Virtual Compton Scattering in the Nucleon Resonance Region

Luminita Anca Todor

Old Dominion University

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VIRTUAL COMPTON SCATTERING IN THE NUCLEON RESONANCE REGION

by

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A Dissertation submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirement for the Degree of

DOCTOR OF PHILOSOPHY

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December 2000

Approved by:

Charles Earl Hyde-Wright (Director)
Pierre Yves Bertin
Gail Dodge
Mark Havey
Stephen Olariu
Winston Roberts
ABSTRACT

VIRTUAL COMPTON SCATTERING IN THE NUCLEON RESONANCE REGION

Luminita Anca Todor
Old Dominion University, 2000
Director: Dr. Charles Earl Hyde-Wright

This thesis is a study of virtual Compton scattering reaction in which a virtual photon is elastically scattered off a proton $\gamma p \rightarrow \gamma p$. The $p(e, e'p)\gamma$ reaction was measured in the nucleon resonance region for the first time in the experiment E93-050 in Hall A at Thomas Jefferson National Accelerator Facility. To lowest order in $\alpha_{QED}$, this reaction is a coherent superposition of radiation from the incident or scattered electron in elastic $ep$ scattering (Bethe-Heitler) and exclusive production of a photon on the proton, by absorption of a virtual photon (virtual Compton scattering). In E93-050, we measured a resonance excitation scan for nine central values of $s$ from 1.3 GeV$^2$ to 3.6 GeV$^2$, with the coincidence angular kinematics centered on $\theta_{\gamma\gamma} = \pi$. Part of the data analysis was also calibration of the Hall A spectrometer pair optics. Besides the experimental data analysis, a theoretical calculation of this cross-section in a phenomenological Lagrangian approach was setup. Preliminary experimental cross-section results and present status of theoretical calculation are shown here.
Dedication:

To all my friends,

for which I'll name Michael and Laura

Motto: from the poem *Glossa*, by Mihai Eminescu

Vreme trece, vreme vine,
Toate-s vechi si noua toate;
Ce e rau si ce e bine,
Tu te-ntrebaba si socioate;
Nu spera si nu ai teama,
Ce e val ca valul trece;
De te-ndeamna, de te cheama
Tu ramii la toate rece.

Days go past and days come still,
All is old and all is new,
What is well and what is ill,
You imagine and construe;
Do not hope and do not fear,
Waves that leap like waves must fall;
Should they praise or should they jeer,
Look but coldly on it all.
Acknowledgment

First I thank my parents Rodica and Dumitru Stoian, that gave me a good start in life and bore supportively the long separation caused by my living in USA. I learned from them curiosity and high work ethics and these are the basis of the joy and satisfaction I find in experimental nuclear physics. Doina, my sister, set the achievement level higher every time and always 'on my side'.

I greatly benefited from the interaction with all the professors at Old Dominion University, their competence, passion for knowledge and teaching talent. It was a privilege to be trained by a group of scientists that are the core of the TJNAF Theory group. Last and not least they put up with my insistent questions - believe it or not it helped me to understand 'everything'. Thank you Professor Radyushkin, Professor Van Orden, Professor Schiavilla, Professor Copeland, Professor Balitsky, Professor Hoy. Above all Professor Roberts deeply influenced my formation during these years. He took a chance accepting me in the ODU Graduate Program in 1994, he was my teacher in class twice and he trusted my capacity and took time to guide me through a theoretical calculation for the process I was studying. Our numerous discussions and his prompt answers have pushed me to heighten the level of theoretical knowledge and ability to cross the bridge between experiment and theory.

The best part of the graduate life was being part of the TJNAF experimental team. For 5 years, in all kind of experimental activities, I enjoyed working with all the Hall A people: Bob Michaels, John LeRose, Eddy Offermann, Bogdan Wojsekhowski, Jack Segal, Ron Gilman, Kees de Jager, Arun Saha, Javier Gomez, J.P. Chen, Eugen Chudakov, Ed Brash, Charles Pedrisat, Vina Punjabi, Steffen
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Due to the European part of the VCS collaboration I got to travel places I only dreamed about. Our collaboration was a challenging experience that ended with the conclusion that my work style matches the American approach better. Professor Pierre Bertin impressed me with his profound grasp of experimental physics and great jovial spirit. I will never forget the through, high quality work of Helene Fonvieille. Merci beaucoup Roger Van de Vyver, Luc Van Hoorebeke, Rachele Di Salvo, Yves Roblin, Vincent Breton, Nicole d’Hose, Jacques Marroncle, Pascal Vernin, Dominique Marchand. The E93-050 analysis progressed due to the efforts of all the graduate students working on this experiment: Natalie Degrande, Stephanie Jaminion, Geraud Laveissiere, Christophe Jutier. Bon courage a tous!

It is hard to imagine a better team to work with than the Nuclear Experimental group at Old Dominion University. Thank you Andi Klein for all the computer resources you built up for our group and all the small and big things you forced us to take time to learn. Thank you Sebastian Kuhn for offering me the opportunity to take part to the SLAC experiment E155. Thank you Larry Weinstein for the way you brought up new dimensions on any subject and the caring 'ritual' of the morning coffee! Thank you Paul Ulmer for getting me involved in MCEEP development and assisting my simulation work (including my errors analysis!). There is somebody for whom 'Thank you!' might be just too little - Gail Dodge. Organized and focused, Gail always amazed me by the way she
can clarify a discussion, by the remarkable talent to pull the thread from details and direct the matter towards solution. She made it possible for me to take part in two experiments at NIKHEF and her explanations and attentive supervision made me get the most from the situation. She definitely stood by me for the whole period of my graduate studies and helped me with pertinent advice. After any discussion with her I felt appreciated and motivated to try to do the best.

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Thanks to all the thesis committee members for taking the time getting familiar with my work, correcting and completing my explanations with their observations. I will name only those that were not mentioned before: Professor Mark Havey and Professor Stephen Olariu.

Getting my PhD. degree wouldn’t have been possible without my husband Dorin’s support and the understanding of my wonderful children, Horia and Alina. I hope better, easier times lie ahead for us. Kind support has come to me from the warm friendship of Vickie and Walt Hooks.

There was one 'little-big' thing that kept me going through, when due to the long time getting my degree, I doubted if I should continue: that was being awarded a SURA Fellowship. It made a world of difference to feel appreciated (since the fellowship is awarded competitively) and I can only hope that the work done these years confirmed my selection.
I will end by thanking my adviser Professor Charles Hyde-Wright. With solid and universal knowledge, a lot of courage and patience, he faced the challenge of a demanding student as I was. Eventually we both succeeded and if I don't expect to get any other PhD degree, I can only hope he will have many other fortunate students to advise. You know, Charles, I never could follow you in the long imbricate calculations you talked about while driving together to and from CEBAF. Thank you, Charles, for everything, from correcting my grammar, to explaining to me the physics picture of DVCS. Thank you for all the time you spent with me, for bearing all my questions, dealing with my impatient nature, for believing in me and above all for teaching me Experimental Nuclear Physics.
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<td>63</td>
<td>The $p(e,e'p)\gamma$ data $\phi$ distributions versus $W$ in $Q^2$ and $\cos \theta_{p_xq_y}$ bins. Each color represents a different kinematic setting. The columns present the three $Q^2$ bins, 1,2,3 from left to right. The rows present the four $\cos \theta_{p_xq_y}$ bins, 1,2,3,4 from top to bottom (see Figure 62).</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>64</td>
<td>The $p(e, e'p)\gamma$ simulation $\phi_{pq}$ distribution versus $W$. The bins are the same as in Figure 63.</td>
</tr>
<tr>
<td>65</td>
<td>The $p(e, e'p)\gamma$ (black) and $p(e, e'p)\pi^\circ$ (blue) $Q^2$ vs $W$ distribution.</td>
</tr>
<tr>
<td>66</td>
<td>The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of 0.85-0.95 GeV$^2$. Errors are statistical only. A 5% systematic error is estimated. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.</td>
</tr>
<tr>
<td>67</td>
<td>The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of 0.95-1.05 GeV$^2$. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.</td>
</tr>
<tr>
<td>68</td>
<td>The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of 1.05-1.15 GeV$^2$. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.</td>
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<tr>
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</tr>
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<td>The two fold differential experimental cross-section for the bins with $0.95 &lt; \cos \theta_{ep} &lt; 1.0$.</td>
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<td>75</td>
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</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Review of Compton Scattering

Compton Scattering refers to the phenomenon of scattering a photon off of a target; the characteristics of the scattered photon reflect the electromagnetic nature of the target. In 1923 the American physicist Arthur H. Compton discovered the scattering of X-rays on electrons. In his experiment, Compton directed an x-ray beam of wavelength $\lambda_0$ towards a block of graphite. The scattered x-rays had a slightly bigger wavelength $\lambda$ than the incident x-rays, and hence the energies of scattered rays are smaller. The decrease in energy depends on the angle $\theta$ at which the X-rays were scattered. The change in wavelength, called Compton shift, is given by

$$\Delta \lambda = \lambda - \lambda_0 = \frac{h}{m_0c}(1 - \cos \theta), \quad (1)$$

where $m_0$ is the electron rest mass, $c$ is the speed of light in vacuum, and $h$ is Planck's constant. This change can be fully understood if we treat the x-ray as particles, photons. The explanation of Compton scattering on electron is a typical argument for the wave-particle duality.

Compton scattering on the nucleon has a long history in nuclear physics research. Early theoretical studies in the 50's were motivated by the desire to probe the mesonic structure of the nucleon ([1] and[2]). In 1954, it was realized ([3]) that
one could use dispersion relations to relate Compton scattering to other photonuclear reactions. Compton scattering is related to the total hadronic photon cross-section through the optical theorem and dispersion relations. The optical theorem states that the total scattering cross-section is proportional with the imaginary part of the forward symmetric scattering amplitude. It was subsequently proven that in the low energy limit ([4]), the Compton scattering amplitude contains a frequency independent term (Thompson amplitude) plus a spin dependent term linear in the frequency proportional with the nucleon polarizabilities. In the early seventies, it was assessed that high energy photons are expected to interact like vector-mesons ([5], [6],[7]).

Interest in Compton scattering was renewed by the possibility of using perturbative quantum chromodynamics as a framework for discussing Compton scattering at high momentum transfer. In the nineties the Compton scattering investigation of the proton was divided into two nuclear reaction:

- Real Compton Scattering (RCS) in which a real photon is scattered off of a proton;

- Virtual Compton Scattering (VCS) in which a virtual photon is absorbed on a proton, leading to a real photon in the final state.

RCS experiments were performed starting in the fifties on many accelerators around the world in different energy range (see [8] for a review). Measurements at and below pion threshold (see [9],[10],[11] [12], [13]) allow one to measure two fundamental structure parameters of the proton namely the electric and magnetic polarizabilities ($\alpha$ and $\beta$). The polarizabilities are the proportionality coefficient between the deformation of the nucleon in external electromagnetic fields and field intensity.

Each polarizability is formally a coherent sum over all excited states of the proton. Real and virtual compton scattering in the nucleon resonance region will tend to highlight individual nucleon resonances. Real comtpons scattering measurements in the nucleon resonance region were reported in [14], [15], [16]. The advent of new high duty cycle facilities, and developments in dispersion theories
linking compton scattering, photo-production, and the nucleon polarizabilities have stimulated intense new experimental effort in real compton scattering in the resonance region, particularly near threshold ([17],[18],[19], [20]).

Above the nucleon resonance region, the forward compton scattering process was measured by R.L. Anderson ([21]) and A.M. Breakstone 1981([16]). The only measurement to date of high energy compton scattering at wide angles is reported in [22]. This experiment will soon be extended by E99-114 at Jefferson lab. An untagged bremsstrahlung beam will be used in Hall A, the recoil proton and scattered photon being detected in coincidence. The electron will be detected in one of the high resolution spectrometers, and the photon will be detected in a large area, multi-segment Pb-Glass calorimeter.

Experimentally VCS is accessible in the electro-production of photons $p(e, e'p)\gamma$. VCS gives access to new degrees of freedom in terms of the pair of electromagnetic transitions involved in the Compton scattering. For example, in the low energy limit for RCS we can have only two combinations of initial and final electromagnetic transitions $E1-E1$ and $M1-M1$. These are the electric and magnetic polarizabilities, respectively. In the case of a low energy (but arbitrary wavelength) initial virtual photon, there are 10 possible combinations of electromagnetic transitions, of which 6 are independent. These generalized polarizabilities describe the response of the proton (final state radiation) to the most general types of electromagnetic excitation (incoming virtual photon) as defined in [23] and [24].

The first measurement of VCS was done in an experiment at Mainz ([25]). The recently published results present the generalized polarizabilities at $Q^2 = 0.33$ GeV$^2$. Another experiment ran in March 2000 at MIT Bates at $Q^2 = 0.05$ GeV$^2$. The VCS experiment E93-050 that ran in the spring on 1998 at Thomas Jefferson National Accelerator (TJNAF) Facility Newport News is the subject of the present work.

VCS, as a tool for investigating the proton seems to have stirred, in the last couple of years, a lot of interest in the nuclear physics community. This has been accelerated by the development of Deeply Virtual Compton Scattering (DVCS)
DVCS is the diffractive production of a real photon in deeply inelastic scattering (DIS). The interest in DVCS comes from its potential to access the skewed parton distributions (SPD’s) of the proton. SPD’s quantify two-particle correlations in the proton, and are a generalization of the usual parton densities measured, for example, in inclusive DIS. A first observation of DVCS was reported ([29]) at DESY in Germany. There are plans for more detailed measurements of DVCS in several laboratories, e.g. CERN (COMPASS collaboration), DESY (HERMES, as well as H1 and ZEUS), and at TJNAF (E00-110).

The present thesis presents Jefferson Lab Experiment 93-050 and the analysis of virtual compton scattering data in the nucleon resonance region. The nucleon resonances are excited states of the nucleon. In the constituent quark model (see for example [30]), the proton and neutron are made of three constituent quarks, $uud$ and $udd$, respectively. The $u$- and $d$-quarks carrying charge $+2/3$ and $-1/3$, respectively, and spin and isospin $1/2$. In Quantum Chromodynamics (QCD), the constituent quark is described as a single elementary quark surrounded by a cloud of gluons and quark–anti-quark pairs. However, this complicated structure is modeled in the CQM by the mass of the constituent quarks, and by a simple interaction potential. The ground state (proton) and excited states (resonances) in the constituent quark model are the different allowed wave functions of the three quark system in its confining potential.

The spectroscopy of the excited states of the nucleon has been studied with hadronic probes for many years. The electromagnetic spectroscopy of these states remains a very active area. One of the key motivations is to look in the electromagnetic channels for the states predicted by the constituent quark model, but not observed in $\pi N$ scattering.

Real Compton Scattering in the nucleon resonance region was calculated by S. Capstick and B.D. Keister in the constituent quark model ([31], [32]). These calculations predict a coherent contribution (due to their common parity) of the “missing resonances” to the Compton cross section.

Between the quark models and QCD (see [33]), theories have calculated the characteristics of the resonances (position, width, quark structure, spin, parity,
FIG. 1: Electron scattering in the one-photon exchange approximation. Notations used for four-momenta are indicated.

etc). Measurement via various reaction channels of the baryon spectra can validate different theories and push forward the understanding of the building blocks of matter and how the stay together. TJNAF is with the high resolution and large acceptance detection capabilities, with its continuous electron beam energy range, the best research laboratory to investigate the nucleon resonances region that corresponds a 1-3 GeV center of mass energy.

1.2 Notations and Definitions

Electron scattering is a powerful tool for studying the structure of the nucleus. The electron-photon interaction is well described by quantum electrodynamics (QED), so consequently electron scattering provides a well understood probe of nuclear structure.

Electron scattering on a nucleus is an interaction described by the exchange of one or more virtual photons between the lepton (electron) and hadron currents. The exchange of more than one photon is supressed by the electromagnetic
coupling constant (\(e^2/4\pi = \alpha = 1/137\) in \(\hbar = c = 1\) units). Therefore, the 'one-photon exchange approximation' (Figure 1) is usually adopted, and processes with more than one virtual photon are treated as corrections.

The virtuality of the photon of energy \(\nu\) and momentum \(\vec{q}\) comes from its negative mass squared \(\nu^2 - q^2 < 0\). For a real particle the positive definite mass is an identifying feature. A real photon has mass 0. Detecting the scattered electron and knowing the characteristics of the incident beam, the four-momentum and polarization of the virtual photon are determined (see Figure 1). The existence of the virtual photon is limited in time and space by Heisenberg inequalities. An important characteristic of the virtual photon is the possibility of a longitudinal polarization, in which the electromagnetic field is parallel to the propagation direction.

In TJNAF experiments, electron energy is in the range of GeV, so its mass is negligible; this means that electron energy equals the magnitude of momentum. Kinematically, the interaction \(e \to e' + \gamma\) is described by the energy momentum four-vector relation:

\[
k_1^\mu = k_2^\mu + q^\mu.
\]

where \(k_1^\mu = (k_1, \vec{k}_1)\) refers to the incident electron and \(k_2^\mu = (k_2, \vec{k}_2)\) to the scattered electron. The four momentum transfer squared from the electron to the nucleus is given by:

\[
Q^2 = -q^2 = -(\nu^2 - |\vec{q}|^2) = 4k_1 \cdot k_2 \sin^2 \left(\frac{\theta}{2}\right),
\]

where \(\theta\) is the angle between the incident and scattered electron. The virtual photon polarization \(\varepsilon_v\) is

\[
\varepsilon_v = \frac{1}{1 + 2\frac{|q|^2}{Q^2} \tan^2 \left(\frac{\theta}{2}\right)}.
\]

The incident and scattered electron determine a plane named the leptonic or scattering plane. The virtual photon is then absorbed by the target nucleus and some final state particles emerge. The virtual photon and final particles determine the hadronic or reaction plane (see Figure 2). The two planes have in common the virtual photon direction. To continue a more specific kinematic definition, we
CHAPTER 1. INTRODUCTION

consider now the target nucleus being the proton and the second final particle of an unspecified nature (X).

The target nucleus is assumed to be at rest initially. The incident electron energy and momentum are considered known. The scattered electron and recoil proton trajectories and momenta are usually measured. We can write:

\[ \begin{align*}
\text{Initial particles,} & \quad k_1^\mu = (k_1, \vec{k}_1), \quad p_1^\mu = (M, \vec{0}) , \\
\text{Final detected,} & \quad k_2^\mu = (k_2, \vec{k}_2), \quad p_2^\mu = (E_p, \vec{p}_2).
\end{align*} \] (5)

where \( M \) is the mass of the proton. The four momentum of the undetected particle can be reconstructed as

\[ p_X^\mu = (k_1 + M - k_2 - E_p, \vec{k}_1 - \vec{k}_2 - \vec{p}_2) = (E_X, \vec{p}_X). \] (6)

We can then calculate the mass of the particle \( X \)

\[ M_X^2 = E_X^2 - p_X^2 , \] (7)

FIG. 2: Kinematics for the \( p(e,e'p)X \) reaction in the laboratory reference system. The out-of-plane angle \( \phi \) is defined as the angle between the leptonic plane and the hadronic plane.
and so identify its nature: photon, pion, etc. This treatment of the process is referred to as the missing mass technique.

The Mandelstam variables for the hadronic reaction $\gamma_{\nu} + p_1 \rightarrow \gamma + p_2$ are Lorentz invariants, defined by

$$s = (p_1^\mu + q_\nu^\mu)^2 = (p_2^\mu + p_X^\mu)^2 ,$$  \hspace{1cm} (8)

$$t = (p_1^\mu - p_X^\mu)^2 = (q_\nu^\mu - p_X^\mu)^2 ,$$  \hspace{1cm} (9)

$$u = (p_1^\mu - p_2^\mu)^2 = (q_\nu^\mu - p_2^\mu)^2 .$$ \hspace{1cm} (10)

It is useful to discuss the kinematics of a process not only in the laboratory frame, but also in the center of mass (CM) frame of the hadronic part of the reaction. The Lorentz boost is characterized by the speed of the CM frame relative to the laboratory frame $\beta$ given by

$$\beta = \frac{q}{\nu + M} .$$ \hspace{1cm} (11)

In the CM frame the initial particles go towards each other and the scattered particles are emitted back to back as shown for the VCS process $\gamma_{\nu} + p_1 \rightarrow \gamma + p_2$, in Figure 3.

FIG. 3: Momenta of particles in $\gamma_{\nu} + p_1 \rightarrow \gamma + p_2$ reaction, in center of mass reference system.

The energies and momenta of the protons and photons in the CM frame for the VCS reaction can be calculated if the Lorentz invariants $Q^2$ (four-momentum...
transfer) and \( s \) (center of mass frame energy) are known. The relations are:

<table>
<thead>
<tr>
<th></th>
<th>CM Energy</th>
<th>CM momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual photon</td>
<td>( q^0 = \frac{s-Q^2-M^2}{2\sqrt{s}} )</td>
<td>(</td>
</tr>
<tr>
<td>Initial proton</td>
<td>( p_1^0 = \frac{s+Q^2+M^2}{2\sqrt{s}} )</td>
<td>( p^{CM}_1 = -\vec{q}^{CM} )</td>
</tr>
<tr>
<td>Real photon</td>
<td>( q_r^0 = \frac{s-M^2}{2\sqrt{s}} )</td>
<td>(</td>
</tr>
<tr>
<td>Final proton</td>
<td>( p_2^0 = \frac{s+M^2}{2\sqrt{s}} )</td>
<td>( p^{CM}_2 = -\vec{q}^{CM}_r )</td>
</tr>
</tbody>
</table>

(12)

A very useful relation gives the angle between the virtual photon direction and final real photon direction in the CM frame by quantities in laboratory system:

\[
\cos \theta_{CM}^{\gamma\gamma} = \frac{\nu + M}{q} \frac{s + M^2}{s - M^2} - \frac{2E_p s}{s - M^2}.
\]

(13)

If \( s < 1.1 \text{ GeV}^2 = (M + m_r)^2 \), the final state particle \( X \) may only be one or more photons. Once \( s \) is bigger than this threshold a pion can be produced in the final state. The pion is the lightest particle. Increasing the center of mass energy \( s \), other reaction channels become energetically possible: two pion production, rho meson production, etc.

1.3 Overview of the Experiment

The study presented in this thesis is based on the experiment E93-050 that ran in March-April 1998 in Hall-A at TJNAF. The main objective of the experiment was the study of Virtual Compton Scattering. The experiment was one of the commissioning experiments of Hall A; this means that, in addition to the physics objectives, the experimental team performed calibration and development activities towards the full capacity and knowledge of the instrumentation.

The VCS data were collected with two goals:

- extracting the generalized polarizabilities for \( Q^2 = 1 \text{GeV}^2 \) and \( Q^2 = 1.9 \text{GeV}^2 \) below the pion threshold;
- extraction of the VCS cross-section in the resonance region at \( Q^2 = 1 \text{GeV}^2 \).
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During the experiment, besides VCS data, spectrometer optics calibration data were collected using fixed solid targets or sieve slit collimator with the liquid hydrogen target. The electron beam interacted with the hydrogen nucleus. The scattered electron and the recoil proton were detected in coincidence in the Hall A high resolution spectrometers. Their four momenta and trajectory at the reaction point were reconstructed. In the phase space accessible in the experimental set-up were possible two undetected particles: a photon or a neutral pion. The high resolution detection allowed the separation of the events belonging to the two different reactions $p(e,e'p\gamma)$ and $p(e,e'p\pi^0)$ with the missing mass technique. This thesis focuses on VCS in the nucleon resonances region. The measurements were done for a squared four-momentum transfer of $Q^2 = 1 \text{ GeV}^2$; the 9 kinematic settings (see Table I) cover a center of mass energy $s$ between 1.3 GeV$^2$ and 3.6 GeV$^2$. The kinematics were chosen to keep the angle between the virtual and real photon for VCS centered at $\theta_{TT}^{CM} = 180^\circ$ in the CM frame. This means that hadron arm spectrometer was placed along the virtual photon direction. From one experimental setting to another, we changed the electron position and momentum to keep $Q^2 = 1 \text{ GeV}^2$, as illustrated in Figure 4. The experimental apparatus is presented.

FIG. 4: Schematic representation of the experimental set-up. The dashed lines indicate a separate setting of the spectrometers for another kinematical setting.

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TABLE I: Central momentum and angle settings for the spectrometers for resonance VCS data. The spectrometer nominal (central) angles $\theta_e$ and $\theta_h$ are defined relative to the beam direction. The four momentum transferred was $Q^2 = 1 \text{ GeV}^2$.

<table>
<thead>
<tr>
<th>Generic name</th>
<th>$s$ (GeV$^2$)</th>
<th>Electron</th>
<th>Hadron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_2$(GeV)</td>
<td>$\theta_e$</td>
<td>$p_2$(GeV)</td>
</tr>
<tr>
<td>res130</td>
<td>1.30</td>
<td>3.282</td>
<td>15.77°</td>
</tr>
<tr>
<td>res150</td>
<td>1.50</td>
<td>3.176</td>
<td>16.04°</td>
</tr>
<tr>
<td>res175</td>
<td>1.75</td>
<td>3.043</td>
<td>16.39°</td>
</tr>
<tr>
<td>res200</td>
<td>2.00</td>
<td>2.909</td>
<td>16.76°</td>
</tr>
<tr>
<td>res225</td>
<td>2.25</td>
<td>2.776</td>
<td>17.16°</td>
</tr>
<tr>
<td>res250</td>
<td>2.50</td>
<td>2.642</td>
<td>17.59°</td>
</tr>
<tr>
<td>res280</td>
<td>2.80</td>
<td>2.482</td>
<td>18.15°</td>
</tr>
<tr>
<td>res320</td>
<td>3.20</td>
<td>2.269</td>
<td>18.99°</td>
</tr>
<tr>
<td>res360</td>
<td>3.60</td>
<td>2.056</td>
<td>19.96°</td>
</tr>
</tbody>
</table>

in Chapter 2. Chapter 3 outlines the optics calibration. In Chapter 4 we detail the data analysis. The theoretical aspects of the $p(e,e'p)\gamma$ cross-section calculation are presented in Chapter 5. In Chapter 6 the results are displayed and discussed.
Chapter 2

Experimental Apparatus

The present work analyzes data from the experiment *Nucleon Structure Study by Virtual Compton Scattering* [34] performed in March-April 1998, in the experimental Hall A at Thomas Jefferson National Accelerator Facility (TJNAF). The Continuous Electron Beam Accelerator Facility (CEBAF) was renamed "The Thomas Jefferson National Accelerator Facility" or "Jefferson Lab" in 1996 to mark the end of the construction phase of the facility. TJNAF is an electron scattering laboratory in Newport News, Virginia, USA built to probe the nucleus of the atom, to learn more about the quark structure of matter. The goal of this experiment was to study the reaction $\gamma_\nu + p_1 \rightarrow p_2 + \gamma$, where $\gamma_\nu$ is a virtual photon. This reaction is accessible by measuring the cross-section of the process $p(e, e'p)\gamma$. The electrons from the beam moving close to the speed of light hit the cold protons from a liquid hydrogen target. The scattered electron and recoil proton were detected in coincidence (within a 100 ns time window) by a pair of magnetic spectrometers. The emerging photon is not detected, but identified using the missing mass technique. In this chapter we present the experimental apparatus.
2.1 The Electron Beam at TJNAF

TJANF accelerator, in the spring of 1998, produced CW ("continuous wave", 100% duty factor) unpolarized electron beams of up to 4 GeV and beam current of up to 200 μA to the three experimental Halls A, B and C. The beam has a microstructure of short bursts of beam (of duration 1.67 ps) at 1497 MHz. For the unpolarized beam used in this experiment, the source of the injector was a 100 keV thermionic gun. In the injector, the beam passes through a 499 MHz chopper which sends every third beam bunch to one of three apertures (one for each experimental hall). The width of these slits determines the beam current that is delivered to the corresponding hall. The switchyard delivers the sequential bursts to the experimental halls ABCABC..., etc. The acceleration of electrons is done in superconducting radio frequency (RF) cavities. These cavities operate...
at 2 K in thermo-insulated tanks called cryo-modules. Each cavity is separately powered by a 5 KW klystron. The accelerating gradient is about 5 MV/m. In the injector cavities the electrons in the beam reach an energy of 45 MeV. They then enter the North linac (see Figure 6), a series of accelerating RF cavities which increases the electron energy by up to 400 MeV. At the end of the North Linac, the electrons are separated by magnetic fields according to their energy and then make a 180° turn guided by arc magnets. Then the electron bunches are recombined and proceed in the South Linac, where the energy is again increased by up to 400 MeV. After any complete pass through the two lines, the beam can be separated, using a combination of magnets and 500 MHz RF separators, in the beam switch-yard for use in the experimental halls. The maximum number of accelerating passes that the beam can make is five, so the nominal maximum energy of the beam during this experiment was 4045 MeV. By extracting the beam after 1, 2, 3 or 4 passes, the electron energy was 845 MeV, 1645 MeV, 2445 MeV and 3245 MeV respectively. The beam has high quality: the beam emittance is \( \approx 2 \cdot 10^{-9} \) mrad, the transverse size at the target is about 200\( \mu \)m FWHM and the fractional energy spread \( \Delta E/E \approx 10^{-4} \).

The electron beam direction and focus is controlled by numerous magnets. The position of the beam along its accelerating path and in the experimental halls is monitored by beam position monitors (BPMs). The electrons travel through evacuated pipes (pressure about \( 10^{-10} \) bar). If accidentally, the radiation level increases (for instance the beam halo interacts with the surface of the pipes), the ionization chambers placed along the electron path would immediately report high count rates and the beam is shut down automatically.

There are three beam parameters that have to be accurately known in this experiment: the beam position at the target with a precision of 100 \( \mu \)m, the current intensity of the beam to better than 1\%, and the beam energy to at least same precision as the precision of the momentum determination in the spectrometers (few \( 10^{-4} \)). For the first two there are dedicated instruments whose use and calibration is described in Section 2.3. At the time of the experiment there was no working method and instruments for direct measurement of the beam energy.
2.2 Hall A Layout

Hall A is a circular underground concrete room with a 53 m diameter. A schematic layout of Hall A is presented in Figure 6.

The beam is incident on the target through the evacuated beam-line. In the middle of the hall is the target. Most of the electrons pass through the target suffering only atomic scale interactions (associated with a momentum variation in the keV/c range) and cross the hall to a shielded beam dump. A small fraction of the electrons will interact with the nuclei in the target undergoing different nuclear reactions. The charged particles that result from a nuclear reaction can be measured with a pair of magnetic spectrometers. The spectrometers are traditionally...
TABLE II: Hall A High Resolution Spectrometers design values from the Conceptual Design Report [36].

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bend angle (vertical)</td>
<td>45°</td>
</tr>
<tr>
<td>Bend radius</td>
<td>8.40 m</td>
</tr>
<tr>
<td>Optical length</td>
<td>23.4 m</td>
</tr>
<tr>
<td>Momentum range</td>
<td>0.3 to 4.0 GeV/c</td>
</tr>
<tr>
<td>Momentum acceptance</td>
<td>±5%</td>
</tr>
<tr>
<td>Momentum dispersion</td>
<td>12.4 cm/ %</td>
</tr>
<tr>
<td>Range of scattering angle</td>
<td>12.5° to 165° (E)</td>
</tr>
<tr>
<td></td>
<td>12.5° to 130° (H)</td>
</tr>
<tr>
<td>Horizontal angular acceptance</td>
<td>± 30 mrad</td>
</tr>
<tr>
<td>Vertical angular acceptance</td>
<td>± 65 mrad</td>
</tr>
<tr>
<td>Transverse length acceptance</td>
<td>± 5cm</td>
</tr>
</tbody>
</table>

named electron arm (HRSE) and hadron arm (HRSH). The design parameters of the spectrometers are presented in Table II.

The two spectrometers are essentially identical in terms of magnetic components and by changing the polarities of the magnets their roles can be interchanged. Particles passing through the magnetic system of the spectrometers, are selected and ordered according to their momenta. After passing the magnetic system, their trajectories and other characteristics are determined by an array of detectors located in a shielded detector hut at the top of each spectrometer. In Figure 6, the detectors are shown rolled out of the shielding huts, for service.

2.3 Beam Instrumentation in the Hall

The intensity of the current used during the experiment was between 2 and 100 \( \mu A \). We surveyed the central beam position on the target to stay within 0.25mm in the plane perpendicular to beam propagation.

There are many instruments on the Hall A beam line upstream of the target:
the Compton polarimeter, beam current monitors (BCM's), beam position moni­tors (BPM's), magnets for focusing and rastering the beam, the e-p beam energy measuring apparatus, and the Möller polarimeter. A schematic representation of the beam instrumentation is shown in Figure 7. We shall focus on those elements that were employed during the experiment.

2.3.1 The Raster System

We used a liquid hydrogen target in this experiment. If we had maintained the beam on a steady trajectory through the target, the liquid hydrogen would have rapidly boiled and the density would have decreased dramatically. This phe­nomenon is the result of the energy transferred by collision of beam electrons with the atomic electrons in the target. To avoid this effect, the beam spot itself is moved (rastered) by small rapidly varying magnetic fields in a pattern covering a rectangle of up to 1 cm in height and width. The beam position on the target is determined by this fast raster and by variations in the orbit of the beam in the accelerator (primarily 60 Hz noise).

The fast raster system is located 23 m upstream from the target on the beam line. This system is called the fast raster because, initially a slow raster was installed after the target. Just as with the fast raster, the slow raster steered the beam with small varying magnetic fields in order to protect the beam dump. Presently instead of the slow raster, there is a diffuser just before the beam dump to serve this purpose. The fast rastering system consists of two pairs of steering electromagnets, one vertical and one horizontal. The current in the magnets varies sinusoidally. The frequencies were chosen such as to not form a stable Lissajoux figure - 18.3 kHz horizontally and 24.63 kHz vertically.

The current in the raster coils is read out by a Pearson probe that is presented in detail in [37]. The Pearson probe generates a voltage proportional to the raster current with a sensitivity of 1 V per 10 A.
FIG. 7: Schematic representation of the beamline instrumentation in Hall A. The upper line continues with the lower line on beam direction.
2.3.2 Beam Position Monitoring in Hall A

There are two beam position monitoring devices in Hall A, located 1.286 m (beamline element code IPM1H03B, or simpler BPMB) and 7.524 m (beamline element code IPM1H03A, or simpler BPMA) upstream of the target. The BPMs consist of a four-wire antenna array inside a resonant cavity. The four wires run parallel to the beam axis. The size and shape of the resonant cavity, as well as the material from which it is made, determine the frequency of the transverse electric and magnetic (TEM) modes possible in the cavity [35]. The mechanical parameters of the cavity are chosen (or tuned) such that the frequency at which the accelerator operates is also a resonant frequency of the cavity. This electron beam passing through the cavity excited these TEM resonant modes. The readout electronics is shown schematically in Figure 8.

The measurement does not disturb the beam, so the beam position can be continuously monitored. The information from the BPM’s is recorded on an event by event basis reflecting the rastering effect. Also, integrated values of beam position over 0.3 s are logged into the EPICS database every 4 s. EPICS (experimental physics and industrial control system) is a control system development environment used to develop the interfaces needed for remote control and monitoring of numerous devices and parameters related to TJNAF. The EPICS development environment consists of a collection of C codes and MOTIF programs, originally developed by Los Alamos National Laboratory and Argonne National Laboratory, but now in use in more than 90 independent projects, including particle and nuclear physics, astronomy and industrial applications [38]. The EPICS beam position logs only the variation of the orbit and does not include the raster effects.

Electron beam passage through the BPM induces in each antenna a signal that varies linearly with the distance between the beam and the respective wire, when the beam is near the center of the cavity. From the recorded signals \((X, Y)_{p,m}\), the beam coordinates \(X_{\text{BPM}}, Y_{\text{BPM}}\) are reconstructed as follows:

\[
X_{\text{BPM}} = \kappa X (18.87 \text{ mm}) \frac{X_p - X_{p,\text{off}} - \alpha_X (X_m - X_{m,\text{off}})}{X_p - X_{p,\text{off}} + \alpha_X (X_m - X_{m,\text{off}})}
\]  

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Hall A BPM Readout

FIG. 8: Hall A BPM read-out electronics for E93-050. The electronics for the X coordinate are not shown, but it is multiplexed into an EPICS signal as shown. The p and m are conventionally named so according to the influence on the reconstructed value of the beam position.
TABLE III: Calibrated values of offsets and relative gain of BPM.

<table>
<thead>
<tr>
<th>BPM</th>
<th>$\alpha_X$</th>
<th>$X_{p,off}$</th>
<th>$X_{m,off}$</th>
<th>$\alpha_Y$</th>
<th>$Y_{p,off}$</th>
<th>$Y_{m,off}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPM03A</td>
<td>0.975</td>
<td>-695</td>
<td>-656</td>
<td>1.109</td>
<td>-206</td>
<td>-190</td>
</tr>
<tr>
<td>BPM03B</td>
<td>0.983</td>
<td>-519</td>
<td>-464</td>
<td>1.119</td>
<td>-440</td>
<td>-415</td>
</tr>
</tbody>
</table>

\[ Y_{BPM} = \kappa_Y(18.87 \text{ mm}) \left( \frac{Y_p - Y_{p,off} - \alpha_Y (Y_m - Y_{m,off})}{Y_p - Y_{p,off} + \alpha_Y (Y_m - Y_{m,off})} \right) \quad (15) \]

The amplitude parameter of 18.87 mm is the nominal calibration of the displacement of the beam for a 100% asymmetry in the $p, m$ signals. The additional factor $\kappa_{X,Y}$ accounts for the attenuation of the signals due to the filters in Figure 8, as described below. The $X_{m,off}$ offset corrections were measured by sending a 1.5 GHz signal on one of the Y wires in the absence of the beam, with unity gain. Prior to E93-050, the relative gain of the two signals was not known or calibrated. The EPICS electronics was calibrated with an unrastered beam. We calibrated the relative gain of data acquisition electronics by comparison with the EPICS values that represent the central position of the beam without the raster deviation. The asymmetry parameters $\alpha_{X,Y}$ to first order influence the mean position of the beam raster, as measured by the BPM. We adjusted the data acquisition values of $\alpha_{X,Y}$ until the mean beam position recorded by the physics events agreed with the EPICS values (from the same time slice of data). The resulting calibrations are listed in Table III. The presence, at the end of the electronic chain of the 30 KHz low-pass filters, induced an attenuation of the signals as recorded by the experimental acquisition system. We estimated theoretically these attenuations and calibrated them against special measurements made with an EPICS chain measurement called B-scopes. The results of these calibrations are summarized in Table IV.
TABLE IV: Calibration of the magnitude of displacement from the center in BPM.

<table>
<thead>
<tr>
<th>Device</th>
<th>Direction</th>
<th>Theoretical value of attenuation $\kappa_t$</th>
<th>Experimental calibrated value of attenuation $\kappa_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPMA</td>
<td>X</td>
<td>1.1714</td>
<td>1.2171</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>1.2938</td>
<td>1.2521</td>
</tr>
<tr>
<td>BPMB</td>
<td>X</td>
<td>1.1714</td>
<td>1.2571</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>1.2938</td>
<td>1.3194</td>
</tr>
</tbody>
</table>

2.3.3 Beam Position Phase Correction

One of the technical problems in knowing the beam position on the target comes from the fact that the BPM readout is delayed in time versus the physical event time. We describe the generic beam position ($x$ or $y$) in one of the beam monitoring chains BPMA or BPMB, by

$$B = B_c(t) + A \cdot \cos(\omega \cdot t),$$

(16)

where $A$ is the amplitude of the deflection due to the fast raster, and $B_c$ is the slow time dependent central position of the beam (mostly 60 Hz noise). The BPM signals read out has an inherent delay. What we reconstruct is, in fact,

$$B = B_c(t - \delta t) + A \cdot \cos[\omega \cdot (t - \delta t)] .$$

(17)

Since the raster deflection has sinusoidal time variation in the plane perpendicular to the beam propagation, this time delay ($\delta t$) induces a fixed phase offset ($\phi_{offset}$) between the raster deflection measured value and the value at the moment of the event

$$\phi_{offset} = \omega \cdot \delta t .$$

(18)

However the fast raster readout is basically simultaneous with the physical event. It has intrinsic information about the momentary phase of the beam deflection:

$$I = I_{max} \cdot \cos(\omega X \cdot t).$$

(19)
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We do then a phase shift correction of the BPM readouts against the raster readout. To remove ambiguity in determining the deflection phase (up to a $2\pi$ term) of any event we use also the sign of the derivative of the raster current recorded for each event,

\[ \phi_{\text{event}} = -\text{sign}\left(\frac{dI}{dt}\right) \cdot \cos^{-1}\left(\frac{I}{I_{\text{max}}}\right) \]  \hspace{1cm} (20)

\[ B_{\text{corrected}} = B - A \cdot \cos(\phi_{\text{event}} - \phi_{\text{off}}) + A \cdot \cos(\phi_{\text{event}}). \]  \hspace{1cm} (21)

The raster current and raster induced deflection amplitudes are determined from histograms of the values recorded during data taking. The effect of re-phasing is illustrated in Figure 9.

The x and y beam position, determined in this way, for BPMB and BPMA, is then forward projected to the target zone.

2.3.4 Beam Current Monitoring

Measurements of the beam current must be stable and non-destructive. Two types of current monitors are used in Hall A: resonant cavities (BCM) and a
parametric DC current transformer (known as the Unser monitor after the name of its inventor).

The design of the BCM's is similar to that of BPM's but the resonant mode selected (the $TM_{101}$ mode) is most sensitive to the beam current and relatively insensitive to the beam position. The material and shape of the cavity can be chosen to adjust the quality factor $Q$ (the ratio of stored energy to dissipated power, weighted by the resonant frequency). This quality factor is sensitive to temperature fluctuations and this will directly affect the current measurement. To alleviate this problem the whole beam current measurement system is thermally insulated from the outside world and the temperature is actively stabilized.

The BCM's were calibrated against the Unser monitor. The advantage of the Unser monitor is that it has an extremely stable absolute gain. However, the zero offset of the Unser monitor can have large drifts over relatively short periods of time (hours) so it is not a reliable instrument for integrating the beam charge. The BCM's have a wide dynamic range (1-100 μA) and are stable over periods of days and weeks.

![FIG. 10: Current steps during the calibration of the BCM's against Unser.](image)

The calibration of the cavities versus the Unser monitor was done by special runs once every day during the experiment. In these special runs the current was alternately turned on and off in 5 steps as represented in Figure 10. The difference between the high and low averaged Unser signals served as an absolute reference.
for the calibration of the resonant cavities. The Unser was calibrated before the experiment with a current sent through a wire along the beam axis, simulating the effect of the electron beam.

The signals from the cavities are processed and recorded in two ways [39]:

- the HP multimeter generates a digital signal equal to a 1 second average of the cavity signal. This digital value is logged by EPICS and injected into the data stream asynchronously once every 4 seconds. Thus this database samples 25% of the charge.

- An integrated charge measurement results from generating an analog signal equal to the RMS of the cavity output. This analog signal is converted into a logic signal (VTOF) whose frequency is proportional to this analog level. The output pulses are then counted and included in the data file as part of the special scaler readout event. This is a continuous integral of the beam charge, but subject to another level of calibration. Furthermore the voltage to frequency conversion was nonlinear for $I < 10 \mu A$.

The BCM readout system is represented in Figure 11 and largely explained in [40] and [41]. Studying the stability of the cavity calibrations and the calibration of the VTOF readout against the EPICS information, we came to the conclusion that the charge for this experiment is given with an accuracy better than 0.5%, by using the following relationships [42] for the upstream cavity:

\[ Q = c_{\text{average}}(\alpha VTOF + \beta T) \quad (22) \]
\[ \alpha = 1.0194 \cdot 10^{-5} V; \]
\[ \beta = 1.77 \cdot 10^{-2}; \]
\[ c_{\text{average}} = 24.43 \mu A/V. \quad (23) \]

where $Q$ is the integrated charge, $\alpha$ and $\beta$ are coefficients, $VTOF$ is the value of the respective scaler and $T$ is the time.
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FIG. 11: Diagram of the Hall A current monitoring system.
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Beam

FIG. 12: Target ladder. Most of data was taken using the 15cm filled with liquid hydrogen target cell. For calibration purposes other targets were put in beam line by a vertical slide of the target ladder.

2.4 Target

The target assembly used during E93-050 consisted of three loops that can be filled with cryogenic target material (hydrogen for this experiment, but the target was used previously with liquid deuterium and later with helium) and a series of solid or empty cell targets used for different calibrations (see Figure 12). The ladder movement was remotely controlled allowing changes within a few minutes of the target in the beam, according to the run plan. The information presented in this section is based on [43].

The distinctive characteristics of this cryo-target are:

1. open ballast tank at room temperature;

2. forced circulation of cryogen with fans;

3. active coolant (cold He gas) flux control via Joule Thompson valves;
4. temperature stabilization with heaters.

2.4.1 Scattering Chamber

The main purpose of the scattering chamber is to protect, mechanically and ther­mally the delicate liquid and solid targets used in experiments. The scattering chamber has three sections. The bottom section is attached to the pivot of the Hall. The middle section is at the height of the beam-line and is the place where the beam interacts with the material from the target cells. The beam entrance and exit pipes are connected directly to this section. The scattered particles can reach the spectrometer after passing through the aluminum exit window that covers 93% of the circumference. Exit windows are made of 16 mil thick aluminum sheet. The upper section of the scattering chamber contained space for the cryogenic plumbing and instrumentation. The vacuum in the chamber was maintained at a level of $10^{-6}$ Torr.

2.4.2 The Cryo-Target

The cryogenic portion of the Hall A target consists of three target loops, each loop having two target cells of 15 cm and 4 cm. We used only the 15 cm H$_2$ target cell during E93-050. The main components of each loop are

- the heat exchanger,
- the loop pump (also called fan),
- the cell block,
- the heaters,
- the temperature thermometry.

Inside the heat exchanger there are three concentric cylinders filled with fin-tubing where the cooling agent circulates. The cooling agent is gaseous $^4$He produced by the End Station Refrigerator (ESR). The cooling flows at a nominal
rate of 10 g/s with a temperature of 15 K and a pressure of 12 atm. One gram of refrigerant per second supplies roughly 50 W of cooling power. The flux to a loop can be adjusted according to the cooling need, by changing the opening of a Joule-Thompson Valve. The coolant passing the valve flows through the tubing in the heat exchanger and returns to ESR at 20 K and 2.6 atm.

The pump in each loop circulates the target cryogen (liquid hydrogen) through the target cells and then through the heat exchanger, around the fin-tubing. The flow speed of the cryogen depends upon the rotation frequency of the circulating fluid. During E93-050 production running, the fan frequency was between 60-72 Hz.

The target cells are built in pairs (one cell 4 cm long and another cell 15 cm long) in cell blocks. The cells are cylinders made of aluminum. We consider the energy loss by the incoming electron and the scattered particles in the target end-caps and walls when we reconstruct the reaction kinematics.

The heaters preserve thermal equilibrium in the target. There is a 'high' power heater used to regulate the temperature of the cryogen during periods when the electron beam is off. The 'low' power heater is used to compensate for small variations of temperature in a feed-back loop.

In the cryogenic loops, temperature is monitored by several types of temperature gauges, the most accurate sensors being the Cernox temperature sensors, CX-1070-SD.

The target control system was developed on the EPICS software platform. The hardware is monitored and controlled via graphic interfaces written with TCL/TK. The remote control is performed via an Ethernet connection to an input/output controller. Information regarding cryo-target functioning is logged into a database.

The operating parameters for hydrogen are:

- temperature 19.0 K;
- pressure 26.0 psia;
The target density enters directly into the cross section calculation. The error in determining the density without beam comes from the error in measuring the temperature and pressure and the uncertainty in our knowledge of the relative amounts of ortho versus para molecules in the liquid hydrogen. All these factors combined lead to an operating density of $0.07230 \pm 0.00007 \text{ g/cm}^3$ in absence of the beam. The target material density with beam on depends on the beam intensity, the fan frequency and raster size.

During the E93-050 experiment, for 100 $\mu$A beam, the power deposited in the target was over 500 W.

### 2.4.3 Dummy and Solid Targets

On the cryo-target ladder are three aluminum dummy targets and five solid targets (see Figure 12). These targets are used for different calibrations. The three dummy targets are empty target cells, which have a length of 4 cm, 10 cm and 15 cm. We used the dummy targets for the spectrometer optics calibration (Chapter 3). The uppermost solid target on the ladder is an aluminum target with two small holes (1 mm and 2 mm square) in it. This target was designed to be used for the fast raster calibration. Further down there are a carbon and aluminum target foil. Imaging the carbon target in the spectrometers is essential in calibrating small displacements of the vertical drift chambers (VDC’s), and thus ensuring appropriate use of the trajectory information to reconstruct the reaction in the target. This is a fundamental calibration run that has to be performed and studied by each experiment whenever the VDC’s are moved inside or outside the detector hut. Next on the solid target ladder is the beryllium-oxide target. When the beam is incident on BeO, it makes the target glow brilliantly. This target is used for a visual check (via a camera) of the presence and position of the beam. At the bottom of the target ladder is an aluminum target with a circular hole in the middle. This so called empty target is used for ‘target-out-of-beam’ situations, the beam going then straight to the dump. The movement of the ladder up or down to position different targets on the beam line is done with a small motor.
remotely controlled via the target software from the Hall A counting house.

2.5 Hall A High Resolution Spectrometers

The main equipment in Hall A is the pair of nominally identical High Resolution Spectrometers (HRS). A spectrometer consists of a magnetic system and a detector package. The role of the spectrometers is to select, record and identify particles emerging from the target within an acceptance region. The main momentum selection is achieved by the curvature the magnetic dipole field imposes on the particle trajectories, invers proportional to their momenta. The effect is similar to light decomposition according to the wave length after passing through a glass prism, hence the name spectrometers. The dipole can be set for any central momenta value between 0.3 and 4.0 GeV/c. The trajectory of a particle entering the system with the reference momentum and in the direction of the spectrometer axis, has a path length of 23.4 m and a 45° bend. This trajectory is called the spectrometer central ray and is the reference for mid-plane symmetry of the system. The whole magnetic system consists of three superconducting quadrupoles and one dipole, in a QQDQ arrangement (see Figure 13). Each quadrupole has a focusing effect in one direction and defocusing in the other direction. The standard tuning for the HRS is point-to-point in the dispersive (vertical) plane and mixed focusing point-to-point (for a good position resolution) and parallel-to-point (for good angular resolution) in the transverse direction. This tune results in high momentum resolution (accuracy of determining the deviation from the central momentum value $\delta p/p \approx 10^{-4}$ for point like target), a momentum acceptance of $\pm 4.5\%$ around the central momentum value, an angular acceptance of $\pm 60$ mrad vertical and $\pm 30$ mrad horizontal, and extended target acceptance projected perpendicular on the spectrometer axis of $\pm 5$cm.

The magnetic system effect is similar to an optic system: the target is the object and the image is recorded by the detector stack (VDC, scintillators, etc.). The central momentum set for the magnetic system plays a similar role to the wave-length for an optic system. The 'central ray' is similar with the optical axis.
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FIG. 13: Representation of each Hall A spectrometer.
TABLE V: Hall A HRS forward calculated 1st order optical properties (from target to the focal plane). For instance $\partial x_{fp}/\partial \delta=11.89\text{m}/100\%$.

<table>
<thead>
<tr>
<th></th>
<th>Hadron Spectrometer</th>
<th></th>
<th>Electron Spectrometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{tg}$</td>
<td>$\theta_{tg}$</td>
<td>$y_{tg}$</td>
<td>$\phi_{tg}$</td>
</tr>
<tr>
<td>m</td>
<td>rad</td>
<td>m</td>
<td>rad</td>
</tr>
<tr>
<td>-2.170</td>
<td>-0.203</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>-0.101</td>
<td>-0.469</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>-0.595</td>
<td>-1.241</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.342</td>
<td>-0.968</td>
</tr>
</tbody>
</table>

concept. According to [44] the first order optical properties are presented in Table V.

The electron arm detector stack is illustrated in Figure 14. The VDC's determines the particle track after it passed through the magnetic system. Based on this trajectory information, the 3-momentum vector of the particle at the target can be reconstructed. The slow read-out of the VDC requires a trigger, supplied by the coincidence between the two scintillator planes. The Čerenkov detector and pre-shower-shower detectors are used to identify electron from other negative charged particles. For an exclusive reaction, the kinematics alone were sufficient to isolate electrons.

The hadron arm detectors package illustrated in Figure 15 has identical VDC chambers and scintillator planes. The hadron arm does not have the gas Čerenkov and lead-glass shower counters. Instead, there is installed a focal plane polarimeter in the hadron arm, however it was not used during E93-050.
In the following subsections I will present the detectors emphasizing the calibration issues since the constructive characteristics and description were extensively treated in the previous Hall A theses [40], [41], [45], [46], as well as in the Hall A Operating Manual [47].

2.5.1 Scintillator Hodoscope

As a high energy particle passes through a medium, it excites some of the atoms along its path to higher atomic levels. In the scintillators, these atoms decay to their ground state with the emission of a visible photon - scintillation light. This light is collected via total internal reflection in the scintillator and associated light guide. The light reaching the end of the light guide is converted to an electrical pulse and amplified by a photomultiplier (PMT).

In each spectrometer there were two planes of trigger scintillators, S1 and S2. The two planes are separated by 1.933 m in the Electron arm and 1.854 m in the Hadron arm. In each plane the dispersive coordinate \( x \) is segmented with scintillator paddles, made of Bicron BC-408 plastic. Adjacent paddles overlap by \( \approx 5 \) mm. Two Burle 8575 two inch PMTs view each scintillator one on each end, at opposite extremes of the transverse coordinate \( y \). The area of the paddles are 36 cm (transverse) \( \times \) 29.3 cm (dispersive) for S1 scintillator paddles and 60 cm (transverse) \( \times \) 37 cm (dispersive) for S2 scintillator paddles with each 0.5 cm thickness.

The plane of each scintillator array is perpendicular to the spectrometer central ray. The scintillators are used, either alone or in combination with another detector (e.g. the gas Čerenkov), to provide the physics event triggers (see Section 2.7). In addition to trigger, the scintillators provide timing and coarse tracking information. The electronics is set-up to have a master time signal in each spectrometer arm (common stop for all TDC's) always defined by S2 plane right PMT (as right is defined standing up right and looking downstream along the central ray) signal. All other times are defined relative to this common stop, and are determined by the time of particle passage together with the relative offset of
FIG. 14: Electron arm detector package as used during E93-050 experiment.

FIG. 15: Hadron arm detector package as used during E93-050 experiment.
different signals (signal delays in detector, electronics delay, cabling, etc).

The time it takes the particle to travel between the two scintillator planes allows calculation of the speed of the particle (we actually use the ratio between the particles speed and speed of light $\beta = v/c$). To do this besides understanding the signal offsets relative to the start signal, there are two other parameters to be calibrated and understood: the average speed of light through the scintillators (geometry dependent) and the pulse height correction.

A crucial experimental characteristic inferred based also on scintillator timing is the coincidence time between the particles detected in the two spectrometers. This allows us to separate between the true $H(e, e'p)X$ coincidences and the random $H(e, e')$ and $H(e, p)$ coincidence recorded in the same time window.

### 2.5.2 Vertical Drift Chambers

In order to determine the position and angle of incidence of particles passing through the detector package, each spectrometer has two VDC's separated by approximative 50cm on the particle paths. For a complete discussion of the VDC's, see Reference [48]. The VDC's are mounted on permanent rails on the floor of the shield house, between the spectrometer Q3 exit window and the detector space frame. As shown in Figure 16, each VDC has two wire planes, perpendicular to each other, in a standard $UV$ configuration (the wires of each plane are oriented at $90^\circ$ to one another and at $45^\circ$ to the $x, y$ axes). This configuration ensures complete determination of the position of the particle when passing the VDC plane and two angles associated with its trajectory. The position resolution of each plane is approximately 225 $\mu$m (FWHM) and the angular resolution is estimated at 0.3 mrad (FWHM). The active area of each VDC is 211.8 cm $\times$ 28.8 cm in the dispersive and transverse directions, respectively. Each wire plane contains 368 signal wires. The signal wires are 20 $\mu$m in diameter and are made of gold-plated tungsten. Distance between adjacent wires is 4mm. Each wire plane is oriented at $45^\circ$ with respect to the spectrometer central ray. Inside each VDC, there are three planes of negative high voltage ($-4000$ V nominal), surrounding the two planes.
FIG. 16: Schematic of the two VDC's of the electron spectrometer. The two in the hadron spectrometer are identically configured.
FIG. 17: Particle track in VDC. The geodetic paths are approximate drift path of the earliest arriving electrons on each wire. The ovals mark the approximate boundary between the region of x radial fields near wire and the region with approximate linear field.
of grounded signal wires. The chamber gas is a mixture of 65% argon (Ar) and 35% ethane (C₂H₆). When a charged particle passes through the VDC, it produces ions in the gas mixture. The electrons drift along the electric field lines defined by the high voltage planes and the signal wires. The large electric fields near the signal wires give rise to ionization avalanches resulting in detectable signals. A discriminator on each wire provides a timing pulse to start the input of a Fastbus multi-hit TDC. The TDC is stopped by the event trigger. This records the elapsed time between the initial ionization and an above threshold signal being induced on a signal wire. This drift time, combined with the electron drift velocity, yields the drift distance (see Reference [46] for details). If adjacent wires yield signals in an event, during analysis they are grouped in a cluster and their timing information analyzed together to learn the particle track (see Figure 17). The perpendicular distances (dot-dashed lines) to each hit wire are fit to reconstruct the particle's path (solid line) through the chambers. The ovals indicate the boundary between the regions of uniform electric field and the regions of radial electric field.

The trajectory information obtained from the VDC is the basis for the reconstruction of the particle trajectory and momenta at the target. Although very robust and efficient, the VDC's have to have some elements calibrated for accurate functioning as follows:

- The shortest valid TDC signal (measuring the delay between valid VDC pulse and trigger) is known as $T₀$; this timing threshold is common for a group of 16 VDC wires cabled together and has to be calibrated considering also the electronics and cabling timing offsets.

- The drift speed of the charged particles through the gaseous mixture had to be calibrated. (it is roughly $50 \, \mu$m/ns).

- The drift time to drift distance transfer function has to be understood. A combination of the geometry and the non-uniform field result in a larger effective drift speed close to the wire as shown in Figure 17. The translation from the recorded time to the distance from the anode wires where primary ionization appeared at the particles passage, has to consider this
phenomenon for the case of short time. For this time to distance translation we used a parameterization. A look-up table can be generated and used instead.

2.6 Coordinate Systems and Spectrometer Miss-Pointing

In order to describe and analyze the data of an experiment it is essential to accurately define convenient reference systems and precisely know how to pass from one reference system to another (see [49]). The position of instrumentation in Hall A is surveyed relative to the Hall A coordinate system (HCS). The origin of this system is a fixed point in the Hall A, equal to the nominal center of the target (but not defined with respect to the target since for different experiments different targets positions can be used).

\[ \hat{x} = \hat{y} \times \hat{z} \]

FIG. 18: Hall A coordinate system plan view.

The \( z \) axis of HCS points downstream along the beam-line. The \( y \) axis of the HCS points upward. The \( x \) axis is determined then as \( \hat{x} = \hat{y} \times \hat{z} \). The central
axes of the spectrometers lie in the x-z plane of the Hall A coordinate system. The central axes of the electron and hadron arms make angles $\theta_e > 0$ and $\theta_h < 0$, respective with the z axis.

Experimental kinematic characteristics are fixed by settings of spectrometer angles and fields. The spectrometers move around the pivot at the center of the experimental hall. The spectrometer positioning was done automatically, remote from the counting house. A camera mounted on each spectrometer images a set of marks painted on the floor by the Jefferson Lab survey group. The marks are on an arc at 10m radial distance from the center of the hall and spaced by 0.5°. A ruler attached to the bottom of the dipole in the field of view of a camera, serves as vernier and is used to interpolate between two marks. This determination is good to better than 1 mm and gives an angular precision better then 0.1 mrad.

However the axis of the spectrometers might not go through the Hall A coordinate system origin. Such miss-pointing induces a shift on angle and acceptance. To measure this miss-pointing a linear variable differential transformer (LVDT) was attached between the collimator box in front of Q1 quadrupole and the scattering chamber. The LVDT is used to measure the off radial distance of the spectrometer axis $d$ (Figure 19). Spectrometer miss-pointing $d$ can be in either side of the Hall A coordinate system origin and is of order of magnitude of millimeters. Based on accurate determination of miss-pointing the set angle is corrected to the real value.

To reconstruct based on the trajectory information collected by VDC's the path of the particle back to the entrance of the magnetic system, the target path is expressed in a curvilinear reference system called transport coordinate system (TCS) (see Figure 20). For each spectrometer we have a separate TCS. In principal, The z axis of the TCS is along the optical axis of the spectrometer. The x axis of TCS is defined in the momentum dispersion direction. At the target it is vertically down. The y axis of TCS forms with z axis and x axis defined before a right-handed cartesian system ($\hat{y} = \hat{z} \times \hat{x}$). The origin of the TCS is defined by a survey to a fixed fiducial point beneath the dipole.

After going through the magnetic system the particle is detected in the 2 pairs
FIG. 19: Spectrometer positioning and mis-pointing.
of VDC's. They record the particle trajectory. We specify this trajectory in a detector coordinate system (DCS). Its origin is defined by intersection of wire 184 of the VDC u1 plane and the perpendicular projection of wire 184 of the VDC v1 onto the plane u1. The y axis is parallel with the short symmetry axis and x axis is parallel to the long symmetry axis of the lower VDC pointing away from the center of curvature of the dipole. Based on the 4 intercept points of the particle trajectory with the VDC planes \(p_{u1}, p_{v1}, p_{u2}, p_{v2}\) one can calculate the detector vertex (see [52]):

\[
\tan \rho_1 = \frac{p_{u2} - p_{u1}}{d_2} \\
\tan \rho_2 = \frac{p_{v2} - p_{v1}}{d_2} \\
\tan \theta_{det} = \frac{1}{\sqrt{2}}(\tan \rho_1 - \tan \rho_2) \\
\tan \phi_{det} = \frac{1}{\sqrt{2}}(\tan \rho_1 + \tan \rho_2)
\]
These formulae are based on the assumptions that the VDC wires are at 45° to the wire frame, wires are positioned in the planes, the wire planes are parallel and separated by known distances \((d_1=13\text{ mm and } d_2=350\text{ mm as shown in Figure 21})\) and the location of the centers of the wire planes are known. Deviation from these assumptions lead to offsets that needs to be considered when calculating the coordinates of the focal plane vertex.

The focal plane coordinate system (FCS) is essential in the spectrometer analysis. This coordinate system is obtained by rotating DCS around its y axis with an angle \(\rho\) (see Figure 22). In the dispersive direction \(x\), a local central ray is defined as the trajectory of a particle with \(\theta = \phi = 0\) at the spectrometer entrance, so position is determined only by the momentum deviation from the central set value. The local ray for the momentum equal to the central momentum of the magnetic system is rotated by 45° with respect to the detector plane. This angle \(\rho\) is locally defined as the angle between the local central ray and the \(z\) axis of DCS.

The transformation from DCS to FCS including corrections for offsets or misalignment in VDC package is:

\[
x_{\text{det}} = \frac{1}{\sqrt{2}}(p_{u1} + p_{v1} - d_1 \tan \rho_1) \tag{28}
\]

\[
y_{\text{det}} = \frac{1}{\sqrt{2}}(-p_{u1} - p_{v1} - d_1 \tan \rho_1). \tag{29}
\]

In the determination of \(y_{fp}\) and \(\phi_{fp}\) from detector measured coordinates, we do not use the local angle \(\rho\), but we apply small corrections polynomial in \(x_{fp}\). In the case of the focal plane trajectory angle \(\theta_{fp}\), we have:

\[
\tan \rho = \tan \rho_0 + \sum_i t_{i000} x_{fp}^i
\]
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FIG. 21: Detector Coordinate System.

FIG. 22: The rotation from detector coordinate system ($z_{det}$) to focal plane coordinate system ($z_{fp}$).
\[ \tan \theta_{fp} = \frac{\tan \theta_{det} + \tan \rho}{1 - \tan \theta_{det} \tan \rho} \] (34)

The polynomial in \( x_{fp} \) corrections, with coefficients \( y_{000}, p_{000} \) and \( t_{000} \) are due to small shifts and tilts of the VDC chambers and are the subject of a special calibration done with solid fixed target, as part of the detector calibrations.

### 2.7 Trigger Electronics

When a particle passes through the spectrometer and is detectable, the signals from all the detectors need to be recorded to gain maximum information about the particle. The data acquisition system is activated by a signal formed by the trigger electronics, based on the fast response of the scintillators and other detectors to the passage of a particle in one or both spectrometers. If the trigger electronics enables the readout, CODA (the data acquisition system, see section 2.8) forms an event buffer with the signals from all detectors, that is written in the data file. However once this event recording is initiated, for approximately 700 µs, no other particle can be considered for read-out. Trigger electronics functions are more complex than transmitting a recording signal:

- logical processing of the scintillator information allows to decide about the passage of particles through both scintillator planes, in a pattern compatible with a trajectory through the spectrometer;
- identification of correlated particles in the two spectrometers for a time window compatible with a common reaction origin - coincidence events;
- even if the data acquisition electronics is busy with readout from an event, the particles coming through each spectrometer are counted (including all coincidences);
- to favor a certain type of event (the one in electron arm or coincidences) for each event class can be set a prescale coefficient \( N \). This means that only the \( N^{th} \) event of that type is considered for being recorded, the rest are only counted in scalers.
CHAPTER 2. EXPERIMENTAL APPARATUS

• singles events are delayed, so that coincidence events can be build and recorded as coincidence.

There are six basic types of triggers (T1, T2, T3, T4, T5, T8). T8 is a random trigger fired by a prescaled 1024 Hz clock. The other trigger types appear as a consequence of seemingly a particle going through one or both spectrometers.

T1 and T3 are the "good" electron and hadron arm singles triggers, respectively. T5 is the coincidence trigger (T1 in coincidence with T3 within a 100 ns window). T2 and T4 are the "junk" electron and hadron arm singles triggers, respectively.

A T1 trigger is formed in the electron arm by a coincidence between the two scintillator planes S1 and S2, in the S-ray configuration. A valid hit in S1 or S2 requires a coincidence between the left and right PMT of an individual scintillator paddle (there are 6 paddles in each plane). The S-ray configuration requires that the paddle number in S2 be the same or adjacent paddle number as in S1. If we label the right and left PMT logic signal from the $i^{th}$ paddle in S1 as $S1R(i)$ and $S1L(i)$ then the trigger logic can be represented symbolically as:

$$S1(i) = S1R(i) \cap S1L(i)$$

$$S2(i) = S2R(i) \cap S2L(i)$$

$$T1 = \bigcup_{i=1}^{6} \left( S1(i) \cap \bigcup_{j=-1}^{+1} S2(i+j) \right)_{0<i+j<6} \quad (35)$$

A paddle hit $S1(i)$ or $S2(j)$ is considered by the trigger logic when in a 40ns time window there are signals in both left and right phototubes of the paddle.

The trigger type 2 is the so-called 'electron junk' trigger and appears when either the S-ray pattern is not respected, or when one of the scintillator planes doesn't have a hit paddle. In the latter case a valid signal in the Čerenkov detector is required in coincidence. Symbolically:

$$S1 = \bigcup_{1 \leq i \leq 6} S1(i)$$

$$S2 = \bigcup_{1 \leq j \leq 6} S2(j)$$
\[ T_2 = (\neg T_1) \cap [(S_1 \cap S_2) \cup (S_1 \cap C) \cup (S_2 \cap C)] \] (36)

The trigger type 3 is the hadron arm correspondent of trigger 1. The trigger type 4 is 'hadron junk' is also similar to the trigger 2 except that in the hadron arm there is no Čerenkov. Trigger 4 is generated by any valid [left-right coincidence] hit in either S1 or S2 in the hadron arm.

\[ T_4 = (\neg T_3) \cap (S_1 \cup S_2) \] (37)

If a trigger type 1 and a trigger type 3 appeared in a 100ns time window they form a coincidence trigger type 5.

A simplified diagram of the HRSE, HRSH and coincidence trigger (from [40]) is shown in Figure 23.

### 2.8 Data Acquisition

#### 2.8.1 Data Acquisition Software

The software for data acquisition was build by the TJNAF data acquisition group, for Hall A from CEBAF online data acquisition (CODA), a dedicated toolkit. Data acquisition systems with different degrees of complexity can be setup using CODA routines. The data acquisition software version during E93-050 was 1.4. Standard components of a CODA system include:

- ROC (readout controller) interface to detector systems;
- EB (Event builder) collects ROC data fragments and assemble them into an event;
- ANA/DD (analyzer/ data distribution) user controlled events analysis and/or filter to disk or tape.

Figure 24 shows the way in which hardware and software are integrated in the Hall A data acquisition. When particles pass through the detector package of either the electron or hadron spectrometers, they can generate signals in the
FIG. 23: Trigger electronics.
scintillators and Čerenkov detector. If the pattern of detector signals is recognized by the trigger electronics as one of the allowed trigger types, the trigger supervisor (TS) causes an event to be recorded by the data acquisition. First, the Read-Out Controllers (ROC's) are read out. There are four ROC's in the Hall A DAQ. The ROC's are single board computers in the Fastbus and VME crates. These crates contain the ADC's, TDC's and scalers which contain the physics event information. The fragments of information from the ROC's are then collected by the event builder (EB) and put together into the CODA event format, including header and identifying information. After the event is built by the EB, it is written to disk for later analysis by the Hall A analysis program ESPACE, which is described in Section 4.1.

While data are being acquired, they can be examined using the Coda data distribution (DD) system. The DD system maintains a real-time event buffer which can be accessed by software tools. During this experiment, the program
DHIST [50], which utilized the DD system to access the data in real-time, was used to make diagnostic histograms so that the quality of data could be monitored online.

### 2.8.2 Raw Data Structure

There is much more running information than the events record needed for successful interpretation of an experiment. The data file contains:

1. event data recorded with each trigger;
2. scalers events;
3. EPICS information events;
4. special events - which are inserted once or infrequently during a run, such as prescale information and detector map.

The data come in the form of an array of 32-bit words, in the case of event data, or character data, in the case of some of the special events. In general, the first part of the event structure is header information, indicating how long the event is, what the event type is, etc.

For physics event, after the header, the data are organized in banks corresponding to each ROC: ROC1 = Fastbus on E-arm, ROC2 = Fastbus on H-arm for VDC's and scintillators, ROC3 = Fastbus on H-arm for focal plane polarimeter, ROC14 = VME crate for BPM and raster data.

The scalers are counters fed with useful information relative to the hit rates in the scintillators and Čerenkov photo-multipliers, but also useful normalization information related to the total number triggers of each type, integrated beam charge, etc. Scaler data are inserted into the data stream as an event type 140, typically every 10 seconds, but not synchronized to any other event, with hadron and electron arm alternating. These counters are accurate indicators for the quality of data both on-line and off-line. Inside the scaler events, the information is organized in banks (groups) of 16. Regardless if the information comes manly
from hadron or electron arm, the fifth bank contains normalization data. All scalers are set to 0 at the beginning of a run.

Data from various EPICS databases are periodically inserted into the data-stream relative to the beam current, position monitors, magnetic fields in our spectrometers, as well as some beamline elements, the beam energy, collimator positions, and several other things. Approximately every 30 seconds, a long list of information from the accelerator and from Hall A are inserted as event type 131. Approximately every 4 seconds a shorter list of information, for which rapid updates are desirable, are inserted as event type 131.

Prescale factors as text are part of event type 133 and is usually inserted once at the beginning of a data file. Most of the time among the first 100 events in a file there is the event 135 that contains another text buffer with the detector map. This contains information relative to the data acquisition set-up and content; this knowledge is essential for decoding the events for analysis.
Chapter 3

Spectrometer Optics Calibration

3.1 Problem Definition

The magnetic system of the spectrometers acts as an optical system for charged particles ([51]). Calibration of the spectrometer optics means establishing the relation between the measured quantities in the detectors and a particle momentum and direction at the reaction vertex position. In optics language it means measuring the source (particle) based on its image in the detectors. The coefficients of the expansion of the target coordinates in the spectrometer coordinate system \((y_{tg}, \theta_{tg}, \phi_{tg}, \delta)\), as polynomial sums of the focal plane coordinates in the transport coordinate system \((x_{fp}, \theta_{fp}, y_{fp}, \phi_{fp})\) were established during the Hall A Commissioning for a point-like target for a central momentum in the spectrometers of \(\approx 800\) MeV [52]. These expressions are

\[
y_{tg} = \sum_{i,j,k,l} Y_{ijkl} x_{fp}^{i} \theta_{fp}^{j} y_{fp}^{k} \phi_{fp}^{l}, \tag{38}
\]

\[
\theta_{tg} = \sum_{i,j,k,l} T_{ijkl} x_{fp}^{i} \theta_{fp}^{j} y_{fp}^{k} \phi_{fp}^{l}, \tag{39}
\]

\[
\phi_{tg} = \sum_{i,j,k,l} P_{ijkl} x_{fp}^{i} \theta_{fp}^{j} y_{fp}^{k} \phi_{fp}^{l}, \tag{40}
\]

\[
\delta = \sum_{i,j,k,l} D_{ijkl} x_{fp}^{i} \theta_{fp}^{j} y_{fp}^{k} \phi_{fp}^{l}. \tag{41}
\]
Mid-plane symmetry of the magnetic system requires that the exponent combinations used in the expansion be constrained in the following manner: \( k + l \) is odd for \( ytg \) and \( \phi tg \) expansions and \( k + l \) even for \( \theta tg \) and \( \delta \).

In the first calibration we assumed that the reaction vertex coordinate \( xtg \) in the Hall A coordinate system is 0 (or small and fixed). Also the coordinate \( ztg \) of the reaction vertex in the Hall A coordinate system is known and fixed. The difference between solving the calibration problem with a point like target \([52]\) versus an extended target and with rastered beam, appears from the fact that some simplifying hypotheses made for the former are unacceptable for the latter. In the first case all one needs to determine are \( ytg, \theta tg \) and \( \phi tg \) and \( \delta \) (percent deviation of the particle momentum from a reference value - central momentum) from the measured focal plane coordinates \( xfp, yfp, \theta fp \) and \( \phi fp \). It is a 4x4 determined problem.

For an extended target and rastered beam, the beam position \( xbeam \) and \( ybeam \) (in the Hall A coordinate system) are non-zero and vary from event to event. We need, therefore, to establish all three reaction vertex coordinates \( (xv, yv, zv) \) in addition to the angles \( (\theta tg, \phi tg) \) and \( \delta \). The additional information necessary to completely determine these 6 quantities, comes from taking the position of the beam at the reaction as measured by the BPM devices. In this way, two of the three coordinates of the reaction vertex are fixed and the problem is again reduced to a 4x4 determination.

<table>
<thead>
<tr>
<th>Measured quantities</th>
<th>Reconstructed quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{det} )</td>
<td>( x_{fp} )</td>
</tr>
<tr>
<td>( y_{det} )</td>
<td>( y_{fp} )</td>
</tr>
<tr>
<td>( \phi_{det} )</td>
<td>( \phi_{fp} )</td>
</tr>
<tr>
<td>( \theta_{det} )</td>
<td>( \theta_{fp} )</td>
</tr>
<tr>
<td>( x_{beam} )</td>
<td>( z_{tg} )</td>
</tr>
<tr>
<td>( y_{beam} )</td>
<td>( x_{tg} )</td>
</tr>
</tbody>
</table>

Special experimental data were collected for determining the optics of the
spectrometers, i.e. the coefficients $Y_{ijkl}, T_{ijkl}, P_{ijkl}, D_{ijkl}$. For optimizing the position reconstruction along the beam axis ($z$ in Hall A coordinate system), we used quasi-elastic data taken with foil targets, $^{12}$C($e,e'$) and $^{27}$Al($e,e'$). For optimizing angle reconstruction we used elastic $H(e,e')$ data taken with a special collimator called a sieve slit. To optimize the vertex and momentum ($\delta$) reconstruction for the entire detection area, the position in the focal plane of the elastic scattering events was moved by setting the spectrometer field at values of -4%, -2%, 0% (nominal), 2% and 4%.

In the case of an extended target, the estimation of the references used in calibration involves the use of some reconstructed target coordinates. It is then necessary to loop a few times between the $Y_{ijkl}, P_{ijkl}$ and $T_{ijkl}, D_{ijkl}$ calibrations.

### 3.2 Singular Value Decomposition Indicator

In each calibration, we try to find a set of coefficients that will minimize:

$$\chi^2 = \sum_{\text{events}} \left( \sum_{ijkl} X_{ijkl} x_{ij}^i y_{ij}^j z_{ij}^k - x_{0}^{\text{event}} \right)^2.$$  \hspace{1cm} (42)

$X_{ijkl}$ can be any of the tensors in Equations 38-41 and $x_{0}^{\text{event}}$ is the corresponding coordinate of known value. We can assume that if a set of such coefficients exists then for any considered value $X_{IJKL}$, taking the derivative with respect to that coefficient we obtain an equation

$$\sum_{\text{events}} \sum_{ijkl} X_{ijkl} x_{ij}^{i+I} y_{ij}^{j+J} z_{ij}^{k+K} - x_{0}^{\text{event}} = \sum_{\text{events}} X_{0}^{\text{event}} x_{ij}^{i} y_{ij}^{j} z_{ij}^{k}.$$  \hspace{1cm} (43)

Putting together the equations for each $IJKL$ combination considered we obtain a linear system

$$AX = B,$$  \hspace{1cm} (44)

where $X$ is the vector of the coefficients $X_{IJKL}$. Stability analysis of this linear system using the Singular Value Decomposition (SVD) indicates which terms in
the expansion should be used for tradeoff between stability and sensitivity (see [53] and [54]).

The Singular Value Decomposition theorem states that for the real \( m \times n \) matrix \( A \), there exists an orthogonal \( m \times m \) matrix \( U \), and an orthogonal \( n \times n \) matrix \( V \) such that

\[
U^T A V = \text{diag}(\sigma_1, ..., \sigma_p) = \Sigma,
\]

where \( \sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n \geq 0 \), \( \Sigma \) is an \( m \times n \) matrix and \( p = \min(m, n) \). The \( \sigma_i \) are the singular values of \( A \). The vectors \( u_i \) and \( v_i \) are the \( i \)th left singular vector and the \( i \)th right singular vector, respectively. By comparing the columns in the equations \( AV = US \) and \( ATU = V\Sigma^T \) we establish

\[
A u_i = \sigma_i u_i,
\]
\[
A^T u_i = \sigma_i v_i,
\]

where \( i = 1 : \min(m, n) \).

In the case of our linear system \( m = n \). Then we can write

\[
A = \sum_{i=1}^{n} \sigma_i u_i v_i^T = U \Sigma V^T.
\]

The solution of the linear system can be written as

\[
X = A^{-1} B = (U \Sigma V^T)^{-1} B = \sum_{i=1}^{n} \frac{u_i^T B}{\sigma_i} v_i.
\]

This expansion shows that small changes in \( A \) or \( B \) can induce relatively large changes in \( X \) if \( \sigma_n \) is small. We can quantize the impact of the variation in \( A \) and \( B \) resulting from using different ensemble of events, relative to the set of coefficients \( X_{ijkl} \) used.

To define and estimate the sensitivity of the linear system \( AX = B \), we consider the parameterized system

\[
(A + \epsilon F) X(\epsilon) = B + \epsilon f.
\]

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The relative errors in $A$ and $B$ are
\[ \rho_A = |c| \frac{\|F\|}{\|A\|}, \quad \rho_B = |c| \frac{\|f\|}{\|B\|}, \]  
where $\| \|$ stands for matrix 2-norm (for matrix norm definition see [53]).

It can be proved [53] that the relative error in $X$ is the product of the relative errors in $A$ and $B$ with the condition number $\kappa(A)$ defined as
\[ \kappa(A) = \|A\| \|A^{-1}\| = \frac{\sigma_1(A)}{\sigma_n(A)}. \]

The condition number $\kappa$ quantifies the sensitivity of the system. The smaller this number, the more stable is the solution of the linear system. In this mathematical view, the optimization can be done in phases in order to achieve a tradeoff between stability and sensitivity. We do want as many terms in the optimization as necessary to reach the desired resolution and avoid unwanted correlations. However we try to keep the condition number $\kappa$ as low as possible.

This procedure does not limit the number of coefficients that can be used in the expansion. It recommends an order or rather which of coefficients can and should be determined directly by solving the linear system $AX = B$. Based on the SVD analysis we choose a minimal representative set. In a second optimization phase, we add more coefficients for removing undesired correlation and aberrations, doing a conjugate gradient minimization for the additional ones, without perturbing the ones determined exactly. In Appendix A are listed the results of the SVD analysis of the polynomial expansion coefficients of $y_{tg}, \theta_{tg}$ and $\phi_{tg}$ as function of the focal plane coordinates (see also [55]).

### 3.3 Vertex Position Optimization

When the beam was not rastered, the position optimization was done on $y_{tg}$ which is perpendicular to the the spectrometer axis. The beam going through a foil target is a point-like interaction source. The projection of the vertices on the $y_{tg}$ direction are a point. In the case of a rastered beam, the beam hits the foil target within the rectangular raster shape, that projects as a range in $y_{tg}$. 

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CHAPTER 3. SPECTROMETER OPTICS CALIBRATION

From event to event, correlated with the beam position, the expected value of $y_{tg}$ changes. For this reason, we fit the vertex position along the beam line, $z_v$ in the Hall A coordinate system; we expect all events to have the same $z_v$ corresponding to the target foil position.

![Diagram](image)

FIG. 25: Experimental set-up for optimization along beam line.

For the optimization of the vertex reconstruction we used the carbon ($^{12}$C) foil and the empty aluminum ($^{27}$Al) targets of 4 cm, 10 cm, 15 cm length, each of which looks like two foils along the beam line, approximately symmetric to the Hall A coordinate system origin as shown in Figure 25.

The relations between $z_v$ and $y_{tg}$ are

$$z_v = -[y_{tg} + d] \frac{\cos \phi_{tg}}{\sin(\theta_0 + \phi_{tg})} + x_{beam} \cot(\theta_0 + \phi_{tg}),$$  \hspace{1cm} (52)

where $\theta_0$ is the spectrometer angle of the spectrometer axis with the $z$ axis in the Hall A coordinate system, $d$ is the mis-pointing of the spectrometer (the distance between Hall A origin and the spectromter axis, as shown in Figure 19). The coefficients are determined from a $\chi^2$ minimization procedure in which the events are reconstructed as close as possible to the known position of the foil from which they originate.

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To estimate the resolution achieved, we can check how accurately the surveyed position of the solid targets is reproduced as well as the reconstructed width of the solid targets (see in Figure 26 the improvement due to the optimization done). The position of the peaks was reproduced with an accuracy better than the actual position survey uncertainty of 0.5 mm. Expressing the reconstructed width in the beam direction is misleading because it depends on the spectrometer angle. Projecting the width back into the $y_{tg}$ direction in the spectrometer coordinate system (to remove the spectrometer angle weighting), we achieved a resolution of 0.60-0.64 mm (RMS) in electron arm, and of 1.3-1.8 mm (RMS) in the hadron arm after optimization [56].

### 3.4 Sieve Slit Plane Optimization

The sieve slit is a collimator with a 7×7 grid of holes (see Figure 27). The hole diameters are 2 mm, except for the central and one off center hole, which have 4 mm diameter. The distance from the Hall A coordinate origin to the sieve
CHAPTER 3. SPECTROMETER OPTICS CALIBRATION

Collimator is 1.1834 m for the electron arm and 1.1745 m for the hadron arm. To calibrate the trajectory angles at the target (\( \phi_{tg} \) and \( \theta_{tg} \)) we optimize the trajectory reconstruction in the sieve slit collimator plane. \( P_{ijkl} \) and \( T_{ijkl} \) are fitted using sieve slit elastic data \( H(e, e') \) for 5 different \( \delta \),

\[
\begin{align*}
y_{\text{sieve}} &= y_{tg} + l \times \tan \phi_{tg}, \\
x_y^2 &= \sum_{\text{events}} (y_{\text{sieve}} - y_{0\text{sieve}})^2, \\
x_{\text{sieve}} &= -y_{\text{beam}} + y_{\text{sp.offset}} + (z_{\text{sieve}} - z_{\text{vertex}}) \tan \theta_{tg}, \\
x_z^2 &= \sum_{\text{events}} (x_{\text{sieve}} - x_{0\text{sieve}})^2,
\end{align*}
\]

where \( l \) is the distance from spectrometer optical point (spectrometer coordinate system origin) to the sieve collimator plane. The reference positions \( x_{0\text{sieve}} \) and \( y_{0\text{sieve}} \) are inferred from the constructive description of the sieve slit collimator and survey data about the collimator position relative to the spectrometer axis. The quantity \( y_{\text{sp.offset}} \) represents the vertical offset (out of plane) of the spectrometer axis in a given position. In Figures 29 and 30 it is represented the collimator image built with the reconstructed events before and after optimization. The lines mark the nominal holes position. We see that two aspects of the this reconstruction improved as consequence of the optimization: the holes position and the focusing of the events around around the nominal position of the holes. We achieved an angular resolution in \( \theta_{tg} \) of about 1 mrad RMS in the electron arm and 2 mrad RMS in the hadron arm. For \( \phi_{tg} \), the resolution is about 0.7 mrad RMS in the electron arm and 1.2 mrad RMS in the hadron arm. The optics calibration from E93-050 data analysis is summarized in reference [56].

3.5 Momentum Reconstruction Optimization

In principle, the momentum optimization is done with elastic events of known momentum which span the focal plane (with the magnetic field adjusted in a series of runs). The quantity optimized is \( \delta \) which represents the deviation from the central ray momentum, of the momentum of the particle after exiting the target,
FIG. 27: Sieve Slit is used for angular calibration of the spectrometer optics.
corrected for spectrometer aberrations and for the nuclear recoil momentum.

$$\delta = \frac{p - p_c}{p_c},$$

(57)

where $p_c$ is the momentum of the central ray, and $p$ is the particle momentum at the spectrometer entrance.

The quantity $\delta$ is also reconstructed as a polynomial expansion of the focal plane coordinates:

$$\delta = \sum_{i,j,k,l} D_{ijkl} x_{ij}^i y_{jk}^j u_{kl}^k v_{lp}^l \phi_{fp}^l.$$

(58)

For elastic data, we fit the polynomial coefficients by minimizing the deviation of this quantity from the calculated value:

$$\delta_{\text{calculated}} = \frac{dp}{p} = \frac{p_{\text{elastic}} - p_c}{p_c},$$

(59)

where

$$p_c = B \ast \Gamma.$$

(60)
CHAPTER 3. SPECTROMETER OPTICS CALIBRATION

FIG. 29: Hadron Sieve Slit Data before (left) and after (right) calibration.

FIG. 30: Electron Sieve Slit Data before (left) and after (right) calibration.
$B$ is the magnetic field of the spectrometer. The magnetic field in the dipole is measured with pairs of NMR (Nuclear Magnetic Resonance) probes. $\Gamma$ is the spectrometer constant; $\Gamma_{\text{electron}} = 269.4$ MeV/kG, $\Gamma_{\text{hadron}} = 270.1$ MeV/kG (calibrated on $^{12}$C(e, e'p)), to be applied to the low NMR dipole field reading [61]. For a elastic scattering of angle $\theta_{\text{scat}}$ from a target of mass $M$, the expected momentum at exit from target is

$$p_{\text{elastic}} = \frac{E_0 - e.\text{loss}1}{1 + \frac{2(E_0 - e.\text{loss}1)}{M} \sin^2 \theta_{\text{scat}} / 2} - e.\text{loss}2 .$$ \hspace{1cm} (61)

$E_0$ is the incident electrons energy. $\theta_{\text{scat}}$ is central nominal angle of the spectrometer. $M$ is the mass of the target nucleus in (MeV). $e.\text{loss}1$ and $e.\text{loss}2$ are the average energy loss before the reaction and from the reaction point to the spectrometer entrance respectively. The relation between $\theta_{\text{scat}}$ and the detection angles $\theta$ and $\phi$ is given by

$$\cos \theta_{\text{scat}} = \frac{\cos \theta_0 - \tan \phi \sin \theta_0}{\sqrt{1 + \tan^2 \phi + \tan^2 \theta}} ,$$ \hspace{1cm} (62)

where $\theta_0$ is the nominal position of the spectrometer in HCS.

The hydrogen elastic data we collected in E93-050 are not well suited for performing a $\delta$ optimization, because Coulomb multiple scattering in the extended target is important, leading to significant natural kinematic broadening of the momentum distribution. Particle momentum optimization is best performed on thin targets of high atomic mass (to reduce the dependence of momentum upon scattering angle). This part of the optimization was performed on data taken as part of other experiments. To estimate the overall performance of the reconstruction, we look at coincidence kinematic quantities. The missing energy in quasielastic scattering reaction $^{12}$C(e, e'p)$^{11}$B is the difference between the energy of the incident electron plus the rest mass of carbon nucleus and the sum of the recoil nucleus energy, the emerging proton and the scattered electron corresponding to an excitation energy. In the C quasi-elastic coincidence from the E93-050 data, we can see the resolution of reconstructing the $p_{1/2}$ excitation line (see Figure 31). The FWHM of the line is 2.5 MeV which includes both spectrometers momentum resolution. Since the added energies of detected particles are greater than

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FIG. 31: The missing energy for the $p_{1/2}$ excitation in $^{12}$C($e,e'p)^{11}$B.

4.5 GeV, the resolution is $5 \times 10^{-4}$. 

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Chapter 4

Data Analysis

The goal of this data analysis is to determine the experimental cross-section value and estimate the error we make in this determination. The cross-section is the bridge between theory and experiment; it essentially gives a measure of the probability for a reaction to occur. Theoretically, it may be calculated if the form of the basic interaction between the particles is known. Experimentally the differential cross-section for a certain process is

\[ \frac{d\sigma}{dT^I}(\text{process}) = \frac{1}{F} \frac{dN(\text{process})}{dt} \],

where \( \sigma \) is the cross-section, \( dT^I \) is the \( I \)-th dimensional elementary phase-space (solid angle of particles, momenta, etc), \( \text{process} \) specifies the event selection conditions, \( \frac{dN(\text{process})}{dt} \) is the measured average process rate, and \( F \) is the density of potential scattering centers. This density is given by the number of incident particles per unit time, multiplied by with the number of target nuclei per unit area

\[ F = \frac{I \, \rho \, L \, N_A}{e \, A} \],

where \( I \) is the average current of the beam, \( \rho \) is the target density, \( L \) is the target length, \( N_A = 6.022 \times 10^{23} \text{ nuclei/mol} \) is Avogadro's constant, \( e = 1.6 \times 10^{-19}\text{C} \) is the magnitude of the electron charge, and \( A \) is the atomic weight. For hydrogen, \( A = 1.00794 \text{ g/mol} \). The cross-section has dimension of area and it is usually expressed in units of barns (1 barn= \( 10^{-28} \text{ m}^2 \)).
During this experiment the data acquisition focused on recording candidate events from the reaction $p(e, e'p)X$ ($\approx$100 GByte of data were collected in the resonance region). The data analysis encompasses the following steps:

1. instrument calibrations (most of them were described in Chapter 2) done before direct analysis:
   
   - scintillator timing optimization;
   - vertical drift chamber calibrations;
   - beam position monitor calibrations;
   - beam current monitor calibration;
   - optics optimization.

2. event selection and evaluation of the detection efficiency:
   
   - event analysis and selection;
   - trigger efficiency;
   - dead time correction;
   - target density correction;
   - background and accidentals subtraction.

3. physics extraction:
   
   - phase space volume estimation via simulation;
   - calculation of cross-sections;
   - error analysis;
   - comparison with theoretical models and interpretation of results.

Using the Hall A analysis software ESPACE (Section 4.1), the raw detector information (ADC and TDC values, etc.) is transformed into meaningful focal plane trajectory characteristics, deposited energy, speed of particles, etc. We use the processed detector information to identify the nature of a particle (electron,
CHAPTER 4. DATA ANALYSIS

proton, pion, etc.) and to estimate its emerging trajectory and momentum. Finally the whole event can be reconstructed. We select, from the analyzed events, the ones that match the process of interest, as shown in Section 4.2. Meanwhile the efficiency of detection is estimated and a correction applied to compensate for the lost events (see Section 4.3). On the other hand, we have to avoid overcounting and subtract the expected amount of random coincidences in time or space, namely the accidentals and background (see Section 4.6). The yield (integrated number of counts) is determined in bins convenient defined for the physics analysis.

The integrated luminosity estimation (eq. 64) is an important normalization aspect. Usually the beam current time integral is considered, i.e. the total charge. The target density varies as consequence of heat deposited by the beam and we need to include corrections to account for this. These corrections are described in Section 4.4.

The phase space volume is understood by simulating the experimental conditions (see Section 4.5). The simulation also allows radiative corrections to be estimated.

In assembling all this information for calculating the experimental cross section there are some systematic and statistical uncertainties. The uncertainty in the cross-section determination is discussed in the end, Section 4.7.

4.1 The Analysis Software ESPACE

ESPACE [62] is the Hall A general purpose analysis software. It was developed starting in 1992, first at the University of Mainz, Germany, and later at MIT and Jefferson Lab. This analysis software was originally adapted for Hall A by Dr. Eddy Offerman, but it has been continuously modified and improved by many users (see reference [63]). Most Hall A experiments to date have used it as the basis of their physics analysis.

ESPACE combines a CODA event decoder, tracking and reconstruction routines, the COOLHANDS histograming package, and the KUIP user interface in
a single program. In addition, ESPACE provides facilities for gain and timing calibrations ('optimizations'). It is designed to analyze the data from the two Hall A High Resolution Spectrometers (HRS), which consist of several scintillator planes, vertical drift chambers, gas and aerogel Cherenkov and shower counters.

Besides the raw data file (output of CODA) the following inputs are necessary to analyze experimental data:

- \textit{analysis kumac}, a command file which controls ESPACE execution;
- \textit{header file}, which contains run-dependent constants, such as magnet settings, reaction type, spectrometer optic point position in the Hall A coordinate system, target type and some dimensional characteristics, detected particles average energy loss in target walls, etc.;
- \textit{detector map file}, which contains the correspondence between the event readout electronics and the physical detector output;
- \textit{database file}, which contains basic calibration constants for the detectors, such as gains, offsets, physical positions, etc., and also the set of coefficients used for reconstructing the target variables of a detected particle based on the measured focal plane coordinates.

The output of ESPACE consists of ntuples (a PAW [64] format, consisting of a list of identical data structures one for each event) and histograms for the analyzed events, optionally cut with various logical conditions. These outputs can be produced for all levels of the analysis, including the raw ADC and TDC information, the position of an event within the detectors, the vertex trajectory and momentum of a particle, and the undetected (missing) final particle mass for coincidence events.

4.2 Event Selection

To identify coincidence events we apply a coincidence time cut. To the events selected by this time coincidence cut, we apply a vertex cut. We select physics
events corresponding to the $p(e, e'p)\gamma$ or the $p(e, e'p)\pi_0$ reaction, by considering the events whose missing mass square is value is around the one expected (0 for photons and $m_{\pi}^2$ for pions in final state).

In the experiment E93-050 we analyzed events in which the scattered electron and recoil proton were detected in coincidence. Their momenta and energy are reconstructed. We considered known the incident electron energy and momenta and we consider the target proton to be at rest. From the four momenta of the incident and scattered electron, initial and recoil proton, we can calculate the undetected (missing) energy and momentum for a reaction model $p(e, e'p)X$. The corresponding missing mass gives an indication of the nature of the undetected particle or particles. Within the experimental acceptance we expect the undetected (missing) final particle to be a photon ($\gamma$) with mass 0 or a neutral pion ($\pi^0$) with mass 134.9764 MeV. However in the final state can be two or more photons or one pion and one or more photons. These additional radiations have the consequence that the missing mass histogram has the peaks at $m_{\gamma}^2 = 0$, $m_{\pi^0}^2$ lowered, with tails at higher $m_{\pi}^2$.

The missing mass peak reflects some alteration of the momenta of the spectrometer detected particles due to additional interactions besides the reaction vertex. For instance the energy loss by the incoming electron and the emerging particles in the target material varies with the length of trajectory inside the target, but it also has an intrinsic distribution (modeled for this energy range with a Landau distribution). In the analysis we account (correct) only for the average value of this energy loss and the additional fluctuation have the effect of make the missing mass histogram peaks wider. The missing mass peak width is also very sensitive to the detection resolution in the spectrometers; the better reconstruction of the momenta of detected particles the sharper the peak in missing mass. Until the last decade, the resolution of electron scattering experiments around the world did not allow measuring and separating the $m_{\gamma}^2 = m_{\pi^0}^2$ and $m_{\pi}^2 = 0$ peaks from reactions. This is why exclusive VCS is a new subject with no previous data.
4.2.1 Coincidence Time-Of-Flight Cut

For a coincidence event, the time between the two spectrometer triggers corresponds to the difference in the flight times through the spectrometers of the detected electron and proton. The coincidence TOF is measured by starting a TDC when a trigger is observed on the HRSE and stopping it when a trigger is observed on the HRSH. In Figure 32) for a real coincidence event we represent the trajectories of the particles through the spectrometers. For a real coincidence we expect to observe a certain time difference. There is one beam 05 every 2 ns. Coincidences within 100 ns (50 beam bursts), trigger the data acquisition. We can have coincidence events that are the result of detecting particles that do not originate from same reaction. These kind of coincidences are called accidental coincidences (or shorter accidentals). The accidentals can come from the same beam burst, or any pair of bursts within the 100 ns relative window.

FIG. 32: Electron and hadron trajectories through the spectrometers. The time difference between the moment in which the electron hits the scintillator planes and the moment in which the hadron hits the respective scintillator planes is a selector for true coincidences.

Since the particles are relativistic, their speed is a fraction of the speed of light,
very close of the speed of light in the case of the electrons. The travel time for the
\( \approx 23 \text{ m} \) from the target to the scintillator planes is tens of nano-seconds. The
single arm trigger is determined by the right hand side (seen from the direction
of traveling particle) photomultiplier TDC for the scintillator plane S2. When
coincidence time is evaluated, the average of the left and right phototube TDCs
is taken, to diminish the fluctuation with the location of the particle along the
scintillator paddle. Applying corrections to account for the differences in the flight-
path due to momentum differences (location in the dispersive direction from one
paddle to the next), the coincidence time was further refined. In general, the TOF
for a particle traveling along the central ray of the spectrometer is given by

\[
t_0 = \frac{l_0}{\beta_0},
\]

where \( l_0 \) is the path length along the central ray and \( \beta_0 \) is the velocity of the
particle. For a particle whose path differs from the central ray by \( \Delta l \), the flight-
time would be

\[
t = \frac{l_0 + \Delta l}{\beta},
\]

where \( \beta \) is the velocity of the particle that depends of its position in the focal
plane. The path difference and beta variation according to the position in the
detection plane, are accounted for by using a polynomial expansion similar with
the spectrometer optics

\[
\Delta l = \Sigma_{i,j,k,m} L_{ijkm} x_i^j y_j^k z_k^m.
\]

The absolute value of \( t \) is irrelevant; it involves also the calibrated time offsets
for different scintillator paddles and the pulse height correction for the signals in
the phototubes. These aspects are extensively treated in reference [41]. In prac-
tice, the flight time for each particle is corrected for these effects. Then the time
difference between electron and proton is reconstructed. The significant aspect is
the width of the corrected TOF spectra. Scattered particles from reactions ini-
tiated by beam electrons from different beam bunches can be distinguished (see
Figure 33).
The real coincidences are the events in which both the electron and the proton are the result of the same nuclear reaction. To select the true coincidences we can apply a cut around the coincidence peak. The way we account for the accidental coincidences that exist in the true coincidence peak is described in Section 4.6.

4.2.2 Vertex Cut

We refine the selection of valid coincidence events by imposing a space coincidence at the reaction vertex. From the focal plane trajectory coordinates the charged particles trajectory at the target can be reconstructed. If the particles appeared as a consequence of a single nuclear reaction, their two paths should cross at the reaction vertex. The beam position in the $xy$ plane at the $z$-vertex (HCS) should also match the vertex coordinates. We chose as a measure of this spatial coincidence, the difference between the horizontal position of the vertex determined by the crossing of the two reconstructed trajectories (noted Twoarmx or 2x) and the beam position in $x$ direction (HCS noted beam$_x$) as shown in Figure 34. In the $y$ (HCS vertical) direction, the position and momentum reconstruction are entangled in vertex determination, so we must limit our spatial coincidence test.
FIG. 34: Geometric scheme of the difference between the vertex as crossing of the two trajectories reconstructed at target and the beam position.

FIG. 35: Vertex reconstruction precision.
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75
to one dimension.

The accuracy of this cut is determined by both the resolution of reconstructing the trajectory at the target in each spectrometer and the calibration of the beam position reconstruction. For the elastic calibration data the Full Width Half Maximum (FWHM) of this difference distribution is \( \approx 1.5 \) mm, centered in 0 for a beam rastered 10 mm peak-to-peak.

For all experimental settings, the FWHM of this difference stays below 2 mm. A typical distribution of the difference between the vertex determined as crossing of the reconstructed trajectories and the beam position is shown in Figure 35. The cut we used in data analysis was \( \pm 3 \) mm around the peak. To be in the right time and space coincidence window is a necessary but not sufficient condition for a true event. It might exist events that satisfy the conditions, but yet are only the result of a random matching of the conditions. These kinds of events form the background; the way to account for this aspect is presented in Section 4.6.

4.2.3 Missing Mass Cut

The true coincidences from the E93-050 experimental data were mainly due to two reactions: \( p(e, e'p)\gamma \) (the investigated process) and \( p(e, e'p)\pi^0 \). The latter falls into the experimental acceptance and has a significantly bigger cross-section, so it dominates over the entire kinematic range. An important goal in the analysis is to achieve sufficient missing particle mass reconstruction resolution to have the best possible separation between the two reactions. As we can see in Figure 36, the events with pions in final state and the events with photons in the final state can be differentiated, even for the settings in which the ratio of pion to photon events is more than 100:1 (the settings with \( s=3.2 \) GeV\(^2\) and \( s=3.6 \) GeV\(^2\)). In this work we required for \( p(e, e'p)\gamma \) events the missing mass squared to be bigger than -3000 GeV\(^2\) and smaller than 7000 GeV\(^2\), and for the \( p(e, e'p)\pi \) events the missing mass squared to be bigger than 10000 GeV\(^2\) and smaller than 40000 GeV\(^2\).
FIG. 36: Histograms of missing particle square for all kinematic settings.
4.3 Detection Efficiency

4.3.1 Prescaling and Deadtime Correction

To estimate the correct number of events with certain physics characteristics associated with a data acquisition period (file), the number of events identified in the file has to be corrected for prescaling and deadtime. Monte-Carlo simulation done by M. Liang illustrated that different triggers can have different deadtime corrections, reference [65]. The prescaler reduces the number of events of a certain trigger type. A prescale factor $p_{si}$ means that, of the events of type $i$, only one in $p_{si}$ is attempted to be recorded. A second reduction in the number of physics events recorded in a data file occurs because data acquisition system is dead for 700 $\mu$s after an event, and can not attempt record another event during that time. To account for this loss of events due to prescale and deadtime we calculated some multiplicative correction factors. We use $T_1$, $T_2$, $\ldots$, $T_5$ for the number of events of the indicated trigger type found in a data file ($T_1$, $T_2$, $\ldots$, $T_5$ stand for trigger type 1, 2, $\ldots$, 5), $S_1 - S_5$ are the total number of raw triggers as recorded by the scalers for the same data file. The scaler will count all triggers including the ones prescaled or the ones that data acquisition can not record due to dead-time. A simple definition of the deadtime multiplicative factor is the ratio between the expected number of events in the data file ($S_i/p_{si}$) and the number of events actually found in the data file ($T_i$). However we must also take into account the fact that trigger 5 (coincidence) is formed when the single triggers 1 and 3 appear in a 100 ns time window. Thus coincidence events are counted in $S_5$, but also in $S_1$ and $S_3$ totals. We avoid double counting when estimating the trigger 1 and 3 deadtime multiplicative correction. Trigger types are exclusive: a T5 event is neither a T1 nor a T3 event. Consequently:

$$dt_1 = \frac{S_1 - S_3}{p_{s1} \cdot T_1}, \tag{68}$$

$$dt_2 = \frac{S_2}{p_{s2} \cdot T_2}, \tag{69}$$

$$dt_3 = \frac{S_3 - S_5}{p_{s3} \cdot T_3}, \tag{70}$$

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These deadtimes corrections are used for both the cross section evaluation and the trigger efficiency estimation. The deadtime correction is specific to each run and determined based on scalers blocks over the portions of the runs used in the analysis (see Section 4.4 where run tailoring is discussed).

4.3.2 Trigger Efficiency

It is necessary to apply corrections to the data to account for loss of events because of the inefficiency of the scintillators and the trigger logic. This correction must be applied either as an efficiency correction, to the acceptance studies (VCSSIM, MCEEP, etc.) or as an event-by-event weight $1/\epsilon(i)$ in the data. We used the latter method.

The efficiency $\epsilon$ (in percentage) of each single arm trigger is defined as

$$\epsilon(\%) = 100 \cdot \frac{\text{No.Det}}{\text{No.Potential}}$$

where

- \textit{No.Det} means the number of valid triggers recorded (detected) in the CODA data file. \textit{No.Det} includes analysis cuts and corrections for prescale factors, deadtime, and tracking normalization (as presented in Section 4.3.5). For the electron arm, \textit{No.Det} includes both T1 and T5 events; for the hadron arm, \textit{No.Det} includes both T3 and T5 events.

- \textit{No.Potential} is the “potential” number of valid triggers. \textit{No.Potential} is \textit{No.Det} plus the number of events identified in the ‘junk’ trigger (T2 or respective T4) that should have formed a valid trigger. Events included in \textit{No.Potential} must pass all the same analysis cuts as for \textit{No.Det}. \textit{No.Potential} includes the same correction factors as \textit{No.Det}.
To identify the ‘junk’ (T2) electron trigger events that should have formed a valid trigger, we used wide cuts on Čerenkov ADC sum and on the energy deposited in the shower and pre-shower (Figure 37 displays data after cuts). We also exclude from T2 (T4) the non S-ray $S1 \cap S2$ coincidence T2 (T4) triggers, since we know that these are either bad tracks or random coincidences between $S1$ and $S2$. We applied the same cuts on the Čerenkov ADC sum and on the energy deposited in the shower and pre-shower for the valid trigger events (T1 and T5).

We did not treat the events with multiple tracks in one spectrometer, but we did a tracking normalization. For the purposes of the trigger efficiency estimation, we assume that the multiple track events include a primary particle with same distribution as the single track events. So we analyzed the single track events and then normalize to the total number of events with tracks (one or many). Formally
we can then write (for the electron arm)

\[ No.\text{Det} = N_{\text{good}T_1} \cdot f_1 + N_{\text{good}T_2} \cdot f_2, \]

\[ No.\text{Potential} = No.\text{Det} + N_{\text{good}T_3} \cdot f_3, \]

where \( N_{\text{good}T_i} \) is the number of events of trigger type \( i \) with one VDC track and passing the detector and geometry cuts and the factors \( f_i \) are

\[ f_i = dt_i \cdot \text{track}_i \cdot ps_i. \]

Here,

- \( dt_i \) is the dead time correction of trigger \( i \),
- \( \text{track}_i \) is the ratio of the number of events of type \( i \) with at least one track to the number of events of type \( i \) with only one track,
- \( ps_i \) is the prescale factor for events of type \( i \).

To study the good events of triggers 2 and 4 (in addition to the Čerenkov and shower-energy cuts) we use the trajectory projection onto the scintillator planes. If there is a track in the VDC we can analyze the coordinates of the crossing point of the VDC track with each scintillator plane. As long as these coordinates stay in the physical limit of the scintillators planes we consider the event good.

The correction factor that we apply in the cross section calculation is

\[ \text{corr} = \frac{No.\text{Potential}}{No.\text{Det}}(x_{\text{det}}, y_{\text{det}}). \]

This factor should be implemented as a weight applied to each individual event according to its \((x, y)\)-coordinate crossing with the second scintillator plane. We choose S2, because the observed efficiency variations are most clearly correlated with the geometry of the S2 paddles (see Figure 38). The correction factor also depends upon run number (actually a time dependence).

The error in estimating the efficiency (or equivalent trigger efficiency correction coefficient) is mainly of a statistical nature. By estimating the efficiency correction
FIG. 38: Projection of the VDC tracks in the scintillator planes, for the good T2 events in Electon Arm; eys1det and exs1det are the coordinates in the scintillator plane 1 (in m) and eys2det and exs2det are the coordinates in the scintillator plane 2 (in m). The upper two histograms plot the events that have no hit in scintillator plane 1. The lower two histograms plot the events that have no hit in scintillator plane 2.
coefficients in a collection of runs, we increase the number of events of each type analyzed and lower the statistical error.

We trace now the error propagation. For the $f_i$ coefficients there is no error on the prescale factors $p_{si}$ and a very small statistical error, less than $2 \cdot 10^{-4}$ in a typical run, for the dead-time correction. The error induced by the normalization factor related to the number of tracks identified in an event is:

$$
\delta_{\text{track}} = \frac{N(\text{track} \geq 1)}{N(\text{track} = 1)} \sqrt{\frac{1}{N(\text{track} \geq 1)} + \frac{1}{N(\text{track} = 1)}}
$$

(78)

Consequently, the error on $f_i$ is

$$
\frac{\delta f_i}{f_i} = \sqrt{\left[\frac{\delta_{\text{track}}}{\text{track}}\right]^2 + \left[\frac{\delta_{dt_i}}{dt_i}\right]^2}
$$

(79)

The corrected number of good events of trigger type $i$ in a kinematic bin $j$ (coordinate $(x, y)$ in the scintillator plane) is

$$
N_i(j) = N_{\text{good}Ti}(j) \cdot f_i,
$$

(80)

where $N_{\text{good}Ti}(j)$ is the actual number of good (one VDC track) events in bin $j$ of trigger type $i$. The error we make on $N_i(j)$ in the trigger efficiency calculation is then

$$
\frac{\delta N_i(j)}{N_i(j)} = \sqrt{\frac{1}{N_{\text{good}Ti}(j)} + \left[\frac{\delta f_i}{f_i}\right]^2},
$$

(81)

where $i$ refers to 1, 2, 5 for the electron arm (respectively 3, 4, 5 for hadron arm trigger efficiency study).

We rewrite the electron- and hadron-arm trigger efficiency corrections for bin $j$ in each arm as

$$
corr_e(j_e) = 1 + \frac{N2(j_e)}{N1(j_e) + N5(j_e)},
$$

$$
corr_h(j_h) = 1 + \frac{N4(j_h)}{N3(j_h) + N5(j_h)}.
$$

(82)

When estimating $corr_e$, we do not make a correction for the hadron arm trigger efficiency in $N5$, and vice versa for the $corr_h$. This introduces a small systematic
error into the calculations. The statistical errors on the trigger efficiency correction factors are

\[
\left[ \frac{\delta \text{corr}_e(j)}{(\text{corr} - 1)} \right]^2 = \left[ \frac{\delta N 2(j)}{N 2(j)} \right]^2 + \frac{\delta N 1(j)}{N 1(j) + N 5(j)}^2 + \frac{\delta N 5(j)}{N 5(j)}^2
\]

\[\text{(83)}\]

\[
= \left[ \frac{1}{N_{\text{goodT}}^2(j)} + \left( \frac{\delta f 2}{f 2} \right)^2 \right]
+ \left[ \frac{N 1(j)}{N 1(j) + N 5(j)} \right] \left[ \frac{1}{N_{\text{goodT}}^1(j)} + \left( \frac{\delta f 1}{f 1} \right)^2 \right]
+ \left[ \frac{N 5(j)}{N 1(j) + N 5(j)} \right] \left[ \frac{1}{N_{\text{goodT}}^5(j)} + \left( \frac{\delta f 5}{f 5} \right)^2 \right]
\]

\[\text{(84)}\]

We bin the correction coefficients in the scintillator plane coordinates. While the track correction (and consequently \( f_i \)) refers to the global quantities, the \( \text{No.Det} \) and \( \text{No.Potential} \) are bin related and the statistics in each bin determines the error in estimating the trigger efficiency. The scintillator plane is bigger than the acceptance limits. In some bins, the statistics is very poor and the error is very large. It is then reasonable to use, for correction of these bins, the efficiency (correction) of the nearest bin with reasonable statistics in valid events.

Summarizing, the trigger efficiency correction \( \text{Corr}^{\text{eff}} \) is a characteristic for each event according to its position in scintillator plane 2 of each spectrometer arm

\[
\text{Corr}^{\text{eff}} = \text{corr}_e(j) \cdot \text{corr}_h(j'),
\]

\[\text{(85)}\]

where \( j \) stands for the \((x, y)\) bin in scintillator plane 2 (it was proven sufficient to use only bins in \( x \) for the hadron arm).

### 4.3.3 Electron Arm Trigger Efficiency in E93-050

From the point of view of electron scintillator phototubes the experiment E93-050 can be split into two parts. Up until March 24, 1998 (run 1943) the phototubes of paddle 4 in scintillator plane 2, for electron arm, functioned worse than all the others (see Figure 38).
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Figure 38 displays the VDC track projection on the scintillator planes for the T2 (junk trigger) events during the early part of E93-050 run. In addition to paddle 4 inefficiency, we observed edge effects at the paddle overlap resulting in an increase of 2-4% in inefficiency. If the paddle edges were butted together, this would be a natural consequence of a track not fully hitting either scintillator. However, adjacent paddles overlap by 5 mm, specifically to avoid this problem. Therefore, the origin of these boundary inefficiencies is not understood. We studied the distribution of the 'good' type 2 events, for the case in which there is no hit in plane 1 or there is no hit in plane 2 (see Figure 38); we noticed that the bad paddle (phototubes) are in plane 2 while the edge effects seem better localized in plane 1. This might suggest that there are gaps between the scintillator paddles in the electron arm scintillator plane 1. To test this hypothesis we looked at the distribution of the multiple scintillator hit events for the valid triggers T1 and T5; if an event goes through a paddle overlap it should generate hits in two paddles.

FIG. 39: Distribution of the valid events (T1 or T5) with two scintillator paddles hit in S1(up) or S2(down); the events grouped along the paddles' overlap confirm the physical coverage.

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FIG. 40: Trigger efficiency correction along paddle 4 black star, and along paddle 1 blue triangle in the first part of the experiment.

in that plane.

In addition to an uniform spread for real multiple particle events, we clearly see the paddles' edges (see Figure 39). This indicates that the paddles are overlapping although the trigger efficiency in the overlap region is lower than the rest of the paddle.

After March 24 (when the tubes from paddle 4, scintillator plane 2, electron arm were changed), there was a period in which the efficiency was good over the entire electron scintillator plane. However, towards the end of the experiment the efficiency of the paddle 6, scintillator plane 1, electron arm dropped significantly.

Due to a bad phototube at the end of one paddle there is not only an $x$ dependence in the variation in the correction coefficient for the trigger efficiency, but also a $y$ dependence. As Figure 40 shows, the efficiency varies in the $y$ direction for the least efficient paddle more than for a normally functioning paddle.

The definition of the trigger efficiency correction can be done as a function of
FIG. 41: Trigger efficiency correction across the paddles for the first part (established for the runs 1800-1836) on top and the second (bottom) part of the experiment (established for the runs 2281-2389). For each x2scintillator bin there are 16 y2scintillator bins; points for all the bins are represented on the plot function of only x2scintillator; the width of this overlap is due to the efficiency variation along y coordinate.
the first or second scintillator coordinates. Ideally we would do it based on all four coordinates \((x\) and \(y\) from each scintillator plane). However the finer the bins we use, the less T2 events we obtain per bin, and we end up by doing a fine grained correction with a big error. As one can see in Figure 38 shows the bad paddle correction is more clearly indicated in scintillator plane 2, while the edge effects are better localized in scintillator plane 1. However, the position in S1 and S2 are highly correlated, so we are relatively free to make a correction in based on the VDC track projection in just one plane. We defined for electron arm efficiency correction bins of 1 cm in \(x\) and 2 cm in \(y\) in scintillator plane 2. Figure 41 shows the correction coefficient we apply to our data to compensate for the scintillator, and consequently the trigger, inefficiency.

4.3.4 Hadron Arm Trigger Efficiency in E93-050

The hadron arm efficiency does not show as strong localized effects as in the electron arm. There is, however, a smooth variation in the efficiency correction, determined in the same way as for the electron arm (see Figure 42). For the hadron arm efficiency correction it was not necessary to bin in both \(x\) and \(y\), so we used bins of 1 cm in \(x\) only. We did not observed a time evolution of the scintillator efficiency. However we evaluated the correction coefficients over a large collection of runs to have enough trigger 4 events to evaluate them. At the extreme the coefficients seem to increase sharply; however we never had good events in those edge bins. The paddle overlapping is not so obvious in the case of hadron arm. This is due to the fact that in case of the hadron arms the 'junk' events of trigger T4 are formed with only one scintillator plane hit.

4.3.5 Tracking Efficiency

The VDC track reconstruction quality has a major impact on the experimental data analysis because the coordinates of the track in the VDC are the raw information based on which the particle trajectories and momenta at the reaction point are reconstructed. There are two aspects that we addressed relative to VDC
FIG. 42: Hadron arm efficiency vs x coordinate of the first hadron arm scintillator plane. We did not observe a variation of the trigger efficiency correction coefficients along y scintillator coordinate (along the paddle) for the hadron arm, so we used one dimensional bins.

tracking:

1. How we deal with the multiple track events;

2. How we can ensure the best possible quality of the track reconstruction.

More than one particle can be detected in the same acquisition time window for either spectrometer. In the VCS resonance coincidence data 7-15% of the events (depending on the setting) had multiple tracks in the electron arm, and 2-4% of the events had multiple tracks in the hadron arm. Although the individual tracks are reconstructed with a satisfactory algorithm in ESPACE, we decided that we do not want to confront the pairing uncertainty (which of the tracks really comes from the reaction of interest). We assume that the probability to have good events in the multiple VDC track events is the same as in the single track events. So we remove the multiple track events and finally we renormalize the number of counts.
applying a correction:

\[
Corr_{VDC}^{\text{track}} = Corr_{VDC}^{Etrack} \cdot Corr_{VDC}^{Htrack} = \frac{N_{VDC}^E + N_{\geq 1\text{track}}^E}{N_{\text{track}}^E} \times \frac{N_{VDC}^H + N_{\geq 1\text{track}}^H}{N_{\text{track}}^H}
\]

This correction coefficient is specific to each run used in the analysis.

Most VDC clusters have 4-6 wires. However tracks can be reconstructed also from only 3 wires and there are events with 7 or more wires. Studying the \(\chi^2\) of track fitting in each plane as function of the number of wires in the cluster and the angle difference between the angles fit in one plane or based on intercepts in the two planes, we came to the conclusion that it is most reliable to keep in the analysis only those events that have 4-6 wires in the cluster in each plane.

In Figure 43 (left) it is a diagram of a typical VDC track. On the right side we have a multi-wire event that suffered a scattering at the sense wire; it is likely that the consequence of such path bending is to increase the number of VDC wires hit for that particle track. We account for them in the same manner as for the multiple tracks, by omitting these 3 wires and 7 or more wires events from our analysis in order to ensure best quality of VDC track reconstruction. We then correct for the omitted events assuming that their distribution is the same as for
selected events

\[ \text{Corr}_{VDC}^{\text{multiplicity}} = \text{Corr}_{VDC}^{E\text{multiplicity}} \times \text{Corr}_{VDC}^{H\text{multiplicity}} = \frac{N_{\text{any} - m}^E}{N_{\text{ideal} - m}^E} \times \frac{N_{\text{any} - m}^H}{N_{\text{ideal} - m}^H}, \quad (87) \]

where \(\text{any} - m\) label stands for any multiplicity, and \(\text{ideal} - m\) stands for ideal multiplicity (4-6 hits per plane). This correction coefficient is determined for each run.

### 4.4 Flux Normalization

As shown at the beginning of this chapter (Eq. 63), to determine the cross section we divide the rate of observed processes with the rate of possible interactions between the beam electrons and target nuclei. In practice we use the time integrated values of these quantities over the interval of time in which the data acquisition was actively creating data files. We count the events that satisfy desired conditions as shown in Section 4.2, for the duration of a data acquisition period (run). We evaluate then

\[ \mathcal{F} = \int F \, dt = \int \frac{I \rho(I) L N_A}{e A} \, dt \approx \frac{\rho(I) L N_A}{e A} Q. \quad (88) \]

Assuming that the target density is constant, the time integral of the luminosity is the product of charge, density (corrected for the variation due to heat deposited by the beam) and target length \(L\).

When the beam goes off in the middle of a run, there is a lack of confidence about the charge measurement, because the cavity current monitoring is unreliable for currents below 10 \(\mu\)A. When the beam comes back on, the target suddenly gets a heat load, and it takes a few tens of seconds until stability is reached via the feedback mechanisms. For this reason we decided to tailor the data files using the scaler blocks as a reference. The scalers in the 5th bank of the hadron arm scaler event (we used the hadron arm scaler because it contained more information) contained:
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Channel Scaler
1-5 T1-T5
8 clock (T8)
13 accepted-triggers (all)
14 upstream BCM
15 Unser
16 downstream BCM

The channel 13 content is correlated with the event number in the data file. If the beam went off during a data acquisition period (obvious from monitoring the VTOF rate obtained by dividing the variation of scaler 14 or 16 by the time interval as indicated by the channel 8 clock), we remove the events recorded just before the current dropped and 4 scaler blocks after the beam comes back on, using the recorded event counter from channel 13. We then calculate $F$ for each good subsection of a run, and for all portions of the run used in the analysis.

In section 2.3.4, the hardware used in the current, and consequently the charge, measurement was presented in detail. We concluded that the best (precision better then 0.5%) measurement of the charge comes from the VTOF scaler.

Although the target was controlled to function at a stable temperature and pressure, due to the way heat load is spatially deposited and dissipated, the target density in the beam line has small variations. The parameters on which it depends are the beam current, the raster size (surface) and the fan frequency. The main dependence is on the beam current; we made an ad-hoc study of this dependence using the single arm event rates in the electron and hadron arms. The single arm event rate varies not only due to heat effect but is also strongly influenced by any beam position variation. To isolate for the pure heat effect we compute the average of the electron and hadron single rate variation with the current. The beam current intensity was constant during the data taking of a certain kinematic setting and only for one setting we do have beam intensity variation between 20-80 $\mu$A. Surveying all the data from all settings we concluded that, in the target density, we have to apply a correction

$$\rho(I) = \rho_0 (1 - c_{\text{beam}} \times I),$$

(89)
where \( c_{\text{boil}} = 0.00030 \pm 0.00005 \text{ A}^{-1} \). All numbers used run by run for charge measurement and normalization are listed in Appendix C.

### 4.5 Simulations

#### 4.5.1 MCEEP

MCEEP [66] is a Monte Carlo simulator for \((e, e'p)\) experiments. MCEEP employs a uniform random sampling method (as opposed to the importance sampling method) to populate the experimental acceptance. An event is defined as any combination of variables which completely specifies the reaction in the laboratory. For each event, weights which correspond to the relevant observables (cross sections, polarizations, etc.) are computed according to a user selected physics model. The histograms represent the accumulation of these weights.

MCEEP can perform calculations for elastic scattering, to bound states of the residual system or in the (unrestricted) continuum according to the user’s choice. We used the bound state case, for which MCEEP performs a five-dimensional integral, wherein the ejectile momentum and the values of the other five randomly selected kinematical variables (the electron momentum and in-plane and out-plane angles and the ejectile in-plane and out-of-plane angles) are calculated from the bound state missing mass (specified in the input file). The program allows simulation of the Hall A spectrometers by a series of "spectrometer elements" specified in the input, which act on a Transport (see [67]) vector based on the laboratory coordinates directly sampled in the Monte Carlo event loop. MCEEP includes radiative effects and realistic multiple scattering treatment. Both internal and external radiation are handled. In the case of the bound state, the prescription of Borie and Drechsel [68] is used for the calculation of the radiative tail in the peaking approximation (this prescription is also used for the continuum problem). The routines for external radiative, energy loss and multiple scattering treatment in MCEEP are detailed in Appendix B. The output of MCEEP consists of an Ntuple (for both kinematics variables and Transport variables) whose content and
associated weights are defined via the input file.

MCEEP performs integrals of the partial differential cross sections to compute yields as well as cross sections averaged over the experimental acceptance, by calculating integrals over the appropriate phase space volume.

In general, for an \((e, e'p)\) coincidence measurement with a beam of fixed energy and orientation, there are six variables needed to describe the reaction in the laboratory. These six variables specify the four-momenta of the scattered electron and ejectile (for a given ejectile mass). For scattering to discrete states of the residual nuclear system, in which case the missing mass (i.e. nuclear excitation) is constrained, one effectively has a five-fold acceptance volume. The MCEEP yield is obtained as

\[
y' = \mathcal{F} \int_{\Delta v_i} \frac{d^5 \sigma_{i\text{model}}(k_1, k_2, p_2)}{dk_2 \, d\Omega_e \, d\Omega_p} \, dk_2 \, d\Omega_e \, d\Omega_p ,
\]

where \(\mathcal{F}\) is defined in eq. 88, \(\Delta v_i\) is the multi-dimensional phase space consistent with the experimental acceptance volume (\(i\) indicates that the integral should extend only over the volume of the \(i\)-th kinematical bin), \(dk_2\) is the final electron energy, and \(d\Omega_e\) and \(d\Omega_p\) are the electron and hadron solid angles respectively. The MCEEP cross section for simulated coincidence experiments with known missing particle mass, averaged in a certain (predefined) kinematic bin is

\[
\left\langle \frac{d^5 \sigma}{dk_2 \, d\Omega_e \, d\Omega_p} \right\rangle_i = \frac{\int_{\Delta v_i} \frac{d^5 \sigma_{i\text{model}}}{dk_2 \, d\Omega_e \, d\Omega_p} \, dk_2 \, d\Omega_e \, d\Omega_p}{\int_{\Delta v_i} 1 \, dk_2 \, d\Omega_e \, d\Omega_p} ,
\]

where the denominator is the phase space volume for kinematic bin \(i\). If we simulate also radiative effects then

\[
Y_i = \mathcal{F} \int_{\Delta v_i} \frac{d^5 \sigma_{i\text{model}}(\tilde{k}_1, \tilde{k}_2, \tilde{p}_2)}{dk_2 \, d\Omega_e \, d\Omega_p} \otimes f_R(q'') \, dq'' \, dk_2 \, d\Omega_e \, d\Omega_p
\]

where \(dq''\) is the radiated energy, and \(\tilde{k}_1, \tilde{k}_2\) and \(\tilde{p}_2\) are the modified vertex kinematic variables. We use \(\otimes f_R\) to show the convolution of the model cross section \(d^5 \sigma\) with the radiative tail. The average cross-section in the bin, in this case is

\[
\left\langle \frac{d^5 \sigma_i}{dk_2 \, d\Omega_e \, d\Omega_p} \right\rangle_i = \frac{\int_{\Delta v_i} \frac{d^5 \sigma_{i\text{model}}}{dk_2 \, d\Omega_e \, d\Omega_p} \otimes f_R \, dk_2 \, d\Omega_e \, d\Omega_p}{\int_{\Delta v_i} 1 \otimes f_R \, dq' \, dk_2 \, d\Omega_e \, d\Omega_p} ,
\]
Practically the experimental cross section values we obtain

\[
\frac{d^5\sigma_i}{dk_2 d\Omega_e d\Omega_p} = \left\langle \frac{d^5\sigma_i}{dk_2 d\Omega_e d\Omega_p} \right\rangle_{\text{model}} \times \left( \frac{d^5\sigma_i}{dk_2 d\Omega_e d\Omega_p} \right)_{\text{experiment}}.
\]

\[\mathcal{F} \int_{\Delta \nu} \frac{d\sigma^\text{model}(k_1, k_2, \vec{p}_2)}{dk_2 d\Omega_1 d\Omega_p} \otimes f_R(q'') \frac{d\nu}{dk_2 d\Omega_e d\Omega_p} \]

Regardless what cross-section model is used, the process goes through few iterations, by replacing the cross section model with previous obtained values of the experimental cross-section.

### 4.5.2 Photon Flux Model

In the resonance model, we used two simple cross section models to simulate the experiment: A constant (5-fold) differential cross section; and the virtual photon flux times a constant photo-absorption cross section. In the future, we intend to use the theoretical model described in Chap. 6.2 to simulate the experimental acceptance.

In the one photon impulse approximation, the unpolarized coincidence electroproduction cross section has the form

\[
\frac{d^3\sigma}{d^3k_2 d\Omega_{p_2}} = \frac{d^3\Gamma}{dk_2 d\Omega_{k_2}} \frac{d^2\sigma}{d\Omega_{p_2}}.
\]

The first factor is called the virtual photon flux and has the formula

\[
\frac{d^3\Gamma}{dk_2 d\Omega_{k_2}} = \frac{\alpha}{2\pi^2} \frac{k_2}{k_1} \frac{s - M^2}{2MQ^2} \frac{1}{1 - \epsilon},
\]

where \(\alpha\) is the fine structure constant and \(\epsilon\) is the virtual photon polarization

\[
\epsilon = \frac{1}{1 + 2Q^2 \tan^2 \left( \frac{\theta_{\text{scattering}}}{2} \right)}.
\]

The Rosenbluth decomposition of the two-fold differential cross-section is:

\[
\frac{d^2\sigma}{d\Omega^\text{CM}_{p_2}} = \frac{d^2\sigma_T}{d\Omega^\text{CM}_{p_2}} + \epsilon \frac{d^2\sigma_L}{d\Omega^\text{CM}_{p_2}} + \sqrt{\epsilon(1 + \epsilon)} \cos \phi \frac{d^2\sigma_{LT}}{d\Omega^\text{CM}_{p_2}} + \epsilon \cos(2\phi) \frac{d^2\sigma_{TT}}{d\Omega^\text{CM}_{p_2}}.
\]
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where:
- $\sigma_T$ is the cross-section for absorption of transverse polarized photons;
- $\sigma_L$ is the cross-section for absorption of longitudinal polarized photons;
- $\sigma_{LT}$ is the interference between the two amplitudes;
- $\sigma_{TT}$ is also called the linear polarization asymmetry.

Virtual Compton Scattering is the electro-production of the photon. However in our experiment we measure the $p(e, e'p)\gamma$ cross-section that is a coherent sum of the VCS and BH amplitudes (see Section 5.1). The dynamics of the BH amplitude does not allow a separation of the virtual photon flux factor. However, in part of the kinematics, the VCS amplitude dominates over the BH process. In this case the decomposition of Eq. 98 is appropriate. Furthermore the reaction $p(e, e'p)\pi^0$ is included in our acceptance. This reaction can be described exactly by Eq. 98.

The dynamics of the BH do not allow a separation of a virtual photon flux factor. However the VCS process dominates within the experimental acceptance and we estimated also the pion electro-production cross section that is suited for the factorization presented in equation (95). We implemented in the MCEEP package routines for setting the cross-section model equal to the virtual photon flux. In order to extract a differential cross section from the data, the experimental acceptance must be weighted by some model of the 5-fold differential cross section. For a given model, the experimental cross section weighted over kinematic bin $i$ (using MCEEP as the simulator) is:

$$\frac{d^5\sigma}{dk_2^{ab}d\Omega_{k_2}^a d\Omega_{p_2}^{CM}} \bigg|_{\text{Exp}} = \frac{d^5\sigma_{\text{Model}}}{dk_2^{ab}d\Omega_{k_2}^a d\Omega_{p_2}^{CM}} \times \frac{1}{\int d\Omega_{k_2}^a d\Omega_{p_2}^{CM}} \int d\Omega_{k_2}^a d\Omega_{p_2}^{CM} \otimes f_R(q^\gamma) dq' dq'' dq'''}$$

As the least biased measure of the average cross section, we first used

$$\frac{d^5\sigma_{\text{Model}}}{dk_2^{ab}d\Omega_{k_2}^a d\Omega_{p_2}^{CM}} = \frac{1}{1 \text{ GeV sr}^2} \mu b$$

$$\frac{d^5\sigma_{\text{Model}}}{dk_2^{ab}d\Omega_{k_2}^a d\Omega_{p_2}^{CM}} = \frac{1}{1 \text{ GeV sr}^2} \mu b$$

(100)
Where appropriate (see Chap. 6.2) we used the photon flux model to weight the acceptance. In this case

\[
\frac{d^2\sigma^\text{Model}}{dk_2^{lab}d\Omega_{k_2}^{lab}d\Omega_{p_2}^{CM}} = \frac{d^3\Gamma}{dk_2^{lab}d\Omega_{k_2}^{lab}d\Omega_{p_2}^{CM}} \frac{d^2\sigma(\gamma, X)^\text{Model}}{d\Omega_{p_2}^{CM}}
\]

(101)

\[
\left\langle \frac{d^2\sigma(\gamma, X)^\text{Exp}}{d\Omega_{p_2}^{CM}} \right\rangle = \frac{d^2\sigma(\gamma, X)^\text{Model}}{d\Omega_{p_2}^{CM}} \times \int_i \frac{d^5\sigma^\text{Model}}{d\Omega_{k_1}^{lab}d\Omega_{k_2}^{lab}d\Omega_{p_2}^{lab}d\Omega_{p_2}^{CM}} \otimes f_R(q''')dq''dk_2^{lab}d\Omega_{k_2}^{lab}d\Omega_{p_2}^{lab},
\]

(102)

with \( X = \gamma, \pi^0 \). In the actual acceptance weighting for the Photon Flux model, we use \( d^2\sigma(\gamma, \gamma)^\text{Model}/d\Omega_{p_2}^{CM} = 1\mu b/sr \).

### 4.5.3 Beam Energy

During experiment E93-050 there was no direct measurement of the beam energy. The nominal value of the beam energy was of 4045 MeV. Today two direct beam energy measurements are functional in the experimental Hall A (arc measurement and \( ep \) measurement). Also, new calibrations have obtained a more precise determination of the spectrometer constants \( \Gamma_e \) and \( \Gamma_h \) and consequently of the central momenta. For our analysis we determine the beam energy by matching data with the simulated position of the missing mass peaks. The matching is illustrated in Figure 44. The extracted beam energy ranged from 4030 to 4035 MeV.

### 4.6 Accidentals and Background Subtraction

The quality of the coincidence time (as shown in Figure 45) makes simple the subtraction of the accidental coincidences. We choose to initial select as good the events appearing in the time peak and in the nearest neighbor bunches. Within this time window there is the same probability to have accidental coincidences as for any the other beam bunches. To sample accidental coincidences, we choose time windows before and after the true coincidence time intervals covering an integer number of 2 ns (delay between 2 beam bunches). In defining the accidental
FIG. 44: Beam energy is determined such as to obtain match between the photon missing mass peak in simulation (solid line) and data (dashed line). Top half: the photon missing mass peak; Bottom half: the pion missing mass peak.
cuts we left one beam bunch to separate the good events window and the accidental window as shown in Figure 45. We analyze and bin the events in these accidental coincidence time intervals with exactly the same physics cuts and conditions as the good events (in the true coincidence window). We also estimate the errors as for the good events (see Section 4.7). We weight the yield of the accidentals with the ratio of the width of the accidentals and true coincidence time window. To calculate the experimental cross section, we subtracted from the true event yield the weighted yield of accidentals. To illustrate the effect of the accidentals, Figure 46 shows the overlay of the missing mass squared histograms for the good events (solid line) and for an accidental window (dashed line) of the same width as the good coincidence. To subtract the background we focus on the vertex cut. There are three phases in the background subtraction for which we analyze all the runs in a setting. In the first phase we reject all events with $|2x - beam_x| > 3$mm, as these events are clearly poorly reconstructed or junk. In the second phase we estimate how many mis-identified events remain in the $|2x - beam_x| < 3$mm window.
FIG. 46: Missing mass squared histograms for true events and accidentals.

FIG. 47: Cut for studying background events.
FIG. 48: Missing mass squared histograms for true events and symmetric bad vertex cuts. The inner background cut \( w_1 > 0.0045 \) and the outer background cut is here \( w_2 < 0.0075 \).

We select (as shown in Figure 47) a window \( 4 \text{mm} < |2x - beam_x| \) symmetrically outside of the good vertex reconstruction cut labeled as 'outer vertex events'. The width of the outer vertex cut is selected (as shown in Figure 48) so that the outer vertex events accounts for 100\% of the yield observed for \( |2x - beam_x| < 3 \text{mm} \) at \( M_X^2 << 0 \). We assume that the events outside of the vertex cut might be due to a bad position reconstruction and we should renormalize for them. We calculate the yield using the following formula:

\[
Y_i = BGN_{setting} \sum_{\text{runs}} [(N_i^b - N_i^{ba}) - (N_i^b - N_i^{ba})]
\]  

(103)

\[
BGN_{setting} = \frac{\sum_{\text{runs}} N^g}{\sum_{\text{runs}} N^g - \sum_{\text{runs}} N^b} \frac{\sum_{\text{runs}} (N^t - N^{ta})}{\sum_{\text{runs}} N^g}.
\]

(104)

The notations are:

- \( Y_i \) the yield in the kinematic bin \( i \) used in cross-section calculation;
- \( BGN_{setting} \) background renormalizing factor evaluated for each setting;
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- $N_{i}^{g}$ the efficiency corrected number of counts in the kinematic bin $i$, that pass the coincidence time and vertex cut - 'good' events;

- $N^{g}$ the efficiency corrected number of counts in all the kinematic bins, that pass the coincidence time and vertex cut, and a missing mass cut $-5000\,MeV^{2} < M_{X}^{2} < 40000\,MeV^{2}$ covering the physical region for both $p(e, e'p)\gamma$ and $p(e, e'p)\pi$ events;

- $N_{i}^{ba}$ the accidentals in the kinematic bin $i$, that pass the vertex cut;

- $N_{i}^{b}$ the efficiency corrected number of counts in the kinematic bin $i$, that pass the coincidence time and the outer vertex cuts;

- $N^{b}$ the efficiency corrected number of counts in all the kinematic bins, that pass the coincidence time and the outer vertex cuts;

- $N_{i}^{ba}$ the efficiency corrected number of accidentals corresponding to the outer vertex cuts in the kinematic bin $i$;

- $N^{ba}$ the efficiency corrected number of accidentals corresponding to the outer vertex cuts in all the kinematic bins;

- $N^{t}$ the efficiency corrected number of counts in all the kinematic bins, that pass the coincidence time without vertex cut;

- $N_{i}^{ta}$ the efficiency corrected number of accidentals in all the kinematic bins, without vertex cut;

- $N_{i}^{ta}$ the accidentals without vertex cut in the kinematic bin $i$;

- $N^{ta}$ the accidentals without vertex cut in all the kinematic bins.
Setting $s=2.25\text{GeV}$, $w_1=0.004$, $w_2=0.0072$

FIG. 49: Missing mass squared histograms for all runs of one setting to illustrate the background subtraction procedure. $w_1$ and $w_2$ are the respective inner and outer limits of the background windows as shown in Figure 48. $w_1 < \text{abs}(\text{twoarm}_x - \text{beam}_x) < w_2$

To illustrate the procedure in Figure 49 we plotted for one setting:

- upper left the missing mass distribution, for the good events (black) and the corresponding accidentals (blue, flat line below);
- upper right the missing mass distribution for the good events after subtracting the accidentals;
- middle left the missing mass distribution, for all events, no vertex cut (black) and the corresponding accidentals (blue, flat line below);
- middle right the missing mass distribution for all events, no vertex cut after subtracting the accidentals;
- bottom left the missing mass distribution, for the outer cut events (black) and the corresponding accidentals (blue, flat line below);
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- bottom right the missing mass distribution for the outer cut events after subtracting the accidentals;

In the final analysis, the yield in the lower right is subtracted bin by bin from the yield in the upper right. Following the evolution of these histograms setting by setting gives interesting ideas about nature of the out of cuts events and their possible treatment (Figure 50 and Figure 51).

The summary of the table of the background corrections per setting for the photon final particle and pion final particle are summarized in table VI.

TABLE VI: Background renormalization coefficient for all runs for each kinematic setting.

<table>
<thead>
<tr>
<th>Setting</th>
<th>$\gamma$ and $\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-5000 &lt; mm^2 &lt; 40000$</td>
</tr>
<tr>
<td>$s = 1.30 GeV^2$</td>
<td>1.14915</td>
</tr>
<tr>
<td>$s = 1.50 GeV^2$</td>
<td>1.16023</td>
</tr>
<tr>
<td>$s = 1.75 GeV^2$</td>
<td>1.22881</td>
</tr>
<tr>
<td>$s = 2.00 GeV^2$</td>
<td>1.12577</td>
</tr>
<tr>
<td>$s = 2.25 GeV^2$</td>
<td>1.11028</td>
</tr>
<tr>
<td>$s = 2.50 GeV^2$</td>
<td>1.09382</td>
</tr>
<tr>
<td>$s = 2.80 GeV^2$</td>
<td>1.12390</td>
</tr>
<tr>
<td>$s = 3.20 GeV^2$</td>
<td>1.14685</td>
</tr>
<tr>
<td>$s = 3.60 GeV^2$</td>
<td>1.12457</td>
</tr>
</tbody>
</table>

In the lower $s$ settings the accidentals are small to negligible. The signal to noise ratio allows a clean subtraction of the cross-section. At the higher values of missing mass squared at the low $s$ settings we notice a small rise due probably to the tail of the pion peak.

4.7 Error Analysis

We determined the average experimental cross-section in a kinematic bin, by dividing the efficiency corrected number of counts in that bin, by the MCEEP
FIG. 50: Background correction and renormalization procedure - part 1. The outer cut events have $w_1 < \text{abs(twoarm}_z - \text{beam}_z) < w_2$. 

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FIG. 51: Background correction and renormalization procedure - part 2. The outer cut events have $w_1 < \text{abs}(\text{twoarm}_x - \text{beam}_x) < w_2$. 

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yield. The result is affected by systematic and statistical errors. Although a part of the systematic errors have been studied carefully (e.g. charge and density determination, efficiency corrections) the acceptance studies are still subject to further improvement. There are also other systematic error sources that need to be investigated (such as beam energy sensitivity, optics calibration uncertainty, etc). We estimate the relative systematic error from all sources to be 5%, based on studies of elastic $p(e, e'p)$ scattering. In this thesis we shall apply only statistical error to cross section plots.

In the followings we use $\delta$ for absolute error and $\epsilon$ for relative error. For each count identified to belong to a certain bin, we apply the efficiency corrections, namely dead time, trigger efficiency for both arms, track renormalization (due to the use of only one track and 4-6 wire hits in the VDC events) and target density. We analyze all data files (runs) from each kinematic setting together. Using the notation:

$$y_{\text{event}} = dt 5 \cdot \text{Corr}^e \cdot \text{Corr}^\text{track} \cdot \text{Corr}^\text{multiplicity} \cdot (1 - \epsilon_{\text{boil}}(I)), \quad Y_{\text{exp}} = \sum_{\text{runs}} \sum_{\text{events}} y_{\text{event}},$$

the statistical error on the experimental yield is

$$\delta Y_{\text{exp}} = \sqrt{\sum_{\text{runs}} \sum_{\text{events}} y_{\text{event}}^2}.$$  \hspace{1cm} (105)

We discussed in Section 4.6 the necessity to subtract from the efficiency corrected number of counts in each bin the estimated accidentals and background yields. For the background and accidentals, the yields ($Y_{\text{acc}}, Y_{\text{bg}}$) and errors on the yields have been determined in same way as for true counts:

$$Y_{\text{exp}}^{\text{cor}} = Y_{\text{exp}} - Y_{\text{acc}} - Y_{\text{bg}}, \quad \delta Y_{\text{exp}}^{\text{cor}} = \sqrt{\delta Y_{\text{exp}}^2 + \delta Y_{\text{acc}}^2 + \delta Y_{\text{bg}}^2}. \hspace{1cm} (106)$$

The number of counts generated by the Monte Carlo simulation is limited by the computer time, because of the efficiency of generating counts and the need to manipulate big data structures. This error can eventually be lowered. To
evaluate the Monte Carlo simulation error we start from the following definitions [69]. For a set of uniformly randomly sampled set of points $x_i, i = 1, n,$ in a multidimensional volume $V$

$$\int f dV \approx V < f > \pm V \sqrt{\frac{< f^2 > - < f >^2}{N}},$$

$$< f > \equiv \frac{1}{N} \sum_{i=1}^{N} f(x_i),$$

$$< f^2 > \equiv \frac{1}{N} \sum_{i=1}^{N} f^2(x_i).$$

The function we want to integrate is the cross-section model. The product of this integrated quantity with the luminosity is the Monte Carlo yield. The kinematic variables describing the physics bins and the kinematic variables describing the experimental acceptance have a complex inter-relation. For this reason we integrate over a simple trial volume $W$ that encloses the entire experimental acceptance. When evaluating the Monte Carlo yield in a bin, if a simulated event satisfies the bin conditions we add it, and if not we consider its respective cross section to be 0, so we reject it.

After running a MCEEP simulation we have an output for a set of simulated events with a weight for each event:

$$N_i^{+1} = \sigma_i \cdot v_i,$$

where $\sigma_i$ is the cross-section of the simulated model and $v_i$ is the elementary phase space volume per event. The Monte Carlo yield results from summing up the weights for the simulated events in a certain bin.

$$Y = \sum_{\text{events}} N_i^{+1}$$

The error on the Monte Carlo yield is

$$\delta Y = \sqrt{\sum_{\text{events}} (\sigma_i \cdot v_i)^2 - \frac{1}{N_{\text{total}}} (\sum_{\text{bin}} \sigma_i \cdot v_i)^2}.$$ 

The relative error on the average cross-section in a kinematic bin is the quadratic sum of the relative errors of the experimental yield and the Monte Carlo yield.
Chapter 5

Theory

Inelastic scattering is an important tool for investigating the nucleon resonances. Many exclusive channels can be studied, including

\[
ep \rightarrow \begin{cases} 
  eN\pi \\
  eN\eta \\
  eN\Phi \\
  eN\rho \\
  eN\omega \\
  eN\gamma 
\end{cases}
\]

The study of these reactions can provide different information according to the strength of the couplings with the specific final particles. In this perspective Virtual Compton Scattering (VCS) has the beauty of being a symmetric process in the sense that we track only the electromagnetic couplings of nucleon resonance states.

In the absence of any theoretical calculation of VCS cross section in the resonance region, and due to the complication relative to the coherent Bethe-Heitler (BH) process, we calculated the \( p(e,e'p)\gamma \) cross section in a phenomenological Lagrangian approach. In this work, we have included the diagrams considered at tree level and treat particles as 'point-like'. The vertex form factors account for structural effects. The calculation can be used as input in a simulator like the MCEEP ([66]) package.
5.1 Bethe-Heitler and Virtual Compton Scattering Amplitudes

In the $p(e, e'p)\gamma$ process to lowest order in the electromagnetic coupling $\alpha$, there are two possible reaction mechanisms that start from the same initial state and yield the same final state: Bethe-Heitler (BH) and Virtual Compton Scattering (VCS). In BH processes the real photon is emitted by the electron either before or after the interaction with the proton via a virtual photon (see Figure 52).

For a spin one-half particle with four-momentum $p = (p_0, \vec{p})$ and spin projection $s$ along a specified $z$-axis we use for the fermion spinor the notation $u(p, s)$. For a photon with four-momentum $q$ and photon polarization $\epsilon$, the transversality condition is $\epsilon \cdot q = 0$. In addition for real photon, $q^2 = 0$ and it is common notation in case of virtual photon $q^2 = -Q^2$. The BH amplitude can be exactly calculated in QED assuming the proton elastic form factors $F_1$ and $F_2$ are known (see Section 5.6). The amplitude is

$$i\mathcal{M}_{\text{BH post}} = \bar{u}(k_2, s'_e) (-ie\gamma_\mu e_\nu^\mu) \frac{i(k_2 + \frac{q_r}{2} + m}{(k_2 + q_r)^2 - m^2} (-ie\gamma_\nu) u(k_1, s_e) i\gamma^\alpha \frac{\Delta^\alpha}{t}$$

$$\times \bar{u}(p_2, s'_p)(ie) \left[F_1(-t)\gamma_\alpha + iF_2(-t)\frac{\sigma_{\alpha\nu}\Delta^\nu}{2M}\right] u(p_1, s_p) , \quad (115)$$
\[ i \mathcal{M}_{BH} = \bar{u}(k_2, s') (-ie\gamma_\nu) \frac{i(k_1 - q_r + m)}{(k_1 - q_r)^2 - m^2} (-ie\gamma_\sigma) \bar{e}_\nu^\sigma u(k_1, s) \frac{i}{2} \left( g^{\nu\sigma} - \frac{\Delta^\nu\Delta^\sigma}{t} \right) \]
\[ \times \bar{u}(p_2, s_p')(ie) \left[ F_1(-t)\gamma_\alpha + iF_2(-t) \frac{\sigma_{\alpha\rho} \Delta^\rho}{2M} \right] u(p_1, s_p) , \]  

where $\Delta = p_2 - p_1$, $m$ is the electron mass, $M$ is the proton mass and $e$ is the absolute value of electron charge. In addition $F_1$ is Pauli proton form factor, $F_2$ is Dirac proton form factor, and $\sigma_{\alpha\rho} = \frac{i}{2} (\gamma^\rho \gamma^\sigma - \gamma^\sigma \gamma^\rho)$. Starting from these complete forms, we note that the term $\frac{\Delta^\nu\Delta^\sigma}{t}$ in the photon propagator vanishes when contracted with the photon-proton elastic vertex, and then using the Dirac equation for the proton. The BH amplitudes can then be compactly written as

\[ i \mathcal{M}_{BH} = \frac{ie^3}{t} \bar{u}(k_2, s') (\gamma_\mu (k_2 + q_r)^2 - m^2) \gamma_\nu + \gamma_\nu (k_1 - q_r)^2 - m^2) \bar{e}_\nu^\sigma u(k_1, s) \]
\[ \times \bar{u}(p_2, s_p')(F_1(-t)\gamma_\nu + iF_2(-t) \frac{\sigma_{\nu\rho} \Delta^\rho}{2M}) u(p_1, s_p) . \]  

In the VCS process, the real photon is emitted by the hadron.

![Diagram](image)

**FIG. 53:** Virtual Compton Scattering process: virtual photon is scattered off the proton.

The expression of the amplitude for this process depends on the intermediate hadronic state. The intermediate hadronic state can be the proton (case in which the process is known as Born term) or resonant states ($N^*$ or $\Delta^*$). The diversity of forms nested in the diversity of possible hadronic states will be treated in detail.
in Section 5.4. The VCS amplitudes can then be written in a most general form as

\[ i\mathcal{M}_{VCS} = \overline{u}(k_2, s'_e)(-ie\gamma^\mu)u(k_1, s_e)\frac{-i(g_{\mu\lambda} - \frac{(g_{\mu\lambda})}{g^2})}{Q^2} \overline{u}(p_2, s'_p)\mathcal{O}^{\nu\epsilon^\dagger}_{\nu\epsilon}u(p_1, s_p). \] (118)

Starting from this complete form, with \( q = k_1 - k_2 \), applying the Dirac equation for electron, the term \( \frac{e^2}{q^2} \) from the photon propagator disappears. Then we can write

\[ i\mathcal{M}_{VCS} = T_\mu -i\frac{1}{Q^2}\overline{u}(p_2, s'_p)\mathcal{O}^{\nu\epsilon^\dagger}_{\nu\epsilon}u(p_1, s_p), \] (119)

\[ T_\mu = \overline{u}(k_2, s'_e)(-ie\gamma^\mu)u(k_1, s_e). \] (120)

### 5.2 VCS Amplitude Expansion

We now focus on the hadronic part of the VCS amplitude. If we consider all the possible independent spin values of the initial and final particles in the hadronic reaction \( p\gamma_v \rightarrow p\gamma_r \) and the parity conservation, we count 12 degrees of freedom. At least 12 products of Lorentz invariant scalars multiplied with covariant operators are then necessary to completely describe the current operator \( \mathcal{O}^{\mu\nu} \)

\[ O^{\mu\nu} = \sum_i a_i \mathcal{O}_i^{\mu\nu}. \] (121)

Considering that \( \epsilon_r \cdot q_r = 0 \), and the Dirac equations for protons

\[ \begin{align*}
\not{p}^1 u_1(p_1, s_p) &= M u_1(p_1, s_p), \quad (122) \\
\overline{u}_2 \not{p}_2(p_2, s'_p) &= M \overline{u}_2(p_2, s'_p). \quad (123)
\end{align*} \]
we can use the following 20 operators that form an overcomplete basis, for expanding $O^{\mu\nu}$.

$$
O_{10}^{\mu\nu} = M^3 g^{\mu\nu}, \quad O_{11}^{\mu\nu} = M^3 g^{\mu\nu} g_r,
$$

$$
O_{12}^{\mu\nu} = M p_1^\mu p_1^\nu, \quad O_{13}^{\mu\nu} = p_1^\mu q_\nu g_r,
O_{14}^{\mu\nu} = M p_1^\mu \gamma_\nu g_r,
O_{15}^{\mu\nu} = M q_1^\mu p_1^\nu, \quad O_{16}^{\mu\nu} = q_1^\mu q_\nu g_r,
O_{17}^{\mu\nu} = M q_1^\mu \gamma_\nu g_r,
O_{18}^{\mu\nu} = M^2 \gamma_\mu p_1^\nu, \quad O_{19}^{\mu\nu} = M \gamma_\mu q_\nu g_r,
O_{20}^{\mu\nu} = M^3 \gamma_\mu \gamma_\nu g_r.
$$

The mass factors are included for dimensional reasons. Conservation of current (gauge invariance) requires that

$$
q_r O^{\mu\nu} = 0. \quad (125)
$$

From Eq. (125) we have

$$
a_1 M^2 q_r^\mu + a_11 M^2 q_r^\nu g_r + a_2 M (q_r.p_1) p_1^\mu + a_{12} (q_r.p_1) p_1^\mu g_r +
a_3 M (q_r.q) p_1^\mu + a_{13} (q_r.q) p_1^\mu g_r + a_4 M^2 p_1^\mu + a_{14} M p_1^\mu g_r +
a_5 M q_r^\mu (q_r.p_1) + a_{15} q_r^\mu (p_1.q_r) g_r + a_6 M q_r^\mu (q_r.q) + a_{16} q_r^\mu (q_r.q) g_r +
a_7 M^2 q_r^\mu g_r + a_{17} M q_r^\mu g_r + a_8 M^2 \gamma_\mu (q_r.p_1) + a_{18} M \gamma_\mu (q_r.p_1) g_r +
a_9 M^2 \gamma_\mu (q_r.q) + a_{19} M \gamma_\mu (q_r.q) g_r +
a_{10} M^3 g_r^\mu + a_{20} M^2 \gamma_\mu g_r = 0. \quad (126)
$$

Consequently, using $g_r \cdot g_r = q_r \cdot q_r = 0$ we obtain

$$
M q_r^\mu [a_1 M^2 + a_5 (q_r.p_1) + a_6 (q_r.q)] = 0, \quad (127)
q_r^\mu g_r [a_{11} M^2 + a_{15} (q_r.p_1) + a_{16} (q_r.q) + a_7 M^2] = 0, \quad (128)
M p_1^\mu [a_2 (q_r.p_1) + a_3 (q_r.q)] = 0, \quad (129)
$$

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These constraints allow us to reduce the number of terms in the $O^\mu\nu$ expansion (Eq. (121)) from 20 to 14. For further reducing the number of terms to 12, one has to try to reduce the number of covariant forms. We used 14 operators, so description is not minimal, although correct. From Eq. (127)-(132) can be extracted some coefficients as linear combination of the others:

\[
\begin{align*}
a_1 &= -\frac{1}{M^2} [a_5(q_r \cdot p_1) + a_6(q_r \cdot q)] , \\
a_{11} &= -\frac{a_{15}}{M^2} (q_r \cdot p_1) + \frac{a_{16}}{M^2} (q_r \cdot q) + a_7 , \\
a_2 &= -\frac{a_3}{(q_r \cdot p_1)} , \\
a_4 &= -\frac{1}{M^2} [a_{12}(q_r \cdot p_1) + a_{13}(q_r \cdot q)] , \\
a_8 &= -\frac{a_9}{(q_r \cdot p_1)} , \\
a_{10} &= -\frac{1}{M^2} [a_{18}(q_r \cdot p_1) + a_{19}(q_r \cdot q)] .
\end{align*}
\]
5.3 Isospin Considerations

To describe a non-strange baryon, besides the spin and parity, another quantum number is needed, namely the isospin. The most common baryons, the proton and the neutron, have near equal mass so they seem to be instances of the same particle generally called 'the nucleon'. Since it is a doublet a quantum number 1/2 is appropriate. The third projection of this number allows us to differentiate between the proton and neutron: +1/2 for proton and −1/2 for neutron. There is a direct relation between this third projection of the isospin (i_3) and the particle charge: \( Q = i_3 + \frac{1}{2} \). In the case of composite systems (nuclei for instance) the isospin is the sum of the isospins of their constituents. Isospin sums as angular momentum. To describe a particle in terms of isospin we need two numbers: total isospin \( I \), that will tell us how many particles with similar masses exist (2I+1), and third isospin projection that indicate the particle charge. In terms of isospin the nucleons wave-function has besides the spinor another two dimensional component for isospin.

For projecting the three components of the isospin we have the Pauli matrices acting in the isospin space:

\[
\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

From these matrices can be built the raising and lowering isospin operators that will transform a neutron to a proton and a proton to a neutron, for instance.

\[
\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.
\]

If we consider now the vertex of an incoming isospin \( (\frac{1}{2}^+) \) particle and outgoing proton and photon \( R \left( \frac{1}{2}^+ \right) \to p\gamma \), the transition amplitude has the form (see Section 5.4):

\[
\bar{u}(p_p, s_p) i e^\gamma_\mu (t_1 \gamma_\mu + it_2 \frac{\sigma_{\mu\nu}q^\nu}{M + M_r}) u(p_R, s_R).
\]

(140)

Considering the isospin behavior we see that \( t_1 \) and \( t_2 \) have to be a linear combination of the form \( \alpha I + \beta \tau_3 \) (where \( I \) is the identity matrix in the isospin...
space). These combinations are operators in the isospin space, to account for the photon behavior as a combination of isospin 0 (scalar type) and 1 (vector type). The term $a_I$ is called isoscalar component and the term $b_I$ is called isovector component. For the isospin part of the wavefunction we have

$$ \left( \begin{array}{c} p \\ n \end{array} \right) = \left( \begin{array}{c} \alpha \\ 0 \\ 1 \\ \beta \\ 0 \\ -1 \end{array} \right) \left( \begin{array}{c} p \\ n \end{array} \right) = p^I(\alpha + \beta)p + n^I(\alpha - \beta)n \quad (141) $$

Note that the coefficients $\alpha$ and $\beta$ are functions of $Q^2$. The most comprehensive form for the $R \rightarrow p\gamma$ vertex is

$$ -ie\bar{u}(p_f, s_f)e^{*}_p \gamma_\mu \gamma_\nu \frac{1}{2} (a_s(Q^2) + a_v(Q^2)\tau_3)u(p_i) + $$

$$ +e\bar{u}(p_f)\epsilon^{*}_p \gamma_\mu \frac{\sigma_\mu q^\rho}{M + M_r} (b_s(Q^2) + b_v(Q^2)\tau_3)u(p_i). \quad (142) $$

For an excited spin one-half resonance $R$, the limit conditions are

$$ a_s(Q^2 = 0) = 0; \quad a_v(Q^2 = 0) = 0 $$

$$ b_s(Q^2 = 0) = \kappa^R_s; \quad b_v(Q^2 = 0) = \kappa^R_v $$

$$ \kappa^R_s + \kappa^R_v = \kappa_R; \quad \kappa^R_s - \kappa^R_v = \kappa_{nR}. \quad (143) $$

If particle $R$ is the proton itself, then $a_s(Q^2) = a_v(Q^2) = \frac{1}{2} F^R_s(Q^2)$ and $b_s(Q^2) + b_v(Q^2) = F^R_v(Q^2)$.

Since the electromagnetic interaction must conserve the third component of the isospin (the charge), for the $R \rightarrow p\gamma$ vertex, the isospin projection of particle $R$ will be always $\frac{1}{2}$ as for the proton regardless of the total value of the isospin (1 or $\frac{3}{2}$). This means that only one Clebsh-Gordan coefficient is needed; we absorb it in the vertex coupling constants, to shorten the expressions for amplitude. However these coefficients should be considered when comparing the coupling constant values with other calculations or values extracted from other processes.

$$ p\gamma \rightarrow R(I = \frac{3}{2}, I_3 = \frac{1}{2}) : \quad < \frac{3}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} | 1,0 > = \sqrt{\frac{2}{3}}, \quad (144) $$

$$ p\gamma \rightarrow R(I = \frac{1}{2}, I_3 = \frac{1}{2}) : \quad < \frac{1}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} | 0,0 > = 1, \quad (145) $$

$$ < \frac{1}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} | 1,0 > = -\sqrt{\frac{1}{3}}. \quad (146) $$

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5.4 VCS Reaction Channels

The electron current $T_\mu$ (see Eq. (120)) remains the same in expressing the VCS amplitude for all possible reaction channels and intermediate state resonances. In this work, we include only spin $\frac{1}{2}$ and $\frac{3}{2}$ intermediate states. We factorized the amplitudes in terms of operators (see section 5.2) using Mathematica and evaluated their contribution to the $a_i$ coefficients. The elegance in the approach lies in the fact that adding new resonances to the calculation requires only knowledge of their mass, width and coupling constants all the rest of the code being re-usable. The coupling constants depend on the four-momentum squared of the photon. In

![Diagram of VCS reaction channels]

FIG. 54: VCS hadron part: the $s$ and the $u$ channels.

The intermediate state for $s$ and $u$ reaction channels (Figure 54) we can have the proton or other resonant states $N^*$ or $\Delta^*$. We use the notation $\mathcal{M}_{A,N}$ for $s$ channel and $\mathcal{M}_{B,N}$ for $u$ channel, where $N$ is the intermediate state spin. To shorten the expression we use $\bar{u}(p_2, s_p') = \bar{u}_2$ and $u(p_1, s_p) = u_1$. The intermediate state has mass $M_r$ and width $\Gamma_r$. In principle the width depends of the invariants mass in the intermediate state $\Gamma_r = \Gamma_r(s)$; in particular, below threshold $\Gamma_r(s) = 0$ for $s < (M + M_r)^2$.

In the case of the $s$ reaction channel, for an intermediate state with spin $\frac{1}{2}$,
positive parity, the amplitude is
\[ i\mathcal{M}_{A^{1+}} = \frac{-ie^2}{Q^2} \tilde{u}_2 \gamma^\mu [t_1(0)\gamma_\mu + it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{i(p_1 + q + M_r)}{(p_1 + q)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \langle e^2 \rangle [t_1(Q^2)\gamma^\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu \]
\[ = \frac{-e^2}{Q^2} \tilde{u}_2 \gamma^\mu [t_1(0)\gamma_\mu + it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{(p_1 + q + M_r)}{(p_1 + q)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \langle e^2 \rangle [t_1(Q^2)\gamma^\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu . \] (147)

For the reaction channel, spin $\frac{1}{2}$ particle, positive parity, the amplitude is
\[ i\mathcal{M}_{B^{1+}} = \frac{-e^2}{Q^2} T_\mu \tilde{u}_2 \gamma_5 [t_1(0)\gamma_\mu + it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 + q + M_r}{(p_1 + q)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu \]
\[ = \frac{-e^2}{Q^2} \tilde{u}_2 \gamma^\mu [t_1(0)\gamma_\mu - it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 + q - M_r}{(p_1 + q)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu . \] (148)

If the intermediate state is a spin $\frac{1}{2}$, negative parity particle, the $s$ reaction channel amplitude is
\[ i\mathcal{M}_{A^{1-}} = \frac{-e^2}{Q^2} \tilde{u}_2 \gamma_5 [t_1(0)\gamma_\mu + it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 - q - M_r}{(p_1 - q_r)^2 - M_r^2 + i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu \]
\[ = \frac{-e^2}{Q^2} \tilde{u}_2 \gamma^\mu [t_1(0)\gamma_\mu - it_2(0)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 - q + M_r}{(p_1 - q_r)^2 - M_r^2 + i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 T^\nu . \] (149)

For the intermediate state spin $\frac{1}{2}$, negative parity particle, the $u$ reaction channel amplitude is
\[ i\mathcal{M}_{B^{1-}} = \frac{-e^2}{Q^2} T_\mu \tilde{u}_2 \gamma_5 [t_1(Q^2)\gamma_\mu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 - q - M_r}{(p_1 - q_r)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 \]
\[ = \frac{-e^2}{Q^2} T_\mu \tilde{u}_2 [t_1(Q^2)\gamma_\mu - it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] \frac{p_1 - q + M_r}{(p_1 - q_r)^2 - M_r^2 - i\Gamma_r M_r} \]
\[ \gamma_5 [t_1(Q^2)\gamma_\nu + it_2(Q^2)\frac{\sigma_{\mu\nu}q_\nu}{M + M_r}] u_1 . \] (150)
In the case in which the intermediate state has spin $\frac{3}{2}$, the most general vertex between the virtual photon, proton and resonance is characterized by three coupling constants. In case of the real photon one of them vanishes, so the vertex $R\rho\gamma$ is characterized by two coupling constants. In the case of a spin $\frac{3}{2}$ positive parity, the amplitude in $s$ reaction channel is

$$i\mathcal{M}_{A,\frac{3}{2}^+} = \frac{-e^2}{Q^2} \bar{u}_2 \gamma_5 \Lambda_{\rho \mu} D^{\mu \delta} \gamma_5 \Lambda_{\nu \delta \rho} u_1 T^\rho, \quad (151)$$

where

$$\Lambda_{\rho \mu} = t_{r1}\theta_{\rho \mu} \gamma_5 (\epsilon^\rho q^\mu - \epsilon^\mu q^\rho) + t_{r2}\theta_{\rho \mu} (p_1 + q)_\rho (\epsilon^\rho q^\mu - \epsilon^\mu q^\rho), \quad (152)$$

$$\Lambda_{\nu \delta \rho} = t_{v0}(Q^2) g_{\delta \rho} + t_{v1}(Q^2) \theta_{5 \rho} \gamma_5 (g^{\rho \mu} q^\delta - g^{\rho \delta} q^\mu) + t_{v2}(Q^2) \theta_{5 \rho} (p_1 + q)_\rho (g^{\rho \mu} q^\delta - g^{\rho \delta} q^\mu), \quad (153)$$

$$\theta_{\mu \alpha} = g_{\mu \alpha} + \left(\frac{1}{2} (1 + 4V) A + V\right) \gamma_{\mu} \gamma_{\nu}, \quad (154)$$

$$D^{\mu \delta} = \frac{p_1 + q + M_r}{(p_1 + q)^2 - M_r^2 + i\Gamma_r M_r} \left\{ g^{\mu \delta} - \frac{1}{3} \gamma^\mu \gamma^\delta - \frac{1}{3M_r} \left[ \gamma^\mu (p_1 + q)^\delta - \gamma^\delta (p_1 + q)^\mu \right] - \frac{2}{M_r^2} (p_1 + q)^\mu (p_1 + q)^\delta \right\}. \quad (155)$$

The tensor $D$ is the form of a Rarita-Schwinger propagator for $\frac{3}{2}$ spin particles. In the expression of the tensor $\theta_{\mu \alpha}$, $A$ is a parameter that reflects a freedom in the choice of the Lagrangian in the limit of respecting some formalism bounds, such as invariance under point transformation; $V$ is the 'off-shellness' parameter, measuring the 'off-shell' of the vertex [73]. Note that $t_{r1} = t_{v1}(Q^2 = 0)$, $t_{r2} = t_{v2}(Q^2 = 0)$ and $t_{r0} = t_{v0}(Q^2 = 0) = 0$. In the following expressions for the VCS amplitudes in $s$ and $u$ reaction channel with a spin $\frac{3}{2}$ particle intermediate state equivalent definitions for tensors $\Lambda_{r}, \Lambda_{\nu}, D$ are considered.

For $u$ reaction channel(B), a spin $\frac{3}{2}$ positive parity particle, the amplitude is

$$i\mathcal{M}_{B,\frac{3}{2}^+} = \frac{-e^2}{Q^2} T^\rho \bar{u}_2 \Lambda_{\nu \rho \mu} D^{\mu \delta} \Lambda_{\tau \delta \mu} u_1. \quad (156)$$

If the intermediate state is a spin $\frac{3}{2}$, negative parity particle, the amplitude corresponding to the $s$ channel reaction is

$$i\mathcal{M}_{A,\frac{3}{2}^-} = \frac{-e^2}{Q^2} \bar{u}_2 \gamma_5 \Lambda_{\rho \mu} D^{\mu \delta} \gamma_5 \Lambda_{\nu \delta \rho} u_1 T^\rho. \quad (157)$$
Finally for the intermediate state is a spin $\frac{3}{2}$ negative parity particle, the amplitude corresponding to the reaction channel is

$$iM_{B,\frac{3}{2}} = -\frac{e^2}{Q^2} T^\rho u_2 \gamma_5 \Lambda_{\mu\nu\rho} D^{\mu\nu} \gamma_5 \Lambda_{\sigma\delta} \gamma_5 u_1 .$$

In the $t$ channel diagram the virtual photon couples to a pion emitted by the proton.

For this process the amplitude is

$$iM'_{o} = -Q^2 u_T^2 \gamma_5 \Lambda_{\mu\nu\rho} D^{\mu\nu} \gamma_5 \Lambda_{\sigma\delta} \gamma_5 u_1 .$$

A similar calculation was done by M. Vanderhagen for the case below the pion threshold. In [24], the nucleon pion cloud was approximated by $t$-channel exchange of a scalar $\sigma$ meson.

\[ \text{FIG. 55: VCS hadron part: the } t \text{ channel.} \]

\[ iM_{\pi^0} = -\frac{e^2}{Q^2} \frac{p_1 - p_2}{(p_1 - p_2)^2 - M_{\pi^0}^2} \gamma_5 g_{\pi\gamma\gamma} \epsilon_{\alpha\beta\gamma\mu} g^{\alpha\rho} q^\rho q^\mu' u_1 T^\rho , \]
5.5 Cross-Section Calculation

To evaluate the cross-section of $p(e, e'p)\gamma$ we need to consider the total amplitude (coherent sum of BH and VCS), square it and integrate within the appropriate kinematic constraints.

We start with assembling the $|\mathcal{M}|^2$. Since the polarization states are not detected, we need to sum over the possible final state polarizations and average over the initial state polarization. Initially there are 2 fermions (the electron and the proton) and therefore there are 4 equiprobable spin state combinations (so for averaging we divide with 4). The process is described by the Lorentz invariant quantity $|\mathcal{M}|^2$, given by

$$|\mathcal{M}|^2 = \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}_{BH} + \mathcal{M}_{VCS}|^2$$

where $\mathcal{M}_{BH}$ and $\mathcal{M}_{VCS}$ are given by the equations (117) and (118).

To shorten the formulas we use the simplified notation for the Bethe-Heitler amplitude

$$i\mathcal{M}_{BH} = \frac{ie^3}{\hbar} \overline{u}(k_2, s_e) L_{\mu\nu} \gamma_{\nu} u(k_1, s_e) \overline{u}(p_2, s_{e'}) \gamma'_{\mu} u(p_1, s_p).$$

(161)
Therefore we obtain
\[
\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}_{BH}|^2 = \frac{1}{4} \sum_{\text{spins}} \frac{e^6}{t^2} \bar{u}(p_1, s_p) H_{2\nu} u(p_2, s_{\nu'}) \bar{u}(p_2, s_{\nu'}) H_{\nu} u(p_1, s_p) \\
\times \bar{u}(k_1, s_e) L^\nu_{\nu'} \varepsilon_{\nu'} u(k_2, s_{\nu'}) \bar{u}(k_2, s_{\nu'}) L_{\mu\nu} \varepsilon^\mu_{\nu'} u(k_1, s_e) .
\] (162)

With standard techniques of contracting the wave functions when summing over the spin states, equation (162) can be reduced to a product of traces, in the form
\[
\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}_{BH}|^2 = -\frac{e^6}{4q^4} Tr[(\gamma_1 + M) H_{\nu'}(\gamma_2 + M) H_{\nu}] Tr[k_1 L_{\mu\nu} k_2 L_{\mu\nu}] .
\] (163)

With a similar procedure, using for the VCS amplitude the notation
\[
i\mathcal{M}_{VCS} = \frac{ie^3}{Q^2} \bar{u}(k_2, s_{\nu}) \gamma_\mu u(k_1, s_e) \bar{u}(p_2, s_{\nu'}) O_{\mu\nu}^{*} \varepsilon^\mu_{\nu'} u(p_1, s_p) .
\] (164)
we obtain for the modulus square of the VCS amplitude the expression
\[
\frac{1}{4} \sum_{\text{spins}} |i\mathcal{M}_{VCS}|^2 = -\frac{e^6}{4Q^4} Tr[(\gamma_1 + M) O^{*}_{\nu',\mu}(\gamma_2 + M) O_{\mu\nu}] .
\] (165)

In the next step we express the operator $O_{\mu\nu}$ in terms of the operator basis described in section 5.2. Using equation (121) we can rewrite
\[
Tr[(\gamma_1 + M) O^{*}_{\nu',\mu}(\gamma_2 + M) O_{\mu\nu}] = \sum_{ij} Tr[(\gamma_1 + M) a_i^* O^{*}_{\nu',\mu}(\gamma_2 + M) a_j O_{\mu\nu}] .
\] (166)

We now introduce the notation
\[
K_{\mu\nu}^{ij} = Tr[(\gamma_1 + M) a_i^* O^{*}_{\nu',\mu}(\gamma_2 + M) a_j O_{\mu\nu}] ,
\] (167)
\[
N_{\mu\nu}^{ij} = Tr[k_1 \gamma^\mu k_2 \gamma^\nu] .
\] (168)

Then we can write
\[
Tr[(\gamma_1 + M) O_{\mu\nu}(\gamma_2 + M) O_{\mu\nu}] = \sum_i (|a_i|^2 K_{\mu\nu}^{ij} + \sum_{j<i} 2 Re(a_i^* a_j) K_{\mu\nu}^{ij}) .
\] (169)

Using equations (165) - (169), we can write
\[
\frac{1}{4} \sum_{\text{spins}} |i\mathcal{M}_{VCS}|^2 = -\frac{e^6}{4Q^4} N_{\mu\nu}^{ij} \left( \sum_i (|a_i|^2 K_{\mu\nu}^{ij} + \sum_{j<i} 2 Re(a_i^* a_j) K_{\mu\nu}^{ij}) \right) .
\] (170)
Note that in this sum we have products of quantities that are the same regardless of the resonances considered - $\frac{e^6}{4\alpha^2} N^\mu_\mu K^\mu_{ij}$ (diagonal $O_i O_i$) and $\frac{e^6}{4\alpha^2} N^\mu_\mu K^\mu_{ij}$ (off diagonal $O_i O_j$). The $a_i$ coefficients are constructed by adding the contribution of each diagram considered.

The remaining contribution to be considered is the BH and VCS interference term. It is important to note that even when the VCS or BH amplitudes are very different in magnitude, this interference term can still be significant. Note the difference between the four-momentum of the virtual photon in BH type of process $q_{BH} = p_2 - p_1 = \Delta$, versus the VCS type process where $q_{VCS} = k_1 - k_2$.

In the following formula we use $Q^2 = -q^2_{VCS}$

$$\frac{1}{4} \sum_{\text{spins}} 2 \text{Re}(M^*_{BH} M_{VCS}) = \sum_{\text{spins}} 2 \text{Re} \left( \frac{e^6}{4t Q^2} \bar{u}(k_1, s_e) L^\mu^\nu u(k_2, s_\nu) \right)$$

$$\bar{u}(k_2, s_\nu) \gamma_\mu u(k_1, s_e) \bar{u}(p_1, s_p) H^*_{\nu'} u(p_2, s_{\nu'}) \bar{u}(p_2, s_{\nu'}) O^\mu \nu u(p_1, s_p) \varepsilon_{\nu'} \varepsilon_{\nu'} \right).$$

Using the same techniques described above, the interference term can be expressed as

$$\frac{1}{4} \sum_{\text{spins}} 2 \text{Re}|M^*_{BH} M_{VCS}| =$$

$$2 \text{Re} \left( \frac{e^6}{4t Q^2} \text{Tr}[k_1 L^\mu^\nu \ k_2 \gamma_\mu] \text{Tr}[(\not{\nu} + M) H_{\nu'} (\not{\nu} + M) O^\mu \nu] \right).$$

We now introduce the notations

$$B^\nu_{\nu'} = \text{Tr}[k_1 L^\mu^\nu \ k_2 H_{\mu}], \quad (173)$$

$$A^\mu \nu = \text{Tr}[\not{\nu} + M) H_{\nu'} (\not{\nu} + M) O^\mu \nu]. \quad (174)$$

Then we can write

$$\frac{1}{4} \sum_{\text{spins}} 2 \text{Re}(M^*_{BH} M_{VCS}) = -2 \text{Re} \left( \frac{e^6}{4t Q^2} \sum_i a_i B^\nu_{\nu'} A^\mu \nu \right). \quad (175)$$

In the same manner as for $|M_{VCS}|^2$, in equation (175) we have a part independent of the different diagrams considered in the VCS amplitude, and the coefficients $a_i$ that comes from each diagram considered.
The differential cross section for a process with two ingoing particles (incident electron and target proton) and three outgoing particles (scattered electron, recoil proton and emitted real photon) is

$$d\sigma = \frac{(2\pi)^4\delta^4(k_e + p_1 - k_e' - p_2 - q_r)|M|^2}{4\sqrt{(k_e p_1)^2 - k_e'^2 p_1^2}} \frac{d^3 k_e'}{2k_e'(2\pi)^3} \frac{d^3 p_2}{2p_2^0(2\pi)^3} \frac{d^3 q_r}{2q_r^0(2\pi)^3}.$$  \hspace{1cm} (176)

For our kinematics, the electron is ultra-relativistic and we can neglect the electron mass everywhere except in the poles of BH propagator. The energy and momentum of the proton are related by $E_p = p^0 = \sqrt{p^2 + M^2}$. Since the initial proton is considered at rest ($p_1 = (M, 0, 0, 0)$), we can replace the Lorentz invariant quantity

$$\sqrt{(k_e \cdot p_1)^2 - k_e'^2 p_1^2} = |k_e^{lab}|M.$$  \hspace{1cm} (177)

In the laboratory frame, the integration over the final electron is

$$d\sigma = \int \frac{d^3 k_e'}{2k_e'} \frac{d\Omega_{k_e'}}{2k_e'}.$$  \hspace{1cm} (178)

We can rewrite the cross section differential in final electron coordinates as

$$d\sigma = \int \frac{(2\pi)^3|k_e'|^2|k_e^{lab}|^2}{8k_e M} \delta^4(p - p_2 - q_r) \frac{d^3 p_2}{2p_2^0} \frac{d^3 q_r}{2q_r^0},$$  \hspace{1cm} (179)

where $p = k_1 + p_1 - k_2$. It is useful to consider that

$$\int \delta^4(p - p_2 - q_r) \frac{d^3 p_2}{2p_2^0} \frac{d^3 q_r}{2q_r^0} = \int \frac{d\Omega_{p_2}^{CM}}{8p_2^0} \sqrt{\lambda(p_2^2, p_2^2, q_r^2)},$$  \hspace{1cm} (180)

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ is the Källen function and CM means that $d\Omega_{p_2}$ is evaluated in the center of mass system of the final proton and real photon. In our calculation, using $q_r^2 = 0$ and $p_2^2 = M^2$, we obtain

$$\sqrt{\lambda(p_2^2, p_2^2, q_r^2)} = \sqrt{p^4 + p_2^4 - 2p^2p_2^2} = (p^2 - M^2).$$  \hspace{1cm} (181)

Note also that $p = p_2 + q_r$ so that $p^2 = s$ ($s$ is the Mandelstam variable for the final proton+real photon). The final expression for the $p(e, e'p)\gamma$ cross-section is:

$$d\sigma = \frac{(2\pi)^{-5}|M|^2}{64M} \frac{k_e s - M}{k_e s}.$$  \hspace{1cm} (182)
5.6 Proton Form Factors

The proton form factors are involved in both the calculation of BH amplitude and VCS amplitude for the contributions in which the intermediate state is a proton. For this reason we paid special attention in considering the influence of different parametrizations as function of respective 4 momentum transferred by the virtual photon. We also note that for same initial and final state if we considered VCS process for central kinematics we have \( q = (k_1 - k_2)^2 = -1 \text{ GeV}^2 \). However if we consider BH mechanism \( q^2 = (p_2 - p_1) \) is taking for the nine kinematic settings values between \(-1.28 \text{ GeV}^2\) to \(-3.22 \text{ GeV}^2\).

The \( p\gamma \rightarrow p \) vertex has the form

\[
\Gamma_\mu = F_1(Q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}\kappa_p}{2M}F_2(Q^2)q^\nu,
\]

where \( \kappa_p \) is the proton magnetic moment, \( F_1 \) is the helicity conserving Dirac form factor, and \( F_2 \) is the helicity non-conserving Pauli form factor. The Dirac and Pauli form factors are simply related to the electric, \( G_E \) and magnetic, \( G_M \), form factors by

\[
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}\kappa_pF_2(Q^2),
\]

\[
G_M(Q^2) = F_1(Q^2) + \kappa_pF_2(Q^2).
\]

In the Breit frame, \( G_E \) and \( G_M \) are the Fourier transforms of the proton charge and current distributions.

Using the available world data, P. Bosted [70] fit the electric and magnetic form factors of the proton as inverse polynomial functions of \( Q \)

\[
G_{Ep} = \frac{1}{1 + 0.62Q + 0.68Q^2 + 2.8Q^3 + 0.83Q^4},
\]

\[
G_{M\mu_p} = \frac{1}{1 + 0.35Q + 2.44Q^2 + 0.50Q^3 + 1.04Q^4 + 0.34Q^5}.
\]

Another parametrization considered was the simple fit to the proton form factor values measured in Hall A [59]. In this fit set-up by Sonja Dieterich and
Steffen Strauch, the measured ratio \( y = \mu_p(G_E^p/G_M^p) \) values are fit as function of \( x = Q^2 \) with the form:

\[
y = 1 + \frac{ax^3}{1 + bx^2}.
\] (188)

Using this form, we have the known value 1 of the ratio for the real photon point \( (Q^2=0) \) and the dependence becomes a straight line for \( Q^2 \gg 1 \text{ GeV}^2 \). The fit values (see Figure 57) are

\[
a = -0.45 \pm 0.23 \\
b = 3.39 \pm 1.87.
\] (189)

In Figure 57 we see that Bosted parametrization over-estimate the ratio of electric and magnetic form factors of proton as function of \( Q^2 \). However Bosted
gives a parametrization for $G_M$. We can use equations 187 and 188 for $G_{M\rho}$ and $\mu_{\rho}(G_E^\rho/G_M^\rho)$ respectively.

5.7 The Calculated Born Cross-Section

The intermediate state in the VCS amplitude can be the proton or any nucleon resonance state. The resonances with $M_R^2 \approx s$ will contribute most strongly. The VCS amplitude with just the nucleon intermediate state is the Born term.

One way of looking at the cross-section is as a function of the angle between the real and virtual photon $\theta_{\gamma\gamma}^{CM}$ in the center-of-mass frame of the hadronic reaction (CM). To simplify the description, a convention has been adopted, in which if $\phi=0^\circ$ the angle $\theta_{\gamma\gamma}^{CM}$ varies from $0^\circ$ to $180^\circ$, while if $\phi=180^\circ$, then $\theta_{\gamma\gamma}^{CM}$ changes sign taking values from $0^\circ$ to $-180^\circ$. In general, however $0 < \theta < 180^\circ$ and $0 < \phi < 360^\circ$. In Figure 58, we note the BH peaking in the direction of incident and

![Graph showing Bethe-Heitler plus Born cross-section.](image)

FIG. 58: Bethe-Heitler plus Born cross-section. Kinematics was defined by incident electron energy 4.045 GeV, $Q^2=1.0$ GeV$^2$ and Mandelstam variable for hadronic reaction $s=1.5$ GeV$^2$. 

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scattered electron. The Born (VCS) contribution to the cross section dominates for $\theta_{CM}^< -50^\circ$. Since VCS is the primary reaction of interest the experimental kinematics are chosen centered on $\theta_{CM}^< -180^\circ$ so that within acceptance we have events with $\theta_{CM}^< -140^\circ$.

The combined BH plus Born amplitude expresses the contribution of the static ground state state of the proton to the $p(e, e'p)\gamma$ reaction. The BH and Born amplitudes are the first order radiative correction to the elastic scattering. These radiative corrections are often approximated due to the strong peak the BH amplitude has in the direction of the incident and scattered electrons ('peaking approximation'). In the present study the full angular dependence is considered: we measured the $p(e, e'p)\gamma$ process away from the BH peaks. The Born term represent the radiative tail contribution from proton Bremsstrahlung. This was described by L. Maximon as the 'kinematic recoil correction'([58]).

5.8 Resonances Contribution to $p(e, e'p)\gamma$

Cross-Section

The electromagnetic coupling constants for the real photon case can be extracted based on the listed ([75]) helicity amplitudes, as shown in Appendix D for the $\Delta$ resonance. We estimate the virtual photon coupling constants to have a dipole type dependence:

$$t_{iv} = t_{ir} \frac{1}{\left(1 + \frac{Q^2}{0.71\text{GeV}}\right)}$$

for $i=1,2$. In case of the virtual photon there is an additional coupling $t_{0v}$ to be considered. Since it vanishes for $Q^2=0$, we can start using a form

$$t_{0v} = -\frac{Q^2}{M_A + Q^2} \left(\frac{1}{1 + \frac{Q^2}{0.71\text{GeV}}}\right)^2$$

with a free $M_A$ parameter (we took it being 1 GeV as a try value for the plots presented here). The freedom in choosing the form of the $Q^2$ dependence for the coupling constants can be reduced by fit against experimental data.
At all values of invariant mass $s$ of the hadronic reaction, all possible resonances contribute to the VCS process in the $s$ and $u$ channels. Above the pion threshold, individual resonances can appear as on-shell intermediate states in the $s$-channel. Seventeen known resonances can appear on shell for the center of mass energy up to 3.6 GeV$^2$ (E93-050 kinematic span). They are characterized by their spin and isospin value as presented in Table VII.

TABLE VII: Resonances that can be produced as intermediate states in VCS $s$ reaction channels, $N^*$ isospin 1/2 and $\Delta^*$ isospin 3/2.

<table>
<thead>
<tr>
<th>$N^*_2(2J)$</th>
<th>$\Delta^*_2(2J)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}(1440)$</td>
<td>$P_{33}(1232)$</td>
</tr>
<tr>
<td>$D_{13}(1520)$</td>
<td>$P_{33}(1600)$</td>
</tr>
<tr>
<td>$S_{11}(1535)$</td>
<td>$S_{31}(1620)$</td>
</tr>
<tr>
<td>$S_{11}(1650)$</td>
<td>$S_{31}(1620)$</td>
</tr>
<tr>
<td>$D_{13}(1675)$</td>
<td>$D_{33}(1700)$</td>
</tr>
<tr>
<td>$F_{15}(1680)$</td>
<td>$P_{31}(1750)$</td>
</tr>
<tr>
<td>$D_{13}(1700)$</td>
<td>$S_{31}(1900)$</td>
</tr>
<tr>
<td>$P_{11}(1710)$</td>
<td>$P_{33}(1750)$</td>
</tr>
<tr>
<td>$P_{13}(1720)$</td>
<td>$S_{31}(1900)$</td>
</tr>
<tr>
<td>$P_{13}(1900)$</td>
<td>$S_{31}(1900)$</td>
</tr>
</tbody>
</table>

The most preeminent contribution is given by the $\Delta(1232)$. This dominates the region around $W=1232$ MeV. For the moment this is the only contribution implemented in the calculation in addition to the BH and Born terms.

One important aspect of the experimental data that this theoretical calculation helped to understand, is the contribution of the BH and VCS amplitude for different values $\phi$ angle between the leptonic and hadronic reaction planes (as shown in Figure 2 from Introduction). The incident and scattered electron trajectories determine the leptonic plane; it turns that out acceptance for the final hadron peaks if the $\phi$ angle is $\pm90^\circ$.

We noticed (Figure 60) that for positive $\theta_{\tau\tau}$ angle the cross-section is enhanced by the interference with the fast rising BH contribution. At negative $\theta_{\tau\tau}$ the
The coupling constants as function of $Q^2$ must be fit by comparison with the data. Future extension of this work will encompass comparison of other electro-production reactions ($\pi^0$, $\eta$, $\pi\pi$, etc) within the same formalism. We will test the hypothesis that VCS is predictable by using the information (coupling) of all other reaction channels.
FIG. 60: Bethe-Heitler plus Born cross-section as function of $W = \sqrt{s}$. The angle $\theta_{\gamma\gamma} = \pm 171^\circ$ kinematics was chosen because it is in the center of the bin with biggest number of reactions recorded events for all the experimental settings, as explained in section 6.1. The red stars correspond to $\theta_{\gamma\gamma} = 171^\circ$ and the blue circles correspond to $\theta_{\gamma\gamma} = -171^\circ$. 

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FIG. 61: Bethe-Heitler plus Born and Delta cross-section as function of $W = \sqrt{s}$. The angle $\theta_{\gamma\gamma} = \pm 171^\circ$ was chosen because it is in the center of the bin with biggest number of reactions recorded for all the experimental settings, as explained in section 6.1. The red star corresponds to $\theta_{\gamma\gamma} = 171^\circ$ and the blue circle correspond to $\theta_{\gamma\gamma} = -171^\circ$.
Chapter 6

Experimental Results

In a coincidence experiment with known undetected particle identity, the cross-section is 5-fold differential (as shown in Section 5.5), so we need 5 kinematic variables to describe it. We have chosen to express it differentially in the magnitude of the momentum and the solid angle for the scattered electron and the solid angle of the final proton.

6.1 Definition of The Kinematic Bins

We average the experimental cross-section results over appropriate kinematic bins. We have chosen three kinematic variables to define the bins: $Q^2$ the four momentum transfer, $\cos \theta_{pq}^{CM}$ the cosine function of the angle between the virtual photon and emerging proton in the center of mass of the virtual photon and target proton, and $W = \sqrt{s}$ the CM invariant mass. We bin the data in the following three $Q^2$ intervals: $(0.85, 0.95)$ GeV$^2$, $(0.95, 1.05)$ GeV$^2$ and $(1.05, 1.15)$ GeV$^2$. For all settings the $Q^2$ distribution is similar. For $\cos \theta_{pq}^{CM}$ we used four bins: $(0.80, 0.85)$, $(0.85, 0.90)$, $(0.90, 0.95)$ and $(0.95, 1.00)$. At higher and higher $s$ settings the coverage in $\theta_{pq}^{CM}$ becomes smaller around $180^\circ$. We used 20 MeV bins in $W$. For each setting there are between 8 and 15 $W$ bins with data. Bin selection and distribution of each event in the experiments kinematic settings are illustrated in Figure 62. However, for a complete description, two more dimensions are necessary for
the bins. Since $Q^2$ and $W$ uniquely determine $k_2$ and $\theta_{k_2}$, and $\theta_{pq}^{CM}$ is one of the used kinematic binning variable, the remaining un-described dimensions are $\phi_{k_2}$ and $\phi_{pq}$. If we consider the electron scattering plane defined by the trajectories of incident and detected scattered electron, the angle $\phi$ between the leptonic and hadronic planes would be enough to describe out-of-plane dependence. Figure 63 shows the distribution of $\phi$ as a function of $W$ in the twelve $Q^2$ and $\cos \theta_{pq}^{CM}$ bins. Figure 64 shows the same distributions as in Figure 63, for the Monte Carlo simulated events used to calculate the acceptance. Comparing these coverages, we conclude that only in the case of the kinematic bin with $Q^2 \in (0.95, 1.05)$ and $\cos \theta_{pq}^{CM} \in (0.975, 1.0)$ we do have a $2\pi$ coverage in $\phi$. For the other bins the average cross section depends on the specific $\phi$ acceptance.

We note that in most of the three dimensional kinematic bins ($Q^2$, $\cos \theta_{pq}^{CM}$ and $W$) we have events from more than one setting. However the average cross section extracted from different settings in the same bin will not necessarily agree.

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FIG. 63: The $p(e,e'p\gamma)$ data $\phi$ distributions versus $W$ in $Q^2$ and $\cos\theta_{CM}^{\gamma}$ bins. Each color represents a different kinematic setting. The columns present the three $Q^2$ bins, 1, 2, 3 from left to right. The rows present the four $\cos\theta_{CM}^{\gamma}$ bins, 1, 2, 3, 4 from top to bottom (see Figure 62).
FIG. 64: The \( p(e, e'p) \gamma \) simulation \( \phi_{pq} \) distribution versus \( W \). The bins are the same as in Figure 63.
since the $\phi$ coverage is different for each kinematic setting.

One more observation is that the distribution of the reactions in the same bin for the $p(e,e'p)\gamma$ cross-section and $p(e,e'p)\pi^o$ cross-section are different. In $Q^2$ versus $W$ distribution at the lower $s$ settings one can see (Figure 65) that the coverage is shifted for the pion reactions. The kinematics was defined to be centered on VCS process for $Q^2 = 1 \text{ GeV}^2$ and angle between real and virtual photons $180^\circ$.

FIG. 65: The $p(e,e'p)\gamma$ (black) and $p(e,e'p)\pi^o$ (blue) $Q^2$ vs. $W$ distribution.
6.2 The Average $p(e, e'p)\gamma$ Cross Sections Using Phase Space Simulation

The cross section extraction using no model is the least biased method of obtaining an experimental cross section. We started with a uniform cross section model in the simulation. We obtained a first set of results for the average cross section in each bin. If our cross section is smoothly varying, the results may be very close to the final values. If, however, the cross section varies rapidly, the radiative effects convolute neighboring bins in the simulation into a given bin of the experimental results. We made, from the first set of results, a look-up tables to use as a cross section model in the simulation and obtained a second set of cross section estimates. Once the simulated yield matched the data, we reached the experimental cross section value.

Figures 66, 67 and 68 present the average 5 fold cross sections differential in scattered electron energy, the solid angle of electron, and the solid angle of the recoil proton. The statistical error bars are also displayed. Each color and symbol represents the cross-section values from a different kinematic setting.
FIG. 66: The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of 0.85-0.95 GeV$^2$. Errors are statistical only. A 5% systematic error is estimated. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.
CHAPTER 6. EXPERIMENTAL RESULTS

$\rho(e, ep)\gamma_{\text{experimental}} 0.95 < Q^2 < 1.05 \text{ GeV}^2$

$0.8 < \cos(\theta_p) < 0.85$

$0.85 < \cos(\theta_p) < 0.9$

$0.9 < \cos(\theta_p) < 0.95$

$0.95 < \cos(\theta_p) < 1$

FIG. 67: The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of 0.95-1.05 GeV$^2$. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.
$p(e, ep)\gamma$ experimental $1.05 < Q^2 < 1.15 \text{ GeV}^2$

FIG. 68: The five fold differential experimental cross-section averaged in bins within a $Q^2$ bin of $1.05-1.15 \text{ GeV}^2$. Each color and symbol represents a different kinematic setting. Values are tabulated in Appendix E.
6.3 The Average $p(e, e'p)\gamma$ Cross Sections Using Photon Flux Model in the Simulation

As we discussed in Section 4.5.2, the electro-production processes can be factorized as a photon flux multiplied with a two fold differential cross-section in the solid angle of the recoil proton. Due to the presence of Bethe-Heitler radiation, this factorization is improper to the $p(e, e'p)\gamma$ cross section. We notice that there is a higher density of data recorded around $\phi = \pm 90^\circ$ (see Figure 63). Our theoretical calculation also pointed out that for $\phi = \pm 90^\circ$ BH amplitude is practically 0 and so it is a pure electro-production of a photon. It is then justified in this bin to treat $p(e, e'p)\gamma$ as an electro-production reaction. The cut around $\phi = \pm 90^\circ$ is illustrated in Figure 6.3.

![Typical \(\phi\) distribution of the data (here it is plotted data of the kinematic setting \(s=1.75\ \text{GeV}^2\)). The cuts used for \(\phi\) bin definition are shown. In this bin we can consider the process \(p(e, e'p)\gamma\) electro-production of the photon.](image)

We used then the virtual photon flux model for the purpose to account for the electron scattering part. We can then make qualitative comparisons of $\frac{d^2\sigma}{dt^2\Omega}$ with
FIG. 70: The two fold differential experimental cross-section for the bins with $0.95 < \cos \theta_{\gamma p} < 1.0$. 

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the strength and structure of other electro-production processes, particularly with the $p(e, e'p)\pi^0$ cross-section that we can also extract from same data. We iterate again with look-up tables to achieve stability of the results. Results are plotted in Figure 70.

### 6.4 $p(e, e'p)\pi^0$ Experimental Cross Section

In the data collected during E93-050 a big part of the coincidences we recorded were the result of a $p(e, e'p)\pi^0$ reaction. The pion electro-production reaction was extensively studied and there are well establish theoretical calculations available. We have applied the same data analysis techniques for pion data and have used the MCEEP simulation, with the MAID ([71]) a unitary isobar model for pion electroproduction on the proton, to test our analysis method. In the following graphs 71 the experimental results are presented together with the lines representing the simulated cross-section. There is generally good agreement in the region of $\Delta(1232)$ resonance.
FIG. 71: The two fold average differential experimental cross-section for pion electroproduction $p(e, ep)\pi^0$. The $\phi$ coverage is restricted to $70^\circ < |\phi| < 110^\circ$. Each color and symbol represents a different kinematic setting. The continuous line is the result of MAID simulation.
Chapter 7

Conclusions

The experiment E93-050 was a challenge from many points of view. It was one of the Hall A commissioning experiments and the team analysing E93-050 experiment made major contributions to the understanding and calibration of the Hall A equipment. There was no theoretical calculation of the VCS cross-section in the resonance region prior to this work.

It is interesting to focus in one kinematic bin and try to correlate the cross section variation with the known nucleon resonances position (Figure 72). We note the shift in the position of the $\Delta$ peak due to the interference of BH and VCS amplitude. We also can see that for some resonances this interference can be destructive. We see a clear signal in the region of $S_{11}(1535)$ and $D_{13}(1520)$ resonances.

We have measured for the first time the differential cross section for $p(e,e'p)\gamma$. In the kinematic settings centered on $Q^2=1$ GeV$^2$ and for $s$ between 1.3 and 3.6 GeV$^2$, we have been able to isolate the $p(e,e'p)\gamma$ signal from the $p(e,e'p)\pi^0$ reaction using missing mass technique, due to the high resolution spectrometers in Hall A at TJNAF. We have checked the validity of the data analysis by extracting from the same set of data $p(e,e'p)\pi$ cross-section and comparing them with other measurements and models available.

VCS ($\gamma p \rightarrow \gamma p$) is another tool to study resonances, beyond the meson electro-production that can yield complementary information. The degree to which the
0.95 < \cos(\theta_w) < 1.0, \quad 0.95 < Q^2 < 1.05

$P_{11}, \quad P_{33}, \quad S_{11}$

$1000 \quad 1200 \quad 1400 \quad 1600 \quad 1800 \quad 2000$

$W$(MeV)

FIG. 72: The five fold differential experimental cross-section.
VCS cross section can be predicted from meson electro-production data is an important test of our understanding of the resonance physics.

We developed a first theoretical calculation $p(e, e'p)\gamma$ cross section in the resonance region using phenomenological Lagrangian approach at tree level. By fitting the coupling constants related to the proton, virtual photon, and different resonant states vertex on the final cross-section values, we expect to learn how the photon electro-production completes and complement our understanding of the resonance physics. Neutral pion electro-production is an interesting reference in the study of VCS. Using the form factors fitted to pion electro-production data using the same theoretical approach, we expect the calculated cross section $p(e, e'p')\gamma$ to be in good agreement with the experimental cross section. We can extract the experimental cross sections ratio $p(e, e'p')\gamma/p(e, e'p')\pi^0$ to higher precision than extracting each cross-section alone.

From both a theoretical and data analysis point of view the team effort continues.
Appendix A

Singular Values and Optics Coefficients

Section 3.2 detailed the singular value decomposition of the spectrometer optics transport matrix. The singular values and the corresponding coefficient are listed in the following tables. The results are similar for electron and hadron arm and do not depend of the number of events considered (same results with 15000 and 150000 good events). We want to consider all necessary coefficients but not to add instability to the linear system by adding un-necessary higher order terms.

Keeping only the coefficients from Table VIII the condition number $\kappa$ of the linear system used for determining the expansion coefficients lowered from $10^{13}$ (with the coefficients from Table IX) to $10^8$.

Keeping only the coefficients from Table X, the condition number $\kappa$ of the linear system used for determining the $\phi$ expansion coefficients lowered from $10^{12}$ (with the coefficients from Table XI) to $10^5$.

We needed to use all the coefficients from XII for obtaining a good fit, without aberations in $\theta$ expansion.

For doing the singular value analysis for the $\delta$ expansion we needed to remove the $x_{tg}$ in spectrometer coordinate system equal $y_{beam}$ in Hall A coordinate system influence before doing the SVD analysis. Taking the first order transport matrix
elements (see [44]) we built the following system of equations:

\[
\begin{align*}
x_{fp} &= -2.181 x_{tg} - 0.198 \theta_{tg} + 11.901 \delta \\
y_{fp} &= -0.100 x_{tg} - 0.496 \theta_{tg} + 1.967 \delta
\end{align*}
\] (192, 193)

Eliminating \( \theta_{tg} \) we can write:

\[
\delta = F(x_{fp}, y_{fp}) + 0.1926 x_{tg}
\] (194)

The functional \( F \) is actually the polynomial expansion and we can write now a \( \chi^2 \) minimisation exactly as before. However \( x_{tg} \) is a term that varies on a event by event basis and has to be removed. SVD analysis allowed again to identify a core group of coefficients. We fitted these first and than we added in the ESPACE optimisation as many coefficients as we felt needed to remove any undesirable correlation. For the moment we can not deconvolute the influence of \( x_{tg} \) on stability so we do not present the list of the SVD of the delta polynomial coefficients.
TABLE VIII: Coefficients considered significant for $Y$ (target coordinate) polynomial expansion as function of the focal plane coordinates.

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TABLE IX: Coefficients considered less significant for $Y$ (target coordinate) polynomial expansion as function of the focal plane coordinates.

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TABLE X: Coefficients considered significant for $\phi$ (target coordinate) polynomial expansion as function of the focal plane coordinates.

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TABLE XI: Coefficients considered significant for $\phi$ (target coordinate) polynomial expansion as function of the focal plane coordinates.

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TABLE XII: Coefficients $\theta$ (target coordinate) polynomial expansion as function of the focal plane coordinates.

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Appendix B

External Processes in MCEEP

In this section are presented the formula and methods I employed in development of the routines for treating energy loss, multiple scattering and external Bremsstrahlung incorporated by Paul Ulmer in MCEEP.

B.1 Hadron Energy Loss by Collision

The basic formula used for calculating the average energy loss is known as Bethe-Bloch formula[74].

\[ \frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z z^2}{A \beta^2} \left[ \ln \left( \frac{2m_e r_e^2 v^2 W_{\text{max}}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right] \]  

(195)

with

\( dE/dx \): the mean rate of energy loss
\( r_e \): classical electron radius 2.818 fm
\( m_e c^2 \): rest energy of the electron 0.511MeV
\( N_a \): Avogadro's number = 6.022 x 10^{23} \text{ mol}^{-1}
\( I \): mean excitation potential
\( Z \): atomic number of absorbing material (target)
\( A \): atomic weight of absorbing material
\( \rho \): density of absorbing material
\( z \): charge of incident particle in units of e
APPENDIX B. EXTERNAL PROCESSES IN MCEEP

\[ \beta: \nu/c \text{ of the incident particle} \]

\[ \gamma: 1/\sqrt{1 - \beta^2} \]

\( \delta: \) density correction

\( C: \) shell correction

\( W_{\text{max}}: \) maximum energy transfer in a single collision

\[ 2\pi N_e r^2_m c^2 = 0.1535 \text{ MeV cm}^2/\text{g}. \]

The maximum energy transfer is produced by a head-on or knock-on collision.

For an incident particle of mass \( M \gg m_e \) can be used the approximation

\[ W_{\text{max}} = 2m_e c^2 \beta^2 \gamma^2 \]

The mean excitation potential of the target material was taken from the tables for the material known to be used in the Hall A experiments. However for other materials some empirical formula [74] were set up to give at least a good approximation of this quantity.

The density and shell corrections to the Bethe-Bloch formula are important at high and low energies respectively. The density effect arises from the fact that the electric field of the particle also tends to polarize atoms along its path. Because of this polarization, electrons far from the path of the particle will be shielded from full electric field intensity. Collisions with these outer lying electrons will therefore contribute less to the total energy loss than predicted by the Bethe-Bloch formula.

Values for \( \delta \) are given by a formula due to Sternheimer [76] for \( X = \log_{10}(\beta \gamma) \)

\[
\delta(X) = \begin{cases} 
\delta_0 10^{2(X - X_0)} & X < X_0 \\
4.6052X + C_0 + a(X_1 - X)^m & X_0 < X < X_1 \\
4.6052X + C_0 & X_1 < X
\end{cases}
\]

The quantities \( X_0, X_1, C_0, a \) and \( m \) depend on the absorbing material. The parameter \( C_0 \) is corrected for the difference between the material density \( \rho_0 \) at which these parameters were fitted and the density at which they are used.

The shell correction accounts for effects which arise when the velocity of the incident particle is comparable or smaller than the orbital velocity of the bound
electrons. At such energies, the assumption that the electron is stationary with respect to the incident particle is no longer valid and the Bethe-Bloch formula breaks down. The correction is generally small and to calculate it is used the following empirical formula[74], valid for \( \eta = \beta \gamma < 0.1 \) and \( I \) in eV:

\[
C(I, \eta) = (0.422377\eta^{-2} + 0.034043\eta^{-4} - 0.00038106\eta^{-6})10^{-6}I^2 \\
+(3.850190\eta^{-2} - 0.1667989\eta^{-4} + 0.00157955\eta^{-6})10^{-9}I^3. \quad (197)
\]

Due to the small variation of these corrections in the kinematic acceptance window it is enough to be evaluated once for the central ray kinematics.

### B.2 Electron Energy Loss by Collision

The basic mechanism of collision loss for electrons is the same as for the heavy charged particles. However, Bethe-Bloch formula has a different form to take into consideration on one hand the fact that the maximum energy transfer \( W_{\text{max}} = T_\varepsilon/2 \) (where \( T_\varepsilon \) is the kinetic energy of the incident electron), and on the other hand the indistinguishability of the final electrons.

\[
-F = -\frac{\beta}{\gamma} - s < 0 \quad (198)
\]

where \( \tau \) is the kinetic energy of the particle in units of \( m_\varepsilon c^2 \), and

\[
F(\tau) = 1 - \beta^2 + \frac{\tau^2/8 - (2\tau + 1)\ln 2}{(\tau + 1)^2}
\]

### B.3 The Energy Loss Distribution (Straggling)

The previous presented formulas represent the mean energy loss suffered by charged particles when passing through a thickness of matter. For any given particle, the amount or energy lost will not, in general, be equal to this mean value because of the statistical fluctuations which occur in the number of collisions suffered and the energy lost in each collision. An initially monoenergetic
APPENDIX B. EXTERNAL PROCESSES IN MCEEP

beam, after passing through a fixed thickness of material, will show a certain distribution in energy that in average is shifted down with the value given by the Bethe-Bloch formula.

Theoretical calculation of these distributions have yield results for different regions of $0$. To choose between these theories it is used the parameter $\kappa$ representing the ratio between the mean energy loss and the maximum energy transfer allowable in a single collision ([77]). Consequently:

\[
\begin{cases}
\kappa \leq 0.01 & \text{Landau distribution thin absorber} \\
0.01 < \kappa \leq 10.0 & \text{Vavilov distribution} \\
10.0 < \kappa & \text{Gaussian distribution}
\end{cases}
\]

**B.4 Multiple Scattering Through Small Angles**

Most of the small angles deflections suffered by a charged particle traversing a medium is due to Coulomb scattering from nuclei. The Coulomb scattering distribution is well represented by the theory of Molière, but for small deflection angles it is roughly a Gaussian. The Moliere scattering formulas are expressed in terms of two angles - the characteristic angle $\chi_c$ and the screening angle $\chi_\alpha$ which are defined as follows [78]:

\[
\begin{align}
\chi_c^2 &= 0.157[Z(Z + 1)X/A][z/(p\beta)]^2 \\
\chi_\alpha^2 &= 2.007 \times 10^{-5}Z^{2/3}[1 + 3.34(Zz\alpha/\beta)^2]/p^2
\end{align}
\]

where $p$ is the momentum in MeV/c, $X$ is the path length in g/cm$^2$, $Z$ and $A$ are the charge and atomic weight of the scattering material, $z$ is the charge of the projectile, $\beta = v/c$, and $\alpha = 1/137$. A good representation of the Moliere distribution is a Gaussian distribution with the width $\sigma$ calculated in the following way:

\[
\begin{align}
u &= 0.5\frac{\chi_c}{\chi_\alpha}(1 - F) \\
\sigma &= \frac{\chi_c^2}{1 + F^2}\left[\frac{1 + v}{v}\ln(1 + v) - 1\right]
\end{align}
\]

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APPENDIX B. EXTERNAL PROCESSES IN MCEEP

For the parameter $F$ between 0.9 and 0.995 the accuracy of representing the Moliere distribution is better than 2%. This is the accuracy with which we can calculate the width of the gaussian distribution centered in 0 of the multiple scattering suffered by a charged particle traversing a medium ([77]).

### B.5 External Bremsstrahlung

The incident and scattered electron lose, in addition to their ionization loss, energy by interaction with the Coulomb field of the nuclei of the traversed medium. If the charged particles are decelerated in the Coulomb field of the nucleus, a fraction of their kinetic energy will be emitted in form of photons (bremsstrahlung). In estimating this process it is convenient to measure the thickness of the material in units of the radiation length $X_0$. This is the mean distance over which a high energy electron loses all but 1/e of its energy by bremsstrahlung. To calculate the parameter $b=1/X_0$ we use the following formula[74]

$$b = \frac{1}{X_0} = 4\alpha r_e \frac{N_A}{A} [Z^2(L_{\text{rad}} - f(Z)) + ZL'_{\text{rad}}]$$

(203)

For $A=\text{g/mol}$, the quantity $4\alpha r_e \frac{N_A}{A} = (716.408 \text{ g/cm}^2)^{-1}$. The parameters $L_{\text{rad}}$ and $L'_{\text{rad}}$ are tabulated or can be calculated with the empirical formulas:

$$L_{\text{rad}} = \ln(184.15 Z^{-1/3}), L'_{\text{rad}} = \ln(1194Z^{-2/3}).$$

(204)

The function $f(Z)$ is an infinite sum but can be very well represented for $a = \alpha Z$ by:

$$f(Z) = a^2 \left( \frac{1}{1 + a^2} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6 \right).$$

(205)

The probability distribution for a particle of momentum $E$ to radiate an energy $\Delta E$ when traversing $t$ radiation length of material is:

$$I_{\text{ext}}(E, \Delta E, t) = \frac{bt}{\Gamma(1 + bt)} \left( \frac{\Delta E}{E} \right)^{bt} \left( \frac{1}{\Delta E} \left( 1 - \frac{\Delta E}{E} + \frac{3}{4} \left( \frac{\Delta E}{E} \right)^2 \right) \right)$$

(206)

For generate this external radiation distribution I implemented in MCEEP an accept-reject technique in the same manner as in AEEXB[79]. Consider $\varepsilon$ the

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envelope function of the target distribution $I_{\text{ext}}$

$$\epsilon(\Delta E) = 1.1 \frac{bt}{\Gamma(1 + bt)E^{bt}} \Delta E^{bt-1}$$  \hfill (207)

To create a sampled distribution having $\epsilon(\Delta E)$ as its shape, two random numbers $0. < r_1, r_2 < 1.$ are generated. With their values a trial value for energy radiated is calculated

$$\Delta E_{\text{trial}} = E_1^{1/bt}.$$  \hfill (208)

If with this $\Delta E_{\text{trial}}$ the following test is passed

$$r_2 \epsilon(\Delta E_{\text{trial}}) < I_{\text{ext}}(E, \Delta E_{\text{trial}}, t)$$  \hfill (209)

the generated $\Delta E_{\text{trial}}$ follows the desired bremsstrahlung distribution. If not the process is restarted with other random numbers $r_1$ and $r_2.$
Appendix C

The Run-by-Run Normalization Factors

For the runs used in the analysis the charge, boiling correction and deadtime correction were determined only for the pieces used in the analysis. If a run number appears twice it is because the run had to be split to recover due to some data acquisition problems.
### TABLE XIII: The run-by-run normalization factors (part 1).

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TABLE XIV: The run-by-run normalization factors (part 2).

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### APPENDIX C. THE RUN-BY-RUN NORMALIZATION FACTORS

#### TABLE XV: The run-by-run normalization factors (part 3).

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Appendix D

$\Delta p\gamma$ and $\pi^0\gamma\gamma$ Coupling Constants

D.1 $\Delta(1232)$ Coupling Constants

To determine the values of the $\Delta \to p\gamma$ coupling constants, we calculated the width of $\Delta$ photo-decay. This cross-section and the helicity amplitudes of the $\Delta$ photo-decay are listed in the Particle Data Book [75].

![Diagram](image)

FIG. 73: $\Delta \to p\gamma$ diagram representation.

The amplitude for the $\Delta \to p\gamma$ transition is

$$iM_{\Delta\gamma} = i\epsilon\bar{u}_p \gamma_5 \Lambda^\mu u_\mu , \quad (210)$$

where

$$\Lambda_\mu = t_{1\nu}\theta_{\mu\nu} \gamma_5 (\epsilon^{\alpha\beta} q^\alpha - \epsilon^{\alpha\beta} q^\beta) + t_{2\nu}\theta_{\mu\nu} \epsilon_\delta (\epsilon^{\alpha\beta} q^\alpha - \epsilon^{\alpha\beta} q^\beta) , \quad (211)$$

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\[ \theta_{\mu\alpha} = g_{\mu\alpha} + \left[ \frac{1}{2} (1 + 4V) A + V \right] \gamma_{\mu} \gamma_{\nu} \simeq g_{\mu\alpha}. \] (212)

Here \( t_{1r} \) and \( t_{2r} \) are the coupling constants, \( \bar{u}_p \) is the final nucleon spinor, \( u_\mu \) is the \( \Delta \) spinor and \( \epsilon^* \) is the outgoing photon polarization. Therefore we can write

\[ i\mathcal{M}_{\Delta N} = i\epsilon_{\mu\nu} \gamma_5 (t_{1r}(\vec{p}_\Delta \cdot q - \vec{p}_\nu) + t_{2r}((p_\Delta \cdot \epsilon^*)q_\mu - (p_\Delta \cdot q)\epsilon^*_\mu))u^\mu \] (213)

\[ = i\epsilon_{\mu\nu} \gamma_5 (t_{1r} \, q + t_{2r}(p_\Delta \cdot \epsilon^*)q_\mu u_\mu - [t_{1r} \, q - t_{2r}(p_\Delta \cdot q)]\epsilon^*_\mu u^\mu. \] (214)

We now consider the process in the center of mass frame (CM) in which initial \( \Delta \) is at rest. The four-momenta that describe the particles are \( p_\Delta = (M_\Delta, \vec{0}) \), \( p_p = (E_p, \vec{p}_p) \) and \( q = (p, \vec{p}_\gamma) \). Therefore

\[ i\mathcal{M}_{\Delta N} = i\epsilon_{\mu\nu} \gamma_5 (t_{1r} \, q - (\vec{p}_\Delta - \vec{p}_p) t_{1r} \epsilon^*_\mu - t_{2r} M_\Delta q_\mu \epsilon^*_\mu)u^\mu \]

\[ = i\epsilon_{\mu\nu} \gamma_5 (t_{1r} \, q - (M_\Delta - M) t_{1r} \epsilon^*_\mu - t_{2r} M_\Delta q_\mu \epsilon^*_\mu)u^\mu. \] (215)

The spinor \( u_\mu \) can be expressed in terms of a Dirac spinor for a spin \( \frac{1}{2} \) particle and a polarization vector of a spin \( 1 \) particle. Starting from

\[ \epsilon^{(+1)} = -\sqrt{\frac{1}{2}} (0, 1, i, 0), \]

\[ \epsilon^{(0)} = \frac{1}{M_\Delta} (0, 0, 0, M_\Delta), \]

\[ \epsilon^{(-1)} = \sqrt{\frac{1}{2}} (0, 1, -i, 0), \] (216)

(217)

for the spin \( 1 \) particle, and

\[ u \left( \frac{1}{2} \right) = \sqrt{E_\Delta + M_\Delta} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, u \left( -\frac{1}{2} \right) = \sqrt{E_\Delta + M_\Delta} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \] (218)

for the spin \( \frac{1}{2} \) particle, the spin \( \frac{3}{2} \) particle spinors are

\[ u \left( \frac{3}{2}, \frac{3}{2} \right) = u \left( \frac{1}{2} \right) \epsilon^{(+1)}, \] (219)
APPENDIX D. $\Delta P\gamma$ AND $\pi^0\gamma\gamma$ COUPLING CONSTANTS

(220)

$u\left(\frac{3}{2}, \frac{1}{2}\right) = \sqrt{\frac{2}{3}} u\left(\frac{1}{2}\right) \epsilon(0) + \sqrt{\frac{1}{3}} u\left(-\frac{1}{2}\right) \epsilon(+1)$,  

(221)

$u\left(\frac{3}{2}, -\frac{1}{2}\right) = \sqrt{\frac{2}{3}} u\left(-\frac{1}{2}\right) \epsilon(0) + \sqrt{\frac{1}{3}} u\left(\frac{1}{2}\right) \epsilon(-1)$,  

(222)

$u\left(\frac{3}{2}, -\frac{3}{2}\right) = u\left(-\frac{1}{2}\right) \epsilon(-1)$.

FIG. 74: $\Delta$ photo-decay process in the center of mass frame.

In the CM frame the $\Delta$ spin is quantized with respect to an arbitrary $z$-axis. Final particles are emitted back to back (see Figure 74). We define the $xz$ plane determined by the final particles momenta and $z$ axis. In $xz$ plane, $\theta$ is defined as the angle from $z$ to the direction in which the photon is emitted. The photon four-momentum is then $(p, p\sin(\theta), 0, p\cos(\theta))$, while the nucleon four-momentum is $(E_p, -p\sin(\theta), 0, -p\cos(\theta))$. In this frame we can write:

\[
p = \frac{M_\Delta^2 - M^2}{2M_\Delta}, \quad E_p = \frac{M_\Delta^2 + M^2}{2M_\Delta}, \quad \sqrt{E_\Delta + M_\Delta} = \sqrt{2M_\Delta}.
\]

\[
\sqrt{E_p + M} = \frac{M_\Delta + M}{\sqrt{2M_\Delta}}, \quad \frac{p}{E_p + M} = \frac{M_\Delta - M}{M_\Delta + M} = \kappa_\Delta.
\]

The outgoing real photon polarization states are then

\[
\epsilon(+1) = -\sqrt{\frac{1}{2}}(0, \cos(\theta), i, -\sin(\theta)),
\]

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\( \varepsilon(-1) = \sqrt{\frac{1}{2}}(0, \cos(\theta), -i, -\sin(\theta)) \). \quad (224)

The nucleon spinors are

\[
\begin{align*}
    u_p \left( \frac{1}{2} \right) &= \sqrt{E_p + M} \begin{pmatrix}
        \cos\left(\frac{\theta + \pi}{2}\right) \\
        \sin\left(\frac{\theta + \pi}{2}\right) \\
        \frac{\sigma \cdot p}{E_p + M} \cos\left(\frac{\theta + \pi}{2}\right) \\
        \frac{\sigma \cdot p}{E_p + M} \sin\left(\frac{\theta + \pi}{2}\right)
    \end{pmatrix}, \\
    u_p \left( -\frac{1}{2} \right) &= \sqrt{E_p + M} \begin{pmatrix}
        -\sin\left(\frac{\theta + \pi}{2}\right) \\
        \cos\left(\frac{\theta + \pi}{2}\right) \\
        \frac{\sigma \cdot p}{E_p + M} \left(-\sin\left(\frac{\theta + \pi}{2}\right)\right) \\
        \frac{\sigma \cdot p}{E_p + M} \left(\cos\left(\frac{\theta + \pi}{2}\right)\right)
    \end{pmatrix}.
\end{align*}
\]

Considering \( \bar{u} = u^\dagger \gamma_0 \) we have

\[
\begin{align*}
    \bar{u}_p \left( \frac{1}{2} \right) &= \frac{M_\Delta + M}{\sqrt{2}M_\Delta}(-\sin\left(\frac{\theta}{2}\right), \cos\left(\frac{\theta}{2}\right), \kappa_\Delta \sin\left(\frac{\theta}{2}\right), -\kappa_\Delta \cos\left(\frac{\theta}{2}\right)), \\
    \bar{u}_p \left( -\frac{1}{2} \right) &= \frac{M_\Delta + M}{\sqrt{2}M_\Delta}(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right), \kappa_\Delta \cos\left(\frac{\theta}{2}\right), \kappa_\Delta \sin\left(\frac{\theta}{2}\right)).
\end{align*}
\]

We use the notation

\[
\begin{align*}
    M_0 \left( \frac{3}{2} \right) &= -e(M_\Delta^2 - M^2)(t_{1r} + \frac{(M_\Delta^2 - M^2)t_{2r}}{2}), \\
    M_0 \left( \frac{1}{2} \right) &= M_\Delta^2 - M^2 \frac{2}{2\sqrt{3}M_\Delta} (-2t_{1r}M + t_{2r}M_\Delta(M_\Delta - M)).
\end{align*}
\]

We label the transition amplitudes with \( \lambda_\Delta, \lambda_p, \lambda_\gamma \) according to \( \Delta \) spin projection along \( z \) axis (quantization axis), and the helicities of the proton and photon,
respectively. For the 16 possible combinations $\mathcal{M}_{\lambda\sigma, \lambda\sigma, \lambda\sigma}$, we obtain the expressions:

\[
\begin{align*}
\mathcal{M}_{\frac{3}{2}, \frac{3}{2}, 1} &= \frac{1}{2} \sqrt{3} \cos^2 \theta \sin^2 \phi, \\
\mathcal{M}_{\frac{3}{2}, -\frac{3}{2}, 1} &= -\frac{1}{2} \sqrt{3} \sin^2 \theta \cos^2 \phi, \\
\mathcal{M}_{\frac{3}{2}, \frac{3}{2}, -1} &= \mathcal{M}_{\frac{3}{2}, -\frac{3}{2}, -1} = \mathcal{M}_{\frac{3}{2}, \frac{3}{2}, 1} = \mathcal{M}_{\frac{3}{2}, -\frac{3}{2}, 1} = \frac{1}{2} \sqrt{3} \sin^2 \theta \cos^2 \phi, \\
\mathcal{M}_{\frac{1}{2}, -\frac{1}{2}, 1} &= \mathcal{M}_{\frac{1}{2}, -\frac{1}{2}, -1} = \mathcal{M}_{\frac{1}{2}, \frac{1}{2}, 1} = \mathcal{M}_{\frac{1}{2}, \frac{1}{2}, -1} = \frac{1}{2} \sqrt{3} \sin^2 \theta \cos^2 \phi, \\
\mathcal{M}_{-\frac{1}{2}, \frac{1}{2}, 1} &= \mathcal{M}_{-\frac{1}{2}, \frac{1}{2}, -1} = \mathcal{M}_{-\frac{1}{2}, -\frac{1}{2}, 1} = \mathcal{M}_{-\frac{1}{2}, -\frac{1}{2}, -1} = \frac{1}{2} \sqrt{3} \sin^2 \theta \cos^2 \phi.
\end{align*}
\]

There are only 8 distinct expressions, the rest being connected by the relation

\[
\mathcal{M}_{-\lambda\sigma, -\lambda\sigma, -\lambda\sigma} = (-1)^{\lambda\Delta + \lambda\sigma + \lambda\tau} \mathcal{M}_{\lambda\sigma, \lambda\sigma, \lambda\sigma}.
\]

Using the notation $\chi = \lambda\sigma - \lambda\tau$, the matrix elements are

\[
\mathcal{M}_{\lambda\Delta, \chi} = \mathcal{M}_0(\lambda |\chi|) d^2_{\lambda\Delta, \chi},
\]

where the angular parts of these amplitudes are the Wigner $d$-functions ([75]).

The decay width is given by ([75])

\[
d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|p|}{M^2} d\Omega.
\]

Integrating in CM frame over solid angle, averaging over initial spin states and summing over the final, then using the relation

\[
\int |d^l_{m, n}|^2 d(cos(\theta)) = \frac{2}{2J + 1},
\]

we obtain

\[
\Gamma_{1/2} = \frac{p|M_0(\frac{1}{2})|^2}{32\pi M^2_\Delta},
\]

\[
\Gamma_{3/2} = \frac{p|M_0(\frac{3}{2})|^2}{32\pi M^2_\Delta}.
\]
As function of helicity amplitudes ([75]), the decay width is

$$\Gamma_\gamma = \frac{p^2}{\pi} \frac{2M}{(2J+1)M_\Delta} [|A_{1/2}|^2 + |A_{3/2}|^2].$$  (236)

Comparing equations (235) and (235) with (236), we have

$$\Gamma_\lambda = \frac{\left|M_0(\lambda)\right|^2 p^2}{16\pi M_\Delta^2} = \frac{p^2}{\pi} \frac{M}{2M_\Delta} |A_\lambda|^2,$$  (237)

and therefore

$$\left|M_0(\lambda)\right|^2 = \frac{8pM_\Delta M|A_\lambda|^2}{4(M_\Delta^2 - M^2)M|A_\lambda|^2}. $$  (238)

Starting from the world present known values [75] for \(\Delta\) photo-decay helicity amplitudes \(A_{1/2} = -0.140 \pm 0.005\) GeV\(^{-1/2}\) and \(A_{3/2} = -0.258 \pm 0.006\) GeV\(^{-1/2}\) we obtain the following possible values of the coupling constants: \(t_{1r} = 2.27554, t_{2r} = -1.42447\).

Another way to determine these constants is by using the multipole expression ([72]). The helicity amplitudes are related to the electric quadrupole transition \(E_2\) and the magnetic dipole transition \(M_1\) by

$$A_{1/2} = -\frac{1}{2}(M_1 + 3E_2),$$  (239)

$$A_{3/2} = -\frac{\sqrt{3}}{2}(M_1 - E_2),$$  (240)

and therefore \(M_1 = 0.293435\) GeV\(^{-1/2}\) and \(E_2 = -0.00447818\) GeV\(^{-1/2}\). The expressions of multipole transition using our coupling constants definitions are

$$E_2 = \sqrt{\frac{3}{2}} \frac{3 - e}{3} \frac{p}{M_\Delta + M} \left[\frac{pM_\Delta}{M}\right]^{1/2} [t_{1r} + t_{2r}M_\Delta],$$  (241)

$$M_1 = \sqrt{\frac{3}{2}} \frac{3 - e}{26} \left[\frac{p}{M_\Delta M}\right]^{1/2} \left[t_{1r}(3M_\Delta + M) + t_{2r}M_\Delta(M_\Delta - M)\right],$$  (242)

and therefore \(t_{1r} = 2.27554\) and \(t_{2r} = -1.42447\). Considering the error in the helicity amplitudes we conclude \(t_{1r} = 2.27554 \pm 0.30283\) and \(t_{2r} = -1.42447 \pm 0.04790\).
D.2 Pion Decay Constant

We calculated \( g_{\pi \gamma \gamma} \) from the width of pion decay. According to ([75]) the mean life-time of \( \pi^0 \) is \( \tau = (8.4 \pm 0.6) \times 10^{-17} \) sec. Lifetime is the inverse of decay rate \( \Gamma \). The pion decays into two photons in \( \Gamma_{2\gamma}/\Gamma = 98.798 \pm 0.032 \)% of cases.

\[
\begin{align*}
&\pi_0 \\
&\gamma(q_1) \\
&\gamma(q_2)
\end{align*}
\]

FIG. 75: \( \pi \rightarrow \gamma\gamma \) diagram representation.

The decay rate of the process \( \pi^0 \rightarrow \gamma\gamma \) (with the four momentum conservation \( p_\pi = q_1 + q_2 \)) is

\[
\Gamma_{2\gamma} = \frac{1}{2m_\pi} S \int \frac{d^3q_1}{(2\pi)^3|q_1|} \int \frac{d^3q_2}{(2\pi)^3|q_2|} (2\pi)^4 \delta^4(p_\pi - q_1 - q_2) \sum |\mathcal{M}(\pi\gamma\gamma)|^2,
\]

where \( S \) is a statistical factor (the product of \( 1/j! \) for each group of \( j \) identical particles in the final state, i.e. due to the 2 final photons \( S=1/2 \)). The matrix element of the decay is

\[
i\mathcal{M}(\pi \rightarrow \gamma\gamma) = e^2 g_{\pi\gamma\gamma} \varepsilon_{\alpha\beta\mu\nu} \varepsilon_{q_1}^\alpha \varepsilon_{q_2}^\beta q_1^\mu q_2^\nu,
\]

and therefore

\[
\sum |\mathcal{M}|^2 = \sum e^4 g_{\pi\gamma\gamma}^2 \varepsilon_{\alpha\beta\mu\nu} \varepsilon_{q_1}^\alpha \varepsilon_{q_2}^\beta q_1^\mu q_2^\nu \varepsilon_{q_1}^{\alpha\prime} \varepsilon_{q_2}^{\beta\prime} q_1^{\mu\prime} q_2^{\nu\prime} = e^4 g_{\pi\gamma\gamma}^2 \left[ (q_1 q_2)^2 - q_1^2 q_2^2 \right] = e^4 g_{\pi\gamma\gamma}^2 \frac{m_\pi^2}{4}.
\]
We used in the previous calculation \( p_x^2 = (q_1 + q_2)^2 = 2q_1q_2 = m_x^2 \). Then, we have

\[
\Gamma_{2\gamma} = \frac{e^4}{64\pi^2 m_x} \int d\Omega_{q_1} \int d|k_1|\delta(2|k_1| - m_x) g_{\pi\gamma\gamma}^2 (q_1q_2)^2
\]

\[= \frac{e^4}{64\pi^2 m_x} g_{\pi\gamma\gamma}^2 (4\pi) \frac{m_x^4}{4} = \frac{e^4 g_{\pi\gamma\gamma}^2 m_x^3}{64\pi} = \frac{\alpha^2 g_{\pi\gamma\gamma}^2 m_x^3}{4}. \tag{248}
\]

We can then extract the value of \( g_{\pi\gamma\gamma} \) as:

\[g_{\pi\gamma\gamma} = \sqrt{\frac{4\Gamma_{2\gamma}}{\alpha^2 \pi m_x^3}} = \sqrt{\frac{4\Gamma_{2\gamma}/\Gamma_h}{\alpha^2 \pi m_x^3}} = 2.74359 \times 10^{-4} \text{MeV}^{-1} \tag{249}
\]

In the vertex \( \pi\gamma\gamma \) at the VCS \( t \)-channel diagram, one of the photons is virtual, and therefore we assume a \( Q^2 \) dependence also for this coupling constant. However, since the amplitude of this process is orders of magnitude smaller than the \( s \) and \( u \) channel contributions, it is unlikely to have sensitivity to this dependence.
Appendix E

Experimental $p(e, e'p)\gamma$

Cross-section

Reading this table one should keep in mind the kinematical bins definition from section 6.1. The tabulated values for $Q^2$, $\cos \theta_{pq}$ and $W$ are the middle of the bins. The five differential cross-section values (in $\text{sr}^{-1}\text{MeV}^{-1}$) are averaged over the kinematical bins.
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## APPENDIX E. EXPERIMENTAL $P(E, E'P)\gamma$ CROSS-SECTION

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## APPENDIX E. EXPERIMENTAL $P(E, E')\gamma$ CROSS-SECTION

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Luminita Anca Todor (born Stoian) from Bucharest, Romania, graduated High School Nicolae Balcescu in June 1981 and became a physics student. In June 1986 she graduated with an Engineer Physicist (M.S. equivalent) degree from Bucharest University. She is married and has two children Horia and Alina. In January 1995 she entered Old Dominion University graduate program. The focus of her research during graduate studies were electron scattering experiments. She took an active part in the installation, commissioning and first experiments in Hall A at Thomas Jefferson National Accelerator Facility. She received a Masters of Physics in August 1996 and defended her thesis in July 2000. She has accepted a Research Associate position with Carnegie Mellon University and will be working in Experimental Nuclear Physics.

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