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Higher-Order Effects in Biaxial Flexure of GFRP I-Section Beams

Zia Razzaq and Faridoon Z. Razzaq

***Abstract* — A theoretical study of Glass Fiber Reinforced Polymer (GFRP) beams subjected to biaxial bending moments is presented with a focus on the influence of higher-order effects on maximum normal stresses. It is shown that the biaxial bending type of loading causes a dramatic increase in the maximum normal stress for a GFRP beam when induced torsional effects are included. The study demonstrates that the traditional first-order theory can grossly underestimate the maximum normal stress in a GFRP beam. Based on the numerical results presented using a higher-order theory which also accounts for induced warping normal stresses, the maximum normal stress is found to be about two to three times larger than that determined using the first-order theory.**

***Keywords* — Biaxial Flexure, Glass Fiber Reinforced Polymer (GFRP), Higher-Order Effects, I-Section Beams.**

I. INTRODUCTION

Glass Fiber Reinforced Polymer (GFRP) I-section beams can be subjected to simultaneous bending about both major and minor cross-sectional axes. The traditional approach to finding the maximum normal stress in such a beam is to superimpose the first-order normal stress values about the major and minor axes while neglecting higher-order effects such as those due to induced torsional deformations that cause significant warping deformations. The Pre-Standard for Load and Resistance Factor Design (LRFD) of Pultruded Fiber Reinforced Polymer (FRP) structures by American Society of Civil Engineers [1] neglects induced torsional effects for biaxial bending. Razzaq and Sirjani [2], [3] have previously investigated the elastic behavior of unsymmetrically or biaxially loaded FRP angle sections. Gyebi and Razzaq [4] have also presented a theoretical and experimental study of angle section members under biaxial bending. Sirjani and Razzaq [5] have also studied the behavior of FRP I-section members when both biaxial bending and externally applied torsion are present. Galambos [6] explained the importance of induced torsional effects in biaxial bending of steel beams. Knorowski *et al.* [7] investigated the lateral-torsional instability and biaxial bending of FRP I-beams. The present paper is based on the outcome of a theoretical investigation of the significance of higher-order effects on the maximum normal stress in a GFRP I-section beam with associated warping stresses due to induced torsional deformations.

II. PROBLEM STATEMENT

Fig. 1 shows the schematic of a Glass Fiber Reinforced Polymer (GFRP) I-section beam subjected to biaxial bending moments M_{ox} and M_{oy} about the major and minor axis, respectively. The longitudinal centroidal axis of the beam is represented by z as shown in Fig. 1 while x and y are the cross-sectional major and minor axes as indicated in Fig. 2. When the beam is acted upon by M_{ox} and M_{oy} at the supports, a given cross section at any location z experiences displacements u and v in the x and y directions, respectively, in addition to an angle of twist Ø as shown in Fig. 2. The induced twist is owing to the higher-order effects while also giving rise to warping normal stresses. The problem addressed herein is to appraise the influence of these effects on the total maximum normal stress in a GFRP beam.

III. THEORETICAL BASIS

The three simultaneous flexural and induced torsional differential equations in Reference 6 for steel beams when modified for GFRP beams take the form given in (1)-(3).

$$E_L I_x v'' + M_{oy}\phi = -M_{ox} \tag{1}$$

$$E_L I_y u'' + M_{ox}\phi = -M_{oy} \tag{2}$$

$$E_L I_w \phi''' - G_{12} K_T \phi' + M_{ox} u' + M_{oy} v' = 0 \tag{3}$$

In (1), (2), and (3), u and v represent displacements along the x and y axes, and ϕ is the induced angle of twist due to higher-order effects as shown in Fig. 2. The primes represent differentiation relative to z. Furthermore, the terms $E_L I_x$ and $E_L I_y$ represent the cross-sectional flexural rigidities about the x and y axes, respectively, whereas $E_L I_w$ and $G_{12} K_T$ are the cross-sectional warping and St. Venant torsional rigidities, respectively.

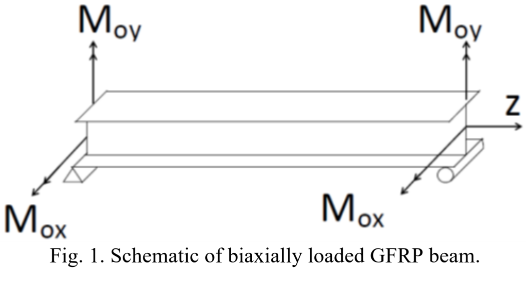

Fig. 1. Schematic of biaxially loaded GFRP beam.

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By differentiating (3) once and substituting v'' and u'' from (1) and (3), one can obtain the following fourth-order differential equation:

$$\phi'''' - \lambda_1\phi'' - \lambda_2\phi = \lambda_3 \qquad (4)$$

In (4), λ_1, λ_2, and λ_3 are functions of M_{ox} , M_{oy} and both material and cross-sectional properties of the GFRP beam. The solution to (4) is given in (5).

$$\phi = C_1\sinh(k_1 z) + C_2\cosh(k_1 z) + C_3\sin(k_2 z) + C_4\cos(k_2 z) - \lambda_3/\lambda_2 \qquad (5)$$

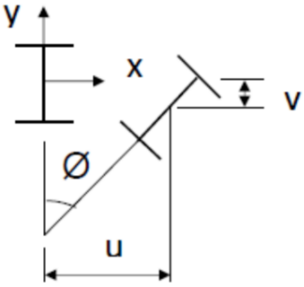

Fig. 2. GFRP beam section and deflections.

In (5), the constants of integration C_1 through C_4 can be found using the torsionally pinned boundary conditions, and k_1 and k_2 are functions of λ_1 and λ_2. Including the higher-order effects accounted for by the induced angle of twist ϕ, the absolute maximum value of the normal flange tip stress occurs at the beam midspan, that is, at z = 0.5L and is given by (6).

$$\sigma_{max} = \sigma_{bx} + \sigma_{by} + \sigma_w \qquad (6)$$

In (6), the right-side terms are defined as (7), (8) and (9).

$$\sigma_{bx} = \left[M_{ox} + |\phi|\, M_{oy}\right]/S_x \qquad (7)$$

$$\sigma_{by} = -\left[M_{oy} - |\phi|\, M_{ox}\right]/S_y \qquad (8)$$

$$\sigma_w = E_L w_n\, |\phi''| \qquad (9)$$

The absolute signs used in (7)-(9) ensure that (6) provides the maximum normal tensile stress at one of the flange tips. Also, S_x and S_y are the section moduli about the x and y axes, respectively, and w_n is the normalized unit warping which is maximum at the cross-sectional flange tips [6].

IV. NUMERICAL RESULTS

Based on the equations presented in the previous section, and for a $M_{oy}/\ M_{ox}$ ratio of -0.10, Table I presents the outcome of a numerical study conducted on a GFRP I-section beam with the following geometric and material properties: beam length = 120 in; section depth = 4.0 in; flange width = 2.0 in., flange thickness = web thickness = 0.25 in.; E_L = 3000 ksi; and G_{12} = 400 ksi. In Table I, σ_{max1} is based on the first-order theory, that is, with ϕ and ϕ'' assumed to be equal to zero in (7)-(9), and σ_{max2} is calculated with the higher-order theory. The last column in Table I shows that the higher-order theory results in 1.88 times to almost three times the stress estimated using the first-order theory. Fig. 3 shows the nonlinear nature of σ_{max} versus M_{ox} relationship corresponding to M_{oy}/M_{ox} = - 0.10 when the higher-order theory is used as compared with the first-order linear theory.

TABLE I: RESULTS WITH M_{oy}/M_{ox} = - 0.10

Mox (kip-in.)	**σmax1** (ksi)	**σmax2** (ksi)	**σmax2/ σmax1**
0.0	0.0	0.0	-
5.0	3.74	8.28	2.21
7.5	5.62	10.11	1.79
10.0	7.49	13.61	1.82
11.0	8.24	15.52	1.88
12.5	9.36	19.83	2.12
14.0	10.48	31.18	2.97

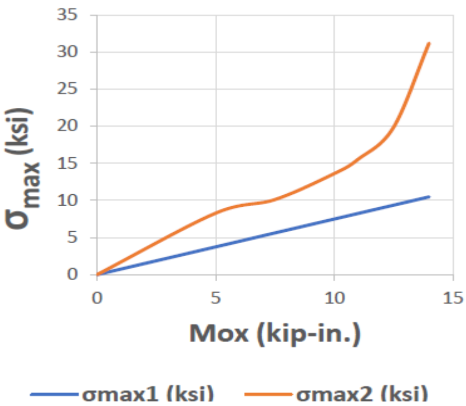

Fig. 3. σ_{max} versus M_{ox} with M_{oy}/M_{ox} = - 0.10.

V. CONCLUSIONS

The study presented shows that biaxially loaded GFRP beams develop dramatically higher maximum normal stresses when higher-order effects are included, that is, when the induced torsional effects causing warping deformations are accounted for. Conversely, the first-order theory grossly underestimates the maximum normal stresses.

CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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