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Hedonic games and Monte Carlo simulation

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Abstract

Hedonic games have applications in economics and multi-agent systems where the grouping preferences of an individual is important. Hedonic games look at coalition formation, amongst the players, where players have a preference relation over all the coalition. Hedonic games are also known as coalition formation games, and they are a form of a cooperative game with a non-transferrable utility game. Some examples of hedonic games are stable marriage, stable roommate, and hospital/residence problem. The study of hedonic games is driven by understanding what coalition structures will be stable, i.e., given a coalition structure, no players have an incentive to deviate to or form another coalition. Different solution concepts exist for solving hedonic games; the one that we use in our study is core stability. From the computational perspective, finding any stable coalition structure of a hedonic game is challenging. In this research, we use Monte Carlo methods to find the solution of millions of hedonic with the hope of finding some empirical points of interest. We aim to explore the distribution of the number of stable coalition structures for a given randomly generated hedonic game and to analyze that distribution using Cullen and Frey graph approach.

Keywords
Cooperative game theory, Hedonic game, Core stability, Monte Carlo methods

1. Introduction
Game theory uses mathematical models to analyze scenarios where self-interested agents interact [1]. The rational outcome is the key concern in game theory. Game theory was introduced by Cournot in the 1800s [2] and popularized by Von Neumann and Morgenstern, their famous book “Theory of games and economic behavior” [3]. It has gained popularity among researchers and expands into different areas like psychology, biology, war, politics, business, computer, social studies, and, especially, economics. Game theory has two main types: cooperative, non-cooperative. Cooperative games focus on coalition formation and, thus, focus on games involving more than two players while a non-cooperative game generally only focus on only two players. Solving a game with more than two players is generally more difficult than solving two players' games, and, as such cooperative game theory is less common in the academic literature. Cooperative game theory has two major sub-categories; transferable utility games (TU), which refer to games where the utility can be transferred between players in each coalition, and non-transferable utility games (NTU) where no utility can be transferred between players.

In this research, we conducted an empirical investigation into hedonic games, a form of NTU cooperative game, using Monte Carlo methods. The intention behind this empirical investigation is to understand the properties of cooperative game theory. Our focus is on properties that could not be resolved using current analytical methods, i.e., the distribution of the core size of hedonic games.

The breakdown of a cooperative game is as follows: players, coalitions, and coalition structure. Players referred to any entities that want to make decisions. Coalitions are groups of players that have formed for some purpose; note that a player can be in a singleton coalition, i.e., on their own. A coalition structure is a partition of all players into coalitions.

Hedonic games look at coalition formation where players have preferences about which coalition they would like to join. Hedonic games were first introduced by Bannerjee et al. [4] and Bogomolnaia & Jackson [5], who created mathematical models of games described by Dreze and Greenberg [6]. A hedonic game is defined by a finite set of players and preference relation for each player over every coalition they could be a member. The outcome of a hedonic game is a coalition structure that consists of disjoint coalitions that cover all the players. The hedonic game has a strong application in economics problems where concentration is on achieving stable outcomes in sufficient conditions, like formation of societies, groups [7]. The mathematical description of hedonic game describes as follow:

Consider sets of players as N = {1, 2, 3, …, n}, and a player $i \in N = \{1,2,3, \dots, n\}$, $\aleph_i$ is the collection of all subsets of N that contain i, so $\aleph_i = \{c \subseteq N | i \in c\}$. If ρ is a coalition structure and $i \in N$ then ρ_i is a coalition in ρ where i is a member [8].

The structure of the hedonic game is $G = (N, \succeq_1, \dots, \succeq_n)$ where the relation $\succeq_i \subseteq \aleph_i \times \aleph_i$ is a complete, reflexive and transitive preference relation over the possible coalitions where i is a member [8]. The intended interpretation is that if $C_1 \succeq_i C_2$, then player i prefers to be in coalition C_1 at least as much as in coalition C_2. The games consider in this paper use strict preferences, that its if $C_1 \succeq_i C_2$, then $C_2 \nsucceq_i \nsucceq C_1$. We denote strict preferences by $\succ_i$.

There are different interruptions as to what constitutes stability in a hedonic game, as such different solution concepts exist for solving a hedonic game, namely: core stability, Nash stable, individually rationality, individually stability and contractually individually stability [5]. The idea of a different solution concept is that they capture a different aspect of rationality in the outcome. As we mentioned before, the key concept in game theory is what is a rational outcome. The solution concept used in this research is core stability. Core stability related to one of the key solution concepts used in cooperative game theory, which is called the core [9]; the core stability is effectively the same concept as the core but for hedonic games [10]. The stable core of a game is a coalition structure in which no subset of players has an incentive to form a new coalition. The mathematical description of this statement is as follow:

Consider player $i \in N$ and partition ρ if $\nexists S \subseteq N$, s.t. $\forall i \in S$, $S \succ_i \rho_i$, then ρ is core stable.

Our Monte Carlo approach is to generate a random hedonic game and find its core; we repeat this process millions of times to generate empirical distributions. For solving a random hedonic game, we consider the same algorithm as our previous research [11]. By increasing the number of players in the hedonic game, the computational aspect of it becomes more complicated.

There have been some studies about hedonic games that incorporated the Monte Carlo simulation in their research [12-15]. These papers are practically focused looking at problems like radiofrequency sharing for swarms of Unmanned Aerial Vehicles (UAV) [15] and dynamic network selection for wireless internet access [12]. These practical papers primarily use Monte Carlo simulation to validate the hedonic game models they have constructed and to confirm their findings. Our use of Monte Carlo simulation is generated millions of random games, which we solve. We discuss some other applications of Monte Carlo simulation to Hedonic games below.

Cao and Wei proposed a model in a wireless network to promote the eavesdropper's performance by analyzing cooperative behavior amongst selfish eavesdroppers and jamming relays. Hedonic games used to frame cooperative behavior, and they focused on the Nash stability solution concept. They considered a novel job-hopping preference for each player in the game based on the coalition formation algorithm. Their model evaluates by Monte Carlo simulation. The results show that in a non-cooperative situation, their proposed model can improve each eavesdropper performance and provide adoption to dynamic changes in the wireless network environment[12].

Shin, Jang, and Tsourdos study of the problem of dividing radio bandwidth amongst swarms of heterogeneous unmanned aerial vehicles (UAV). Communication between each UAV's coalition defined by frequencies named channel. For satisfying situational network awareness, individual UAVs need to communicate with other UAVs through a complete mission, but because of bandwidth limitation, it is not possible to have communication with all UAVs. Based on this restriction, a preference for coalitions that provide enough situational awareness specified. They used the Nash stable solution concept and the validity of their approach done by a Monte Carlo simulation [15]. The author also applies a similar approach to robotic swarms [14].

As we mentioned, all the literature review considers Nash stability as a solution concept to their problem while we consider core stability for this research. Checking Nash stability is much easier and has less computational complexity than core stability; as such, core stability is less common in applications of hedonic games. We hope the approach outlined in this paper will help researchers apply core stability within their practical research.
The structure of the rest of the paper is as follows; in section 2, we introduce methods of conducting this research. In section 3, we discuss the outcome of the research. In section 4, we describe our conclusions.

2. Method

As we mentioned in the previous section, we use the algorithm in our previous research to conduct Monte Carlo experiment for generating one million runs of random hedonic games for each game of size three to twelve players to find the core of those games. Games of one and two players were ignored due to their simplicity. The preferences for players over each coalition allocated based on each player has a preference relation over 2^{n-1} coalitions that contain them. The random generator number used was Mersenne Twister [16]. The core for each hedonic game determined, and the results show in the results and discussions section.

We illustrate all 10 million random hedonic games outcome into Cullen and Frey graph to understand different options of which distributions fit our data. Cullen and Frey's graph firstly introduced by Cullen et al., which works based on skewness-Kurtosis relations [17]. The skewness shows the symmetry in distribution, and Kurtosis shows the tail existence in the distribution. Cullen and Frey graph method used when the distribution of the data is unknown. It illustrates which distribution is a better fit to the data. For normal, uniform, logistic, and exponential, the distribution shows as a single point because only one skewness and kurtosis value described for them. Other distributions, like lognormal and gamma, are represented by a line. For the beta, a larger area considered for showing the distribution.

3. Results and Discussions

The size of the core calculated for 10 million random hedonic games by the application of the Monte Carlo method. Since core stability is a criterion, it is possible that, for a given game, there exists more than one partition that satisfies this condition (or, even, none at all). The core is the collection of all these partitions, and it is the core size (or cardinality) that is the focus of our results. Table 1 represents the empirical results of core size for games of one to twelve players (while games of 1 and 2 players calculated analytically without conducting the Monte Carlo method).

Table 1: Monte Carlo simulation Core size result for games of 1 to 12 players

	Core Size									
Players	0	1	2	3	4	5	6	7	8	9^{+}(9-15)
1	-	1 M	-	-	-	-	-	-	-	-
2	-	1 M	-	-	-	-	-	-	-	-
3	7,170	952,117	40,713	-	-	-	-	-	-	-
4	19,566	878,328	96,731	5,253	121	1	-	-	-	-
5	33,593	798,176	150,855	16,193	1,115	65	2	1	-	-
6	46,235	722,188	194,521	3,228	4,235	520	63	9	1	-
7	56,991	656,425	225,495	49,667	9,410	1,656	296	56	4	-
8	64,401	599,949	246,387	67,854	16,532	3,786	862	183	38	8
9	68,351	553,564	259,432	85,142	24,574	6,546	1,780	441	133	37
10	70,890	514,170	266,404	100,528	33,469	10,180	3,045	903	267	144
11	72,094	483,108	270,061	112,414	41,453	13,973	4,628	1,480	529	260
12	71,102	455,585	270,138	123,530	49,954	18,914	6,937	2,483	873	484

There are a number of observations that can be drawn from this data. Firstly, we notice that most portion of the game's results belongs to the core size one; that is, there only exists one partition of the game; this is core stable. This is the most preferable result because (1) it shows a solution exists and (2) there is only one solution to choose from. The second most common solution is an empty core. An empty core has always been a challenge for researchers [4] because the solution method does not produce an "answer.". The empirical results show that the empty cores occur with games of three players or more. Although the number of empty cores increased till games with eleven players, start decreasing from games of twelve players. So, it can be assumed that as long as the number of players increases, the probability of having an empty core decreases which would be a good result. In other words, the more the game becomes complicated, the less chance it has of having an empty core. To be more precise about this finding, further researches with games of more than 12 players required to be conducted to demonstrate that this empty core decreasing trend will continue for games of larger players or not. We limit our research to twelve players because finding a core size for more than that becomes unreasonably time-consuming. Thirdly, by analyzing the represented data, it seems that by increasing the number of players in the game, there is a growth in the core size of more than two. It can be considered that maybe for games with much more than twelve players, core size one will no longer be the most populated. Considering all these three findings, we hypothesis that as the number of players increases, finding an empty core is less of a concern, and games always have at least one solution.

The key concern of our research is to find what statistical distribution the core size distributions are similar. To achieve this investigation, we construct a Cullen and Frey graph, shown Figure 1

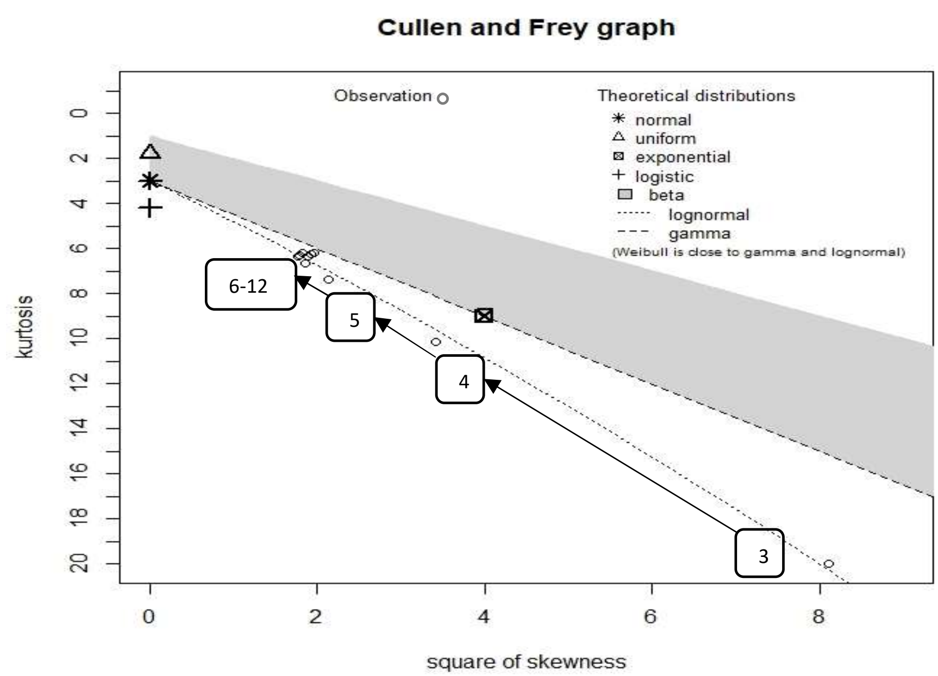

Figure 1: Cullen and Frey for games of 3 to 12 players

What Figure 1 shows us is that for most of the games, especially for games of ten players and more, fitting distributions of our empirical dataset are converging to one of Gamma, Weibull, and lognormal. It means that most of the games' core size distributions have very close skewness and kurtosis value. These two factors represent that most of our empirical data have symmetry in their distributions because non-zero value of skewness and the tail existence confirm in compare to the normal distribution, which has kurtosis value equal 3. Because most of the games kurtosis data value is close to six so the data set will not follow the normal distribution. Maximum Likelihood estimators were used to better determine the distance between the distributions and these analytics confirmed our results.

4. Conclusion

Research conducted and presented in this paper focused on the core size properties of particular cooperative games called hedonic games. Our finding reveals that in all kinds of games, which is differentiated based on the number of players in the game, core size one is the most common one. This is a useful result because having a unique solution is the most desirable outcome of a game. Our results also show that a core distribution for the more complicated game, i.e., ten plus players, fits a gamma distribution. Also, the core size has grown to the number of players in the game. In other words, for games of more than twelve players, maybe a core size one will no longer be the common one. In other words, the probability of having an empty core decrease while the number of players increases.

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