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Understanding Angle and Angle Measure: A Design-Based Research Study Using Context Aware Ubiquitous Learning

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Mobile technologies are quickly becoming tools found in the educational environment. The researchers in this study use a form of mobile learning to support students in learning about angle concepts. Design-based research is used in this study to develop an empirically-substantiated local instruction theory about students' develop of angle and angle measure. This local instruction theory involves real-world connections and mobile technologies through a sub category of mobile learning called context-aware ubiquitous learning. Through a process of anticipation, enactment, evaluation, and revision, the local instruction theory was developed to include a theoretical contribution of how students come to understand angle and angle measure using context-aware ubiquitous. A set of instructional activities was also developed as an embodiment of that theory. The findings from clinical interviews indicate that context-aware ubiquitous learning is a valuable mathematical context for introducing students to angle and angle measure.

1 INTRODUCTION

Geometry is a complex subject incorporating many challenging mathematical concepts. Angle concepts are particularly difficult for students of elementary age to grasp (Battista, 2007; Clements, 2004; Clements and Battista, 1992; Lindquist and Kouba, 1989; Piaget and Inhelder, 1948/1967). Understanding angle concepts requires the apperception of the physical properties of angle, including the static (configurational) and dynamic (moving) aspects (Scally, 1986). Many teaching approaches and resources are not always effective, such as prototype diagrams that can lead students to consider non-relevant attributes (Battista, 2009; Clements and Battista, 1992). Furthermore, angle measure requires students to consider measure as the relationship between two components (rays) in a dynamic turn, which is different than the linear measure they have typically encountered (Clements and Sarama, 2009).

Despite the difficulties many children may encounter when learning about angle and angle measure, elementary students display many skills towards this understanding, and Clements and Sarama (2009) suggested that these skills should be fostered and angle concepts need to be taught within the elementary years. Researchers (viz., Browning and Garza-Kling, 2009; Clements and Burns, 2000; Fyhn, 2007; Lehrer, Jenkins and Osana, 1998; Mitchelmore, 1998; Mitchelmore and White, 2000) have explored various pedagogical strategies to provide opportunities for students to develop an understanding of angle and angle measure. Two recurring trends emerged from the research; the use of real-world connections and the use of technology as supportive pedagogical components to promote students' understanding of angle concepts.

Mathematicians and governments have advocated for connections to mathematics in the real world (viz., Bartolini-Bussi, Taimina and Isoda, 2010; Gainsburg, 2008; Hiebert and Carpenter, 1992; NCTM, 2000; National Research Council, 1990). There have been a number of researchers who have reported positive results from using Dynamic Geometry Environments (DGEs) to support the understanding of angle and angle measure (e.g. Noss and Hoyles, 1996; Sarama and Clements, 2002: Zbiek, Heid, Blume and Dick, 2007). Context-aware ubiquitous learning (context-aware u-learning; Hwang, Wu and Chen, 2007; Yang, 2006) is a sub category of mobile learning that refers to mobile technologies being utilised while connecting with real world phenomenon.

The purpose of this research was to use design-based research to develop a local instruction theory for students' learning about angle. The local instruction theory consists of a learning process and a means for supporting that process. The learning process is an empirically-based instruction theory of how students come to understand angle, and to support that process a set of exemplary instructional materials were devised to be an embodiment of that instructional theory.

2 THEORETICAL FRAMEWORK

How Children Come to Understand Angle and Angle Measure

In this study, the van Hiele model (van Hiele, 1957/1984) of geometric thinking was explored in relation to how students come to understand angle and angle measure. The van Hiele model highlights students' development through five levels of geometric thought, from gestalt-like unanalyzed viewing, to a highly complex level of thinking. Scally (1990) used the van Hiele model and developed a set of level indicators that focus specifically on angle. The overall descriptions are: First level: In general, the student identifies, characterises, and operates on angles according to their appearance. Second level: In general, the student establishes properties of angles and uses properties to solve problems. Third level: In general, the student formulates and uses definitions, gives informal arguments that order previously discovered properties, and follows and gives deductive arguments. The van Hiele levels adapted by Scally (1990) are utilised in this study.

Support for Learning about Angle and Angle Measure

Real-world connections.

Using real-world connections in mathematics has many recorded benefits, such as enhancing students' understanding of the mathematical concepts (De Lange, 1996; Steen and Forman, 1995), amplifying students' ability to think mathematically outside the classroom (Lehrer and Chazan, 1998), and motivating students to learn about mathematics (National Academy of Sciences, 2003). There have been a number of studies to determine the affordance of teaching angle concepts with real-world connections.

There are those who have used real-world objects; for example Piaget and Inhelder (1948/1967) used tongs, and Mitchelmore and White (2000) used adjustable models of wheels, doors, scissors, and fans. Others used real-life physical situations; for instance, Munier, Devichi and Merle (2008) had students determine angles in a playground experience, Fyhn (2007) used a climbing project for the students to study angles made by body formations during climbing activities, and Clements, Battista, Sarama and Swaminathan (1996) began their study by having students use their experience of body movements to consider angle and help them mathematise their physical experiences.

Battista (2009) lamented that "geometry instruction and curricula generally neglect the process of forming concepts from physical objects and instead focus on using diagrams and objects to represent formal shape concepts" (p. 97). Consequently, students connect irrelevant attributes of the diagram or object to the geometric concept (Clements and Battista, 1992), for example, the orientation or the length of angle rays. Understanding salient criteria needed for judging angles is a common difficulty or misconception students possess. In the study conducted by Munier et al. (2008), the researchers conclude that real-world situations enable students to invalidate the idea that length is an appropriate way to compare angles.

Dynamic geometry environments.

DGEs are a more recent type of computer program credited with supporting students' developing understanding of angle concepts. DGEs can help avoid the common difficulties and misconceptions students have. As the name suggests, it is also a program that provides dynamic images that may assist students in recognising that angle measure is based on a turn. Having the ability to create and manipulate objects assists students in perceiving the angles as geometric entities, rather than just visual objects (Zbiek et al., 2007). Therefore, students are more likely to reflect on the appropriate properties to determine the categorisation of the angles, as they are able to simultaneously take into account the specific and grounded with the abstract and generalised (Clements and Battista, 1994). In other words, DGEs support students in understanding the abstract nature of angles while understanding salient criteria for judging angles. DGEs expand the repertoire of representations available, beyond the prototypical angles often displayed in textbooks (Clements and Battista, 1992; Zbiek et al., 2007).

Mobile learning: Context-aware ubiquitous learning.

The theories and empirical findings surrounding the teaching and learning of angle and angle measure advocate for the use of real-world connections (viz., Bartolini-Bussi et al., 2010) and Dynamic Geometry Environments (DGEs; viz., Zbiek et al., 2007) to support learning. There are scholars (viz., Sarama and Clements, 2009) who have made the connection between the two supports as they describe how designers of mathematical computer programs have sought to mathematise the world by adding real-world referents.

Mobile learning (m-learning) has provided a new phase in the evolution of technology enhanced learning. M-learning is defined as "learning across multiple contexts, through social and content interactions, using personal electronic devices" (Crompton, 2013, p. 4). Scholars have developed a sub category of m-learning that makes a connection between technology and real-world learning; that sub category is referred to as context-aware ubiquitous learning (context-aware u-learning; Lonsdale, Baber, Sharples and Arvanitis, 2004). Context-aware u-learning is a situation in which the student is interacting with a real-world environment while using a mobile technology to support his or her learning.

Dynamic geometry environments are a type of computer program credited with supporting students' developing understanding of angle concepts (Zbiek et al., 2007). Sketchpad Explorer (2012) is a type of dynamic geometry environment that is now available on mobile devices. With this application, specific add-ons allow the students to interact with the real world by taking photographs of physical objects in the environment and then using the dynamic tools within the program to measure the angles. Sketchpad Explorer was utilised as part of the context-aware ubiquitous learning activities used in this study.

Context-aware u-learning has been used in other studies in mathematics, for example, Elisson and Ramberg (2012) used this form of learning to have students learn about volume. However, from an in-depth review of the literature, there have been no studies to date that use context-aware u-learning to have students study angle concepts. This study adds to the scholarly understanding in this area. As another unique addition, the context-aware u-learning used in this study involved the use of a dynamic geometry environment, as well as the real-world context. To ensure transfer occurs, other activities are combined with the contextual activities to ensure that the students are connecting the contextual activities outside the classroom to decontextualise activities in the classroom.

3 METHODS

Participants

Two teaching experiments were carried out, one with each class of fourth grade students from a school in the southeastern United States. Two fourth grade teachers chose to participate in the study, which determined the classes from which students participated. There were 30 students in each class, for a total of 60 student participants in the study. Eight of the 60 students completed the pre and post instruction clinical interviews. The eight students were made up of four randomly selected students from each class.

Design-Based Research Protocol for this Study

The design-based research selected for this study was developed by Gravemeijer and colleagues (Gravemeijer, 1994; Gravemeijer and Cobb, 2006; Gravemeijer and van Eerde, 2009). It was designed to connect directly with mathematics education and has been used in mathematical research methodologies within the K-12 environment (e.g., Markworth, 2010).

The study involved two macro cycles with one teaching experiment occurring in each macro cycle. The teaching experiments consisted of seven days of mini cycles of thought and instructional experiments to serve the development of the local instruction theory. One of the two macro cycles for this study is illustrated in Figure 1. Note the occurrence of the three phases within the macro cycle: (a) the design of instructional materials, (b) classroom-based teaching experiments and mini cycle analysis, and (c) the retrospective analysis of the teaching experiments which informed the next macro cycle.

One day prior to the commencement of the teaching experiment, the clinical interview was administered to the four students from the first class. Next, using the instructional materials, the first teaching experiment was conducted in early fall, for seven consecutive school days. During the teaching experiments, the co-researcher and witness observed and took notes on the classroom instruction, and the instruction was videotaped. Students' work was collected at the end of each day. Also, at the end of the day's instruction, the researcher, co-researcher, and witness met to discuss the lesson. The conversations were audio recorded. Following this meeting, the researcher completed a daily reflection journal.

During each daily mini cycle of the teaching experiment, the researcher utilised the collected data to modify the next day's instruction when necessary. The second teaching experiment took place two weeks after the conclusion of the first teaching experiment. There were two retrospective analyses conducted, one at the conclusion of each macro cycle. The local instruction theory came from the final retrospective analysis.

4 DATA SOURCES

Data Collection and Analysis

A distinct characteristic of design-based research methodology is that the researchers develop deep understanding of the phenomenon while the research is in progress. For that purpose, it is crucial that the research team generated a comprehensive record of the entire process (Cobb, Confrey, diSessa, Lehrer and Schauble, 2003). There were several sources of data that were used in this design-based research process. These data sources are; a) pre and post instruction clinical interviews, b) co-researcher and witness classroom observations, c) whole class video recording, d) daily mini cycle reflection audio-recording with research team, e) artifact collection of student classwork, f) researcher's daily reflection journal, and g) retrospective analysis at the end of a macro cycle.

These data sources were utilised during both the daily mini cycle analysis and the retrospective analysis phases at the end of each macro cycle. The data from the final retrospective analysis was used to create a more robust local instructional theory. Figure 1 indicates when each of these data were collected using the diagrammatic representation of the study.

Pre and post instruction clinical interviews.

The pre and post clinical interviews were conducted using an instrument developed by Scally (1990) based on the first three levels of the van Hiele's model of geometric thinking (van Hiele, 1957/1984). The pre instruction interview was administered to the four selected participants one day before the teaching experiment began, and the post instruction interview administered one day following the conclusion of the teaching experiment. The interviews lasted for approximately 30 minutes, although there were no temporal restraints on this procedure.

Co-researcher and witness classroom observations.

While the researcher was conducting the teaching experiment, the respective classroom teacher and co-researcher acted as witnesses to the process. They observed the class and took notes during each of the teaching experiments. The observation notes were collected at the end of each day by the researcher.

Whole class and small group video.

Each teaching episode was video recorded to capture both the instruction and student participation. The transcripts were coded using Scally's (1990) van Hiele level indicators.

Daily mini cycle reflection.

Following each of the seven teaching episodes, the researcher, co-researcher, and teacher meet to discuss the instructional activities of that day and student progress in understanding the angle concepts taught.

Artifact collection.

Hard copies of students' work were collected at the end of each teaching episode. In addition, screen captures were taken of students' work on the iPads and downloaded at the end of each day. The students work was coded using Scally's (1990) van Hiele level indicators.

Researcher reflection journal.

The researcher completed a personal reflection journal for each of the teaching episodes during each mini cycle. The researcher reflection journal completed during each mini cycle was a catalyst for change during the teaching experiment and the retrospective analysis.

Retrospective analysis.

During this study, there were two retrospective analyses, one after each teaching experiment. The data from the first retrospective analysis was used for the next macro cycle, and the data from the final retrospective analysis was used to create a more robust local instructional theory.

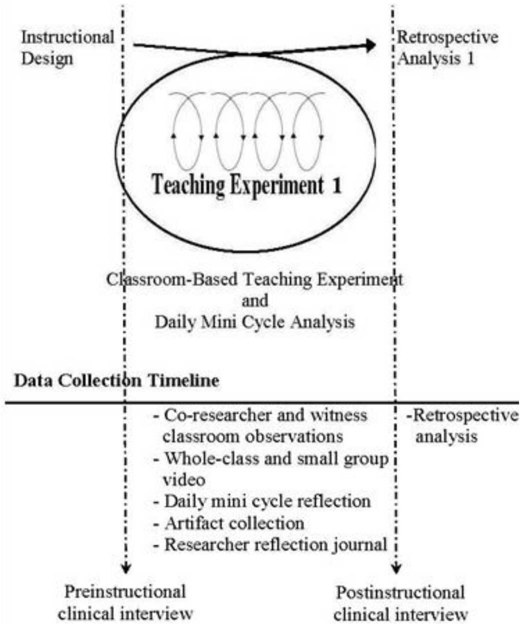

Figure 1 A Diagrammatic Representation of the Study with Points of Data Collection. This Representation Includes One of the Two Macro Cycles in this Study.

5 RESULTS AND CONCLUSIONS

The purpose of this research was to use design-based research to develop a local instruction theory for students' learning about angle. The local instruction theory consists of a learning process and a means for supporting that process. The learning process is an empirically-based instruction theory of how students come to understand angle, and to support that process a set of exemplary instructional materials was devised to be an embodiment of that instruction theory. The discussion of the exemplary instructional materials includes changes to *Measure a Picture*, the mobile application used as well as the lesson plans.

Levels of Geometric Thinking

Findings about students understanding of angle and angle measure in relation to the three van Hiele levels of thinking are now presented along with a discussion on angle and angle measure as applicable. The three levels are followed by the findings of the pre and post instruction interviews for the two macro cycles. These discussions are connected to the six context-aware ubiquitous lessons that are the embodiment of the instructional theory reported in the final section.

Level one: Visual level of geometric thinking.

The objectives for Lessons One and Two were developed to have the students move to working at level two; they were asked to focus on angle properties rather than attending to the visual appearance.

Summary of Lessons One and Two and student responses.

In Lesson One, students were introduced to a set of angles and were required to determine whether the angles are alike or different. Students then went out into the area surrounding the school to identify angles in the real-world setting. The technologies were not introduced in the initial lesson as it was important to not over load students as they learned about a new mathematics concept at the same time they learned a new technology. In Lesson Two, students explored the use of a Dynamic Geometry Environment (DGE) and then used this program to identify angles in the real world using screenshots from Lesson One. Possible angles were discussed with a partner. The lesson was summarised with the students' screenshots shared in class and a discussion about how the students identified angles.

At the beginning of the first lesson, students were given a sheet of angles, asked to work in pairs to study the figures, and asked to answer two questions stated verbally: 1) What can you tell me about these figures from what you have noticed?; and 2) What do all these figures have in common? Data was triangulated from the video and observer comments from teaching experiment one, these data suggest that approximately two thirds of the students in the class described the important attributes of angles to their partners.

In Lesson Two, the students used the Dynamic Geometry Environment (DGE) Measure a Picture (Steketee and Crompton, 2012), the add-on program of Sketchpad Explorer (2012). They used this program with iPad mobile devices to photograph angles they identified in their playground environment. In teaching experiment one, as students went out to find angles in the playground, video evidence, observation notes and students' work show that many of the students gravitated towards natural artifacts to find angles in places such as trees. The students would often find an artifact visually resembling an angle, but if students considered the attributes of angle, such as two straight lines, they would determine that it was not always an angle. For example, in Figure 2 Claire found angle like shapes on a tree stump and marked those as angles with the dynamic protractor. Under the protractor, the lines are distinctly bent and distorted on the natural curves of the wood.

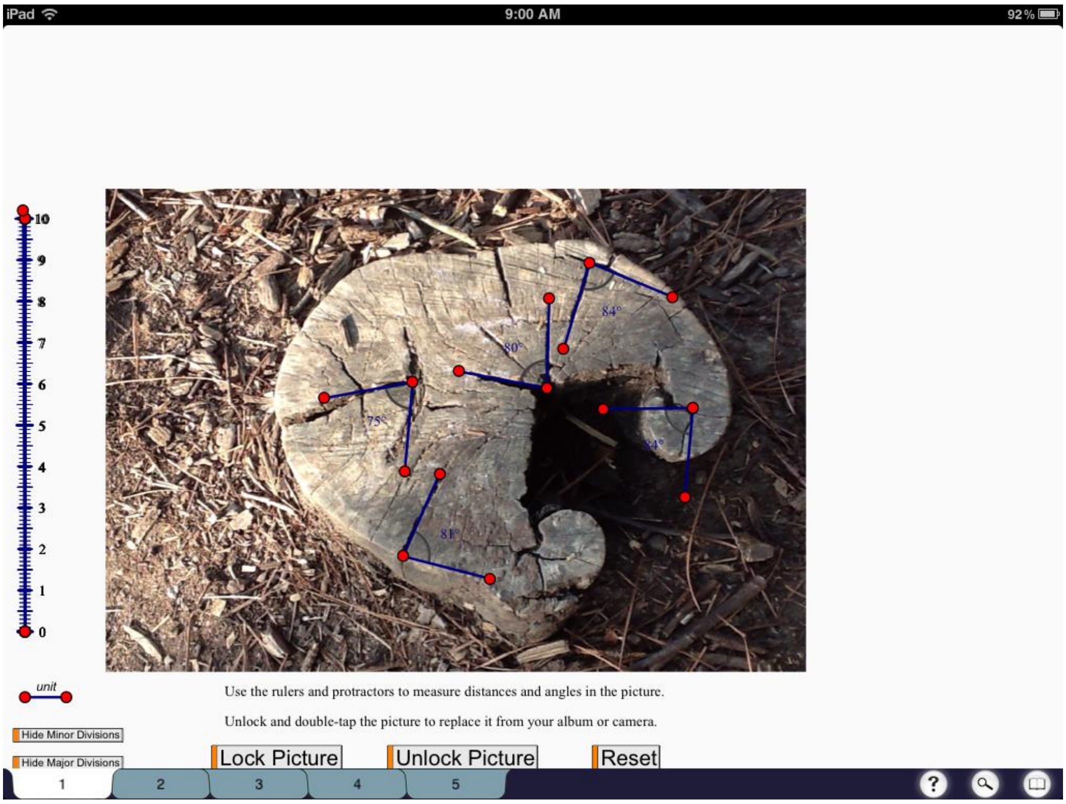

Figure 2 Student Found *Angle Like* Shapes in the Tree Stump

Using mobile devices are beneficial in that they can be taken out into the real-world to have students learn while also having the availability to use the tools, such as the dynamic protractor available. However, this greatly increases the information that students have to cognitively process. Claire was identifying angles based on the visual appearance, searching for shapes that look like angles and was not identifying angles by the properties of angles. While she is actively looking for angles in the real-world, Claire is working within the visualisation level of geometric thinking.

In light of this issue and before the second teaching experiment, the instructional materials were altered to include the instructor conducting a brief class discussion about the best places to look for angles based on salient angle properties. This discussion focused primarily on the point that straight lines are more likely to be found on manufactured artifacts than those found in nature. This discussion was included to encourage students to work towards the analysis level of geometric thinking as they had to consider the properties rather than the gestalt appearance.

During this activity, students were required to take screenshots of the angles they found in both teaching experiment one and teaching experiment two. The screenshots were coded for those pictures that were (actually) angles or were (actually) non-angles. Students often identified more than one angle in the screenshot, although there were no more than five potential angles identified on a screenshot. From the observations and the mini cycle reflections it was evident that the use of the application on the iPads was providing a way for students to mathematize the real world. Instead of students looking through a text book to find individual instances of angles in traditional formats, the students were using the technology to see that there were angles in multiple forms even in one photograph taken with the application. For each angle identified a code was given (i.e., example of angle or not an angle). This was completed for both teaching experiments and the results are presented in Table 1.

	Teaching Experiment 1 (*n* = 30)	Teaching Experiment 2 (*n* = 30)
Angle	26 (28%)	55 (87%)
Non-Angle	68 (72%)	8 (13%)

Table 1 Real-World Angle Identification

Note. There were 30 students in each class; however, each student may have identified between one and five angles on each screenshot.

In teaching experiment one, 30 students took screenshots of angles and identified them using the dynamic protractor. Of the 94 potential angles found by the students, 28% were examples of angles with 72% not being examples of angles, i.e., non-angles, as they did not have the relevant attributes required to be an angle. In experiment two, 30 students took screenshots of angles and identified them using the dynamic protractor. Of the 63 potential angles identified by the students, 87% were examples of angles and 13% were not examples of angles, i.e., non-angles. This was evidence that there was a change between the two teaching experiments in students' ability to identify angles in real-world contexts.

It would appear from the findings summarised on Table 1 that after a discussion about finding man made angles was implemented in teaching experiment two this was helpful as fewer non-angles were identified than in teaching experiment one. However, even in teaching experiment two some students were still working at level one at the end of Lesson Two. For example, Matthew believed that he had found an angle in Figure 3.

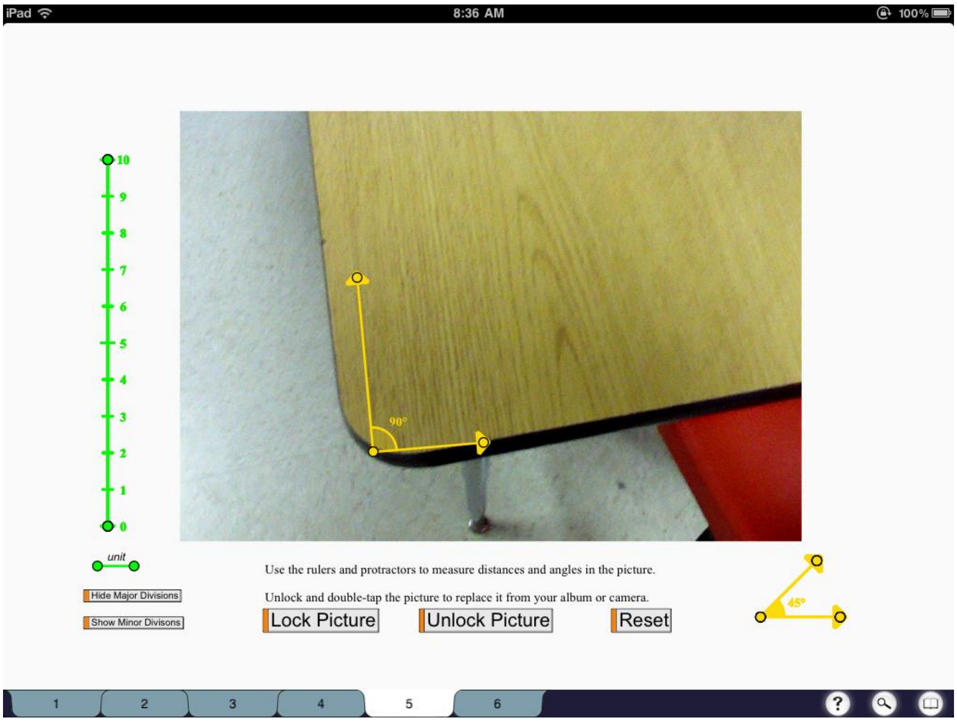

Figure 3 Searching for Real-World Angles

This is an extract from a conversation following Matthew's potential angle find.

Teacher: In your screenshot where is the angle Matthew?

Matthew: There (Pointing to the angle indicated on the screenshot).

Teacher: How do you know that is an angle?

Matthew: This is the corner of the table and angles are corners.

In the van Hiele level indicators for the visualisation level, one of those indicators describes the way that a student can exclude relevant angle properties. As Matthew chose this potential angle, he has failed to consider relevant angle attributes, i.e., that the two lines need to be straight lines and that the two lines should meet at one end point.

Level two: Analysis level of geometric thinking.

In the sequence of six lessons, it was conjectured that the students would be working at level two during Lessons Three and Four and begin moving into level three during Lesson Five.

Summary of Lessons Three and Four and student responses.

The objective of Lesson Three was for students to recognize acute, obtuse, right and straight angles in different contexts (viz., real-world and paper and pencil).

Level one thinking beyond the first two lessons.

The objective of Lesson Three was to recognize and compare angles based on size using non-standard and standard language (acute, obtuse and right angle). The students made triangles using wooden coffee stirrers cut to different lengths. Then, working in groups, the students sorted those angles into similar groups. The students had to determine their own groups using what they had learned about salient and non-salient angle attributes.

Triangulating the data by using the video and the video transcripts coded using Scally's van Hiele level indicators, as well as observer notes, these data show that four-fifths of the students in teaching experiment one class were moving into level two. However, the other one-fifth, represented as two groups of three students, was working at the visualisation level. Although students appeared to be able to find angles with different ray lengths in the real-world with the iPad's, when students were asked to transfer this knowledge to wooden sticks many of the students went back to thinking that the length of the sticks (the ray length) determined the size of the angles. This finding led to a modification to the add-on program Measure a Picture. In the initial program, the dynamic protractor did not have adjustable ray lengths. The rays appeared more like line segments with another end point. Modifications were made for the ray to have an arrow and for the length to be adjustable, see Figure 4. In addition, the colour of the rays was changed to make the protractor more visible on photographs.

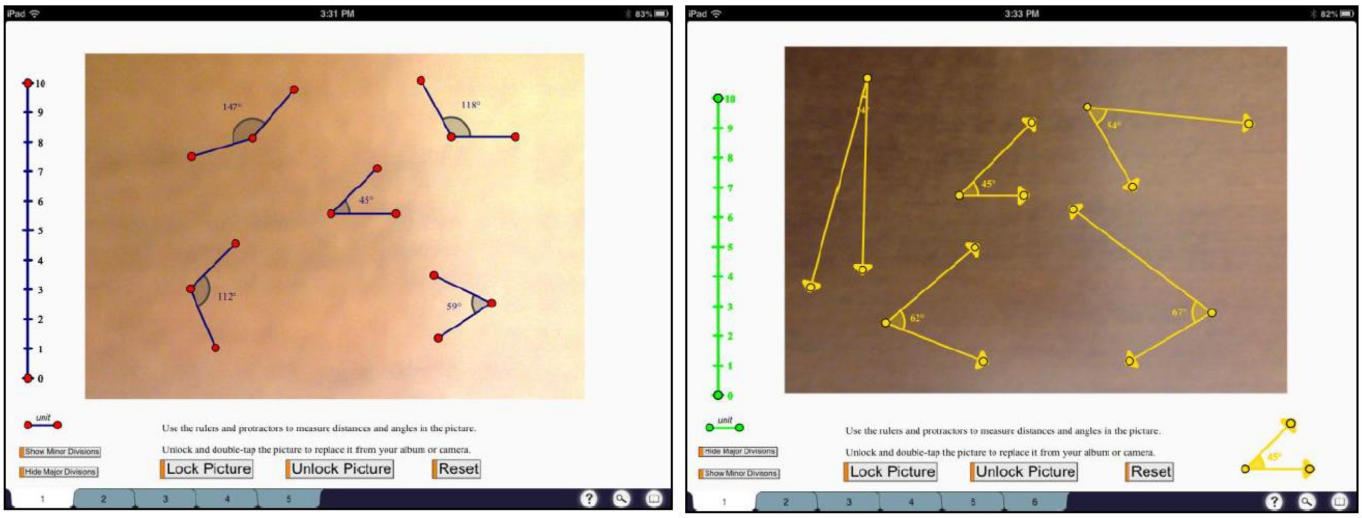

Figure 4 Modifications to Measure a Picture

Level two thinking in Lessons Three and Four.

From the angle sorting activity, using data from the student work artifacts, video evidence, and observation notes it appears that students in teaching experiment two were analysing and comparing angles in terms of their properties and were able to formulate and use generalisations about properties of angles in problem solving situations. This is congruent with the van Hiele level two indicators for thinking about angles.

The changes to the DGE program appear to have also supported students earlier in the instructional sequence. During Lesson Two, as the students in teaching experiment two found angles using the modified program, from the video evidence and observational notes it appears that students were focused on salient angle attributes with 87% of the angles found by students in teaching experiment two correctly identified in comparison to the 28% correctly found by the students in teaching experiment one, see Table 1. In addition, students often made the rays of different lengths to point out that the length of the rays were non-salient attributes. For example, Catrin took this screenshot of angles, see Figure 5, and the following discussion ensued.

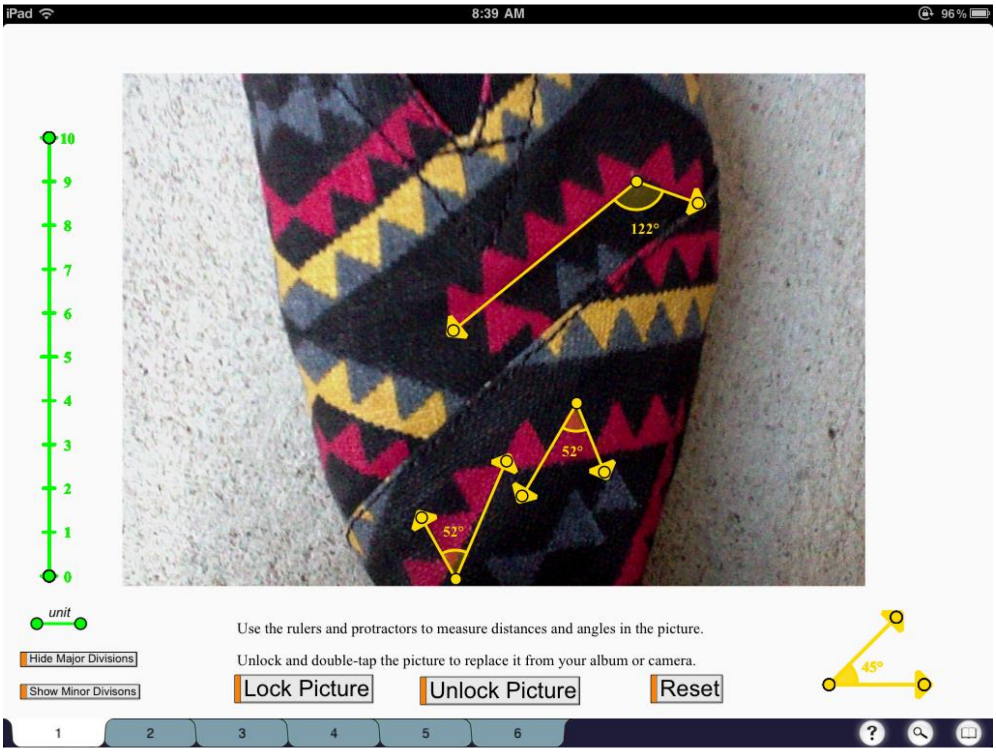

Figure 5 Rays are a Non-salient Angle Attribute

Teacher: I notice that the rays are of different lengths.
Catrin: Because, that does not matter. I have put the rays against where I see the angles, like there (pointing to the top angle), that is only short and that is long, but it does not make a difference to the angle size as it is not measuring the length of the lines.

Catrin's screenshot and response is indicative of a student working within the second level of geometric thinking as she has analysed the angles based on their properties rather than the gestalt appearance.

Level three: Informal deduction level of geometric thinking.

In the sequence of six lessons, it was conjectured that the students would begin working at level three during Lesson Five and Six.

Summary of lessons five and six and student responses.

The objectives for Lesson Five required students to understand that angles can be measured with reference to a circle and that angles are fractions of a circle. The lesson used an adapted version of Browning, Garza-Kling, and Hill-Sundling (2007) and Millsaps' (2012) wedge activity. The students used a folded paper circle to create a wedge to measure various angles on paper and real-world objects. The objectives for Lesson Six required the students to recognize that angle size can appear different based on different visual perspectives. The activity for this objective was to have the students taking photographs of angles from various positions. The photographs were taken within the DGE and students then use the tools to measure the angles and discuss their findings.

Level two thinking during Lessons Five and Six.

During Lesson Five students had to complete a worksheet during which they had to estimate the size of nine angles and categorised the angles as acute, obtuse, right and straight angles. All 12 students from teaching experiment two got all nine answers correct, which was double the amount in teaching experiment one.

One of the changes made to the measurement activity was to provide the name reflex angle to students when asked. Observational notes show that during teaching experiment one and teaching experiment two students asked what the name of this category was as they began to consider a full turn as 360°. Students understood 1-89° was an acute angle, 90° a right angle, 91-179° an obtuse angle and 180° a straight angle. As the dynamic protractor continued beyond 180° students asked the name of this other category. This change was not based on student's achievement, but on the basis of just-in-time learning, that the students had identified that a category was missing from their understanding and they wanted to know the answer to fill this gap in their learning.

Evidence of level three thinking in Lessons Five and Six.

Triangulated data, gathered from the video recording, classroom observations and collectively the daily mini cycle reflections did not highlight any issues with Lessons Five and Six. In teaching experiment two, the video and observation data show that students were typically working within van Hiele level two as the students often demonstrated the ability to list the salient properties of angle.

Interviews

Macro cycle one.

The four students interviewed in teaching experiment one began working between the visual and the analysis level for drawing, identifying, and sorting angles. For angle measure and relations the students were working within the visual level. For the post instruction interviews, the four students in teaching experiment one improved and moved from the visual to the analysis level. The pre and post instructional scores can be seen in Table 2. The majority of the students were working fully within the analysis level (level two) at the end of the macro cycle.

	Pre Instruction					Post Instruction				
	V	VA	A	AI	I	V	VA	A	AI	I
Draws Angles		4						4		
Identifies Angle		4					1	3		
Sorts Angle		4						4		
Angle Measure	4						1	3		
Angle Relations	4							4		

Table 2 Teaching Experiment One: Pre and Post Instruction Interview Summary

Note. V indicates that those students are working at the visual level; A indicates that those students are working at the analysis level, and I indicates that those students are working at the informal deduction level. Two letters indicate that those students are working between two levels. Dominance in one level is not denoted on this table. The numbers represent the students working at that level. Table adapted from "The impact of experience in a Logo learning environment on adolescents' understanding of angle: a van Hiele-based clinical assessment," by S. P. Scally, 1990, Unpublished doctoral dissertation, Emory University, Atlanta, Georgia.

Macro cycle two.

Students in teaching experiment two predominantly scored within the visual level in the pre instruction interview with some students working partially between the visual and analysis level. One student was working in the analysis level for sorting angles during the pre instruction interview. For the post instruction interview, the majority of the students moved into the analysis level of geometric thinking, however, for drawing angles and angle relations three of the four students were working between the analysis level of thinking and the informal deduction level. These pre and post instructional scores can be seen in Table 3.

	Pre Instruction					Post Instruction				
	V	VA	A	AI	I	V	VA	A	AI	I
Draws Angles	3	1						1	3	
Identifies Angle	1	3						4		
Sorts Angle		3	1					4		
Angle Measure	4						1	3		
Angle Relations	3	1					1		3	

Table 3 Teaching Experiment Two: Pre and Post Instruction Interview Summary

Note. V indicates that those students are working at the visual level; A indicates that those students are working at the analysis level, and I indicates that those students are working at the informal deduction level. Two letters indicate that those students are working between two levels. Dominance in one level is not denoted on this table. The numbers represent the students working at that level. Table adapted from "The impact of experience in a Logo learning environment on adolescents' understanding of angle: a van Hiele-based clinical assessment," by S. P. Scally, 1990, Unpublished doctoral dissertation, Emory University, Atlanta, Georgia.

In the post instruction interview, these data show that students were still lacking in certain understandings, specifically that:

- Angle is developed by a turn and angles are measured by the degree of that turn.
- Benchmark measures can assist students in estimating the measure of an angle.
- Practice in spatial reasoning is needed to gain these skills.

Changes were made to the instructional plans to have students label the benchmark to support students in internalizing these benchmark measures. Further discussion on angle as a turn were included using the dynamic protractor to support this understanding. For the spatial reasoning difficulties, students will need ongoing practice and this will need to be considered a skill to be practiced by students on a regular basis. As spatial reasoning is not a mathematical skill pertinent to angle and angle measure, changes were not addressed in the instructional sequence.

Exemplary Instructional Materials

Using a cyclical iterative process of anticipation, enactment, evaluation, and revision (Gravemeijer and van Eerde, 2009), a final set of activities were developed. Due to space considerations, the full set of activities can be found here http://bit.ly/SHJpBE . Researchers found that students learning about mathematical concepts using technology were often not able to transfer the knowledge from the technology to paper and pencil representations (Clements et al., 1996). To ensure that the students can transfer the information from the context-aware ubiquitous learning activities to angles drawn on paper, the lessons include a mix of contextualized and complementing decontextualized activities.

The pre and post interviews show a positive improvement in the small study group. Specifically, in the first teaching experiment, during the pre interview the students interviewed were working primarily within the visual level of the van Hiele levels of geometric thought with some movement into the analysis level. For the post interview, the majority of the students were working well within the analysis level. In the second teaching experiment, during the pre interview the majority were working within the visual level with only a few showing indications of working towards the analysis level. For the post interview, the majority of the students were working in the analysis level for all angle understandings and students were also provided evidence of working towards the informal deduction level.

The findings indicate that context-aware u-learning is a valuable mathematical context for introducing students to angle and angle measure. From these data, it also appeared that common misconceptions about angle can be avoided. For example, as the students studied angles in the real world they were presented with angles with rays of different lengths and in various orientations, this avoided the misconception that these were salient attributes of angles. Furthermore, the dynamic geometry environment enabled the students to measure angles they had photographed; this provided them with additional information about the angle without having to move from the real-world setting. The extendable rays also avoided the misconception that the length of the rays made a difference to the size of the angle and the movement of the dynamic protractor supported students in thinking about angle as a turn rather than a static shape.

Scientific and Scholarly Significance

This study is significant as it appears at a time when mathematics teachers are being required to reassess their mathematical practices with the implementation of the Common Core State Standards in North America and other similar standards across the world. Furthermore, the promise and potential of using mobile devices is now rapidly becoming apparent and there is widespread interest amongst parents, students, principals, and teachers. One significant challenge to this implementation is the lack of teacher training and knowledge on how to successfully implement such technological tools. This study provides a list of core understandings for learning about angle and angle measure, plus a set of exemplary instructional materials that utilize context-aware u-learning for learning about these concepts that can be adapted for use in other fourth grade classrooms. Curriculum designers can also use these materials to develop other technology enhanced environments using context-aware u-learning.

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