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A criterion for vortex breakdown

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A criterion for the onset of vortex breakdown over a wide range of the Reynolds number is proposed. Based upon previous experimental, theoretical, and numerical studies, as well as a new numerical study, an appropriately defined local Rossby number is used to delineate the region where breakdown occurs. Comparisons are made with previously suggested criticality parameters and the unique features of the proposed Rossby number parameter are shown. A number of previous theoretical studies concentrating on inviscid standing-wave analyses for trailing wing-tip vortices are reviewed and reinterpreted, along with the previous numerical and experimental studies, in terms of the Rossby number parameter. For the case of trailing wing-tip-type vortices, it is shown that previous numerical studies were performed at lower Reynolds numbers than the corresponding laboratory experiments. Utilizing a consistently defined Reynolds number, Rossby number–Reynolds number plots of these previous vortex breakdown studies, for both trailing wing-tip and leading-edge vortices, are obtained. A criticality condition is identified for both types of vortices.

I. INTRODUCTION

Vortices can be generated in many ways. Of specific interest are vortices generated by a finite plate or sharp-edged body at a nonzero angle of attack. These vortices are often highly stable structures characterized by a strong axial flow. Other examples of vortices with a strong axial velocity component include tornadoes and waterspouts, intake vortices, and swirling flow in pipes and tubes.

Leading-edge vortices shed from a delta wing induce a velocity field that results in increased lift and stability of the wing. However, under certain conditions related to the angle of attack of the wing, these vortices can undergo a sudden and drastic change in structure known as vortex breakdown. This breakdown can adversely alter the aerodynamic characteristics of the wing. A similar vortex bursting phenomena has been observed for trailing wing-tip vortices, which is desirable because these vortices represent a hazard to smaller aircraft in areas of dense air traffic. The fundamental difference between these two classes of vortices lies in their circumferential velocity distributions. Far downstream, as was shown by Batchelor,[1] the circumferential velocity profile of the wing-tip vortex behaves like the two-dimensional Burgers' vortex. However, Hall[2] has shown that the circumferential velocity distribution of the leading-edge vortex can be approximated using the concept of a viscous subcore very near the axis surrounded by an inviscid rotational conical flow region. Thus the radial gradients of the circumferential velocity near the axis of the leading-edge vortices are much larger than those of the wing-tip vortices.

The ability to control these vortical structures is an important and active area of research. For example, it is desirable to delay the process over a delta wing and accelerate it for trailing-tip vortices. Unfortunately, a comprehensive scheme to describe the breakdown process and the parameters affecting it is presently lacking, although several theories have been proposed.

Vortex breakdown was first observed experimentally by Peckham and Atkinson.[3] They observed that vortices shed from a delta wing at high angles of attack appeared to "bell out" and dissipate several core diameters downstream from the trailing edge of the wing. Since then, vortex breakdown has been observed in swirling flows in straight pipes, nozzles and diffusers, combustion chambers, and tornadoes. Seven types of breakdown have been identified experimentally,[4] ranging from a mild "spiral"-type to a strong "bubble"-type breakdown. Observations in the early 1960's spurred considerable effort to develop a theoretical explanation for the vortex breakdown phenomena. Three different classes of phenomena have been suggested as the cause or explanation of breakdown. These are (1) the concept of a critical state,[5–8] (2) analogy to boundary-layer separation,[2,9] and (3) hydrodynamic instability.[10–12]

The critical state theory is based upon the possibility that a columnar vortex can support axisymmetric standing waves. The supercritical state has low swirl velocities and the flow is unable to support these waves. Subcritical flows have high-swirl velocities and are able to support waves. Conditions favorable for the occurrence of vortex breakdown can be related to the ability of the flow to sustain standing waves.

In Hall's[2] theory, the breakdown phenomena is taken to correspond to a failure of the quasicylindrical approxima-

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tion. The idea being that when streamwise gradients in the vortex become large the quasicylindrical approximation must fail, thus signaling breakdown. This is considered to be analogous to the failure of the boundary-layer equations, which signals an impending separation.

Stability theory only allows one to investigate the amplification or decay of infinitesimally small disturbances imposed on the base vortex flow. Breakdown is then assumed to be analogous to laminar-turbulent transition. Of course, as pointed out by Leibovich,[13] breakdown can occur with little sign of instability and a vortex flow may become unstable and not undergo breakdown.

In this paper, we first reexamine the previous theoretical studies and identify the common basis among them. A review of these inviscid studies indicates that an appropriately defined Rossby number is the relevant controlling parameter. Next, the previous numerical studies are reexamined. The susceptibility of these results to breakdown in close proximity to the inflow boundary is analyzed and the need for a controlling parameter is identified. In conjunction with a consistently defined Reynolds number, a Rossby number–Reynolds number plot of the previous numerical and experimental studies for bubble-type breakdown is presented. The prediction capability of this plot is substantiated by one of the author's (RES)[14] recent numerical studies. In addition, a similar Rossby number–Reynolds number plot of some experimental studies for leading-edge-type vortices is presented. Finally, a comparison of the Rossby number–Reynolds number parameter basis presented here, with previously suggested parameter bases, is made. The mathematical and physical consistency of the Rossby number–Reynolds number definitions, used in this study, relative to previous suggestions is emphasized.

II. THEORETICAL RESULTS REEXAMINED

Throughout the remainder of this paper we use a cylindrical polar coordinate system, (r,θ,z), and the corresponding velocity components: U in the radial (r) direction, V in the circumferential (θ) direction, and W in the axial (z) direction. In discussing previous work, we adopt the (r,θ,z) convention.

Squire[8] appears to be the first to have performed a theoretical analysis of vortex breakdown. He suggested that if standing waves were able to exist on a vortex core then small disturbances, present downstream, could propagate upstream and cause breakdown. This is analogous to the earlier work of Taylor[15] on the stability of circular Couette flow. There, a linear stability analysis was performed to ascertain the ability of the base flow to support axisymmetric standing-wave disturbances. In all of the cases studied, Squire assumed that the vortex flow was inviscid and axisymmetric. He then sought to determine conditions under which an inviscid, axisymmetric, steady perturbation to the flow could exist. This condition, which was necessary for the existence of a standing wave, was taken to mark the transition between subcritical and supercritical states. Two of the cases studied by Squire are relevant to the present study.

In the first case W was taken to be a constant. Here V was taken to be that of a solid body rotation inside a core of unit radius and that of a potential vortex outside. That is,

$$V = V_0 r, \quad 0 \leqslant r \leqslant 1, \qquad V = V_0/r, \quad r \geqslant 1, \tag{1}$$

with V_0 a constant. He found that for standing waves to exist a swirl parameter k, the ratio of the maximum swirl speed to the axial speed, had to satisfy a criterion

$$k = V_{\max}/W \geqslant 1.20. \tag{2}$$

When $k = 1.20$ the wave is infinitely long but has a finite wavelength for $k > 1.20$.

In the second case W was also taken to be a constant, but

$$V = (V_0/r)(1 - e^{-r^2}), \tag{3}$$

with V_0 a nondimensional parameter. Again, Squire found that there was a condition on the swirl parameter k for the existence of a standing wave. The condition was

$$k = V_{\max}/W \geqslant 1.00, \tag{4}$$

where we note that

$$V_{\max} = 0.638 V_0. \tag{5}$$

Benjamin[5] examined this phenomena from a different point of view. He considered vortex breakdown to be a finite transition between two dynamically conjugate states of flow. There is subcritical flow, which is defined as the state that is able to support standing waves, and a conjugate supercritical flow that is unable to support standing waves. In this context the work of Squire[8] gives a condition marking the interface between these two states. As in the work of Squire, a universal characteristic parameter was defined that delineates the critical regions of the flow. This parameter, denoted by N, is the ratio of the absolute phase velocities of long wavelength waves which propagate in the axial direction, i.e.,

$$N = (C_+ + C_-)/(C_+ - C_-). \tag{6}$$

Here C_+ and C_- are the phase velocities of the waves which propagate with and against the flow, respectively. For $N > 1$ the flow conditions are supercritical and for $N < 1$, subcritical.

Benjamin applied this theory to a specific vortex flow, defined by W a constant, and

$$V = V_0 r, \quad 0 \leqslant r \leqslant 1, \qquad V = V_0/r, \quad 1 \leqslant r \leqslant R. \tag{7}$$

If $R \to \infty$, this is just the combined vortex studied by Squire. Benjamin found that the critical condition was of the same form as Squire's,

$$V_{\max}/W = \text{const.} \tag{8}$$

The precise value of the constant depends on the value of R but lies between 1.92 when $R = 1$, and 1.20 when $R = \infty$. Thus Benjamin (although starting from a different perspective) arrived at the same critical condition for a combined vortex as did Squire.

As a variation of the phase velocity criterion of Benjamin, Tsai and Widnall[16] examined a group velocity criterion that follows more directly from the view that the breakdown occurs as a result of a wave trapping mechanism.[17] Their investigation was of swirling pipe flows where the radial and axial velocity distributions can both be fit to exponential profiles. They used the least squares fit of Garg and Leibovich[18] to calculate the dispersion relation from linear parallel stability theory. The group velocity associated with the various flow profiles was then calculated. The results showed that upstream of breakdown the group velocity of both the symmetric and asymmetric modes was directed downstream. Even though their criticality condition of zero group velocity proved an accurate guide for the various types of breakdown, they were unable to establish a relationship between vortex breakdown and wave trapping.

Finally to complete this brief review of previous theoretical studies, a recent paper by Ito, Suematsu, and Hayase[19] is considered. There, both stationary and unsteady vortex breakdown were examined. They considered the stability to small amplitude disturbances of a vortex with solid body rotation in a streamtube. The disturbances can be axisymmetric as well as asymmetric and steady or unsteady. Their analysis yields a criterion for breakdown from the requirement for the existence of solutions to their disturbance equations. A comparison of these results with those of Benjamin[5,6] for the same case of a finite-radius pipe containing a rigid-body rotation gives the same criterion for breakdown. The important aspect of the Ito *et al.*[19] work lies in their interpretive criterion. Their nondimensionalization leads to the Rossby number as the relevant parameter. For example, in the case of swirling pipe flow consisting of a solid body rotation, the relevant scales are the axial velocity W, pipe radius r^*, and constant angular velocity of the flow Ω.

It is advantageous to summarize this section by placing these theoretical analyses into perspective. As has been shown there is quantitative agreement among the results of Squire,[8] Benjamin,[5] and Ito *et al.*[19] for the various test problems that have been examined. These analyses have been constrained by either the scope of the analysis (linear, parallel, inviscid) or the narrow class of flows that have been considered. In the study of Tsai and Widnall,[16] the calculation of group velocity is an added task. This is generally not feasible in engineering applications where a criterion based solely on mean quantities may be necessary. Nevertheless, these analyses indicate that a criterion for vortex breakdown is available. In Sec. V, this Rossby number criterion is applied to a variety of computational and experimental, confined and unconfined flows, and its range of applicability as a function of Reynolds number is examined. However, before proceeding with this analysis it is instructive to examine the large number of numerical studies that have been performed.

III. ANALYSIS OF PREVIOUS NUMERICAL STUDIES

Numerical simulations of vortex breakdown abound.[20–23] The purpose of the computational experiments was to obtain additional information concerning the structure of the breakdown bubble as well as identifying the various parameters affecting its development. In all of the studies known to us, the flow was restricted to have axial symmetry and, as will be shown in the next section, a relatively low range of Reynolds numbers when compared to experiments. Nevertheless, geometries and boundary conditions were chosen to hopefully reflect experimentally observed flows. A possible criticism[24] of these numerical experiments is that when breakdown occurred, it invariably did so in close proximity to the inflow plane. There are two reasons why this proximity breakdown near the inflow boundary may be suspect: the first is related to the question of numerical accuracy, in general, and the second is related to the insensitivity of the flow to the choice of inflow boundary conditions.

In the case of incompressible flow calculations, and irrespective of the formal accuracy of the numerical algorithm, the accuracy of a numerical solution of the full *Navier–Stokes equations* near and at a computational boundary is dictated by the accuracy with which the boundary conditions approximate the physical flow situation. At freestream boundaries, sufficiently far away from the dominant dynamic regions of the flow, inaccuracies in boundary condition specifications are not that critical to the overall accuracy of the numerical solution. This is because of the lack of significant source dynamics and momentum or vorticity flux at these boundaries. However, at outflow or inflow boundaries the situation is much more volatile. Clearly, at outflow boundaries the inability to properly handle momentum or vorticity flux will soon destroy the global accuracy of a numerical solution. Even in the best of circumstances, it is exceedingly difficult, if not impossible, to obtain completely "transparent" boundary conditions at outflow boundaries for incompressible finite Reynolds number flows. At inflow boundaries, a similar situation to the outflow boundary arises; although, because of the dominance of the mean flow, the situation is not nearly as critical. Nevertheless, for the numerical solution of the Navier–Stokes equations for incompressible flows, it is once again extremely difficult to choose the "correct" inflow boundary condition. Specifically, if the inflow boundary condition does not exactly satisfy the Navier–Stokes equations, there must be some spatial adjustment range over which the solution adjusts to a solution of the Navier–Stokes equations. The spatial extent of this adjustment region varies depending on the physical problem at hand, but it does exist, and in flows which may be close to some criticality threshold, this region of adjustment may trigger unwanted and unrealistic numerical solutions. Clearly, the conclusion that can be drawn from the discussion of "numerical" boundary layers induced by numerical boundary conditions is to be cautious of the physical significance of results obtained near computational boundaries. As previously indicated, there is another reason why vortex breakdown near the inflow boundary may be suspect.

A review of the previous computational studies has revealed that, irrespective of the type of inflow boundary conditions, breakdown invariably occurred near the inflow boundary in these studies. This is somewhat surprising. Even though breakdown may occur well upstream near the turning vanes of the generating apparatus in some experiments, there are several cases where it occurs well downstream within the test section. In addition, Leibovich[24] has pointed out that the internal structure of the numerically generated breakdown bubbles is not consistent with the structure observed experimentally. The question that arises is as follows: "Why the apparent insensitivity to inflow conditions?" As examples of the types of boundary conditions that have been used in the numerical studies, consider the studies of Kopecky and Torrance[20] and Grabowski and Berger.[21] The numerical work that followed[22,23] these studies appears to be redundant as far as types of boundary conditions and results are concerned.

Kopecky and Torrance[19] solved for the unsteady axisymmetric swirling flow through a cylindrical streamtube with uniform streamwise velocity. The exponential distribution of swirl velocity, assumed at inflow, was continuous (along with its radial derivatives) and behaved as a solid body near the axis and a potential vortex away from the axis. The inflow profiles were not a solution to the finite Reynolds number Navier–Stokes equations. A parametric study was performed with the Reynolds number, based on mean axial velocity in the streamtube and streamtube radius, ranging from 50 to 500; and the swirl ratio (defined in terms of streamtube radius) ranging from 0.4 to 10.0. The development of a recirculation zone was demonstrated as the swirl was increased for fixed Reynolds number and viscous core diameter. Similar results were obtained when the core diameter and swirl ratio were fixed while the Reynolds number was increased. In all cases, the breakdown occurred within one streamtube radius of the inflow boundary.

As an example of a different type of inflow boundary condition consider the work of Grabowski and Berger.[21] They solved the steady axisymmetric Navier–Stokes equations for a free vortex embedded in an irrotational flow. This was approximated by a two parameter family of assumed inflow velocity distributions. The velocity profiles were the polynomial profiles given by Mager[25] in his integral analysis and are assumed to be approximations to experimentally measured profiles. The axial velocity profiles were allowed to range from wakelike to jetlike profiles. It is worth noting that at inflow the vorticity distribution was continuous; however, the radial gradient of vorticity was discontinuous at the interface between the free vortex and the irrotational flow. Once again the inflow velocity profiles were not a solution to the finite Reynolds number Navier–Stokes equations. Solutions with vortex breakdown were obtained for Reynolds numbers based on free-stream axial velocity and core radius of up to 200. These solutions were obtained with inflow conditions that were, in many cases, subcritical. The results showed that breakdown was enhanced by increasing the swirl and was relatively Reynolds number independent. In the cases studied (Reynolds number of 200), breakdown appeared to occur within two core radii downstream of the inflow boundary.

These two numerical studies, Kopecky and Torrance[20] and Grabowski and Berger,[21] serve as examples of the vulnerability of the vortex to breakdown near the inflow boundary, irrespective of the details of the inflow boundary conditions. These results, along with those of the other numerical studies,[22,23] suggest that there must be another alternative to the formulation of these flow problems. If this is not the case, then there appears to be no way of obtaining numerical results that yield breakdown away from an inflow boundary. From the previous discussion on computational boundary layers and computational boundary conditions, the restriction would seriously curtail the usefulness of numerically simulated vortex breakdown.

IV. BREAKDOWN CRITERION

It is apparent from previous theoretical work that a criticality condition, in terms of a Rossby number, can be established for the onset of breakdown. This choice is motivated by the fact that both the Squire[8] and Benjamin[5] studies can be reinterpreted in terms of this parameter and the recent work of Ito *et al.*[19] explicitly expresses the result in terms of a Rossby number.

The Rossby number (or inverse swirl ratio) must be defined in a consistent manner with respect to the basic type of vortex flow being considered. It is defined as

$$\mathrm{Ro} = W/r^*\Omega\,, \tag{9}$$

where W, r^*, and Ω represent a characteristic velocity, length, and rotation rate, respectively. For the velocity profiles consistent with swirling flows, leading-edge, and trailing wing-tip vortices we define r^* as the radial distance at which the swirl velocity is a maximum. As pointed out by Leibovich,[13] this is a characteristic viscous length scale appropriate for swirling flows. Here W represents the axial velocity at r^*. This is justified by the fact that it is a consistent velocity scale for both uniform and radially varying axial velocity profiles, and it is also consistent with the "swirl velocity scale" implied by $r^*\Omega$. A characteristic property of trailing wing-tip vortices is the solid body rotation occurring near the vortex centerline. This rotation rate is considered to be the characteristic rate Ω of the vortex.

The swirl velocity component of wing-tip vortices is often described in terms of the two-dimensional Burgers' vortex, which is given by

$$V(r) = (K/r)[1 - \exp(-ar^2/2\nu)]\,, \tag{10}$$

where a is an adjustable constant associated with the core size, ν is the kinematic viscosity, and K is proportional to the circulation. Here, Ω is the limit of V/r as $r \to 0$, i.e.,

$$\Omega = \lim_{r\to 0}(V/r) = aK/2\nu\,. \tag{11}$$

The characteristic length is taken as

$$r^* = 1.12\sqrt{2\nu/a}\,, \tag{12}$$

which is the radius of maximum swirl velocity. For the case of the combined vortex considered by Squire[8] and Benjamin[5] the characteristic radius r^* is 1. Note that the parameter k given by Squire for the combined vortex is the inverse of the Rossby number (or swirl ratio), since the characteristic rate of rotation is given by the solid body rotation of the vortex core V_0.

For consistency, the Reynolds number is defined here in terms of the viscous length scale r^* and the axial velocity W at the radius r^*. A third parameter associated with such flows is the Eckman number, or a "rotational" Reynolds number, but in this context it is not an independent parameter. Based on this discussion, it appears that a consistent parameter basis for characterizing these flows is the Rossby number–Reynolds number set. The relationship of this set to other parameter bases, proposed earlier, will be examined in the next section; however, first this duo will be applied to both trailing wing-tip and leading-edge vortices.

V. APPLICATION OF CRITERION

Figure 1 is a plot of the Rossby number versus Reynolds number for a variety of numerical and experimental studies of swirling flows and trailing wing-tip vortices. Throughout the figure, the open symbols denote no breakdown and the solid symbols denote breakdown. For these computational and confined experimental studies, breakdown is defined as stagnation of the axial velocity on the axis. For the unconfined experimental studies, breakdown is defined as a rapid expansion of the core coupled with a strong deceleration of the axial velocity. The data in the figure show that the numerical work to date, expressed in terms of the viscous core radius r^*, has been performed at relatively low Reynolds numbers compared to the experimental studies. With the exception of one of the Kopecky and Torrance[20] data points, all the numerical studies consistently delineated the critical region. An examination of the numerical results, presented in their study (Fig. 2a, p. 295), did not reveal any axial stagnation point. Both numerical and/or graphical resolution restrictions may have precluded such a representation for this threshold case. Since such an evaluation was outside the intent of the present investigation, it will suffice to include this data point as a case representing no breakdown. Since the results are Reynolds number dependent in the range of computational test cases, direct application of inviscid theory in this range is invalid.

The authors have performed numerical calculations using a numerical algorithm in which no assumption of axisymmetry has been made. The algorithm is the three-dimensional extension of the earlier work of Gatski, Grosch, and Rose,[26] using vorticity–velocity variables and a compact discretization of the Navier–Stokes equations. Application of this algorithm to the numerical study of the breakdown phenomena for a variety of flow conditions and parameters is presented in Ref. 14. For a Reynolds number of 200 and a Rossby number of 0.5, breakdown occurred at the inflow plane. For the same Reynolds number and a Rossby number of 0.64, a decrease in axial velocity occurred near inflow, but did not result in breakdown. In addition, for a Reynolds number of 50 and Rossby number of 0.5, breakdown occurred at inflow.

The experimental studies have been conducted for both confined and unconfined flows at higher Reynolds number. Figure 1 shows the results for the confined flows of Garg and Leibovich,[18] which are characteristic of the wing-tip class of vortices. Since the data were fit to Burgers' vortex, the Rossby number is easily obtained. Here the Reynolds numbers ranged from 1288 to 2150. The mean position of the upstream stagnation point of the breakdown bubble ranged from 2.3 to 5.6 cm downstream from the beginning of the diverging section of the duct. The data points were all taken 0.2 cm downstream from the beginning of the diverging section. This location corresponded to a range of r^* values between 0.29 and 0.38 cm. The points representing the spiral form of breakdown were taken within 3 cm of the breakdown initiation point, with a corresponding r^* value of ~ 0.44 cm. This initiation point occurred further downstream (~ 15 cm) than the upstream stagnation point of the bubble-type breakdown. A single set of data was available from the study of Uchida and Nakamura.[27] This is a confined flow with axisymmetric breakdown occurring at a Rossby number of 0.64. The data point from Singh and Uberoi[28] is for an unconfined trailing wing-tip vortex of a laminar flow wing. In this case the minimum axial velocity rapidly decreases to $0.3 W_\infty$, which suggests vortex breakdown.

Figure 2 displays the relationship between Rossby number and Reynolds number for the leading-edge class of vortices. The experimental data were obtained from reports by Owen and Peake,[29] Anders,[30] Verhaagen and Kruisbrink,[31] and Pagan and Solignac.[32] Once again open symbols denote no breakdown and closed symbols denote breakdown.

In the study of Owen and Peake,[29] axial core blowing was introduced into vortices shed from delta wings in order to study its effect on breakdown. The symbols in Fig. 2 representing this data are variations based on a blowing coefficient C_μ, at fixed streamwise stations $z/c = 3$ and $z/c = 4$ (c is the chord length of the delta wing). As C_μ increases, the corresponding axial velocity increases and the Rossby number increases past critical. They state that breakdown occurs for the case $C_\mu = 0.0$, while for $C_\mu = 0.05$ and 0.12 the flow is stabilized and no breakdown occurs. For the study of Anders,[30] the variation of the data in Fig. 2 is parametrized by the angle of attack of the delta wing. The results for the

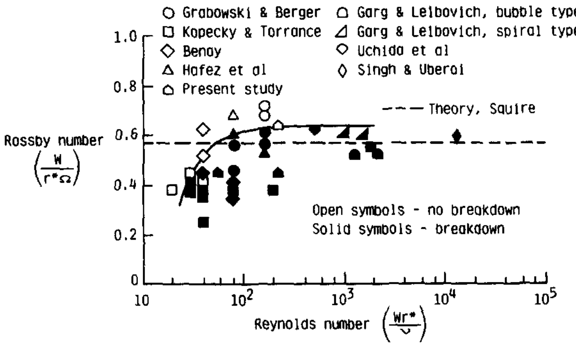

FIG. 1. Rossby number dependence of wing-tip vortices.

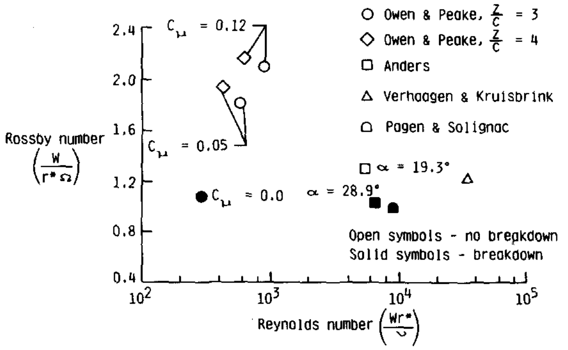

FIG. 2. Rossby number dependence of leading-edge vortices.

two angles of attack, $\alpha = 19.3°$ and $\alpha = 28.9°$, at essentially the same downstream location, are shown in the figure. As shown the higher angle of attack causes breakdown to occur closer to the wing leading edge. Verhaagen and Kruisbrink[31] measured the flow properties of the core to support and validate the development of mathematical models. They report that no breakdown occurred. The final study that was examined for the leading-edge-type vortices was by Pagan and Solignac.[32] They generated a leading edge vortex from a delta wing at an angle of attack of 19.3°. An adverse pressure gradient was imposed in their wind tunnel, by a set of adjustable flaps downstream of the air duct, to facilitate breakdown. The vortical structure upstream of the breakdown (~ 0.09 chord length of the delta), but downstream of the trailing edge of the wing (~ 0.375 chord lengths), was nearly symmetric. At this location, the local Rossby number was slightly less than 1.0 with a corresponding Reynolds number of ~ 9800. It is important to note that for this class of vortices, as well as for the trailing wing-tip vortices, the Reynolds number range over which the Rossby number criterion holds is significant.

Although the evaluation of the Rossby number is approximate, one may conclude that vortex breakdown for leading-edge vortices occurs at a higher Rossby number than for trailing wing-tip vortices. This may be because of the fact that the swirl velocity profiles are of a different type. Far downstream, the flow outside the core of a trailing wing-tip vortex is nearly irrotational. For a leading-edge vortex, the flow at the edge of the core is rotational and nearly inviscid. In addition, the leading-edge vortex contains a narrow viscous subcore where the radial gradients of the circumferential velocity are extremely large. In contrast, the wing-tip vortex approaches a solid body rotation as the axis is approached. Upstream of breakdown both types of vortices can generally be approximated as quasicylindrical. The authors can find no analyses that seeks standing-wave solutions to profiles applicable to leading-edge vortices. If these were available, an analytic Rossby number criterion could be obtained. Based on experimental results, it should be near unity.

It is necessary to compare the present criterion for vortex breakdown with previous attempts at establishing a criticality condition. Sarpkaya[33] proposed a parameter basis of Reynolds number–circulation number as a means of predicting the location of the onset of breakdown. These parameters were defined in terms of the mean axial velocity in the vortex tube, the imposed circulation, and the characteristic diameter of the apparatus. Escudier and Zehnder[34] extended this parameter basis to include a third parameter, the ratio of radial to tangential velocity. Both of these studies were for confined flows and, as indicated, the scaling parameters were based on apparatus characteristic scales. No local flow measurements were taken in the vortex breakdown region, thus precluding inclusion of this data in Fig. 1. Extension of the criterion established in either of these studies to unconfined flows is not straightforward as a result of the defining characteristic scales used. Nevertheless, these studies suggest that breakdown location is dependent on Reynolds number. Leibovich[13] points out that this is probably not the case, since the breakdown process is essentially governed by inviscid dynamics. Certainly there is a threshold where viscous effects must have some impact; however, the point of the objection concerns regimes where the Reynolds number is above this threshold value. Leibovich[13] identifies the problem in these studies[33,34] as the choice of characteristic length scale. The variation of the characteristic diameter changes the value of the Reynolds number, but it also affects the magnitude of swirl and vortex core vorticity (i.e., circulation can be kept constant). Therefore, even for fixed circulation number, the Reynolds number is not an unambiguous measure of the importance of viscous forces in the flow. As Leibovich[13] has suggested, the correct choice of characteristic length scale is the vortex core diameter, which is the choice used [cf. Eq. (9)] in the Rossby number–Reynolds number set of the present study. For example, for the case of the trailing wing-tip-type vortex breakdown, Fig. 1 clearly shows that the breakdown dynamics is essentially "inviscid" for Reynolds numbers greater than about 200.

Another attempt at establishing a criticality condition for vortex breakdown was made by Kopecky and Torrance.[20] This numerical study has been reviewed earlier in connection with breakdown occurring near the inflow computational boundary and has also been included in the data set of Fig. 1. The parameter basis proposed in their study was the swirl ratio–Reynolds number set. At first this appears to be the same basis as proposed in the present study (recall that the swirl ratio can be interpreted as simply the inverse of the Rossby number); however, closer examination reveals there are two differences. The first difference is the choice of characteristic length scale. Kopecky and Torrance[20] choose the streamtube radius (height of computational domain) as their characteristic length rather than the core radius; thus the results are vulnerable to the same physical misinterpretation as the Sarpkaya[33] and Escudier and Zehnder[34] results. The second, related difference is the inclusion of a parameter B that adjusts the radius of the viscous core at the inflow boundary. Their results on a criticality condition are somewhat misleading since the swirl ratio and Reynolds number, as defined, need to be supplemented with a conversion factor $B^{1/2}$, to take into account the effect of viscous core size. In addition, application of this swirl ratio–Reynolds number basis set to experimental results would be complicated by the need to extract out a parameter B from the experiments.

It suffices to point out, finally, that these previous studies did not attempt to extend their parametrization bases to breakdown of leading-edge-type vortices. The inclusion of this type of vortex breakdown in the present study further validates the applicability of the proposed Rossby number–Reynolds number criterion. It also shows that the criticality condition for the leading-edge vortices is different (slightly higher) than the criticality condition for the trailing wing-tip vortices.

VI. CONCLUSIONS

The results shown in Fig. 1 make it apparent that experimentally, analytically, and computationally, the critical Rossby number for the symmetric form of trailing wing-tip vortex breakdown for Reynolds numbers greater than 100 is about 0.65. For lower Reynolds numbers, the value of the critical Rossby number is lowered, undoubtedly owing to the increased damping effects of viscosity on the wave motions.

Figure 1 sheds light on the proper way to perform computational experiments. The inflow profile should correspond to a Rossby number greater than the critical value. This prevents the possibility of wavelike solutions near the inflow thus precluding breakdown. A mechanism, either inherent in the dynamics of the flow or externally imposed, must then modify the local Rossby number as the flow evolves in the streamwise direction. For example, the decay of a jetlike axial flow due to viscosity or the imposition of an adverse pressure gradient might be sufficient to lower the local Rossby number. Once the critical condition is achieved the possibility of wavelike solutions arises. One would expect a standing wave to originate at this location. If, on the other hand, the Rossby number at inflow is less than the critical value, axisymmetric waves can be expected to originate at the inflow boundary. Here, the velocity profiles are fixed, thus acting as an "artificial" critical condition. Thus breakdown occurs at this point.

This scenario for numerical computations corresponds to the way in which experiments conducted in tubes have been carried out. A supercritical flow is drawn toward critical as it evolves downstream as a result of the slight expansion of the tube. At the critical station, breakdown occurs.

The theoretical analyses of Squire,[8] Benjamin,[5,6] and Ito *et al.*[19] reduce to a criterion for the existence of axisymmetric standing waves based on a Rossby number. The exponential profile [Eq. (10)] that most closely models experimental flows yields a critical Rossby number of 0.57. This value is shown as a dashed line in Fig. 1. The experimental data of Garg and Leibovich,[18] interpreted in terms of a Rossby number, shows that breakdown occurs when the local Rossby number falls in the range of 0.57 to 0.63. Numerical experiments reveal a high Reynolds number limit ($\mathrm{Re} > 50$) of about $\mathrm{Ro} = 0.6$ for breakdown to occur. For lower Reynolds numbers, a lower Rossby number is required to initiate breakdown.

The case for the breakdown of the leading-edge vortices is shown in Fig. 2. The data for this case are more difficult to cast in terms of the Rossby number and Reynolds number than the data for the trailing wing-tip vortices. This is because of the small viscous subcore characteristic of the type of vortex. However, the data shown in Fig. 2 were obtained from a more diverse set of flow conditions than the trailing wing-tip vortices. For example, there was a case where vortex core axial velocity blowing was used,[29] a case where variation of the angle of attack of the delta wing was used,[30] and a case where an adverse pressure gradient was imposed.[32] Nevertheless, in these studies, the data was consistent with the concept of a Rossby number criterion.

The comparison, between the Rossby number–Reynolds number parameter basis, proposed in the present study, and the parameter bases proposed previously,[20,33,34] showed that the present formulation is both physically consistent and less ambiguous than the previous studies.[20,33,34] The choice of characteristic scales associated with the vortex itself allows for the application of the established criterion to both confined and unconfined vortical flows, as well as trailing wing-tip and leading-tip-type vortices.

It is apparent from the results of this paper that retarding or precluding vortex breakdown is a practical and viable objective. This altering of the vortex characteristics can be accomplished by either reducing the characteristic rotation rate of the vortex or enhancing the streamwise velocity. The rotation rate can be reduced, for example, by imposing transverse pressure gradients, or the streamwise velocity can be enhanced by imposing streamwise pressure gradients. In either approach the effective measure is the Rossby number.

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