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Wake Vortex Pair Formation as an Analog for Dust Devil and Tornado Genesis

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In 1966, meteorologist R.S. Scorer attempted to explain how large-scale oceanic tropical depressions become hurricanes or typhoons. His model was based on the idea that when these large-scale tropical depression structures begin to rotate, mostly due to Coriolis effects, an annular outer portion of that structure changes suddenly to a potential vortex segment, with the same outer radial limit as the low-pressure structure, but with an inner radius that conserves the overall system angular momentum and kinetic energy. By analogy with the "jump" instability describing sudden buckling of a vertical column, this paper shows that his conjecture merits additional consideration. If valid, the Scorer model implies that the controlling large-scale flow is essentially an inviscid Rankine vortex. While hurricanes can sustain this Rankine vortex "eye structure" over warm ocean, over land smaller-scale tornadoes and dust devils cannot draw from a similar sustaining energy source. Scorer's model implies that, without additional energy, the outer inviscid vortex region should force the rotating inner cylindrical region to collapse as the overall inviscid structure proceeds toward the rotational axis. That vortex evolution requires additional energy—from an unknown source.

This paper utilizes Scorer's finite vortex domain hypothesis on an evolving aircraft wake vortex pair, and his assertion that the inviscid vortex pair is the controlling flow, to generate turbulent non-equilibrium vortex cores and by extension explain how tornadoes and dust devils form from rotating atmosphere.

I. Nomenclature

$\dot{\mathbf{e}}_{r,\theta}(r)$	=	radially-varying shear strain rate for a potential vortex
h	=	height
k	=	Scorer's circulation parameter
M	=	Mach number
R_{Max}	=	outer radius of rotating atmosphere
R_{Min}	=	inner "jump" radius
r	=	radial distance from rotational axis
S	=	wingspan
s_o	=	semi-major axis (of wake vortex ellipse)
U_∞	=	cruise velocity
V_{axis}	=	induced initial negative velocity resulting from equal-but-opposite potential vortices
$V_\theta(r)$	=	radially-varying azimuthal velocity

$\mathbf{V}(x,y) = u(x,y)\mathbf{i} + v(x,y)\mathbf{j}$, velocity vector

W	=	weight
ε	=	turbulent eddy viscosity
$\dot{\varepsilon}_{r\theta}$	=	shear strain rate

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η	=	Ratio of spontaneous vortex circulation to maximum rotational circulation limit
η_p	=	pressure relaxation coefficient (seconds)
μ	=	dynamic viscosity
ρ	=	density
Γ	=	circulation
Ω	=	angular velocity of rotating atmosphere (radians/s)

II. Introduction

Vortex-based lifting line methods have been utilized in the study of aerodynamics for more than a century [1],[2]. Models utilizing a lattice of discreet axial vortices, neglecting viscous and turbulent effects have enabled surprisingly accurate estimates of wing loads and lift for realistic airfoil geometries, in spite of the mathematical fact that these discreet axial vortices incorporate non-physical "infinite" centerline rotational velocities and total kinetic energy limits. Even with these non-physical limits, the coalescence and roll up of the left and right wing-vortex-systems into a trailing line vortex pair have been modeled to yield useful initial circulation-based trailing line strength estimates helping to characterize the long-lived *and hazardous* trailing-line vortex pair. Away from the ground, this equal-but-opposite circulation pair usually stabilizes vortex spacing, while inducing a constant downward descent rate. Lord Kelvin (*circa* 1868) determined that a fully-formed, two-dimensional equal-but-opposite line vortex pair produced an ellipse-shaped flow regime that could be isolated from its inviscid surroundings [3]. Initial circulation and aircraft wake vortex spacing can be estimated, based on total weight, wingspan and flight speed. Greene [4] utilized heuristic arguments to justify modeling that shed trailing line vortex pair during cruising flight, as a sort of descending ellipse-shaped object with a frontal area based on Lord Kelvin's descending vortex pair model,,but subjected to pressure drag. His model produced useful insights while implying that the descending elliptical volume of fluid could be isolated from its surroundings.

While airplane wings and boat oars usually shed stable quasi-two-dimensional equal-but-opposite axial vortex pairs; occasionally multiple dust devil and tornado vortices are observed, but rarely, if ever, do the atmospheric vortices form equal-but-opposite circulation pairs. Both weather-driven local phenomena evolve from volumes of buoyant atmosphere that begin to rotate rigid-body-like in their turbulent surroundings. Under the right conditions, ambient turbulence is believed to organize these buoyancy-driven, rotating plume regions into one or more Rankine-like axial vortices [5],[6]. Unlike the "isolated" elliptical cylinder that delineates a descending wake vortex pair, cylindrical volumes of buoyant atmosphere rotating about an axis, whether with a constant angular velocity or with a potential-vortex-like velocity distribution, are constrained by total available energy limitations. It is possible to convert some atmospheric thermal energy into mechanical energy due to local variations in temperature and density (natural convection). While ambient temperature differences within these organizing volumes aren't large, converting the thermal energy available from a 1 ^{o}C temperature drop to mechanical energy corresponds to a 100 m elevation increase, or a 1000 m^2/s^2 increase in kinetic energy. Substantially more mechanical energy can be released from the rising volumes of warm humid air associated with tornadoes, when phase change energy is released by condensing water vapor into liquid droplets or ice particles. Regardless of rotational velocity type, Carnot efficiency limitations restrict large-scale thermal to mechanical energy conversions. *Rotating cylindrical volumes of atmosphere must have finite, mechanical energy-based radial limits.*

The actual metamorphosis process by which a rotating turbulent volume of buoyant atmosphere transforms itself into a dust devil or tornado is not understood. While Ref. [5] and Ref. [6] simulated small-scale approximations of these processes experimentally, many more experimental studies were unsuccessful. Currently, there are no instability models to explain this sort of "inviscid jump".

Lord Kelvin [7] studied the inviscid stability of a rigidly rotating fluid column, identifying a stability criterion resulting from a finite centerline pressure requirement. The year before, Lord Rayleigh had also examined the inviscid instability of this rigidly-rotating fluid column [8], but it was much later that he evolved what is now called *Rayleigh's discriminate*. He was able to demonstrate that if the radial gradient of the square of circulation [$\left(2\pi\Omega r\right)^2$, for a rotating fluid cylinder] was greater than zero, the column was stable with respect to axisymmetric perturbations [9]. Rayleigh's discriminate is silent when the circulation is constant, i.e. for a potential vortex, and we have no logical way currently to explain a fluid mechanical "jump" from a rigid-body-like rotating fluid column to a potential vortex. The fundamental question is how a simple inviscid axial vortex model can describe so many aspects of large-scale rotating flows while imposing a ridiculous centerline velocity limit and requiring impossibly large quantities of mechanical energy.

In 1995, we attempted to summarize the current state of vortex stability theory knowledge [10]. While progress has been made, none appear to provide a mechanism by which a translating column of atmosphere, rotating about a vertical axis with some angular velocity, Ω, can quickly transform itself into a dust devil or tornado.

Meteorologist R.S. Scorer attempted to explain how hurricanes form over tropical-latitude-oceans when buoyant large-scale rotating low-pressure regions release latent heat as near-surface atmosphere cools during ascent [11]. Recognizing that Coriolis forces can facilitate the evolution of organized rotating columns, Scorer tried to address the process by which such rotating columns could transform themselves into large-scale, potential-vortex-like hurricanes and typhoons. He reasoned that since rotating fluid columns and potential vortices both require impossibly-large quantities of mechanical energy at large, but finite radial limits, they must have finite radial limits. When this physically-based mechanical limit is reached, he proposed that a rotating cylindrical column of atmosphere could "jump" to a potential vortex velocity distribution as long as total angular momentum and total kinetic energy were conserved. He avoided the non-physical infinite centerline vortex velocity enigma by supposing that this sudden jump from one frictionless velocity distribution to another only involved an outer annular region. His "mechanism" did not provide an explanation regarding simultaneous maintenance of an inner rotating cylindrical region while creating this outer potential vortex region.

Recently, we have been exploring non-equilibrium behavior of incompressible fluid structures; one aspect of which demonstrated how a *pressure relaxation coefficient*, based on measurable frequency-dependent aero-acoustic properties, could be employed to forecast the severity of aircraft wake vortex core encounters based on local weather data [12]. That 2017 Aviation Forum paper showed how an incompressible potential vortex creates unsustainable strain rates in the vicinity of the vortex axis, and at radial distances well outside of any compressibility limit. As those extreme strain rates are encountered, the vibrational and rotational energy distributions for the nitrogen, oxygen and water molecules in air must depart from their normal isotropic Boltzmann energy distributions [13]. This departure from continuum flow behavior is known to occur in normal shock waves[2]. If one assumes that the inviscid vortex flow field is the dominant continuum fluid structure—certainly the case, in terms of this inner non-equilibrium region of influence—the unsustainable strain rate zone is maintained by the outer potential vortex flow regime. Non-equilibrium theory implies that this cylindrical zone radiates sound while sustaining a "steady-state" non-equilibrium pressure zone around the rotational axis. The exact solution is a continuous Rankine vortex solution [14], and is identical with the Burnham and Hallock aircraft wake vortex correlation [15]. If Scorer's *rotating atmosphere "jumping to a typhoon" conjecture* is correct, there is an outer limit to the parent potential vortex, based on conservation of angular momentum and energy requirements. While a complete theoretical model for Scorer's conjecture is lacking, the equal-but-opposite vortex pair trailing behind aircraft in cruising flight can provide additional insight. That will be a focus herein.

III. Scorer's Conjecture for Hurricanes, Tornadoes and Dust Devils

One way to interpret Scorer's conjecture [11], is to assert that the rotating, moisture-laden, turbulent fluid column, responsible for tornado genesis, grows as it moves with the parent storm front. At some point that rotating structure (called a *wall cloud* by meteorologists) reaches a maximum mechanical energy limit. A meteorologically-defined maximum radius threshold (not yet isolated) will be achieved. At that limit, depending on the characteristic wall cloud turbulence [5], [6], a potential vortex velocity distribution with overall angular momentum and total mechanical energy matching the evolving wall cloud suddenly becomes the most stable sustainable mean velocity distribution[3]. While this is certainly not consistent with the fluid flow stability models known to this author, *Euler column* instabilities, observed when an axially-loaded vertical column suddenly buckles is a classical instability mechanism in solid mechanics, and is fundamental to column buckling theory [16].

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[2] From Mott-Smith's Boltzmann solution, the estimated thickness of a weak normal shockwave (Mach 1.05, T=300 K) is 100 mean-free-paths, i.e. $O[10\ \mu m]$. The associated velocity jump is approximately 11.5 m/s, thus representing a *normal* strain rate ($\Delta V / \Delta x$) of $O[2x10^6\ s^{-1}]$. A potential vortex, generated by a light aircraft creates similar *shearing* strain rates at a radius of approximately 1 mm—small, but approximately 15,000 mean-free-paths.

[3] Incompressible atmospheric processes occurring within these rotating *wall clouds* can convert substantial quantities of phase-change energy into turbulence and organized mechanical energy, as the rotating column evolves. The degree to which this conversion of water vapor into rain, snow or hail is converted to non-random kinetic and potential energy is limited by Carnot efficiency and depends on the local variations in wall cloud temperature and water concentration. A mechanical energy-based radial limit will be reached based on the actual wall cloud composition.

Scorer had no explanation, other than "physical impossibility", for why the sudden jump from a rotating column to a potential vortex required a "Rankine vortex" core. He simply avoided the non-physical azimuthal velocity region by assuming that the suddenly-evolving potential vortex zone was an annular volume with a finite inner radius. Employing our non-equilibrium theory, it is possible to gain additional insights related to Scorer's conjecture. In order to do this, it is first necessary to examine a generalization of Scorer's hypothesis: *A finite rotating cylinder of inviscid fluid can, under some conditions, suddenly organize itself as a potential vortex.* Employing a slightly modified version of Scorer's notation, we consider a rigidly-rotating fluid column with azimuthal velocity $V_\theta(r) = \Omega r$.

Assuming there is an annular region within that rotating column (with inner radius, $a = R_{Min} = R_i$), and an outer, kinetic-energy-limited radius, dictated by the particular weather system, $b = R_{Max} = R_o$. At any height (z) within the rotating annular column, it is necessary to conserve the angular momentum during this transformation. That is,

$$\rho(z)\int_{R_{Min}}^{R_{Max}} 2\pi r\left[rV_\theta(r)\right]dr = 2\pi\rho(z)\int_{R_{Min}}^{R_{Max}}\Omega r^3 dr = \frac{\pi\Omega}{2}\rho(z)\left[R_{Max}{}^4 - R_{Min}{}^4\right] = \text{Constant.} \tag{1}$$

Scorer did not specify the circulation for the resulting potential vortex. He simply assumed that $V_\theta(r) = \frac{k}{r} \Rightarrow k = \frac{\Gamma_o}{2\pi}$. It is logical to assume that the resulting potential vortex circulation cannot be greater than that produced by the angular velocity at the radial limit of the parent atmospheric unit as it evolves. That is,

$$k = \frac{\Gamma}{2\pi} \le \Omega R_{Max}^2\text{; therefore } \Gamma_o \le 2\pi\,\Omega R_{Max}^2 . \tag{2}$$

With that in mind, conservation of mechanical energy requires:

$$\int_{R_{Min}}^{R_{Max}} 2\pi r\frac{V_\theta^2}{2}dr = \pi\int_{R_{Min}}^{R_{Max}} r\Omega^2 r^2 dr = \pi\Omega^2\int_{R_{Min}}^{R_{Max}} r^3 dr = \pi\left(\frac{\Gamma_o}{2\pi}\right)^2\int_{R_{Min}}^{R_{Max}}\frac{dr}{r} = \pi\left(\frac{\Gamma_o}{2\pi}\right)^2\ln\left(\frac{R_{Max}}{R_{Min}}\right),$$ implying that:

$$\frac{\pi\Omega^2}{4}\left[R_{Max}{}^4 - R_{Min}{}^4\right] = \pi\Omega\left(R_{Max}^2 + R_{Min}^2\right)\frac{\Omega}{4}\left(R_{Max}^2 - R_{Min}^2\right) = \Gamma_o\frac{\Omega}{4}\left(R_{Max}{}^2 + R_{Min}{}^2\right) =$$

$$= \frac{\Gamma_o^2}{4\pi}\ln\frac{R_{Max}}{R_{Min}} \Rightarrow \Omega\left(R_{Max}{}^2 + R_{Min}{}^2\right) = \frac{\Gamma_o}{\pi}\ln\frac{R_{Max}}{R_{Min}} \Rightarrow$$

$$\Gamma_o = \frac{\pi\Omega R_o^2\left[1+\left(\frac{R_i}{R_o}\right)^2\right]}{\ln\frac{R_o}{R_i}} = \Gamma_{o,Max}\frac{\left[1+\left(\frac{R_i}{R_o}\right)^2\right]}{2\ln\frac{R_o}{R_i}} \Rightarrow \frac{\Gamma_o}{\Gamma_{o,Max}} = \frac{\left[1+\left(\frac{R_i}{R_o}\right)^2\right]}{2\ln\frac{R_o}{R_i}} = \frac{\left[1+\left(\frac{R_i}{R_o}\right)^2\right]}{\ln\left(\frac{R_o}{R_i}\right)^2} . \tag{3}$$

Now, if we relate the vortex circulation to its rotating parent, we have:

$\Gamma_o = 2\pi\,\Omega R_{Max}^2\eta$, where η is the conversion efficiency, i.e. for $\eta \le 1$.

$$\eta = \left[1+\left(\frac{R_{Min}}{R_{Max}}\right)^2\right]\Big/\ln\left(\frac{R_{Max}}{R_{Min}}\right)^2 . \tag{4}$$

This relation indicates the degree to which the spontaneous vortex circulation, resulting from Scorer's jump instability, is linked to the outer circulation limit of the rotating parent. Employing more compact notation,

$$0<\left(R_{Max}-R_{Min}\right)/R_{Max}<1 \Leftrightarrow 0<\left(R_o-R_i\right)/R_o<1,$$

to measure the annular span, the "rotational conversion efficiency" is related to the span of the annular zone, as shown in Figure 1. Obviously, there is insufficient rotational energy available to sustain a potential vortex in the limit, $r \rightarrow 0$. At the same time, it is important to note that these adjusted vortex circulation levels, based on annular span, can create a non-physical velocity jump discontinuity between the rotating inner cylinder region and the inner boundary of the annular vortex.

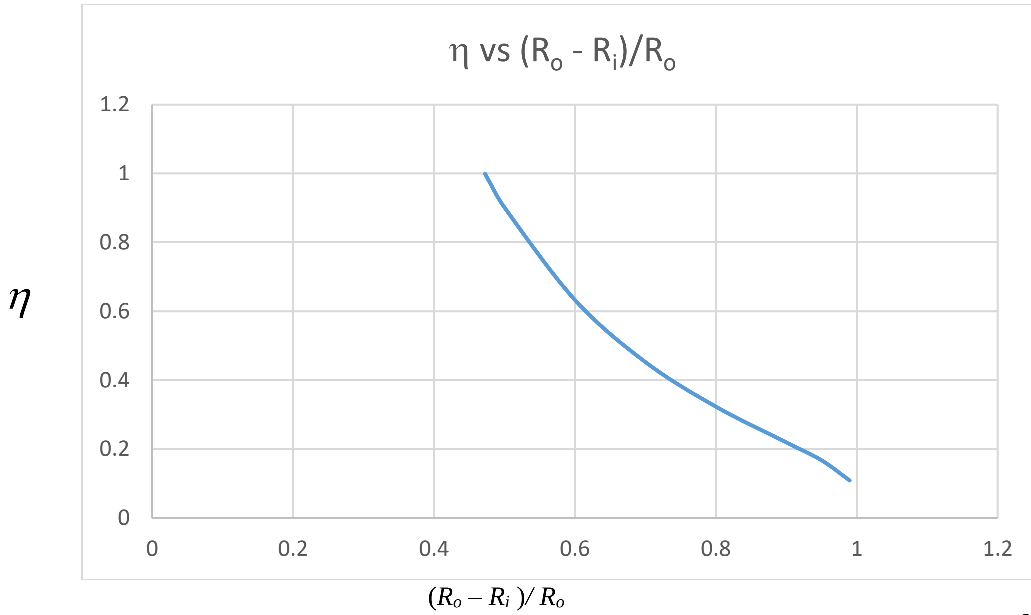

Figure 1. Influence of annular vortex extent on circulation efficiency ($\Gamma_{Max} = 2\pi R_o^2 \Omega$).

Scorer [11] seems to ignore the inner rotating cylindrical column. However, since it wasn't altered, we only note here that this "Rankine vortex" can now have a velocity slip interface boundary in addition to the strain rate discontinuity. In air, we have shown that the limiting inner radius for a potential vortex is imposed physically by excessive strain-rates, well-before compressibility effects are encountered [14]. With Scorer's interface, there is, in addition to departures from equilibrium, a strong mechanism for generating anisotropic turbulence[4].

Scorer's Rankine vortices can't be sustained over land, i.e. for tornadoes and dust devils. When a "Scorer jump" occurs, the potential vortex circulation level will be established, defining an eye region, but without a warm ocean reservoir, the resulting Rankine vortex flow field lacks an adequate energy supply to sustain an eye. Even though tornadoes can continue to convert latent heat energy to mechanical energy from rising moist atmosphere (dust devils can't), their smaller size inhibits that mechanism[5]. Scorer's initial Rankine-like potential vortex eye must somehow evolve rapidly toward its dominant overall potential vortex structure—until impossible strain rates bound the inner radial limit. Clearly, these evolutionary meteorological processes require additional mechanical energy, but from where?

IV. Scorer's Conjecture applied to aircraft wake vortices

Unlike Scorer's rotating atmosphere metamorphoses, cruising aircraft produce an equal-but-opposite pair of axial trailing line vortices with predictable circulation levels. Utilizing a simple wing load model, the shed wake vortex pair have circulations, $\pm\Gamma_o$, that can be estimated based on aircraft weight, W, atmospheric density, ρ, wingspan, S, and cruising flight speed, U_∞, as:

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[4] That turbulence appears to be approximated by a simple eddy viscosity turbulence model.

[5] EF-4 or 5 tornadoes may sustain eyes, when their observed path widths (core diameters) are substantially greater than 100 m. However, multiple tornado funnels within these large "eye regions" is an equally plausible explanation.

$$\left|\pm\Gamma_o\right| = \frac{4W}{\rho\pi S U_\infty} \quad . \tag{5}$$

In addition, the nominal spacing between the vortex axes, $2s_o$, can be related to the wingspan, S using:

$$2s_o = \frac{\pi S}{4} \, . \tag{6}$$

Assuming fixed lateral spacing [17], potential flow theory predicts that the vortex pair descends at a constant velocity [3]. Utilizing Eqs. (5) and (6), with $\mathbf{V}(x, y) = u(x, y)\mathbf{i} + v(x, y)\mathbf{j}$, the induced velocity of the vortex axes is:

$$V_{axis} = v\left(-\pi\frac{S}{8}, 0\right) = v\left(+\pi\frac{S}{8}, 0\right) == -\frac{\Gamma_o}{2\pi\left(2\pi\frac{S}{8}\right)} = -0.2026\frac{\Gamma_o}{S} \, . \tag{7}$$

Based on Kelvin's calculations, the descending elliptical cross section shown in Figure 2, is established. His streamwise semi-major axis was $1.73s_o$, and the corresponding spanwise semi-major axis was $2.09s_o$, i.e., for a downward-moving wake-vortex pair, an x, y coordinate system moving with the descending ellipse, is given by:

$$\left(\frac{x}{2.09a}\right)^2 + \left(\frac{y}{1.73a}\right)^2 = 1 = \left(\frac{x}{0.821S}\right)^2 + \left(\frac{y}{0.679S}\right)^2 \, . \tag{8}$$

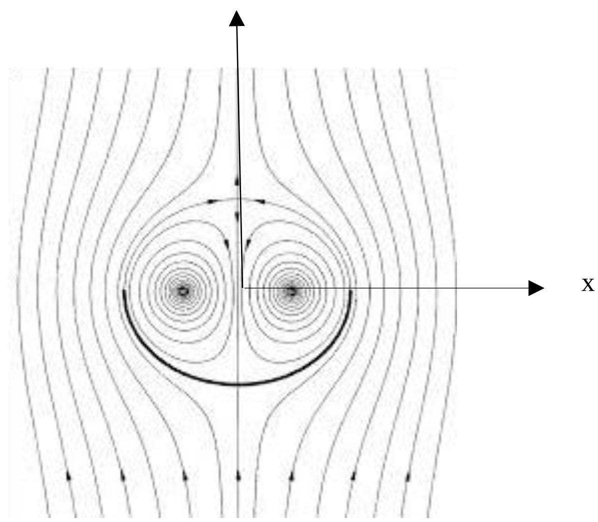

Figure 2. *Inviscid vortex pair streamlines in a uniform vertical flow*

The purely potential flow streamlines are shown in Figure 2 (by superimposing uniform vertical potential flow, $-V_{axis}$).

If these aircraft-generated potential-flow vortices control the primary, large-scale flow physics, and Scorer's finite potential flow domain conjecture applies to wake vortices, then the finite elliptic volume whose cross section, defined by (Eq.8), can be isolated from its surroundings. While this may seem like a trivial assertion, the idea that inviscid *potential flow structures of finite extent* control the large-scale physics of actual flows is significant.

Transient finite element methods (so-called acceleration methods [18]) for achieving steady-state solutions to incompressible Euler equations is an ongoing area of research [19]. Those methods rely on a variety of techniques to compensate for severe numerical accuracy problems resulting from the very large differences between acoustic propagation speeds and characteristic incompressible flow speeds—usually in mathematically unbounded flow domains. More recently, entropy-consistent approaches have emerged [20]. While computational fluid dynamics (CFD) plays a major role in contemporary research, these approaches appear to ignore the physics as CFD achieves very-fine-scale numerical resolution. More than 50 years ago, Mott-Smith [13] developed Boltzmann equation solutions for normal shock waves in air. His calculations showed that normal shockwaves become thicker as the weak shock limit ($M \rightarrow 1$) is approached (spanning more than 10,000 mean-free-paths near weak shock limits). In 2009, we showed how incompressible non-equilibrium pressure could occur in an incompressible flow as the result of excessive strain rates, predicting normal shockwave thicknesses based on our pressure relaxation coefficient estimates that were consistent with Mott-Smith's calculations, up to M = 5 [21].

Unlike the CFD approaches, we recognized that when subjected to excessive strain rates, a fluid like air, made up primarily of diatomic molecules (and triatomic H_2O), departs from simple continuum behavior, *while remaining incompressible*. We developed that model employing Hamilton's Principle of Least Action with provision for molecular-level distribution function departures from vibrational and rotational equilibrium. A quasi-isentropic constraint was employed concurrently. Our approach defined a *pressure relaxation coefficient*, (η_p, seconds) based entirely on cataloged aero-acoustic properties.

Scorer's conjecture implies that the elliptic-volume-vortex-pair, produced by an airplane flying in cruise can be bounded by a finite pair of inviscid vortices whose flow domain is limited to approximately one wingspan. That outer elliptic shell, for all intents and purposes, behaves like Kelvin's vortex pair. When allowance is made for acoustic radiation from the inner non-equilibrium cylindrical zones surrounding the potential vortex axes, the azimuthal velocity profile of a single, undisturbed vortex is given by [14]:

$$V_\theta(r) = \frac{\Gamma_o}{2\pi}\frac{r}{r_{core}^2 + r^2} = 2V_{\theta,Max}\frac{(r/r_{core})}{(r/r_{core})^2 + 1} \tag{9}$$

where

$$r_{core} = \frac{\Gamma_o}{4\pi}\sqrt{\rho\frac{\eta_p}{\mu}}\text{, for laminar flow; } = \frac{\Gamma_o}{4\pi}\sqrt{\frac{\eta_p}{\varepsilon}}\text{, employing eddy viscosity, } \varepsilon\text{, for turbulent flow,} \tag{10}$$

and

$$V_{\theta,Max} = \frac{\Gamma_o}{4\pi r_{core}} = \sqrt{2\frac{\varepsilon}{\eta_p}}\text{, for the turbulent case.} \tag{11}$$

Note also that:

$$V_{\theta,Max}r_{core} = \frac{\Gamma_o}{4\pi}\text{, which is independent of the pressure relaxation coefficient and eddy viscosity.} \tag{12}$$

For a purely rotational velocity profile, the shearing strain rate is given by:

$$\dot{\varepsilon}_{r\theta} = \frac{r}{2}\frac{d}{dr}\left(\frac{V_\theta}{r}\right) = -\frac{\Gamma_o}{2\pi r^2} \equiv \dot{e}_{r,\theta}(r)\text{, for a potential vortex,} \tag{13}$$

and

$$\dot{\varepsilon}_{r\theta} = \dot{e}_{r,\theta}(r)\cdot\left[\frac{(r/r_{core})^2}{1+(r/r_{core})^2}\right]^2 \text{ for the non-equilibrium velocity profile (Eq. 9).} \tag{14}$$

Scorer asserted that the finite potential flow domain was the controlling large-scale flow. If correct, that would imply that the inviscid flow depicted in Figure 2 confines our non-equilibrium core effects to the elliptical zone. An interesting test of that conjecture could be addressed by answering the following:

If the potential flow ellipse is the controlling large-scale fluid motion in a wake vortex, the initial induced downward velocity will be given by V_{axis} (Eq. 7), rather than:

$$V'_{Axis} = \frac{\Gamma_o}{2\pi}\frac{2(0.821S)}{r_{core}^2 + [2(0.821S)]^2}\text{, \textit{the velocity induced by the distorted potential vortex.}}$$

Although little data is available, a NASA wake vortex encounter flight campaign conducted in the mid-1990s can be employed to assess the feasibility of such a "Scorer assertion test."

NASA's 1995 wake vortex encounter flights

In late fall, 1995, NASA flew a series of wake encounter experiments in the vicinity of NASA Wallops Flight Facility [22]. A Lockheed-Martin C-130, with wing-tip smoke generators, was flown at a constant speed and altitude

(140 kts., 7,000 ft.), to enable an instrumented North-American Rockwell OV-10 airplane [23] to fly across and through the generated wake vortices. The data obtained during one flight, designated *Flight 705 (Run 37)*, was thoroughly documented and will be examined in the context of Scorer's finite vortex conjecture. NASA estimated the C-130 wake circulation to be 200 m^2/sec. At the time of that flight, the ambient (at altitude) pressure and temperature were, 0.772 atmospheres, and 2° C (275.25 K), respectively. The estimated relative humidity at that altitude was 10 %, permitting a direct estimation of the pressure relaxation coefficient [21] (η_p = 0.691 μsec.), while the kinematic viscosity was 1.715x10^{-5} m^2/s. In addition, the C-130 wingspan was 40.4 m, and its in-flight weight varied from 113,000 to 95,000 lb_f. Ref [22] recorded a maximum azimuthal velocity ($V_{\theta,Max}$) of 150 ft/s, with an associated core radius of 0.4 ft.; a second maximum combination was $V_{\theta,Max}$ = 125 ft/s with r_{core} =1.8 ft. From Ref. [14],

$$\Gamma\left(r_{core}\right) = \Gamma_o / 2 = 2\pi r_{core} V_{\theta,Max} \Rightarrow 754 \text{ ft}^2/\text{s} \ (70 \text{ m}^2/\text{s}) \le \Gamma_o \le 2827 \text{ ft}^2/\text{s} \ (262 \text{ m}^2/\text{s}).$$

From Eq.(5), the estimated initial circulation should have been: $183 \le \Gamma_o \le 218$ m^2/s. For our purposes, we will assume that the initial circulation is 200 m^2/s. If we use $V_{\theta,Max}$ = 125 ft/s (38 m/s) with Γ_o = 200 m^2/s, then the "corrected" core radius is 0.4 m (1.4 ft., vs 1.8 ft., measured). The potential vortex-induced initial velocity would be:

$$V_{axis} = v\left(-\pi\frac{S}{8}, 0\right) = v\left(+\pi\frac{S}{8}, 0\right) = -\frac{\Gamma_o}{2\pi\left(2\pi\frac{S}{8}\right)} = -0.2026\frac{\Gamma_o}{S} = \text{-1.0 m/s}.$$

The corresponding non-equilibrium descent velocity would be:

$$V'_{Axis} = -\frac{\Gamma_o}{2\pi}\frac{2\left(0.821S\right)}{r_{core}^2 + \left[2\left(0.821S\right)\right]^2} = -\frac{\Gamma_o}{S}\frac{0.821S^2/\pi}{r_{core}^2 + \left[2\left(0.821S\right)\right]^2} = -0.0969\frac{\Gamma_o}{S} = \text{- 0.48 m/s}.$$

While difficult to measure, employing high-resolution ground-based lidar, flying at a low altitude (just out of ground effect), it should be possible to measure vortex core radii, initial vortex spacing and induced velocities. If the vortex core radii correlate with the humidity-based pressure relaxation estimates, and the descent velocity agrees with Scorer's assertion, these tests would go a long way toward validating two surprising results.

V. Conclusions

In 1966, R.S. Scorer (1919-2011) attempted to describe the process by which weather events evolved suddenly from rotating-cylinder-like turbulent formations into cyclones or hurricanes. He discussed his "theory" at an international conference in Ann Arbor, MI, but the organizers decided to forgo published conference papers in favor of rather brief summaries of the speakers' contributions in the *Journal of Fluid Mechanics*. Subsequently, Scorer published "Origin of Cyclones", in *Science Journal* [11], but that journal is extremely difficult to retrieve. In "Origin of Cyclones", he asserted that the potential vortices that appear to be ubiquitous in nature are the dominant over-riding flow structures which when formed become difficult to extinguish. In addition, he reasoned that nature did not conform with the mathematically-elegant assumption that a potential vortex should occupy all of space, i.e. $V_\theta\left(r\right) = \frac{\Gamma}{2\pi r}$, $0 \le r < \infty$. On that basis, he asserted that the controlling potential flows observed in nature, though quite large, are finite. Furthermore, he postulated that the domains describing natural potential vortices were annular volumes, thereby avoiding both non-physical mathematical inner and outer limits.

This work has explored Scorer's conjecture. His annular vortex formation concept can explain why hurricanes have well-defined eyes while tornadoes and dust devils don't. However, it is difficult to relate his hurricane birth assertion to quantitative measurements because he failed to define his imposed circulation. This work shows how the span of the potential vortex annulus can be related to the diameter and angular rotation rate of the parent atmospheric disturbance, but can go no farther.

On the other hand, an aircraft in cruising flight generates a well-defined vortex pair, and the circulation level can be estimated reliably. The descending equal-but-opposite potential vortex pair produces a bound elliptic cross section that can be isolated from the surrounding inviscid fluid, avoiding Scorer's circulation ambiguity. Furthermore, the trailing vortex pair can be probed to determine whether Scorer's inviscid flow unit is the controlling flow (with non-equilibrium pressure cores isolated by their inviscid parents), and/or whether descent rate and vortex core size and strength are controlled by pressure relaxation. Since Scorer's assertion enables a thermodynamically plausible explanation for why tornadoes and dust devils lack well-defined eyes, it would be prudent to either validate or disprove our non-equilibrium pressure theory concurrently since it predicts that the pressure deficit on the rotational axis of a tornado is twice as large as the simple, incompressible Bernoulli-based maximum pressure deficit.

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