2010

Comment on "Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades"

Jeffrey Yepez

George Vahala

Linda L. Vahala
Old Dominion University, lvahala@odu.edu

Min Soe

Original Publication Citation

Yepez, J., Vahala, G., Vahala, L., & Soe, M. (2010). Comment on "Superfluid turbulence from quantum Kelvin wave to classical Kolmogorov cascades.
Physical Review Letters, 105(12), 1. doi: 10.1103/PhysRevLett.105.129402

Yepez *et al.* Reply: We agree with Krstulovic and Brachet [1] that the k^{-3} power law, in the energy spectrum for a linear vortex, marks the presence of a vortex core, using the standard kinetic energy definition, $\int dx^3 \frac{1}{2} m \boldsymbol{v}(x)^2 |\varphi(x)|^2$. Yet, the k^{-3} power law also marks the presence of a vortex tangle with a Kelvin wave (KW) cascade, provided it occurs with a $k^{-5/3}$ power law at small $\boldsymbol{k}$.

Our initial vortices had winding number $n = 6$, equivalent to 6 overlapping $n = 1$ vortices, a highly unstable configuration as illustrated in Fig. 1. We used ray tracing to image surfaces around the nodal lines $\varphi = 0$.

Consider an $L^3 = 2048^3$ simulation with initial vortex wave number $k_\xi = 40$ and vortex-vortex separation $\ell \sim \sqrt{\frac{L^3}{\mathcal{L}_v}} = \frac{2048}{\sqrt{72}} \approx 241$, using a total vortex length $\mathcal{L}_v = 12nL$. In the initial linear vortex spectrum, the transitional wave number between k^{-1} and k^{-3}_{linear} related to the inverse coherence length, k_ξ^{linear}, is pronounced. In contrast, in the quantum turbulence spectrum with clean $k^{-5/3}$ and k^{-3}_{tangle} power laws, the transitions related to the inverse Kolmogorov scale, $k_{\text{outer}} = k_\ell \sim \ell^{-1}$, and an inner scale, $k_{\text{inner}}^{\text{tangle}}$, are both pronounced. This is seen in Fig. 2 with $k_\xi^{\text{linear}} \equiv \frac{\sqrt{3}}{2} \frac{L}{\xi} = 40$ at $t = 0$ (no KWs) and with $k_\ell^{\text{tangle}} \approx 40$ at $t = 20\,000$ in a KW cascade. Thus, we find $k_\xi^{\text{linear}} \approx k_\ell^{\text{tangle}}$, and this similarity also occurred for the 5760^3 simulation reported in our Letter [2]. We identified the classical to quantum transition region as $k_{\text{outer}} \lesssim k \lesssim k_{\text{inner}}$, and identified the outer scale with the Kolmogorov length ($k_{\text{outer}} \approx k_\ell$) and the inner scale with the coherence length. When the k^{-3} spectrum is absent or significantly diminished, temporarily due to intermittency [3], we do not see a vortex tangle with a KW cascade. When the k^{-3} spectrum at high $k \gtrsim k_{\text{inner}}$ is present (along with a $k^{-5/3}$ Kolmogorov spectrum at small $k \lesssim k_{\text{outer}}$ marking a vortex tangle), we see distorted vortices supporting KWs undergoing kelvon-kelvon couplings, including at $k > k_\xi^{\text{linear}}$.

We believe there is essential dynamics at high wave numbers $k > k_\xi$. The $L^3 = 5760^3$ grid simulation we reported has $\sim 10^{11}$ microscopic (bit) particles, and a single vortex can contain hundreds of thousands of grid points. The unitary algorithm $\Psi' = U\Psi$ employs a tensor product state $\Psi = \psi(x)^{\otimes L^3}$ separated over the L^3 points of the system, where each local ket $\psi(x)$ is a 2-spinor. This gives an exact quantum simulation modulo the lattice cutoff $\ll \xi$ that accurately solves the Gross-Pitaevskii equation. A

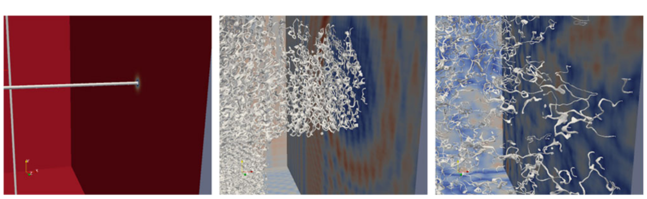

FIG. 1 (color online). Two initially nearly intersecting rectilinear $n = 6$ vortices on a portion of a 4032^3 grid (left). By $t = 4000$, many $n = 1$ vortices are subject to the KW instability by mutual interaction (middle). By $t = 57\,500$, a vortex tangle is evident (right).

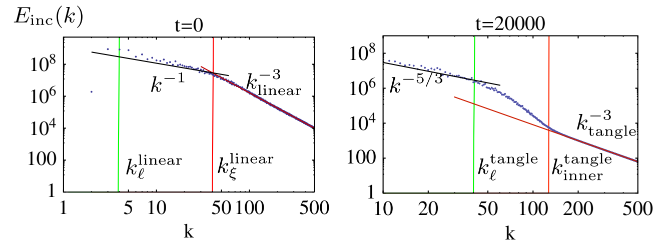

FIG. 2 (color online). Incompressible kinetic energy spectra with 12 linear $n = 6$ vortices at $t = 0$ (left) and during turbulence with a KW cascade at $t = 20\,000$ (right) for a 2048^3 grid. Low-$\boldsymbol{k}$ power-law regression fits: $k^{-1.00}$ (left) and $k^{-1.67}$ (right). High-$\boldsymbol{k}$ power-law fits: $k^{-3.16}$ (left) and $k^{-3.03}$ (right). Initially, the wave number cutoff is $k_\xi \approx 40$ (red vertical line). Later at $t = 20\,000$, we find $k_{\text{outer}} = k_\ell \approx 40$ (green vertical line) and $k_{\text{inner}} \approx \pi k_{\text{outer}} = 127$ (red vertical line).

fluctuating part of $\psi(x)$ are quasiparticles

$$\delta\psi(x) \cong \varepsilon \begin{pmatrix} u(\boldsymbol{x}) e^{-i\omega t} \\ -v^*(\boldsymbol{x}) e^{i\omega t} \end{pmatrix}$$

governed by the Bogoliubov–de Gennes (BdG) equations,

$$i\hbar \begin{pmatrix} \partial_t u \\ -\partial_t v \end{pmatrix} = \begin{pmatrix} \mathcal{L} & -g\varphi_v^{*2} \\ -g\varphi_v^{*2} & \mathcal{L} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix},$$

with a spatial operator $\mathcal{L} \equiv -\frac{\hbar^2}{2m}\nabla^2 + 2g|\varphi_v|^2 - \mu$. High $\boldsymbol{k}$-space resolution, especially at large $\boldsymbol{k}$, is vital to ensure these fluctuations are numerically represented inside the cores. Finally, high-$\boldsymbol{k}$ kelvons are known experimentally [4], and such kelvons have been verified numerically at the BdG level [5,6]. The cutoff $r_c < \xi$ is inside the core with a modified KW dispersion relation [6].

Jeffrey Yepez,[1,*] George Vahala,[2] Linda Vahala,[3] and Min Soe[4]

[1]Air Force Research Laboratory, Hanscom Air Force Base Massachusetts 01731, USA
[2]Department of Physics, William & Mary Williamsburg, Virginia 23185, USA
[3]Department of Electrical and Computer Engineering Old Dominion University, Norfolk, Virginia 23529, USA
[4]Department of Mathematics and Physical Sciences Rogers State University, Claremore, Oklahoma 74017, USA

Received 29 April 2010; published 13 September 2010
DOI: 10.1103/PhysRevLett.105.129402
PACS numbers: 47.37.+q, 03.67.Ac, 03.75.Kk, 67.25.dk

*To whom correspondence should be addressed.

[1] G. Krstulovic and M. Brachet, preceding Comment, Phys. Rev. Lett. **105**, 129401 (2010).
[2] J. Yepez *et al.*, Phys. Rev. Lett. **103**, 084501 (2009).
[3] G. Vahala *et al.*, Proc. SPIE Int. Soc. Opt. Eng. **7702**, 770207 (2010)
[4] V. Bretin *et al.*, Phys. Rev. Lett. **90**, 100403 (2003).
[5] T. Mizushima, M. Ichioka, and K. Machida, Phys. Rev. Lett. **90**, 180401 (2003).
[6] T. P. Simula, T. Mizushima, and K. Machida, Phys. Rev. Lett. **101**, 020402 (2008).

