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Using Mobile Learning to Support Students' Understanding in Geometry: A Design-Based Research Study

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ABSTRACT

The use of mobile learning offers new affordances to teaching and learning. In this study, students from two fourth grade classes used iPads in dyads and groups to learn about angle. Using a design-based research methodology, which included observations, video, researcher journals, and artefact collection, a local instruction theory was developed on how students can learn about angle concepts through mobile learning activities. The local instruction theory is comprised of two components: (a) a seven lesson curriculum for 4th grade students on developing an early understanding of angle utilizing a mobile learning approach, and (b) additions to the scholarly theories, by providing a revised set of indicator behaviours for van Hiele levels of geometric thinking in regards to angle.

Keywords

Angles, Angle Measure, Design-based research, Geometry, Mobile learning

Introduction

Educators and governments have advocated for the use of digital technologies in classroom instruction (Bereiter & Scardamlia, 2006; Common Core State Standards Initiative, 2010). Digital technologies can be used to support mathematics teaching and learning. For example, technology offers the opportunity for students to actively participate and reorganize the way they see mathematical concepts (Stohl-Lee, Hollenbrands, & Holt-Wilson, 2010) and various mathematical representations can reveal different methods to solve problems (Heid, 2005). With technological attributes, such as the graphical capabilities, technology enhanced environments were identified for facilitating the construction of geometric understanding (Clements & Battista, 1989).

In mathematics, angles are particularly difficult concepts for students to grasp and students often develop many misconceptions and difficulties (Clements & Battista, 1989; Mitchelmore, 2002). A review of the literature reveals two strategies that appear to have been successful in supporting students with angle concepts, these are the use of Dynamic Geometry Environments (DGE; e.g., Vitale, Swart, & Black, 2014) and real-world connections (e.g., Gainsburg, 2008). DGEs provide the students with figures (e.g., lines, points, circles) and basic tools to create composite figures. Various dynamic transformations can also be performed, with the ability to trace the path of the movements for later visual inspection. Empirical evidence shows that DGEs support learning about angle as they: expand the repertoire of representations available, beyond those provided in textbooks; are a cognitive technology (Pea, 1987) acting as an external aid to amplify students' cognitive capacities during thinking, learning, and problem solving (Lajoie & Azevedo, 2006); and provide students with a way to access the underpinning mathematical features in geometry.

There have been a number of studies to determine the affordance of teaching angle concepts with real-world connections. Researchers have used real-world objects; for example, Mitchelmore and White (2000) used adjustable models of wheels, doors, and scissors. Real-life physical situations have also been used; for instance, Fyhn (2007) used a climbing project for the students to study angles made by body formations during climbing activities. Mobile learning can provide a way of bringing these two strategies (DGE and real-world connections) together. The study of students learning angles through the use of DGE has not yet been examined. Digital technologies are constantly evolving and becoming more personalized. The use of mobile technologies is becoming ubiquitous throughout today's society. These digital technologies are also seeping into educational establishments. Mobile learning offers new affordances to teaching and learning, such as learning that is contextualized, personalized, and unrestricted by temporal and spatial constraints (Crompton, 2013), which can provide a way for students to learn about angle concepts in a more comprehensible form.

There are two research questions that guided this study:

- Are there additions to the indicators for the van Hiele levels of geometric thinking when the students are involved in mobile learning activities?
- How can mobile learning be used to facilitate students' understanding of angle and angle measure?

Design-based research (DBR) was chosen as a method to enable the researchers to answer these two questions from the development of a local instruction theory. A local instruction theory is composed of two parts, the first part is a contribution to the theory of students learning about angle to specifically understand what additional indicator behaviours students' exhibit when they are involved in mobile learning activities. The second part is that a mobile learning curriculum is developed for teaching angle based on the theory presented. These contributions to educational theories are important as they are highly focused towards having students learn these particular mathematical concepts. In this case, that focus is on students learning about angle.

Theoretical framework

Design-based research

Design-based research (DBR) is "a series of approaches, with the intent of producing new theories, artifacts, and practices that account for and potentially impact learning and teaching in naturalistic settings" (Barab & Squire, 2004, p. 2). The specific DBR selected for this study was developed by Gravemeijer and van Eerde (2009), as they developed a method for creating a set of exemplary instructional activities for students learning particular concepts in mathematics (Nickerson & Whitacre, 2010). Anderson and Shattuck (2012) highlighted seven characteristics of this methodology which are all used in this study. The research is; (1) situated in a real educational context, (2) focuses on the design and testing of a significant intervention, (3) uses mixed methods where appropriate, (4) involves multiple iterations, (5) involves a collaborative partnership between researchers and practitioners, (6) involves the evolution of design principles, and (7) provides practical impact on practice. Through the process of DBR, a conjectured local instruction theory is modified and strengthened.

Geometry

School geometry involves interlinked concepts, axiomatic representational systems and ways of reasoning that mathematize spatial objects, relationships, and transformations. Although geometry forms the foundation of learning in mathematics and other academic subjects, it is it is a difficult subject in mathematics to learn due to the abstract nature of angles (Battista, 2007; Clements & Battista, 1992) and the multiple ways in which angles can be represented (Smart, 2009). The use of digital technologies has appeared to be beneficial in mathematics; yet, mathematics teachers are often resistant to students using technologies for learning (Crompton, 2011). This opposition is often due to a lack of understanding and training in how to use technology in teaching. The mathematical theoretical framework used in this study is van Hiele's levels of geometric thinking (van Hiele, 1984). This is used to analyse students' geometric thought from an overarching view of shapes to a highly complex level of thinking. For example, students working at the first van Hiele level would be able to recognize a square, but could not explain why they knew that shape was a square. There are five levels altogether in the theory, but only the first three levels are used for the purpose of this study. Although the levels are not directly related to the age of the student, students of elementary age typically do not move beyond the second level of the framework. Van Hiele believed that students' levels of geometric thought are achieved largely as a result of effective geometry instruction.

The initial indicators for each of the van Hiele levels of geometric thinking (van Hiele, 1984) were adapted by Scally (1990) and provide a criteria by which the students are matched to a particular van Hiele level of thinking in angle. The researchers in this study use these indicators to determine if they can be extended when the students are involved in mobile learning activities.

Mobile learning

Mobile learning is "Learning across multiple contexts, through social and content interactions, using personal electronic devices" (Crompton, 2013, p. 4). This definition includes the four central constructs of mobile learning which are learning pedagogies, technological devices, context, and social interactions (Crompton, 2013), that have been used to extend the boundaries of traditional learning. The term context refers to the subject content and the environment in which the learning takes place. Learning can take place seamlessly across multiple environments with the portability of the device. Therefore, students can learn in the real-world in which they live, connecting typically decontextualized subjects, often taught with text books, to tangible contextualized concepts.

Educators have been taking students on field trips for centuries, but in the last few years those field experiences are changing due to the technological supports that can provide additional supports to the students. Mobile devices can sense the situation of learners and provide adaptive supports to the students (Shih, Kuo, & Liu, 2012) as well as just-in-time questions, instant feedback (Hung, Hwang, Su, & Lin, 2012) and access to the Internet for further exploration of mathematical concepts. When the students are working with others, mobile computer supported collaborative learning allows for an active, motivating, dynamic environment as the device can scaffolds the formation and coordination of the members of the group while the small portable form do not inhibit direct face-to-face interaction (Zurita, & Nussbaum, 2007).

Yin, Ogata, Tabata, and Yano (2010) contextualized learning as they had students using mobile device as a dynamic support in learning aspects of a foreign language. Yin et al. (2010) had the mobile device provide just-in-time scaffolding in respect of different situations in the real world. Yin and collegues (2013) went on to develop the scaffolding participatory simulation for mobile learning (SPSML). This supportive framework adopts an experiential learning approach with five cyclical steps: the initial stage, concrete experience, observation and reflection, abstract conceptualization, and then testing in new situations. This approach is similar to the DBR approach used in this study.

Eliasson and Ramberg (2012) conducted a study that had students using a mobile software application which measured the distance between two mobile devices via Global Positioning System (GPS) to learn about area and volume. DGEs are now available on mobile devices, such as Sketchpad Explorer (2012). With this application, specific add-ons, for example, Measure a Picture (Steketee & Crompton, 2012), allow the students to interact with the real world to take photographs of physical objects in the environments and use tools within the program to measure those angles. A small number of researchers have used mobile learning to study geometry in the real world (e.g., Eliasson & Ramberg, 2012), and at this time there are none who have studied angle concepts.

Researchers have investigated students’ learning mathematics with DGE (e.g., Vitale, Swart, & Black, 2014), real-world environments (e.g., Gainsburg, 2008), and mobile devices (e.g., Shih, Kuo, & Liu, 2012). However, none have studied DGE, real-word environments, and mobile devices combined to support students learning angles. In this study, students used a DGE called Measure a Picture and that program is on an iPad. The activities took place in the school grounds. In this study, DBR is being used for its intended purpose– to bridge the gap between research and practice and provide a curriculum that teachers can use to teach angle concepts using mobile learning.

Methods

Participants

A total of 62 participants were involved in this study; two fourth grade teachers and 60 students. The fourth grade students were between 9-10 years old and 52% were males. All 60 students had participated in basic training in how to use the iPad 7 weeks before the lessons began and during that time they had also used the iPad for various applications. None of the students had used the “Measure a picture” application before this study. The participants were a convenience sample as this was one of a few schools in the Southeastern United States to have class sets of iPads. This particular grade was chosen as the Common Core State Standards requirements state that teachers should formally begin teaching angle concepts in fourth grade.

Measure a picture

As the students worked in pairs on the iPads, they primarily used an add-on program to Sketchpad Explorer. The add-on program was called Measure a Picture (Steketee & Crompton, 2012) which is a free program available on Sketch Exchange (Sketchexchange.keypress.com). The program has two measurement tools in the bottom right of the screen; the dynamic protractor and an adjustable ruler. Examples of real-world images are provided as part of the program but this image can be replaced by a photograph that is taken within the program. Real-world photographs can be taken and then the tools can be used to measure that image. Figure 1, provides a screenshot of the program with a real-world image that has two angles indicated by the placement of two dynamic protractors.

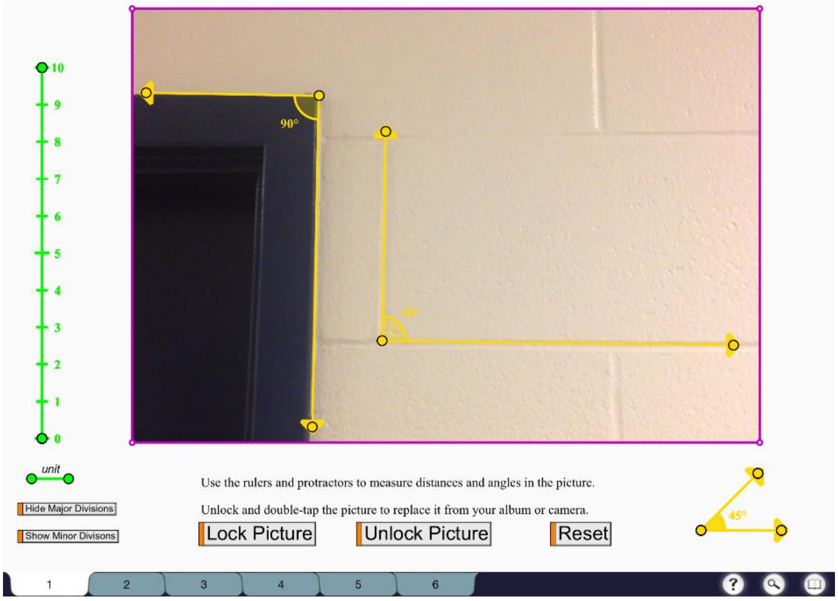

Figure 1. Measure a picture program

Design-based research protocol for this study

This DBR study consisted of two macro cycles with one teaching experiment occurring in each macro cycle. Each teaching experiment involved seven days of mini cycles of instructional experiments and reflection. The primary researcher acted as the teacher in both of the teaching experiments which is not uncommon in the DBR process (e.g., Cummings-Smith, 2010). The macro cycles for this study are illustrated in Figure 2. Note the occurrence of the three phases within each macro cycle: (a) the design of the instructional materials, (b) classroom based teaching experiments and mini cycle analysis, and (c) the retrospective analysis of the teaching experiments which informed the next macro cycle.

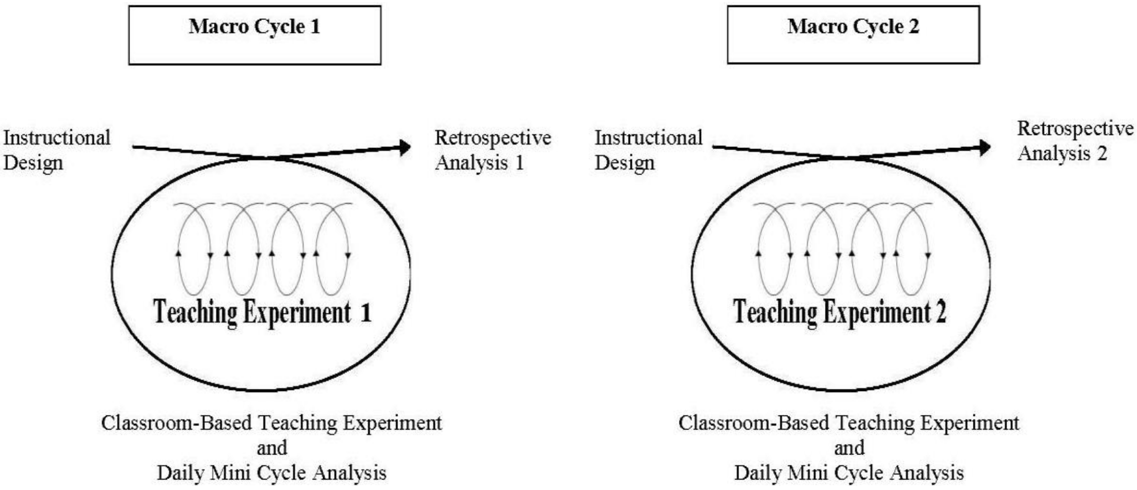

Figure 2. A diagrammatic representation of the study

Using the initial mobile learning curriculum developed from the review of the literature, the first teaching experiment was conducted over seven consecutive school days. The teaching experiment was the implementation of the curriculum developed in the initial stage of the macro cycle. During the teaching experiments, the co-researcher and witness observed and took notes on the classroom instruction, and the instruction was videotaped. Students' work was collected at the end of each day. At the end of the day's instruction, the researcher, co-researcher, and witness met to discuss the lesson and these conversations were audio recorded. Following this meeting, the researcher completed a daily reflection journal. During the retrospective analysis, in the final stage of the macro cycle, all these data were reviewed. The conjectured local instructional theory was then revised before repeating the entire process again in a different class for the second macro cycle. The local instruction theory came from the final retrospective analysis.

Data sources

One of the distinct characteristics of DBR methodology is that the researcher's develop a deeper understanding of the phenomenon while the research is in progress. Therefore, it is essential that the research team collect a comprehensive record of the entire process (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). A list of the various data components is provided in Table 1. The horizontal headers show when these data were collected and the vertical headers are the types of data collected. These data were used in the mini cycle daily reflections and the retrospective analysis from macro cycle one and two. Scally's (1990) angle indicators were used to code the observation notes, video, student artefacts, and researcher reflection journals.

Table 1. Data sources and when these data were analyzed

Select students for interviews	Daily mini cycle analysis	Retrospective Analysis 1 Macro Cycle 1	Retrospective Analysis 2 Macro Cycle 2
Co-Researcher and witness classroom observations	✓	✓	✓
Whole-class Video		✓	✓
Daily mini cycle reflection	✓	✓	✓
Artifact collection		✓	✓
Researcher reflection journal	✓	✓	✓

Results and discussion

The purpose of this study was to answer the two research questions guiding this study. To that end, a local instruction theory was developed to extend the theory of van Hiele levels of geometric thinking (the theory) and how mobile learning can be used to facilitate students' understanding of angle concepts (the curriculum). However, to present the results, the short mobile learning curriculum is presented first as the discussion of the theory connects with some of the activities from that curriculum.

Curriculum

A total of seven angle lessons were designed as part of this study. In Table 2 an overview of the Instructional Sequence can be found. This will give the reader some idea of the mobile learning activities and how the real-world settings were included in these lessons. The activities were a blend of mobile learning activities and in class activities to ensure that the students were able to translate what they had learned in the real-world (the contextualized) with the decontextualize activities that took place inside the classroom.

Table 2. Overview of the instructional sequence

Lesson	Learning progression	Instructional activity
1	Recognize angles as geometric shapes that are formed whenever two rays share a common endpoint. Identify angles in a real-world setting.	Students are introduced to the concept of angle via projected images of different examples of angles in different orientations with sides of different lengths. The term angle is introduced. Students look for angles in the real-world.
2	Identify angles in a real-world setting. Begin to recognize that there are an infinite number of angles.	Students are introduced to the application Sketchpad Explorer and taught how to use the DGEs to take photographs, screenshots, and how to use the self-copying dynamic protractor. Students take photographs of angles in the playground and use the dynamic protractors to highlight the angles found.
3	Recognize and compare angles based on size using standard language (right, obtuse, acute, and straight angles).	Students introduced to the terms: right, obtuse, acute, and straight angles. Students are then involved in an activity where they use quick response (QR) codes in the playground to test to see if they are naming the angles correctly. Students then find angles in the playground and test their partner to see if they

		can correctly name the angles.
4	Recognize acute, obtuse, right, and straight angles in different contexts (real-world and paper and pencil. Recognize salient attributes of angle.	Students work in groups to categorize acute, obtuse, right, and straight angles. Class discussion to create a table of important and non-important attributes of angles.
5	Understand that angles can be measured with reference to a circle and that angles are fractions of a circle. Experience using a nonstandard unit of measure (a wedge). Recognize that the attribute being measured is the space between the two line segments caused by the turn of the line segment. Understand that angles are measured by units called degrees.	Wedge activity to create benchmarks. Using the wedges to measure a set of materials such as a coat hanger, books, scissors, and a car ramp, noting that the latter two can be changed to vary angle size. (This lesson is based on the work by Browning and Garza-Kling (2009).)
6	Recognize that the same angle can appear to be a different size depending on different visual perspectives (positions). Understand that angles are defined by particular attributes which involve angle as a turn (e.g., "two rays, the common endpoint, the rotation of one ray to the other around that endpoint, and measure of that rotation"; Clements and Sarama (2009, p.186)).	Using Measure a Picture, students work in pairs to photograph and measure angles from different positions. Class discussion with the dynamic protractor in Measure a Picture to demonstrate angles going beyond 180° to 360° Work in groups to create a poster to define angle to students who have not yet studied angle.

Changes were made to the instructional materials throughout macro cycle 1 and macro cycle 2 from the findings during the teaching experiments and the interviews.

Summary of changes made to instructional materials in macro cycle 1

- Mathematical language reduced in Lesson One.
- Mathematical journaling was added.
- One discussion added about angles found in manufactured or natural settings.
- An infinite angle discussion was included that utilized the dynamic protractor.
- Discussion included about the importance of beginning at zero measure.

Summary of changes made to instructional materials in macro cycle 2

- After Lesson Two, the instructional plans have the teacher telling the students to just focus on one angle in their photograph, not multiple angles.
- An additional emphasis on having students discussing the mathematical concepts when working in pairs and not just point at the mobile device to explain to their partners.

Extending the theories

The additions to the van Hiele level indicators (Scally, 1990) are presented divided into five sections: drawing angles, identifying angles, sorting angles, angle measure, and angle relations. Within each section, a figure is provided in each section to articulate the revised van Hiele level indicators for angle. These revised indicators were developed from the findings of this research.

Drawing angles. Many of the students could draw figures that resembled angles, but referred to the angles by focusing on the visual characteristics. For example, one student drew a straight line angle and described the measure as about 2" as this was approximately the length of the rays. This is a common misconception that the length of the rays is salient angle attribute. Another common misconception found early in the macro cycle was that angle orientation was a salient angle attribute. For example, one student drew lines as he considered them to be angles and the lines were drawn in different directions to separate them from each other. Students worked within the visualization level of geometric thinking as they were challenged with these misconceptions.

Within the instructional sequence, students were required to consider salient and non-salient angle attributes during all of the activities. From the observations, video, and researcher reflections diary, on the last two days of instruction students did not exhibit any of these misconceptions during the activities. Furthermore, unsolicited, the students would often point out that the length of the rays and the orientation were non-salient angle attributes. As the students drew angles they would intentionally draw the rays of different lengths or draw angles in different orientations to make this point.

As the instruction began, the students often described angle categories such as right, acute and obtuse angles, but when questioned, the students did not have any further understanding beyond rote learned names and did not believe that there were different angles that could fit into those categories. Following changes and additions to the instructional plans, specific discussions were added to have students think about how many angles were in a circle and also to begin to understand that there could be a fraction of a degree. With these skills, the majority of students moved from the visualization level one to the analysis level and to indicate a possibility of drawing an infinite number of angles, this was a level of geometric thinking situated within the informal deduction level (level three) and three students demonstrated this skill in the second class.

Early in the process, another problem highlighted in through the data was that the students struggled to find words for what they wanted to describe. Students found it hard to articulate the differences between the angles they had drawn. It was a twofold issue as students did not know the vocabulary to express those meanings and also did not fully understand the mathematical concepts which made it even more difficult for the students to articulate the differences and the similarities between the angles. These skills were developed during the instructional sequence and students began to describe the angles by the degrees of measure, salient properties, and angle categories. In addition, students were also beginning to provide justifications for these descriptions. In Figure 3, the revised level indicators for drawing angles are provided. This information is based on the findings of this study as applicable to drawing angles.

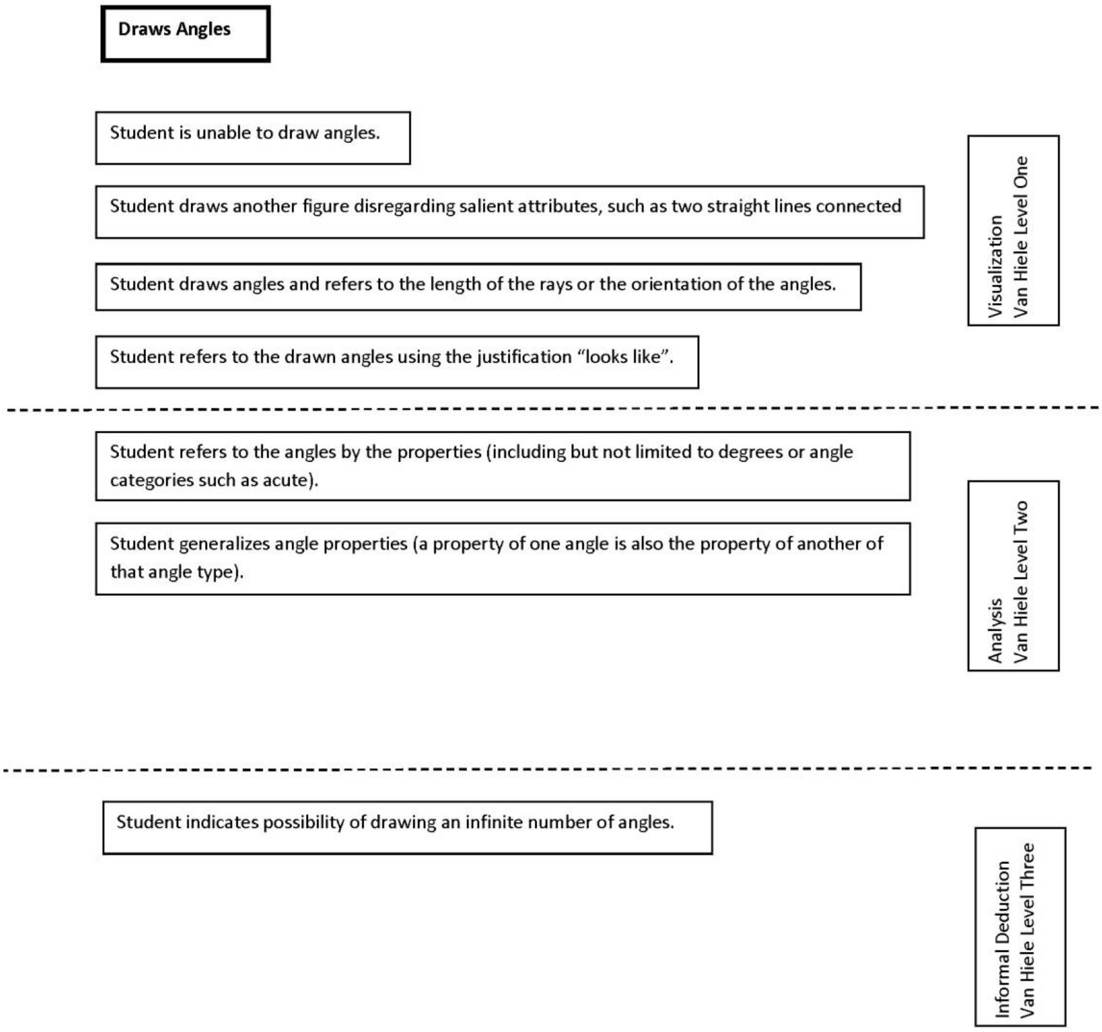

Figure 3. Revised level indicators for drawing angles

Identifies angles. The first activity of the instructional plans had students decomposing angles into their salient components. The findings of this activity laid the foundations for understanding angle and completing the other activities in the sequence. This initial activity primarily has students working towards the analytical level two of the van Hiele levels of geometric thinking. The findings showed that a few of the students were not working at this level as they began these tasks.

Using these data collected from the two retrospective analyses, students were working within van Hiele level two. Indicators of this thinking were students' ability to identify angles based on salient angle attributes, and to recognize non-salient attributes, such as length of the rays and orientation. In addition, students were able to generalize salient angle properties across all angles. Battista (2007) and Smart (2009) described how angles were difficult to learn due to their abstract nature and the multiple ways that angles can be represented. In the instructional sequence there were multiple opportunities provided for the students to practice using what they had learned about angle attributes to find angles in various real-world environments as well as on paper. Identifying angles in the real-world cognitive challenged the students as they had to recall and utilize this new information as they scanned across the vast amount of visual information to find the angles.

As the students searched for angles in the real world they would move back and forth between van Hiele level one and two. Initially, students used visualization (level one), as they scanned for objects that looked like angles. Students then began to determine angles by their properties moving into level two. Figure 4 provides the revised level indicators for identifying angles. This information is based on the findings of this study as applicable to identifying angles.

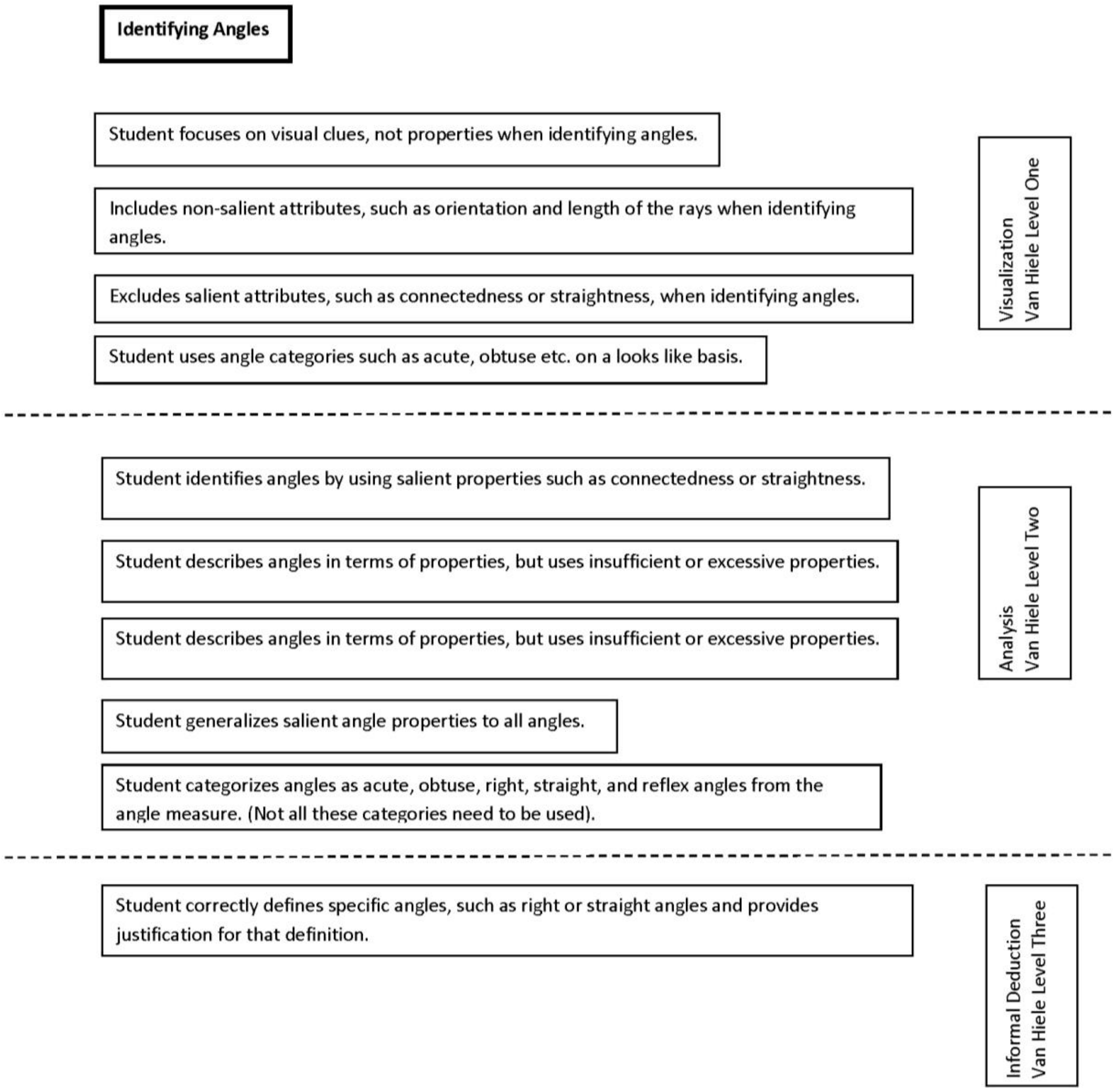

Figure 4. Revised level indicators for identifying angles

Sorts angles. The instructional plans included a number of angle sorting activities to have students consider appropriate ways to sort angles. The initial sort required students to create a set of angles and then sort those angles into groups. Students were required to consider what they had learned about angle properties and sort by those salient attributes. Many of the students did consider these properties and provided evidence of working at level two. A few of the students sorted by non-salient attributes which is indicative of thinking at van Hiele level one.

The instructional activities included angle sorts based on angle categorization. Students based those categorizations on acute, obtuse, straight and right angles. When students specifically requested further information reflex angles were also mentioned, but this additional categorizations was not utilized in the instructional activities. From the observations and video it appears that the majority of the students were able to correctly name examples of a particular category. In other words, the students were able to identify obtuse angles that were between 120° and 175°, but when the measure was 100° students would often revert to level one thinking and determine that it looked like a right angle without checking its properties.

Figure 5 provides the revised level indicators for sorting angles. This information is based on the findings of this study as applicable to sorting angles. Although students did begin to provide some justifications for sorting angles, further work is needed to have the students working within van Hiele level three of geometric thinking.

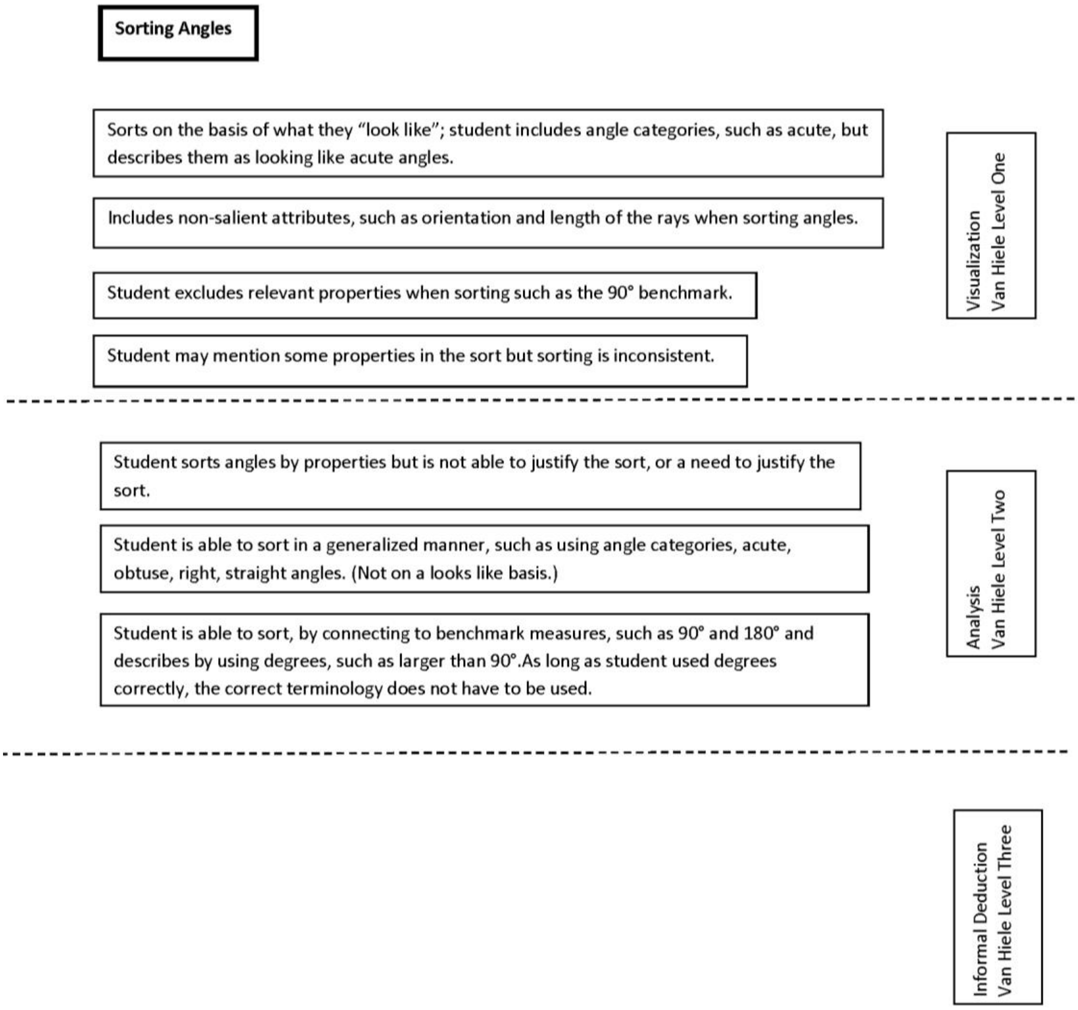

Figure 5. Revised level indicators for sorting angles

Angle measure. Although angle measure did not appear until later in the instructional sequence, foundational skills related to angle measure were developed from the beginning of the sequence, such as angle categorizations and discussions focused on what was being measured. In addition, the dynamic protractor was used from day two giving the students chance to build some understanding of how the turn or the ray created the angle. One of the key measurement objectives of the instructional plans was to have students internalizing benchmarks. This skill would support students in many of the measurement activities.

As expected, many of the students were working at level one at the start of instruction and this was still evident as students moved into the measurement lesson on day six. It was helpful to remind the students of certain skills they had developed as they learned about linear measurement. For example, in TE2 a discussion was added to have students remember to begin their measure at zero. This meant lining up the paper wedge along one side of the angle and looking to the other line for the measure. Another addition in TE2 was to label the 90° and 180° to support the students to internalize these benchmark measures. The revised indicators can be found in Figure 6. This information is based on the findings of this study as applicable to angle measure.

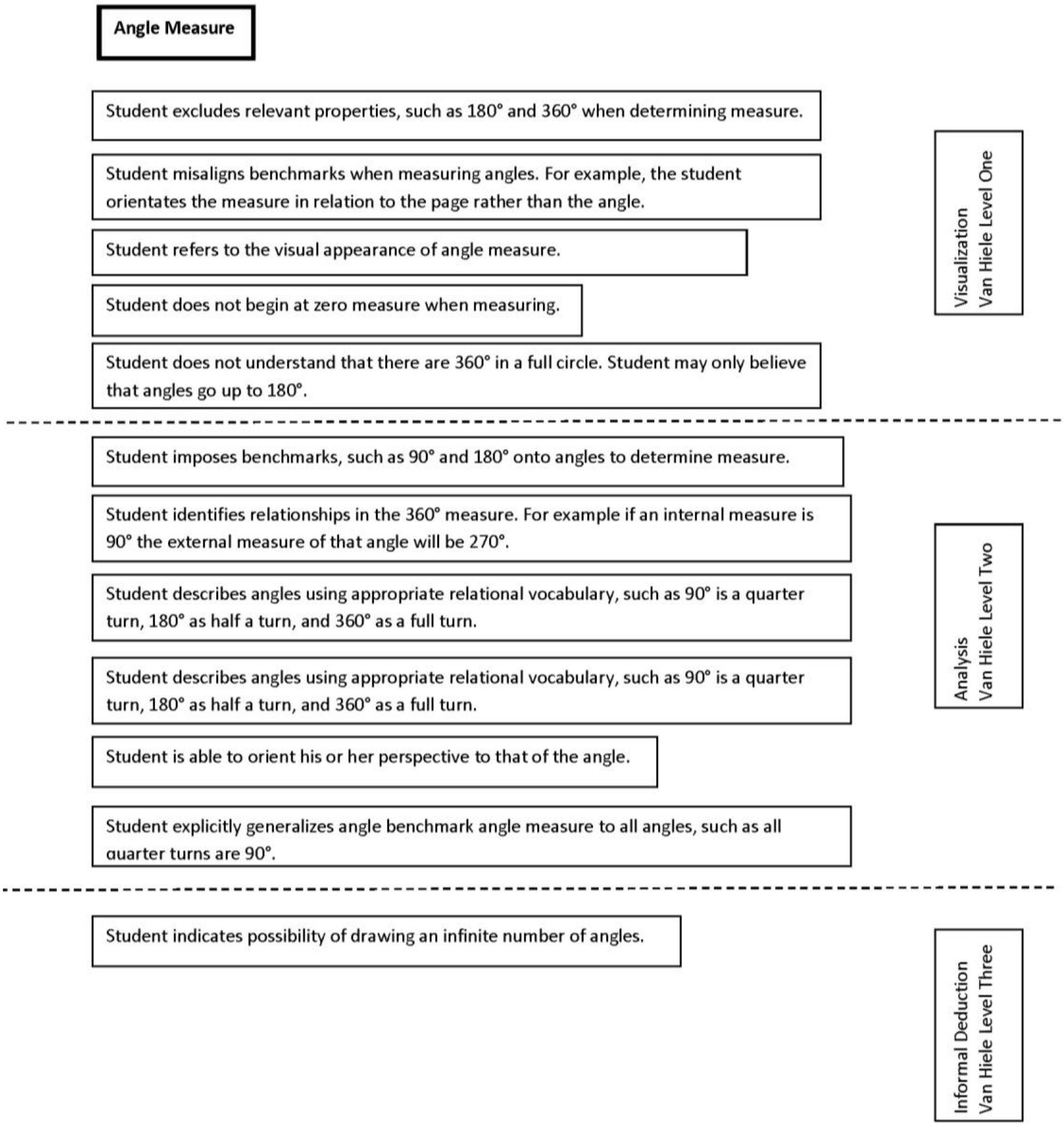

Figure 6. Revised level indicators for angle measure

Angle relations. There are two main skills encompassing angle relations, those are spatial reasoning and angle definitions. Spatial reasoning was discussed in the last section on angle measure as students struggled with spatial orientation and spatial visualization. The other skill is to understand the properties of the angles and formulate complete definitions. The instructional plans included a number of activities to have students thinking about salient and non-salient angle attributes. It also included opportunities to have students create definitions of angle.

From the observations and video, as the initial macro cycle began it was clear that some students held misconceptions to do with angle orientation. For example, one student said that a 90° angle in the orientation of an L was a left angle as it pointed towards the left. Both angle orientation and length of the rays were common misconceptions students had at the beginning of each macro cycle. Journaling was added following discussions to have the students considering what an angle is and to describe the various categories of angle. Students also had opportunities to work as a group to consider the salient properties of angle.

Over the instructional sequence, students came to understand what attributes were salient and those that were not. Students were also able to generalize properties across angles and other properties to specific angles. For example, angles between 91° to 179° are obtuse angles. Figure 7 provides the revised level indicators for angle relations. This information is based on the findings of this study as applicable to angle relations.

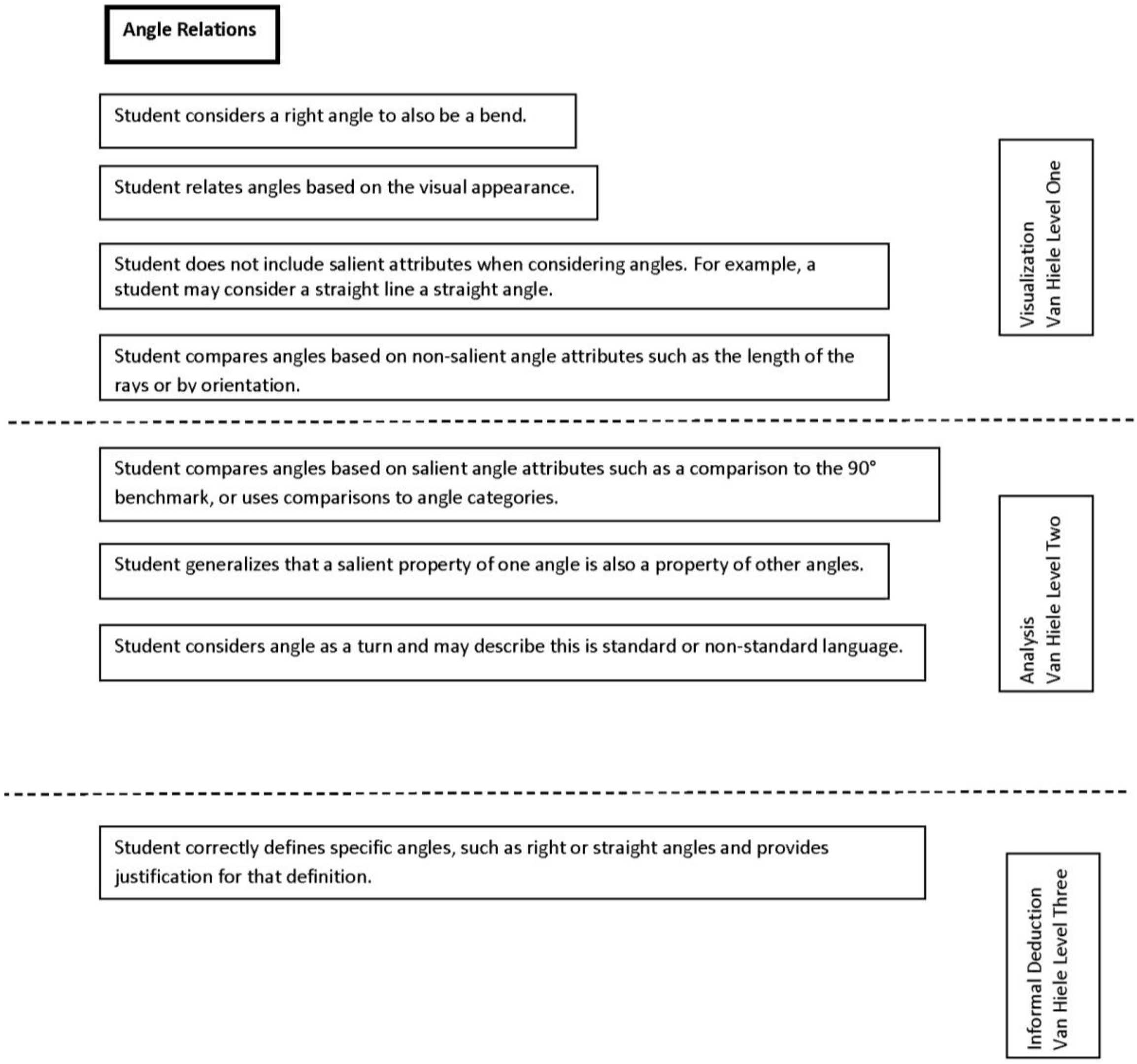

Figure 7. Revised level indicators for angle relations

In summary, the instructional sequence was effective in developing students understanding of angle and angle measure. Students in a short space of time showed significant progress across the van Hiele levels of geometric thinking in regard to drawing, identifying, and sorting angles as well as angle measure and angle relations. However, these seven days of instruction were only the initial steps and further instruction is needed if students are going to fully understand angle and angle measure. These additions to the theory on how students learn angle may be used in future research and by practitioners to develop a rich understanding of students' understanding and use this to develop future teaching direction.

Limitations and future research

A limitation to this study is the initial definition of angle used in the first day of the curriculum. The angle definition developed by Clements and Sarama (2009) was designed for angles viewed in two-dimensions only. Future researchers could develop a robust three-dimensional angle definition and conduct further examination on the use of this definition with real-world mobile learning activities. Future researchers may also examine the van Hiele level of geometric thinking of pre-teachers and if programs are adequately preparing these future teachers to themselves understand angle in the way we are trying to get students to understand angle.

Conclusion and significance

In this study, the researchers used DBR to develop a local instruction theory of how 4th grade students come to understand about angle through mobile learning. Aligned to the DBR methodology, the local instruction theory is comprised of two components: (a) a mobile learning curriculum, and (b) additions to the scholarly theories. Using a cyclical iterative process of anticipation, enactment, evaluation, and revision (Gravemeijer & van Eerde, 2009) over two macro cycles, a sequence of instructional materials were developed and additions were made to van Hiele's levels of geometric thinking (van Hiele, 1984), specifically in Scally's (1990) angle indicators.

The activities used a mobile learning approach which had the students making real-world connections to mathematics with the use of iPads and the Sketchpad Explorer (2012) app which is a DGE. This curriculum and the apps used in this study are freely available for educators to adapt to the needs of the students in their classrooms. The curriculum can provide a springboard for educators to begin to understand the affordances of mobile learning in a mathematics classroom and develop other mobile learning activities, enabling students to contextualize mathematics to make sense of difficult concepts. The development of the level indicators are particularly useful to practitioners and researchers to go beyond students initial numerical responses to assess the students level of angle understanding in the choices they make and the vocabulary they use.

This study is significant as it appears at a time when mathematics teachers are being required to rethink their mathematical practices to go beyond the textbook to connect with real-world mathematics while using additional technological supports to extend and enhance students thinking. The promise and potential of using mobile devices is now rapidly becoming apparent and there is widespread interest amongst parents, students, principals, and teachers. One significant challenge to this implementation is the lack of teacher training and knowledge on how to successfully implement such technological tools. This study provides a list of angle indicators and a curriculum outline for learning about angles that can be used in other fourth grade classrooms.

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