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Quark-hadron duality in spin structure functions $g_1^p$ and $g_1^d$


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New measurements of the spin structure functions of the proton and deuteron $g_1^p(x, Q^2)$ and $g_1^d(x, Q^2)$ in the nucleon resonance region are compared with extrapolations of target-mass-corrected next-to-leading-order (NLO) QCD fits to higher energy data. Averaged over the entire resonance region ($W < 2 \text{ GeV}$), the data and QCD fits are in good agreement in both magnitude and $Q^2$ dependence for $Q^2 > 1.7 \text{ GeV}^2/c^2$. This “global” duality appears to result from cancellations among the prominent “local” resonance regions: in particular strong $\sigma_{1,2}$ contributions in the $\Lambda(1232)$ region appear to be compensated by strong $\sigma_{1,2}$ contributions in the resonance region centered on $1.5 \text{ GeV}$. These results are encouraging for the extension of NLO QCD fits to lower $W$ and $Q^2$ than have been used previously.

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The theoretical description of particle interactions has utilized quark-gluon degrees of freedom at high energies and hadronic degrees of freedom at low energies. With suitable averaging over resonant excitations, the two approaches have been found in several cases to be nearly equivalent, a phenomenon referred to as quark-hadron duality. These cases include $e^+e^-$ annihilation, semi-leptonic decays of heavy mesons, electron-pion scattering, semi-inclusive deep-inelastic scattering, and both spin-averaged and spin-dependent inclusive lepton-nucleus scattering [1], the subject of the present investigation. Pragmatically, understanding the limitations and applicability of quark-hadron duality in this process is useful to define the kinematic region in which parton distribution functions (PDFs) can be reliably extracted.

In lepton-nucleon scattering, the low- and high-energy regimes have traditionally been separated using $W$, the invariant mass of the hadronic final state, and $Q^2$, the four-momentum transfer squared. A region of prominent nucleon resonances is observed for $W < 2 \text{ GeV}$ and $Q^2 < 10 \text{ GeV}^2/c^2$, whereas for higher $W$ or $Q^2$ there is no longer any obvious resonance structure. Historically, quark-hadron duality was first observed in 1970 by Bloom and Gilman [2] in the spin-averaged lepton-nucleon process. They noted that the inclusive structure function $F_2(W, Q^2)$ averages smoothly at low $W$ and $Q^2$ to the scaling function $F_2(W, Q^2)$ measured at high energy, using an empirical scaling variable in place of the original Bjorken $x$ scaling variable. Subsequently, Georgi and Politzer [3] found that quark-hadron duality is exhibited down to $Q^2 \sim 1 \text{ GeV}^2/c^2$ using the Nachtmann [4] scaling variable $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$, which approximates the purely kinematic higher twist corrections arising from the nonzero nucleon mass $M$. More recently, explicit target-mass (TM) corrections have been derived in the framework of QCD for both unpolarized and polarized structure functions [5] that obviate the need for an approximate scaling variable.

To explain quark-hadron duality theoretically, de Rújula, Georgi, and Politzer [6] employed a perturbative operator product expansion of QCD structure function moments. In this framework, quark-hadron duality implies a small net...
effect from higher twist contributions, once the kinematic TM contribution is taken into account. In a simple QCD picture, the additional higher twist contributions (which are proportional to powers of $1/\sqrt{Q^2}$) are due to quark-quark and quark-gluon correlations. Close and Isgur [7] provided an interesting explanation in the constituent quark model in terms of cancellations from resonance contributions with opposite parity. A recent theoretical QCD study [8] of both polarized and unpolarized structure functions incorporates many of these concepts, with the addition of a careful treatment of so-called high-$x$ resummation corrections. The authors concluded that higher twist corrections are suppressed more for the unpolarized structure function $F_2$ than for the proton polarized structure function $g_1^p$, where a sizable negative contribution is observed. A comprehensive review of quark-hadron duality from both the experimental and theoretical perspective was also published recently [1].

Unpolarized structure function data exhibit excitation-energy-averaged scaling averaged not only over the entire resonance region ($M < W < 2$ GeV), referred to as “global duality,” but also in each of several restricted regions in $W$, corresponding to the three prominent resonance regions centered on $W = 1.23, 1.5,$ and 1.7 GeV, a phenomenon referred to as “local duality.” This was demonstrated experimentally using high-statistical-accuracy data from Jefferson Lab [9] and interpreted theoretically by Carlson and Mukhopadhyay [10] using the expected pQCD $Q^2$ dependence of nucleon transition form factors.

The new data presented here augment previously available results for $g_1^p$ from SLAC [11,12], DESY [13], and JLab [14, 15] with higher statistical precision and an expanded range of $Q^2$. This allows us to experimentally examine local duality for $g_1^p$ much more accurately than was previously possible. The addition of a considerable body of deuteron $g_1^d$ data allows the first examination of the isospin dependence of global duality in $g_1$.

When testing duality there is an intrinsic uncertainty as to which Deep Inelastic Scattering (DIS) curves to use for comparison with the averaged resonance region data. In this paper, we choose the average of two representative next-to-leading order (NLO) QCD fits [16,17] to polarized structure function data above the resonance region. The NLO evolution is considered to be reasonably reliable down to $Q^2$ values of order 1 GeV$^2$/c$^2$. We choose to use fits with NLO evolution, rather than LO or purely empirical fits to data, to give the best possible estimate of the $Q^2$ dependence of $g_1$. The high-energy data used in the NLO QCD fits have relatively large errors compared to those for unpolarized structure functions, particularly at the high values of $x$ that tend to correspond to our resonance region data. Since precise error bands in our kinematic region are not available, we ascribe a very approximate relative error of 10% (20%) to the $g_1^d$ ($g_1^p$) DIS fits, independent of $x$. The errors are likely larger than this for $x > 0.6$, but for $Q^2 > 0.8$ GeV$^2$ (the lowest $Q^2$ for which we plot PDF predictions), $x = [(W^2 - M^2)/Q^2 + 1]^{-1} < 0.6$ for $W > 1.18$ GeV, which corresponds to the great majority of the experimental data points of this study. The error on the average deuteron DIS fit is larger owing to the much larger relative contribution of negatively polarized quarks in the neutron compared to the proton. We take kinematic TM corrections into account using the prescription of Blümlein and Tkabladze [5]:

$$g_1^{TM}(x, Q^2) = \frac{x}{\xi(1 + \gamma)^{3/2}} g_1^{QCD}(\xi, Q^2)$$

$$+ \frac{(x + \xi)\gamma}{\xi(1 + \gamma)^2} \int_{\xi}^{1} \frac{du}{u} g_1^{QCD}(u, Q^2)$$

$$- \frac{\gamma(2 - \gamma)}{2(1 + \gamma)^{3/2}} \int_{\xi}^{1} \frac{du}{u} \int_{\xi}^{1} \frac{dv}{v} g_1^{QCD}(v, Q^2), \quad (1)$$

where $\gamma = 4M^2x^2/Q^2$. This prescription is not unique, and in particular it has the drawback of resulting in nonzero values of $g_1$ at $x = 1$. An approach that avoids this problem has been worked out for $F_2$ [18], but it is not yet available for $g_1$. We note that if one were to use the very simple approximation $g_1^{TM}(x, Q^2) = g_1^{QCD}(\xi, Q^2)$ the PDF predictions would be raised by 30% (20%) at the lowest (highest) $Q^2$ of this study. The main source of increase would come from setting $x/\xi(1 + \gamma)^{3/2}$ in Eq. (1) equal to unity, while the two integral terms are of secondary importance.

The calculation of high-$x$ resummation corrections is theoretically more complicated [19] and technically more challenging than TM corrections. Rather than attempting these calculations ourselves, we simply note that Ref. [8] finds enhancements of order 10% to 20% for the proton averaged over the full resonance region, roughly independent of $Q^2$ for $0.5 < Q^2 < 5$ GeV$^2$/c$^2$. These corrections could well be different for the deuteron and for individual “local” regions in $W$.

The analysis is based on recently published data [20] from Jefferson Lab. Very briefly, in this experiment the CEBAF Large Acceptance Spectrometer [21] in Jefferson Lab’s Hall B was used to measure spin asymmetries in the scattering of longitudinally polarized electrons from longitudinally polarized protons and deuterons. The data were collected in 2001 using incident energies of 1.6 and 5.7 GeV. Beam currents ranged from 1 to 5 nA, and the beam polarization averaged 70%. The detector package [21] allowed clean identification of electrons scattered at polar angles between 8 and 45 degrees. Ammonia, polarized via dynamic nuclear polarization [22], was used to provide polarized protons and deuterons, by using the $^{15}$NH$_3$ and $^{15}$ND$_3$ isotopes, respectively. The average target polarization was about 75% for the proton and about 25% for the deuteron. The data were divided into 40 bins in $Q^2$, equally spaced on a logarithmic scale between 0.01 and 10 GeV$^2$/c$^2$.

Values of $g_1(x, Q^2)$ were determined from the ratios of $g_1/F_1$ presented in [20] by using recent fits to proton [23] and deuteron [24] data to evaluate the unpolarized structure function $F_1(x, Q^2)$. The resulting values of $g_1(x, Q^2)$ for both the proton and the deuteron are plotted (scaled by $x$) as a function of $x$ for four representative $Q^2$ bins in Fig. 1. The three arrows on each panel correspond to the three prominent resonance regions at $W = 1.7, 1.5,$ and 1.23 GeV, from left to right. We compare the data to the extrapolations of DIS fits (as described earlier), represented by the hatched bands.

It can be seen in Fig. 1 that the data in fixed $Q^2$ bins indeed exhibit oscillations in $x$ compared to the smooth behavior
FIG. 1. Present data for proton $g_1^p(x, Q^2)$ (left panels) and deuteron $g_1^d(x, Q^2)$ (defined to be per nucleon, right panels) at four representative values of $Q^2$. The errors include statistical and systematic contributions added in quadrature. The three arrows on each plot indicate the central kinematic position of the three prominent resonance regions at $W = 1.7$, 1.5, and 1.23 GeV from left to right. The hatched band represents the range of $g_1$ predicted by modern NLO PDF fits (GRSV [17] and AAC [16]) to high-energy data, evolved to the $Q^2$ of our data and corrected for TM as described in the text.

of the DIS curves. In addition, the averaged data become increasingly commensurate with the models with increasing $Q^2$, as expected if quark-hadron duality is valid for $g_1$. In closer detail, one can also observe that the experimental data for both the proton and the deuteron lie below the curves in the $\Delta(1232)$ region and above in the $W = 1.5$ GeV [$S_{11}(1535)/D_{13}(1520)/P_{11}(1440)$] region. This is not surprising at low to moderate $Q^2$, where resonant contributions dominate over nonresonant contributions. Recall that $g_1$ is proportional to $\sigma_{3/2} - \sigma_{1/2}$. The $N \rightarrow \Delta(1232)$ transition is known to be dominated by $M1$ strength [25] over the $Q^2$ range of the present study, which results in a virtual photon cross section $\sigma_{3/2}$ about three times larger than $\sigma_{1/2}$, corresponding to negative values of $g_1$. In contrast, in the DIS limit of incoherent scattering from massless quarks, $g_1$ must be positive at large $x$ because of helicity conservation. In the second resonance region, the $S_{11}(1535)$ and $P_{11}(1440)$ transitions can only contribute to $\sigma_{1/2}$, and recent studies [26] show that the $N \rightarrow D_{13}(1520)$ transition changes from dominantly $\sigma_{3/2}$ at low $Q^2$ to dominantly $\sigma_{1/2}$ above $1$ GeV$^2/c^2$. The three resonances together therefore are expected to result in large positive values of $g_1$ above $1$ GeV$^2/c^2$ (potentially larger than the DIS limit).

To clarify these observations with respect to both local and global duality, we have averaged over $x$ both data and models for $g_1$ over a $Q^2$-dependent interval corresponding to four specific regions in $W$. The $x$-averaged values of $g_1$ for the entire resonance region (scaled by $Q^2$) are plotted as a function of $Q^2$ in Fig. 2 for both targets. The proton averages for four smaller regions in $W$ are plotted in Fig. 3. Specifically, the averages were determined as

$$\langle g_1(Q^2) \rangle = \frac{\int_{x_l}^{x_h} g_1(x, Q^2) dx}{x_h - x_l},$$

where $x_l$ and $x_h$ correspond, respectively, to the maximum and minimum values of $W$ in the interval considered, at the given value of $Q^2$ [with the definition $x^{-1} = 1 + (W^2 - M^2)/Q^2$]. The systematic errors on the averages are evaluated by assuming that the systematic errors on the individual points are purely overall scale factors, which is a reasonable approximation for this data set at a given value of $Q^2$. The TM-corrected NLO PDF parametrizations shown in Fig. 1 were averaged over the same $x$ ranges as the experimental data.

The averages displayed in Fig. 2 test “global” duality by averaging $g_1$ over $x$ for the entire region from pion threshold
to $W = 2$ GeV. The data for both targets exhibit a power-law-type deviation from the DIS curves at low $Q^2$, but they essentially agree with them above $Q^2 = 1.7$ GeV$^2/c^2$, within the systematic errors of the data and models. This is somewhat higher in $Q^2$ than for the unpolarized $F_2$ structure, supporting the conclusions of Ref. [8].

Turning to the examination of “local” duality for the proton, we can see in the upper left panel of Fig. 3 that in the “first” resonance region, dominated by the $\Delta(1232)$ resonance, the data have the opposite sign of the extrapolations of DIS models at low $Q^2$. This is allowed by the spin-3/2 nature of the $\Delta(1232)$ and is expected owing to the dominance of the $M1$ transition strength [25], as discussed in the foregoing. What is interesting is that, although the data change sign at $Q^2$ near 1 GeV$^2/c^2$, the averaged values are significantly below the models even to the highest $Q^2$ of the present experiment, in spite of the fact that the $N \rightarrow \Delta(1232)$ transition form factor (FF) decreases more rapidly with $Q^2$ than, for example, the elastic FF or the $N \rightarrow S_{11}(1535)$ transition FF [25,27] (a phenomena sometimes referred to as the “disappearing $\Delta$”). It is evident that the $\Delta(1232)$ has not yet completely disappeared at $Q^2 = 5$ GeV$^2/c^2$.

In the second resonance region, two of the three known resonances [$P_{11}(1440)$ and $S_{11}(1535)$] contribute only to $\sigma_{1/2}$, while the third [$D_{13}(1520)$] contributes more to $\sigma_{1/2}$ than to $\sigma_{3/2}$ above 1 GeV$^2/c^2$ [26]. Therefore, it is not surprising that the proton data lie significantly above the DIS extrapolations in this narrow region of $W$.

In the “third” resonance region centered on 1.7 GeV, the $F_{15}(1680)$ resonance is dominant at low $Q^2$, but above about 1 GeV$^2/c^2$ other resonances are also important [26], such as the $S_{11}(1650)$, $S_{11}(1620)$, and $D_{13}(1700)$. The $F_{15}$ contributes mainly to $\sigma_{3/2}$ at low $Q^2$ (i.e., negative $g_1$), but it switches to $\sigma_{1/2}$ dominance at higher $Q^2$ [26]. The average over all of these resonances plus nonresonant background produces very good agreement between data and DIS models in this region, as might be expected from the parity-averaging arguments of Close and Isgur [7].

For completeness, we have also studied a fourth region centered on 1.9 GeV (for which there are numerous poorly established resonances, which are difficult to distinguish from nonresonant contributions). In this case, the data lie slightly below the DIS models, although the significance is marginal when systematic errors are taken into account. It appears that much of the good agreement between data and models observed in the entire resonance region comes about from pairing the “first” and “second” resonance regions together, with further improvement from including the “fourth” region. This lends further support to the Close-Isgur model.

Again following Close and Isgur [7], one might expect DIS and resonance region data to converge at lower values of $Q^2$ if the ground-state elastic contribution is also included in the global duality averaging. The open circles in Fig. 2 include the elastic (quasi-elastic) contributions to the $g_1$ averages for the proton (deuteron), given by $G_E(Q^2)g_1(Q^2) + \tau G_M(Q^2)/2(1 + \tau)(x_h - x_l)$, where $\tau = Q^2/4M^2$. To evaluate the nucleon electric and magnetic form factors $G_E(Q^2)$ and $G_M(Q^2)$, we used the parametrization of Ref. [28], which uses polarization transfer measurements of $G_E^p/G_M^p$, appropriate for evaluations of $g_1$. Using a fit such as Ref. [29], which uses Rosenbluth measurements of $G_E^p/G_M^p$, results in 3% to 5% higher relative values of the $g_1$ averages. For both the proton and the deuteron, the $Q^2$ dependence with the elastic contribution more closely resembles the $Q^2$ dependence of the models, down to values of $Q^2$ as low as 0.7 GeV$^2/c^2$, which is already pushing below the expected region of validity of 1 GeV$^2/c^2$ for the PDF fits. However, the magnitude of the data is of order 10% to 20% higher than the models. The difference is reduced if the unphysical contributions below pion threshold, generated by the TM formalism that we are using, are included in the shaded bands. To a good approximation, the relative increase in the magnitude of the shaded bands for both proton and deuteron is given by $10\%/Q^2$. 

FIG. 2. The $Q^2$ dependence of $Q^2g_1(x, Q^2)$, averaged over a region in $x$ corresponding to 1.08 < $W$ < 2 GeV (solid circles) for (a) the proton and (b) the deuteron. The inner error bars reflect only statistical contributions; the outer error bars include statistical and systematic components added in quadrature. The open circles represent our data after adding the contribution from ep elastic (ed quasi-elastic) scattering at $x = 1$ for the proton (deuteron). For clarity these results are slightly displaced in $Q^2$, and the error bars include only statistical contributions. The hatched bands represent the range of the averages calculated from extrapolated NLO DIS fits, as in Fig. 1 (see text for details).
FIG. 3. The $Q^2$ dependence of $Q^2 g_1(x, Q^2)$ for the proton, averaged over various regions of $x$. At each $Q^2$, the $x$ range over which $g_1$ was averaged is determined by the corresponding range in $W$ as indicated in each panel (see text). Symbols and curves are as in Fig. 2.

[with $Q^2$ in units of GeV$^2$], which reduces the difference between data and models by roughly a factor of 2 at low $Q^2$.

As already mentioned, high-$x$ resummation corrections might also account for some of this difference.

The result of pairing the $\Delta(1232)$ resonance (the lowest spin-3/2 ground state) with the elastic contribution (the lowest spin-1/2 ground state) is illustrated in the upper left panel of Fig. 3 for the proton. The elastic contribution, in the way we have treated it, overcompensates, resulting in power-law deviations at low $Q^2$ that lie well above the data, rather than well below. It appears that including the elastic contribution with the entire resonance region works much better than pairing it with the single $\Delta(1232)$ resonance.

In summary, we have used data for both the proton and deuteron to examine both “local” and “global” quark-hadron duality in $g_1$. As was determined in previous studies [1,8], $g_1^p$ in the resonance region oscillates around extrapolations of NLO PDF fits to higher energy data, especially when TM corrections are taken into account. Averaged over the traditional resonance region ($W < 2$ GeV), the data and fits agree within errors for $Q^2 > 1.7$ GeV$^2/c^2$, a slightly higher value than observed for the spin-averaged structure function $F_2$ [8,9]. Including the elastic contribution may extend the region of agreement to $Q^2 = 0.7$ GeV$^2/c^2$, after consideration of the uncertainties resulting from high-$x$ resummation. A similar effect was found in the unpolarized case [1,9]. We find results for the previously unexamined $g_1^d$ structure function similar to those for $g_1^p$, indicating no large effects from different isospin projections. In terms of “local” duality, we find that in the $\Delta(1232)$ region, the proton data lie below the PDF fits, even for $Q^2$ values as large as 5 GeV$^2/c^2$, whereas the region centered on 1.5 GeV lies above the PDF fits for all $Q^2$ values studied. It appears that global duality is largely realized by summing over the four lowest mass resonances.

Since the $Q^2$ dependence for the PDF fits and the data are remarkably similar above $Q^2 = 1.7$ GeV$^2/c^2$, we conclude that, from the practical point of view, it is not unreasonable, with suitable averaging over $W$, to include data with $W > 1.58$ GeV and $Q^2 > 1.7$ GeV$^2/c^2$ in future global NLO PDF fits, as long as the effects of TM and high-$x$ resummation corrections are taken into account. In the near future, high-statistical-accuracy data from the present experiment with 2.5 and 4.2 GeV electron energies, and additional data at 5.7 GeV, presently under analysis, will allow more precise studies, particularly for the deuteron. Recently published [30] separated proton results for $g_1$ and $g_2$ near $Q^2 = 1.3$ GeV$^2/c^2$
have also helped clarify the experimental situation, which will be further improved when the deuteron data from this experiment are finalized.

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[24] I. Niculescu, private communication for fit to data in Ref. [9].