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Generalized Predictive Control of Ship Coupling Motions Using Active Flume Tanks

Abdulkarim Mohammed Alotaiwi

Old Dominion University

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GENERALIZED PREDICTIVE CONTROL OF SHIP COUPLING MOTIONS USING ACTIVE FLUME TANKS

by

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A Dissertation Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirement for the Degree Of

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OLD DOMINION UNIVERSITY
DECEMBER 2003

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ABSTRACT

GENERALIZED PREDICTIVE CONTROL OF SHIP COUPLING MOTIONS USING ACTIVE FLUME TANKS

Abdulkarim M. Alotaiwi
Old Dominion University, 2003
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This dissertation uses the Generalized Predictive Control (GPC) approach to design a control system for a ship rolling motion coupled with the sway and yaw using an activated flume tank. GPC is a strategy based on system output prediction over finite horizon known as the prediction horizon. GPC controller is designed from the coefficients of the Autoregressive model with exogenous input (ARX) that are computed directly from input and output data. It computes the future control input based on the cost function with weighted input and output. System identification approach is implemented on the system to find the ARX coefficients parameters.

A mathematical model of the anti-rolling flume tank and the ship coupling model are derived to be in the state space form. The time domain model of the ship motions has been extended to predict the coupling motions of sway, yaw and roll. Also, the disturbance model is generated as irregular waves. Analyses for the ship rolling and coupling models, with and without the anti-rolling flume tank, are presented.

A numerical simulation using the MATLAB program is implemented. The numerical simulation indicates that there are three factors that affect the ship motions: sea state conditions, wave attack angle and ship control system. The simulation result shows that the passive control system using an anti-rolling flume tank is able to reduce the ship rolling angle up to fifty percent.
In comparison, simulation result of the actively controlled system using GPC shows that the ship rolling angle can be mitigated up to eighty percents. The GPC approach is tested on the ship model in different weather conditions. The numerical simulation is implemented to evaluate the controller performance and investigate the benefit of the GPC in the ship coupling motions. The numerical results show that the coupling model of roll, sway and yaw can affect each other simultaneously. The roll motion can be affected by the sway force more than yaw moment. The effect and performance of the GPC in controlling the ship roll motion in different wave's disturbances and sea state conditions are discussed.
In the memory of my parents
ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Jen-Kuang Huang, for his thoughtful, guidance, care, and motivation. The other members of my Doctoral committee, Dr. Sebastian Bawab, Dr. Keith Williamson and Dr. Cheng Lin are especially appreciated for all the help they gave me.

This dissertation would not have been possible without the encouragement of my brothers and sisters helping me to overcome many obstacles. I would like to thank them for their patience and constant support in the good and bad times during the last years of my graduate study.

Finally, I would like to thank the Cultural Mission of the Saudi Arabian embassy in Washington D.C for their financial support and academic advising.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACKNOWLEDGEMENT</td>
<td>vi</td>
</tr>
<tr>
<td></td>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>NOMENCLATURE</td>
<td>xiii</td>
</tr>
<tr>
<td>I.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.1 Background and Problem Statement</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 Dissertation objective</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1.3 Dissertation outline</td>
<td>4</td>
</tr>
<tr>
<td>II.</td>
<td>LITERATURE REVIEW</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.1 Introduction</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.2 Roll stabilization devices.</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.3 Active anti-rolling flume tank</td>
<td>12</td>
</tr>
<tr>
<td>III.</td>
<td>MATHEMATICAL MODELING</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>3.1 Introductions</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>3.2 Dynamic model of the anti-rolling flume tank</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>3.3 Ship roll model and stimulator</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>3.4 Ship coupling model</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>3.4.1 Equation of motion of the coupled model</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>3.4.2 Ship coupling combined model</td>
<td>29</td>
</tr>
</tbody>
</table>
III. WAVE DISTURBANCE MODEL

3.5 Wave disturbance model ................................................. 30
   3.5.1 Generation of the sea wave .................................. 31
   3.5.2 Excitation force and momentums of the wave .......... 34
   3.5.3 Sea state conditions ........................................... 35

IV. GENERALIZED PREDICTIVE CONTROL (GPC) 36

4.1 Introduction .......................................................... 36
4.2 Fundamental of GPC .................................................... 37
4.3 System identification .................................................. 41
4.4 Multi step Prediction .................................................. 42
4.5 Derivation of the control law ........................................ 46

V. SIMULATION OF THE NUMERICAL MODEL 48

5.1 Introduction .......................................................... 48
5.2 The ship numerical model .......................................... 49
5.3 Ship rolling model in different sea state ....................... 51
5.4 Numerical simulation of the coupling model ................. 52
   5.4.1 Ship coupling model under the excitation of sway, roll
        and yaw ................................................................ 53
   5.4.2 Effective of the wave disturbance ......................... 58

VI. SHIP ROLL MITIGATION USING GPC 61

6.1 Introduction .......................................................... 61
6.2 Passive control system .............................................. 61
   6.2.1 Numerical Model of the passive system ............... 63
6.2.2 Simulation of the passive control system ...................... 65
6.3 Active control system using GPC........................................ 69
  6.3.1 Observer Markov Parameters......................................... 69
  6.3.2 Tuning of the weighting factors...................................... 71
  6.3.3 Numerical simulation of the coupling model using GPC 75
6.4 Performance of the GPC in the roll motion.............................. 78

VII DISCUSSION AND CONCLUSION........................................ 81
  7.1 Result and discussion.................................................... 81
  7.2 Contributions.............................................................. 82
  7.3 Further extension of the research....................................... 82

APPENDIX............................................................................ 84
REFERENCE........................................................................... 89
CURRICULUM VITA.............................................................. 93
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 3.1</td>
<td>The description of the sea state code (Price and Bishop, 1974)</td>
<td>84</td>
</tr>
<tr>
<td>Table 5.1</td>
<td>The dimensional model of the 1-container ship</td>
<td>85</td>
</tr>
<tr>
<td>Table 5.2</td>
<td>The normalization variable used for the prime system and Bis-system</td>
<td>86</td>
</tr>
<tr>
<td>Table 5.3</td>
<td>Model parameters of the a L- container ship (hull only)</td>
<td>87</td>
</tr>
<tr>
<td>Table 6.1</td>
<td>Numerical values of flume tank</td>
<td>88</td>
</tr>
</tbody>
</table>

LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>Ship motions in six degree of freedom</td>
<td>3</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Block diagram of the ship model and the stimulator</td>
<td>3</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>Schematic diagram of the active anti rolling tank</td>
<td>20</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Characteristic of wave traveling</td>
<td>33</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Wave spectrums with one peak</td>
<td>33</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Wave attack angle on the ship</td>
<td>35</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>Closed loop system of MPC</td>
<td>40</td>
</tr>
<tr>
<td>Figure 5.1</td>
<td>The locations of the poles of the ship model</td>
<td>50</td>
</tr>
<tr>
<td>Figure 5.3</td>
<td>Ship rolling angle in sea state 3</td>
<td>51</td>
</tr>
<tr>
<td>Figure 5.4</td>
<td>Ship rolling angle in sea state 5</td>
<td>52</td>
</tr>
<tr>
<td>Figure 5.5</td>
<td>Ship response in sway, roll and yaw when $\beta=30$, sea state 5</td>
<td>54</td>
</tr>
<tr>
<td>Figure 5.6</td>
<td>Ship response in sway, roll and yaw when $\beta=60$, sea state 5</td>
<td>55</td>
</tr>
<tr>
<td>Figure 5.7</td>
<td>Ship response in sway, roll and yaw when $\beta=90$, sea state 5</td>
<td>55</td>
</tr>
</tbody>
</table>
Figure 5.8: Ship response in sway, roll and yaw when $\beta =$120, sea state 5 ..... 56
Figure 5.9: Ship response in sway, roll and yaw when $\beta =$150, sea state 5 ..... 56
Figure 5.10: Ship response in sway, roll and yaw when $\beta =$180, sea state 5 ..... 57
Figure 5.11: Ship responses in different wave attack angles $\beta$ ................. 57
Figure 5.12: The ship coupling model under the effect of the yaw and roll disturbance .......................................................... 60
Figure 5.13: The ship coupling model under the effect of the sway and roll disturbance .......................................................... 60
Figure 6.1: bode plot of the ship rolling model ........................................... 63
Figure 6.2: poles and zeros location of the passive system ......................... 65
Figure 6.3: ship rolling angle and water head in the passive control ............. 67
Figure 6.4: Sway, roll and yaw with and without passive control ............. 68
Figure 6.5 the norm and variance of the prediction .................................. 70
Figure 6.6 : Controlled ship roll angle $\lambda =0.2$, $R=4$ .......................... 72
Figure 6.7: Controlled ship roll angle $\lambda =0.3$, $R=4$ .......................... 72
Figure 6.8: Controlled ship roll angle $\lambda =1$, $R=1$ ............................ 73
Figure 6.9: Controlled ship roll angle $\lambda =0.2$, $R=7$ .......................... 73
Figure 6.10 ships roll angle under the wave excitation and water head in the tank with the GPC controller.
Figure 6.11: Ship coupling response with the GPC in sea state 3 ............... 76
Figure 6.12 Ship coupling response with the GPC in sea state 5 ............... 76
Figure 6.13: Ship coupling response with the GPC in sea state 6 ............... 77
Figure 6.14: Comparison of uncontrolled, passive and Active system ........... 77
Figure 6.15 ship roll angle at R=1 and $\dot{\lambda} = 1$ ......................................................... 79
Figure 6.16 ship roll angle at R=3 and $\dot{\lambda} = 7$ ......................................................... 79
Figure 6.17 ship roll angle at R=10 and $\dot{\lambda} = 0.1$ ......................................................... 80
Figure 6.18 ship roll angle at R=1 and $\dot{\lambda} = 0.3$ ......................................................... 80
NOMENCLATURE

$A_{tank}$  
Tank area

$A_{pipe}$  
Pipe area

$I$  
moment of inertia of ship about x-axis

$L_{stimu}$  
distance between the centerlines of the tanks

$L_x, L_y, L_z$  
Distance of the center of the cross pipe to the center of gravity of the ship  
In the $X_0$-, $Y_0$-, and $Z_0$-direction

$H_{tank}$  
nominal height of the water in each tank from the center of the cross pipe

$M$  
Roll moment act on the ship by sea

$M_{stimu}$  
Moment action between stimulator and ship

$M_{wave}$  
Sea wave moment act on the ship

$P$  
Pressure

$R$  
Position vector of center of gravity of ship with respect to inertial ref  
frame on ship fixed frame.

$b_f$  
Volume friction coefficient of water motion in the flume tank

$F$  
Force act on fluid in the flume tank

$f_{acc}$  
Force due to acceleration

$f_{fri}$  
Force due to friction

$f_{grav}$  
Force due to gravitational force

$G$  
Gravitational acceleration

$h_{tank}$  
change of head from its nominal height of the water in the tank

$i,j,k$  
Unit vector along x-, y-, and z-axis

$R$  
position vector of water in the flume tank measured from ship fixed frame

$U$  
Relative water velocity vector in the flume tank with respect to ship

$u_s$  
Unit vector along stimulator
\( x_{\text{ship}}, y_{\text{ship}}, z_{\text{ship}} \) Ship displacement on ship fixed frame

\( \rho \) Water density 1000 Kg/m³

\( \omega \) Ship angular velocity vector

\( \phi \) Roll angle

\( \theta \) Pitch angle

\( \psi \) Yaw angle

\( y \) Sway displacement

\( \omega_{n_{\text{ship}}} \) Ship roll natural frequency

\( \zeta_{\text{ship}} \) Linear damping ratio of ship roll motion

\( \nu \) Kinematics viscosity of fluid flow

\( m', m'_x, m'_y \) Added mass in the x and y direction

\( J'_x, J'_z \) Added moment of inertia in the x and z direction

\( \alpha'_y \) Denoted the x coordinate of the center of \( m'_y \)

\( I'_z, I'_y \) Denoted the Z coordinate of the center of \( m'_x, m'_y \)

\( X'_{\text{sway}} \) The hydrodynamic force on the sway

\( K'_{\text{roll}} \) The hydrodynamic moment on the roll

\( N'_{\text{yaw}} \) The hydrodynamic moment on the yaw

\( x_G \) the distance of C.G in front of the midship

\( L \) Ship length

\( B \) Breadth

\( d \) Draft

\( W \) Displacement Volume

\( K_M \) Height from keel to transverse metcenter

\( K_B \) Height from Keel to center of buoyancy

\( C \) Block Coefficient

\( A_{i} \) Wave component
$S(\omega_i)$ Wave spectral density

$\omega_i$ is the wave frequency of wave component

$\Delta \omega$ is a constant difference between successive frequencies

$k_i$ one single wave component

$\lambda_i$ Wave length

$H_i$ Wave height

$\beta$ Angle between the ship heading and the direction of the wave

$s_i(t)$ Wave slope

$m$ Number of output

$r$ Number of input

$\beta_j$ Observer Markov parameter

$\alpha_j$ Observer Markov parameter

$y(k)$ Output prediction at time $k$

$u(k)$ Input data at time $k$

$\varepsilon$ Error Different between the desired response and prediction response

$\bar{Y}$ Observer Markov Parameters matrix

$P$ System Order

$u_{id}$ Random Excitation

$u_c$ Closed Loop Control input

$J$ Objective function or cost performance index

$Q$ Weighting Factor

$R$ Weighting Factor

$\lambda$ Weighting Factor

$h_p$ Prediction horizon

$h_c$ Control horizon

$A, B, C, D$ Open loop system matrices of state space form
**ABBREVIATION**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SID</td>
<td>System Identification</td>
</tr>
<tr>
<td>OMP</td>
<td>Observer Markov parameters</td>
</tr>
<tr>
<td>SMP</td>
<td>System Markov parameters</td>
</tr>
<tr>
<td>ARX</td>
<td>Autoregressive model with exogenous input</td>
</tr>
<tr>
<td>MPC</td>
<td>Model predictive control</td>
</tr>
<tr>
<td>GPC</td>
<td>Generalized predictive control</td>
</tr>
<tr>
<td>DPC</td>
<td>Deadbeat Predictive Control</td>
</tr>
<tr>
<td>SISO</td>
<td>Single input single output system</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Input multi output system</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1 Background and problem statement

Ship rolling is considered to be one of the most promising areas in dynamics and control systems research. The fluctuation of the ship is one of the most unsatisfactory characteristics of marine vehicles. It leads to the ship instability, sets up stress in their structure, causes distress to both passenger and crew, and increases the cost of operation. Roll motion also makes passengers uncomfortable and makes the process of loading and unloading difficult. Suppression of ship rolling will help passengers and crew on board to be in great performance, avoid sea sickness, and increase their safety.

The ship has six degrees of freedom which allows it to move when forces and moments act upon it. This freedom includes translation and rotation, moving about three axes. Linear motions include surge (forward or backward), sway (either side), and heave which translation is (up and down). Angular motions include roll which is the rotation about the surge axis, pitch which is the rotation about the sway axis, and yaw which is the rotation about the heave axis as shown in Figure 1.1. Forces and momentums that act upon the ship come from thrusters which may include propeller forces, control surfaces such as rudder forces, and environmental forces such as waves, wind, and ocean currents.

The ship has restoring forces that counter the effects of the roll motions, which enable the ship to oscillate in sea condition. Various methods have been used to suppress ship roll motion. For example, active systems include active anti-rolling flume tanks,

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1 The journal model adopted for this dissertation is AIAA
gyroscopes, active fins, and rudders. Passive systems may consist of passive tanks and bilge keel.

The active anti-rolling flume tank is considered to be one of the useful methods for ship roll cancellation. Figure 1.2 shows the block diagram of the anti-rolling flume tank and ship models. When ship loading and weather conditions change, the stimulation can change the amplitude and/or period of the stimulation moment generated in order to compensate for changes in environmental conditions and maintain the desired roll amplitude. There are two sensors available for the stimulator control design. A gyroscope sensor is located on the ship to provide the ship roll angle (output signal), and a pressure sensor is located at the bottom of each flume tank to measure the water head difference.

As shown in the figure, the ship model could be affected by the wave disturbances. The anti-rolling flume tank works as an actuator, which could generate moments that balance the rolling angle of the ship. The input signal generates the pressure of the pump inside the water tank, which allows the water, or the fluid inside the tank to move from one direction to another. In order to control the roll motion, GPC has been selected to design the controller of the system. For a closed loop system a gyroscope sensor is suggested to observe the output data of the ship rolling angle. The sensor reads the output signal of the ship roll angle beside a sensor that read the water head inside the tank which is a pressure sensor. These two sensors read the historical data of the input and output of the system. The GPC controller is designed based on the historical output data of the system. The controller will be able to maintain the movement of the water inside the tanks. Thus, the ship rolling motion will be suppressed and controlled using a passive control system and GPC controller.
Figure 1.1 Ship motions in six degree of freedom

Figure 1.2 Block diagram of the ship rolling model and anti-rolling flume tank with the GPC controller
1.2 Dissertation Objective

In this dissertation, an active anti-rolling flume tank (stimulator) is used as an actuator for the controller design. The main objective is to design a control system to mitigate the ship's roll motion on the basis of the generalized predictive control theory. The designed controller is simulated and investigated to see its' effect in the coupling motions of the sway, roll, and yaw. In order to achieve this goal, certain objectives have to be considered. First, a mathematical model of the ship model and stimulator is introduced and analyzed. Also, construct a multidimensional model for a container ship, which include couple roll, yaw and sway. The ship behavior in the sea is studied and investigated in different weather conditions. Second, system identification technique is implemented in order to solve for the Observer Markov Parameters (OMP). Third, the multi step prediction equation is constructed based on the GPC algorithm. Fourth, the GPC gain matrix is calculated by tuning the weighting factors of the performance index. Finally, a numerical simulation of ship coupling model is implemented under the excitations of the wave disturbance to determine whether the GPC controller would suppress the ship's roll motion. The GPC is tested on the coupling behavior of the sway and yaw.

1.3 Dissertation outline

Chapter II presents a historical background of the anti-rolling flume devices with a brief description of its mechanisms and efficiency. Also, it explains in details the anti-rolling flume water tank which is used as an actuator in this work. Moreover, it presents a brief history of the related research that controls the marine vehicles.
Chapter III introduces a mathematical model of the systems in a state space form. The mathematical model is divided into three sections. The first section deals with the modeling of the stimulator and ship roll model. The second section presents the coupling model in sway, roll and yaw. The third section presents the wave disturbance model. The wave disturbance equations of the ocean environment are described.

Chapter IV proposes the Generalized Predictive Control (GPC) technique that is used as a control method for suppressing ship roll motion. Descriptions of the GPC are presented in a detailed manner. The fundamental concept of the GPC and the algorithm used in this work are introduced. Moreover, the optimization techniques and tuning of the weighting factors of the performance index are discussed.

Chapter V provides a numerical model of a container ship that has been used for this simulation. The numerical simulations of the ship coupling model are presented in different sea state conditions. The behavior of the uncontrolled ship in sway, roll and yaw is simulated and discussed.

Chapter VI verifies the new control design algorithms by a numerical simulation of the results. The numerical simulation of the new controller is presented with and without the environmental disturbance. The controller result presents the effectiveness of the tuning of the weighting factors in the roll mitigation.

Finally, Chapter VII provides a discussion and conclusions of the dissertation results. This chapter provides the dissertation main contributions and the prospects for the extension of this research.
CHAPTER II
LITERATURE REVIEW

2.1 Introduction

Reducing or controlling the ship roll motions has a long history dating back to the nineteenth century; several methodologies have been proposed and implemented. A historical account of this subject is given in Bennett. Passive methods appeared first, including bilge Keels, anti-rolling flume Tanks, Moving Weight, and gyroscopic methods. Following the development of feedback control theory, active methods began to emerge, many of which were inspired by or modified from the passive ones, including fin stabilizers, active tanks, controlled moving weight, and active gyroscope methods.

As control theory has progressed and ship dynamic models have improved, new control strategies have been brought to bear on this problem. For example, an adaptive control is used to control a wing actuator (similar to fin stabilizers) based on gain scheduling and neural networks. A good collection of recent development on the topic of sea-going vehicles, such as autopilots and ship positioning, is provided in a textbook by Dr. Fossen.

In the next section of this chapter, a history of the control of the ship rolling is provided. The literature review includes roll stabilization devices of the ship, and background of the anti-rolling flume tank and the control application.

2.2 Roll Stabilization Devices

Since the 1800s, different methodologies have been presented to control and suppress the ship rolling in the water. All methods add weight to the ship and most of...
them occupy space, which could be used for commercial or military purposes, if stabilization were not required. Generally, but not always, the period of the roll-damping mechanism should be the same as that of the ship. There should always be a definite relation between these two periods.

To date, roll stabilizations device put into practical use such as bilge keels, gyroscope stabilizers, moving weight, fin stabilizer, rudder rolls stabilizer system, vertically roll stabilizer system, air fin and anti-rolling flume water tank. These all have impressive records of achievement as roll stabilization systems. A full description of the various types of apparatus, which have been proposed and developed for ship stabilization and works containing such information's in greater detail is outlined and performance compared.

Ship roll stabilizers, which have been used for ship roll cancellation, can be divided into three categories, each defined by its use of power. A passive system can be self operated and does not need power. A semi-active system uses little power to operate. An active system uses power to generate a moment that will cancel the roll motion.

Active systems are divided into two categories. The first type can generate an internal moment by itself and work at zero to low speeds. The second will use force, due to relative motion of the fluid and the ship, with high velocity. These forces will be small or zero when relative motion is small or zero. Passive equipment is usually left alone on board except for tuning operations which are needed for a change in ship loading conditions. The sophisticated control unit is needed in active control, but it can operate in wide ranges and higher performance. The histories of each method of the roll stabilization devices are presented in detail.
1) Bilge keels

Bilge keels are fins in planes approximately perpendicular to the hull at or near the turn of the bilge. Ever since their effectiveness to reduce rolling was first demonstrated, around 1870, bilge keels have been installed on nearly all ocean-going vessels, both commercial and military.

In 1894 and 1895, White and Sir W.H gave the results of rolling tests on a model with and without bilge keels. The tests show a great damping effect of the bilge keels. The effectiveness of bilge keels was also very strikingly illustrated by the experience of the British navy with the Royal Sovereign class of battleships.

In 1936, Drs. Saints and Russo showed that the effectiveness of bilge keels is materially greater when the ship is in motion ahead than it is when it is empty. Bilge keels have very low values of aspect ratio. Therefore, it shows poor lift values for the speed and angle of attack. Their aspect ratio could be improved by leaving out parts of them, thus converting them to two series of hydrofoils having individually greater aspect ratio values. Such discontinuous bilge keels will probably produce greater roll-damping effect than continuous bilge keels of the same area.

In 1937, Mandelkorn, R.S and Miller, W.R showed the evidence to support this, which reports the results of experiments with models of bilge keels and hydrofoils mounted on boards in a wind tunnel.

2) Gyroscope stabilizers

The earliest use of the gyroscope to reduce or prevent the rolling of ships was considerably later than the use of bilge keels and anti-rolling flume tanks for this purpose.
The gyroscope possesses certain advantages as a ship-stabilizing agency over both bilge keels and anti-rolling flume tanks. The earliest installations were of the Schlick type in Germany, but the most recent ones have been the Sperry stabilizers manufactured in the United States.

The first recorded construction of the gyroscope is usually credited to C.A Bohneriberger in 1810 while the first electrically driven gyroscope was demonstrated in 1890 by G. M. Hopkins. The development of the electrically driven gyroscope was motivated by the need of more reliable navigation systems for steel ships and underwater warfare. A magnetic compass as opposed to a gyrocompass is highly sensitive for magnetic disturbances, which are easily generated within steel ships and submarines, equipped with electrical devices. In parallel works, Dr. H. Anschutz of Germany and Elmer Sperry of the USA both worked on a practical application of the gyroscope.

In 1908, Dr. H. Anschutz patented the first North seeking gyrocompass while Elmer Sperry was granted a patent for his ballistic compass including vertical damping three years later. The work on the gyrocompass was further extended to ship steering and closed-loop control by Elmer Sperry (1860-1930) who constructed the first automatic ship steering mechanism in 1911. This device is refereed to as the "Metal Mike" and it was capturing much of the behaviors of a skilled pilot or a helmsman. "Metal Mike" did compensate for varying sea states using feedback control and automatic adjustments. Later in 1922, Nicholasorsky presented a detailed analysis of a position feedback control system where he formulated a three-term control law, which today is refereed to as Proportional Integral-Derivative (PID) control, Minorsky. Observing the way in which a helmsman steered a ship motivated these three different behaviors.
3) Jet Flaps

Jet flaps have the fixed fin attached to the hull, so it can withstand a higher load as compared to the moving fin. This works well at low frequency. It consists of pumping water to force flow to the end of fin. The directed flow of water will generate a lift force or moment. The lift force depends on jet angle, ship velocity, jet velocity, and water mass flow rate.

4) Moving Solid Weight

The system consists of large weights transversely across a ship. They move with same ship roll period and lag ship roll motion by 90 degrees. This system is suitable for small ships. Weight should not be greater than 5 percent of a ship is displacement. Drawbacks include large weights and weight mounting.

Saeki\textsuperscript{13} studied ship roll cancellation in the sea states 5, with wave height equal to 4m, by using a pendulum of mass driven by a motor. This actuator is developed for damping devices on main tower bridges and high-rise buildings.

5) Fin stabilizer

This system reduces rolling by way of a fin installed at the curve at the center of the hull to generate a moment that resists rolling. A high rolling reduction rate is obtained during cruising by controlling the fin return angle by means of control signals from a roll sensor. However, the system is ineffective when the ship is stopped.

A patent for stabilizing fins was granted to John I. Thornycroft in 1889, but there is no record of any actual installation until after World War I. It appears that, without any knowledge of Thornycroft’s patent, Dr. Motora developed the same device and
installations of his ship design were made on several Japanese ships. At almost the same time, Denny Brown stabilizer, operation on the same principle, was developed in England and one experimental installation was made.

6) Rudder rolls stabilizer system

This system reduces rolling of the ship's hull by a rotation moment generated between the lifting force generated by operation of the rudder during cruising and the center of gravity of the hull.

Rudder roll damping was first suggested in the late seventies. Research in the early eighties showed that it was indeed feasible to control a ship's heading with the rudder(s) while simultaneously also using the rudders for roll damping. Research groups in Holland, Sweden, and Denmark were pursuing results in this area. In Denmark, MB and co-workers developed a rudder-roll damping (RRD) autopilot (Blanke et.al, 1989) that was later implemented by the Danish Navy on their Standard Flex 300 Class, consisting of 14 ships (Munk and Blanke, 1987). Results from sea trials with the first ship showed convincing agreement with theory and the simulations made during the design. Later ships in the series, however, experienced significantly less efficiency of the RRD system. An investigation was made to disclose the factors behind the discrepancy result.

Parametric investigations showed that cross-couplings between steering and roll might give rise to problems with performance robustness for the RRD controller, (Blanke and Christensen, 1993). Later sea trials and identification of ships in several loading conditions showed clear changes in the dynamics between the first and later ships in the series and confirmed that a robustness problem does exist. Reasons included changes in loading conditions and in rudder shape (Blanke, 1996).
Research by Stoustrup, Niemann and Blanke treated the RRD design problem from a robust control outset. It was shown that a separation result exists that makes it possible to make separate roll and steering specifications and optimize the two controllers independently, (Stoustrup, Niemann and Blanke 1995). Collaboration with the Danish Maritime Institute (DMI) and the ship owner A. P. Moller led to results from a test facility at DMI where coupling effects can be measured in model scale and a comparison with full-scale trial results. This gave a detailed insight into the phenomena, (Blanke and Jensen, 1997). Collaboration with the University of Pavia, Italy, led to a new approach in identification of steering-roll models, (Tiano and Blanke, 1997).

Recent work has included H∞ control of the roll damping loop, (Yang and Blanke, 1997) and qualitative feedback theory (QFT) applied to solve the combined RRD-heading control problem with due regard of model uncertainty, (Hearns and Blanke, 1998). Experience from sea was reported in (Blanke, Adrian, Larsen and Bentsen, 2000).

2.3 Active anti-rolling flume tanks

The active anti-rolling flume tank is a tank full of water and installed to reduce the roll angle of a ship. The shifting of water or other liquid in the tank reduces rolling motions. Currently, there are three types of anti-rolling flume tank are used; passive, semi active and active anti-rolling flume water tank.

The passive type anti-rolling flume tank is a U-shaped water tank installed in the ship and rolling is reduced by the action of the movement moment of the tank liquid created by rolling of the ship. The volume of liquid must be 2 to 3 percent of the ship's displacement. The passive type of anti-rolling flume tank is the most economical type available without a control system. The passive type anti-rolling flume tank reduces
rolling by shifting the liquid in the tank which naturally uses the difference in height of the left and right sides of the hull produced by the rolling of the ship instead of controlling the period of movement of the liquid in the tank.

The semi-active type anti-rolling flume tank has the same shape as the passive type, but the bottom duct is split and the period of movement of the tank liquid is controlled by a damper, so a wide range of periods of movement can be handled. A tilt sensor, computer, damper and drive system, and air duct opening/closing system control the period. The semi-active type anti-rolling flume tank was developed to respond to a wide range of period changes by splitting the bottom duct of the passive type and installing a damper in the duct.

The active type anti-rolling flume tank uses power to forcefully shift the liquid in the tank to obtain a rolling moment. Products used together with a Hill compensator have been developed and put to practical use.

The free surface type anti-rolling flume tank was developed in 1880. In 1910, H. Frham developed the current U-pipe type ART. The earliest installation of tanks was made about 1874, only a few years after the earliest use of bilge keels. At this time W. Froude and others were assiduously studying ocean waves and the rolling of ships. Earliest applications of anti-rolling flume tanks used compartments in the upper part of the ship where free water could be carried. The reduction in metacentric height, due to the free surface of the water, caused a lengthening of the ship's period of rolls and, if the ship were previously rolling in synchronism with the waves, destroyed the equality of periods. Furthermore, transfer of water to the low side of the ship created moments that balance the ship rolling and increase the ship roll damping ratio.
In 1936, a similar type of anti-rolling flume tanks were installed on the S.S. Kdnigen. This installation, however, has major differences from Froude's water chambers which overcome the latter's undesirable features.

All other anti-rolling flume tanks were evolved from the U-tube type. In 1910, the first anti-rolling flume tanks were developed. Most of the early Frahm anti-rolling flume tanks were located with the horizontal leg of the U-tube above the ship's center of gravity. This came about for two reasons. It was easier to find available space for the tanks above the machinery space. Also, when the horizontal leg is above the ship's center of gravity, the moment of the force due to the horizontal acceleration of the water therein acts in the same direction as the moment of the water in the vertical legs.

Frahm has contributed a general rule in the design of anti-rolling flume tanks which states that the moment of inertia of the free surface about the centerline of the ship divided by the volume of displacement of the ship should be 40 percent of the met centric height.

Later anti-rolling flume tank installations of the Frahm type have no horizontal leg of the U-tube, and no air cross-connection installed on the steamships described by Hansa, Deutschland and Hamburg. The tanks are open at the bottom to the sea, which takes the place of the bottom leg of the U-tube on the Ypiranga, and are vented to the top, the atmosphere from the tops of the vertical legs.

The most recent installations of Frahm anti-rolling flume tanks on large ships are those of the steamships Bremen and Europe. In these ships, the installation consists of four tanks located lowdown in the ship, necessitating a return to the U tube type. Hort describes an experimental installation of activated anti-rolling flume tanks in 1934.
In 1963, Japan's first practical anti-rolling flume tank developed jointly by NKK and Professor Motoyoshi of Tokyo University was installed in the patrol boat Shikine. In 1983, NKK developed a multi period anti-rolling flume tank and installed the first unit in the survey ship Takuyo.

In 1990, on feedback control, the latest ship stabilizers are capable of both heel and roll control using water tanks. The stabilizer is equipped with a roll indicator which is a microprocessor based computer that constantly calculates the root mean square roll, the heel and the average apparent roll period (Honkanen, 1990).

In 1988 Webster, Dalzel and Bar discussed a prediction method for evaluating the performance of free flooding ship anti-rolling flume tanks. A study of the effectiveness of these free-flooding tank systems was done for the USS midway. The equations for the prediction method are presented in the frequency domain where the tank system influences the ship in only the sway, roll and yaw degrees of freedom.

In 1990, Yamaguchi and Ogawara proposed a feedback control system of ship rolling motion by active anti-rolling flume tank. They presented a numerical method of analyzing a feedback control system of rolling motion of a ship in waves by a U-section type active anti-rolling flume tank. Their method consists of a simulation technique of liquid motion in tank with impellers and of a design procedure of a feedback control system. The design procedure is based upon a theory of a control system root locus methods method.

In 1992, Sellers and Martian presented a comparison of various roll stabilization systems. Several considerations were made in reference to the use of the passive anti-rolling flume tank in the ship and the effectiveness of the system in the ship stabilization.
In 1994, an experimental study on actively controlled anti-rolling flume system was developed by Koike, Eiki and Masao with a view of reducing the rolling motion by the movement of the mass controlled by actuator. This system consists of sliding mass on the rail shaped in acicular arc and compact, passive pendulum mechanism is realized that it does not require a suspension structure such as a simple pendulum or spring mechanism. The driving force to control the movement of the damper mass is imparted from the electric motor through reduction gearing connected to a gear and pinion mechanism. The LQR control theory has been adopted for controlling the damper mass. Results showed that the rolling was reduced to about 1/3 in beam seas under the condition that the ship was stationary.

In 1995, designs method of a control system to reduce a ship’s rolling motion was proposed on the basis of the adaptive control system in the anti-rolling flume tank by Satoru Yamaguchi and Akiji in Japan. They have concluded that, "The system was confirmed to be a useful one for the ship greatly varied in loading condition".

In 1998, Aiichiro, Hiroak, Masao and Yuji proposed a new type of anti-rolling flume device developed for the oceanographic research vessel “MIRIA”. The Hybrid system combines the advantages of both active and passive mechanisms which allow the device to be compact and achieve high performance to reduce rolling during cruising and drifting under various loading condition and sea states. They concluded that it reduces the ship rolling by 50 percents and its effect could be obtained when the ship is drifting, decelerated operation and when the ship decreased due to the head sea.

A time domain numerical study of a passive and active anti-rolling flume tank was conducted in 1998 by the Virginia Polytechnic Institute and State University. A
PID control was used for the active anti-rolling flume tank. Results of the time domain simulation indicate the effect of the anti-rolling flume tank.

In 2000, Kleefsman in the Department of the Mathematics at the University of Groningen published a numerical simulation of the ship motion stabilization by an active U tube anti-rolling flume tank\textsuperscript{22}. A mathematical model of the anti-rolling flume tank motion was presented based on Navier Stoike’s equation. The coupling motion of 2 DOF have been considered.

In 2001, Gawad and Nayfeh published a technical report of roll stabilization of the anti-rolling flume tank\textsuperscript{23}. Also, in 2002, a design of a passive anti-rolling flume tanks for roll stabilization in a nonlinear range was presented by Dr.Nayfeh\textsuperscript{24}.

Design-guides and the effects of those in real sea conditions have already been reported. Active researches are thought to be finished. Fin stabilizers have also been studied and so many compact products to be fitted with small yachts are on the markets. Japanese researchers contributed an advance works on the ship rolling systems. There are other research papers that relate to this field, but which are printed or published in languages other than English. For example, Korea, Japan, Norway and Denmark are conducting advanced experimental and theoretical research in this area.

To improve the efficiency of the tank optimized for the natural frequency roll motion, pumps or blowers are adopted to move the fluid in the tanks into other frequencies. Active control routines in the anti-rolling flume tanks are to control the power of pumps or blowers considering the level of the fluid in the tanks and the roll motion. The effect of the active anti-rolling flume tank in the coupling motion is an interesting field to investigate.
Most of the research available on the coupling motion is applied based on the rudded stabilization fan. Much of the anti-rolling flume tank research also deals with water motion behavior in the tank in perspective of the fluid mechanics. Some other researchers have worked on the control analysis of the ship rolling system. To date, many different methods have been used such as frequency response analysis, root locus optimal control and adaptive control theory. Yet, system identification or model predictive control has not been used in this application field. The application of the generalized predictive control (GPC) techniques on the ship roll stabilization is a new method to be studied and investigated. Thus, using control methods to control the rolling angle coupled with the others degree roll motions deserves more study and analysis in different sea conditions.
CHAPTER III

MATHEMATICAL MODEL

3.1 Introduction.

One of the most important parts in the control design is the mathematical model. In the control system the designer must be able to model and analyze the dynamic characteristics of the system. Mathematical modeling can be defined as “a set of equations that represents the dynamics of the system accurately or at least fairly well”\textsuperscript{25}. In this chapter, the mathematical models of the stimulator, ship rolling and coupling model are presented. The dynamic equations are obtained in a state space form. The wave equations that generate the disturbances are presented. Moreover, a full detailed description of the sea state condition is described. Notice that the definition of the symbols of the equation in this chapter is defined at the beginning of this dissertation (see nomenclatures section).

3.2 Dynamic model of the anti-rolling flume tank

This section represents the mathematical model of the anti-roll flume tank (stimulator). In order to form a mathematical model for the stimulator, the physical behavior of the stimulator must be considered. A schematic diagram of the active anti-rolling flume tank is shown in Figure 3.1. It consists of two container systems made up of two water tanks mounted in the outboard container cells and connected to a bow thruster inside a pipe. The water, driven by an axial pump, has a varying flow rate and is used to generate forces and moments acting on the ship.
Figure 3.1 schematic diagram of the active anti-rolling flume tank

Normally, a ship has multi anti-rolling flume devices which connect to each other. The location of the anti-rolling flume tank is different from one ship to another. It depends on the ship design and other factors like type of the ship, weight, and length of the ship.

In the reference [14] and [15], the equation of motion of the stimulator has been driven in two different forms. The first derivation was based on the Euler equation and the second one is based on the Navier-Stokes equation, which is going to be used in this work. For the stimulator model, there are two dynamic equations that need to be
considered. These equations are the water motion in the tank and the moment that is
generated by the stimulator.

1) Water motion inside the tank

The first equation is the water motion in the tank which is forced by either ship
motion or by water pump in the stimulator. Water motion in the anti-rolling flume tank
can generate force acting on the ship. The fluid inside the tank is assumed to be one
dimensional motion. Also, the velocity of the water inside the duct is equal, the velocity
in the duct, and has no effect of flow rate at the connection between the duct and tank.

Based on the reference [15], the governing dynamic equation for water motion in
the anti-rolling flume tank can be expressed:

\[
\begin{align*}
\left(2H + \frac{A_{\text{tan}k}}{A_{\text{pipe}}} L_{\text{stimu}}\right) \ddot{h} & = \frac{1}{\rho} \Delta p - L_{\text{stimu}} g \sin \phi - 2gh \cos \phi \\
+ \dot{\psi}_{\text{ship}} L_{\text{stimu}} & + 2h \dot{z}_{\text{ship}} - 2(\dot{\phi}^2 - \dot{\theta}^2) Hh + 2\dot{\phi} \psi_{L_X CM \ tan k} H + \dot{\phi} L_{\text{stimu}} h \\
+ \dot{\phi} \psi_{L_X CM \ tan k} L_{\text{stimu}} & + \dot{\theta} \psi_{L_Z CM \ tan k} L_{\text{stimu}} - \dot{\phi} L_{\text{stimu}} H + 2\dot{\phi} L_{X CM \ tan k} H \quad (3.1)
\end{align*}
\]

Each term of this equation is defined in the nomenclatures section. Equation
(3.1), represents the fluid motion of the water inside the flume tank in terms of the
dimensions of the tank and the pipe beside the water density and viscosity of the fluid
inside the tank.
2) Moment generated by the Stimulator

The second important equation is the moment that is generated by the stimulator.

The moment generated by the stimulator to roll the ship can be described as:

\[ M_{stimu} = M_{stimu,acc} + M_{stimu,grav} + M_{pump} \]  

where \( M_{stimu,acc} \) is the generated moment due to the acceleration of fluid in the stimulator, \( M_{stimu,grav} \) is the generated moment due to gravitational force acting on the fluid, and \( M_{stimu,pump} \) is the moment due to pressure difference between two sides of the pump. Equations (3.2) can be expressed to be:

\[
M_{stimu,acc} = -\rho \left( H A_{tank}^k L_{stimu} + L_{stimu} A_{tank}^k L_{Z,CM}^k \tan k \right) \ddot{h}
\]

\[
-\frac{1}{2} \rho L_{stimu} A_{tank}^k \left( 2(\dot{\phi}^2 - \dot{\theta}^2)H h - 2\dot{\phi}\dot{\phi} L_{X,CM}^k \tan k H - \theta \dot{\phi} L_{stimu} h \right)
\]

\[
+ \rho L_{Z,CM}^k A_{pipe} \left( \dot{\phi} \dot{L}_{X,CM}^k \tan k L_{stimu} + \dot{\phi} L_{Z,CM}^k \tan k L_{stimu} \right)
\]

\[
- \frac{1}{2} \rho A_{tank}^k L_{stimu} \left( \ddot{\phi} L_{stimu} H - 2\dot{\phi} L_{X,CM}^k \tan k H \right)
\]

\[- \rho A_{pipe} L_{Z,CM}^k \tan k \left( \dot{\phi} L_{Z,CM}^k \tan k L_{stimu} - \ddot{\phi} L_{X,CM}^k \tan k L_{stimu} \right)
\]

\[
+ \rho L_{stimu} A_{pipe} L_{Z,CM}^k \tan k \ddot{y}_{ship} + \rho \dot{h}_{ship} A_{pipe} L_{stimu}
\]  

Also, the moment generated due to the gravitational force acting on the fluid can be written:

\[ M_{stimu,grav} = -L_{stimu} \rho \sin \phi L_{Z,CM}^k \tan k A_{pipe} \rho - g \dot{h} \cos \phi L_{stimu} A_{tank}^k \rho \]  

Moment due to the pressure generated by the pump can be written:

\[ M_{stimu,pump} = -A_{pipe} L_{Z,CM}^k \tan k \Delta P \]
3.3 Ship rolling model and the stimulator.

This section shows the state space form of the ship roll model and the anti-rolling tank.

The state space model of the combined system can be expressed as:

\[
\begin{bmatrix}
\dot{h}_{\text{tank}} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-c_{a2} \left( \frac{a_{21}}{a_{11}} a_{13} - a_{23} \right) & -c_{a2} a_{24} & -c_{a2} \left( \frac{a_{21}}{a_{11}} a_{15} - a_{25} \right) & -c_{a2} a_{26} a_{16} \\
-c_{a4} \left( \frac{a_{21}}{a_{12}} a_{13} - a_{23} \right) & -c_{a4} a_{24} & -c_{a4} \left( \frac{a_{21}}{a_{11}} a_{15} - a_{25} \right) & -c_{a4} a_{26} a_{16}
\end{bmatrix}
\begin{bmatrix}
\dot{h}_{\text{tank}} \\
\dot{\theta}
\end{bmatrix} + 
\begin{bmatrix}
-c_{a2} \left( \frac{a_{21}}{a_{11}} A_{\text{tan} k} L_{Z, \text{CM, stimu}} - 1 \right) & 0 & 0 & 0 \\
0 & c_{a2} / a_{11} & 0 & 0 \\
-c_{a4} \left( \frac{a_{22}}{a_{12}} A_{\text{tan} k} L_{Z, \text{CM, stimu}} - 1 \right) & 0 & c_{a4} / a_{12} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta p \\
M_{\text{wave}}
\end{bmatrix},
\tag{3.6}
\]

Where is:

\[
a_{21} = \rho \frac{m_{s2}}{A_{\text{tan} k}} a_{22} = m_s \rho 
\]

\[
a_{24} = \rho \left( 2 HA_{\text{tan} k} + A_{\text{tan} k} L_{\text{stimu}} \right)
\]

\[
a_{25} = \rho L_{\text{stimu}} g
\]

\[
a_{26} = 0
\]

\[
a_{11} = \left( I - \frac{1}{2} \rho L_{\text{stimu}} A_{\text{pipe}} H m_s + \frac{1}{A_{\text{tan} k}} \rho A_{\text{pipe}} \left( L_{Z, \text{CM, tan k}} - H \right) m_1 \right)
\]

\[
a_{12} = \rho m_s
\]

\[
a_{13} = \rho g L_{\text{stimu}} A_{\text{tank}}
\]

\[
a_{14} = 0
\]

\[
a_{15} = \omega_{\text{ship}}^2 + \rho g L_{\text{stimu}} A_{\text{pipe}} L_{Z, \text{CM, tank}}
\]

\[
a_{16} = 2 \omega_{\text{ship}} \omega_{n, \text{ship}}
\]

\[
c_{a2} = \left( \frac{1}{a_{11}} a_{12} a_{13} - a_{22} \right)^{-1}
\]

\[
c_{a4} = \left( \frac{1}{a_{12}} a_{22} a_{11} - a_{21} \right)^{-1}
\]
3.4 Ship coupling model

In order to have an accurate study for the ship motions, the coupling model of the roll along with other degrees of freedom has to be considered. In the actual seaway, ships experience all six degree of freedom of motion coupling with each other. Unfortunately, a study of these coupling motions is rather difficult, and investigations are often restricted to the following coupled motions:

1. heave and pitch
2. yaw and sway,
3. sway, roll and yaw
4. roll, yaw, and pitch.

Since the main concern of this work is the rolling motions, of these four coupled motions, the coupling model of the roll is considered. The roll motion is always coupled with the yaw and either the sway or the pitch. From the previous research, the pitch movement has less effect to the roll than the sway. Thus, in this dissertation, the coupling model of the sway, roll and yaw will be considered.

3.4.1 Equation of motion of the coupled model

There are many different approaches that could lead to the coupling model. There are different theoretical approaches, such as the energy-formulation and strip theory, which form the coupling equation of motions. A mathematical model for a single-screw high speed container ship in surge, sway, roll and yaw has been presented by Son and
Nomoto in [18] and [19]. The model has been adapted to be used in this work and modified, so it is capable of describing the ship response in sea waves. The fundamental equations of sway yaw and roll coupled motion is expressed by the following non linear equation:

\[(m + m_y)\ddot{y} + m_y\alpha_y\dot{y} - m_yl_y\dot{\phi} = X_{\text{sway}}\]  \hfill (3.7)

\[(I_x + J_x)\ddot{\phi} - m_yl_y\dot{\phi} + W_G\dot{M}\phi = K_{\text{roll}}\]  \hfill (3.8)

\[(I_x + J_x)\dot{\phi} + m_y\alpha_y\dot{y} = N_{\text{yaw}} - Y_{x_G}\]  \hfill (3.9)

Where, \(m_x, m_y, J_z, J_x\) denoted the added mass and added moment of inertia in the x, and y directions and about the z, and x axis respectively. Also, \(\alpha_y\) denoted the x-coordinates of the center of \(m_y\) and \(I_x, I_y\) the z-coordinates of the center of \(m_x\) and \(m_y\), respectively.

The hydrodynamics force and moment \(Y, K, N\) of a linearized system are:

Where, \(Y\) denoted the hydrodynamic forces of the sway (hydro-inertial forces) in the y direction. \(N\) is the hydrodynamic yaw moment about the midship, \(x_G\) which is the distance of C.G in front of the midship. \(K\) is the hydrodynamic roll moment about the CG

\[Y_{\text{sway}} = Y_v\dot{y} + Y_p\dot{\phi} + Y_\varphi\phi\]  \hfill (3.10)

\[K_{\text{roll}} = K_v\dot{y} + K_p\dot{\phi} + K_\varphi\phi\]  \hfill (3.11)

\[N_{\text{yaw}} = N_v\dot{y} + N_p\dot{\phi} + N_\varphi\phi\]  \hfill (3.12)
By expanding the dynamic equation of motion of a coupled model in the sway, roll and yaw; it can be written:

\[(m + m_y)\ddot{y} + m_y \alpha_y \dddot{y} - m_y l_y \dddot{\phi} - Y_v \dot{y} - Y_r \dot{r} - Y_p \dot{\phi} - Y_y \phi = F_{\text{sway}}\]  

\[(I_x + J_x)\ddot{\phi} - m_y l_y \dddot{y} + WGM\phi - K_v \dot{y} - K_r \dot{r} - K_p \dot{\phi} - K_y \phi = M_{\text{roll}}\]  

\[(I_x + J_z)\dddot{\psi} + m_y \alpha_y \dddot{y} - N_v \dot{y} - N_r \dot{r} - N_p \dot{\phi} - N_y \phi = M_{\text{yaw}}\]  

Where \(F_{\text{sway}}, M_{\text{roll}}, M_{\text{yaw}}\) are the excitation forces and momentum due to the environmental disturbance such as a wave or wind. Thus, the coupled equations can be written in matrix form:

\[
\begin{bmatrix}
(m + m_y) & -m_y l_y & m_y \alpha_y \\
-m_y l_y & (I_x + J_x) & 0 \\
m_y \alpha_y & 0 & (I_z + J_z)
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{\phi} \\
\dot{\psi}
\end{bmatrix}
+ \begin{bmatrix}
-Y_v & -Y_p & -Y_r \\
-K_v & -K_p & -K_r \\
-N_v & -N_p & -N_r
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{y} \\
\phi
\end{bmatrix}
+ \begin{bmatrix}
0 & -Y_y & 0 \\
0 & WGM - K_y & 0 \\
0 & -N_y & 0
\end{bmatrix}
\begin{bmatrix}
\dot{r} \\
\dot{\psi} \\
\phi
\end{bmatrix}
= \begin{bmatrix}
F_{\text{sway}} \\
M_{\text{roll}} \\
M_{\text{yaw}}
\end{bmatrix}
\]
Since we consider that:

\[ M = \begin{bmatrix}
(m + m_y) & -m_y l_y & m_y \alpha_y \\
m_y l_y & (I_x + J_z) & 0 \\
m_y \alpha_y & 0 & (I_z + J_z)
\end{bmatrix} \]

\[ C = \begin{bmatrix}
-Y_v & -Y_p & -Y_r \\
-K_v & -K_p & -K_r \\
-N_v & -N_p & -N_r
\end{bmatrix} \]

\[ K = \begin{bmatrix}
0 & -Y_\phi & 0 \\
0 & WGM - K_\phi & 0 \\
0 & -N_\phi & 0
\end{bmatrix} \]

\[ U = \begin{bmatrix}
F_{\text{sway}} \\
M_{\text{roll}} \\
M_{\text{yaw}}
\end{bmatrix} \]

Since the general state space form is:

\[ \dot{X} = AX + BU \]
\[ Y = CX + DU \]

One can represent the dynamic equation of the ship coupling motions of the container in state space form. From this dynamic equation (3.16), matrix M, C, K and U is already defined. Thus, the numerical model of the state space is represented in terms of M, C and K. Notice that, the terms in equations (3.16) represent the ship parameters such as the hydrodynamic forces and momentums of the ship.
Thus, the numerical state space equation will be:

\[
\begin{bmatrix}
\dot{y} \\
\dot{\phi} \\
\dot{\psi} \\
\dot{h}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y \\
\phi \\
\psi \\
h
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
U_{4 \times 1}
- [M]^{-1}V
\]

\[
\begin{bmatrix}
y \\
\phi \\
\psi \\
h
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y \\
\phi \\
\psi \\
h
\end{bmatrix}
\]
3.4.2 Combined the coupling model with the stimulator

In order to control the ship motions in the seaway; the roll stabilization system has to be considered. Once the ship roll model is derived in state space form, the ship stabilization device is considered to describe the ship coupling model. The main function of the anti-rolling flume tank is to generate pressure difference inside the pipe which will generate rolling moment. The moment will stabilize the ship rolling. Thus, the governor dynamic equation of the fluid motion inside the anti-rolling flume tank and stimulator moments must be considered. By recalling equation (3.1) and (3.2) and assuming that:

1) The motions in the heave, pitch and surge are ignored.

2) The rolling angle in the tank is very small. Thus; sin $\phi = \phi$, and $\cos \phi = 1$

3) The nonlinear term is ignored

Thus, the governor dynamic equation of the fluid motion inside the anti-rolling flume and the moment generated by the stimulator can be expressed:

$$\Delta p = \rho \left( 2H + \frac{A_{\text{tan} k}}{A_{\text{pipe}}} L_{\text{stimu}} \right) \dot{h} + \rho L_{\text{stimu}} g s \phi + 2 \rho g h - \rho L_{\text{stimu}} \dot{y}_{\text{ship}} +$$

$$+ (\rho L_{\text{stimu}} H + \rho L_{Z, \text{CM, tan} k} L_{\text{stimu}}) \ddot{\phi} - \rho L_{X, \text{CM, tan} k} L_{\text{stimu}} \dot{\phi} +$$

$$\rho b \left( 2H + \frac{A_{\text{tan} k}}{A_{\text{pipe}}} L_{\text{stimu}} \right) \dot{h}_{\text{tan} k}$$

$$M_{\text{stimu, acc}} = -\rho \left( HA_{\text{tan} k} L_{\text{stimu}} + L_{\text{stimu}} A_{\text{tan} k} L_{Z, \text{CM, tan} k} \right) \ddot{h}$$

$$- \left( \frac{1}{2} \rho A_{\text{tan} k} L_{\text{stimu}}^2 H + \rho A_{\text{pipe}} L_{Z, \text{CM, tan} k} L_{\text{stimu}} \right) \dot{\phi}$$

$$+ \rho A_{\text{pipe}} L_{Z, \text{CM, tan} k} L_{X, \text{CM, tan} k} L_{\text{stimu}} \dot{\phi} + \rho L_{\text{stimu}} A_{\text{pipe}} L_{Z, \text{CM, tan} k} \dot{y}_{\text{ship}}$$

$$- L_{\text{stimu}} g L_{Z, \text{CM, tan} k} A_{\text{pipe}} \rho \phi - g L_{\text{stimu}} A_{\text{tan} k} \rho h$$

$$- A_{\text{pipe}} L_{Z, \text{CM, tan} k} \Delta P$$

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From the above equation, the equation of motions of the coupled model with the stimulator can be written in the matrix form: This equation can be rewritten in a matrix

\[
\begin{bmatrix}
(m+m_y -m_y x_1) & y & y & 0 \\
-y_x & (I_x + J_y) & (I_y + J_y) & 0 \\
-m & y & y & 0 \\
-\rho & L & (H + \rho A L_2) & -\rho \text{pipe} & L_2 & L_3 \end{bmatrix}
\begin{bmatrix}
y \\
\phi \\
\psi \\
\hat{h}
\end{bmatrix}
\]

3.4 Wave disturbance model

In the previous section, ship rolling and coupling model have been presented. In the practical condition, certain environmental aspects are considered to be an important factor. In this section, we will look further into details on the modeling aspects in terms of the environmental disturbances models. There are three major environmental disturbances in the marine that are to be normally considered. These disturbances are the wave function, the wind function and the ocean current. In general, these disturbances will be additive and multiplication to the dynamic equations of motion. Normally, the wave function is generated based on the wind speed and the ocean current. Thus, in this section we will focus only on the wave function and ignore the other two factors which the wind and ocean current. For most of marine application it is assumed that the
principle of the superposition can be applied. A full detailed history of the wave function formulation was presented by Dr. Fossen textbook\(^{10}\).

### 3.5.1 Generation of the sea wave

The wave disturbance is a very important factor in the marine control applications. The process of wave generation is due to wind starts with small wavelets appearing on the water surface. This will increase the drag force which, in turn, allows short waves to grow. This short wave continues to grow until they finally break and their energy is dissipated. It is absorbed that a developing sea or storm starts with high frequencies creating a spectrum with peak at relative high frequency. A storm which has been blowing for long time is said to create fully developed seas. After the wind has stopped blowing, low frequency decaying sea or swell is formed. These long waves form a wave spectrum. If the swell forms one storm interacts with the waves from another storm, a wave spectrum with two peak frequencies may be observed. For simplicity, we will only consider wave spectra with one peak frequency. Wind generated waves are usually represented as a sum of a large number of waves components.

The earliest spectral formulation is due to Neumann\(^{16}\) who proposed the one parameters spectrum. Newman\(^{16}\) presented the wave amplitude \(A_i\) of wave component; \(i\) is related to the wave spectral density function \(S(\omega_i)\) as:

\[
A_i^2 = 2S(\omega_i)\Delta\omega
\]

\(\omega_i\) is the wave frequency of wave component \(i\) and \(\Delta\omega\) is a constant difference between successive frequencies. Let the wave number of one single wave component be denoted by \(k_i\), Hence it can be represented as a function of the wave length \(\lambda_i\).
Figure 3.2 shows the wave elevation of long crested irregular sea propagation along the positive x-axis can be written as a sum of wave components.

Since the 1950, the standard wave spectrum are presented in so many different forms based on an experimental testing in the ocean environment. In this work, the Pierson-Mokowitz spectrum is selected to be used to generate the wave disturbance in irregular sea wave condition.

In 1963, the Pierson and Mokowitz developed a wave spectral formulation for a fully developed wind–generated seas from analyses of wave spectra in North Atlantic Ocean. The Pierson-Mokowitz spectrum is written:

\[
S(\omega_i) = A \omega^{-5} \exp(-B \omega^{-4}) \quad (m^2/s) \quad (3.19)
\]

Where \(A\) and \(B\) can be represented as a function of the gravity constant and the wind speed \(V\) or the wave height \(H_s\).

\[
A = 8.110^{-3} g
\]

\[
B = 0.74\left(\frac{g}{V}\right)^4 \quad \text{or} \quad B = 0.032\left(\frac{g}{H_s}\right)^2 \quad (3.20)
\]

The wave slope for wave component \(i\) can be computed according to

\[
s_i = A_i K_i \sin(\omega_{ei} t + \phi_i) \quad (3.21)
\]

Where \(\omega_{ei}\) is the encounter frequency corresponding to wave component \(i\).
\[ \lambda = \text{wavelength} \]

H = Wave height
A = amplitude

Figure 3.2: Characteristic of wave traveling

Figure 3.3: Wave Spectrum with one peak

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3.5.2 Excitation force and momentums of the wave

In order to simulate the motion of ocean vehicles in the presence of irregular waves, the 1st order wave disturbances will be presented. The response of an ocean vehicle in a seaway is usually computed by applying the principle of the superposition. Each force or moment acting on the ship will be considered individually. If we consider the wave slope $s_i$ for a wave component the wave disturbance can be formed. Thus, the wave disturbance in the coupling model of the sway, roll and sway will be calculated as:

\[
F_{\text{sway}} = \sum_{i=1}^{N} -\rho g B L T \sin \beta s_i(t) \tag{3.22}
\]

\[
M_{\text{sway}} = \frac{1}{24} \rho g B (L^2 - B^2) \sin 2\beta s_i^2(t) \tag{3.23}
\]

Where, $\beta$ is the angle between the ship heading and the direction of the wave or the wave attack angle. The symbol $s_i(t)$ is the wave slope which can be computed by:

\[
s_i(t) = A K_i \sin(\omega_c t + \phi_i) \tag{3.24}
\]

Where $\omega_c t$ is the encounter frequency corresponding to the wave component $i$.

The wave disturbance of the roll angle can be computed by:

\[
M_{\text{roll}} = \frac{2}{3} \rho g s_i(t) \sin(\beta) \tag{3.25}
\]

It seems from the equations that the wave is generated based on the ship geometry and the wave attack angles. The most important part of the three equations is the angle between the ship heading and the direction of the wave or the wave attack angle. Figure 3.4 shows a definition of the ship’s heading angle. The value of the wave heading angle, $\beta$, is changing from 0 to 180 degrees.
3.5.3 Sea state conditions

Sea state conditions are considered to be one of the major effects on the ship motion in the seaway. The sea state conditions change based on the weather environment. One of the major effects on the state condition is the wind velocity which affects the wave height. There is a constant relationship between the wave height and the wind speed which is expressed on equation (3.20). The sea state code is classified to ten different levels starting from 0 until ten. The sea state code zero represents a calm sea whereas sea state 9 represents a phenomenal sea state level. Table 4.1 (see appendix) shows all the sea state codes based on the definition represented by the price and Bishop in 1974. The percentage probability for sea states 0, 1 and 2 are summarized\(^\text{10}\). In this dissertation, different sea state levels will be considered.
4.1 Introduction

Generalized Predictive Control (GPC) is just one of the techniques of the Model Predictive Control (MPC). MPC originated in the late seventies and since then has developed considerably. It was first introduced in the chemical industries to control the chemical processes. Predictive controllers were introduced in the chemical industries for controlling chemical process and have found applications in a wide variety of industrial processes. Predictive control refers to a strategy wherein the decision for the current objective function that involves the prediction of the system is expressed in term of the response at some number of time steps into the future. One of the definitions of the Model Predictive control is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The concept of the predictive control was introduced simultaneously by Richalet, Cutler, and Ramaker in the late seventies. Predictive control belongs to a class of design concepts. GPC is considered to be one of the useful controller methods for designing the controller.

GPC has made a significant impact on industrial control engineering. Recently, it has been applied in other sectors of process industry. According to J.M Maciejowski, the main reasons for its success in these applications are its ability to be applicable in engineering designs and research. GPC handles multivariable control problems easily. It takes into account actuator limitations and allows operation closer to constraints, which frequently leads to more profitable operations. Control update rates are relatively low in these applications, so that there is a sufficient time for necessary on-line computations.
The theory of the Model Predictive Control is easy to understand and can be applied to work in the real practices situations.

A variety of MPC controllers have been proposed like GPC, DPC, LPR, DMC, and MMAR. All MPC algorithms possess common elements and different options can be chosen for one of these elements:

- Prediction Model.
- Objective Function.
- Obtaining the Control law.

All MPC algorithms share the same philosophy, but they are different in their details.

4.2 Fundamental of the GPC

The classical papers which present Generalized Predictive Control (GPC) are references [27] and [28]. In Ref [27], the basic theory and algorithm of GPC is presented along with some interpretations. In Ref. [28], the interoperations are expanded and additional filters are introduced into the GPC algorithm and their use is explained. In Ref. [31], a GPC algorithm is presented with guaranteed theoretical stability, although this algorithm is rather computationally intensive for adaptive control. In Ref. [30], the state space derivation of deadbeat predictive control is presented. The deadbeat predictive controller is further developed to include an extended horizon and presented in both state space and polynomial form.

In order to design the controller, an input/output model of the plant must be identified. This model is the transfer function from the actuator to the error sensor.
Typically, methods of system identification are used to determine the parameters of this model. Theses methods include batch least squares and recursive least square. Generally, any system identification technique can be used which will return an auto-Regressive with exogenous input (ARX) model of the plant. Based on the ARX model obtained using a system ID technique, a controller can be designed.

Several computational algorithms are presented to compute the Model Predictive Control. Generalized Predictive Control (GPC) may be used to regulate a plant based on an identified model. Another technique for achieving control is Deadbeat Predictive Control (DPC). For both GPC and DPC, a finite difference model is used. The form of this model, commonly called ARX, is shown below

$$y(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \ldots + \alpha_p y(k-p) + \beta_0 u(k) + \beta_1 u(k-1) + \ldots + \beta_p u(k-p)$$

It is the task of the system identification technique to produce estimates of $\beta_j$ and $\alpha_j$ where $j = 1, 2 \ldots p$ and $p$ is the ARX model order. The ARX Model is used to design the controller and leads to a control law.

The Generalized Predictive Control (GPC) may be used to regulate a plant based on an identified model. This control technique may be tuned to the desired balance between performance and robustness. If an increased level of performance is desired, the control penalty may be decreased as long as stability is maintained. On the other hand, if a more robust controller is desired, the controller penalty may be increased at the expense of the performance.

The Deadbeat Predictive Control (DPC) is similar to the GPC in that it is a receding horizon controller but differs from the GPC in that the current control output is
designed to be zeros. Also, it does not require a weight factors to be tuned. It is assume that the weighting value is already considered. The prediction horizon is assumed to equal or greater than the system order.

Generalized Predictive Control was introduced in 1987 and has received a notable attention by researcher. GPC is a time domain multi-input – multi-output (MIMO) predictive control method that uses a linear difference equation to describe the input-output relationship of the system. The input-output equation is used to form a multi-step output prediction equation over a finite prediction horizon while subject to control imposed over finite control horizon. The control to be imposed at the next step is determined by minimizing the deviation of predicted controlled plant outputs from the desired (or target) outputs, subject to a penalty on control effort.

The essential feature of the adaptive control process used in the present GPC investigation is depicted in the Figure (4.2). The system has r control inputs u, m measured output y, and is subjected to unknown external disturbance d. There are two fundamental steps involved:

1) Identification of the system.
2) Use of the identified model to design a controller.

A linear differential equation or model is used to describe the relationship between the input/output measurements. The input - output model used in the present work takes the form of what is referred to as an ARX (autoregressive with exogenous input) model. The ARX model is used for both system identification and controller design. System identification is done online but not at every time points in the presence of a disturbance that acting on the system. In this way, an estimate of the disturbance
model is reflected in the identified model separately. This approach represents a case of feedback with embedded feedforward control parameters. There is no need for measurement of the disturbance information embedded in the feedforward the disturbance signal.

The parameters of the identified model are used to compute the predictive control law. A random excitation $u_{id}$ (sometimes called dither) is applied initially with $u_c$ equal to zero to identity the open-loop system. Dither is added to the closed-loop control input $u_c$ If it is necessary to re-identify the system while operating in the closed-loop model.

![Figure 4.2 Closed loop system of GPC](image-url)
4.3 System Identification

System identification in the presence of the operational disturbances acting on the system is the first of the two major computational steps. The external disturbances acting on the system are assumed to be unknown (immeasurable). The number of control inputs is \( r \) and the number of measured outputs is \( m \). The system is excited with band-limited white noise for SID. These random excitations are input to all \( r \) control inputs simultaneously and \( m \) responses are measured. The digitized input and output time histories \((u \text{ and } y)\) at \( l \) time points are then used to form the data matrices \( y \) and \( V \) in the equation.

\[
y = yV
\]

\[
y = \begin{bmatrix} y(0) & y(1) & y(2) & \ldots & y(p) & \ldots & y(l-1) \end{bmatrix}
\]

And

\[
V = \begin{bmatrix} u(0) & u(1) & u(2) & \ldots & u(p) & \ldots & u(l-1) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

Equation 4 and 5 follow from writing the discrete-time state space equations for a linear time-invariant system at a sequence of time steps \( k=0,1,2,\ldots,l-1 \) and grouping them into matrix form. The vector \( v(k) \) appearing in the data matrix \( V \) is formed from the vectors \( u(k) \) according to:
\[ v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix} \quad (4.4) \]

The order of the ARX Model \((p)\) and the number of time steps \((l)\) must be specified by the user. Some guidelines for their selection are given later. The sizes of the vectors are noted.

In forming the matrices given in equation 4 and 5, it has been assumed that the state matrix \(A\) is asymptotically stable. The SID process yield OMP rather than system Markov parameters (SMP), because of the inclusion of an observer. A complete Discussion of these aspects of the development may be found in reference \([\ldots]\).

\[ \bar{Y} : \text{is the observer Markov parameters matrix that is to be identified and has the form} \]
\[ \bar{Y} = \begin{bmatrix} \beta_0 & \beta_1 & \alpha_1 & \beta_2 & \alpha_2 & \beta_3 & \alpha_3 & \ldots & \beta_p & \alpha_p \end{bmatrix} \quad (4.5) \]

The solution for \(\bar{Y}\) is obtained by solving the equation for \(\bar{Y}\) according
\[ \bar{Y} = \nu \nu^\top = \nu \nu^\top \left[ \nu \nu^\top \right]^{-1} \quad (4.6) \]

4.4 Multi-step Prediction

The input output relationship of a linear system, even a nonlinear system, is commonly described by a finite difference model. Given a system with \(r\) inputs and \(m\) outputs, the finite difference equation for the \(r \times 1\) input \(u\) \((k)\) output \(y\) \((k)\) at time \(k\) is

\[
y(k) = \alpha_0 y(k-1) + \alpha_1 y(k-2) + \ldots + \alpha_p y(k-p) + \beta_0 u(k) + \beta_1 u(k-1) + \beta_2 u(k-2) + \ldots + \beta_p u(k-p) \]

This simply means that the current output can be predicted by the past input and output time histories. The finite difference model is also often referred as the ARX model.
where AR refers to the Autoregressive part and X refers to the exogenous part. The coefficient matrices, $\alpha_i$ (i=1, 2, 3 ...p) of $m \times m$ and $\beta_i$ (i=0, 1, p) of $m \times m$ are commonly referred to as the observer Markov parameters (OMP) or the ARX parameters. The matrix $\beta_0$ is the direct transmission term. By shifting a time step, one obtains

$$y(k+1) = \alpha_1 y(k) + \alpha_2 y(k-1) + \ldots + \alpha_p y(k-p+1) + \beta_0 u(k+1) + \beta_1 u(k) + \beta_2 u(k-1) + \ldots + \beta_p u(k-p+1)$$

(4.7)

Define the following quantities

\[\begin{align*}
\alpha_1^{(1)} &= \alpha_1 \alpha_1 + \alpha_2 \\
\alpha_2^{(1)} &= \alpha_1 \alpha_2 + \alpha_3 \\
&\vdots \\
\alpha_p^{(1)} &= \alpha_1 \alpha_p \\
\beta_0^{(1)} &= \alpha_1 \beta_0 + \beta_1 \\
\beta_1^{(1)} &= \beta_1 \alpha_1 + \beta_2 \\
\&\vdots \\
\beta_p^{(1)} &= \alpha_1 \beta_p
\end{align*}\]

(4.8)

Substituting $y(k)$ from Eq. into Eq. (2) yields

$$y(k+1) = \alpha_1^{(1)} y(k-1) + \alpha_2^{(1)} y(k-2) + \ldots + \alpha_p^{(1)} y(k-p) + \beta_0^{(1)} u(k+1) + \beta_1^{(1)} u(k) + \beta_2^{(1)} u(k-1) + \ldots + \beta_p^{(1)} u(k-p)$$

(4.9)
The output measurements at time step \( k+1 \) can be expressed as the sum of the past input and output data with the absence of the output measurement at time step \( k \). By induction, one may be express the output measurement at time step \( k+1 \)

\[
y(k + j) = \alpha_1^{(j)} y(k - 1) + \alpha_2^{(j)} y(k - 2) + \ldots + \alpha_p^{(j)} y(k - p) \\
+ \beta_0 u(k + j) + \beta_1^{(j)} u(k + j - 1) + \ldots + \beta_p^{(j)} u(k) \\
+ \beta_1^{(j)} u(k - 1) + \beta_2^{(j)} u(k - 2) + \ldots + \beta_p^{(j)} u(k - p)
\]

(4.10)

Where

\[
\begin{align*}
\alpha_1^{(j)} &= \alpha_1^{(j-1)} \alpha_1 + \alpha_2^{(j-1)} \\
\alpha_2^{(j)} &= \alpha_1^{(j-1)} \alpha_2 + \alpha_3^{(j-1)} \\
&\vdots \\
\alpha_{p-1}^{(j)} &= \alpha_1^{(j-1)} \alpha_{p-1} + \alpha_p^{(j-1)} \\
\alpha_p^{(j)} &= \alpha_1^{(j-1)} \alpha_p
\end{align*}
\]

\[
\begin{align*}
\beta_1^{(j)} &= \alpha_1^{(j-1)} \beta_1 + \beta_2^{(j-1)} \\
\beta_2^{(j)} &= \alpha_1^{(j-1)} \beta_2 + \beta_2^{(j-1)} \\
&\vdots \\
\beta_{p-1}^{(j)} &= \alpha_1^{(j-1)} \beta_{p-1} + \beta_{p-1}^{(j-1)} \\
\beta_p^{(j)} &= \alpha_1^{(j-1)} \beta_p
\end{align*}
\]

(4.11)

Note that \( \alpha_i^{(0)} = \alpha_i \), and \( \beta_i^{(0)} = \beta_i \) for any possible integer \( 1, 2 \ldots \)including 0 if applicable. With some algebraic operation, Eq.(8) can also be expressed by

\[
\begin{align*}
\beta_0^{(0)} &= \beta_0 \\
\beta_0^{(k)} &= \beta_k + \sum_{i=1}^{k} \alpha_i \beta_0^{(k-i)} \quad \text{for} \quad k = 1, \ldots, p \\
\beta_0^{(k)} &= \sum_{i=1}^{k} \alpha_i \beta_0^{(k-i)} \quad \text{for} \quad k = p + 1, \ldots, \infty \\
\alpha_0^{(0)} &= \alpha_1 \\
\alpha_1^{(k)} &= \alpha_{k+1} + \sum_{i=1}^{k} \alpha_i \alpha_1^{(k-i)} \quad \text{for} \quad k = 1, \ldots, p - 1 \\
\alpha_1^{(k)} &= \sum_{i=1}^{p} \alpha_i \alpha_1^{(k-i)} \quad \text{for} \quad k = p, \ldots, \infty
\end{align*}
\]

(4.12)

(4.13)
Observation of Eq. (9) and (10) reveals that $\beta_s^{(j)}$ and $\alpha_i^{(j)}$ for $J > p$ is a linear combination of its past $p$ parameters weighted by the parameters $\alpha_1, \alpha_2, \ldots, \alpha_p$. This property is very useful in developing predictive control design. The quantities $\beta_0^{(i)} (i = 0, 1, \ldots)$ are, in fact, the pulse response sequence which will be shown later. On the other hand, the quantities $\alpha_i^{(i)} (i = 0, 1, \ldots)$ are the observer gain Markov parameters which can be used compute an observer for state estimation.

Let the index $j$ be $j = 1, 2, \ldots, q, q+1, \ldots, s-1$ Equation (7) produces the following matrix equation.

$$y_s(k) = u_s(k) + \beta u_p(k - p) + Ay_p(k - p)$$  \hspace{1cm} (4.14)

Where:

$$y_s(k) = \begin{bmatrix} y(k) \\ y(k + 1) \\ \vdots \\ y(k + q) \\ y(k + s - 1) \end{bmatrix}, \quad u_s(k) = \begin{bmatrix} u(k) \\ u(k + 1) \\ \vdots \\ u(k + q) \\ u(k + s - 1) \end{bmatrix}$$  \hspace{1cm} (4.15)

$$y_p(k - p) = \begin{bmatrix} y(k - p) \\ y(k - p + 1) \\ \vdots \\ y(k - 1) \end{bmatrix}, \quad u_p(k - p) = \begin{bmatrix} u(k - p) \\ u(k - p + 1) \\ \vdots \\ u(k - 1) \end{bmatrix}$$  \hspace{1cm} (4.16)

The quantity $y_s(k)$ represents the output vector with total of $s$ data points for each from the time step $k$ to $k+s-1$, whereas $y_p(k - p)$ includes the $p$ data from $k-p$ to $k-1$. 

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4.5 Derivation of the control Law

The Predictive control law is obtained by minimizing the deviation of the predictive controlled response (as computed from multi-step output prediction equation) from a specified target response over a prediction horizon $h_p$. To find this end, one first defines an error function that is the difference between desired (target) response $y_T(k)$ and Predicted response $y_{hp}(k)$:

$$
\varepsilon = y_T(k) - y_{hp}(k) = y_T(k) - u_{hp}(k) - \beta u_p(k-p) - Ay_p(k-p)
$$

(4.17)

An objective function $J$ quadratic in the error and the unknown future controls is then formed

$$
J = \varepsilon^T R \varepsilon + u_{hc}^T Qu_{hc}
$$

(4.18)
Two weighting matrices are included in the objective function. Q (symmetric and positive definite) is used to weight the control effort and stabilize the closed-loop system. R (symmetric and positive semi-definite) is used to weight the relative importance of the differences between the target and predicted response. Typically, Q and R are assumed to be diagonal and for Q to have the same values $w_c$ along its diagonal and R to have the same value $w_r$ along its diagonal. Minimizing $J$ with respect to $u_{hc}(k)$ and solving for $u_{hc}(k)$ gives

$$u_{hc}(k) = -(\tau_c^\top R \tau + Q)^{-1} \tau_c^\top R(-y_T(k) + Bu_p(k-p) + Ay_p(k-p))$$

(4.19)

$$u_{hc}(k) = \gamma (-y_T(k) + Bu_p(k-p) + Ay_p(k-p))$$

The control sequence is applied to the system over next $h_c$ time steps. However, only the first $r$ values are applied to the $r_c$ control inputs, the remainder are discarded, and a new control sequence is calculated at the next time step.

$$u_c(k) = \text{the first } r_c \text{ rows } [\gamma (-y_T(k) + Bu_p(k-p) + Ay_p(k-p))]$$

(4.20)

The prediction and the control horizon are set to be according to these relations

$$h_p \geq p \quad h_c \leq h_p$$

In some engineering book the value of Q can be expressed to be $\Lambda \lambda$. In this dissertation the value of Lambda will be considered to weight the cost function. It is very important to consider the right values for Lambda that optimize the performance index.
CHAPTER V

SIMULATION OF THE NUMERICAL MODEL

5.1 Introduction

In this chapter, the numerical model for a single container ship coupled in a sway, roll and yaw is presented. The linear model for the ship is presented according to the data of a large container. The linear equation of motion of the ship in the sway, roll and yaw is used. The non dimensional data of the container ship is normalized to fit in our control application. It is assumed that the ship is in a stationary position. The numerical simulation in this chapter is concentrated on the ship coupling model without a controller. The result in this chapter simulates the container ship in different sea state levels and different wave attack angles.

5.2 The Ship Numerical Model.

The numerical parameters of the ship are used based on table 5.1 which is extracted from an experimental data presented by Son and Nomoto\(^ {18-19}\). Also, the dynamic parameter of the selected ship is used based on testing of ship models which are explained more in Fossen book’s\(^ {10}\). The data given in this section are non dimensional data that need to be normalized in order to simulate the ship reality. The non dimensional parameters have to be changed to a dimensional data such that the model parameters can be treated as corresponding with stimulator parameters.

In the ship designing system, there are three normalization forms for ship equation of motion which are the prime-system I, the prime system II and the Bis-system. The most commonly used normalization form for the ship equation of motion is the prime
system I, which is selected in our ship numerical model. The normalization variable for the prime system and Bis-system is presented in table 5.3 (see appendix). The normalization technique is used to change the in table 5.2 (see appendix) to be in a dimensional form.

By recalling the state space form of the ship model in chapter three; the normalized numerical state space model of the ship coupling model is:

$$A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & -0.0120 & 0 & -0.0031 & -0.0132 & 0.0108 \\
0 & -0.00430 & 0 & 0.0003 & -0.0028 & -0.0817 \\
0 & -0.003 & 0 & -0.0001 & -0.0014 & -0.0145
\end{bmatrix}$$

$$B = 10^{-7} \begin{bmatrix}
0 \\
0 \\
0 \\
0.2983 & 0.0179 & -0.0007 \\
0.0179 & 0.0069 & -0 \\
-0.0007 & -0 & 0.0001
\end{bmatrix}$$

$$C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

; $D= [Zeros]$
From the numerical state space model one can analyze the ship model. Figure 5.1 shows the mapping of the poles of the system in continuous form. The figure indicates that there are 2 poles in the original axis which means that the system stability is undefined. There are 2 poles that are close to the imaginary axis which mean that the system has faster response and less damping ratio. Our objective is to stabilize the system to be asymptotically stable.
5.3 Ship rolling model in different sea state level

This part of the simulation is concerned with the effect of sea state levels on ship behaviors. As explained in Chapter three, a sea state level is based on wind speed. The ship roll angle is increase when the sea state level is increase. For example, in Figure 5.3 at sea state level 3 which is consider to be slight weather; the ship roll angle oscillates and reaches up to 15 degrees. Because of the lack of controller or actuator the ship oscillating constantly. At the same time, in a rough weather like sea state 5, the rolling angle will increase up to 30 degrees. It is considered a normal phenomenon due to the wind speed and waving height.

![Figure 5.3 ship rolling angle in sea state 3](image-url)
5.4 Numerical simulation of the ship coupling model.

The next subsection will show the simulation of the ship coupling model under the excitations of the wave disturbance. The main objective of that simulation is to understand ship’s behaviors when it is excited under different attack angle. The wave’s excitation forces and moments used in this simulation are based on the mathematical equation which is introduced in Chapter 4. By recalling that equation it is assumed that:

1) Sea state level is considered to be 5. This is considered to be a rough condition as it shows in table 4.1. The wave range height is between 2.5 to 4 meters. The main reason for selecting this level is because the size of the container ship is large compared to the other marine vehicles.
2) The range of the wave attack angles are changing from 30-180 degrees. As an example, when the wave attacks the ship from the front, it is considered being zero attack angles. When the wave attacks the ship perpendicular from the side, it is considered to be 90 attack angles.

3) The disturbances wave is considered to be an irregular wave disturbance.

4) The sampling time of the simulation is set to be 0.2 seconds.

In this chapter the numerical simulation is implemented based on those factors and playing with the input angle of the wave attack angle. Then, the ship response in sway, roll and yaw were simulated. In order to identify which one of these three disturbances has a major impact on the ship response, the wave angles were changed and some of the wave disturbances were ignored in some part of the simulation.

5.4.1 Ship coupling model under the excitation of sway, roll and yaw

Figure 5.5 through Figure 5.11 represent the numerical simulation for the container ship in a different wave angle ($\beta$), and the excitation force and momentum generated by the sea waves. The wave excitation contains the force acts on the sway, and moments act on the roll and yaw. The reaction of the ship is different based on the wave attack angle. For example, in Figure 5.5 the ship sway displacement moves from zero to 120 meters within 800 seconds. At the same time the rolling angle is excited and reaches 40 degrees. The yaw is rotated to 60 degrees and back again to the 30 degree angle. However, this is not the case when the wave attacks the ship at 90 degree angle. At this angle, the sway displacement reaches the optimal movement and so does the roll angels as shown in Figure 5.7.
In the remaining figures, the wave attack angles have increased which means the wave excitations are changed. The maximum rolling angle occurs at 90 degree angle. Also, the minimum wave attack angles that almost have no impact on the ship are angle zeros or 180. When the waves attack the ship at zero degree angle, the ship will response as if it hit at 180 degree angle. This is a normal response which relates to the geometry or a curve shape of the ship, which plays a major role on the wave excitation.

The sway displacement responds faster than the roll or the yaw angle. Also, the yaw angle response is based on the attack angle, and rotates back to its same position. The rolling angle, which is our main concern, needs to be damped faster, which enhances the idea of adding the stimulator. The maximum rolling angle at sea state 5 is almost 30 degrees as shown in Figure 5.6.

![Figure 5.5 Ship response in sway, roll and yaw when $\beta =30$, sea state 5](image-url)

Figure 5.5 Ship response in sway, roll and yaw when $\beta =30$, sea state 5

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Figure 5.6 Ship response in sway, roll and yaw when $\beta = 60$, sea state 5

Figure 5.7 Ship response in sway, roll and yaw when $\beta = 90$, sea state 5
Figure 5.8 Ship response in sway, roll and yaw when $\beta = 120$, sea state 5

Figure 5.9 Ship response in sway, roll and yaw when $\beta = 150$, sea state 5
Figure 5.10 Ship response in sway, roll and yaw when $\beta = 180$, sea state 5

Figure 5.11 Ship responses in different wave attack angles $\beta$
Figure 5.11 summarizes the ship response behavior in the sway, roll and yaw. The x-axis represents the attack wave angle in different degrees and y-axis represents the sway displacement in meters, yaw angles and roll angles in degrees. Figure 5.11 selected the maximum response from the previous simulation. It seems that the wave disturbance is critical when it attacks the ship from the 90 degree angle. Thus, our next simulation will be concerned within these wave angles. It should be understood which one of the wave disturbances has a major impact on the ship response. Therefore, the simulation in the next subsection will answer this question.

### 5.4.2 Effective of the wave disturbance

Because we dealing with the coupling model, the ship response will be different than if it is uncoupled. Normally, the coupling model has an impact on each other. The numerical simulation in the previous section does not explain which one of sway and yaw disturbance has an impact on the roll angle. In order to understand this, the sway or the yaw disturbances will be ignored and repeated the simulation will be repeated, which is sea state 5, and the wave will attack the ship at a 90 angle degree.

Figure 5.12 shows the wave disturbance that is generated by the yaw and roll only. The response of the ship is different than in the previous section in Figure 5.7. Even though there is no sway disturbance, the sway reacts to the other disturbances generated by the yaw and roll. Of course, it has less impact than Figure 5.7, but this would not be the case in an uncoupled model. Also, the roll angle has less rolling than before. The yaw response has not changed at all.
In the same concept, we ignored the yaw disturbance in Figure 5.13. At this time, there is no major effect in the roll angle or the sway displacement. The yaw angle responded steadily which is different than Figure 5.7. From these two figures, one can conclude that the sway disturbance plays a major effect in the simulation. Also, the coupling parameters of the sway roll affect each other more than the yaw and roll. One may notice that the roll angle is damped faster in Figure 5.12, which results in an opposite moment generated by the yaw disturbance.
Figure 5.12 the ship coupling model under the effect of the yaw and roll

Figure 5.13 the ship coupling model under the effect of the sway and roll

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CHAPTER VI

SHIP ROLL MITIGATION USING GPC OF COUPLING MODEL

6.1 Introduction

This chapter provides the ship roll mitigation using passive control and the Generalized Predictive Control (GPC). The GPC controller is used for the ship model and the result is simulated. Before the controller is designed, certain steps must be considered. The anti-rolling flume tank (stimulator) that suitable for the ship model must be selected. The combined numerical model of the stimulator and ship coupling model is presented in state space form. The system identification approach is applied to calculate the Observer Markov Parameter (OMP). The Generalized Predictive Control (GPC) is applied to calculate the control gain input matrix. This chapter presents the numerical results and the simulation of the passive and the active GPC control system. A comparison among uncontrolled, passive and active system is given. The controller performance is studied and discussed.

6.2 Passive Control system

The passive control system is the anti-rolling flume tank which will shift the water to both sides of the tank when the pump shuts off. The passive type anti-rolling flume tank is the most economical type available without a control system. The passive type of anti-rolling flume tank reduces rolling by shifting the liquid in the tank naturally. Instead of controlling the period of movement of the liquid in the tank, it uses the difference in height of the left and right sides of the hull produced by the rolling of the ship.
The parameters of the anti-rolling flume tank have to be defined. The anti-rolling flume tank can be designed effectively with some understanding of the ship’s basic parameters and the magnitude of the rolling angle to be induced. The ship’s natural frequency is one of the most important factors that need to be considered. It is very important to know this in order for it to be stabilized. The ship’s natural frequency must be close to the stimulator’s natural frequency so that the stimulator is able to generate momentum, which cancels the rolling motion. Stabilization of the ship is critical when the natural frequency of the anti-rolling flume tank is about to be the same as the frequency of the waves. Normally, the frequency of the waves of the sea will not be the same as the ship’s natural frequency.

From the bode plot of the ship roll model shown in Figure 6.1, the ship roll natural frequency is \( \omega_n = 0.207 \) (rad/sec). This means that the ship rolling period is

\[
T_r = \frac{2\pi}{\omega_n} = 30.35 \text{ seconds.}
\]

The stimulator natural frequency can be found to be:

\[
\omega_{n\_stimu} = \frac{(2g)^{1/2}}{\left(2H_{\text{tank}} + \frac{A_{\text{tank}}}{A_{\text{pipe}}}L_{\text{stimu}}\right)^{1/2}}
\]

From that equation we can identify the dimension of the anti-rolling flume tank. Table 6.1 (see appendix) shows the numerical values of anti-rolling flume tank. This data is assumed to be used for 2 anti-rolling flume tanks installed on the ship. It is assumed that the anti-rolling flume tanks are installed to be in parallel position located in the center of the ship.
Figure 6.1 shows the bode plot of the ship rolling model

6.2.1 Numerical model of the passive system

By recalling the mathematical models in chapter three and using the ship and stimulator data, the numerical model of the passive system is presented. The state parameters are increased to eight parameters. The new parameter is the height of the water head in the tank. The ability to read all the state output will be based on the number of sensors. Because the roll angle is the main objective to be reduced, a gyroscope sensor is used to read the output angle and a pressure sensor reads the water height in the tank. The numerical state space model can be found to be:
The passive system can be analyzed by mapping the poles and zeros of the single input single output system open loop system. The poles' mappings are presented on Figure 6.2 which shows that all poles are shifted to the left side of the imaginary axis. This indicates that the model has more stability than it did before adding the stimulator. There are two poles that are still in the original axis which means that the passive controller does not affect all the system parameters.
6.2.2 Simulation of the passive control system

This section illustrates the effect of the passive control system in the roll angle and the other coupling motions of the sway and yaw. The passive control system demonstrates an effective result for reducing roll angle. On the other hand, there was no major effect of passive system to the yaw and sway. Based on our previous simulation, realized that the maximum roll excitation will be at a wave attack angle of 90 degrees. The same simulation has been repeated, but this time with the existence of the anti-rolling flume tank.

Figure 6.3 shows a comparison result between the passive control system and uncontrolled system of the rolling angle. The figure shows the wave disturbance models...
that excited the ship at a 90 degree angle. The rolling angle at a passive control system is overshooting less and damping faster than the uncontrolled system. The water head in the anti-rolling flume tank is moving to each side by 5 meters. This means that the anti-rolling flume tank can work effectively as passive roll mitigation.

Figure 6.4 demonstrates the coupling models when the passive control system is used. The sway displacement is moving in the other direction up to 200 meters and the yaw angle is overshooting and returning to the same position. It seems that there are no major impacts on the sway and yaw as it does for the roll motion.

The stabilization of the roll motion is guaranteed with the passive system. The only comment on the passive system is the overshooting and the time to be settled. It appears that the roll angle is overshooting for a while and then damping after 400 seconds. This problem can be solved by using the controller or the active controlled system. By considering the GPC, the water will pump to the tank immediately and the roll angle will be damped faster than if it is passive. In this case, the control and the time consuming effort will be less. The anti-rolling flume tank will be optimal when the pump is turned on, so that the water will move from one side to another. The pump in this situation functions based upon the GPC control system that will turn the valve on and off each time.
Figure 6.3 Ship roll angle with water with and without stimulator.
Figure 6.4 Sway, roll and yaw with and without passive control.
6.3 Active control system using GPC

The active control system uses GPC methods to compute the predictive control law. The GPC is based on the relationship between the input and output digitized histories of the system know as ARX model. The coefficients of that model are Observer Markov parameters (OMP). Once the OMP is calculated the multistep output prediction equation will be constructed over a prediction horizon. In order to start the GPC algorithm, certain assumptions have to be considered. The system is considered to be a single input single output system (SISO) which means r and m is set to be one. The disturbance input is set to be a random signal. The system order is p = 8, and the control horizon \( h_c \) is set to be equal to the prediction horizon \( h_p = 20 \). The Weighting factor R and \( \lambda \) will be tuned in order to have the optimized control performance.

6.3.1 Observer Markov Parameters

In this part, the generalized predictive control is applied to the ship combined model. In order to design the control gain, the OMP has to be calculated. The system identification technique is applied to determine the OMP. The OMP can be defined as the coefficient of the ARX model based on the input and output data. If the OMP of the system is known, the future outputs may be predicted with a recursive relationship which is explained in chapter four.

In the ship model we have to deal with the system input data as a random signal, so we could simulate our output signal. Figure 6.5 shows the norm and variance of the prediction error. The first figure indicates that prediction error between the ship model parameters and the prediction parameters during the time histories. It seems that the prediction error is very small which indicates that it is close to the true system. The
second figure shows the variance of each of the parameters. In this case, the system order is \( p = 8 \) which means that we expected to have 16 parameters that need to be identified. The variance is changing from 1.5 to 2.5 to each parameter. Thus, by solving the OMP, the multi-step prediction equation can be found.

The system observer parameters for a single input single output system are:

\[
\beta = 1.0 \times 10^{-9} \times \begin{bmatrix} 0 & 0.0352 & -0.1745 & 0.3108 & -0.1689 & 0.1756 & 0.3098 & -0.1708 & 0.0340 \end{bmatrix}
\]

\[
\alpha = \begin{bmatrix} 1.0000 & -7.9545 & 27.6928 & -55.1111 & 68.5729 & -54.6271 & 27.2087 & -7.7470 & 0.9654 \end{bmatrix}
\]

![Graph showing norm and variance of prediction error](image)

![Bar chart showing variance](image)

Figure 6.5 the norm and variance of the prediction error
6.3.2 Tuning the weighting factor

One of the best features of the GPC is the optimization of the cost function. In this part of the result, the tuning of the weighting factors will be considered. The system is simulated with a sampling time of 0.2 per second. The Weighting factor $\lambda$ is changing from $0.1$ to $2$. The weighting factor $R$ is fixed to be $4$. The values of $\lambda$ are supposed to be bigger than zero. If the value of lambda $\lambda$ is equal to zero that means the system will be unstable which indicates the prediction matrix is poor. The more the lambda increases the more the system will be stable. The higher the value of lambda, the more stable the system is. At the same time, we do not want lambda to be higher than it should, so the system may responded in an unrealistic manner.

In fact, the value of lambda has to be tuned between minimum and maximum values. The simulation result shown in figure 6.6 through figure 6.8, illustrate the system response with different weighting factors. The reason the weight factor $R$ is set to be fixed, is because it does not have as big an impact on the simulation as the lambda does. Figure 6.6 shows that the system responded faster with an overshooting of the rolling angle up to 3 degrees. Thus, when value of $\lambda$ is increased the rolling angle is decreased as shown in Figure 6.7 and Figure 6.8.

Once the weighting factor is tuned, the ship roll model is simulated under a random wave excitation as it shown in Figure 6.10. The figure shows that the roll angle is cancelled due to the effect of the new controller. Also, the figure illustrates the water head in the anti-rolling flume tank.
Figure 6.6 Controlled ship roll angle at $\lambda=0.2$, $R=4$

Figure 6.7. Controlled ship roll angle at $\lambda=0.3$, $R=4$
Figure 6.8. Controlled ship roll angle at $\lambda=1$, $R=4$

Figure 6.9. Controlled ship roll angle at $\lambda=0.2$, $R=7$
Figure 6.10 ships roll angle under the wave excitation and water head in the tank with the GPC controller.
6.3.3 Numerical simulation of the coupling model using GPC

In this section, the numerical simulation for the ship coupling model will be resimulated using the GPC. The sea state conditions will be changed from level 3 to 4 in order to prove the effectiveness of the controller. The control horizon is set to be equal to the predicting horizon is $h_p = h_c = 30$ and the weighting factors are $R=4$ and $\lambda=1$. The system order is $p=8$ and is considered to be a single input and multioutput system with three wave disturbances. Because the GPC is based on the data history, the control input will be more accurate. The closed loop system of the coupling model is controlling the input data generated from the momentum in the anti-rolling flume tank. Therefore, the control input is considered to be the input signals that generate the pressure input.

The controlled system is tested under different sea state conditions. Figure 6.11 through 6.13 shows that the rolling angle is almost canceled under the wave disturbances. As sway displacement decreases, so does the yaw angle. Figure 6.14 shows a comparison among the uncontrolled system, passive system and active controlled system in sea state 5. The comparison figure shows that the rolling angle is oscillated during the time history of the simulation. The passive has cancels the roll angle with an overshooting 25 degrees and a settling time of 200 seconds. The GPC controller has mitigated the roll angle with an overshooting of 3 degrees and a settling time of 50 seconds. As illustrated in the figures, the sway and the yaw have not been affected by the passive control system. Whereas GPC control has canceled the sway displacement and the yaw angle effectively. A comparison simulation among uncontrolled, passive and active system using GPC is illustrated on Figure 6.14.
Figure 6.11 Ship coupling response with the GPC in sea state 3

Figure 6.12 Ship coupling response with the GPC in sea state 5
Figure 6.13 Ship coupling response with the GPC in sea state 6

Figure 6.14 Comparison of uncontrolled, passive and Active system of the ship coupling motion
6.4 Performance of the GPC in the roll motion

The main objective of the feedback control design is to guarantee the stability of the system and to satisfy certain performance criteria. In the roll motion, the overshooting and time settling are important factors for the successful design controller. In the GPC there are so many decisions that have to be considered in the design process. Good performance will be based on decisions such as selecting the prediction horizon, control horizon, or the weighting factors.

The previous results show a good effect for roll motion. The next simulation will focus on the ship roll model presented chapter three. The roll model using the anti-rolling tank is simulated under a random sine wave. The main objective of this simulation is to clarify the importance of the weighting factors on the performance of the GPC.

Figure 6.15 through figure 6.18 simulates the ship roll angle when it is controlled by the GPC. The figures show that the higher values of lambda, the more accurate result. The good performance of the GPC is based on selecting the right weighting factors. As explained on the previous section in the coupling model, the controller function accurately when the value of lambda increase. Also, in this case the value of R plays major role in terms of the overshooting. It seems that the less value of R, the less overshooting is. The value of R and lambda has to be selected in order to reach optimal performance. The prediction horizon and the control horizon are recommended to be at least in the same values.
Figure 6.15 ship roll angle at $R=1$ and $\lambda=1$

Figure 6.16 ship roll angle at $R=3$ and $\lambda=7$
Figure 6.17 ship roll angle at $R=10$ and $\lambda=0.1$

Figure 6.18 ship roll angle at $R=1$ and $\lambda=0.3$
CHAPTER VII
DISCUSSION AND CONCLUSIONS

7.1 Discussion

In this dissertation, GPC control was used to control the ship roll motion using anti-rolling flume tanks. Numerical Simulation was presented to simulate the ship’s behaviors under the excitation of the disturbance which was generated to simulate the sea waves. The result of the GPC has mitigated the ship roll angle with almost 80%. The effect of the controller is based on the selection of the weighting factors used to optimize the performance index or the cost function.

In addition to the GPC controller, passive controller is presented. The numerical simulation for the two controllers are presented and compared. Even though the passive control system using an anti-rolling flume tank has demonstrated a positive result in roll cancellation, it did not affect the other coupling motion of the sway and roll. Passive control system reduced the ship roll angle up to fifty percents. In other words, the GPC active controller system is able to control the ship coupling motion in roll, yaw and sway. The rolling angle is mitigated up to eighty percents due to the new controller.

The ability of the GPC to tune the cost function was the key issue of the successful result of the GPC. The simulations showed that the control system can mitigate the ship rolling effectively. The GPC approach is tested on a single input single output system. The weighting factors are tuned in order to optimize the cost function. The numerical simulation is implemented to evaluate the controller performance and
investigate the benefit of the GPC in the ship coupling motion in different sea state conditions.

7.2 Contributions

This dissertation has presented a Generalized Predictive Control (GPC) to control the ship rolling motion using an active anti-rolling tank. The contributions of this dissertation are divided into five parts. First, deriving a mathematical model for the anti-rolling flume tank. Second, construct a multidimensional model for a container ship, which include coupled roll, yaw and sway. Third, Design an active control system using Generalized Predictive Control. Fourth, apply the numerical simulations to evaluate the controller performance. Fifth, investigate the benefits of using the anti-rolling flume tank for passive and active control in the ship rolling.

7.3 Further extension of the research.

Dynamic of the marine vehicle is one of the most important applications in the control area. The dynamic behavior of ship is an interesting topic to research. This work can be expanded and studied further more. The availability of marine control literature will help to extend this research. In the following section, some suggestions for the work extension are proposed.

First, the mathematical Model of the stimulator can be presented in many different forms. The ship coupling model can be expanded to six degrees of freedom and investigate the effective of the heave, pitch and surge. Also, a nonlinear coupling model would be more accurate and effective to be studied.
Second, an experimental testing for a ship coupling model can be conducted and the system identification techniques can be used to identify the system parameters. A prototyping model of the container ship will help ensure an accurate model and reduce the chance of errors. Experimental study will help to simulate the reality of dynamic motion of the marine vehicles.

Third, the numerical data of the container ship can be replaced by other types of ships either bigger or smaller based on the purpose of the research. The anti-rolling flume tanks could be good for mitigation of the roll angle for the container ship. At the same time, it could not work for other types of ships especially when the ship cursing at a certain speed. Having said that, the actuator could be replaced by any other type of stabilizers likes fins stabilizers or rudder fan … etc.

Fourth, the control method used in this dissertation can be replaced by any other control method like optimal control, adaptive control or $H_\infty$. Having different controllers will enhance the result and open broad discussion of the effect of the model in the dynamic motion of the ship. Also, the GPC method can be studied in terms of the tuning and optimizing the cost functions compared with other methods.
### APPENDIX

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Description</th>
<th>Wave height ((H_s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Calm (glassy)</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>Calm (rippled)</td>
<td>0.0 - 0.1</td>
</tr>
<tr>
<td>2</td>
<td>Smooth (Wavelets)</td>
<td>0.1 - 0.5</td>
</tr>
<tr>
<td>3</td>
<td>Slight</td>
<td>0.5 - 1.25</td>
</tr>
<tr>
<td>4</td>
<td>Moderate</td>
<td>1.25 - 2.5</td>
</tr>
<tr>
<td>5</td>
<td>Rough</td>
<td>2.5 - 4</td>
</tr>
<tr>
<td>6</td>
<td>Very rough</td>
<td>4.0 - 6</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>6.0 - 9</td>
</tr>
<tr>
<td>8</td>
<td>Very high</td>
<td>9.0 - 14</td>
</tr>
<tr>
<td>9</td>
<td>Phenomenal</td>
<td>Over 14</td>
</tr>
</tbody>
</table>

Table 3.1 the description of the sea state code
<table>
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<th>Ship (L)</th>
<th>175 cm</th>
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<tbody>
<tr>
<td>Breadth (B)</td>
<td>25.40 m</td>
</tr>
<tr>
<td>Draft fore (d_f)</td>
<td>8.00 m</td>
</tr>
<tr>
<td>Draft aft (d_A)</td>
<td>9 m</td>
</tr>
<tr>
<td>Draft mean (d)</td>
<td>8.50 m</td>
</tr>
<tr>
<td>Displacement Volume</td>
<td>21,222 m$^3$</td>
</tr>
<tr>
<td>Height from Keel to mtransverse (KM)</td>
<td>10.39 m</td>
</tr>
<tr>
<td>Height from keel to center of buoyancy (KB)</td>
<td>4.6154 m</td>
</tr>
<tr>
<td>Block coefficient (C_b)</td>
<td>0.559</td>
</tr>
<tr>
<td>Water sea density is</td>
<td>1025 kg/m$^3$</td>
</tr>
</tbody>
</table>

Table 5.1 the dimensional model of the l-container
<table>
<thead>
<tr>
<th>Unit</th>
<th>Prime- system I</th>
<th>Prime- system II</th>
<th>Bis-system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$L$</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>Mass</td>
<td>$\frac{\rho L^3}{2}$</td>
<td>$\frac{\rho L^2T}{2}$</td>
<td>$\mu \rho \mathcal{N}$</td>
</tr>
<tr>
<td>Inertia moment</td>
<td>$\frac{\rho L^3}{2}$</td>
<td>$\frac{\rho L^4T}{2}$</td>
<td>$\mu \rho \mathcal{N}L^2$</td>
</tr>
<tr>
<td>Time</td>
<td>$\frac{L}{U}$</td>
<td>$\frac{L}{U}$</td>
<td>$\frac{\sqrt{L}}{\sqrt{g}}$</td>
</tr>
<tr>
<td>Reference</td>
<td>$L^2$</td>
<td>$LT$</td>
<td>$\mu^2 \frac{\mathcal{N}}{L}$</td>
</tr>
<tr>
<td>Position</td>
<td>$L$</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>Angle</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Linear Velocity</td>
<td>$U$</td>
<td>$U$</td>
<td>$\sqrt{Lg}$</td>
</tr>
<tr>
<td>Angular Velocity</td>
<td>$\frac{U}{L}$</td>
<td>$\frac{U}{L}$</td>
<td>$\frac{\sqrt{g}}{\sqrt{L}}$</td>
</tr>
<tr>
<td>Linear acceleration</td>
<td>$\frac{U^2}{L}$</td>
<td>$U$</td>
<td>$g$</td>
</tr>
<tr>
<td>Angular acceleration</td>
<td>$\frac{U^2}{L^2}$</td>
<td>$\frac{U^2}{L^2}$</td>
<td>$\frac{g}{L}$</td>
</tr>
<tr>
<td>Force</td>
<td>$\frac{\rho U^3L^2}{2}$</td>
<td>$\frac{\rho U^2LT}{2}$</td>
<td>$\mu \rho g \mathcal{N}$</td>
</tr>
<tr>
<td>Moment</td>
<td>$\frac{\rho U^2L^3}{2}$</td>
<td>$\frac{\rho U^2L^2T}{2}$</td>
<td>$\mu \rho g \mathcal{N}L$</td>
</tr>
</tbody>
</table>

Table 5.2 the normalization variable used for the prime system and Bis-system
\[
m' = 0.00792 \quad Y_p = 0.0 \quad N'_{v\phi} = -0.019058
\]

\[
m_x' = 0.000238 \quad Y'_{\phi} = -0.000063 \quad N'_{v\phi\phi} = -0.0053766
\]

\[
m_y' = 0.007049 \quad Y'_{vv} = -0.109 \quad N'_{rr\phi} = -0.0038592
\]

\[
P'_x = 0.000176 \quad Y'_{rr} = 0.00177 \quad N'_{r\phi\phi} = 0.0024195
\]

\[
J'_z = 0.00034 \quad Y'_{rr\phi} = 0.0214 \quad K'_{\phi} = 0.0003026
\]

\[
J'_z = 0.000419 \quad Y'_{rrr} = -0.0405 \quad K'_{r} = -0.0003026
\]

\[
x'_y = 0.05 \quad Y'_{v\phi} = 0.00304 \quad K'_{\phi} = -0.000021
\]

\[
P'_x = 0.0313 \quad Y'_{rr\phi} = 0.009325 \quad K'_{rr} = 0.0002843
\]

\[
K'_{x} = 0.527 - 0.455J \quad N'_{v} = -0.0038545 \quad K'_{rrr} = -0.000462
\]

\[
X'_{uu} = -0.0004226 \quad N'_{r} = -0.00222 \quad K'_{r\phi} = 0.000558
\]

\[
X'_{rr} = -0.00311 \quad N'_{r} = 0.000213 \quad K'_{rr} = 0.0010565
\]

\[
X'_{rr} = -0.00386 \quad N'_{\phi} = -0.0001424 \quad K'_{v\phi} = -0.0012012
\]

\[
X'_{rr} = 0.00020 \quad N'_{v\phi} = 0.001492 \quad K'_{v\phi\phi} = -0.000793
\]

\[
X'_{rr\phi} = -0.00020 \quad N'_{rr} = -0.00229 \quad K'_{rr\phi} = -0.000243
\]

\[
Y'_{v} = -0.0116 \quad N'_{rr} = -0.0424 \quad K'_{rr\phi} = 0.0003569
\]

\[
Y'_{r} = 0.00242 \quad N'_{rr} = 0.00156
\]

Table 5.3 Model parameters of the a L- container ship (hull only)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank cross sectional area per side</td>
<td>290 ft²</td>
<td>26.94 m²</td>
</tr>
<tr>
<td>Minimum water height (pump cavitations limit)</td>
<td>11 ft</td>
<td>3.35 m</td>
</tr>
<tr>
<td>Maximum water height (safety margin)</td>
<td>21 ft</td>
<td>6.4 m</td>
</tr>
<tr>
<td>Average operation</td>
<td>16 ft (± 3 ft)</td>
<td>4.88 m (± 0.91 m)</td>
</tr>
<tr>
<td>Maximum operating head</td>
<td>3 ft</td>
<td>0.91 m</td>
</tr>
<tr>
<td>Maximum volume transferred</td>
<td>± 880 ft³</td>
<td>± 24.92 m³</td>
</tr>
<tr>
<td>Maximum mass transferred</td>
<td>± 54,780 lbs</td>
<td>± 24,920 Kg</td>
</tr>
<tr>
<td>Maximum flow rate</td>
<td>240,000 gpm</td>
<td>15.14 m³/sec</td>
</tr>
<tr>
<td>Minimum period to move fluid</td>
<td>6.48 sec</td>
<td>6.48 sec</td>
</tr>
<tr>
<td>Operating water capacity</td>
<td>80,000 gal</td>
<td>302.8 m³</td>
</tr>
<tr>
<td>Pipe cross sectional area (average) A&lt;sub&gt;pipe&lt;/sub&gt;</td>
<td>66.96 ft²</td>
<td>6.22 m²</td>
</tr>
<tr>
<td>Distance between centerline of the tank L&lt;sub&gt;stimu&lt;/sub&gt;</td>
<td>56 ft</td>
<td>17.1 m</td>
</tr>
<tr>
<td>Nominal height of the water in the tank H&lt;sub&gt;tank&lt;/sub&gt;</td>
<td>16 ft</td>
<td>4.88 m</td>
</tr>
<tr>
<td>Distance below the pipe to the c.g. of the ship L&lt;sub&gt;Z, CM, tank&lt;/sub&gt;</td>
<td>6 ft</td>
<td>1.83 m</td>
</tr>
<tr>
<td>Maximum rate of change of head h&lt;sub&gt;max&lt;/sub&gt;</td>
<td>1.84 ft/sec</td>
<td>0.562 m/sec</td>
</tr>
</tbody>
</table>

| Table 6.1 Numerical values of the anti-rolling flume anti-rolling flume tank |
REFERENCE


15. Huang, and Tongchia, “Feasibility analyses of using the ship roll stimulation system as a roll mitigation device”, Old Dominion University, 2002.


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