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UNIFORM $L^1$ BEHAVIOR IN CLASSES OF INTEGRODIFFERENTIAL EQUATIONS WITH CONVEX KERNELS

RICHARD NOREN

Introduction. We consider families $\mathcal{R}$ of functions such that each $a$ in $\mathcal{R}$ satisfies

$$\int_0^1 a(s)ds < \infty,$$

(1.1)

$a$ is nonconstant, nonegative, nonincreasing, convex, and $-a'$ is convex.

We will show, for certain such families, that

$$\int_0^\infty \sup_{a \in \mathcal{R}} |u(t; a)| dt < \infty,$$

(1.2)

where $u(t) = u(t; a)$ is the solution of the scalar problem

$$u'(t) + \int_0^t a(t - \tau)u(\tau)d\tau = 0, \quad u(0) = 1, \quad t \geq 0, \quad a \in \mathcal{R}.$$

(1.3)

When $\mathcal{R} = \{ \lambda a_0(t) : 0 < \lambda_0 \leq \lambda < \infty \}$, (1.2) is true. These and similar results were proved in [1, 2, 4, 5, 10 and 11]. The technique of proof relies on the methods of Shea and Wainger [13].

The estimate (1.2) was used in [1, 4, 5 and 11] to estimate the resolvent kernel

$$U(t) = \int_{\lambda_0}^\infty u(t; \lambda a_0) \, dE_{\lambda},$$

of the problem

$$y'(t) + \int_0^t a_0(t - s) \, Ly(s)ds = f(t), \quad y(0) = y_0,$$

(1.4)

in a Hilbert space $\mathcal{H}$. The operator $L$ is a densely defined self-adjoint linear operator with spectrum contained in $[\lambda_0, \infty)$ ($\lambda_0 > 0$), $y_0$ and