2014

A Heuristic for the Analysis of Truncated Standard Normal Distribution Assemblies

John Walter Ralls

Cesar Ariel Pinto
Old Dominion University

Shannon R. Bowling

Resit Unal
Old Dominion University

Charles B. Daniels
Old Dominion University

See next page for additional authors

Original Publication Citation
Ralls, J. W., Pinto, C. A., Bowling, S. R., Unal, R., & Daniels, C. B. (2014). A heuristic for the analysis of truncated standard normal distribution assemblies. In S. Long, E.H. Ng, & C. Downing (Eds.), *35th International Annual Conference of the American Society for Engineering Management - Entrepreneurship Engineering: Harnessing Innovation, ASEM 2014, Virginia Beach, Virginia, United States, 15-18 October 2014.* (pp. 658-665). American Society for Engineering Management.

Proceedings of the American Society for Engineering Management 2014 International Annual Conference
S. Long, E-H. Ng, and C. Downing eds.

A HEURISTIC FOR THE ANALYSIS OF TRUNCATED STANDARD NORMAL DISTRIBUTION ASSEMBLIES

John Walter Ralls
Professional Engineer
Email: rallsjw@yahoo.com

C. Ariel Pinto
Associate Professor
Old Dominion University, Norfolk, VA 23529, Email: cpinto@odu.edu

Shannon R. Bowling
Dean, School of Engineering Technology and Computer Science
Bluefield State College, Bluefield, WV 24701, Email: sbowling@bluefieldstate.edu

Resit Unal
Professor
Old Dominion University, Norfolk, VA 23529, Email: runal@odu.edu

Charles B. Daniels
Lecturer
Old Dominion University, Norfolk, VA 23529, Email: cbdaniel@odu.edu

---

Abstract
The analysis of data sets and process conditions commonly assume the use of standard normal distributions and truncated standard normal distributions (TSND). The use of these distributions has application to various engineering disciplines along with numerous other industries (e.g., financial industries, medical fields, management, etc.). For engineering managers, the use of truncated standard normal distributions has particular relevance when evaluating process conditions commonly associated with assembly tolerances, manufacturing, and associated measures of quality.

This article summarizes a heuristic approach for the analysis of assembly-level truncated standard normal distributions and associated research from a recent dissertation (Ralls, 2014). This article provides a cursory review of the literature presented by that research, briefly reviews key analysis equations, and provides a heuristic procedure from that research. The approach presented summarizes TSND assembly analysis utilizing a distributions characteristic function and an inversion factor for a single doubly truncated standard normal distribution is also reviewed. Applications, research recommendations, and future investigations for engineering managers in the following areas of truncated distribution analysis are proposed: heuristic improvement, distributions expansion, simulation expansion, and further application to storage and part allocations.

Keywords
Truncated, Doubly Truncated, Standard Normal Distribution, Heuristic, Assembly, Assemblies, Engineering Management, Manufacturing

Introduction
Engineering managers, manufacturing engineers, quality practitioners, risk managers, and numerous other industries deal with the relationships associated with assemblies. These relationships could include the assembly of machined parts having an upper and lower specification, part binning philosophies, process specification

tolerances, and numerous other applications that deal with the assessment of assemblies, particularly truncated standard normal distributions.

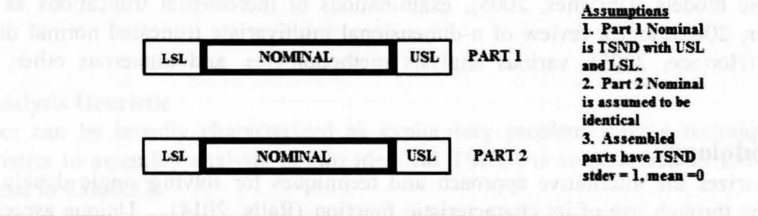
Assemblies may have numerous parts; however, the subassembly portions can generally be simplified and reduced to a manageable size. In its simplest form an assembly consists of at least two parts. This article summarizes a heuristic approach for the analysis of part assemblies through the use of a baselined inversion factor and the unique properties of a distributions characteristic function (Ralls, 2014). As a result, decision makers can utilize this heuristic as a tool to aid in analyzing assembled truncated standard normal distributions for two TSNDs (e.g., two parts). Furthermore, this article has application to part assembly, part binning strategy analysis, manufacturing specification limits, and numerous other engineering applications dealing with assembly analysis.

This article provides engineering managers and other practitioners with an initial framework to perform an analysis of truncated assemblies. Specifically, the framework identified in this article summarizes the heuristic approach presented by Ralls (2014). While this article focuses on heuristics associated with identical truncated standard normal distributions, future research implications and applications could be adapted to other distributions with additional investigation and research. Applications of its use are also discussed in limited context.

Background

The analysis of assembly-level truncated standard normal distributions presented by this article summarizes the research presented in recent dissertation (Ralls, 2014). This dissertation examined a compendium of scholarly works that included selective assembly methods, part binning philosophies, heuristics, and various works directly related to truncated standard normal distributions. "While an assembly may have numerous parts, the subassembly portions can generally be simplified and reduced to a manageable size. In their simplest form assemblies should be able to be reduced into at least two parts (Ralls, 2014)." A simplified example of a part assembly model is shown in Exhibit 1.

Exhibit 1. Truncation Assembly-level Example (Ralls, 2014)



Selective Assembly is a means by which high-precision assemblies may be fabricated from relatively low precision components (Pugh, 1986). Conference proceedings on selective assembly introduce the idea of partitioning a component population into groups prior to random assembly (Pugh, 1986). Selective assembly works by dividing component distributions into two or more groups, randomly choosing components and limiting their group creation by discarding groups beyond three standard deviations (Pugh, 1986). Further studies in the area of selective assembly identified the use of statistical selective assembly as a means to produce high-precision assemblies from relatively low-precision components (Pugh, 1992). While elements of selective assembly focus on the creation of high-precision assemblies formulations and heuristics associated with assembly analysis of these and other methods were lacking.

TSND assembly analysis and their associated heuristics have application to the area of subassembly partitioning (Ralls, 2014). Authors (Lee and Saitou, 2007) in the area of subassembly partitioning identified a design approach relative to critical dimension adjustment as part of subassembly partitioning. TSND assembly analysis has application to the area of part binning (Ralls, 2014).

Heuristics is defined as follows (Merriam-Webster.com, 2014): "Involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-an-error methods." Heuristics is a very broad knowledge base that aids in effective problem solving and serves as a way to "frame new problems" (Michalewicz & Fogel, 1998). The research and the heuristics summarized (Ralls, 2014) were baselined against analysis methods and equations identified by Equations 1 through 5 (Khasawneh, Bowling, Kaewkuekool, & Cho 2005). Equations 1 through 5 are summarized as follows:

$$f_T(z) = \int_{z_L}^{z} \frac{f(z)}{\left(\int_{z_L}^{z_U} f(z)dz\right)} dz, \; z_L \le z \le z_u \tag{1}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} \tag{2}$$

$$f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} dz \tag{3}$$

$$z = \frac{x-\mu}{\sigma} \tag{4}$$

$$\mu_{T_1}(z) = \int_{z_{L_1}}^{z_{U_1}} z f_{T1}(z) dz \tag{5}$$

Research in the area of truncated distribution analysis was lacking (Ralls, 2014). An alternate method to analyzing a single doubly truncated standard normal distribution using computer models was identified (Johnson and Thomopoulos, undated), however, this method did not address assembly level analysis. As part of the research effort performed during a recent dissertation (Ralls, 2014) an application and exhaustive literature review was performed. Examples of this review included the assessment of moments of truncated distributions in dummy endogenous variable models (Dhrymes, 2005), examinations of incremental truncations as a method of DNA pairing (Ostermeier, 2003), and a review of n-dimensional multivariate truncated normal distributions for one-sided truncations (Horrace, 2005), various analysis methodologies, and numerous other applications within industry.

Analysis Methodology

This article summarizes the alternative approach and techniques for solving single doubly truncated standard normal distributions through use of its characteristic function (Ralls, 2014). Unique aspects of a distributions characteristic function are leveraged as part of the analysis of truncated standard normal distributions assemblies. A "characteristic function φ uniquely determines the measure of μ it comes from (Billingsley, 1995)." As a result, an inversion formula can be used to identify the result of two doubly truncated normal distributions. This article summarizes an evaluation method to compute the result of two assembled identical truncated standard normal distributions through the use of the inversion of the combination of their respective characteristic functions (Ralls, 2014).

Application of a distributions characteristic function to the area of TSND assembly analysis investigations (Ralls, 2014) included a compendium of scholarly works (Shephard, 1992; Kawata, 1969; Bernadic & Candel 2012; and Abate & Whitt, 1991). Unique properties of a distributions characteristic function were essential to the analysis of TSNDs. Specifically, Equation 6 (as adapted) identifies that the sum of two characteristic functions is the product of their respective characteristic functions (Billingsley, 1995).

$$\varphi_{X+Y} = \varphi_x \varphi_y = \varphi_z \tag{6}$$

Given Equation 7 (Srinivasa Varadhan, 2000) and Equation 8 (Abadir and Magdalinos, 2002) the resulting inversion baseline from Equation 9 was formulated by baselining the results against those TSND results from Equation 1 (Ralls, 2014). In equation 8, F_x (b) represents the cumulative distribution function (CDF) of a normal distribution for a given USL and F_x (a) represents the same CDF for a given LSL.

$$\varphi(t) = \int_a^b f_x(u) e^{iu\tau} du = e^{iu\tau - \frac{\sigma^2 t^2}{2}} \tag{7}$$

$$\varphi_y(\tau) := \frac{1}{F_x(b) - F_x(a)} \int_a^b f_x(u) e^{iu\tau} du \tag{8}$$

$$f_t(x) \approx \frac{1}{2\pi}(C_{Tc}) \left(\frac{e^{-\frac{\sigma^2 x^2}{2}}}{F_x(b) - F_x(a)} \right), \text{ where } C_{TC} = \frac{1}{\sqrt{2\pi}} \tag{9}$$

Using Equations 6 through 10 and other in process research steps (Ralls, 2014) a final assembly formula, Equation 11, was formulated based on the initial baseline inversion factor documented in Equation 9. A high-level summary of the corresponding analysis heuristic is shown in Exhibit 2.

$$\varphi_z(\tau) := \left(\frac{1}{F_x(b) - F_x(a)} \int_a^b f_x(u) e^{iu\tau} du \right) \left(\frac{1}{F_y(b) - F_y(a)} \int_a^b f_y(u) e^{iu\tau} du \right) \tag{10}$$

$$f_t(x)_{assy} \approx \frac{1}{2\pi}(C_{Tc}) \left(\frac{\left(e^{-\frac{\sigma^2 t^2}{2}} \right)_{d1} \left(e^{-\frac{\sigma^2 t^2}{2}} \right)_{d2}}{(F_x(b) - F_x(a))_{d1} * (F_x(b) - F_x(a))_{d2}} \right), \text{ where } C_{TC} \ \frac{1}{\sqrt{2\pi}} \tag{11}$$

TSND Assembly Analysis Heuristic

Heuristics techniques can be broadly characterized as exploratory problem solving techniques (Ralls, 2014). Application of heuristics to assembly analysis of two identical TSNDs is summarized in Exhibit 2. Simplified results are summarized in Exhibit 3.

Exhibit 2. TSND Assembly CF Inversion Heuristic (Simplified)

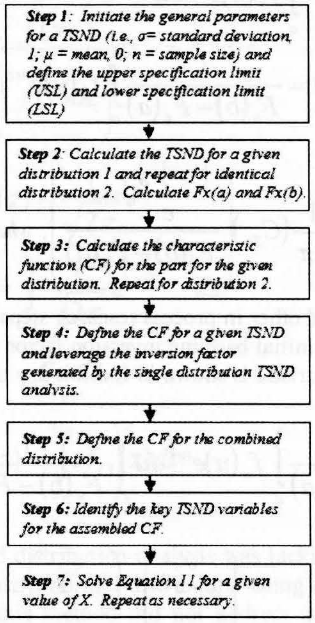


Exhibit 3. TSND Assembly Analysis Results (Ralls, 2014)

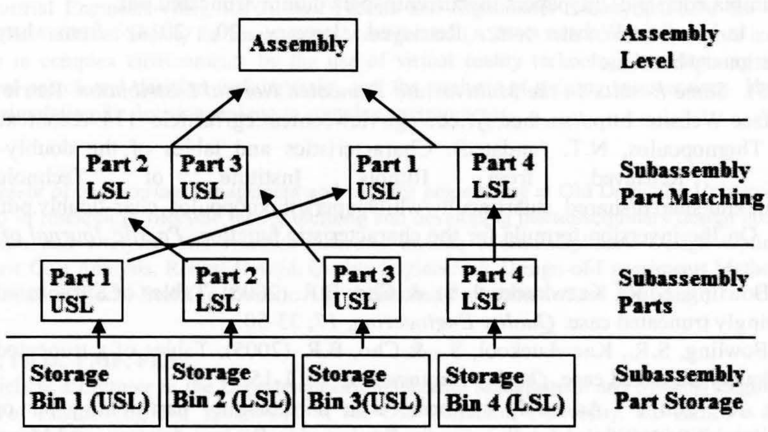
DISTRIBUTION 1 $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)				**DISTRIBUTION 2** $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)				**DISTRIBUTION ASSEMBLY** $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)		
X	*pdf*	$\varphi_z(w\|\mu\&\sigma)$	*ft(z)*	*X*	*pdf*	$\varphi(w\|\mu\&\sigma)$	*ft(x)*	*X*	$\varphi_z(w\|\mu\&\sigma)$	*ft(z) - ASSY*
-4	0.00013383	0.00033548	0.00013384	-4	0.00013383	0.00033548	0.00013384	-8	1.1255E-07	4.49E-08
-3.5	0.00087268	0.00218763	0.00087274	-3.5	0.00087268	0.00218763	0.00087274	-7	4.7857E-06	1.909E-06
-3	0.00443185	0.0111097	0.00443213	-3	0.00443185	0.0111097	0.00443213	-6	0.00012343	4.924E-05
-2.5	0.0175283	0.04393972	0.01752941	-2.5	0.0175283	0.04393972	0.01752941	-5	0.0019307	0.0007702
-2	0.05399097	0.13534386	0.05399439	-2	0.05399097	0.13534386	0.05399439	-4	0.01831796	0.0073078
-1.5	0.1295176	0.32467303	0.1295258	-1.5	0.1295176	0.32467303	0.1295258	-3	0.10541258	0.0420535
-1	0.24197072	0.60656908	0.24198605	-1	0.24197072	0.60656908	0.24198605	-2	0.36792605	0.1467813
-0.5	0.35206533	0.88255281	0.35208763	-0.5	0.35206533	0.88255281	0.35208763	-1	0.77889945	0.3107359
0	0.39894228	1.00006335	0.39896755	0	0.39894228	1.00006335	0.39896755	0	1.0001267	0.3989928
0.5	0.35206533	0.88255281	0.35208763	0.5	0.35206533	0.88255281	0.35208763	1	0.77889945	0.3107359
1	0.24197072	0.60656908	0.24198605	1	0.24197072	0.60656908	0.24198605	2	0.36792605	0.1467813
1.5	0.1295176	0.32467303	0.1295258	1.5	0.1295176	0.32467303	0.1295258	3	0.10541258	0.0420535
2	0.05399097	0.13534386	0.05399439	2	0.05399097	0.13534386	0.05399439	4	0.01831796	0.0073078
2.5	0.0175283	0.04393972	0.01752941	2.5	0.0175283	0.04393972	0.01752941	5	0.0019307	0.0007702
3	0.00443185	0.0111097	0.00443213	3	0.00443185	0.0111097	0.00443213	6	0.00012343	4.924E-05
4	0.00013383	0.00033548	0.00013384	4	0.00013383	0.00033548	0.00013384	8	1.1255E-07	4.49E-08

Application Extension

Application of TSND assembly analysis has broad applications for engineering managers, quality practitioners and manufacturing engineers. Specifically, the identification of an alternate means to approximate and/or analyze the relationship of two-part assembly combinations can aid in the optimization of more complex assemblies. Additionally, a broader application of this approach could be applied to part binning strategies in addition to facility and storage optimization techniques. While the approach summarized by this article focuses on the analysis of two identical TSND further refinement of the equations would be useful in part binning applications.

Exhibit 4 depicts a simplified part-binning assembly approach for two-part pairs. Two-part assemblies can be expanded to develop larger assemblies. In this example subassembly parts are mated with parts that enable the overall assembly level part specification to be met. Expansion of the TSND approach from this paper to part binning could aid in the development of an alternate part binning assembly approach from commonly accepted methods.

Exhibit 4. TSND Assembly Analysis Part Binning Simplification



Conclusions

For engineering managers, risk managers and quality practitioners, understanding and evaluating data sets, assembly tolerances, and process specification limits is relevant to numerous manufacturing or decision making policies. This article summarizes an "alternative approach to the analysis of TSNDs using an inversion factor" and the relationship of truncated distributions assemblies (Ralls, 2014). A simplified heuristic procedure was presented along with key equations (Ralls, 2014).

In general, the approach presented has application to part-binning philosophies, fabrication and tolerance settings, and the summary is expected to provide a foundational framework to future analysis of other distributions relevant to engineering managers and other practitioners.

Recommendations

Elements of this article summarize future research opportunities in the area of truncated distribution assembly analysis. The research summarized by this work focused on the analysis of truncated standard normal distributions, however application of these heuristics and approaches to other distributions could be considered. This research could also be further expanded as follows:

- Enhancement and improvement of the heuristics (Ralls, 2014).
- Evaluation of the application of normalization concepts to this TSND assembly analysis approach (Ralls, 2014).
- Investigation into the inversion factors for alternative distributions (Ralls, 2014).
- Expansion to part binning and storage assembly of truncated piece parts (Ralls, 2014).
- Theoretical expansion to analyze two different TSNDs.
- Benchmarking and optimization from a known TSND baseline.
- Expansion of heuristic approach to include search techniques such as Tabu, beam, and/or other heuristic techniques (Ralls, 2014).

References

Abadir, K., & Magdalinos., T. (2002). The characteristic function from a family of truncated normal distributions. *Econometric Theory*, 18(5). 1276-1287.

Abate, J., & Whitt, W. (1991). *The Fourier-Series Method for Inverting Transforms of Probability Distributions*. Retrieved from the Pennsylvania State University, CiteSeer$^{X}{}_{\beta}$ Website: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.94.9058&rep=rep1&type=pdf

Bernadic, M., & Candel, J. (2012). The doubly truncated function of indices on discrete distributions. *Stat Papers, 53*, 177-193.

Billingsley, P. (1995). Probability and Measure (3rd ed.). New York, NY: John Wiley and Sons, Inc.

Dhrymes, P. J. (2005). *Moments of truncated (normal) distributions*. Retrieved from the Columbia University, Department of Economics Web site: http://www.columbia.edu/~pjd1/mypapers/mycurrentpapers/dummytruncated.pdf

Heuristic. (2014). In Merriam-Webster.com. Retrieved January 20, 2014, from http://www.merriam-webster.com/dictionary/heuristic

Horrace, W.C. (2005). *Some Results on the Multivariate Truncated Normal Distribution*. Retrieved from Syracuse University, Surface Website: http://surface.syr.edu/cgi/viewcontent.cgi?article=1149&context=ecn

Johnson, A.C., & Thomopoulos, N.T. (undated). Characteristics and tables of the doubly-truncated normal distribution. Retrieved from Illinois Institute of Technology, Website: http://business.iit.edu/shared/shared_stuartfaculty/whitepapers/thomopoulos_char-doubly.pdf

Kawata, T. (1969). On the inversion formula for the characteristic function. *Pacific Journal of Mathematics, 31*, 81-85.

Khasawneh, M.T., Bowling, S. R., Kaewkuekool, S., & Cho, B.R. (2005). Tables of a truncated standard normal distribution: a singly truncated case. *Quality Engineering, 17*, 33-50.

Khasawneh, M.T., Bowling, S.R., Kaewkuekool, S., & Cho, B.R. (2005). Tables of a truncated standard normal distribution: a doubly truncated case. *Quality Engineering, 17*, 1-15.

Lee, B., & Saitou, K. (2007). Assembly synthesis with subassembly partitioning for optimal in-process dimensional adjustability. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing, 21*, 31-43.

Michalewicz, Z., & Fogel, D. (1998). *How to Solve it: Modern Heuristics* (2nd ed.). New York, NY: Springer.

Ostermeier, M. (2003). Theoretical distribution of truncation lengths in incremental truncation libraries. Biotechnology and Bioengineering, 82(5), 565-577.

Pugh, G.A. (1986). Partitioning for selective assembly. *Computers & Industrial Engineering, 11* (1-4), 175-179.

Pugh, G.A. (1992). Selective assembly with components of dissimilar variance. *Computers and Industrial Engineering, 21*(1-4), 487-491.

Ralls, J. W. (2014). An investigation into the analysis of truncated standard normal distributions using heuristic techniques.(Order No. 3580485, Old Dominion University). ProQuest Dissertations and Theses, , 246. Retrieved from http://search.proquest.com/docview/1535275140?accountid=12967. (1535275140).

Shephard, N.G. (1992). From characteristic function to distribution function: a simple framework for the Theory. Econometric Theory, 7, 519-529.

Srinivasa Varadhan, S.R. (2000). Characteristic Functions. Retrieved from New York University, Mathematics Department Website: http://www.math.nyu.edu/faculty/varadhan/course/PROB.ch2.pdf

About the Author(s)

John W. Ralls, Ph.D., PE, PMP

John W. Ralls is currently employed as an engineering manager. He earned a Ph.D. in Engineering Management from the Old Dominion University, a Master of Engineering Management (MEM) from The George Washington University and a Bachelor of Science in Marine Engineering Systems from the U.S. Merchant Marine Academy. Dr. Ralls is a licensed professional engineer in the state of Virginia.

C. Ariel Pinto, Ph.D.

C. Ariel Pinto is an Associate Professor of Engineering Management and Systems Engineering at Old Dominion University. He earned a Ph.D. in Systems Engineering from the University of Virginia. His research focus includes a focus on multi-disciplinary approaches to risk management in engineered systems and disruptions in the continuity of organizations and information systems.

Shannon R. Bowling, Ph.D.

Shannon Bowling received his Ph. D. degree in industrial engineering with a focus in human factors engineering from Clemson University, USA in 2003. He received his M. Sc. degree in engineering technology with an emphasis in quality management (2000) from East Tennessee State University, USA, and his B. Sc. degree in electrical engineering technology (1998) from Bluefield State College, USA. He also has a graduate minor in experimental statistics from Clemson University and management from East Tennessee State University. He is the Dean of Engineering Technology and Computer Science at Bluefield State College. He teaches topics relating to modeling and simulation including operations research, optimization methods, systems dynamics, and agent-based modeling. He was a principle investigator (PI) on a project involved in a large scale modeling effort for the Department of Homeland Security. He has various papers published and/or submitted in the area of modeling and simulation. He has over 70 refereed conference and journal publications on various topics. He is a member of the Institute of Industrial Engineers (IIE), the Human Factors and Ergonomics Society (HFES), the American Society for Quality (ASQ), and the American Society for Engineering Management (ASEM). His research interests include the modeling of human performance in complex environments by the use of virtual reality technology, ascertaining factors that influence performance in visual search and decision making tasks, and the analysis of training interventions. He also is interested in using modeling and simulation for decision making in complex environments.

Resit Unal, Ph.D.

Resit Unal is a professor of engineering management and systems engineering at Old Dominion University, Norfolk, Virginia. Dr. Unal is involved in research in lifecycle model building and developing multidisciplinary design optimization approaches for design integration and analysis. His research interests include Multidisciplinary Design Optimization, Design for Reliability, Parametric Cost Analysis, Robust Design, Quality Engineering, Design-of-Experiments Methods, Response Surface Methods for Approximation Model Building, Genetic Algorithms and Probabilistic Risk Analysis.

Charles B. Daniels, Ph.D., PMP, PEM

Dr. Charles B. Daniels is a Lecturer in the Department of Engineering Management and Systems Engineering Department in the Batten College of Engineering and Technology of the Old Dominion University. He earned a Ph.D. in Engineering Management from the Old Dominion University, a Master of Engineering Management (MEM) from the George Washington University and a Bachelor of Science degree from the State University of New York.