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Keywords: lateral-torional buckling, I-section FRP, ASCE-LFRD pre-standard for FRP structures.

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Lateral-Torsional Buckling of FRPI-Section Beams

Mojtaba B. Sirjani [α] & Zia Razzaq [σ]

Abstract- This paper presents the outcome of an experimental and theoretical investigation into the load-carrying capacity of Fiber Reinforced Polymer (FRP) I-section beams subjected to four-point loading. The overall lateral-torsional buckling, web and flange local buckling as well as material rupture load estimates are also made using the American Society of Civil Engineers' Load and Resistance Factor Design (ASCE-LRFD) Pre-Standard for FRP Structures. Lateral-torsional buckling failure mode is found to govern for each of the beams studied. The study also revealed that the height of applied loads relative to the shear center has a very significant influence on lateral-torsional buckling load of a beam thus making ASCE-LRFD buckling load estimates over-conservative in a vareity of cases.

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I. Introduction

A Fiber-Reinforced Polymer (FRP) beam subjected to inplane bending moments about its cross-sectional strong axis can develop lateral-torsional buckling. Such a beam will initially deflect normal to the strong axis until the critical value of the bending moment is reached after which lateral and torsional deflections develop. Mamadou and Razzaq [1] investigated the failure modes for I-section Glass Fiber Reinforced Polymer (GFRP) beams with single mid-span web brace in which theoretical predictions were made based on ASCE-LFRD Pre-Standard for Pultruded Fiber Reinforced Polymer (FRP) Structures [2]. It was found that for small and medium I-sections, lateral-torsional buckling failure mode governed while the larger I-sections reached their peak capacity associated with material rupture. Sirjani, Bondi and Razzaq [3] presented the outcome of an experimental and theoretical study on FRP beams with an I-shaped cross section subjected to four-point loading with and without applied torsion. The focus of that study was to identify the significance of lateral bending and warpingstrains due to practical imperfections.

The present paper addresses the influence of vertical location of applied loads with respect to the shear center when estimating the beam lateral-torional buckling strength. Three different applied load locations are considered, namely, when the loads act above, below and at the shear center. In addition, load-carrying capacity predictions are made for various failure modes using the ASCE-LRFD Pre-Standard, and the buckling load estimates compared to those observed experimentally as well as obtained using the buckling formula presented by Razzaq, Prabhakaran, and Sirjani [4].

II. Experimental Study

Figure 1 shows a FRP beam of length L with an I-shaped cross section, and subjected to a pair of gradually increasing applied loads each of magnitude P. Figure 2 shows the experimental test setup. The beam ends were simply supported both flexurally and torsionally. The test procedure,

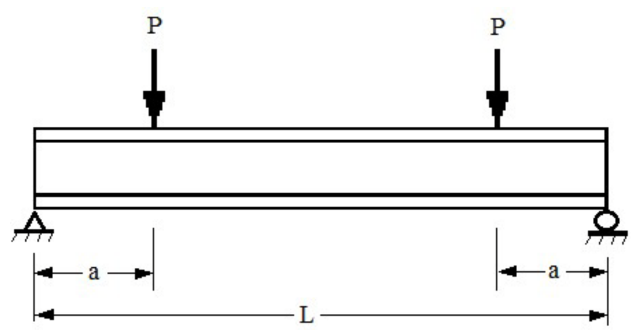

Fig. 1: Schematic of I-Section FRP beam

involved applying the load pair(P, P) in small increments and recording the resulting load-deflection relationship until the peak lateral-torsional buckling load was reached.

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Fig. 2: Exprimental test setup

The experimental and theoretical maximum loads P_e and P_t, respectively, are presented in Table 1 in addition to their ratios for a 4x2x0.25 in. I- shaped FRP cross section withlength L equal to 60, 72, 84, 96 and 108 inches, respectively. The value of (L – 2a), that is, the distance between the two applied loads P and P shown in Figure 1 was kept constant at 24 inches. The Young's (E_{11}) and shear (G_{12}) modulus values of the FRP beam material were 2,550 ksi and 420 ksi, respectively.

Figure 3, shows the applied loading mechanism in which a pair of steel tie rods are used to apply upward vertical load (P/2 per tie rod) placed symmetrically about the shear center, S. the resultant load P is transmitted to a steel bar which pushes a steel shaft against an aluminum loading plate mounted on to the FRP beam. The resultant force P acts at a distance y_o^*below the x-axis but passes through S. The value of y_o^* defines the vertical location of the applied loads. It should be noted that the downward load pair (P, P) shown in Figure 1 was applied in the upward direction by means of two separate sets of the loading mechanism schematically depicted in Figure 3.

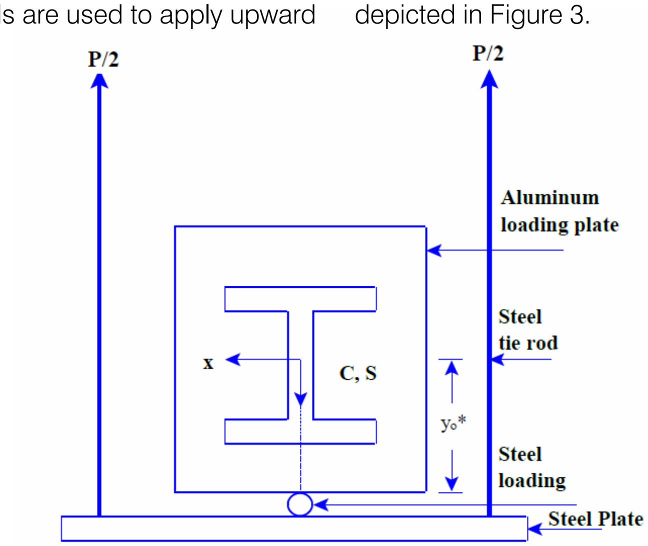

Figure 3: Loading Mechnism

Table 1: Experimental and Theortical Maximum loads

L (in.)	P_e(Lb.) (Experimental)	P_t(Lb.) (Theoretical)	P_t / P_e
60	292	340	1.164
72	190	214	1.126
84	125	150	1.200
96	111	112	1.009
108	77	88	1.143

III. Theoretical Study and Results

For the beam shown in Figure 1, the lateral-torsional buckling load P_{cr} can be found using the following formula presented by Razzaq, Prabhakaran, and Sirjani[4]:

$$P_{cr} = \frac{0.5\left[-f_2 + \sqrt{f_2^2 + 4 f_1 f_3}\right]}{f_1} \quad (1)$$

in which:

$$f_1 = \frac{1}{16}\left[f(a) - \frac{\pi^2 a^2}{L^2} - \frac{2\pi a}{l} g(a)\right]^2 \quad (2)$$

$$f_2 = \frac{\pi^4 E_{11} I_y}{4L^3} y_0^* \sin^2\left(\frac{\pi a}{L}\right) \quad (3)$$

$$f_3 = \frac{\pi^6 E_{11} I_y}{16L^4}\left[\frac{\pi^2 E_{11} I_w}{L^2} + G_{12} K_T\right] \quad (4)$$

$$f(a) = \frac{\pi a}{L}\sin\left(\frac{2\pi a}{L}\right) - \sin^2\left(\frac{\pi a}{L}\right) \quad (5)$$

$$g(a) = \frac{1}{2}\left[\pi\left(1 - \frac{2a}{L}\right) - \sin\pi\left(1 - \frac{2a}{L}\right)\right] \quad (6)$$

In the above expressions, the distances a and L are defined in Figure 1; I_y is the minor-axis moment of inertia; K_T is the St. Venant torsional constant; and I_w is the warping moment of inertia of the cross section.

Table 2 presents the ASCE-LRFD theortical maximum load values with aresistancefactor of ø =0.80for flange or web local buckling, and ø =0.65 for rupture load. Also, this table presents the moment modification factorC_b for unsupported spans with both ends braced corresponding to various L values of the beam shown in Figure 1 with (L – 2a) kept constant at 24 inches.

Table 2: ASCE-LRFD Therotical Maximum loads

Based on ASCE-LRFD					
L (in.)	C_b	P_{LT} (Lb.)	$øP_{fcr}$ (Lb.)	$øP_{wcr}$ (lb.)	$øP_{rupture}$ (lb.)
60	1.087	468	43749	13626	2057
72	1.136	288	43749	13626	1543
84	1.168	195	43749	13626	1244
96	1.190	141	43749	13626	1028
108	1.207	107	43749	13626	881

Table 3 pesents the critical load results for different distance y_o* of applied load about the shear center. The last three columns in Table 3 present the load ratios r_1, r_2, and r_3 defined as P_{LT} divided by P_{cr} correspoinding to y_o* = -2.00 in., 0.0 in., and +2.0 in., respectively.

Table 3: Critical Load for various applied load through shear center

L (in.)	P_{cr}(Lb.)with y_o* equal to			P_{LT} (Lb.)	r_1	r_2	r_3
	-2.0(in.)	0.0(in.)	+2.0(in.)				
60	451	689	1053	468	1.038	0.679	0.444
72	278	408	600	288	1.036	0.706	0.480
84	190	268	378	195	1.026	0.728	0.516
96	139	189	257	141	1.014	0.746	0.549
108	107	140	185	107	1.000	0.764	0.578

IV. Conclusions

Expeimental results are in good agreement with the lateral-torsional buckling load formula presented [4]. Theoretical predications for various beam failure modes are also made using ASCE-LRFD Pre-Standard for FRP Structures. It is found that in all of the cases presented, the I-section beam failure mode was governed by lateral-torsional buckling. The study also clearly reveals that the height of the applied loads relative to the shear center has a very significant influence on the lateral-torsional buckling load of the beam thus making ASCE-LRFD buckling load estimates over-conservative in a number of cases.

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Appendix

This appendix summarizes the ASCE-LRFD Pre-Standard expressions used in arriving at those particular numerical results which were based on the ASCE-LRFD Pre-Standard [2]. The critical stress for the compression flange local buckling is given by:

$$f_{fcr} = \frac{4}{(\frac{b_f}{t_f})^2}\left(\frac{7}{12}\sqrt{\frac{E_{L,f}E_{T,f}}{1+4.1\xi}} + G_{LT}\right) \quad (1)$$

in which:

G_{LT} = characteristic in-plane shear modulus, ksi

v_{LT} = characteristic longitudinal Poison's ratio

b_f = Full width of the flange, in.

h = Full height of the member, in.

t_f = Thickness of the flange, in.

k_r = Rotational spring constant, kip/rad

$E_{L,f}$ = Characteristic longitudinal modulus of the flange, ksi

$E_{L,w}$ = Characteristic longitudinal modulus of the web, ksi

$E_{T,f}$ = Characteristic transverse modulus of the flange, ksi

$E_{T,w}$ = Characteristic transverse modulus of the web, ksi

$$f_{wcr} = \frac{11.1\pi^2}{12(\frac{h}{t_w})^2}\left(1.25\sqrt{E_{L,w}E_{T,w}} + E_{T,w}v_{LT} + 2G_{LT}\right)$$

in which, f_{wcr} is the critical stress for the web local buckling.

There are four nominal moments that are calculated based on the formulae [2] as summarized here. The nominal bending moment M_{LB} due to lateral-torsional buckling is given by:

$$M_{LB} = C_b\sqrt{\frac{\pi^2 E_{L,f} I_y D_J}{L_b^2} + \frac{\pi^4 E_{L,f}^2 I_y C_w}{L_b^4}} \quad (3)$$

A resistance factor $\phi = 0.7$ is used for M_{LB}. The other terms are defined as follows:

C_b = Moment modification factor for unsupported spans with both ends braced

D_J = Torsional rigidity of an open section $= G_{LT}\sum\frac{1}{3}b_i t_i^3$, $kip - in.^2$

C_ω = Warping constant $= \frac{t_f h^2 b_f^3}{24}$, $in.^6$

$$M_{fLT} = f_{fcr}\frac{E_{L,f}I_f + E_{L,w}I_w}{yE_{L,f}} \quad (4\text{-a})$$

$$M_{wLT} = f_{wcr}\frac{E_{L,f}I_f + E_{L,w}I_w}{yE_{L,w}} \quad (4\text{-b})$$

In which, M_{fLT} and M_{wLT} are the nominal flexural strengths due to local instability in the flanges and webs, respectively; the resistance factor $\phi = 0.80$ is used. The other terms are defined as follows:

I_f = Moment of Inertia of the flange(s) about the axis of bending, in^4

I_w = Moment of Inertia of the web(s) about the axis of bending, in^4

y = Distance from the neutral axis to the extreme fiber of the member, in.

$$M_{cr} = \min\left(\frac{F_{L,f}(E_{L,f}I_f + E_{L,w}I_w)}{y_f E_{L,f}}, \frac{F_{L,w}(E_{L,f}I_f + E_{L,w}I_w)}{y_w E_{L,w}}\right) \quad (5)$$

In which, M_{cr} is the nominal flexural strength due to material rupture and the resistance factor ϕ = 0.65 is used. The other terms are defined as follows:

$F_{L,f}$ = characteristic longitudinal strength of the flange (in tension or compression),ksi

$F_{L,w}$ = characteristic longitudinal strength of the web (in tension or compression),ksi

I_f = Moment of Inertia of the flange(s) about the axis of bending, in^4

I_w = Moment of Inertia of the web(s) about the axis of bending, in^4

y_f = Distance from the neutral axis to the extreme fiber of the flange, in.

y_w = Distance from the neutral axis to the extreme fiber of the web, in.

t_w = Thickness of the web, in.

ξ = Coefficient of restraint

Lastly, applying the formula of maximum moment for a simply supported beam with a point load as shown in Figure 1, the respective loads are obtained

$$P_{LT} = \frac{M_{LT}}{a} \quad (6)$$

$$P_{fLT} = \frac{Mf_{LT}}{a} \quad (7)$$

$$P_{wLT} = \frac{Mw_{LT}}{a} \quad (8)$$

$$P_{cr} = \frac{M_{cr}}{a} \quad (9)$$

If $P_{LB} = P_{fLT} = P_{wLT} = P_{cr} = P_c$ is the load-carrying capacity of the member, a LFRD approach is proposed as follows:

$$P_c = \phi P_n \quad (10)$$

in which ϕ = 0.7, 0.8, and 0.65 depending whether the failure is due to lateral torsional buckling, local instability in the flanges and webs, and rupture of the materials.

The C_b values in Table 2 were computed using the following expression:

$$C_b = 12.5M_{max}/(2.5M_{max} + 3M_A + 4M_B + 3M_C) \quad (11)$$

in which Mmax is the maximum bending moment, and M_A, M_B, and M_C are the values of quarter-point moments along the beam length.

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