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SYSTEMS STATISTICAL ENGINEERING – HIERARCHICAL FUZZY CONSTRAINT PROPAGATION

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Abstract
Driven by a growing requirement during the 21st century for the integration of rigorous statistical analyses in engineering research, there has been a movement within the statistics and quality communities to evolve a unified statistical engineering body of knowledge (Hoerl & Snee, 2010). Systems Statistical Engineering research seeks to integrate causal Bayesian hierarchical modeling (Pearl, 2009) and cybernetic control theory within Beer's Viable System Model (S Beer, 1972; Stafford Beer, 1979, 1985) and the Complex Systems Governance framework (Keating, 2014; Keating & Katina, 2015, 2016) to produce multivariate systemic models for robust dynamic systems mission performance. (Cotter & Quigley, 2018) set forth the Bayesian systemic hierarchical constraint propagation theoretical basis for modeling the amplification and attenuation effects of environmental constraints propagated into systemic variability and variety. In their theoretical development, they simplified the analysis to only deterministic constraints, which models only the effect of statistical risks of failure. Imprecision and uncertainty in the assessment of environmental constraints will induce additional variance components in systemic variability and variety. To make causal Bayesian hierarchical modeling more capable of capturing and representing the imprecise and uncertain nature of environments, we must incorporate rough or fuzzy functions and boundaries to model imprecision and grey boundaries to model uncertainty in constraint propagation at each system level to measure the overall impact on the organization variability and variety. This paper sets forth a proposed research method to incorporate rough, fuzzy, and Grey set theories into Systems Statistical Engineering causal Bayesian hierarchical constraints modeling.

Keywords
Fuzzy Hierarchical Systems, Causal Bayesian Hierarchical Models, Systems Statistical Engineering.

Introduction
Many of the world's most challenging statistical problems are large, complex, and unstructured. Statistical engineering has been proposed to guide the integration of multiple statistical methods to address these large, complex, and unstructured problems (Hoerl & Snee, 2010). Statistical engineering was initially defined as: "The study of how to best use statistical concepts, methods, and integrate them with information technology and other relevant disciplines, to achieve enhanced results." (Hoerl & Snee, 2010). It involves the integration of statistical thinking (often at the strategic level) with the application of statistical methods and tools (at the operational level). It has the potential to provide the missing tactical link that will drive the proper application of statistical methods based on a solid understanding of statistical thinking principles. Statistical Engineering typically involves the appropriate selection and use of multiple statistical tools integrated with other relevant tools into a comprehensive approach to solving complex problems.

In literature, (Cotter, 2012) summarized previous studies that led to the proposal for statistical engineering, identified the gaps in knowledge that statistical engineering needs to address, explored additional gaps in knowledge not addressed in the prior works, and proposed a working definition of and body of knowledge for statistical engineering. In this paper, the author mentioned that rapid development in statistics and quality management cause failure to contribute to both academic and industrial domains. Quality experts do not seem to be using the latest published research, and scientists do not sufficiently inscribe the potential problems experienced by practitioners. In addition, (Cotter, 2012) stated that it is vital to verify that the empirical statistical model correctly represents the physics of the practical problem definition and validates the model's predictive capability against actual systemic problem behavior. In other words, the closer the empirical statistical model represents actual systemic behavior, the more accurate and precise its predictive capability.

The Statistical System Engineering Causal Bayesian Hierarchal Model

To achieve this goal, (Cotter, 2015) argued that continuing using the general linear model (GLM) to frame statistical engineering is not the right choice and has several drawbacks. For example, (Tahami et al., 2016) explained that conditional dependencies in statistical engineering could not be addressed in the GLM framework, results in models that are hard to fit or that may not converge to a unique solution. They may not enhance the understanding of causal physical processes in dynamic stochastic systems (Tahami et al., 2019). To address this gap, that is, integrating deterministic engineering models within stochastic models to better capture the uncertainty, the author proposed that causal hierarchical Bayesian networks can be used as a framework to model joint deterministic-stochastic dynamic causal effects in engineering models.

The limitation of (Cotter, 2015) was that the author did not explicitly develop the modeling methodology that can be used to predict the socio-technical systemic performance of statistical engineering models. Using the concept of Hierarchical Bayesian Networks (Cotter, 2016) specified the modeling methodology to address this gap and proposed the initial theoretical foundation for the method. The author stated that after developing such a method of accurately address the economic, environmental, political, social, legal, and technical aspects of any socio-technical system, it is vital to decompose the systemic constraints to subsystems, modules, and components level. In doing so, (Cotter, 2017) stated that to accurately decompose the economic, environmental, political, social, legal, and technical constraints to subsystems and modules and then accurately predict systemic mission performance, the systems statistical engineering dimension should be considered as a hierarchical constraint propagation. Towards this end, the author developed a methodology for the decomposition of systemic models using a causal Bayesian hierarchal modeling approach.

In their joint work, (Cotter & Quigley, 2018) presented constraint propagation theory, systems theory, and Bayesian constrained regression theory related to the problem of systemic hierarchical constraint propagation and established the primary method for their integration into the systems statistical engineering body of knowledge. Their proposed constraint propagation was based on causal Bayesian hierarchal model,

$$\begin{aligned}
&\text{Min } \mathbf{Y}_{\text{Total}} = f(\mathbf{w}'(\mathbf{Y}_{\text{pred}} - \mathbf{T})) \qquad (1)\\
&\text{s.t.}\\
&\mathbf{Y} = \mathbf{F}(pa_{\text{i}}, u_{\text{xi}})\boldsymbol{\beta} + \mathbf{F}(pa_{\text{j}}, u_{\text{zj}})\boldsymbol{\gamma} + \varepsilon\\
&\mathbf{LB}_{\text{X}} \leq \mathbf{F}(pa_{\text{i}}, u_{\text{xi}}) \leq \mathbf{UB}_{\text{X}}\\
\text{possibly} \quad &\mathbf{LB}_{\text{Z}} \leq \mathbf{F}(pa_{\text{j}}, u_{\text{zj}}) \leq \mathbf{UB}_{\text{Z}}
\end{aligned}$$

Where $\mathbf{Y}_{\text{Total}}$ is the vector or matrix of systemic performance variables, $f(\text{w}'(\bullet))$ is a vector or matrix of normalized weighting functions that admit tradeoffs among the ($\mathbf{Y}_{\text{pred}} - \mathbf{T}$) differences, and $\mathbf{T}$ is the vector or matrix of identified systemic mission performance targets. $\mathbf{F}(pa_{\text{i}}, u_{\text{xi}})$ is a matrix of functional relationships of the X predictors, and $\mathbf{F}(pa_{\text{j}}, u_{\text{zj}})$) is a matrix of functional relationships of the $\mathbf{Z}$ within and cross-layer covariates, respectively to the $\mathbf{Y}_{\text{pred}}$ variables performance levels. Where the functional relationship has an unknown form, $f_{\text{i}}(pa_{\text{i}}, u_{\text{xi}}) = \text{x}_{\text{i}}$ observed data and $f_{\text{j}}(pa_{\text{j}}, u_{\text{zj}}) = \text{z}_{\text{j}}$ observed covariate values, the residual error accumulates in the ε term. The $\boldsymbol{\beta}$ response parameters of $\mathbf{Y}_{\text{pred}}$ to $\mathbf{X}$ and the $\boldsymbol{\gamma}$ response parameters of $\mathbf{Y}_{\text{pred}}$ to $\mathbf{Z}$ are constant coefficients to be determined.

Previous studies in the Systems Statistical Engineering domain simplified the analysis to only deterministic constraints, which models only the effect of statistical risks of failure (Tahami & Fakhravar, 2020b). However, imprecision and uncertainty in the assessment of environmental constraints will induce additional variance components in systemic variability and variety that need to be considered. To make causal Bayesian hierarchical modeling more capable of capturing and representing the imprecise and uncertain nature of environments, we must incorporate rough or fuzzy boundaries to model imprecision and grey boundaries to model uncertainty in constraint propagation at each system level to measure the overall impact on the organization variability and variety (Tahami & Fakhravar, 2020a). In other words, what has not been researched is how deterministic and stochastic engineering models can be integrated into state space, non-recursive causal Bayesian hierarchical models of structural, stochastic, fuzzy, rough, and grey components to model the response of purposeful systemic causal variety to environmental constraining causal variety to design robust dynamic systems mission performance. This research sets forth a proposed method to incorporate rough, fuzzy, and Grey set theories into Systems Statistical Engineering causal Bayesian hierarchical constraints modeling.

Proposed Research Modeling Methodology

All assessments of systemic variation and variety require measurement or assessment of $\mathbf{Y}_{\text{pred}}$, $\mathbf{LB}_{\text{X}} \leq \mathbf{F}(pa_{\text{i}}, u_{\text{xi}}) \leq \mathbf{UB}_{\text{X}}$, and $\mathbf{LB}_{\text{Z}} \leq \mathbf{F}(pa_{\text{j}}, u_{\text{zj}}) \leq \mathbf{UB}_{\text{Z}}$ respectively. $\mathbf{Y}_{\text{pred}}$, $\mathbf{F}(pa_{\text{i}}, \bullet)$, and $\mathbf{F}(pa_{\text{j}}, \bullet_{\text{j}})$ may be measured or assessed with some level of accuracy and precision admitting probabilistic models or subjectively admitting some level of fuzzy, rough, or grey uncertainty depending on the data type. $\mathbf{F}(\bullet, u_{\text{xi}})$, and $\mathbf{F}(\bullet, u_{\text{zj}})$ may be assessed only imprecisely or with uncertainty. Definitions of risk, imprecision, and uncertainty are:

> ***Risk*** is observed in those situations in which the potential outcomes can be described by well-known probability distributions.
> ***Imprecision*** is observed in those situations in which the potential outcomes cannot be described by well-known probability distributions but can be estimated by subjective probabilities.
> ***Uncertainty*** is observed in those situations in which the potential outcomes cannot be described by well-known probability distributions and cannot be estimated by subjective probabilities but can be estimated by statements of ambiguity.

Exhibit 1 sets forth a mapping of risk, imprecision, and uncertainty modeling to data type and measurement or assessment type.

Exhibit 1. Mapping of uncertainty type to data type and measurement/assessment.

		Measurement/Assessment	
Uncertainty Type	**Data type**	*Objective*	*Subjective*
Risk	Discrete	Probabilistic	Rough sets
	Continuous	Probabilistic	Fuzzy sets
Imprecision	Discrete	Rough sets	Fuzzy-Rough sets
	Continuous	Grey sets	Fuzzy sets; Grey sets
Uncertainty	Discrete	Rough sets; Grey sets	Fuzzy sets; Grey sets
	Continuous	Gray sets	Fuzzy sets; Grey sets

The problem of constraint propagation within the proposed SSE causal Bayesian hierarchal model is one of integrating these differing forms of uncertainties into a unified variance component representation of equation (1). Generally, there are three possibilities that need to be investigated based on what type of data we are dealing with:

- Risk probabilistic data: Cotter (2018) presented constraint propagation theory, systems theory, and Bayesian constrained regression theory related to the problem of systemic hierarchical constraint propagation and established the primary method for their integration into the systems statistical engineering body of knowledge.
- Fuzzy and rough data: To deal with uncertainty, there are several approaches that can be considered. The first possible method is to consider fuzzy sets. By definition, fuzzy set A is a set of ordered pairs $\{\langle x, \mu_A(x)\rangle | x \in \mathbb{U}\}$ where $\mathbb{U}$ is a universe of discourse and $\mu_A: \mathbb{U} \to [0,1]$ is the membership function of A and $\mu_A(x)$ is the grade of belongingness of x in A. The second approach is to extend fuzzy sets to rough sets. Generally, rough sets take a different route from fuzzy sets in representing uncertainties. It represents an uncertain set by means of approximations in information systems. A rough approach is with regards to ambiguity and a lack of information, whereas a fuzzy approach is more associated with vagueness and a lack of definable boundaries. For defining rough sets, first, we need to define an approximation of a set. (Khuman 2015; Yang and Hinde 2010)
  Let $\Lambda = (\mathbb{U}, A)$ is a given information system, where $\mathbb{U}$ is a non-empty, finite set of objects called the universe, and A is a non-empty, finite set of attributes. Also, consider $X \subseteq \mathbb{U}$ and $B \subseteq A$. The set X is approximate with two sets $B_*(X)$ and $B^*(X)$ as follows:

$$B_*(X) = \bigcup_{x \in \mho} \{B(x) : B(x) \subseteq X\},$$
$$B^*(X) = \bigcup_{x \in \mho} \{B(x) : B(x) \cap X \neq \phi\}.$$

$B_*(X)$ and $B^*(X)$ are a lower and upper approximation set, respectively. The lower approximation is the set of all objects that absolutely belong to set X with respect to B, and the upper approximation is the set of all objects which can be classified as being possible members of set X with respect to B.

Knowing the approximation definition, let the pair $apr = (\mathbb{U}, B)$ be an approximation space on $\mathbb{U}$ and $\mathbb{U}/B$ denotes the set of all equivalence classes of B. The family of all definable sets in approximation space apr is denoted by $Def(apr)$. Given two subsets $\underline{A}, \overline{A} \in Def(apr)$, with $\underline{A} \subseteq \overline{A}$, the pair $(\underline{A}, \overline{A})$ is called a rough set. (Yang & Hinde, 2010)

Various uncertainties in real-world applications can bring difficulties in determining the crisp membership functions of fuzzy sets, and various approaches have been developed to accommodate the uncertainties in fuzzy membership values, such as interval-valued fuzzy sets where the membership of an individual element is characterized as an interval instead of a single value in fuzzy sets, Atanassov intuitionistic fuzzy sets where a degree of membership and degree of non-membership are presented, shadowed sets where the evaluation of membership is scored as either (1), (0) or belonging to the shadowed region [0, 1] and type-2 fuzzy sets where the secondary grade membership function itself is a type-1 fuzzy set. In addition to the aforementioned approaches, the R-fuzzy sets use rough sets to approximate the membership function of fuzzy sets. R-fuzzy sets are an extension of fuzzy set theory that allows for the uncertain fuzzy membership value to be encapsulated within the bounds of an upper and lower rough approximation. (Khuman, 2016)

Utilizing the notion of R-fuzzy sets, by referring to equation 1, the rough sets are used to address the uncertainty in the boundaries ($\mathbf{LB}_X$, $\mathbf{UB}_X$, $\mathbf{LB}_Z$, $\mathbf{UB}_Z$), and fuzzy sets are used for the functions ($\mathbf{F}(pa_i, u_{xi})$, $\mathbf{F}(pa_j, u_{zj})$).

- Grey data: As a different model for uncertainty representation, grey systems provide another route to uncertainty modeling. In grey systems, the information is classified into three categories: white with completely certain information, grey with insufficient information, and black with totally unknown information. When Combining fuzzy sets and grey systems, grey sets were proposed as a general model for uncertainty representation. By definition, for a set $A \subseteq \mathbb{U}$, if the characteristic function value of x with respect to A can be expressed with a continuous grey number $v^{\pm} = [v^-, v^+] \in D[0,1]^{\pm}$ or a discrete gray number $v^{\pm} = \{v^-, v_1, v_2, \ldots, v_k, v^+\} \in D[0,1]^{\pm}$, then A is a grey set. Also, a grey number is a number with clear upper and lower FHboundaries but which has an unknown position within the boundaries. It is different from an interval or a set in that it is a single number represented by an interval or a set.

  In the case of dealing with grey sets, by referring to equation 1, to encapsulate uncertainty, Fuzzy sets are used for the boundaries ($\mathbf{LB}_X$, $\mathbf{UB}_X$, $\mathbf{LB}_Z$, $\mathbf{UB}_Z$), and grey sets are used for the functions ($\mathbf{F}(pa_i, u_{xi})$, $\mathbf{F}(pa_j, u_{zj})$).

Application of Fuzzy Constraints to Engineering Management

The primary contribution of hierarchical fuzzy constraint propagation within the systems Bayesian hierarchical model (1) to engineering management is to EMBOK Domain 6: Quality Management System and Domain 9: Systems Engineering. In general, there are several standard practices to specify qualitative product characteristics such as visual standards or written specifications of two or three-dimensional size or depth or effect on final product functionality. However, currently, practitioners mostly consider deterministic limits to set these visual and written specifications (Erudural, 2006). In their article (Brenneman & Myers, 2003) discussed that qualitative product and process parameters had been measured as 0-1 factors in general. However, due to the inherent complexity and uncertainty that exists in product design and thus in determining the qualitative product characteristics, modeling these parameters as 0-1 factors cannot capture the uncertainty and does not result in accurate outcomes. To address this issue, (Wang & Tong, 2003) utilized grey relational analysis from system theory. They proposed a procedure to consider uncertainty and variation in the quality design of products and eventually determine the qualitative specifications effectively.

In this paper, we proposed a methodology that directly incorporates appropriate fuzzy, rough, and grey sets models in assessing the propagation effects of environmental factors on product performance. We utilized fuzzy set, rough set, or grey set theory within the context of hierarchical Bayesian analysis to model the imprecision and uncertainty nature of environments and to measure the overall impact on the organization's variability and variety. By directly incorporating appropriate fuzzy, rough, and grey sets models, engineering managers of product design and production processes will have a better understanding of the impact of environmental factors on resultant product performance.

Conclusions

Previous studies in the Systems Statistical Engineering domain simplified the analysis to only deterministic constraints, which models only the effect of statistical risks of failure. However, imprecision and uncertainty in the assessment of environmental constraints will induce additional variance components in systemic variability and variety that need to be considered. To make causal Bayesian hierarchical modeling more capable of capturing and representing the imprecise and uncertain nature of environments, we must incorporate rough or fuzzy boundaries to model imprecision and grey boundaries to model uncertainty in constraint propagation at each system level to measure the overall impact on the organization variability and variety. This research sets forth a proposed method to incorporate rough, fuzzy, and Grey set theories into Systems Statistical Engineering causal Bayesian hierarchical constraints modeling.

For future research, the authors are going to develop the mathematical formulation of integrating Fuzzy set theory, Rough set theory, and Grey set theory for the hierarchical mapping of systemic constraints. In addition, in terms of developing Systems Statistical Engineering body of knowledge, as it was stated in Cotter (2018), other aspects could be considered to develop a more accurate representation of the model which are not implemented yet in the literature, such as considering Technical-economic versus human preference constraints or considering systems with non-recursive directed acyclic graph feedback loops.

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