_T)$. Therefore, if there is no change in the leading edge flap, as shown in Figure 3.9, this is another validation of the trimming solution and the numerical computation.
For further validation of the computational method and to demonstrate the nonlinear characteristics of the F-16 model, the time responses of aircraft translational and rotational rates as a result of the three separate step inputs of unit magnitude of the primary control surfaces are presented. Three separate simulations of the vehicle motions are conducted and represented in the following figures. For each of the three cases, the simulation starts from the equilibrium conditions in Equation (3.35) at time equal to zero and the positive step input of the selected control surface is applied after 1 s. The simulation is conducted with the leading edge flap schedule actuated.

Figure 3.7 Translational Velocity Component Response at Symmetrical Straight Level Flight Condition
In the first simulation, the vehicle motion responses to the horizontal stabilizer step input are plotted in Figures 3.10-3.16. In Figure 3.10, as a result of horizontal stabilizer step input, the velocity component $u$ undergoes a very small increase, then slightly drops.
The decrease in the velocity component $u$ is accompanied by small increase in the side velocity component $v$ as shown in Figure 3.11. The asymmetric velocity excitation originates from the engine angular momentum terms in Equations (3.5) and (3.7). Figure 3.12 shows a relative large decrease in the velocity component $w_{A/C}$ as the aircraft starts to climb because of the trade-off between the potential and kinetic energies of the aircraft. Figure 3.13 shows that the pitch rate is suddenly damped directly after the horizontal stabilizer step input. This behavior is due to the high pitch damping characteristics of the F-16 model at the subsonic speeds ($M = 0.4525$). Although the pitch rate is rapidly damped out, it triggers a long period response of the roll rate $P$. This trend is due to the coupling between the longitudinal and lateral motions through the engine spin moment term in the moment equations. Moreover, the yaw rate $R$ is shifted away from its equilibrium value to an almost constant negative value as shown in Figure 3.12. The $\alpha$ and $\theta$ propagations are shown in Figures 3.15 and 3.16, respectively.

The second simulation demonstrates the vehicle motion responses due to flaperon step input, Figures 3.17-3.23. Unlike the vehicle motion responses to the horizontal stabilizer step input, the reduction in the axial velocity component $u$ is larger, as seen in Figure 3.17. In Figure 3.18, it is obvious that the change in the side velocity component $v$ is relatively large in the negative direction of $Y_B$ axis. This effect is because of the negative side force which is due to the negative roll rate that is generated directly after the flaperon step input. From Figure 3.20, one can notice that the vehicle rolls and turns gradually, and maintains stable lateral behavior. Note also that the vehicle roll rate changes between negative to positive and hence a stable turn is expected. Figure 3.23
shows a change of the heading (yaw) angle in the negative direction as a result of negative roll directly after the flaperon positive step input.

In the final simulation, the rudder step input is applied to the vehicle after 1 s and the vehicle motion responses are shown in Figures 3.24-3.29. Notice that the reduction in the axial velocity component $u$ is small compared to the change due to flaperon step input (see Figure 3.24). In contrast to the flaperon step input, the side velocity component $v$ increases in the positive direction as a result of positive side force due to positive rudder deflection, see Figure 3.25. Also, notice that the roll rate as a result of rudder step input changes from positive to negative as shown in Figure 3.27. As the vehicle roll rate changes between positive and negative, a stable turn is expected. Comparing Figures 3.23 and 3.29, one can conclude that the heading angle response, which is a result of the rudder step input, is almost neglected relative to the heading angle response which is due to flaperon step input.

Figure 3.10 Velocity Component $u$ due to Horizontal Stabilizer Step Input
Figure 3.11 Velocity Component $v$ due to Horizontal Stabilizer Step Input

Figure 3.12 Velocity Component $w_{AC}$ due to Horizontal Stabilizer Step Input
3.13a Overall Behavior

3.13b Detail Behavior

Figure 3.13 Pitch, Roll, and Yaw Rates due to Horizontal Stabilizer Step Input
Figure 3.14 Altitude Response due to Horizontal Stabilizer Step Input

Figure 3.15 Angle of Attack Response due to Horizontal Stabilizer Step Input
Figure 3.16 Pitch Angle Response due to Horizontal Stabilizer Step Input

Figure 3.17 Velocity Component $u$ due to Flaperon Step Input
Figure 3.18 Velocity Component $v$ due to Flaperon Step Input

Figure 3.19 Velocity Component $w_{AC}$ due to Flaperon Step Input
Figure 3.20 Roll, Pitch, and Yaw Rate Response due to Flaperon Step Input

Figure 3.21 Plane Motion Response due to Flaperon Step Input
Figure 3.22 Altitude Response due to Flaperon Step Input

Figure 3.23 Heading Angle Response due to Flaperon Step Input
Figure 3.24 Velocity Component $u$ due to Rudder Step Input

Figure 3.25 Velocity Component $v$ due to Rudder Step Input
Figure 3.26 Velocity Component $w_{AC}$ due to Rudder Step Input

3.27a Overall Behavior
3.27b Detail Behavior

Figure 3.27 Pitch, Roll, and Yaw Rates due to Rudder Step Input

Figure 3.28 Plane Motion Response due to Rudder Step Input
Figure 3.29 Heading Angle Response due to Rudder Step Input
CHAPTER 4

FLEXIBLE WING DYNAMIC MODEL

4.1 Wing Model Selection

Flexibility of the wing is the key mechanism on which the forces applied to the wing, either aerodynamic forces or those due to air vehicle motion, can generate stresses in the wing that drive the crack propagation at the critical points. The flexible wing model should have the fidelity to characterize structural deformation, stress-strain behavior, and dynamic motion of the true wing, without being overly complicated in an analytical or numerical sense. Such a model would be sufficiently adequate for the intended application. Further, the flexible wing model should be consistent, in a materialistic, geometric, aerodynamic, and dynamic sense, to the previously chosen flight dynamic model (F-16). The general built-up structure of aircraft wings are composed of skins, spars, and ribs having complex geometries with varying material and size properties throughout. Additionally, the specific F-16 wing has a very thin, low aspect ratio nature with near sandwich-like structure build-up. For these reasons, equivalent cantilevered plate theory is selected to implement the flexible wing model. The wing structure is modeled as an elastic isotropic cantilevered plate of uniform thickness along the span. The data used to define the structural model are a conglomerate of properties related to the F-16 collected from various publications that were publicly accessible.\textsuperscript{24,25} Figure 4.1 shows a schematic of the structural dimensions of the clean starboard (right) wing of the parametric model under investigation.
4.2 Parameterization

The wing planform of the F-16 is effectively that of a cropped delta wing with a 40 deg leading edge sweep. The structure incorporates eleven spars and five ribs with an airfoil section of a four percent thickness-to-chord ratio. The root section of the wing blends with the fuselage, making it impossible to tell where the wing begins and the fuselage ends. The wing thickness increases gradually in the region of the root resulting in a stiffer wing over conventional designs. Lacking structural details, it was decided to model the wing as a plate of isotropic, linear elastic properties with the dimensions of the F-16's wing planform and uniform thickness along the span from the root to wing tip.

Figure 4.1 F-16 Clean Wing Planform
The full scale lumped masses were given at specified spanwise locations in Reference 24. These masses are summed up to be uniformly distributed on the equivalent plate, where the total mass of each wing is approximately 1,039 lbm (471.28 kg). An ad-hoc model updating procedure to estimate the modulus of elasticity $E$, the modulus of rigidity $G$, the Poison's ratio $v$, and the overall thickness of the equivalent plate $h$, was employed to match the natural frequencies of the first four modes of the proposed finite element model to the measured values of the full scale model studied in Reference 25. For the sake of comparison, the two groups of frequencies are listed in Table 4.1, where an eigen-solver algorithm is used to calculate the natural frequencies and corresponding modes, as will be discussed later in this chapter. The mean thickness of the F-16 wing airfoil (NACA-64A204) guided the search for a suitable overall thickness. On the other hand, material properties of real airframe metals, stresses at the wing root, as well as deflection behavior under static loads, guided the search for admissible material strengths and properties. The geometrical and the mechanical properties of the wing under investigation determined by this process are listed in Table 4.2. Figure 4.3 shows a typical representation of the proposed cantilevered plate, which was drawn in ABAQUS.143

<table>
<thead>
<tr>
<th>Table 4.1 Frequencies of the First Four Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1st Bending</td>
</tr>
<tr>
<td>2nd Bending</td>
</tr>
<tr>
<td>1st Torsion</td>
</tr>
<tr>
<td>2nd Torsion</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.2 Thickness and Material Properties of the Proposed Wing Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Value</td>
</tr>
</tbody>
</table>
Figure 4.2 Lumped Mass Distribution of NASTRAN Full Scale F-16 Model

Figure 4.3 Isometric View of Starboard Wing
4.3 Finite Element Model Formulation

The basic assumptions made to formulate the linear finite element model (FEM) can be summarized as follows. It is a well known fact that, in the absence of large axial forces, fixed-free and fixed-sliding plates are approximated as in-extensional members.\textsuperscript{91-93} Therefore, the wing is treated as an in-extensional plate. Although the Von-Karman plate formulation can provide reasonable solutions to large deformation problems of extensional structures, its performance becomes unsatisfactory for the in-extensional case even under relatively small deflections.\textsuperscript{144} Therefore, the strain components due to Von-Karman large deflection assumption are no longer added to the strain components in the case under investigation.

As the order of magnitudes of the wing planform dimensions vary from twenty to fifty times the order of magnitude of the thickness, the wing can not be considered a truly thin plate. Therefore, the classical plate theory (CPT) based on Kirchhoff-Love’s hypotheses, which states that any straight plane transverse to the plate middle surface remains straight, inextensible, and normal to the mid-plane after deformation, is not applicable in the case under investigation. CPT works well for truly thin isotropic plates, but for moderately thick plates and for thin laminated plates, the theory underestimates the deflection and overestimates the natural frequencies (plate stiffness). The reason for the stiffness overestimation is the ignorance of the effects of through-the-thickness shear deformation.\textsuperscript{145} First order shear deformation theory (FSDT) that is based on the Mindlin-Reissner model,\textsuperscript{145} where the constraint that any plane transverse to the mid-surface remains normal to the mid-surface after deformation, is relaxed and transverse shear strain is allowed. FSDT is employed in the finite element formulation of the case under
investigation because the finite elements based on Reissner-Mindlin plate bending theory have advantages over elements derived from the CPT. First, the inclusion of shear deformation extends the range of applicability from thin plates to moderately thick plates. Second, only $C^0$ continuity of the displacement and rotation variables is required by the finite element formulation. This feature is in contrast to the $C^1$ continuity required of Kirchhoff based elements, which are much more difficult to achieve.\textsuperscript{145}

Tessler and Hughes\textsuperscript{94} presented a robust and accurate, three-node, nine degrees-of-freedom triangular plate element based on Reissner-Mindlin plate theory. They concluded that the performance of this element, called MIN3, is excellent for both moderately thick and thin plate regimes. Many low-order, Reissner-Mindlin plate elements become very stiff when used to model thin plates and this phenomenon is called shear locking. In the MIN3 element, anisotropic interpolation functions are used to avoid shear locking, and a shear correction factor is used to avoid excessive stiffness and ill-conditioning for coarse meshes, even for extremely thin plates. Notice that the anisotropic interpolation function simply uses a transverse displacement interpolation function of one degree higher than that used for bending rotation. Based on these reasons, the MIN3 element is used for flexible wing modeling.

\textbf{4.3.1 Element Displacement Relations}

A typical in-extensional triangular thin plate element is shown in Figure 4.4. This element considers three degrees-of-freedom, namely a transverse displacement and two rotations at each node. The element translational displacement functions used in the derivation of the equations of motions are
\[
\begin{align*}
    u_x &= z\psi_y(x, y, t) \\
    u_y &= z\psi_x(x, y, t) \\
    u_w &= w(x, y, t)
\end{align*}
\] (4.1)

where \(u_x, u_y, u_z\) are the three translational displacement components at any point in the element, \(w\) is the lateral displacement of the middle surface, and \(\psi_x\) and \(\psi_y\) are the rotations of the plate normal about the -x and +y axes due to bending only. Note that \(x, y, z\) replace \(X_B, Y_B, Z_B\) in these relations. The element nodal displacement definitions are defined as

\[
\begin{bmatrix}
    w
\end{bmatrix} = \begin{bmatrix}
    w_1 & w_2 & w_3
\end{bmatrix}
\] (4.2)

where the transverse displacement and rotations per element are

\[
\begin{bmatrix}
    w_1 \\
    w_2 \\
    w_3
\end{bmatrix} = \begin{bmatrix}
    \psi_{x1} & \psi_{x2} & \psi_{x3} & \psi_{y1} & \psi_{y2} & \psi_{y3}
\end{bmatrix}
\] (4.3)

\[
\begin{bmatrix}
    w_\psi
\end{bmatrix} = \begin{bmatrix}
    \psi_{x1} & \psi_{x2} & \psi_{x3} & \psi_{y1} & \psi_{y2} & \psi_{y3}
\end{bmatrix}
\] (4.4)

The interpolation functions for the triangular in-extensional element are

\[
\begin{align*}
    w(x, y, t) &= \left( H_w \right) \{ w_1 \} + \left( H_{w\psi} \right) \{ w_\psi \} \\
    &= \begin{bmatrix}
        \xi_1 & \xi_2 & \xi_3
    \end{bmatrix} \{ w_1 \} + \begin{bmatrix}
        L_1 & L_2 & L_3 & M_1 & M_2 & M_3
    \end{bmatrix} \{ w_\psi \}
\end{align*}
\] (4.5)

\[
\psi_x(x, y, t) = \left( H_{w\psi} \right) \{ w_\psi \} = \begin{bmatrix}
    \xi_1 & \xi_2 & \xi_3 & 0 & 0 & 0
\end{bmatrix} \{ w_\psi \}
\] (4.6)

\[
\psi_y(x, y, t) = \left( H_{w\psi} \right) \{ w_\psi \} = \begin{bmatrix}
    0 & 0 & \xi_1 & \xi_2 & \xi_3
\end{bmatrix} \{ w_\psi \}
\] (4.7)

where \(\xi_1, \xi_2, \xi_3\) are the area coordinates. The transformation between \(x, y\) and \(\xi_i\) is accomplished by

\[
\begin{bmatrix}
    1 \\
    x \\
    y
\end{bmatrix} = \begin{bmatrix}
    1 & 1 & 1 \\
    x_1 & x_2 & x_3 \\
    y_1 & y_2 & y_3
\end{bmatrix} \begin{bmatrix}
    \xi_1 \\
    \xi_2 \\
    \xi_3
\end{bmatrix}
\] (4.8)
\[
\begin{bmatrix}
\varepsilon_{s1} \\
\varepsilon_{s2} \\
\varepsilon_{s3}
\end{bmatrix}
= \frac{1}{2A}
\begin{bmatrix}
x_2y_3 - x_3y_2 & y_2 - y_3 & x_3 - x_2 \\
x_1y_3 - x_3y_1 & y_3 - y_1 & x_1 - x_3 \\
x_1y_2 - x_2y_1 & y_1 - y_2 & x_2 - x_1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\] (4.9)

where

\[
A = \frac{1}{2} [(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]
\] (4.10)

and

\[
\begin{align*}
L_1 &= \frac{1}{8}(b_3N_4 - b_2N_6), & L_2 &= \frac{1}{8}(b_1N_5 - b_3N_4) \\
L_3 &= \frac{1}{8}(b_2N_6 - b_1N_5), & M_1 &= \frac{1}{8}(a_2N_6 - a_3N_4) \\
M_2 &= \frac{1}{8}(a_3N_4 - a_1N_5), & M_3 &= \frac{1}{8}(a_1N_5 - a_2N_6) \\
N_4 &= 4\xi_3\xi_2, & N_5 &= 4\xi_2\xi_3, & N_6 &= 4\xi_3\xi_1 \\
a_1 &= x_{32}, & a_2 &= x_{13}, & a_3 &= x_{21} \\
b_1 &= y_{32}, & b_2 &= y_{31}, & b_3 &= y_{12}
\end{align*}
\] (4.11)

\[
x_y = x_i - x_j, \quad y_y = y_i - y_j
\]

\[
\int_A \varepsilon_{s1} \varepsilon_{s2} \varepsilon_{s3}^m dA = 2A \frac{k!m!}{(2 + k + l + m)!}
\] (4.12)

4.3.2 Strain-Displacement Relations

Recall that the in-extensional plate theory is applied and hence no membrane strain components are allowed. Therefore, the strain-displacement relationship is expressed by

\[
\{\varepsilon\} = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \mathbf{z} \{\kappa\}
\] (4.13)

where \{\kappa\} is the curvature vector

\[
\{\kappa\} = \begin{bmatrix}
\psi_{y,x} \\
\psi_{x,y} \\
\psi_{y,y} + \psi_{x,x}
\end{bmatrix}
\] (4.14)
and \( \varepsilon_x, \varepsilon_y \) are strain displacements in the \( x, y \) directions and \( \gamma_{xy} \) is the shear strain displacement in \( xy \) plane.

Therefore, the expression for the strain vector becomes

\[
\{ \varepsilon \} = z \begin{pmatrix}
\psi_{y,x} \\
\psi_{x,y} \\
\psi_{y,x} + \psi_{x,y}
\end{pmatrix}
\]  

(4.15)

The shear strain-displacement relation, under FSDT shown in Figure 4.5, is given by

\[
\{ \gamma \} = \begin{bmatrix}
\gamma_{xx} \\
\gamma_{yy}
\end{bmatrix} = \begin{bmatrix}
w_{y,x} \\
w_{x,x}
\end{bmatrix} + \begin{bmatrix}
\psi_x \\
\psi_y
\end{bmatrix}
\]  

(4.16)

By defining strain and shear strain interpolation matrices used later on

\[
[C_i] = \begin{bmatrix}
H_{w,y} \nabla_{x} \\
H_{w,x} \nabla_{y} \\
H_{w,x} \nabla_{y} + H_{w,y} \nabla_{x}
\end{bmatrix}
\]  

(4.17)

\[
[C_{\gamma x}] = \begin{bmatrix}
H_{w,x,x} \\
H_{w,y,y}
\end{bmatrix}
\]  

(4.18)

\[
[C_{\gamma y}] = \begin{bmatrix}
H_{w,x,y} + H_{w,y,x} \\
H_{w,x,x} + H_{w,y,y}
\end{bmatrix}
\]  

(4.19)

one can write

\[
\{ \kappa \} = [C_i] \{ w_y \}
\]  

(4.20)

\[
\{ \gamma \} = [C_{\gamma x}] \{ w_x \} + [C_{\gamma y}] \{ w_y \}
\]  

(4.21)
4.4a Starboard Wing Finite Element Mesh

4.4b Two Triangular Finite Elements

Figure 4.4 Finite Element Geometry
4.3.3 Constitutive Relations

Customarily, normal stress $\sigma_z$ is considered negligible in comparison with in plane normal and shear stresses $\sigma_x, \sigma_y, \text{and} \tau_{xy}$. Then, for a linearly elastic and isotropic material, the stress-strain relationship in each $z$-parallel layer of the plate is the familiar plane stress expression

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{4.22}$$

$$\begin{bmatrix} \tau_{yz} \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} \tag{4.23}$$

One customarily associates the above-listed stresses with element moments and forces per unit length in the $xy$ plane. For example, an increment of the bending moment acting on the element side parallel to $yz$ plane $M_x$ is $dM_x = z (\sigma_x \, dA)$, where $dA = (1) \, dz$ is an increment of cross-sectional area with a distance $z$ from the deformed mid-surface. Thus, resultant bending moments on the element sides parallel to $yz, xz$ planes, the resultant
twisting moment along the $x$ or $y$ axis, and the resultant shear forces acting on the element sides parallel to $yz, xz$ planes, are

$$
M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z \, dz \quad M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z \, dz \quad M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z \, dz \quad (4.24)
$$

$$
R_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz} \, dz \quad R_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz} \, dz \quad (4.25)
$$

The above-listed integral forms can be cast into matrix forms such that

$$
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
\frac{E h^3}{12(1-\nu^2)}
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\begin{bmatrix}
\psi_{y,x} \\
\psi_{x,y} \\
\psi_{y,y} + \psi_{x,x}
\end{bmatrix}
$$

$$
\{M\} = [D] \{\kappa\} \quad (4.26)
$$

$$
[D] =
\frac{E h^3}{12(1-\nu^2)}
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
$$

$$
\begin{bmatrix}
R_y \\
R_x
\end{bmatrix} =
\begin{bmatrix}
Gh & 0 \\
0 & Gh
\end{bmatrix}
\begin{bmatrix}
\gamma_{yz} \\
\gamma_{xx}
\end{bmatrix}
$$

$$
\{R\} = [A] \{\gamma\} \quad (4.27)
$$

### 4.3.4 Equations of Motion

Finite element equations of motion for an isotropic homogeneous in-extensional cantilevered plate, subjected to aerodynamic loads and root excitation as a result of the air vehicle’s motion, are derived using the principle of virtual work

$$
\delta W_{int} = \delta W_{ext} \quad (4.28)
$$
where $\delta W_{\text{int}}$ and $\delta W_{\text{ext}}$ denote the internal and external virtual works.

The work done by internal forces and moments is

$$
\delta W_{\text{int}} = \int_A \left( \{\delta \kappa\}^T \{M\} + \alpha_{sh} \{\delta \gamma\}^T \{R\} \right) dA
$$

(4.29)

where $\alpha_{sh}$ is the shear relaxation or correction factor that is used to avoid excessive stiffness and ill-conditioning for coarse meshes, and the matrix transpose is assigned the superscript $T$. In case of homogenous isotropic plates, $\alpha_{sh}$ is taken equal to $5/6$.\textsuperscript{146} Note the integral in Equation (4.29) is over the element surface area. From Equation (4.20) and (4.21), one can obtain

$$
\{\delta \kappa\}^T = \{\delta w_v\}^T [C]^{\text{T}}
$$

(4.30)

$$
\{\delta \gamma\}^T = \{\delta w_i\}^T [C_{ri}]^{\text{T}} + \{\delta w_v\}^T [C_{rv}]^{\text{T}}
$$

(4.31)

By expanding the term for work done by internal forces and moments in Equation (4.29), one obtains

$$
\delta W_{\text{int}} = \int_A \left( \{\delta w_v\}^T [C]^{\text{T}} [D][C] \{w_v\} + \alpha_{sh} \left( \{\delta w_i\}^T [C_{ri}]^{\text{T}} + \{\delta w_v\}^T [C_{rv}]^{\text{T}} \right) \right)

\left[ A_s \right] \left( [C_{ri}] \{w_i\} + [C_{rv}] \{w_v\} \right) dA
$$

(4.32)

Further expansion leads to

$$
\delta W_{\text{int}} = \int_A \left( \{\delta w_v\}^T [C]^{\text{T}} [D][C] \{w_v\} + \alpha_{sh} \{\delta w_i\}^T [C_{ri}]^{\text{T}} [A_s] [C_{ri}] \{w_i\} + \alpha_{sh} \{\delta w_v\}^T [C_{ri}]^{\text{T}} [A_s] [C_{rv}] \{w_v\} + \alpha_{sh} \{\delta w_v\}^T [C_{rv}]^{\text{T}} [A_s] [C_{rv}] \{w_v\} \right) dA
$$

(4.33)
Therefore, the linear stiffness and linear shear stiffness matrices are

\[
[k_{w}] = \int_{A} [C_{w}]^T [D] [C_{w}] dA
\]  \hspace{1cm} (4.34)

\[
[k_{i}^s] = \alpha_{sh} \int_{A} [C_{i}]^T [A_{s}] [C_{i}] dA
\]  \hspace{1cm} (4.35)

\[
[k_{i}^{s\gamma}] = \alpha_{sh} \int_{A} [C_{i}]^T [A_{s}] [C_{i}] dA
\]  \hspace{1cm} (4.36)

\[
[k_{i}^{s\gamma}] = \alpha_{sh} \int_{A} [C_{i}]^T [A_{s}] [C_{i}] dA
\]  \hspace{1cm} (4.37)

\[
[k_{i}^s] = \alpha_{sh} \int_{A} [C_{i}]^T [A_{s}] [C_{i}] dA
\]  \hspace{1cm} (4.38)

Expanding the work done by external forces in Equation (4.28), one obtains

\[
\delta W_{ext} = \int_{A} (-\rho h \delta w \ddot{w} + \delta w F_{a} + \rho h \delta w a_{b}) dA
\]  \hspace{1cm} (4.39)

where \( F_{a} \) is the aerodynamic surface traction, while \( a_{b} \) denotes the acceleration of wing root excited by rigid fuselage dynamic motion. Further expansion of Equation (4.39) leads to

\[
\delta W_{ext} = \int_{A} \left( \rho h \left\{ \delta w_{i} \right\}^T \left[ H_{w} \right]^T + \left\{ \delta w_{r} \right\}^T \left[ H_{w} \right]^T \right) \left( \left[ H_{w} \right] \left\{ \ddot{w}_{i} \right\} + \left[ H_{w} \right] \left\{ \ddot{w}_{r} \right\} \right) dA
\]

\[
+ \int_{A} \left( \left\{ \delta w_{i} \right\}^T \left[ H_{w} \right]^T + \left\{ \delta w_{r} \right\}^T \left[ H_{w} \right]^T \right) F_{a} dA
\]

\[
+ \int_{A} \left( \rho h \left\{ \delta w_{i} \right\}^T \left[ H_{w} \right]^T + \left\{ \delta w_{r} \right\}^T \left[ H_{w} \right]^T \right) \left( \left[ H_{w} \right] \left\{ 'a_{b} \right\} + \left[ H_{w} \right] \left\{ 'a_{b} \right\} \right) dA
\]

where \( \left\{ 'a_{b} \right\} = -\left[ \dot{\dot{w}}_{AIC} \dot{w}_{AIC} \ddot{w}_{AIC} \right]^T \) and \( \left\{ 'a_{b} \right\} = \left[ \dot{\dot{\dot{P}}} \dot{\dot{P}} \dot{\dot{P}} -\ddot{Q} -\ddot{Q} -\ddot{Q} \right]^T \)

\[
\delta W_{ext} = -\left\{ \delta w_{i} \right\}^T \int_{A} \rho h \left[ H_{w} \right]^T \left[ H_{w} \right] \left\{ \ddot{w}_{i} \right\} dA
\]  \hspace{1cm} (4.40)
\[-\{\delta w_i\}^T \int_A \rho h [H_w]^T [H_{ww}] \{\ddot{w}_i\} dA\]
\[-\{\delta w_i\}^T \int_A \rho h [H_{ww}]^T [H_w] \{\ddot{w}_i\} dA\]
\[-\{\delta w_i\}^T \int_A \rho h [H_{ww}]^T [H_{ww}] \{\ddot{w}_i\} dA\]
\[+\{\delta w_i\}^T \int_A [H_w]^T F_o dA\]
\[+\{\delta w_i\}^T \int_A [H_{ww}]^T F_o dA\]
\[+\{\delta w_i\}^T \int_A \rho h [H_w]^T [H_{ww}] \{\dot{w}_i\} dA\]
\[+\{\delta w_i\}^T \int_A [H_{ww}]^T [H_w] \{\dot{w}_i\} dA\]
\[+\{\delta w_i\}^T \int_A [H_{ww}]^T [H_{ww}] \{\dot{w}_i\} dA\]
\[+\{\delta w_i\}^T \int_A \rho h [H_{ww}]^T [H_{ww}] \{\dot{w}_i\} dA\]

From the previous equation, mass matrices can be written as

\[ [m_i] = \int_A \rho h [H_w]^T [H_w] dA \]
(4.43)

\[ [m_{ww}] = \int_A \rho h [H_w]^T [H_{ww}] dA \]
(4.44)

\[ [m_{ww}] = \int_A \rho h [H_{ww}]^T [H_{ww}] dA \]
(4.45)

\[ [m_{ww}] = \int_A \rho h [H_{ww}]^T [H_{ww}] dA \]
(4.46)

and the external loading vectors are

\[ \{p_i\} = \int_A [H_w]^T F_o dA \]
(4.47)

\[ \{p_{ww}\} = \int_A [H_{ww}]^T F_o dA \]
(4.48)
With any general virtual displacement, the equation of motion for the plate element
with in-extensional Reissner-Mindlin plate theory is

\[
\begin{bmatrix}
[m_i] & [m_{iw}] \\
[m_{wi}] & [m_w]
\end{bmatrix}
\begin{bmatrix}
\{\ddot{w}_i\} \\
\{\ddot{w}_w\}
\end{bmatrix}
\]

\[+
\begin{bmatrix}
0 & 0 \\
0 & [k_w]
\end{bmatrix}
\begin{bmatrix}
[k^i] & [k^i_{iw}] \\
[k^i_{wi}] & [k^i_w]
\end{bmatrix}
\begin{bmatrix}
\{w_i\} \\
\{w_w\}
\end{bmatrix}
\]

\[+
\begin{bmatrix}
\{p_i\} \\
\{p_w\}
\end{bmatrix}
\begin{bmatrix}
[m_i] & [m_{iw}] \\
[m_{wi}] & [m_w]
\end{bmatrix}
\begin{bmatrix}
\dot{p} \\
\dot{p}
\end{bmatrix}
\]

\[+
\begin{bmatrix}
\{q\}
\end{bmatrix}
\begin{bmatrix}
[m_i] & [m_{iw}] \\
[m_{wi}] & [m_w]
\end{bmatrix}
\begin{bmatrix}
\ddot{q}
\end{bmatrix}
\]

\[=
\begin{bmatrix}
\{\ddot{w}_{AiC}\} \\
\{\ddot{w}_{AiC}\} \\
\{\ddot{w}_{AiC}\}
\end{bmatrix}
\]

where the second term in the right-hand side of the previous equation represents the external force applied to the aircraft wing as a result of excitation originating from the aircraft motion. Recall that \(\dot{w}_{AiC}\) is the aircraft acceleration in the \(Z_B\) direction, \(\dot{p}\) is the roll acceleration, and \(\dot{q}\) represents the pitch acceleration. Several other base motion accelerations exist but were neglected due to their minor role in exciting the flexible wing.

After assembling all the elements and taking into account the kinematic boundary conditions (root lateral and rotational deflections are zero), the system equations of motion in structural node degrees-of-freedom can be expressed as
where \( \{UN\} \) is a column vector with elements of unity and a length of 270 that equals one third of the \( \{W\} \) length. The aerodynamic loads applied to the wing are calculated using the vortex-lattice method that will be discussed in details later in this chapter.

### 4.4 Transient Structural Response Using Modal Analysis

The ability to decompose vibrating systems in terms of modal properties is a very powerful technique that serves one well in both performing analysis and providing more physical insight. The key to normal mode analysis is to develop tools which allow one to reconstruct the overall response of the system as a superposition of the response of the different modes of the system. The modal method allows one to replace a set of coupled linear differential equations with uncoupled equations. Each uncoupled equation represents the motion of the system for that mode of vibration. If natural frequencies and mode shapes are available for the system, then it is simpler to visualize the motion of the system in each mode. The following steps represent the roadmap of the modal solution.

1. Solve the undamped eigen-value problem, which identifies the resonant frequencies and mode shapes (eigenvalues and eigenvectors).
2. Calculate the contribution of each mode to the overall response that allows one to reduce the size of the problem by eliminating modes that cannot be excited or those that have small effect on the overall response. In the current investigation, only the first ten modes are retained, while the modes of higher frequencies that have little contribution to the system at lower frequencies will be eliminated.
3. Use the eigenvectors to uncouple or diagonalize the original set of coupled equations, allowing the solution of a set of reduced number of uncoupled equations.

4. Use the selected eigenvectors to transform the initial conditions and forcing functions defined in the physical coordinates to associated variables defined in the principal (modal) coordinates.

5. Solve the uncoupled 2\textsuperscript{nd} degree differential equations using 4\textsuperscript{th} order Runga-Kutta numerical integration.

6. Transform from the modal coordinates back to the physical coordinates, where the physical displacements will be used to calculate the stress at the desired element.

Simplifying the equation of motion in Equation (4.52), to the case of free oscillation, yields the right-hand side equal to zero.

\[
\begin{bmatrix} M \end{bmatrix}\{\ddot{\bar{W}}\} + \left(\begin{bmatrix} K \end{bmatrix} + \begin{bmatrix} K^s \end{bmatrix}\right)\{\bar{W}\} = \{0\} \tag{4.53}
\]

Using the eigen-solver, calculate the eigenvalues and the corresponding eigenvectors or mode shapes listed in Tables 4.3 and 4.4. Recall that an ad-hoc iterative solution method for the flexible wing parameters (see Section 4.2) was implemented till the first four mode frequencies of the equivalent cantilevered plate become acceptably close to the measured frequencies of the wing model studied in Reference 24 (see Table 4.1). Once the frequency conditions are satisfied, the mechanical properties and plate thickness become identified (see Table 4.2).

Expressing the panel response as

\[
\{W\} = \sum_{\tau=1}^{10} q_\tau^n(t) \{\phi\}' = [\Phi]\{q^n\} \tag{4.54}
\]
where $[\Phi]$ is the matrix of eigenvectors and $\{q^m\}$ is the corresponding vector of modal coordinates.

$$[\Phi] = \begin{bmatrix} \phi^1 & \phi^2 & \ldots & \phi^{10} \end{bmatrix}$$

$$\{q^m\} = \begin{bmatrix} q_1^m & q_2^m & \ldots & q_{10}^m \end{bmatrix}^T$$

Substituting Equation (4.54) in Equation (4.52), one can get the following motion equation.

$$[M][\Phi]\{\ddot{q}^m\} + ([K] + [K^s]) [\Phi] \{q^m\} = \{P(t)\} + [M] \begin{bmatrix} -\dot{w}_{AIC} * \{UN\} \\ \hat{P} * \{UN\} \\ -\dot{Q} * \{UN\} \end{bmatrix}$$

(4.56)

Multiplying both sides of the equation by $[\Phi]^T$ from the left, a set of ten uncoupled equations of motion result

$$[\tilde{M}] \{\ddot{q}^m\} + [\tilde{K}] \{q^m\} = \{\Phi\}^T \{P(t)\} + [\Phi]^T [M] \begin{bmatrix} -\dot{w}_{AIC} * \{UN\} \\ \hat{P} * \{UN\} \\ -\dot{Q} * \{UN\} \end{bmatrix}$$

(4.57)

where $[\tilde{M}]$ and $[\tilde{K}]$ are $10 \times 10$ modal mass and modal stiffness matrices, respectively.

$$[\tilde{M}] = [\Phi]^T [M] [\Phi]$$

$$[\tilde{K}] = [\Phi]^T [K_L] [\Phi]$$

(4.58)

$$[K_L] = [K] + [K^s]$$

(4.59)

Once the set of uncoupled 2nd order differential equations in terms of the principal coordinates is constructed, use the 4th order Runge-Kutta method to get the propagations of these principal coordinates. The initial conditions of the physical displacements, velocities, and accelerations are chosen to be zeros, while the initial conditions of the rigid aircraft contributions to the right-hand side of the equation are the values of the
trimming conditions mentioned in Chapter 3, which are also zero. To calculate the physical displacement at the most critical element on the lower surface of the wing based on the modal displacement, use the back-transformation relation shown in Equation (4.54).

Table 4.3 First Two Bending Modes

<table>
<thead>
<tr>
<th>First Bending Mode at $\omega = 8.023 , \text{Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$X_B (m)$</strong></td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-0.1</td>
</tr>
<tr>
<td>-0.3</td>
</tr>
<tr>
<td>-0.5</td>
</tr>
<tr>
<td>-0.7</td>
</tr>
<tr>
<td>-0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second Bending Mode at $\omega = 26.563 , \text{Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$X_B (m)$</strong></td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
</tr>
<tr>
<td>-4</td>
</tr>
<tr>
<td>-5</td>
</tr>
<tr>
<td>-6</td>
</tr>
<tr>
<td>-7</td>
</tr>
<tr>
<td>-8</td>
</tr>
<tr>
<td>-9</td>
</tr>
<tr>
<td>-10</td>
</tr>
</tbody>
</table>

\[ X_B (m) \quad Y_B (m) \]
Table 4.4 First Two Torsion Modes

First Torsion Mode at $\omega = 31.602 \text{ Hz}$

<table>
<thead>
<tr>
<th>$X_B (m)$</th>
<th>$Y_B (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Second Torsion Mode at $\omega = 47.231 \text{ Hz}$

<table>
<thead>
<tr>
<th>$X_B (m)$</th>
<th>$Y_B (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Modal Lateral Displacement

Graph showing the modal lateral displacement for both modes.
4.5 Vortex-Lattice Model Formulation

As air flows around an aircraft, forces build up. These forces originate from pressure and friction acting on every free surface of the interface between the air and the airframe. The resulting force \( \vec{F} \) acting on the aircraft is given by integrating the distributed normal and tangential pressures \( (P_n, P_t) \) across the interface as shown in Figure 4.6. The vortex-lattice method (VLM) is used to calculate the aerodynamic forces. The vortex-lattice method models the fluid as an inviscid, incompressible, and irrotational flow obtained by the superposition of elementary flows. This method is a linear method with limitations, but is still very useful and is all that is needed for the intended application. The linear domain corresponds to small Mach numbers where compressible effects can be disregarded. The angles of attack are also small to ensure that the lifting surfaces remain well below the stall limit. For the cases under investigation in Chapter 6 where the maximum Mach number is assigned to be sufficiently below 0.6 and the angles of attack are significantly lower than the stall angle of attack, the linear aerodynamic assumption is acceptable for a large extent.

![Figure 4.6 Normal and Tangential Pressures on Air Vehicle Starboard Wing](image_url)
4.5.1 Velocity for Line Vortex

Due to the spatial conservation of vorticity, a vortex filament cannot begin or end abruptly in a fluid. The filament should either form a closed ring or end at infinity or at a solid or free surface. When a vortex line with differential length is considered, a vortex segment, the induced flow field velocity at point $P$ is defined by the law of Biot-Savart, which is represented in Equation (4.60) and Figure 4.7.

\[
d\vec{V} = \frac{\Gamma \left( d\vec{\ell} \times \vec{r} \right)}{4 \pi r^3}
\]  

(4.60)

The previous equation can be integrated to give the induced velocity for a vortex segment of arbitrary finite length which takes the form of the following equation.

\[
\vec{V} = \frac{\Gamma (\vec{r}_1 \times \vec{r}_2)}{4 \pi |\vec{r}_1 \times \vec{r}_2|^2} \left[ \vec{r}_0 \cdot \left( \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} \right) \right]
\]  

(4.61)

In Figure 4.7, the finite segment has end points denoted by $A$ and $B$.

Vortex segments can be utilized to build very intricate vortex systems, such as the meshwork of vortex segments used in vortex-lattice theory. The vortex-lattice method employed in this research uses three vortex segments to construct a vortex horseshoe for every panel discretizing the wing surface.

![Figure 4.7 Finite Segment of a Straight Vortex Filament](image-url)
4.5.2 Velocity for Horseshoe Vortex

The expressions developed from the straight vortex filament will be used to create a horseshoe vortex, which extends from downstream infinity to a point A in the field, then from point A to another point B, and finally from point B downstream to infinity (see Figure 4.8). Notice that both \( \overline{\infty A} \) and \( \overline{B \infty} \) are parallel to the x axis. The \( xyz \) vortex-lattice frame appearing in Figure 4.8 is obtained from the body frame by 180° rotation around the \( y_B \) axis. The velocity induced by this vortex is the summation of velocities from the three horseshoe parts.

\[
\begin{align*}
A &= A(x_1, y_1, z_1) \\
B &= B(x_2, y_2, z_2)
\end{align*}
\]

Figure 4.8 Horseshoe Vortex Geometry

To obtain the expression for the velocity field at a general point \( P(x, y, z) \) due to the specified horseshoe vortex, rewrite the Biot-Savart law in the Bertin-Smith notation, which is given in Figure 4.9. Using the integrated Biot-Savart Equation (4.61) between the bound vortex ends A and B, the induced velocity for this finite segment of the horseshoe is

\[
\vec{V}_{AB} = \frac{\Gamma}{4\pi|\vec{r}_p|} (\cos \theta_1 - \cos \theta_2) \hat{e} \tag{4.62}
\]
where $\vec{e}$ represents the unit vector calculated as follows

$$\vec{e} = \frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|} \quad (4.63)$$

Using vector analysis, one can get

$$\vec{r}_p = \frac{(\vec{r}_1 \times \vec{r}_2)}{|\vec{r}_0|}, \quad \cos \theta_1 = \frac{(\vec{r}_0 \cdot \vec{r}_1)}{|\vec{r}_0||\vec{r}_1|}, \quad \cos \theta_2 = \frac{(\vec{r}_0 \cdot \vec{r}_2)}{|\vec{r}_0||\vec{r}_2|} \quad (4.64)$$

Substituting Equation (4.64) in Equation (4.62), the final form of the induced velocity by the bound vortex AB is

$$\vec{V}_{AB} = \frac{\Gamma |\vec{r}_0|}{4\pi |\vec{r}_1 \times \vec{r}_2|} \left[ \frac{\vec{r}_0 \cdot \left( \frac{\vec{r}_1}{|\vec{r}_0||\vec{r}_1|} - \frac{\vec{r}_2}{|\vec{r}_0||\vec{r}_2|} \right)}{|\vec{r}_1 \times \vec{r}_2|} \right] \vec{r}_1 \times \vec{r}_2 \quad (4.65)$$

or

$$\vec{V}_{AB} = \frac{\Gamma}{4\pi} F_{1AB} F_{2AB} \quad (4.66)$$

where $F_{1AB}$ and $F_{2AB}$ are lengthy expressions. By following the vector definitions, these expressions can be written in Cartesian coordinates as follows.
To calculate the velocity induced by the filament that extends from \( \infty \) to \( A \), first calculate the velocity induced by the collinear, finite length filament that extends from \( C \) to \( A \), as shown in Figure 4.10.

\[
F_{1AB} = \frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|^2} \\
\left\{ \begin{array}{l}
\left[ (y - y_1)(z - z_2) - (y - y_2)(z - z_1) \right]^2 \\
- \left[ (x - x_1)(z - z_2) - (x - x_2)(z - z_1) \right]^2 \\
+ \left[ (x - x_1)(z - z_2) - (x - x_2)(z - z_1) \right]^2 \\
\end{array} \right. \\
= \frac{\left\{ \begin{array}{l}
\left[ (y - y_1)(z - z_2) - (y - y_2)(z - z_1) \right]^2 \\
+ \left[ (x - x_1)(z - z_2) - (x - x_2)(z - z_1) \right]^2 \\
+ \left[ (x - x_1)(z - z_2) - (x - x_2)(z - z_1) \right]^2 \\
\end{array} \right\}}{|\vec{r}_1 \times \vec{r}_2|^2} \\
(4.67)
\]

\[
F_{2AB} = \vec{r}_0 \cdot \left( \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1| |\vec{r}_2|} \right) \\
= \frac{\left[ (x_2 - x_1)(x - x_1) + (y_2 - y_1)(y - y_1) + (z_2 - z_1)(z - z_1) \right]}{\sqrt{\left[ (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \right]}} \frac{\left[ (x_2 - x_1)(x - x_1) + (y_2 - y_1)(y - y_1) + (z_2 - z_1)(z - z_1) \right]}{\sqrt{\left[ (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 \right]}} \\
= \frac{\left[ (x_2 - x_1)(x - x_1) + (y_2 - y_1)(y - y_1) + (z_2 - z_1)(z - z_1) \right]}{\sqrt{\left[ (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \right]}} \\
(4.68)
\]

The key position vectors can be expressed as

\[
\begin{align*}
A &= A(x_1, y_1, z_1) \\
C &= C(x_3, y_1, z_1) \\
P &= P(x, y, z)
\end{align*}
\]

Figure 4.10 Induced Velocity Computation Nomenclature for Left Trailing Vortex
\[
\vec{r}_0 = (x_1 - x_3)\hat{i} \\
\vec{r}_1 = (x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k} \\
\vec{r}_2 = (x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k}
\]

as shown in Figure 4.10. Thus, the corresponding induced velocity is

\[
\vec{V}_{AC} = \frac{\Gamma}{4\pi} \frac{F_{1AC}}{F_{2AC}}
\]

where \(F_{1AC}\) and \(F_{2AC}\) are again lengthy expressions. By following the vector definitions, these expressions can be written in Cartesian coordinates as follows.

\[
F_{1AC} = \frac{[(z - z_1)\hat{j} - (y - y_1)\hat{k}]}{[(z - z_1)^2 + (y - y_1)^2]}(x_3 - x_1)
\]

\[
F_{2AC} = (x_3 - x_1)\left\{ \frac{(x_3 - x)}{\sqrt{[(x - x_3)^2 + (y - y_1)^2 + (z - z_1)^2]}} + \frac{(x - x_1)}{\sqrt{[(x - x_3)^2 + (y - y_1)^2 + (z - z_1)^2]}} \right\}
\]

When \(x_3\) approaches \(\infty\), the first term of \(F_{2AC}\) goes to 1. Therefore, the velocity induced by the vortex filament, which extends from \(\infty\) to \(A\) in a positive direction parallel to the \(x\) axis is given by

\[
\vec{V}_{A\infty} = \frac{\Gamma}{4\pi} \left\{ \frac{[(z - z_1)\hat{j} - (y - y_1)\hat{k}]}{[(z - z_1)^2 + (y - y_1)^2]} \right\} \times \\
\left\{ 1 + \frac{(x - x_1)}{\sqrt{[(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2]}} \right\}
\]
Similarly, the velocity induced by the vortex filament that extends from B to \( \infty \) in a positive direction parallel to the \( x \) axis is given by

\[
\vec{V}_{B\infty} = \frac{-\Gamma}{4\pi} \left\{ \frac{[(z-z_2)]\hat{j} - [(y-y_2)]\hat{k}}{[(z-z_2)^2 + (y-y_2)^2]} \right\} * \frac{(x-x_2)}{\sqrt{[(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2]}}
\]

(4.74)

Note that \( \Gamma \) is contained linearly in each velocity expression, so that the total velocity induced at a general point \( P(x, y, z) \) by the horseshoe vortex is the sum of the velocity components given in Equations (4.65), (4.73), and (4.74).

\[
\vec{V}_p = \vec{V}_{AB} + \vec{V}_{A\infty} + \vec{V}_{B\infty}
\]

(4.75)

### 4.5.3 Velocity for Lattice of Horseshoe Vortices

Now consider a lifting surface discretized by a lattice of panels, as shown in Figure 4.11, where each panel contains a horseshoe vortex. The \( n^{th} \) panel is highlighted in Figure 4.11. Assume there are \( N \) panels for the starboard wing and \( N \) on the port wing, or a total of \( 2N \) panels. The analysis and equations of Section 4.5.2 are applicable here, except a subscript \( n \) is placed on all variables as indicated in Figure 4.11.

Let the point \( P(x, y, z) \) be the control point of the \( m^{th} \) panel, which is designated by the coordinates \((x_m, y_m, z_m)\). The velocity induced at the \( m^{th} \) control point by the vortex representing the \( n^{th} \) panel will be designated as \( \vec{V}_{m,n} \). Examining Equations (4.65), (4.73), and (4.74), note that

\[
\vec{V}_{m,n} = \vec{C}_{m,n} \Gamma_n
\]

(4.76)
where the influence coefficient \( \bar{C}_{m,n} \) depends on the geometry of the \( n^{th} \) horseshoe vortex and its distance from the control point of the \( m^{th} \) panel. Since the governing equation is linear, the velocities induced by the \( 2N \) vortices are added together to obtain an expression for the total induced velocity at the \( m^{th} \) control point.

\[
\bar{V}_m = \sum_{n=1}^{2N} \bar{C}_{m,n} \Gamma_n
\]  

(4.77)

The finite segment bound vortex is typically located at the ¼ chord line from the leading edge, and the control point is located at the ¾ chord point. This selection is known as the “1/4 - 3/4 rule”.

![Horseshoe Vortex Geometry of n^th Panel](image)

\( A = A(x_{1n}, y_{1n}, z_{1n}) \)

\( B = B(x_{2n}, y_{2n}, z_{2n}) \)

Figure 4.11 Horseshoe Vortex Geometry of \( n^{th} \) Panel

To compute the unknown vortex strengths that represent the lifting flow field of the wing, the no-penetration boundary condition given in Equation (4.78) is used. In other words, the surface is treated as a streamline where the resultant flow is tangent to the
wing at each and every control point. If the flow is tangent to the wing, the component of
the induced velocity normal to the wing at the control point balances the normal
component of the free-stream velocity. To evaluate the normal velocity component at this
point, the induced velocity and free-stream velocity are projected on the normal to the
panel. Referring to Figure 4.12, the tangency requirement yields the relation

\[-u_m \sin \delta_m \cos \phi_w - v_m \cos \delta_m \sin \phi_w + w_m \cos \delta_m \cos \phi_w + V_T \sin(\alpha - \delta_m) \cos \phi_w = 0\]  (4.78)

where \(\delta_m\) is the slope of the mean camber line at the control point \(m\), \(\phi_w\) is the dihedral
angle, \(u_m\), \(v_m\), and \(w_m\) are the components of the induced velocity \(\bar{V}_m\) at control point \(m\),
and \(V_T\) is the free stream velocity which is the velocity of the vehicle. Once the vortex
strengths are known, it is possible to determine the resultant induced velocity at any point
in space.

### 4.5.4 Flexible Wing Implementation

The thickness-to-chord ratio of the F-16 wing airfoil (NACA-64A204) almost equals
4% for both the wing root and wing tip. Therefore, the F-16 wing is considered thin and
the airfoil is modeled as a thin lifting mean line (camber line), shown in Figure 4.13. The
whole wing is modeled as a curved surface composed from the camber lines of the airfoil
sections from the wing tip to the root. The discretizing panels (lattices) are projected on
that surface.

When the control surfaces on the F-16 wing deflect (leading edge flap and flaperon),
the vortex points located on these appendages are rotated around the hinge lines, as seen
in Figure 4.14. This rotation causes a motion where the vortex segments on both of the
control surfaces and in the wake changes direction. Therefore, the Cartesian \(z\) coordinates
of these points are updated every time the control surfaces are deflected by an amount
that reflects the slopes of the mean camber line of the control surfaces at each span station. Updating the camber based on the wing flexibility was not considered. If this was to be implemented, it is projected that the effects on the final results would be small due to high stiffness of the wing.

Figure 4.12 Tangency Requirement Nomenclature

Figure 4.13 Normalized NACA-64A204 Airfoil Geometry
The classical vortex-lattice method solution steps, when applied to the flexible wing model, are summarized below.

1. Divide the mean camber surfaces of both starboard and port wings into a lattice of quadrilateral panels, and put a horseshoe vortex on each panel. The mean camber surface of each wing is divided into 18 span-wise $\times$ 14 chord-wise lattices (panels), as shown in Figure 4.15. In other words, each wing is divided into 252 panels to facilitate the accuracy in the calculated aerodynamic loadings. Note that each quadrilateral panel covers two neighboring triangular finite elements, where the largest trapezoidal chord is the common hypotenuse of the two triangular elements.

2. Place the bound segment of the horseshoe vortex on the 1/4 chord element line of each panel.
3. Place the control point on the 3/4 chord point of each panel at the midpoint in the span-wise direction.

4. Assume a flat wake in the usual classical method.

5. Determine the strengths $\Gamma_n$ of each panel vortex required to satisfy the boundary conditions by solving a system of linear equations in terms of the vortex strengths.

6. Once the strength $\Gamma_n$ is calculated, the aerodynamic force can be calculated as listed below in Equation (4.82).

![Figure 4.15 Starboard Wing Vortex-Lattice Mesh](image)

Step 5 requires the satisfaction of the no-penetration boundary condition (recall Equation (4.78)). Introducing a general vehicle state with combined angle of attack and sideslip, and as the dihedral angle of the F-16 wing is zero, Equation (4.78) will take the form

$$-u_m \sin \delta_m + w_m \cos \delta_m + V_T \cos \beta \sin (\alpha - \delta_m) = 0$$

(4.79)
or

$$\sum_{n=1}^{2 \times 252} \left( C_{m,n_1} \sin \delta_m - C_{m,n_k} \cos \delta_m \right) \Gamma_n = V_T \cos \beta \sin (\alpha - \delta_m)$$

(4.80)
where $C_{m,n_i}$ is the value of the $x$ component of the velocity induced by the panel $n$ vortex at the control point of panel $m$ divided by $\Gamma_n$ and $C_{m,n_k}$ is the $z$ component of the velocity induced by the panel $n$ vortex at the control point of panel $m$ divided by $\Gamma_n$.

Applying Equation (4.80) to all panels on both the starboard and port wings results in a set of $2 \times 252$ linear equations. These equations are condensed in the following matrix form that can be solved for the vortex strength vector, $\{\Gamma\}$.

$$
\begin{bmatrix}
C_{1,1'} \sin \delta_1 - C_{1,1''} \cos \delta_1 & \cdots & C_{1,504'} \sin \delta_1 - C_{1,504''} \cos \delta_1 \\
\vdots & \ddots & \vdots \\
C_{504,1'} \sin \delta_{504} - C_{504,1''} \cos \delta_{504} & \cdots & C_{504,504'} \sin \delta_{504} - C_{504,504''} \cos \delta_{504}
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\vdots \\
\Gamma_{504}
\end{bmatrix}
= 
\begin{bmatrix}
\sin(\alpha - \delta_1) \\
\vdots \\
\sin(\alpha - \delta_{504})
\end{bmatrix}
\cos \beta
$$

(4.81)

Once the strength of each panel is known, one can calculate the aerodynamic load for each panel where the load is assumed to be uniform on the panel area. Hence, each two neighboring finite elements of common hypotenuse will be subjected to the same aerodynamic traction, which is calculated as follows.

$$
F_{tr_n} = \rho V_r \cos \beta \cos (\alpha - \delta_n) \frac{\Gamma_n}{C_n}
$$

(4.82)

In Equation (4.82), $F_{tr_n}$ is the traction at the panel $n$, $C_n$ is the panel mean chord length, and $\rho$ is the free-stream density. Also recall the aerodynamic pressure at element $n$, which is used in Equations (4.41), (4.47), and (4.48), can be calculated as

$$
F_{a_n} = F_{tr_n}
$$

(4.83)
4.6 Aerodynamic Test Case

Before using the traction forces calculated from the vortex-lattice model, the validity of the model must be established. The theoretical lift curve that is generated by using the vortex-lattice method is compared in Figure 4.16 with the experimental results reported in Reference 26. To get the theoretical results, the sideslip angle was fixed to zero and the angle of attack was varied from 0° to 40° with an increment of 2°. The program was run at a stream velocity of 540 ft/s and altitude of 3,000 ft. The theoretical lift coefficient, $C_L$, is computed as

$$C_L = \frac{\sum_{n=1}^{2x252} F_{rx} A_n}{0.5 \rho V^2 T S}$$

(4.84)

where $A_n$ is the area of panel $n$ and $S$ is the planform area of the wing. Note that in Figure 4.16, the theoretical total lift coefficient is in good agreement with the experimental data.

To study the effect of camber updating, which is due to the leading edge flap and flaperon deflections on the lift, the same steps of calculating the theoretical total lift coefficient are exercised with four additional cases. In each case, one control surface deflection was fixed to zero while the other was fixed at its mechanical limit (see Table 3.1).

$$-2^\circ \leq \delta_{LF} \leq +21.5^\circ$$
$$-25^\circ \leq \delta_f \leq +25^\circ$$

(4.85)

The theoretical total lift coefficients for the mentioned cases are plotted with the experimental total lift coefficients in Figure 4.17. Notice that moving the flaperon in the positive direction (down) will increase the camber, which in turn will increase the lift on
the wing, while moving up will lead to a reversed behavior, where the lift will decrease. Moving the leading edge flap will lead to similar results, but the change in the lift is more significant than that resulting from the flaperon.

Figure 4.16 Theoretical and Experimental Total Lift Coefficient Comparison

Figure 4.17 Lift Coefficient Comparison with Different Control Surface Deflections
The above observation is one of the main reasons that the leading edge flap is used to change the wing’s lift whenever needed, according to the proposed LEC logic presented in Chapter 6. The other major reason for using the leading edge flap to implement the proposed LEC logic is that this surface is used to influence static properties of the airframe rather than as a maneuvering device. This role is confirmed by the following observations.

1. Droste and Walker stated that “the programmed LEF as a function of the Mach number and angle of attack will cause an automatic variable wing camber. Automatic variable wing camber is employed to maximize the lift/drag ratio.”

2. Wind tunnel tests demonstrated the need for leading edge flap to improve the lift and directional stability at a high angle of attack.

3. In general, the purpose of a leading edge flap system is to delay stall inception or airflow separation over the upper surface of an airfoil by providing an increase in camber in the nose area of the airfoil.

4.7 Critical Stress Model

The main objective of the above-mentioned simulations is to calculate the remote stress to which the pre-cracked panel is subjected. According to Reference 12, the most common positions in the F-16 wing under service loadings for fatigue cracks to appear are the lower surfaces close to the wing root. To confirm this claim, a static test case was computed using the finite element equivalent cantilevered plate model of the flexible wing built in ABAQUS. The loading condition was selected to be static and uniform. Figure 4.18 shows the results of this test. In this figure, the deflected wing is shown with corresponding principle stress contours overlaid. The maximum stress occurs on the
wing's underside at the root location. Further, according to the stress analysis the most stressed elements are close to the trailing edge. Therefore, in LEC simulations to be conducted in Chapter 6, the crack will be located in this critical position: element 451 on lower wing skin.

To calculate the principle stress on each of the triangular elements, the following assumptions are considered.

1. Within each element, lines initially straight remain straight in their displaced positions.

2. The strains $\varepsilon_x, \varepsilon_y$, and $\gamma_{xy}$ are assumed to be constant within each element. Hence, the stresses are also constant within the finite element.

3. An isotropic homogenous thin plate (the case under investigation) is assumed, thus plane stress assumptions and Equations (4.22) and (4.23) are valid.

Using Equations (4.15) and (4.22), the element uniform stress components, shown in Figure 4.19, can be calculated from the following relation.

$$
\begin{align*}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
&= \frac{E}{1-\nu^2}
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} \\
&= \frac{Eh}{2(1-\nu^2)}
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\begin{bmatrix}
\psi_{y,x} \\
\psi_{x,y} \\
\psi_{y,y} + \psi_{x,x}
\end{bmatrix}
\end{align*}
$$

(4.86)

The plate rotation derivatives are evaluated at the element mid point. Assume the crack is aligned normal to the principal stress, $\sigma_1$, shown in Figure 4.20. This assumption is a
worst case scenario that is used in Chapter 6. The principal stresses $\sigma_1$ and $\sigma_2$ and their
directions can be calculated from the stress components $\sigma_x, \sigma_y$, and $\tau_{xy}$ as

$$
\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2},
$$

$$
\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2},
$$

$$
\theta_p = \frac{1}{2} \tan^{-1} \left( \frac{\sigma_y - \sigma_x}{2\tau_{xy}} \right)
$$

(4.87)

where $\theta_p$ is the angle between $\sigma_1$ and the $x$ axis as shown in Figure 4.20.

Figure 4.18 Principal Stress Distribution on Lower Skin of Aircraft Wing
Figure 4.19 Finite Element Stress Components

Figure 4.20 Principal Stress Orientation

Stresses in Given Coordinate System

Principal Stresses

Crack Orientation
CHAPTER 5
FLIGHT CONTROL MODEL

5.1 Control System Selection

The proposed flight control system (FCS) is a simplified version of the Block 25 F-16 FCS introduced in Reference 151 with outer-loop autopilot control loops added. The FCS consists of a 3-axis stability and control augmentation system serving as the inner-loop for an outer-loop autopilot navigation control system. The FCS design is based on the assumption that the aircraft is flying with no extended landing gear or extended flaps and is not in air-refueling mode. The aircraft aerodynamic tables used in this dissertation do not include most of the conditions that the full control system\textsuperscript{151} considers such as landing, gunnery, high angle of attack, and refueling. Therefore, usage of the proposed simplified FCS is justified. An advanced control architecture designed with modern techniques is not needed or warranted here. All that is needed is a control system that will fly a mission trajectory so that LEC can be explored. Further, a simple controller is easy to program, straightforward to integrate with the overall systems, and facilitates autopilot gain computation using a time-consuming iterative optimization method, the Monte Carlo method.\textsuperscript{152} For all the above reasons, the FCS described above was selected for the research. The full FCS is divided into the longitudinal and the lateral directional FCS, the inner-loops of which are discussed in the following sections.

5.2 Longitudinal Stability/Control Augmentation System

The basic architecture of the longitudinal stability and control augmentation system consists of feeding back the angle of attack, $\alpha$, the total velocity, $V_T$, and altitude, $h$, as shown in Figure 5.1. Although $\alpha$, $V_T$, and $h$ feedback loops are present, these variables
are used primarily for leading edge flap scheduling. Stick pitch commands, $F_{pc}$, excite these loops and propagate down the forward path providing a command signal to the horizontal stabilizer and leading edge flap actuators. The stabilator actuator is modeled as a first order lag of 0.0495 s, with a rate limit of 60 deg/s and the surface deflection limit is ± 25 deg.

The leading edge flap is used as a secondary control surface to manipulate the lift distribution along the wing span. The flap servo-actuator is also simplified to 7.35 rad/s first order lag filter which corresponds to the lowest frequency aircraft servo-actuator, with a rate limit of 25 deg/s, and the surface deflection limits are -2 deg and +25 deg. Note Figure 5.1 shows $\delta_{lef}$ is also a function of the LEC logic to be developed in Chapter 6. The flap will be perturbed with an incremental value serving as the output of the implemented LEC logic to mitigate crack propagation.

At this stage there is no augmentation applied to the throttle and speed brake paths. The speed brake actuator is modeled by a first order lag of 0.0495 s, with a rate limit of 120 deg/s and maximum surface deflection limit is 60 deg. The throttle does not have an actuator model, however, the throttle signal must pass through the engine dynamic model residing in the aircraft motion equations.

For the longitudinal stability and control augmentation system, the primary input is pilot pitch command force and the primary output is horizontal stabilizer deflection angle with a secondary leading edge flap deflection schedule. The speed brake and the throttle will be actuated by the velocity-hold autopilot that will be discussed in a later section of this chapter. The pitch command gradient is a nonlinear function of $F_{pc}$ that is graphically
represented in Figure 5.2. Therefore, the functionality between $F_{pc}$ and $\delta_h$ can not be rigorously expressed as a transfer function.

Figure 5.1 Longitudinal Stability/Control Augmentation System Architecture

Figure 5.2 Pitch Command Gradient
5.3 Lateral Stability/Control Augmentation System

The basic layout of the lateral stability and control augmentation system is shown in Figure 5.3. The inputs to the lateral augmentation system are pilot roll command force, \( F_{rc} \), and the pilot yaw command force, \( F_{yc} \). The outputs are differential flaperon angle, \( \delta_f \), and the rudder deflection, \( \delta_r \). The lateral flight control system employs roll rate feedback to the flaperon. The rudder and a flaperon-rudder interconnection use a combination of lateral acceleration, \( A_y \), yaw rate, \( R \), and angle of attack, \( \alpha \), feedbacks. Figure 5.4 shows the nonlinear roll command gradient. The rudder servo-actuator is modeled as a first order lag of 0.0495 s, with a rate limit of 120 deg/s and the surface deflection limit is ± 30 deg. The servo-actuator of the flaperon is modeled as a first order lag of 0.0495 s, with a rate limit of 80 deg/s and the surface deflection limit is ± 21.5 deg.

5.4 Longitudinal and Lateral Autopilot System

Figure 5.5 shows the overall flight control system with the autopilot system, the stability and control augmentation system, and the interfacing with the flight dynamics model. The autopilot consists of the altitude-hold system, the velocity-hold system, and the heading-hold system. Note heading angle and yaw angle are synonymous here. The autopilot system is driven by three commands coming from a nominal mission profile stored in the on-board flight computer, and the system generates pitch, roll, and speed commands that are fed into the pilot pitch-roll force command, throttle, and speed brake signals. At the same time, the pilot yaw force command is frozen at the zero position. The autopilot system receives the three commands from the mission generating logic which provides desired altitude, velocity, and heading angle. At each time step, these three commands are calculated from the nominal flight trajectory, which in turn is generated
based on the required vehicle motion during each section of the mission. The function of
the autopilot is to closely track the desired trajectory.

Figure 5.3 Lateral Stability/Control Augmentation System Architecture

Figure 5.4 Roll Command Gradient
The altitude-hold autopilot consists of three feedback loops starting with $Q$, then $\theta$, and finally $h$. This architecture provides proper pitch damping, pitch response, and altitude tracking. Note the altitude loop incorporates proportional-integral compensation for zero steady state error while the other loops utilize proportional control. The velocity-hold autopilot feeds back total aircraft velocity $V_T$ and operates through a proportional-integral compensator providing throttle commands. Moreover, this autopilot drives the speed brake command, which is triggered once the difference between the desired and current aircraft speed is less than -1.5 ft/s to assist in decelerating the aircraft. Finally, the heading-hold autopilot is based on an inner-loop feedback of $\phi$ and an outer-loop feedback of $\psi$. Proportional compensation is used here since the vehicle is neutrally stable with respect to heading.

In each autopilot, a table look-up calculation is indicated in Figure 5.5. These tables contain the nominal desired trajectories for altitude, velocity, and heading that the autopilot is to track. The trajectory tables are indexed with time and maintained in the computer memory for processing. Generation of the nominal mission is addressed next.

5.5 Mission Profile Design

The remote stress experienced by the cracked wing panel mentioned in Chapter 4 depends on the in-service motion of the F-16 vehicle. In order to generate a realistic simulation that represents the motion of the aircraft, a nominal mission is developed in this section. An air-to-ground mission for a fixed target is designed to simulate a representative mission of the studied aircraft. This mission is a simple strike mission for ordnance delivery or training. The mission consists of several phases including climb, cruise, descent, ordnance release, steep climb to escape enemy defenses, short cruise at
relatively high altitude, descent to and cruise at lower altitude, and then final descent. The whole mission elapse time is approximately 35 minutes and it is developed based on a mission presented in Reference 23. The steering points listed in Table 5.1 represent the instants at which altitude, speed, and heading angle of the aircraft are changed during the nominal mission. The steering points and any combination of altitude and velocity pairs at any instant of the mission are designed to be completely enclosed inside the flight envelope, as shown in Figure 5.6, in order not to violate the structural limit, the stall limit, the propulsive limit, and the atmospheric limit.

The continuous flight path is generated based on the mission profile, where at each time step, the path consists of the triple-command for altitude, total velocity, and heading angle. These sets of three commands are direct input to the autopilot system described in Section 5.4. The autopilot system will follow the prescribed flight path commands through generating the necessary maneuvers for the vehicle. In order to avoid necessary changes to the structure of the velocity-hold and altitude-hold autopilots depending on the operating point within the flight envelope, the speeds at all of the steering points were chosen to be on the front side of the power curve shown in Figure 5.7. On the front side of the power curve, changing the throttle opening primarily leads to a change in the aircraft speed while a change in the horizontal stabilizer deflection will lead to a significant change in the altitude of the aircraft. If the speeds at the steering points were chosen to lie on the back side of the power curve, these response characteristics would be reversed requiring autopilot changes.
During the generation of flight path altitudes, a smoothing function is used to eliminate sharp motions, which are not realistic maneuvers performed by the F-16 model. Equation (5.1) represents the altitude trajectory smoothing function.

5.5a Autopilot and Stability/Control Augmentation System (Part I)
5.5b Autopilot and Stability/Control Augmentation System (Part II)
5.5c Autopilot Stability/Control Augmentation System (Part III)

Figure 5.5 Overall Flight Control System
Table 5.1 Altitude, Velocity, and Heading Angle Steering Points

<table>
<thead>
<tr>
<th>Steering Point</th>
<th>Time (s)</th>
<th>Altitude (ft)</th>
<th>Velocity (ft/s)</th>
<th>Heading Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1,000</td>
<td>540</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>7,100</td>
<td>582</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>350</td>
<td>7,100</td>
<td>640</td>
<td>80</td>
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<td>4</td>
<td>980</td>
<td>7,100</td>
<td>700</td>
<td>183</td>
</tr>
<tr>
<td>5</td>
<td>1,130</td>
<td>1,000</td>
<td>700</td>
<td>212</td>
</tr>
<tr>
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<td>1,180</td>
<td>1,000</td>
<td>700</td>
<td>218</td>
</tr>
<tr>
<td>7</td>
<td>1,330</td>
<td>20,000</td>
<td>723</td>
<td>223</td>
</tr>
<tr>
<td>8</td>
<td>1,380</td>
<td>20,000</td>
<td>744</td>
<td>229</td>
</tr>
<tr>
<td>9</td>
<td>1,490</td>
<td>16,000</td>
<td>750</td>
<td>240</td>
</tr>
<tr>
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<td>1,565</td>
<td>16,000</td>
<td>750</td>
<td>241</td>
</tr>
<tr>
<td>11</td>
<td>1,675</td>
<td>10,000</td>
<td>710</td>
<td>262</td>
</tr>
<tr>
<td>12</td>
<td>1,850</td>
<td>10,000</td>
<td>650</td>
<td>300</td>
</tr>
<tr>
<td>13</td>
<td>2,075</td>
<td>1,000</td>
<td>540</td>
<td>360</td>
</tr>
</tbody>
</table>

\[
h(t) = h(t_s) + \frac{h(t_f) - h(t_s)}{T_j} \left(\frac{t}{T_j} - t_s\right)\]

\[
t_s = \frac{T_j}{2\pi} \sin \left(2\pi \frac{t - t_s}{T_j}\right)
\]

where

- \( h(t) \): altitude at current time \( t \)
- \( h(t_s) \): beginning altitude of current segment \( j \)
- \( h(t_f) \): ending altitude of current segment \( j \)
- \( t \): current time of the mission
- \( t_s \): beginning time of current segment \( j \)
- \( t_f \): ending time of current segment \( j \)
- \( T_j \): duration time of current segment \( j \)

Utilization of the above smoothing function will guarantee the climb and descent rates and their accelerations to be within \( \pm 150 \text{ ft/s}^2 \) and \( \pm 40 \text{ ft/s}^2 \), respectively. Similar smoothing functions are used in generating the total velocity and heading angle of the
flight trajectory, where the heading rate is limited to within ± 0.027 deg/s and the angular acceleration of heading angle is limited to within ± 0.004 deg/s². The altitudes, velocities, and heading angles along the above described flight trajectory are plotted in Figures 5.8, 5.9, and 5.10 respectively.

![Figure 5.6 Nominal Mission Overlaid on Flight Envelope](image)

Figure 5.6 Nominal Mission Overlaid on Flight Envelope

![Figure 5.7 Power Curve for F-16 Model](image)

Figure 5.7 Power Curve for F-16 Model
Figure 5.8 Altitude Along Nominal Mission

Figure 5.9 Total Velocity Along Nominal Mission
5.6 Control System Gain Selection

Numerical values for gains in the inner-loop stability and control augmentation system from Reference 151 are available. However, only a subset of these values appears in Figure 5.5, where the remaining gains appear in their symbolic form. Further, all autopilot gains appearing in Figure 5.5 appear symbolically. Numerical values for this set of control gains which allow the flight control system to track the desired mission are now sought.

The coupled flight motions lead to significant effects on the lateral motions due to the longitudinal loop closures. Conversely, the lateral control loops can affect the longitudinal motions when the entire system is excited. Therefore, selection of the longitudinal and lateral control gains for the stability and control augmentation and
autopilot systems will be conducted concurrently. The objective is to identify a constant gain set for the whole mission so that no gains schedules need to be implemented.

A nontypical design approach, the Monte Carlo method, will be used for gain selection. Monte Carlo methodology dates back to the mid 1940s and the mid 1950s. In recent decades, researchers have realized the power of this technique and adapted it for a wide variety of computational tasks such as image restoration, hierarchical modeling, time series forecasting, and computational biology. The Monte Carlo method is based mainly on the principle of random numbers that can be employed efficiently using a digital computer, wherein arbitrary data is generated for different processes to depict a statistical conclusion. The advantages of the Monte Carlo simulation are the nonlinear systems can be evaluated directly, the results reflect the influences of combinatorial effects of various uncertain parameters, and the uncertain parameters can be determined as physical values. Monte Carlo simulation has become recognized as an effective and powerful tool for system evaluation in the development of aerospace vehicles.

In the current application, a Monte Carlo design method was implemented through selecting the gain values randomly and checking the cost function to be optimized until a minimum solution was reached. The chosen objective function was the summation of weighted errors between the desired and control system derived altitudes, total speeds, and heading angles across the simulation time window respectively. The weights were selected such that the errors are of the same order, where heading angle error weight is 100, the total velocity error weight is 10, and the altitude error weight is 1. The algorithm of the optimization tool is shown in Figure 5.11. For an initial selection of the gain ranges, the algorithm would typically diverge as the gains in the early iterations were
selected randomly. After inspecting the control surface values in these situations, it was found that the horizontal stabilizer would reach its mechanical limit within a few seconds. The cause of this behavior is that, for those selected gains, the nose-down control moment from the horizontal stabilizer is needed to balance the strong inherent nose-up airframe moment. Consequently, the horizontal tail would reach its travel limit, at which time the airplane will lose the stability contribution of the tail and the airplane will diverge in the pitch axis. This behavior is the well-known F-16 departure phenomena.\textsuperscript{27} The major drawback of the iterative Monte Carlo method is that it is highly time-consuming, especially when no accurate initial estimate for the selected ranges of gains are available. After considerable effort, the final gains are listed in Table 5.2.

An evaluation of the designed control system to track the nominal mission is given here. The following figures show the desired and the actual altitudes, total velocities, and the heading angles for the nominal mission along with the time histories of the control surface deflections and different state variables. The variations of altitudes, total velocities, and heading angles with time for the mission are represented in Figures 5.12-5.14 respectively for actual and desired flight trajectories. The variations in the primary and secondary control surfaces are plotted vs. time in Figures 5.15-5.19, while the various state variables are considered in Figures 5.20-5.26. First, Figures 5.12-5.14 clearly demonstrate that the flight control system is working properly with very little tracking error. Figure 5.15 shows that the horizontal stabilizer deflection varies between -1.4 deg and -2.2 deg. The change in the rudder deflection is small as shown in Figure 5.16. Figure 5.17 shows that the change in the flaperon deflection is small until close to the end of the mission where the changes fluctuate abruptly between -0.3 deg and 0.3
deg. The variations in the speed brakes are spiky as they are activated only when the
difference between the desired and actual total velocity is less than -1.5 ft/s to decelerate
the aircraft (see Figure 5.18). Comparing Figures 5.19 and 5.20, one can notice that the
leading edge flap responds to the variation in the angle of attack closely. In Figure 5.21,
notice that the change in the sideslip angle along the mission is very small. The variation
in the pitch angle and the roll angle with time during the whole mission is plotted in
Figures 5.22 and 5.23 respectively. The rates of roll, pitch, and heading are shown in
Figures 5.24, 5.25, and 5.26 respectively.

5.7 Off-Design Mission Evaluation

To investigate the robustness of the selected gains using the iterative Monte Carlo
design method for off-design missions differing from the nominal mission, four
additional missions are considered. The nominal mission is denoted the first mission, and
the additional four off-design missions will be called the second, third, fourth, and fifth
missions.

The four off-design missions are documented in the following Tables 5.3-5.6 and
Figures 5.27-5.90. For each mission, the steering points are listed in a table which is
followed by an overlay plot of the mission on the flight envelope as an indication for not
violating any of the envelope limits. With the fixed gain set, each mission is flown by the
control system and the desired and actual altitude, velocity, and heading angle responses
are plotted, followed by different figures that show the control surface deflections and
various state variables during the whole mission. The desired vs. actual responses for the
four off-design missions demonstrate the fixed gain set and corresponding control
architecture is quite robust at handling various maneuvers and flight conditions associated with a variety of missions.

Figure 5.11 Iterative Monte Carlo Algorithm
Table 5.2 Flight Control System Gains

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<th>Gains</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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<td>lb/ft s</td>
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</tr>
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</tr>
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</tr>
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<tr>
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<tr>
<td>$K_{Pi2}$</td>
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<td>-0.52417</td>
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<td>$K_{psb}$</td>
<td>ft deg/s</td>
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Figure 5.12 Actual and Desired Altitudes vs. Time (Nominal Mission)
Figure 5.13 Actual and Desired Total Speeds vs. Time (Nominal Mission)

Figure 5.14 Actual and Desired Heading Angles vs. Time (Nominal Mission)
Figure 5.15 Horizontal Stabilizer Deflection vs. Time (Nominal Mission)

Figure 5.16 Rudder Deflection vs. Time (Nominal Mission)
Figure 5.17 Flaperon Deflection vs. Time (Nominal Mission)

Figure 5.18 Speed Brake Deflection vs. Time (Nominal Mission)
Figure 5.19 Leading Edge Flap Deflection vs. Time (Nominal Mission)

Figure 5.20 Angle of Attack vs. Time (Nominal Mission)
Figure 5.21 Sideslip Angle vs. Time (Nominal Mission)

Figure 5.22 Pitch Angle vs. Time (Nominal Mission)
Figure 5.23 Roll Angle vs. Time (Nominal Mission)

Figure 5.24 Roll Rate vs. Time (Nominal Mission)
Figure 5.25 Pitch Rate vs. Time (Nominal Mission)

Figure 5.26 Heading Rate vs. Time (Nominal Mission)
Table 5.3 Altitude, Velocity, and Heading Angle Steering Points (Second Mission)

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<tr>
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<th>Heading Angle (deg)</th>
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Figure 5.27 Mission Overlay on Flight Envelope (Second Mission)
Figure 5.28 Actual and Desired Altitudes vs. Time (Second Mission)

Figure 5.29 Actual and Desired Total Speeds vs. Time (Second Mission)
Figure 5.30 Actual and Desired Heading Angles vs. Time (Second Mission)

Figure 5.31 Horizontal Stabilizer Deflection vs. Time (Second Mission)
Figure 5.32 Rudder Deflection vs. Time (Second Mission)

Figure 5.33 Flaperon Deflection vs. Time (Second Mission)
Figure 5.34 Speed Brake Deflection vs. Time (Second Mission)

Figure 5.35 Leading Edge Flap Deflection vs. Time (Second Mission)
Figure 5.36 Angle of Attack vs. Time (Second Mission)

Figure 5.37 Sideslip Angle vs. Time (Second Mission)
Figure 5.38 Pitch Angle vs. Time (Second Mission)

Figure 5.39 Roll Angle vs. Time (Second Mission)
Figure 5.40 Roll Rate vs. Time (Second Mission)

Figure 5.41 Pitch Rate vs. Time (Second Mission)
Figure 5.42 Heading Rate vs. Time (Second Mission)

Table 5.4 Altitude, Velocity, and Heading Angle Steering Points (Third Mission)

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Figure 5.43 Mission Overlay on Flight Envelope (Third Mission)

Figure 5.44 Actual and Desired Altitudes vs. Time (Third Mission)
Figure 5.45 Actual and Desired Total Speeds vs. Time (Third Mission)

Figure 5.46 Actual and Desired Heading Angles vs. Time (Third Mission)
Figure 5.47 Horizontal Stabilizer Deflection vs. Time (Third Mission)

Figure 5.48 Rudder Deflection vs. Time (Third Mission)
Figure 5.49 Flaperon Deflection vs. Time (Third Mission)

Figure 5.50 Speed Brake Deflection vs. Time (Third Mission)
Figure 5.51 Leading Edge Flap Deflection vs. Time (Third Mission)

Figure 5.52 Angle of Attack vs. Time (Third Mission)
Figure 5.53 Sideslip Angle vs. Time (Third Mission)

Figure 5.54 Pitch Angle vs. Time (Third Mission)
Figure 5.55 Roll Angle vs. Time (Third Mission)

Figure 5.56 Roll Rate vs. Time (Third Mission)
Figure 5.57 Pitch Rate vs. Time (Third Mission)

Figure 5.58 Heading Rate vs. Time (Third Mission)
Table 5.5 Altitude, Velocity, and Heading Angle Steering Points (Fourth Mission)

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Figure 5.59 Mission Overlay on Flight Envelope (Fourth Mission)
Figure 5.60 Actual and Desired Altitudes vs. Time (Fourth Mission)

Figure 5.61 Actual and Desired Total Speeds vs. Time (Fourth Mission)
Figure 5.62 Actual and Desired Heading Angles vs. Time (Fourth Mission)

Figure 5.63 Horizontal Stabilizer Deflection vs. Time (Fourth Mission)
Figure 5.64 Rudder Deflection vs. Time (Fourth Mission)

Figure 5.65 Flaperon Deflection vs. Time (Fourth Mission)
Figure 5.66 Speed Brake Deflection vs. Time (Fourth Mission)

Figure 5.67 Leading Edge Flap Deflection vs. Time (Fourth Mission)
Figure 5.68 Angle of Attack vs. Time (Fourth Mission)

Figure 5.69 Sideslip Angle vs. Time (Fourth Mission)
Figure 5.70 Pitch Angle vs. Time (Fourth Mission)

Figure 5.71 Roll Angle vs. Time (Fourth Mission)
Figure 5.72 Roll Rate vs. Time (Fourth Mission)

Figure 5.73 Pitch Rate vs. Time (Fourth Mission)
Figure 5.74 Heading Rate vs. Time (Fourth Mission)

Table 5.6 Altitude, Velocity, and Heading Angle Steering Points (Fifth Mission)

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<td>163</td>
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<td>Steering Point 6</td>
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<td>1,000</td>
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<td>180</td>
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<td>Steering Point 7</td>
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Figure 5.75 Mission Overlay on Flight Envelope (Fifth Mission)

Figure 5.76 Actual and Desired Altitudes vs. Time (Fifth Mission)
Figure 5.77 Actual and Desired Total Speeds vs. Time (Fifth Mission)

Figure 5.78 Actual and Desired Heading Angles vs. Time (Fifth Mission)
Figure 5.79 Horizontal Stabilizer Deflection vs. Time (Fifth Mission)

Figure 5.80 Rudder Deflection vs. Time (Fifth Mission)
Figure 5.81 Flaperon Deflection vs. Time (Fifth Mission)

Figure 5.82 Speed Brake Deflection vs. Time (Fifth Mission)
Figure 5.83 Leading Edge Flap Deflection vs. Time (Fifth Mission)

Figure 5.84 Angle of Attack vs. Time (Fifth Mission)
Figure 5.85 Sideslip Angle vs. Time (Fifth Mission)

Figure 5.86 Pitch Angle vs. Time (Fifth Mission)
Figure 5.87 Roll Angle vs. Time (Fifth Mission)

Figure 5.88 Roll Rate vs. Time (Fifth Mission)
Figure 5.89 Pitch Rate vs. Time (Fifth Mission)

Figure 5.90 Heading Rate vs. Time (Fifth Mission)
The error between the desired and actual altitudes, velocities, and heading angles during the five designed missions are plotted vs. time (see Figures 5.91-5.93) to further investigate the accuracy and robustness of the FCS and the Monte Carlo selected gains. Notice that the errors in altitude are initially significant at the steering points where the aircraft must change from one cruise level to another as seen in Figure 5.91. However, the control system eliminates this error after a short duration. Figure 5.92 shows that the errors in the total velocities are bounded between 4 ft/s and -4 ft/s during all five complete missions. The errors in the heading angles in Figure 5.93 are almost negligible during the first third of each mission but the errors become larger very close to the end of each mission. However, the error size is still tolerable for the intended application.

Figure 5.91 Error Between Desired and Actual Altitudes vs. Time
Figure 5.92 Error Between Desired and Actual Total Speeds vs. Time

Figure 5.93 Error Between Desired and Actual Heading Angles vs. Time
CHAPTER 6
CRACK PROPAGATION MITIGATION

6.1 Overview

In this pinnacle chapter, the feedback control logic to mitigate the propagation of a typical structural crack located at the wing root of the airframe during a flight mission immediately after takeoff to just before landing is investigated. The life extending control (LEC) logic is implemented as a simple modification to the existing flight control system by utilizing the leading edge flap control surface to exploit the nonlinear crack retardation phenomenon. During the flight mission, LEC will inject inputs, based on the current crack state, to modify the aircraft motions that in turn influence the wing deformations and hence stress loading on the crack. The validity of the proposed LEC logic under various loading histories to which the aircraft wing is subjected will be studied. The closed-loop aircraft is flown along the five missions presented in Chapter 5 with the proposed logic on and off to explore the concept. In the cases with LEC activated, the aircraft maneuvers will activate the LEC leading edge flap input when the projected crack has grown beyond the critical length. Gust loadings are added during prescribed periods through the missions to further trigger the proposed logic.

6.2 Life Extending Control Logic

The proposed LEC logic is based on injecting an overload stress to the in-service stress cycles of the wing to mitigate the crack propagation rate that would lead to the crack propagating beyond a specified tolerance value, or value that might lead to catastrophic results, by the end of the flown mission. The overload stress is applied to the crack head as a result of an instantaneous and momentary increase in the wing lift. The
leading edge flap deflection pulse will change the lifting surface camber causing the increase in the wing lift during the mission. The timing and amplitude of this overload injection are critical and are computed by the LEC logic. Hence, an optimal or suboptimal overload is applied to the cracked panel on the lower wing skin. The leading edge flap is selected for implementing the proposed logic because of its dramatic influence on wing lift when compared with the influence from the flaperon, as discussed in Chapter 4. Additionally, perturbing the leading edge flap to implement the LEC logic will minimize any impact on the maneuverability of the aircraft.

The LEC logic and its relationship to the overall flight system is illustrated in Figure 6.1. In an overall sense, a proportional compensator is used to relate the current crack propagation error state to the perturbation in the leading edge flap, however, this signal path typically does not operate in a continuous manner. The gain value of this compensator is denoted \( K_{pLEC} \) and is selected to be equal to \( 2.563 \times 10^3 \) deg s/mm based on an ad-hoc iterative searching technique. The LEC logic resides totally inside the indicated dashed box which inputs the stress signal \((\sigma_t \text{ or } \sigma_l, \text{see Chapters 2,4})\) and outputs the leading edge flap increment \((\delta_{lef-LEC}, \text{see Chapter 5})\). The output of this box was indicated in Figure 5.5 in Chapter 5. In Figure 6.1, note the appearance of all the separate components and models previously developed. The aircraft dynamic model is represented as a separate block, while the vortex-lattice aerodynamic model is enclosed within the finite element wing model where the aerodynamic loads serve to excite the flexibility of the wing along with the wing root excitations due to the linear and angular accelerations from the flight dynamic model.
In the actual implementation, LEC logic would measure the wing stress and compute the crack state, or it would measure the crack state directly. The former set-up is assumed here. Once the stress cycles are obtained from the flexible wing finite element model, a pseudo rain flow counting method is used to define the cycle duration, and the mountains and valleys of each cycle. With this data the dynamic crack propagation model can be initiated to calculate the current crack length $a$, the current crack propagation rate $da/dt$, and the crack length at the end of the mission $a_{\text{final}}$, assuming that the features of the stress cycle will be the same during the remaining duration of the current mission. The life extending control logic will be triggered only when the expected final crack length is greater than the prescribed final crack length. When this happens, the crack rate feedback signal $(da/dt)$ differs from the specified required rate $(da/dt)_{\text{req}}$ and a LEC pulse is generated. The leading edge flap pulse will remain on until the crack propagation rate is arrested by exploiting the crack retardation physics, usually occurring after a very short duration following the initial activation. After this happens, the LEC logic returns to its benign state. All computations were performed with $(da/dt)_{\text{req}} = 0$, but other specified values could be used.

Note the final crack length is computed from a separate nonlinear simulation taken out to mission termination. In other words, at each LEC compute cycle, a complete closed-loop aircraft dynamics, flexible wing, and crack propagation simulation is conducted. This process is a form of predictive control and is also similar to zero effort miss steering laws found in missile guidance. One hidden detail in the LEC logic is the temporal mismatch between discrete cycles and continuous seconds. The crack propagation model outputs $da/dN$ which must be converted to $da/dt$. This conversion
is achieved by dividing the crack extension during the preceding cycle by the duration of this cycle.

All simulations are started with the same initial conditions. For the aircraft dynamics model, the initial conditions correspond to the steering point 1 level symmetric flight conditions. These initial conditions include \( h = 1,000 \text{ ft}, \ V_T = 540 \text{ ft/s}, \ \theta = \alpha = 2.526 \text{ deg}, \ \delta_h = -0.571 \text{ deg}, \ \delta_{lf} = 3.639 \text{ deg}, \ \text{and} \ \theta_{ia} = 14.96\% \). For the flexible wing model, all initial conditions correspond to zero deflection and zero deflection rate. For the crack propagation model, stress is initiated as described in Chapter 2 and crack length is \( a = 14 \text{ mm} \). A value of \( a_{\text{critical}} = 18 \text{ mm} \) is used in all computations. All numerical integration routines employed a time step of 0.05 s which provided a balance between processing time and sampling of the higher frequency dynamics present in the system. A rough model of atmospheric turbulence and/or ordnance delivery was added to each of the five missions for all computations. Dryden turbulence was used in all cases and is applied at three instances along the mission. The first turbulence set was injected after 10 s from the beginning of the mission and lasted for 90 s, where the gust trim speed was \( V_g = 540 \text{ ft/s}, \) the turbulence scale length \( L_g = 600 \text{ ft}, \) and the turbulence intensity \( \sigma_g = 3 \text{ ft/s}. \) The second turbulence set was applied during the first cruise between \( t = 500 \text{ s} \) and \( t = 600 \text{ s}, \) where the characteristics of this turbulence was \( V_g = 600 \text{ ft/s}, \) the turbulence scale length \( L_g = 1,750 \text{ ft}, \) and the turbulence intensity \( \sigma_g = 0.9 \text{ ft/s}. \) The third turbulence was applied as a compensation for the sudden decrease of aircraft weight as a result of weapon release. This last injected turbulence started at \( t = 1,130 \text{ s} \) and lasted for 40 s, where \( V_g = 700 \text{ ft/s}, \ L_g = 2,500 \text{ ft}, \) and \( \sigma_g = 1.5 \text{ ft/s}. \) The first two instances approximately correspond
to the initial climb, and the initial cruise, respectively. The turbulence value was added directly to the $w_{AC}$ aircraft velocity component.

6.3 Nominal Mission Crack Mitigation Results

In this section, the effectiveness of the proposed LEC logic to mitigate crack propagation during airframe operational service is studied. With the computational tool, the F-16 dynamic model, the aerodynamic model, the finite element wing model, the crack propagation model, and the flight control system including the LEC logic are flown along the nominal mission. Two cases are considered. The first case is with LEC deactivated and the second case is with LEC activated. The behavior of the system is analyzed and the final crack length in the presence of LEC logic is compared with the crack length in the absence of the logic.

The first group of figures in this section includes the desired and actual altitudes, total velocities, and heading angles during the mission vs. time with LEC logic turned off. These figures are followed by another group of figures that show, in various overlay formats, the FCS deflection of the leading edge flap, the LEC perturbation in the leading edge flap, the stress, the crack opening stresses, the crack length, and the key aircraft-states that correlate to stress. These two groups of figures will be followed by a similar set of figures corresponding to LEC logic turned on where the leading edge flap is perturbed as a result of the proposed logic.

Figure 6.2-6.4 show the controlled variables during the nominal mission in absence of the proposed LEC logic. Although slight differences exist between these figures and Figures 5.12-5.14 due to the added turbulence, Figures 6.2-6.4 show the tracking performance of the FCS even when disturbed by the atmospheric environment. Figure 6.5
shows the FCS leading edge flap, the LEC leading edge flap (offset for ease of viewing), and stress, while Figure 6.6 shows stress, crack opening stress, and crack length. For this case, the LEC flap perturbation is zero. Note how the stress response correlates with the FCS flap movement arising from the programmed schedule (see Equation (3.34)).

Regarding Figure 6.6, first note that, because of the complex nature of the loading profile, the crack propagation behavior is unlike the behavior in Chapter 2 using a simplistic load profile. During the initial climb between steering points 1 and 2, the maneuver and the added turbulence causes the crack opening stress to rapidly rise and the crack also undergoes rapid growth. As the aircraft levels off and enters the cruise phase (steering points 2-3), the stress falls below the elevated opening stress and the crack is fully arrested. This behavior continues up to 500 s where the aircraft encounters additional turbulence and the stress responds in an oscillatory manner with amplitudes that exceed the opening stress. During this period the crack is growing, but the amount is so small that it can not be seen on the scale used in Figure 6.6. From steering point 4 up to 7 (980 ≤ t ≤ 1,330 s) the aircraft maneuvering and atmospheric disturbances are not sufficient to push the stress above the opening stress value. When the aircraft levels off from the long climb ending at point 7, the stress exceeds the opening stress but it has very low frequency content and the crack growth is again not visible in the figure. However, when the aircraft levels off after the descent preceding steering point 9 (t = 1,490 s), the exceeding stress has very high frequency content with larger amplitudes causing a very significant crack growth acceleration period. After this period, the crack growth again slows due to the mostly nonexceeding stress level. Finally, Figures 6.7-6.9 show some of
the major aircraft state variables that correlate with the stress response. The data shows
the stress is mostly responding to longitudinal motions.

Figure 6.1 Life Extending Control Logic Flow Chart
Figures 6.10-6.12 are similar to Figures 6.2-6.4 except the logic was activated. Notice that these figures show the accurate FCS tracking performance even though the dynamic model is disturbed by the added gust and the leading edge flap is perturbed at the critical time as a result of the LEC implementation. The presence of the LEC logic does not impact the ability of the FCS to track the desired trajectory. The leading edge flap, the leading edge flap LEC perturbation (offset for ease of viewing), and the principal stress are overlaid in Figure 6.13 such that one can easily correlate between the principal stress variation and the deflection in the leading edge flap. The LEC flap perturbation is activated once for the entire mission. The perturbation occurs early in the mission during the initial climb and when the gust is present \( \delta_{\text{LEC}} = 13.9 \text{ deg at } t = 16 \text{s} \). Also notice that the rapid stress change during the early moments of climb is related to the momentary LEC deflection. Figure 6.14 shows the stress, the opening stress, and the crack extension along the mission. In this figure, the opening stress increased beyond 200 MPa as a result of LEC perturbation. Although the stress is fluctuating with different amplitudes and frequencies in the remaining phases of the mission as a result of the injected turbulences and maneuvers, the crack is fully arrested until the end of the mission where all stress peaks are lower than the opening stress. In Figure 6.15, notice that the pitch rate frequencies are similar to those of the stress, where the higher the pitch rate frequencies are, the higher the stress frequencies are also. Notice also that the high stress frequencies are taking place between steering points 8 and 9 \( (1,380 \leq t \leq 1,490 \text{s}) \) and the next trajectory segment that extends from steering point 10 until \( t = 1,675 \text{ s} \).

Figure 6.16 shows the velocity component \( w_{AC} \) and the stress along the mission, while Figure 6.17 shows the variation of the stress with the angle of attack vs. time. In this
latter figure, notice that the change in the stress is very similar to the angle of attack variation. To easily see the momentary rise of the opening stress as a result of the LEC perturbation, during the early moments of the first climb, Figure 6.18 is shown below. To underscore the effectiveness of LEC logic on structural integrity, the crack length extension -when the LEC is activated- is overlaid on the crack length extension in the absence of the logic in Figure 6.19. Even though structural integrity is slightly worse initially, the benefit of LEC by the mission end is evident.

Figures 6.2 Actual and Desired Altitudes vs. Time (Nominal Mission, LEC Off)
Figure 6.3 Actual and Desired Velocities vs. Time (Nominal Mission, LEC Off)

Figure 6.4 Actual and Desired Heading Angles vs. Time (Nominal Mission, LEC Off)
Figure 6.5 Leading Edge Flap and Stress vs. Time (Nominal Mission, LEC Off)

Figure 6.6 Crack Length and Stress vs. Time (Nominal Mission, LEC Off)
Figure 6.7 Pitch Rate and Stress vs. Time (Nominal Mission, LEC Off)

Figure 6.8 Velocity Component $w_{A/C}$ and Stress vs. Time (Nominal Mission, LEC Off)
Figure 6.9 Angle of Attack and Stress vs. Time (Nominal Mission, LEC Off)

Figure 6.10 Actual and Desired Altitudes vs. Time (Nominal Mission, LEC)
Figure 6.11 Actual and Desired Velocities vs. Time (Nominal Mission, LEC)

Figure 6.12 Actual and Desired Heading Angles vs. Time (Nominal Mission, LEC)
Figure 6.13 Leading Edge Flap and Stress vs. Time (Nominal Mission, LEC)

Figure 6.14 Crack Length and Stress vs. Time (Nominal Mission, LEC)
Figure 6.15 Pitch Rate and Stress vs. Time (Nominal Mission, LEC)

Figure 6.16 Velocity Component $w_{A/C}$ and Stress vs. Time (Nominal Mission, LEC)
Figure 6.17 Angle of Attack and Stress vs. Time (Nominal Mission, LEC)

Figure 6.18 Opening Stress and Flap Perturbation vs. Time (Nominal Mission, LEC)
6.4 Off-Design Mission Crack Mitigation Results

To test the effectiveness of the LEC logic beyond the nominal mission, the logic is implemented during the other four off-design flights discussed in Chapter 5. For each mission, a set of two groups of figures are plotted when the logic was activated. The first group contains the desired and actual trajectory variables presented in three figures which include altitudes, total velocities, and heading angles vs. time correspondingly. The second group of figures show, in various overlay formats, the FCS leading edge flap deflection, the leading edge flap perturbation, the stress, the opening stress, the crack length, and the most effective aircraft-states influencing the stress. For benchmarking purposes, another set of two groups of data are shown when the LEC logic was deactivated.
Figures 6.20-6.22 show the desired and actual altitudes, total velocities, and heading angles vs. time during the second mission in the absence of LEC logic, respectively. Slight variations between these figures and Figures 5.28-5.30 are observed during the three durations of gust injection, however, the FCS tracking accuracy is still noticed. Figure 6.23 shows the leading edge flap and stress vs. time while the LEC flap perturbation is zero during the whole mission. The peaks of stress created during the vehicle descent from the second level off \((1,560 \leq t \leq 1,620 \text{s})\) and during the last descent \((1,930 \leq t \leq 2,000 \text{s})\) are larger than the crack opening stress as shown in Figure 6.24. Notice that dramatic crack extensions occur during these periods. The pitch rate, the velocity component \(w_{AC}\), and the angle of attack are plotted with stress vs. time in Figure 6.25-6.27, correspondingly.

Figures 6.28-6.30 are similar and in the same order of Figures 6.20-6.22, except the LEC logic was activated. Note the FCS system tracking accuracy, with turbulence existence and the leading edge flap perturbation. In Figure 6.31, the leading edge flap is perturbed two times, the first is at the beginning of the first climb, while the second perturbation takes place directly after the air vehicle levels off. The opening stress due to the first perturbation is just below 100 MPa which is not sufficient to arrest the crack propagation until the end of the mission. Consequently, another leading edge flap perturbation should and does take place, whereby the opening stress rises above 170 MPa which is larger than all of the following stress peaks except two peaks around \(t = 1,560 \text{ s}\) and \(1,930 \text{ s}\). At these times, the crack starts to propagate but not as dramatically as the former case where the logic was not activated (see Figure 6.24). The key aircraft-states that greatly affect the stress \(\mathcal{Q}, w_{AC}, \alpha\) are plotted respectively with the stress vs. time.
in Figures 6.33-35. Figure 6.36 shows the change in opening stress that is related to the LEC leading edge flap perturbations. Finally, the crack extensions with LEC absence and existence are overlaid vs. time in Figure 6.37. The crack extension is slowed once again by LEC in the long term sense.

Figures 6.38-6.45 represent the third mission data where the LEC is deactivated, while Figures 6.46-6.54 represent the activated logic data. To complete the picture of the third mission, the two crack extension responses -when the logic is activated and deactivated- are plotted vs. time as shown in Figure 6.55. Nearby steering point 7 ($t \approx 1,200 \text{s}$) where the aircraft is about to level off at 20,000 ft, notice that the rapid stress fluctuation above the opening stress value leads to a small reduction in the opening stress value which is accompanied with crack extension, as shown in Figure 6.42. Figure 6.42 also shows a block of high frequency and large amplitude stress between steering points 10 and 11 ($1,550 \leq t \leq 1,600 \text{s}$) which cause an increase in the opening stress value to about 140 MPa. Because the stress amplitudes in this block are significantly larger than the opening stress, the extension in the crack length is dramatic. Figure 6.50, which is similar to the last mentioned figure, shows that the LEC was activated at two instants. The first instant was at an early time for the first climb where the opening stress rises above 150 MPa. At the same time, the crack extended dramatically by 0.1 mm. The second instance is in the middle of the stress block created between the steering points 9 and 10. This LEC perturbation causes a momentary opening stress rise to almost 200 MPa which reduces the dramatic crack extension experienced in the absence of LEC logic, as mentioned in the discussion of Figure 6.42.
Figures 6.56-6.58 show the desired and actual altitudes, total velocities, and heading angles correspondingly vs. time during the fourth mission in the case of logic deactivation. The FCS tracking performance is noticed along the mission and during the turbulence periods except for a deviation away from the desired total velocities during air vehicle descent from the highest cruise level \((1,490 \leq t \leq 1,700 \text{s})\). In this mission, the FCS could not complete the mission beyond 2,000 s. The leading edge flap and the stress are plotted vs. time, while the LEC flap deflection is zero through the mission. The high frequency stress block near 1,700 s is not related to the changes in the key states affecting the stress overlaid correspondingly in Figure 6.61-6.63 vs. time. During this time only the velocity-hold autopilot is having difficulty in following the desired total velocity and may be the cause for this unexpected stress block (review Figure 6.57). The three stress peaks preceding the above-mentioned stress block do not cause any crack propagation and the reason may be that the peaks are not captured by the counting method. For activated LEC logic, Figures 6.64-6.66 show the desired and actual altitudes, total velocities, and heading angles vs. time respectively. Here the three autopilots did not track the desired mission variables from \(t = 1,990 \text{s}\) to the end of mission. The logic is activated at the beginning of the first climb and the beginning of first level segment as shown in Figure 6.67. Figure 6.68 shows the opening stress rise directly after the two LEC perturbations. Figures 6.69-6.71 shows the high frequency changes in pitch rate, velocity component \(w_{AC}\), and angle of attack respectively during the autopilot’s poor performance period. In Figure 6.72, the decrease in the opening stress followed by rise to 200 MPa is noticed at the moment when the LEC was activated for the second time. The figures related to the
fourth mission are concluded with the crack extension comparison in case of activating and deactivating the logic as shown in Figure 6.73.

Finally, a similar group of figures to those mentioned above are drawn for the last mission and are arranged in the same order. Figures 6.74-6.76 show the actual and desired altitudes, total velocities, and heading angles vs. time, correspondingly in absence of LEC logic. Notice that the autopilot behavior near the mission end behaves like in the fourth mission around the same period. Figure 6.77 shows the leading edge flap deflection overlaid with the stress and the zero LEC flap deflection (offset for easy viewing). In Figure 6.78, the crack is repeatedly extended during the mission as the stress peaks are larger than the crack opening stress most of the mission time. Notice also the dramatic crack extension taking place nearby the mission end where the autopilots are poorly acting. Figures 6.79-6.81 show the pitch rate, velocity component $w_{AC}$, and angle of attack overlaid with stress vs. time respectively so that one can insightfully see their effect on stress variations. The last two groups of figures in this section are assigned for the activated LEC logic, where the first group that contains Figures 6.82-6.84 show the desired and actual altitudes, total velocities, and heading angles vs. time. Also note the poor autopilot performance near the mission end in the three figures respectively. In this mission the LEC logic is activated only after the aircraft initially levels off as shown in Figure 6.85. The pulse in the leading edge flap raises the crack opening stress to 200 MPa, as shown in Figure 6.86, which is sufficient to completely arrest the crack propagation until the end of the mission. Figures 6.87-6.89 show the pitch rate, the velocity component $w_{AC}$, and the angle of attack overlaid with stress vs. time, along the mission respectively. Finally, the crack lengths with logic activated and deactivated are
plotted vs. time for comparison, where it is clear the LEC implementation is effective, although it causes an abrupt crack extension beyond the extension happening in absence of LEC logic initially.

Figures 6.20 Actual and Desired Altitudes vs. Time (Second Mission, LEC Off)

Figure 6.21 Actual and Desired Velocities vs. Time (Second Mission, LEC Off)
Figure 6.22 Actual and Desired Heading Angles vs. Time (Second Mission, LEC Off)

Figure 6.23 Leading Edge Flap and Stress vs. Time (Second Mission, LEC Off)
Figure 6.24 Crack Length and Stress vs. Time (Second Mission, LEC Off)

Figure 6.25 Pitch Rate and Stress vs. Time (Second Mission, LEC Off)
Figure 6.26 Velocity Component $w_{AC}$ and Stress vs. Time (Second Mission, LEC Off)

Figure 6.27 Angle of Attack and Stress vs. Time (Second Mission, LEC Off)
Figure 6.28 Actual and Desired Altitudes vs. Time (Second Mission, LEC)

Figure 6.29 Actual and Desired Velocities vs. Time (Second Mission, LEC)
Figure 6.30 Actual and Desired Heading Angles vs. Time (Second Mission, LEC)

Figure 6.31 Leading Edge Flap and Stress vs. Time (Second Mission, LEC)
Figure 6.32 Crack Length and Stress vs. Time (Second Mission, LEC)

Figure 6.33 Pitch Rate and Stress vs. Time (Second Mission, LEC)
Figure 6.34 Velocity Component $w_{AC}$ and Stress vs. Time (Second Mission, LEC)

Figure 6.35 Angle of Attack and Stress vs. Time (Second Mission, LEC)
Figure 6.36 Opening Stress and Flap Perturbation vs. Time (Second Mission, LEC)

Figure 6.37 Crack Length vs. Time (Second Mission, LEC and LEC Off)
Figure 6.38 Actual and Desired Altitudes vs. Time (Third Mission, LEC Off)

Figure 6.39 Actual and Desired Velocities vs. Time (Third Mission, LEC Off)
Figure 6.40 Actual and Desired Heading Angles vs. Time (Third Mission, LEC Off)

Figure 6.41 Leading Edge Flap and Stress vs. Time (Third Mission, LEC Off)
Figure 6.42 Crack Length and Stress vs. Time (Third Mission, LEC Off)

Figure 6.43 Pitch Rate and Stress vs. Time (Third Mission, LEC Off)
Figure 6.44 Velocity Component $w_{A/C}$ and Stress vs. Time (Third Mission, LEC Off)

Figure 6.45 Angle of Attack and Stress vs. Time (Third Mission, LEC Off)
Figure 6.46 Actual and Desired Altitudes vs. Time (Third Mission, LEC)

Figure 6.47 Actual and Desired Velocities vs. Time (Third Mission, LEC)
Figure 6.48 Actual and Desired Heading Angles vs. Time (Third Mission, LEC)

Figure 6.49 Leading Edge Flap and Stress vs. Time (Third Mission, LEC)
Figure 6.50 Crack Length and Stress vs. Time (Third Mission, LEC)

Figure 6.51 Pitch Rate and Stress vs. Time (Third Mission, LEC)
Figure 6.52 Velocity Component $w_{A/C}$ and Stress vs. Time (Third Mission, LEC)

Figure 6.53 Angle of Attack and Stress vs. Time (Third Mission, LEC)
Figure 6.54 Opening Stress and Flap Perturbation vs. Time (Third Mission, LEC)

Figure 6.55 Crack Length vs. Time (Third Mission, LEC and LEC Off)
Figure 6.56 Actual and Desired Altitudes vs. Time (Fourth Mission, LEC Off)

Figure 6.57 Actual and Desired Velocities vs. Time (Fourth Mission, LEC Off)
Figure 6.58 Actual and Desired Heading Angles vs. Time (Fourth Mission, LEC Off)

Figure 6.59 Leading Edge Flap and Stress vs. Time (Fourth Mission, LEC Off)
Figure 6.60 Crack Length and Stress vs. Time (Fourth Mission, LEC Off)

Figure 6.61 Pitch Rate and Stress vs. Time (Fourth Mission, LEC Off)
Figure 6.62 Velocity Component $w_{AC}$ and Stress vs. Time (Fourth Mission, LEC Off)

Figure 6.63 Angle of Attack and Stress vs. Time (Fourth Mission, LEC Off)
Figure 6.64 Actual and Desired Altitudes vs. Time (Fourth Mission, LEC)

Figure 6.65 Actual and Desired Velocities vs. Time (Fourth Mission, LEC)
Figure 6.66 Actual and Desired Heading Angles vs. Time (Fourth Mission, LEC)

Figure 6.67 Leading Edge Flap and Stress vs. Time (Fourth Mission, LEC)
Figure 6.68 Crack Length and Stress vs. Time (Fourth Mission, LEC)

Figure 6.69 Pitch Rate and Stress vs. Time (Fourth Mission, LEC)
Figure 6.70 Velocity Component $w_{A/C}$ and Stress vs. Time (Fourth Mission, LEC)

Figure 6.71 Angle of Attack and Stress vs. Time (Fourth Mission, LEC)
Figure 6.72 Opening Stress and Flap Perturbation vs. Time (Fourth Mission, LEC)

Figure 6.73 Crack Length vs. Time (Fourth Mission, LEC and LEC Off)
Figure 6.74 Actual and Desired Altitudes vs. Time (Fifth Mission, LEC Off)

Figure 6.75 Actual and Desired Velocities vs. Time (Fifth Mission, LEC Off)
Figure 6.76 Actual and Desired Heading Angles vs. Time (Fifth Mission, LEC Off)

Figure 6.77 Leading Edge Flap and Stress vs. Time (Fifth Mission, LEC Off)
Figure 6.78 Crack Length and Stress vs. Time (Fifth Mission, LEC Off)

Figure 6.79 Pitch Rate and Stress vs. Time (Fifth Mission, LEC Off)
Figure 6.80 Velocity Component \( w_{AC} \) and Stress vs. Time (Fifth Mission, LEC Off)

Figure 6.81 Angle of Attack and Stress vs. Time (Fifth Mission, LEC Off)
Figure 6.82 Actual and Desired Altitudes vs. Time (Fifth Mission, LEC)

Figure 6.83 Actual and Desired Velocities vs. Time (Fifth Mission, LEC)
Figure 6.84 Actual and Desired Heading Angles vs. Time (Fifth Mission, LEC)

Figure 6.85 Leading Edge Flap and Stress vs. Time (Fifth Mission, LEC)
Figure 6.86 Crack Length and Stress vs. Time (Fifth Mission, LEC)

Figure 6.87 Pitch Rate and Stress vs. Time (Fifth Mission, LEC)
Figure 6.88 Velocity Component $w_{A/C}$ and Stress vs. Time (Fifth Mission, LEC)

Figure 6.89 Angle of Attack and Stress vs. Time (Fifth Mission, LEC)
Figure 6.90 Opening Stress and Flap Perturbation vs. Time (Fifth Mission, LEC)

Figure 6.91 Crack Length vs. Time (Fifth Mission, LEC and LEC Off)
6.5 Wing Deflection Implications

In this section, the data that includes the maximum lateral deflections of the wing during the studied missions with life extending control logic activated is presented. The displayed data corresponds to maximum absolute deflection at each finite element mid point, regardless of the up or down direction and the specific time during the mission. The data is given in Figure 6.92-6.96, where the \(xyz\) aerodynamic axes with \(x\) pointing aft are employed. Figure 6.92 shows that the maximum deflection of the wing tip is almost 1 ft for the nominal mission. The maximum wing tip deflection during the second mission is almost 1.1 ft as shown in Figure 6.93. The maximum wing tip deflection during the third mission is almost equal to that of the nominal mission, as seen in Figure 6.94. Figures 6.95 and 6.96 show that the maximum wing tip deflection did not cross 25 cm during the fourth and fifth missions respectively. The trailing edge wing tip is consistently the location for maximum deflection. Further, in all cases, a crease of local minimum deflections runs from the leading edge wing tip to the trailing edge wing root. Across all five missions, note that the maximum lateral deflection did not exceed 40 cm. This deflection level is acceptable according to the wind tunnel results given in References 24-25, and provides further validation of the flexible wing model. The main conclusion is that LEC logic can provide a significant influence on crack mitigation without needing excessive wing deformation.

6.6 Aircraft Maneuverability Implications

The effect of implementing the LEC logic on the maneuverability of the aircraft model is investigated through recording the error between desired and actual trajectory variables, such as altitude, total velocity, and heading angle, along the whole mission.
Errors in the presence of and the absence of the proposed logic will be plotted in the same figure vs. time. For each mission, three figures will be plotted corresponding to altitude error, total velocity error, and heading angle error. Figures 6.97-6.111 are the relevant graphs.

Figure 6.92 Wing Maximum Lateral Deflection During Nominal Mission

Figure 6.93 Wing Maximum Lateral Deflection During Second Mission
Figure 6.94 Wing Maximum Lateral Deflection During Third Mission

Figure 6.95 Wing Maximum Lateral Deflection During Fourth Mission
Figure 6.96 Wing Maximum Lateral Deflection During Fifth Mission

Figure 6.97 Altitude Error vs. Time During Nominal Mission (LEC and LEC Off)
Figure 6.98 Total Velocity Error vs. Time During Nominal Mission (LEC and LEC Off)

Figure 6.99 Heading Angle Error vs. Time During Nominal Mission (LEC and LEC Off)
Figure 6.100 Altitude Error vs. Time During Second Mission (LEC and LEC Off)

Figure 6.101 Total Velocity Error vs. Time During Second Mission (LEC and LEC Off)
Figure 6.102 Heading Angle Error vs. Time During Second Mission (LEC and LEC Off)

Figure 6.103 Altitude Error vs. Time During Third Mission (LEC and LEC Off)
Figure 6.104 Total Velocity Error vs. Time During Third Mission (LEC and LEC Off)

Figure 6.105 Heading Angle Error vs. Time During Third Mission (LEC and LEC Off)
Figure 6.106 Altitude Error vs. Time During Fourth Mission (LEC and LEC Off)

Figure 6.107 Total Velocity Error vs. Time During Fourth Mission (LEC and LEC Off)
Figure 6.108 Heading Angle Error vs. Time During Fourth Mission (LEC and LEC Off)

Figure 6.109 Altitude Error vs. Time During Fifth Mission (LEC and LEC Off)
Figure 6.110 Total Velocity Error vs. Time During Fifth Mission (LEC and LEC Off)

Figure 6.111 Heading Angle Error vs. Time During Fifth Mission (LEC and LEC Off)
In the first three missions, the errors between the commands and responses of the aircraft are nearly the same either for the case of engaging the LEC logic or disengaging the logic. The implication is that the LEC logic has not degraded the ability of the aircraft and control system to maneuver and track the desired path. In the fourth and fifth missions, a similar characteristic is observed, at least for most of the mission. However, the error between the desired and actual responses enlarges towards the end of the mission, either with the application of the LEC or the absence of the logic. In this small region, the nominal FCS is having difficulty in tracking the reference trajectory. The discrepancies are not being caused by the LEC logic. A simple refinement to the FCS gains is all that is needed here.
CHAPTER 7

CONCLUSION AND FUTURE INVESTIGATION

7.1 Conclusion

Regarding the objectives listed in Section 1.1, all objectives have been pursued and achieved. A model of crack propagation dynamics has been successfully integrated within a closed-loop aircraft dynamic system that was used to conduct the investigation. With this model, the feasibility of life extending control has been successfully explored. Enhancement to the airframe structural integrity has been successfully quantified in terms of crack growth per mission relative to the baseline case. A practical implementation of the life extending control logic was successfully identified. Finally, an assessment of the impact of life extending control on flight stability and performance was successfully completed.

The proposed LEC system was employed to show quantifiable advantages of utilizing secondary control surfaces to mitigate crack propagation in a vehicle airframe by exploiting nonlinear retardation fatigue behavior. The percentages of reduction in crack propagation, as a result of applying the LEC logic, relative to the baseline case without this logic, are listed in Table 7.1. For the system that was studied, a reduction of 40-75 % in crack extension per mission was found. As a rough estimation for the crack length extension after 500 hrs of in-service operation, and assuming that the growth will be accumulated linearly, the extension in the crack lengths would be approximately the values in Table 7.2. If the safe or tolerable maximum crack size, before repairs are needed, is on the order of 50 mm, Table 7.2 shows that the vehicle structural service life can be significantly enhanced by life extending control. Further, a reduction in expenses
associated with structural repair and periodic inspection can also be achieved. Dissertation results also indicate that using a secondary control surface to implement the proposed logic can reduce the trade between the structural integrity benefit and the cost to aircraft maneuverability. Moreover, the robustness of the augmentation system and the autopilot system will help to reach the desired level of crack retardation without impairing the maneuverability of the aircraft.

<table>
<thead>
<tr>
<th>Mission</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction in Crack Length (%)</td>
<td>43%</td>
<td>55.3%</td>
<td>55.5%</td>
<td>44%</td>
<td>75.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mission</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extension in Crack Length (mm)</td>
<td>LEC On</td>
<td>18.75</td>
<td>40.5</td>
<td>60</td>
<td>36.6</td>
</tr>
<tr>
<td>LEC Off</td>
<td>32.15</td>
<td>90</td>
<td>137.5</td>
<td>64.12</td>
<td>54.4</td>
</tr>
</tbody>
</table>

These conclusions are not without qualification. The proposed logic is high risk since it utilizes overload injection to achieve the desired objective. Further, the results depend on modelling assumptions and fidelity employed. Finally, the dissertation research did not address or answer all questions and issues related to life extending control strategy. Therefore, these concerns should be kept in mind when considering such strategy.

7.2 Future Investigation

From the system integrity point of view, a model representing an online monitoring system of the crack propagation needs to be included in the system to enhance the prediction of the dynamic crack model. More missions covering different patches of the flight envelope need to be investigated with the proposed LEC logic to study the
robustness of the logic. An experimental investigation and in-situ implementation of the proposed logic needs to be conducted to validate the results of the current study. Further, refined gust models need to be considered along with other expected disturbances during an actual mission, especially when the iterative gain searching design technique is employed. The current study needs to be extended to different types of aircraft to study the trade between the desired structural integrity and the aircraft maneuverability.
REFERENCES


VITA

Dr. Mohamed Mostafa Yousef Bassyouny Elshabasy was born in Alexandria, Egypt in 1972. He enrolled in the Department of Mechanical Power Engineering at Alexandria University in September 1991 as an undergraduate student. In June 1995, he was awarded the Bachelor of Science in Mechanical Engineering from Alexandria University. He graduated second in his class of 194 students. Dr. Elshabasy started his graduate academic career in the Applied Mechanics Branch of the Department of Mechanical Power Engineering, Alexandria University soon thereafter. In June 2001, he completed all requirements for the Master of Science in Mechanics of Materials. His thesis was titled "The Inclusion Effect On The Fatigue Strength Of Woven Roving GRP Composite Materials". Dr. Elshabasy entered the doctoral degree program in the Department of Aerospace Engineering at Old Dominion University in August 2002. During fall and spring 2009, he worked as an adjunct instructor at the Department of Mechanical Technology in Old Dominion University. He completed his Ph.D. degree in May 2009. His dissertation is titled "Mitigating Crack Propagation in a Highly Maneuverable Flight Vehicle Using Life Extending Control Logic".