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Measurement of the $g_{\pi NN}(t)$ Form Factor

Kelly Gene Vansyoc
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MEASUREMENT OF THE $g_{\pi NN}(t)$ FORM FACTOR

by

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M.S. May 1998, Old Dominion University

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Old Dominion University in Partial Fulfillment of the
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ABSTRACT

MEASUREMENT OF THE $g_{\pi NN}(t)$ FORM FACTOR

Kelley Gene Vansyoc
Old Dominion University, 2001
Director: Dr. Andreas Klein

Cross sections were measured for the reaction $^1H(e, e'\pi^+)n$ at the energy $W = 1.95$ GeV and momentum transfer $Q^2 = 0.6$ (GeV/c) 2 . At this W and Q^2 , the longitudinal cross section is dominated by t-channel production, giving a unique opportunity to examine the strong coupling form factor $g_{\pi NN}(t)$. The measured cross sections were separated using a method similar to a Rosenbluth separation. For the extraction of $g_{\pi NN}(t)$, the Actor and Körner model [42] and a parameterization of the MAID2000 model [3] were employed to fit the longitudinal cross section. Three parameterizations $g_{\pi NN}(t)$ were used in both models. These fits resulted in a strong coupling constant $g_{\pi NN}(m_\pi^2)$ that is consistent with theoretical predictions. However, this coupling constant leads to a cutoff parameter that is less than 1 GeV.

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Chapter 1

Introduction

Over the past century, nuclear physics has established that the substructure of the atom is more complex than just a nucleus of protons and neutrons surrounded by a cloud of electrons. The protons and neutrons, collectively called nucleons, have also exhibited a substructure. This substructure is presented through the nucleons demonstrating states of excitation and emitting particles, like mesons, when a sufficiently energetic probe is used in examining them. The nucleons and mesons are categorized as hadrons. Hadrons are particles that strongly interact and are made up of elementary particles called partons.

Prior to the 1930's, the potential that binds nucleons together was described by phenomenological models based on spin interactions between the nucleons. However, continuing in a phenomenological manner to produce a more accurate model would not help us to understand the fundamental characteristics of the nucleon-nucleon (NN) interactions. In 1935, Japanese physicist Hideki Yukawa, examined NN interactions in order to calculate a potential that would manifest the nuclear force through the exchange of particles. This potential is based on the Exchange Force Model that predicts something is exchanged between the nucleons, which produces a sort of saturation bond and causes the nucleons to change their characteristics (*e.g.*, for the exchange of a positively charged π meson from a

⁰This dissertation follows the form of *The Physical Review*

proton to a neutron, the proton will become a neutron and the neutron will become a proton.). The saturated bond is based on the fact that a nucleon will only interact with a finite number of other nucleons at the same time.

Based on field theory, the exchanged particles must have integer spin and be charged according to the type of interaction. If the interaction is between proton and a neutron (pn), then the particle needs to be charged. However, if the exchange is pp or nn , then the particle must be charge neutral. Further, the exchange must be limited to an amount of time that is dictated by the uncertainty principle $T < \hbar/m_{\pi}c^2$ in order to mask violations of energy conservation. Here T is the time of the exchange of the particle of mass m_{π} . Such particles are known as virtual particles.

From the Yukawa potential we see that the nuclear force is carried by particles called mesons. If we look at this potential as a function of the interaction range, we can divide it into three regions [1]. The long-range interaction ($r > 2$ fm) is governed by the exchange of the π meson. The intermediate range ($1 < r < 2$ fm) is dominated by two-pion and heavier exchanges, and in the hard core region ($r < 1$ fm), multi-pion and heavier exchanges will occur as well as quantum chromodynamic (QCD) effects.

Yukawa's potential is based on the coupling strength between the meson and the nucleon. For the long-range interaction, the coupling strength between the pion and the nucleon is given by the strong coupling form factor $g_{\pi NN}(t)$.

1.1 Pions

The lightest of the mesons are the pions. The charged pions ($\pi^{\pm}$) have a mass of 140 MeV, where the neutral pion (π^0) has a mass of 134 MeV. These masses are almost one order of magnitude below the typical hadron (1 GeV). This mass and the fact that their total spin is $J = 0$ implies that a pion is made up of a quark-antiquark pair, or a linear combination of quark-antiquark pairs. For the positively charged pion these quarks are the *up* and *antidown* quarks ($u\bar{d}$). The

negatively charge pion is characterized by the *down* and *antiup* quarks ($d\bar{u}$). The neutral pion is described by a linear combination of these quarks $((u\bar{u} + d\bar{d})/\sqrt{2})$.

In QCD, there is a possibility that quark pair condensates exist. If these condensates exist, then the pion would most likely be the lowest energy excitation in the QCD vacuum. Further, the existence of these condensates spontaneously breaks the underlying symmetry created by the smallness of the pion's mass (chiral symmetry breaking). However, because the pion mass is very small, chiral symmetry is still a valid assumption. An effect of chiral symmetry breaking is presented in Appendix A.

The size of a pion is determined by the form factor F_π . F_π can be determined from the cross section for elastic scattering of electrons off pions. Since the pion is unstable outside of a baryon, there are no free pion targets available. This leaves two alternatives for measuring the pion form factor. The first is to perform elastic scattering of energetic pions off atomic electrons to produce cross sections at low momentum transfer $Q^2 < 0.28 \text{ (GeV/c)}^2$ as has been done at CERN [2]. The second possibility is to produce pions elastic scattering electrons off of virtual pions in a proton target, which will give cross sections at much higher Q^2 . The latter process is called pion electroproduction. Both of these processes are very dependent on the coupling strength between the pion and the nucleon.

1.2 $g_{\pi NN}(t)$

The coupling strength between the pion and nucleon is described by the form factor $g_{\pi NN}(t)$, where t is the square of the pion four-momentum in the pion-nucleon center of mass frame. If the pion and nucleon are on shell (not energetically excited), then the square of the pion momentum is equal to the square of the pion mass $t = m_\pi^2$. On the other hand, if the pion and/or the nucleon are off shell (or virtual), then t becomes more complicated and no longer equals the pion mass $t \neq m_\pi^2$. This difference becomes important in calculations using nuclear models *e.g.*, a Born term model. This will be discussed in more detail in chapter 2.

FIG. 1: Second Order Differential Cross Section Prediction for Pion Electroproduction. This prediction is from the MAID2000 model [3] with $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$. It includes the Born terms, the ρ and ω contributions, and all baryon excitations. All multipole scalars were set to one. The line near $t = 0 \text{ (GeV/c)}^2$ represents the pion pole.

If the pion electroproduction cross section is plotted as a function of the pion four-momentum (t), then the response of the cross section as a function of t is governed by the coupling form factor $g_{\pi NN}(t)$. In Fig. 1, we plot the pion electroproduction cross section as a function of the virtual pion momentum t [3]. We see that the cross section increases as we go to smaller absolute values of t . When we get near the pion pole (at $t = m_\pi^2$) the cross section increases rapidly. If we are to extract a meaningful result for $g_{\pi NN}(t)$, then we need to include this region near the pion pole in our measurement of the cross section. With today's technology, this portion of the virtual pion momentum can only be reached through pion electroproduction.

The remainder of this work will present why and how we extracted $g_{\pi NN}(t)$ from the pion electroproduction cross section. In chapter 2 we examine what pion electroproduction is and what is needed to extract $g_{\pi NN}(t)$. We also look at current theoretical predictions, which motivated our experiment. In chapters 3 and 4 we discuss the experiment and the data analysis. Chapter 5 presents the calculation of the cross sections and compares them to a current model. The extraction of $g_{\pi NN}(t)$ and our final results are given in chapter 6 followed by some concluding remarks in chapter 7.

Chapter 2

Theoretical Overview

2.1 Motivation

Current predictions of $g_{\pi NN}(t)$ are from direct calculations based on empirical assumptions or models fit to experimental data. Regardless of the prediction, each assumes that $g_{\pi NN}(t)$ takes on one of the following parameterizations:

$$\textit{Monopole} \quad g_{\pi NN}(t) = g_{\pi NN}(m_\pi^2) \frac{\Lambda_{\pi N}^2 - m_\pi^2}{\Lambda_{\pi N}^2 - t}, \quad (1)$$

$$\textit{Dipole} \quad g_{\pi NN}(t) = g_{\pi NN}(m_\pi^2) \left(\frac{\Lambda_{\pi N}^2 - m_\pi^2}{\Lambda_{\pi N}^2 - t} \right)^2, \quad (2)$$

$$\textit{Exponential} \quad g_{\pi NN}(t) = g_{\pi NN}(m_\pi^2) e^{\frac{(t-m_\pi^2)}{\Lambda_{\pi N}^2}}. \quad (3)$$

In these equations, $g_{\pi NN}(m_\pi^2)$ is the value of $g_{\pi NN}(t)$ at $t = m_\pi^2$, known as the pole value or the coupling constant, and defines the coupling strength. The quantity $\Lambda_{\pi N}$ is the cutoff parameter, and with $g_{\pi NN}(m_\pi^2)$, sets determines the response of the pion electroproduction cross section as a function of t . For an example of how the value of $\Lambda_{\pi N}$ affects the cross section, consider the longitudinal cross section from pion electroproduction (see section 2.2) shown here

$$d\sigma_L \approx -\xi g_{\pi NN}^2(t) \frac{t}{(t - m_\pi^2)^2}, \quad (4)$$

where ξ is a constant which represents the Q^2 and W dependence. Setting the coupling constant to $g_{\pi NN}(m_\pi^2) = 13.5$ and $\Lambda_{\pi N}$ to two different values, we can

FIG. 2: Example of $g_{\pi NN}(t)$ at Two Values of $\Lambda_{\pi N}$. $g_{\pi NN}(m_\pi^2)$ is fixed at 13.5 GeV and ξ is normalized to $0.01 \mu\text{bc}^2$ in Eq. 4. This plot shows the effect of two different values of $\Lambda_{\pi N}$ on the response of the cross section. A smaller value produces a steeper (softer) response than a larger value.

graph the resulting cross section as a function of the virtual pion momentum as is shown in Fig. 2. For this graph we have assumed that ξ is normalized to $0.01 \mu\text{bc}^2$. In Fig. 2 we see that as $\Lambda_{\pi N}$ goes to higher values, the response becomes flatter (stiffer, or harder). This would indicate more high momentum virtual pions in the pionic cloud around the nucleon. Similarly, if the value of $\Lambda_{\pi N}$ is small, then the response would be steeper (softer) and indicate fewer high momentum virtual pions. This is also true for the other forms of $g_{\pi NN}(t)$.

FIG. 3: $\Lambda_{\pi N}$ versus Calculation Method. In the cases of no error bar, the error was not given [4]-[10]. See the text for details.

Usually, theoretical predictions will fix the strong coupling constant, $g_{\pi NN}(m_\pi^2)$, to an empirical value near 13.5 and determine the cutoff parameter, $\Lambda_{\pi N}$, by fitting a model to cross sections or by calculating it directly. Still, some predictions allow both the coupling constant, though constrained using the Goldberger-Treiman Relation (GTR) (see Appendix A), and the cutoff parameter to be floating parameters in a model that is fit to measured cross sections. As a

result, $\Lambda_{\pi N}$ is biased by our choice of the strong coupling constant $g_{\pi NN}(m_\pi^2)$ and is very model dependent. As an example of these variations, Fig. 3 and Fig. 4 show $\Lambda_{\pi N}$ and $g_{\pi NN}(m_\pi^2)$ as determined by various calculations. The data points in these plots correspond to one another (*e.g.*, the hollow square point in Fig. 3

FIG. 4: $g_{\pi NN}(m_\pi^2)$ versus Calculation Method. In the cases of no error bar, the error was not given [4]-[10]. See the text for details.

is related to the hollow square point in Fig. 4. If no point is shown, then no value was given in the literature.). The values under the Skyrme heading are pure calculations of nucleon-nucleon reactions with single or multiple meson exchanges. They are based on simple Born principles and use the monopole version of $g_{\pi NN}(t)$ [4]-[6]. The “other” heading describes a calculation that starts from the Hardtree-Fock theory and is based on nucleon-antinucleon interaction [7]. The QCD Sum Rule result is from T. Meissner [8] who used the Borel Sum Rule to extrapolate from intermediate t to $t = 0$ $(\text{GeV}/c)^2$ in order to determine $\Lambda_{\pi N}$ with a fixed $g_{\pi NN}(m_\pi^2)$. Meissner used the monopole version of $g_{\pi NN}(t)$. K. Liu *et al.*

[9] used a quenched Lattice QCD calculation to determine $g_{\pi NN}(t)$. In this lattice calculation, the Liu group generated cross section predictions based on values of the axial and electromagnetic form factors that matched experimental results to within 10%. This group then fit their model using either the monopole or dipole model of $g_{\pi NN}(t)$ to these predictions. The last point in Figs. 3 and 4, under the “Data” heading, comes from Machleidt *et al.* [10]. The Machleidt group used a very extensive effective meson model to fit nucleon-nucleon cross sections measured at Bonn. This model included multiple meson exchange and final state interactions. The value for $\Lambda_{\pi N}$ was produced by extrapolating this model to the pion pole with a fixed $g_{\pi NN}(m_\pi^2)$. From these plots, we can see, for a consistent $g_{\pi NN}(m_\pi^2)$, $\Lambda_{\pi N}$ varies significantly.

In the literature that was used to collect the values in Figs. 3 and 4, the authors indicated the calculations and fits were based on nucleon-nucleon interactions. This raises the question of having other mesons and baryon excitations in the determination of $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$. These additional contributions to the cross sections will have an effect on the value for $\Lambda_{\pi N}$ at a fixed $g_{\pi NN}(m_\pi^2)$ depending on the energy range (W^2) which is applicable to the model and how these additions interfere with each other. Also, the probe itself will interact with the pion after scattering, which will perturb the results. Another possibility is that in multiple pion exchange the cross section may be larger than what it should be to extract a reasonable result for $g_{\pi NN}(t)$. Clearly, the best way to determine $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$ is to use single pion production in an energy range where contributions from other mesons and excited baryons become highly suppressed in the cross sections. These cross sections should be measured as close to the pion pole as possible in order to take into account the rapid increase in the size of the cross section below momentum transfer of $t = 0.5 \text{ (GeV/c)}^2$. With the advent of high intensity, high duty factor electron accelerators like Jefferson Lab (Newport News) we can probe the nucleon with a high resolution probe to produce pions in such a way that there are no final state interactions with the probe and only a single pion per event will occur. At Jefferson Lab, cross sections can be measured at momentum transfer t of the order $10^{-2} \text{ (GeV/c)}^2$. These measurements will have

FIG. 5: Scattering Representation of the ${}^1\text{H}(e, e' \pi^+)n$ Reaction.

the additional advantage of having large statistics, which has not been previously seen at this momentum transfer. The process used to produce pions in this fashion is called pion electroproduction.

2.2 Pion Electroproduction

The meson electroproduction process describes the scattering of an electron off a target in order to produce a meson. In pion electroproduction the ejected meson is a pion. Fig. 5 schematically shows this process for producing a positively charged pion from a free proton in the laboratory frame (${}^1\text{H}(e, e' \pi^+)n$). The scattering plane is defined by the three-momentum vectors $\mathbf{k}$ and $\mathbf{k}'$ of the incident and scattered electron four-momentum vectors $P_e \equiv (E, \mathbf{k})$ and $P_e' \equiv (E', \mathbf{k}')$ respectively. The electron is scattered through an angle $\theta_{e'}$. A third four-momentum vector, $q \equiv (\omega, \mathbf{q})$, is also in this plane and describes the momentum transfer to the target

by the virtual photon $q = P_e - P'_e$. The square of q , $q^2 = (\omega^2 - \mathbf{q}^2)$, is always negative in electron scattering and we define $-q^2 \equiv Q^2$. The angle at which the virtual photon is produced is described by θ_q . The reaction plane is defined by the three-momentum vectors $\mathbf{p}_\pi$ and $\mathbf{p}_n$ of the four-momentum vectors for the scattered pion $P_\pi \equiv (E_\pi, \mathbf{p}_\pi)$ and the recoiling neutron $P_n \equiv (E_n, \mathbf{p}_n)$. The angles $\theta_{q\pi}$ and ϕ_π describe the scattering angles for the pion relative to the $\mathbf{q}$ vector and the scattering plane respectively.

Other Lorentz invariant variables are the Mandelstam variables s , t and u , defined as:

$$s = (q + P_p)^2 = (P_\pi + P_n)^2 \equiv W^2, \quad (5)$$

$$t = (q - P_\pi)^2 = (P_p - P_n)^2, \quad (6)$$

$$u = (q - P_n)^2 = (P_p - P_\pi)^2 \quad (7)$$

where P_p is the four-momentum of the target proton, and W^2 is the square of the invariant mass.

In the pion-nucleon center-of-mass frame we have

$$t = (\omega^* - E_\pi^*)^2 - (|\mathbf{q}^*| - |\mathbf{p}_\pi^*|)^2 - 4|\mathbf{q}^*||\mathbf{p}_\pi^*| \sin^2 \frac{\theta_{q\pi}^*}{2}. \quad (8)$$

(Please note that the asterisk denotes quantities in the pion-nucleon center-of-mass system.) The lowest value of $|t|$, which is denoted as t_0 , occurs at $\theta_{q\pi}^* = 0$ and is given by

$$t_0 = (\omega^* - E_\pi^*)^2 - (|\mathbf{q}^*| - |\mathbf{p}_\pi^*|)^2, \quad (9)$$

where

$$\omega_{lab} = \frac{s + Q^2 - M^2}{2M}, \quad (10)$$

$$\omega^* = \frac{s + Q^2 - M^2}{2\sqrt{s}}, \quad (11)$$

$$|\mathbf{q}_{lab}| = \sqrt{Q^2 + \omega_{lab}^2}, \quad (12)$$

$$|\mathbf{q}^*| = \frac{|\mathbf{q}_{lab}|M}{\sqrt{s}}, \quad (13)$$

$$E_\pi^* = \frac{s + m_\pi^2 - M^2}{2\sqrt{s}}, \quad (14)$$

$$|\mathbf{p}_\pi^*| = \sqrt{E_\pi^{*2} - m_\pi^2}, \quad (15)$$

M is the proton mass, and m_π is the pion mass.

2.2.1 Differential Cross Sections

In the pion-nucleon center-of-mass frame for the one photon exchange approximation (O.P.E.A.), the five fold differential cross section can be expressed as a product of two rank-2 tensors [11] [12]:

$$\frac{d^5\sigma}{dE'd\Omega'd\Omega_\pi^*} = \frac{\alpha}{2\pi^2} \frac{E'}{EQ^2} \frac{k_L}{1-\epsilon} \frac{d\sigma_v}{d\Omega_\pi^*}, \quad (16)$$

$$\frac{d\sigma_v}{d\Omega_\pi^*} = \frac{\alpha}{16\pi} \frac{|\mathbf{p}_\pi^*|}{MW} \frac{1-\epsilon}{Q^2 k_L} \mathcal{L}_{\mu\nu} \mathcal{M}^{\mu\nu}. \quad (17)$$

In Eq. 16 and Eq.17, α is the fine structure constant, $\mathcal{L}_{\mu\nu}$ is the lepton tensor, and $\mathcal{M}^{\mu\nu}$ is the hadron tensor. The transverse photon polarization parameter ϵ is given by

$$\epsilon = [1 + 2 \frac{Q^2 + (E - E')^2}{Q^2} \tan^2 \frac{\theta_{e'}}{2}]^{-1}, \quad (18)$$

evaluated in the laboratory frame, and k_L , given by

$$k_L = \frac{s - M^2}{2M}, \quad (19)$$

is the energy that a real photon would need in order to excite the hadronic system to center-of-mass energy W .

If an unpolarized incident beam is used, then Eq.17 can be rewritten as

$$\frac{d^2\sigma_v}{d\Omega_\pi^*} = d\sigma_T + \epsilon d\sigma_L + \epsilon d\sigma_{TT} \cos(2\phi_\pi^*) + \sqrt{2\epsilon(\epsilon+1)} d\sigma_{LT} \cos(\phi_\pi^*), \quad (20)$$

where $d\sigma_x = d\sigma_x/d\cos\theta_{q\pi}^*$ are the corresponding differential cross sections of the tensor elements ($x = T, L, TT, LT$). These differential cross sections depend only on Q^2 , W , and $\theta_{q\pi}^*$. The first term is the transverse differential cross section generated by the unpolarized virtual photon. The longitudinal differential cross

FIG. 6: Feynman Diagrams for Pion Production. A) is the direct channel (s-channel), B) is the pole channel (t-channel), C) is the cross channel (u-channel), and D) is the contact Kroll-Ruderman or “seagull” channel.

section ($d\sigma_L$) is produced by the longitudinal component of the virtual photon’s polarization. $d\sigma_{TT}$ is created by the transverse linear polarization component of the virtual photon, and $d\sigma_{LT}$ is the cross section originating from the interference between the transverse and longitudinal components of the photon polarization.

2.2.2 Born Term Model

At a low enough Q^2 , pion electroproduction can be approximated by one photon exchange approximation (O.P.E.A.) between the scattered electron and the nucleon system. The photon can couple to the nucleon’s charge or magnetic field, or it can couple to the electric charge of a virtual pion. These processes are shown in Fig. 6. The Feynman diagrams A and C, referred to as the s- and u-channels, represent the magnetic coupling to a nucleon and produce p-wave pions. There is

also a charge coupling associated with these channels depending on whether or not the incident nucleon has a charge. Diagram B, known as the t-channel, shows the electromagnetic coupling of the photon to the pion current $\mathbf{J}_\pi$. The last diagram, called the Kroll-Ruderman (KR) or “seagull”, represents the electric dipole amplitude, which produces charged s-wave pions. The KR-channel is used depending on how the amplitudes are calculated. If a pseudoscalar model is calculated, then the KR-channel is not used. If a pseudovector model is determined, then the KR-channel is used and contributes almost half the cross section amplitude near the pion pole. Together these diagrams represent the amplitudes of the Born term approximation for pion production. These diagrams can be modeled by using the following Lagrangians to describe the vertex [3]:

$$\mathcal{L}_{\gamma NN} = -e\bar{\Psi} \left[\gamma_\mu \mathcal{A}^\mu F_1^{p,n}(Q^2) + \frac{\sigma_{\mu\nu}}{2M} \partial^\mu \mathcal{A}^\nu F_2^{p,n}(Q^2) \right] \Psi, \quad (21)$$

$$\mathcal{L}_{\gamma\pi\pi} = e \left[(\partial^\mu \pi)^\dagger \times \pi \right]_3 \mathcal{A}^\mu F_\pi(Q^2), \quad (22)$$

$$\mathcal{L}_{\pi NN}^{PS} = ig\bar{\Psi}\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\Psi}\boldsymbol{\pi}, \quad (23)$$

$$\mathcal{L}_{\pi NN}^{PV} = -\frac{g}{2M}\bar{\Psi}\gamma_5\gamma_\mu\boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi}\Psi. \quad (24)$$

In these Lagrangians, Ψ and $\boldsymbol{\pi}$ are the nucleon and pion field operators. $\mathcal{A}^\mu$ is the electromagnetic vector potential. $F_{1,2}^{p,n}$ and F_π are the Dirac and pion electromagnetic form factors respectively. Also included are the Dirac spinors (γ_μ, γ_5) , the Pauli spin vector $\sigma_{\mu\nu}$, and the Pauli isospinor $\boldsymbol{\tau}$. The coupling term g is the strong coupling form factor $g_{\pi NN}(t)$, and e is related to the fine structure constant as $e = \sqrt{4\pi\alpha}$. Eqs. 21 and 22 are associated with the photon-nucleon and photon-pion vertices respectively. Eqs. 23 and 24 refer to the pseudoscalar (PS) or pseudovector (PV) models for the pion-nucleon-nucleon vertices respectively. When we have low pion energies, such as near the pion pole, the PV model is preferred due to its consistency with leading order chiral symmetry and low energy theorems (LET). It also fulfills PCAC at low pion energies. However, at higher pion energies the PV model has a normalization problem and so the PS model should be used.

FIG. 7: π^+ Electroproduction Prediction From Maid2000 [3] calculated at $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$ for the $^1H(e, e'\pi^+)n$ reaction. The dotted curve was produced by considering Born terms only. The solid line was produced by including Born terms plus the ρ and ω exchange along with five baryon excitations. The double curves in the top graph represent two different ϵ settings.

The contributions from the diagrams in Fig. 6 to the cross sections can be shown using Fig. 7. The contributions depend on whether the virtual photon coupling is transverse or longitudinal in nature. The magnetic coupling is transverse and the charge coupling is longitudinal. Since, for the most part, the s- and u-channel represent magnetic coupling between the photon and the nucleon, they will contribute to the transverse portion of the cross section as shown in the $d\sigma_T$, $d\sigma_{TT}$, and $d\sigma_{LT}$ graphs. The t- and KR-channels show charge coupling between the photon and the nucleon, and are attributed to the longitudinal part of the cross sections as shown in $d\sigma_L$ and $d\sigma_{LT}$. The dashed lines in these plots are the Born predictions from the MAID2000 model [3]. The interactive MAID model allows the user to select what contributes to the cross sections from a list containing the Born terms, the ρ and ω meson exchanges, and seven baryon excitations, which include P33(1232), P11(1440), D13(1520), S11(1535), S11(1650), F15(1680) and D33(1700). It also allows the scaling of the multipole contributions for the P33(1232), P11(1440) and S11(1535).

As W increases, other baryon excitations and mesons begin to contribute to the cross sections. These additional contributions can be complicated in nature and will not be discussed in any detail except where they become significant in the analysis. However, the contributions that these excitation and additional meson exchanges give to the cross section can be seen as the solid curves in Fig. 6. From this we see that the longitudinal cross section $d\sigma_L$ is not significantly affected by these additions. This would indicate that the longitudinal cross section is the best selection for extracting $g_{\pi NN}(t)$.

2.3 Rosenbluth Separation

To isolate the longitudinal cross section from the rest of the cross sections in Eq. 20, one needs to first remove the ϕ_π^* dependence. This can be done through integrating over all ϕ_π^* . $d\sigma_T$ and $d\sigma_L$ can be separated by taking data at two different values of the virtual photon polarization parameter ϵ . To do this, one

can use two different incident beam energies, moving the electron spectrometer to settings that produce the same mean values for Q^2 , W and P_π^* at both energies. Once these measurements are done and both are integrated over all ϕ_π^* , then the separation of $d\sigma_T$ and $d\sigma_L$ can be accomplished through the solving of a set of linear equations.

Chapter 3

Experimental Apparatus

3.1 Introduction

In the fall of 1997, the Pion Form Factor experiment (e93-021) was conducted at the Thomas Jefferson National Accelerator Facility in Newport News, Virginia, USA. In conjunction with the Pion Form Factor experiment we conducted the experiment to extract the Pion-Nucleon-Nucleon (πNN) Form Factor $g_{\pi NN}(t)$. The πNN form factor experiment used two accelerator energies, and Hall C's two spectrometers, the High Momentum Spectrometer (HMS) and the Short Orbit Spectrometer (SOS). Data were taken for π^+ electroproduction from a liquid hydrogen target. In this experiment the electrons and pions were detected in the SOS and HMS respectively. A special optic target assembly "Quintar" was also used to produce data for the calibration of the spectrometers and the determination of the background contributions from the target cell walls. The objective of the $g_{\pi NN}(t)$ portion of this experiment was to collect data to produce a benchmark set of cross sections as close to the pion pole as physically possible in Hall C in order to better investigate the $g_{\pi NN}(t)$ form factor. The physical constraints placed on this experiment came from the maximum electron beam energy (see section 3.2), the maximum central momentum of the SOS, the minimum HMS angle, and the minimum closure angle between the HMS and the SOS (see section

TABLE I: Kinematic Settings for e93-021. $W = 1.95$ (GeV/c)

Q^2 (GeV/c) ²	ϵ	E_e (GeV)	θ_e ($^\circ$)	E_e' (GeV)	θ_q ($^\circ$)	P_π (GeV/c)	$ t _{min}$ (GeV/c) ²
0.6	0.37	2.445	38.40	0.567	9.99 ^a	1.856	0.030
0.6	0.74	3.548	18.31	1.670	14.97	1.856	0.030

^a The smallest achievable central HMS angle was 10.5 $^\circ$.

3.4). In order to achieve our goal, two beam energies and spectrometer settings were chosen which maximized the invariant mass energy (W^2) and minimized the momentum transfer (Q^2). These two beam energies allowed us to measure cross sections at two different virtual photon polarization parameter values (ϵ), which was necessary for us to do a Rosenbluth separation of the cross sections. Table I shows the best settings as selected from simulations. These are the settings used in this experiment with the exception of the HMS angle in the first energy setting, which was set to the minimum achievable HMS angle of 10.5 $^\circ$.

FIG. 8: Schematic View of the CEBAF at TJNAF.

In this chapter, the major components of the experimental apparatus will be described. The flow will start at the accelerator and follow the particles into the

test hall, through the target and spectrometers. Finally, the trigger and data acquisition electronics will be discussed. In some areas only a brief overview will be provided while in others a more detailed description is presented. In all cases, references are given for a more in depth discussion.

3.2 Accelerator

During this experiment, the accelerator provided continuous wave (100% duty factor), unpolarized electron beams to all three halls with a maximum current load of $200\ \mu\text{A}$. The accelerator, shown in Fig. 8, consists of an injector, two linear accelerators (Linacs), and two recirculation arcs. The beam is injected at 45 MeV into the North Linac where it is accelerated up to 440 MeV. The beam then passes through the East Recirculation Arc and enters the South Linac where it is accelerated an additional 440 MeV before passing through the West Recirculation Arc or entering the Beam Switch Yard (BSY) on its way to the test halls. The beam can make up to five passes around the accelerator for a maximum energy of 4044 MeV (at the time of this experiment). For the $g_{\pi NN}(t)$ portion of this experiment, electron energies of 2445, and 3548 MeV with a current range between 10 and $100\ \mu\text{A}$ were used.

At the West End of the South Linac the beam enters the BSY to be sent to the three test halls. For Hall C, the beam passes through the BSY and enters the 3C arc where it is monitored by several devices that measure its position, profile, and current. The arc also has several quadrupole and dipole magnets to focus and steer the beam into the hall.

The profile of the beam is measured using the Superharps which are located, as pairs, in three different locations along the beam arc (see Fig. 9). These sites are surveyed with high precision for absolute position measurements. The Superharp consists of fine Tungsten wires strung in a wooden fork (see Fig. 10). The wires are moved back and forth through the beam using a stepper motor, in order to measure the horizontal and vertical beam position with a resolution of $10\ \mu\text{m}$. The

FIG. 9: Schematic View of Hall C Arc.

FIG. 10: Schematic of the Superharp. The wooden fork is strung with Tungsten with which is moved through the electron beam to measure the beam's position and profile.

Superharps are also used to determine the beam width which was found to be 100 μm for this experiment. The measurements from the Superharps are used, with very accurate field maps of the arc bending magnets, to determine the beam's energy and emittance [13]-[15].

There are five beam position monitors (BPM) which provide non-destructive beam position information along the beam line, upstream of the target (see Fig. 9), which are used during data taking. These BPMs are resonant cavities that are placed coaxially along the beam line. Their cavities are tuned in such a way that the electromagnetic TEM modes are excited at the accelerator's frequency. There are four antennas in each BPM arranged in opposing pairs that monitor the resonant modes as a function of the distance between the beam and an antenna. The beam position is then determined by the difference over the sum of the properly normalized signals produced from opposing antennas. Since this is a position relative to the central axis of the beam line, these measurements must be compared to the Superharp measurements to determine the absolute position of the beam during data taking. The precision on the beam location was 1 mm. The two BPMs nearest to the target monitor the position and angle of the beam at the target. More detailed information on the BPMs can be found in Refs. [13], [14] and [16].

The electron beam energy can be measured by using the arc dipole magnets. In this procedure only the bending magnets are used, and the corrector magnets are degaussed and turned off. The position monitors measure the beam's position at the beginning, middle, and end of the arc. The use of the Superharps makes for a more accurate measurement, with an uncertainty on the order of 10^{-4} . During the beam energy measurement, the purpose of the position monitors is to make sure the beam is steered through the central trajectory of the arc. With precisely mapped fields of the dipoles, the beam positions are used to determine the electron momentum from

$$p = \frac{e}{\theta_{arc}} \int B \cdot dl, \quad (25)$$

where p is the electron momentum, e is the electron charge and θ_{arc} is the bend

angle of the arc (43.40°). More detailed information on this is given in Ref. [17]-[19]. For this experiment, the dipoles were cycled in such a way as to reproduce the fields that were used to generate the field maps. Since the Superharps were not working for parts of this experiment, we used the BPMs to monitor the beam positions. For this procedure, the beam was first steered to the central axis of the entrance BPM, and then the current of each dipole was adjusted until the exiting beam was centered on the central axis of the exit BPM. The dipole currents were then recorded and the field integrals were calculated to determine the beam energy with an uncertainty of order 10^{-3} .

There are two beam current monitors (BCM) in the beam line along with an Unser monitor to measure the beam's current (see Fig. 9). The BCMs are resonant cavities similar to the BPMs. They are tuned to electromagnetic TEM modes that are sensitive to the current of the beam and insensitive to the beam position. Though the Unser has a more stable gain than the BCMs, its zero offset drifts significantly over short periods of time. Therefore, the Unser is only used to calibrate the BCM gain. The current from BCM2 was used for this experiment to determine the integrated charge and had an uncertainty of 0.5%. More detailed information on the BCMs can be found in [13], [14] and [16].

During the experiment we used the fast rastering magnets on the beam to reduce possible density changes in the liquid targets due to localized boiling. See Fig. 9 for the position of these magnets. The current in these magnets was driven, in a sinusoidal fashion, at a frequency of 17 kHz in the vertical direction and 24.2 kHz in the horizontal direction. These frequencies were chosen to prevent standing Lissajous patterns. This rastering results in the beam being steered in a square pattern with a side of 2.8 mm. As can be seen in Fig. 11, this pattern creates regions of greater intensity near the edges where the rastering moves the slowest. See Ref. [20] for more detailed information about the fast raster system in Hall C.

FIG. 11: Fast Raster beam pattern on target. The amplitude is 1.4 mm. The fast rastering is used to prevent localized density changes due to heat transfer from the beam to the target.

FIG. 12: Diagram of the Hall C Cryogenic Loop. The liquid target is pumped from a reservoir to the heat exchanger where it is cooled. From the heat exchanger, the liquid is sent through the lower target cell, through the upper target cell, and back to the reservoir.

3.3 Targets

The target assembly consists of three cryogenic target loops and a solid target optics assembly. The cryogenic loops are used to circulate the liquid hydrogen and liquid deuterium targets (see Fig. 12 and Fig. 13) and the optics assembly is used to calibrate the magnetic spectrometers. Each loop consists of a reservoir tank, an axial pump system, and two target cells. The first loop holds a liquid hydrogen target, the second loop is empty, and the third loop contains a liquid deuterium target. The target cells and the optics assembly are housed in an evacuated cylindrical scattering chamber in such a way that it can be raised and lowered to align the desired target cell in the beam. A more detailed description of the targets and the scattering chamber can be found in Refs. [21]-[23].

FIG. 13: Schematic View of the Hall C Target Ladder. The target ladder consists of six liquid target cells (three 4.5 cm and three 12.5 cm long) and a solid target array, called the “Quintar”, at the bottom. The sides, “beer can” and the entrance and exit windows of the target cells are made of aluminum.

3.3.1 Liquid Hydrogen Target

The target for this experiment was liquid hydrogen (LH2) which is in loop 1 of the cryogenic target assembly (See Fig. 12 and Fig. 13). The LH2 is cooled by helium gas maintained at 15 K and 1.9 MPa via a heat exchanger. The liquid hydrogen is maintained at a temperature of 19.00 K and a pressure of 165.5 kPa in order to hold the average density to 0.0723 ± 0.0004 mg/cm³. An error of 50 mK is associated with the temperature read out giving an uncertainty of 0.1% for the target density. The entrance window, exit window, and target cell wall (“beer can”) are made of aluminum with effective lengths of 191.7 mg/cm², 280.8 mg/cm² and 351.0 mg/cm² respectively. Under running conditions, the cold length of the cell is 4.53 ± 0.01 cm along the central axis of the target resulting in an effective length of 0.328 mg/cm² with a 3.0% uncertainty.

3.3.2 Solid Targets

For this experiment we used a new thin target assembly for optics calibration and empty target data. The “Quintar” (shown in Fig. 14) uses five 2.265 mg/cm² thick carbon wafers spaced 3 cm apart to define precise interaction points for use in the calibration of the magnetic spectrometers. It contains two 2.70 mg/cm² aluminum fins, spaced 2.25 cm from target center in either direction as shown in the lower diagram in Fig. 14, which are used to simulate the target windows of the 4.5 cm cryogenic targets. This assembly was designed to allow the user to select which of targets they want to use by choosing the appropriate vertical ladder position. The horizontal position is fixed.

FIG. 14: Schematic View of the Hall C Solid Targets. The target array is called the "Quintar". It consists of six carbon wafers and five aluminum fins. The carbon wafers are used to calibrate the spectrometers, and the aluminum fins are used to simulate the target cell walls of an empty target. The empty target data is used to remove the events resulting from the interaction of the beam with the cell walls.

3.4 Spectrometers

3.4.1 The Spectrometers

The High Momentum Spectrometer (HMS) is made up of three quadrupole magnets (Q1, Q2, and Q3) for focusing the particle beam onto the focal plane, one dipole magnet (D1) for selecting the momentum dispersion, and a detector package housed in the detector hut. The magnets Q1 and Q3 focus in the dispersive direction while Q2 focuses in the non-dispersive direction. Similarly the Short Orbit Spectrometer (SOS) is made of one quadrupole magnet for focusing in the non-dispersive direction, two dipole magnets for determining the momentum dispersion, and a detector package, similar to the package in the HMS, housed in its detector hut. Schematic views of the HMS and the SOS are shown in Figs. 15 and 16. The magnets are tuned point-to-point such that a particle with central momentum p_o (central momentum of the spectrometer) enters the first quadrupole along the optical axis and arrives at the center of the detection plane, which is between the two drift chambers in the detector package (see section 3.5). The magnets are mounted in-line with respect to their optical axes on carriages that are free to pivot about a fixed common central bearing while riding on a rail system. The detectors are mounted on supports attached to the same carriages as the magnets, and their own carriages support the detector huts.

For the HMS, the quadrupole magnets are cold iron superconducting magnets cooled with 4K liquid Helium from the End Station Refrigerator, and the dipole magnet is a warm iron superconducting magnet. The magnetic field of the dipole magnet is monitored with NMR probes placed in the regions of uniform magnetic field and are stable to 10^{-4} . The magnetic fields of the quadrupole magnets are set by current and are also stable to 10^{-4} .

The SOS magnets are non-superconducting and are water cooled by the Low Conductive Water system. The magnetic fields are monitored using Hall probes and are set by the field strength. Additional focusing occurs in the dipoles due to

FIG. 15: Schematic View of the HMS.

FIG. 16: Schematic View of the SOS.

the curvature of the pole tips which makes the optics sensitive to the saturation effects of the iron cores. The magnetic fields are stable to 10^{-4} . The operational specifications for the HMS and the SOS are shown in Table II. A more detailed discussion of the spectrometers can be found in Refs. [13], [22], and [24]-[26].

TABLE II: Specifications for the HMS and SOS.

	HMS	SOS
Maximum Central Momentum	7.4 GeV/c	1.75 GeV/c
Optical Length	26.0 m	7.4 m
Momentum Acceptance	± 10 %	± 20 %
Momentum Resolution	< 0.1 %	0.1 %
Angle Range	10.5° - 90.0°	14.5° - 168.4°
Minimum Closure HMS to SOS	30.5°	
Solid angle	6.7 msr ^a	7.5 msr
Angular Acceptance - In-Plane	± 27.5 mrad ^a	± 57.5 mrad
Angular Acceptance - Out-of-Plane	± 70 mrad ^a	± 37.5 mrad
Angular Resolution - In-Plane	1.0 mrad	2.5 mrad
Angular Resolution - Out-of-plane	2.0 mrad	0.5 mrad
Extended Target Acceptance	± 7 cm	± 1.5 cm
Vertex Reconstruction Accuracy	2 mm	1 mm
^a After HMS-100 tune and using large collimators.		

3.4.2 Repositioning the HMS Q1

Just prior to this experiment, the HMS Q1 and collimator box were moved backwards from the target 40.0 cm in order to allow the detection of particles at more forward scattering angles. In the original configuration, the collimator box (mounted in front of Q1) was 126.2 cm from the center of the target and the HMS had a minimum forward angle of 12.5°. Similar to the SOS, in front of the collimator box is mounted an extension snout which limits the air gap between scattering chamber and the spectrometer to 15 cm. After moving the magnet, the minimum angle became 10.5° and a new snout was installed to maintain the same 15 cm gap. The moving also required that a new tune of the optics be

accomplished.

Spectrometer optics refers to the settings of the magnetic fields in the spectrometer. When these fields are properly set for a central momentum p_0 , a particle with momentum p_0 will follow a path along the optical axis of the first quadrupole magnet through the spectrometer to the center of the detection plane (or focal plane). Once these settings are determined for a particular p_0 , they should not need to be changed unless one or more of the magnets change position. Since we moved Q1 of the HMS back 40 cm from the target, we needed to adjust the settings in order to restore the original point-to-point tune. These new field settings were obtained through iterating a TRANSPORT simulation [27] with the new magnet positions [28]. This new tune was named “HMS-100”.

3.4.3 Setting The Magnetic Fields

HMS

From previous work, we had noted that the convergence point on the focal plane moved in the positive x direction with increasing p_0 . This can be attributed to residual fields in the quadrupole magnets. The original software, which set the quadrupole fields by monitoring the magnet current, assumed the fields to be linear with respect to the current and that there were no residual fields present. However, measurements of the fields, with a Hall probe near the pole tip in the center of the magnets, showed significant residual fields. In order to take these fields into account, the program was updated to determine the field strength using:

$$B^\pm = \pm B_0 + \alpha I, \quad (26)$$

where α is a fitting parameter which has units of Tesla/amp and the sign correlates to the polarity of the magnet current I . With this offset now in the calculation, one needs to make sure that the field is set on the same slope of the hysteresis

curve. The quadrupole magnets are set on the decreasing slope. Therefore, all setpoints need to be approached from a higher absolute current value (*e.g.*, if the original setpoint is at +100 A and the next is at +150 A, then ramp up the current to +250 A and return to 150 A. If the polarity is negative, then simply change the signs of this example.). After a polarity change, the magnets should be degaussed by ramping the current to ± 500 A (depending on the polarity of the next setpoint), and back to 0 A. The new setpoint is then set according to the example. This results in a stable reproducible field on the order of 10^{-4} .

The dipole is set in a similar fashion. However, since the dipole uses a NMR probe to monitor the fields, the new setpoint can be set directly. The dipole is degaussed after polarity changes by ramping up the current to ± 1500 A (the sign depends on the next setpoint), and then the new setpoint is set directly. This takes several minutes, but results in a stable field on the order of 10^{-5} .

SOS

Since the SOS uses Hall probes to monitor the fields, new setpoints can be set directly after degaussing if necessary. Degaussing is necessary when the new setpoint is lower than the previous one, or a polarity change is required. If degaussing is required the current is ramped down to 0 A and, if needed, the polarity is changed. Next the current is ramped up to ± 1000 A and then returned to 0 A. At this point the polarity is changed regardless of the new setpoint and then the current is ramped up to ∓ 200 A and again returned to zero. Now the magnets are degaussed. Finally, the polarity is changed again and the current is ramped to the new setpoint. This procedure ensures the hysteresis curve is reproducible on the upward slope. These magnets are stable on the order of 10^{-4} .

3.4.4 Collimators

Mounted in front of the quadrupoles of each spectrometer are the collimator

FIG. 17: HMS Collimators. Both collimators are made of an alloy called “Heavymet” which has a density of 170.0 mg/cm^3 . This gives the large collimator and the sieve slit an effecting length of 1079 mg/cm^2 and 539.75 mg/cm^2 respectively. The hole size in the sieve slit is 0.508 cm , except for the smaller center hole, which is used for reconstruction resolution measurements. The missing holes are for top-bottom, left-right reconstruction measurements. The dimensions of the large collimator are of the new one, which was installed after Q1 was moved to its experiment position.

boxes that contain three collimators each. Only two of the three collimators in each box were used in this experiment. The first are the sieve slits, which are used for optical studies, and the second are the large octagonal apertures that are used to limit the solid angle. Both collimators are made from an alloy called “Heavymet” which consists of 90% Tungsten and 10% CuNi. Heavymet has a density of 170.0 mg/cm^3 which makes the sieve slits 539.75 mg/cm^2 thick and the large collimators 1079 mg/cm^2 thick. The difference between the HMS and the SOS large collimators is the orientation (see Figs. 17 and 18). The solid angle acceptance of both spectrometers is similar. As for the sieve slit collimators, we notice that the patterns are similar, though the three middle columns in the SOS collimator are spaced closer together than the other columns. The hole sizes are the same for both spectrometers (0.508 cm) with the center hole smaller in order to measure the resolution of the angular reconstruction. Two holes are missing in order to verify top-bottom and left-right reconstruction.

FIG. 18: SOS Collimators. These collimators are made of the same stock as the ones in the HMS. The hole sizes in the sieve slit are the same as that in the HMS sieve slit.

After moving Q1 back, a Monte-Carlo study of the original location and the new location [29] found that in the final position the entrance aperture of Q1 did

not affect the original solid angle acceptance of the HMS. Therefore, in order to match the solid angle acceptance for the original position, the collimator shown in Fig. 17 was designed and built. This new collimator has the same effective thickness as the old one.

3.5 Detector Packages

FIG. 19: Schematic View of the Detector Package. This package consists of two drift chambers (DC1 and DC2), four scintillator planes (S1X, S1Y, S2X and S2Y), a Čerenkov detector, and a lead glass calorimeter. The calorimeter is tilted 5° to prevent particles from passing through the cracks. The detection plane is a fictitious plane centered between DC1 and DC2. The focal plane is another imaginary surface created by the focal point as it moves in response to the momentum distribution of the particle beam. The curvature of the focal plane is exaggerated for visual clarity.

The detector packages used were the Hall C standard equipment. The HMS and the SOS have similar detector packages that consist of two drift chambers (DC1 and DC2), two pairs of scintillator hodoscopes (SX1, SY1, SX2, and SY2), a gas Čerenkov detector, and a segmented lead glass calorimeter. For a more

detailed description of the detectors, please see Ref. [13], [24], [25] and [30]. See Fig 19 for a schematic of the arrangement for these packages.

3.5.1 Drift Chambers

FIG. 20: Typical Drift Chamber Cell. As a charged particle passes through the drift chamber, it ionizes the gas. The electrons are attracted to the positive potential on the sense wire where they collect and change the potential on the wire. This potential change is differentiated and a signal is sent to indicate a charge particle passed through the chamber.

The first detectors that the charged particle will enter are the drift chambers (DC). After the particle enters the DCs, it begins ionizing the gas along its path. When the particle passes an anode wire, the knocked out electrons, in the region of the wire, are attracted to the wire which has a positive potential applied to it (see Fig. 20). As the initially liberated electrons approach the anode wire, they are accelerated, which causes them to produce more ionization. This creates an avalanche effect. The mixture of gases in the chamber contains a quenching gas,

which localizes the avalanche by absorbing most of the photons generated by the avalanche. Eventually, the electrons will build up on the anode wire, which causes a change of the potential on the anode wire. This difference in potential is used to indicate that a charged particle passed by the anode wire and initiates a signal used to start a multi-hit Time-to-Digital Converter (TDC). The TDC stop signal comes from the trigger formed in the trigger logic circuit (see section 3.6). The drift chamber TDCs measure the time between the detection of the electrons on the anode wire and the formation of the trigger.

Using the hodoscope TDCs to determine the time that the charged particle passed the focal plane, we can determine the amount of time it took for the liberated electrons to drift to the anode wire. This time can be converted to the distance of closest approach that the particle made as it passed by the wire. Having this information, along with the same type of information from the other anode wires, the trajectory of the charge particle can be calculated.

The drift chambers in both spectrometers are similar in construction, with some notable differences. The chambers in the HMS are identical, as are the chambers in the SOS. Each chamber contains six detection planes which have wires strung in an alternating fashion starting with a field wire and followed by a sense wire (anode wire) as shown in Fig. 21. The X and X' wires measure the position of the track in the dispersive direction, while the Y and Y' wires measure the transverse position. There are no Y or Y' wires in the SOS chambers. The stereo wires (U,U' and V,V') are rotated $\pm 15^\circ$ ($\pm 60^\circ$ for the SOS) with respect to the X wires. The redundancy of the X and Y wires as well as having the stereo wires provide the ability to resolve left-right ambiguities, and multiple hit resolution. The distance between the chambers in the HMS is 81.2 cm, and for the SOS is 45.7 cm. The focal plane is set at the center between the chambers. The gas in the chambers is a standard mixture of Ar(49.5%), Ethane(49.5%), and Isopropyl Alcohol(1.0%), where the Isopropyl Alcohol is the quenching gas. The resolution of HMS chambers is 150 μm , and 200 μm for the SOS. Just before this experiment, the HMS drift chambers were removed to be re-strung and were replaced to within 1 mm of the original positions. A better description of these

drift chambers can be found in Ref. [31].

FIG. 21: Schematic View of the HMS and SOS Drift Chambers.

3.5.2 Hodoscopes

Both the HMS and SOS are equipped with two pairs of scintillator hodoscopes. A pair of hodoscopes consist of two planes, one plane in the X- and one in the Y-direction (see Fig. 22). Each plane is made up of scintillating bars with light guides and photomultiplier tubes (PMT) on both ends of the bars. In order to provide good coverage with no holes, the paddles in each layer are overlapped by 0.5 cm. The purpose of the hodoscopes is to provide a clean trigger to start the data acquisition (see Sect. 3.6), and to measure the time of flight (TOF) of the particle between the two pairs of hodoscopes for calculation of the particle's

FIG. 22: Hodoscopes. The Hodoscopes on the left are those for the SOS. The front hodoscope is smaller than the one in the back. For the front hodoscope, the x-plane has 9 elements, 36.5 cm x 7.5 cm x 1.0 cm, and the y-plane has 9 elements, 63.5 cm x 4.5 cm x 1.0 cm. The back hodoscope has 16 elements, 36.5 cm x 7.5 cm x 1.0 cm, in the x-plane, and 9 elements, 112.5 cm x 4.5 cm x 1.0 cm, in the y-plane. The hodoscope on the right is the front one for the HMS. The back one is similar to the front. The x-plane has 16 elements, 75.5 cm x 8.0 cm x 1.0 cm, and the y-plane has 10 elements, 120.5 cm x 8.0 cm x 1.0 cm. Each element has a PMT on each end.

velocity, $\beta = v/c$ (see section 4.4 for β calculations). A description of the trigger electronics is given in section 3.6. The timing resolution has been determined to be ~ 100 psec per bar in the HMS and 80-100 psec per bar in the SOS. Since, in this experiment, the HMS was tuned for positively charge particles, β was used to separate the recoiling protons from the π^+ mesons. The use of the β in particle identification is described in section 4.4. A more detailed description of the hodoscopes, TOF calculations, and timing resolutions can be found in Ref. [13].

3.5.3 Gas Čerenkov Detectors

The gas Čerenkov counters operate on the principle that a charged particle will produce an electromagnetic shockwave (Čerenkov radiation) if it travels through a medium faster than the speed of light in that medium. This implies that the production of Čerenkov radiation is heavily dependent on the speed of the charged particle and the index of refraction n of the medium. Also, for a given medium, two charged particles of significantly different masses, but the same momentum, will have different probabilities of creating Čerenkov radiation. So, this principle can be used to differentiate between two species of charged particles, if n is set properly. The HMS Čerenkov detector is filled with Perfluorobutane (C_4F_{10}) at 79 kPa, while the SOS Čerenkov detector is filled with Freon-12 (CCl_2F_2) at 101 kPa. This sets n to 1.0011 in both detectors, which means that the pion detection threshold is 3 GeV/c and that of the proton is 20 GeV/c. However, the electron momentum threshold is 11 MeV/c. Since the electron momentum is in the GeV/c range and the spectrometers have a maximum momentum of 2.6 GeV/c for this experiment, the electrons will produce Čerenkov radiation in both spectrometers, unlike the heavier hadrons. Section 4.4 describes the use of the gas Čerenkov detectors for particle identification in this experiment. A detailed description of these gas Čerenkov detectors can be found in Ref. [25].

3.5.4 Lead Glass Calorimeters

The purpose of the lead glass calorimeters is to measure the energy of the detected charged particles. As a high-energy electron passes through the calorimeter it loses energy through bremsstrahlung radiation. The bremsstrahlung photons then go on to produce electron/positron pairs which, in turn, lose their energy through bremsstrahlung radiation as well. The thickness of the lead glass blocks is such that this process continues until the energy of the initial electron is exhausted. As the showering electron/positron pairs pass through the blocks, they cause scintillation in the plastic fibers that traverse the lead blocks from end to end. The light generated is collected at the end of the blocks by PMTs and the sum of the outputs of the PMTs is proportional to the energy of the initial electron.

In contrast, as a charged hadron passes through the calorimeters, they do not lose energy through bremsstrahlung radiation, but through ionization as described by the Bethe-Bloch equation [32]. A pion would not be stopped in these calorimeter and so not all the pion energy will be deposited (usually only about 300 MeV is deposited) unless it undergoes a charge exchange through the process $n\pi^+ \rightarrow p\pi^0$ or $p\pi^- \rightarrow n\pi^0$. If the pion undergoes a charge exchange, the resulting π^0 will decay into two photons which will cause an electron shower similar to that caused by the bremsstrahlung photons. This charge exchange is the primary cause of particle misidentification in the calorimeters because all of the pion's energy is left in the calorimeter, just like the electron's energy. The use of the calorimeters in particle identification is discussed in section 4.4.

The calorimeters in both spectrometers are almost exactly the same with the SOS calorimeter having fewer blocks in each stack. They consist of four stacks (layers) of thirteen blocks (eleven for the SOS) as shown in Fig. 23. Each block is square with 10 cm on the edge and 70 cm long. The blocks are made of lead with plastic fibers traversing the length of the interior, and they have a radiation length of 9.80 g/cm² which means the electron will deposit most of its energy in the first layer. In order to prevent missed particles which pass through the gaps

FIG. 23: HMS Lead Glass Calorimeter. The SOS Calorimeter is similar, but with 2 less blocks in each stack. Each bar is made of lead with plastic fibers traversing the length of the interior. They have a radiation length of 9.80 g/cm^2 each.

between the blocks, the calorimeters are tilted 5° relative to the beam axis.

3.6 Triggers and Data Acquisition

3.6.1 Pretriggering

FIG. 24: General Schematic of the Trigger Logic.

The HMS and the SOS use similar trigger logic which is shown in Fig. 24. The trigger logic is a single-arm type which generates a pre-trigger (*PRETRIG*) when a particle arrives in the detectors. The HMS and SOS were configured so that a pre-trigger would be generated if an electron was detected in the SOS or a pion was detected in the HMS. The triggering begins with the hodoscopes in

Diagram illustrating the Trigger System for the T2000 experiment.

The system processes an Analog signal from PMT's through a Splitter, a Discriminator (Discr.), and a Lecroy 4564 OR gate. The output of the OR gate is connected to four delay lines (S1X, S1Y, S2X, S2Y) which are connected to the Trigger Electronics. The output of the OR gate is also connected to a TDC (T) and an ADC (A). The output of the ADC is connected to a TDC (T). The output of the TDC is connected to a TDC (T). The output of the TDC is connected to a TDC (T).

Legend:

- = Delay Line
- = TDC
- = ADC
- 86/128 = SOS/HMS
- = Discriminator

Only S1X is shown; others are similar.

Logic for S1X:

S1X1+
.
.
S1XN+
S1X1-
.
.
S1XN-

N = Total Number of PMT's

FIG. 25: Schematic of the Hodoscope Trigger Logic.

FIG. 26: Schematic of the Gas Čerenkov Electronics.

FIG. 27: Schematic of the Lead Glass Calorimeter Electronics.

There are five more inputs to the main trigger logic in Fig. 24 which come from the Gas Čerenkov detector and lead glass calorimeter in each detector hut. The outputs of the Gas Čerenkov's PMTs are summed together and then discriminated to generate two signals for particle identification used in determining which branch of the trigger logic will be used in producing a pretrigger (see Fig 26). These signals are called $\check{C}$ and NOT $\check{C}$. If an electron is detected, then the $\check{C}$ will become active and veto the pion branch. Alternately, if no electron is detected, then the NOT $\check{C}$ will veto the electron branch (see Fig. 24).

The calorimeter produces the last three signals needed in the trigger logic ($PRHI$, $PRLO$, and $SHLO$). In Fig. 27 we see all of the PMTs in each layer are summed together to find the total energy for each layer. The energy for the first layer is used in the creation of $PRHI$ and $PRLO$ which indicate that this energy is between a high and low threshold. Finally, all of the layers are summed together to arrive at the total energy deposited in the calorimeter ($SHSUM$). $SHLO$ indicates that $SHSUM$ is greater than a minimum threshold.

As was stated earlier, the SOS trigger electronics were configured for the detection of electrons. If an electron is sensed in the SOS, then the Gas Čerenkov will send a signal to enable the $ELLO$ gate (see Fig. 28). At this point, in order to get a $PRETRIG$ signal out, the electron would need to have passed through three out of the four hodoscope planes, which also sends the $S1$ and $S2$ signal to the STOF gate, and deposit sufficient energy in the first layer of the calorimeter to exceed the minimum threshold for the that layer ($PRLO$). However, a pretrigger signal will also be generated if any particle passes through three out of four hodoscope planes and delivers enough energy to the calorimeter to overcome the minimum threshold energy for the entire calorimeter, but did not give too much energy to the first layer. This paragraph can be summed up in a single Boolean expression, $PRETRIG \equiv (PRLO \bullet 3/4(S1X \bullet S1Y \bullet S2X \bullet S2Y) \bullet \check{C}) + (PRHI \bullet SHLO \bullet 3/4(S1X \bullet S1Y \bullet S2X \bullet S2Y))$.

The main trigger logic for the HMS was configured in the same manner as that for the SOS. However, the $PRLO$ input was not connected to the $PION1$ gate because a pion would not deliver enough energy to the first layer of the

calorimeter to produce this signal (see Fig. 28). For a pretrigger to be sent to the Trigger Supervisor in this circuit, a pion would need to pass through three out of the four hodoscope planes and deposit the correct energy in the calorimeter to generate the *PRHI* and *SHLO* signals. A pion or a proton would satisfy this last condition. If the particle were a positron, then the requirements to produce a pretrigger would be the same as those described for the electron in the SOS logic. So, to sum this up in Boolean fashion, $\text{PRETRIG} \equiv (\text{PRLO} \bullet \frac{3}{4}(\text{S1X} \bullet \text{S1Y} \bullet \text{S2X} \bullet \text{S2Y}) \bullet \text{NOT } \bar{C}) + (\text{PRHI} \bullet \text{SHLO} \bullet \frac{3}{4}(\text{S1X} \bullet \text{S1Y} \bullet \text{S2X} \bullet \text{S2Y}))$.

FIG. 28: Trigger Logic for Pion Form Factor Experiment TJLAF-e93-021. The top circuit was used for the electron arm (SOS) and the bottom was used for the pion arm (HMS).

TABLE III: 8LM Programming.

Output Name		Triggers or Controls Needed		
<i>HMSPRETRIG</i>	<i>HMSPRETRIG</i>		<i>TSEN1</i>	
<i>SOSPRETRIG</i>		<i>SOSPRETRIG</i>	<i>TSEN1</i>	
<i>COIN.PRETRIG</i>	<i>HMSPRETRIG</i>	<i>SOSPRETRIG</i>	<i>TSEN1</i>	
<i>PED.PRETRIG</i>	<i>PED.PRETRIG</i>	<i>TSGO</i>	$\overline{TSEN1}$	
<i>HMSTRIG</i>	<i>HMSPRETRIG</i>		<i>TSEN1</i>	$\overline{TSBUSY}$
<i>SOSTRIG</i>		<i>SOSPRETRIG</i>	<i>TSEN1</i>	$\overline{TSBUSY}$
<i>COIN.TRIG</i>	<i>HMSPRETRIG</i>	<i>SOSPRETRIG</i>	<i>TS EN1</i>	$\overline{TSBUSY}$
<i>PED.TRIG</i>	<i>PED.PRETRIG</i>	<i>TSGO</i>	$\overline{TSEN1}$	$\overline{TSBUSY}$

The 8LM uses the three control signals to produce eight output triggers in accordance with the program in Table III. Also if a coincidence has occurred between the HMS and SOS pretriggers, it creates a coincidence trigger, which becomes two of the eight outputs. In the 8LM, the pretriggers are split into two paths, one of which leads directly out to scalers and TDCs. The second path goes to control circuits that combine the pretriggers with the control signals to generate the triggers for the TS. These triggers are only sent to the TS, scalers, TDCs and the retiming gates when the conditions shown in Table III are satisfied.

The coincidence pretrigger is created when the SOS PRETRIG and the HMS PRETRIG signals overlap in time. During this experiment both the HMS and the SOS PRETRIG signals were 15 ns wide. This gave a total coincidence window of 30 ns as can be seen in Fig. 30. This figure will be discussed in detail in section 4.6.3.

The TS is a programmable device created by TJNAF to control the timing between the pretriggers of the spectrometers and the spectrometer ADC/TDC readout electronics. When it is not busy, the TS receives up to four triggers from the 8LM. When the 8LM provides the TS with triggers, the TS will process a fraction of these triggers depending on the prescaling values programmed into the

FIG. 30: Typical HMS TDC Output. The shaded channels show the coincidence window. Each channel is 0.1 ns wide. This plot will be explained in detail in section 4.6.3.

TS at the start of the run. The prescaling values are determined by the shift's Run Coordinator who selects them accordingly in order to reduce computer dead time. For this experiment, all of the coincidence triggers were used to read the ADCs and TDCs. The single triggers (those without a coinciding trigger from the other spectrometer) were prescaled as needed.

The output of the TS is two sets of very long gate signals (usually $>100\ \mu\text{s}$ as preprogrammed) used to enable the reading of the ADCs and TDCs in each spectrometer. The length of these signals is set to be the minimum amount of time needed to read the ADCs and TDCs. The gating signals are ANDed with the delayed ("retimed") triggers associated with them which starts the readouts. The delay of the triggers is important because, in the case of a coincidence, the long gate signal is started by whichever trigger arrives at the 8LM last. In this case, the timing of the gating signal would be correct for the last trigger, but too late for the first. This would cause incorrect readings of the ADCs and mistiming of the TDCs that belong to the spectrometer that generated the first trigger.

3.6.3 Data Acquisition

Data acquisition was accomplished using the CODA (CEBAF Online Data Acquisition) [34] software run on Hewlett-Packard 735 Unix workstations. The output of the TS starts the readout of the ADCs and TDCs, which are located in FASTBUS and VME crates inside the detector huts. The ROC (Read Out Controller) CPU's controls these crates and are read directly by the CODA software. The CODA Event Builder consolidates the data, adds a header, and writes the entire package to disk. Also, every 30 seconds, the EPICS software reads out the status of the magnets, the target, the beam position, and the accelerator. For this experiment, 6.6 million coincidence events were used to determine the cross sections for both energy settings.

Chapter 4

Data Analysis

4.1 Introduction

In this chapter, the analysis of the raw data will be discussed. It starts with an overview of the replay engine, which reads the raw data from disk and uses this information to reconstruct the event kinematics at the target. From there, this chapter will present the spectrometer calibrations, particle identifications, background subtractions, and the significant detection efficiencies. In order to aid in the reading, we need to establish some definitions.

The first items to define are the laboratory and spectrometer coordinate systems. The origin of the lab system is at the center of the target with the positive z-axis pointing downstream along the incident electron beam (see Fig. 31). The x-axis points to the right of the beam when facing downstream and the y-axis points to the floor.

The spectrometer coordinate system is rotated 90° clockwise about the z-axis when compared to the lab frame (see Fig. 31), and the z-axis points along the optical axis of the spectrometer with the detector hut downstream. This is the system used to define the focal plane and target coordinates that are reconstructed from the raw data. The labels of these axes are dependent on where the origin is located. The focal plane coordinates are referenced in respect to the intersection

FIG. 31: Coordinate Systems. The z -axes point down stream of the beam. In the case of the laboratory frame, the beam is the incident electron beam. For the spectrometer, the beam is the scattered particle beam. The target coordinates (subscript “tar”) are reconstructed from the focal plane coordinates (subscript “fp”) (see section 4.3.3). The quantity z_{tar} is a projection of y_{tar} onto z_{lab} .

point of the optical axis and the focal plane between the drift chambers and are labeled x_{fp} , y_{fp} , and z_{fp} . The target axes are referenced at the center of the target and are labeled as in x_{tar} and y_{tar} . z_{tar} is somewhat misleading. One would think that it is the orthogonal axis to the x_{tar} and y_{tar} plane, but in reality it is defined as the projection of the y_{tar} variable onto a vector, which is parallel to the z-axis in the lab frame as shown in Fig. 31. z_{tar} can be calculated from:

$$z_{tar} = y_{tar} \sin \theta_1 + \frac{\cos \theta_1}{\tan \theta_2} \quad (27)$$

where θ_1 is the angle of the spectrometer relative to z_{lab} in the laboratory frame and $\theta_2 = \theta_1 - y'_{tar}$ for the HMS and $\theta_2 = \theta_1 + y'_{tar}$ for the SOS. The primed variables, such as x'_{fp} , y'_{fp} , x'_{tar} and y'_{tar} refer to the slope of the particle's trajectory (*e.g.*, dx/dz and so on) at the focal plane or target, and are measured in units of radians.

4.2 Replay Engine

Each event was decoded and reconstructed using the Hall C analysis software (Hall C Replay Engine). This engine is described in detail elsewhere (Refs. [13] and [22]), so only a brief discussion will be given here.

The Hall C Replay Engine extracts the raw data which were stored by CODA during data taking and for each spectrometer, on an event by event basis, determines whether the event is valid (*e.g.*, did the event occur in the fiducial volume?). If the event is valid, then its trajectory, and focal plane and target coordinates (including the primed coordinates) are reconstructed for each spectrometer. Using these reconstructed quantities and the data from the beam line equipment and target monitoring systems, the code calculates the event's kinematics. The program also determines if events in both spectrometers occurred within a user defined coincidence time window and calculates items such as the missing energy and the missing mass. In this experiment the missing mass was that of the neutron. After this information is determined, the engine then calculates the detector efficiencies and outputs the results, in the form of histograms, ntuples, and ASCII

FIG. 32: Correction to the SOS central momentum [28]. This is the result of a decrease in effective field length. The circles are the result from a $^1H(e, e'p)$ analysis of this experiment. The triangles are from experiment TJNAF-e91-003. The solid line is a plot of the parameterization.

files, for use in further analysis.

During the data analysis, we made a few upgrades to the replay engine. These changes consisted of updating the program to include the determination of which spectrometer detected the electron and chooses the appropriate coordinate system, changing the energy loss calculation to be based on the Bethe-Bloch formula [32] which includes relativistic corrections (these losses lie between 0.5 and 3 MeV), and adding a subroutine to take into account saturation effects of the SOS dipoles.

During this experiment, some of the reconstructed central momenta (p_0) of the SOS were actually lower than was expected due to the saturation of the magnets at high momentum settings. Since the magnets are made of iron, there is a value of the field strength beyond which the field shape changes and the effective field length decreases due to the saturation of the iron. Since the reconstruction of the momentum of the particle depends heavily of our knowledge of these characteristics, any change in them will result in the momentum being calculated incorrectly.

The central momentum can be corrected by using the expression:

$$p_0^{corr} = p_0(1 + \epsilon_p), \quad (28)$$

where $\epsilon_p = \epsilon_p(p_0)$ is a correction term. Using the data from this experiment and from the following experiment (TJNAF e91-003), we found the saturation point occurring at the central momentum of $p_0 = 1.5$ GeV/c. These data also showed that ϵ_p can be parameterized as:

$$\epsilon_p(p_0) = a + b(p_0 - 1.5)^2 \quad p_0 \geq 1.5 \text{ GeV/c} \quad (29)$$

$$\epsilon_p(p_0) = c + dp_0 \quad p_0 \leq 1.5 \text{ GeV/c}. \quad (30)$$

The quantities a , b , c , and d are fitting parameters and their values are shown in Table IV. Fig. 32 shows the plot of the correction at central SOS momenta for this and the following experiments.

Similarly, if the central momentum was affected by the saturation of the magnets, then so were all the other momenta in the SOS. The momentum in the spectrometers is reported as δ , which is the difference between the reconstructed momentum of the particle and the central momentum of the spectrometer normalized to the central momentum (*ie.* $\delta = (p_{recon} - p_0)/p_0$, where p_{recon} is the reconstructed momentum of a particle passing through the detector). δ was also corrected by:

$$\epsilon_\delta(p_0) = (x'_{fp})^2 \sum_{i=1}^3 a_i \cdot p_0^i \quad p_0 \geq 1.0 \text{ GeV/c}, \quad (31)$$

$$\epsilon_\delta(p_0) = 0.15(x'_{fp})^2 \quad p_0 \leq 1.0 \text{ GeV/c}, \quad (32)$$

TABLE IV: Coefficients for the Corrections to the SOS p_0 and δ .

ϵ_p	a	b	c	d
	0.002	-0.17	0.006	-0.0027
ϵ_δ	a_0	a_1	a_2	a_3
	0.47	-1.37	1.81	-0.76

where again ϵ_δ is a correction term to $\delta_{corr} = \delta(1 + \epsilon_\delta)$. The a_i s are fitting parameters and their values can be found in Table IV. For the highest values of x'_{fp} ($\pm .1$ mrad) and the $p_0 = 1.715$ GeV/c, ϵ_δ is -0.004. Both corrections are positive for low central momenta and negative for high central momenta [28].

These corrections were added to the replay engine.

4.3 Spectrometer Calibration

The spectrometers used in Hall C are described in chapter 3. In this section, the calibration of the spectrometer optics will be discussed. For a detailed discussion of the calibration procedures please see Ref. [28].

4.3.1 Reconstruction of Target Quantities

The term “reconstructed target quantities” refers to target coordinates x_{tar} , y_{tar} , x'_{tar} , y'_{tar} , and δ which describe the event vertex position in the target and the trajectory of the scattered particles (see section 4.1). These values are determined by projecting the focal plane quantities x_{fp} , y_{fp} , x'_{fp} , and y'_{fp} back to the target. The focal plane quantities are found through the analysis of the trajectory of the particles in the detector stack. The projection to the target can be determined from the focal plane quantities using:

$$\xi_{tar}^i = \sum_{j,k,l,m}^N \mathcal{M}_{j,k,l,m}^i x_{fp}^j x_{fp}^{'k} y_{fp}^l y_{fp}^{'m}. \quad (33)$$

In Eq. 33, the term on the left-hand side (ξ_{tar}^i) represents the target values x_{tar} , y_{tar} , x'_{tar} , y'_{tar} , and δ ; and the index i keeps track of which one is calculated. On the right-hand side, $\mathcal{M}$ is the reconstruction matrix with j , k , l , and m having the values of (1..N) such that $j + k + l + m \leq N$. The value of N is the order of the matrix (5 for the HMS and 6 for the SOS) and is determined, along with the matrix elements, through transport simulation and iterative fitting of experimental data.

FIG. 33: HMS Sieve Slit Reconstruction [28]. Overlaid onto the sieve slit pattern is the large collimator acceptance. This is the result after the iterative process. The lack of events in the corners is due to limited acceptance.

FIG. 34: HMS Quintar and Sieve Slit Reconstruction [28]. Top: Reconstruction of z_{tar} for all five carbon targets in the Quintar, overlaid by the sum of these peaks. Bottom: Reconstruction of the x'_{tar} and y'_{tar} for the sieve slit hole pattern as generated from the central carbon target.

TABLE V: Resolutions of reconstructed target quantities for the HMS and SOS. The HMS was set to 2.2 GeV/c, while the SOS was set to 1.65 GeV/c. The range in values represents the effect of moving from the center of the collimator to the edge.

	HMS	SOS
x'_{tar} (indiv. holes)	1.8 mrad	0.3-0.5 mrad
x'_{tar} (columns)	1.8-2.1 mrad	0.3-0.8 mrad
y'_{tar} (indiv. holes)	0.3-0.7 mrad	2.4-2.7 mrad
y'_{tar} (rows)	0.8-1.0 mrad	3.1-3.3 mrad
y_{tar} (mean)	2.0 mm	0.9-1.1 mm

The initial values of the matrix elements are determined from simulation using the COSY INFINITY program [35]. Then, using data from elastically scattered electrons from the thin ^{12}C targets on the “Quintar” (chapter 3) and the sieve collimators on the SOS and HMS, the target quantities, as projected onto the collimator, are compared to the known hole positions in the collimator. The matrix element values are then adjusted until the reconstructed target values at the collimator are (event by event) reconstructed to the center of the nearest hole. The new matrix is used in the next iteration. This continues until the reconstructed target quantities at the collimator converge with the known hole positions. Figs. 33-36 show results of this procedure.

Table V shows the final resolutions of the target quantities, for the HMS and SOS, as a result of this process. An important note here is that x_{tar} is assumed to be zero in the target reference frame, since the beam is assumed to be centered at $x_{tar} = 0$. A detailed description of this procedure can be found in Ref. [36]. For more detailed discussion of how this was done for this experiment, please see Ref. [28].

FIG. 35: SOS Sieve Slit Reconstruction [28]. Overlaid onto the sieve slit pattern is the large collimator acceptance. This is the result after the iterative process. The lack of events in the corners is due to limited acceptance.

FIG. 36: SOS Quintar and Sieve Slit Reconstruction [28]. Top: Reconstruction of z_{tar} for all five carbon targets in the Quintar, overlaid by the sum of these peaks. Bottom: Reconstruction of the x'_{tar} and y'_{tar} for the sieve slit hole pattern as generated from the central carbon target.

4.3.2 Spectrometer Offsets

Using the target quantities, we can calculate other physical quantities such as W , Q^2 , or the Mandelstam variable t , if we know the spectrometer angles θ_{HMS} and θ_{SOS} , and the incident energy. The incident energy is known to one part in 10^3 , and the spectrometer angles are very well surveyed, although small deviations can still occur in these quantities. Calibration of the spectrometers can minimize these deviations, but not necessarily eliminate them. Most of the variations can be determined using coincidence data from the $^1\text{H}(e,e'p)$ reaction. This reaction is well understood and is kinematically complete. In our experiment, this reaction was measured for all incident beam energy settings. Since this was a two arm experiment we need to use a Monte-Carlo simulation of it to determine the experimental acceptance. The resulting offsets are used in the Monte-Carlo simulation and in the definitions for the data cuts.

In the analysis of the $^1\text{H}(e,e'p)$ reaction, the invariant mass W , the missing energy E_m and the missing momentum $\mathbf{p}_m$ can be examined to determine if there are any offsets in the incident electron beam energy or the settings of the spectrometers. The missing energy and momentum should be equal to zero, and the invariant mass should be equal to the proton mass. Any deviation of the means of these quantities from the noted values indicates that there is some offset in the scattered electron or scattered proton momenta, $\mathbf{p}_{e'}$ or $\mathbf{p}_p$ respectively, their in-plane scattering angles, $\theta_{e'}$ or θ_p respectively, their out-of-plane angles, $\phi_{e'}$ or ϕ_p , or the incident energy E_e (all values are in the laboratory frame). The offsets of these kinematic quantities should be small. For this experiment, our goal was that the angles have an offset no greater than 2 mrad, assuming that the spectrometers are in a common scattering plane, and that offsets for the incident energy and the momenta should be no more than 0.2%. Table VI shows the results of this analysis. We see that the in-plane angles have small offsets while the out-of-plane angles are larger than 2 mrad. The offset in the in-plane angles can be attributed to the setting procedure. The offset in the out-of-plane angles is due to the fact that the spectrometers were sitting on their wheels and not jacked up into a common

TABLE VI: Experimental Offsets. These offsets were found using a $^1H(e, e'p)$ analysis.

Quantity	HMS	SOS
in-plane angle	+1.0 mrad	-0.4 mrad
out-of-plane angle	+2.4 mrad	+2.6 mrad
central momentum	-0.33%	-0.67% to +0.45%
incident energy	-0.15% to +0.14%	

scattering plane. Since there were several spectrometer in-plane angles needed in this experiment, jacking the spectrometers up into a common scattering plane and then jacking them back down to get to the next setting would have been too time consuming. The fact that the spectrometers were not in a common plane requires us to take into account the mechanical offset created by having the spectrometers on the rails. This also means that we must allow for a slightly larger offset limit for the out-of-plane angles. As we can see, the out-of-plane offsets are not that much greater than our desired limit of 2 mrad, which is acceptable. The offset for the incident beam energy is well within our tolerance. The offsets of the central momenta are greater than 0.2% and in the case of the SOS can be attributed to the saturation effect of the SOS magnets (See sect. 4.2). The offset in the HMS momentum can be correlated to the offsets in the incident energy and out-of-plane angle.

FIG. 37: PID HMS Pion versus Proton. These scatter plots show the particle's momentum relative to the spectrometer's central momentum (δ) as a function of the particle's velocity (β). Left: The protons are shown as the group at $\beta = 0.9$ and the pions are in the group at $\beta = 1.0$. No acceptance cuts were used on these data. Right: This plot is the same as the one on the left. However, the acceptance cuts have been used on these data. The line indicates our β cut at $\beta = 0.925$.

4.4 PID

In order to ensure the events that we analyze are real ${}^1\text{H}(\text{e},\text{e}^{\prime}\pi^{+})\text{n}$ events, we need to correctly identify the pions and electrons within the HMS and SOS spectrometers respectively. These particles must also arrive in coincidence at the 8LM within a very narrow 30 nsec window in order to minimize random events. For the most part, these particles are identified through the hardware triggers. However, random coincidences can be recorded by the data acquisition. These events need to be identified and removed from the data during the analysis phase.

4.4.1 Separating Pions from Protons and Positrons in the HMS

In the HMS, protons, muons, and positrons were detected as coincidence or background events. The source of the protons was elastic scattering. The muons came from pion decay in the detector hut. The positrons were created by positron/electron pair production when pions interacted with the collimator or the magnet walls. The proton and the positron have masses that are considerably different from the pions. Due to these differences, each particle will generate a unique signature in the detectors that can be used to separate the protons and positrons from the pions.

In order to separate the pions and protons, we used the difference in their velocities (β) to identify them. The velocity is calculated from

$$\beta = \frac{|\mathbf{p}|}{E}, \quad (34)$$

if we know the particles momentum and energy. However, we can also determine the velocity from

$$\beta_{TOF} = \frac{d}{(\Delta t)c}, \quad (35)$$

which is the velocity based on the particle's time of flight. In Eq. 35, Δt is the particle's time of flight and is measured from the difference in the times recorded by the TDCs of the detecting scintillators in both hodoscopes. This time must be corrected for variations in cable lengths, signal pulse heights, timing offsets between PMTs, and propagation time of scintillation light in the paddles. The quantity d is the distance between the detecting scintillators as determined by the reconstructed track of the particle and the known distance between the hodoscopes. A detailed description of how β_{TOF} is calculated is given in Ref. [21].

In the HMS, a particle must have a momentum within $\pm 10\%$ of p_0 in order to be detected. Since protons and pions are both detected in the HMS, their momenta must be relatively similar. This would make the protons' β less than that for the pions'. Fig. 37 is a sample run showing the HMS δ as a function of

β . There are two distinct concentrations of data shown in the plot to the left. The group at $\beta = .9$ is identified with the protons, and the one at $\beta = 1.0$ with the pions. The data presented in this plot are not subjected to the standard acceptance cuts described in section 4.4.3. The plot to the right shows the same data after applying the standard acceptance cuts. A cut on the data which accepts particles with $\beta > .925$ rejects over 93% of the protons and only 0.3% of the pions after the standard acceptance cuts. The remaining protons make up less than 2% of the accepted events before the background subtractions (see section 4.5).

The separation of positrons from the pions was accomplished via a cut on the Čerenkov data. According to the settings for this detector (see section 3.5.3), only electrons or positrons would create Čerenkov radiation. Fig. 38 shows a sample run where the pions appear as a clean spike at 0 photoelectrons (p.e.). Normally the distinction between the pions and the positrons is not so clear. The rest of the spectrum consists of positrons. An upper limit of 0.2 p.e. in the Čerenkov was chosen in order to remove the positrons. This resulted in more than 99% positron rejection with less than 1% loss of the pions. More of the positrons were removed through background subtraction and those that remained provided much less than 1% of the accepted events.

Since the source of the muons was from pion decay in the HMS spectrometer and the distinction between pions and muons is difficult, we chose to treat the muons as pions and didn't aggressively reject them. Comparing the charged pion laboratory path length in the HMS (21 m) to its $1/e$ decay length ($c\tau = 7.8045$ m) [32], roughly 20% of the pions decay within the HMS spectrometer and only 25% of these pass all cuts. This results in 5% to 6% contamination due to muons.

4.4.2 Separating Electrons from Pions in the SOS

In the SOS, both electrons and negatively charged pions are detected. The pions come from pion production off a neutron in the target cell walls or the

FIG. 38: PID HMS Čerenkov. This is a close-up the HMS Čerenkov spectrum in units of photoelectrons. The HMS Čerenkov was adjusted so that hadrons would not produce Čerenkov radiation within the spectrometer's momentum bite. These event are shown as the spike at 0.0 p.e.. The other events are positrons created earlier in the spectrometer from pions interacting with the collimator or the magnet walls. Events above the line at 0.2 p.e. are rejected for this experiment.

FIG. 39: PID SOS Electron versus Pion. These plots show the events that created Čerenkov radiation in the SOS Čerenkov detector as a function of the normalized energy deposited in the SOS Calorimeter. The Čerenkov units are in photoelectrons. Left: Events in the upper right corner are electrons. The events below the horizontal line at 0.5 p.e. are hadrons. The rest of the events are background or random events. No acceptance cuts were used on these data. Right: This plot is the same as the one on the left. However, the acceptance cuts have been used on these data.

TABLE VII: PID Cut Settings for TJNAF e93-021.

Variable	Value
No. Photo Electrons SOS	≥ 0.5
No. Photo Electrons HMS	≤ 0.2
Calorimeter Sum SOS	≥ 0.7
HMS β	≥ 0.925

vacuum windows. Like in the HMS, pions and electrons can be identified from the spectrum produced by the gas Čerenkov detector. However, the lead glass calorimeter can also be used in PID. In this experiment we used both. From an analysis done by C. Armstrong [21] on elastically scattered electrons in the SOS, the electrons deposit over 70% of their energy in the calorimeter. Typically, the electrons will deposit 100% of their energy in the calorimeter. The pions will leave only a small amount (usually in the range of 300 MeV). In Fig. 39, we see the Čerenkov spectrum plotted as a function of the total normalized electron energy ($E_{norm} = E/p_0$) measured by the calorimeter. The events in the upper right corner are identified as electrons. The pions are visible as a line below 0.5 p.e. The plot on the left shows data that are not subjected to the standard acceptance cuts, while these cuts are applied to the same data in the plot on the right. For this experiment, we chose to allow events that generated more than 0.5 p.e. in the Čerenkov detector and deposited an E_{norm} greater than 0.7. Though this resulted in less than 1% loss in electrons, more than 99% of the π^- 's were rejected. The remaining π^- events in the SOS were reduced through background subtraction. Table VII tabulates the particle identification cuts used in this experiment.

4.4.3 Standard Cuts

The acceptance cuts (or Standard cuts) used in this experiment involved the collimators, target variables, particle momentum, missing mass, coincidence time,

TABLE VIII: Standard Acceptance Cuts for TJNAF e93-021.

Variable	Value
Coincidence Timing (nsec)	$-1.07 < T < 1.25$
HMS $ \delta $	$\leq 8.5\%$
HMS $ x'_{tar} $	≤ 60.0 mrad
HMS $ z_{tar} $	≤ 3.375 cm
SOS $ \delta $	$\leq 15.0\%$
SOS $ z_{tar} $	≤ 6.750 cm
SOS x_{fp} (cm)	$-20.0 < x_{fp} < 22.0$
Missing Mass (GeV)	$0.925 < M_m < 0.96$
Q^2 and W	$Q^2 > 0.79 + ((W - 1.86)(-4.2)$ $Q^2 > 0.59 + ((W - 1.91)(-1.4)$ $Q^2 < 0.62 + ((W - 1.99)(-4.2)$ $Q^2 < 0.79 + ((W - 1.86)(.23)$

and the kinematic variables Q^2 , W , and $-t$. These cuts are shown in Table VIII. The collimator cuts were hexagonal to match the shape and size of the collimator opening. The z_{tar} cuts were set in such a way as to include the entire target. The HMS x'_{tar} was restricted to 60 mrad because the matrix elements (see section 4.3.3) do not reconstruct the event trajectories very well beyond this limit. The cuts on the particle momentum (δ) were set to the Hall C standards. The cut on the x_{fp} quantity was applied to remove a portion of the SOS focal plane that was not well reconstructed. The remaining cuts will be discussed individually.

The coincidence time cut is shown in left histogram of Fig. 40 by the vertical lines. The tall peak contains the real coincidence events and the smaller peaks are random coincidences. Though the cut at $-1.07 < cointime < 1.25$ excludes most of the random coincidences, it doesn't remove these event from the reals peak. How these events are removed from the accepted events is discussed in the next section.

The missing mass spectrum is shown on the right of Fig. 40. The peak contains

FIG. 40: Coincidence Time, Missing Mass, Q^2 and W Cuts. Upper Left: This is a typical coincidence time distribution. The events in the tall peak are real coincidences, and those in the smaller peaks are random coincidences. The lines represent the minimum and maximum values for the accepted timing range $-1.07 < \text{cointime} < 1.25$. See the text for how the random coincidences are removed from the real peak. Upper Right: This is a typical missing mass spectrum. For this experiment the peak is centered about the neutron mass. The lines represent the minimum and maximum values for the accepted missing mass range $0.925 < \text{missmass} < 0.96$. Events outside these values are mostly radiated events. Bottom: This is the phase space in Q^2 and W . The larger diamond represents events from a higher ϵ run, and the smaller diamond represents events from a lower ϵ run. The accepted phase space of the higher ϵ runs are reduced through data cuts to match the phase space of the lower ϵ runs. Only the phase space that has events from both ϵ settings is needed for this analysis. These cuts reduce the number of events in the higher ϵ runs by a factor of 3.

events that reconstructed to the correct missing mass, which was the mass of the neutron. The tails contain events that did not reconstruct to the correct mass due to radiative effects. If the incident or scattered electron lose energy, through, for example, bremsstrahlung radiation, then there will be more energy and momentum missing which results in a larger reconstructed mass. This is also true if the pion loses energy. This is a random energy loss and by making cuts on this spectrum, we can stabilize the cross section in the tail of the momentum transfer $-t$ spectrum. These cuts removed 36% of the events.

A special cut was applied to the phase space in Q^2 and W' . This experiment examined two setting of the virtual photon polarization parameter ϵ . The higher setting covered more phase space than the lower. We applied a cut to the higher ϵ phase space so it would match that of the lower setting. We only needed to examine that portion of space that is filled by both upper and lower ϵ data. This cut can be seen in Fig. 40. The larger diamond is the phase space coverage of the high ϵ data. This space is reduced to the size of the smaller diamond which covers all the phase space for the low ϵ data. The cut removes two thirds of the event from the larger phase space. To compensate for the reduced statistics, we took three time as much data at the high ϵ setting than at the low ϵ setting.

4.5 Background Subtractions

The non-physics background in this experiment came from random unrelated electrons, protons and pions, and coincident electron-pion reactions in the aluminum target cell walls. The random events occur with every electron beam burst, and we can see them as the small peaks in coincidence time spectrum shown in Fig. 40. We can assume that random events are also in the reals peak. These random events can not be identified in the reals peak, but we can estimate them by calculating the weighted average, bin-for-bin, of the three random peaks just to the right of the reals peak shown in Fig. 40. This average random background is then subtracted, bin-for-bin, from the peak. After the standard and PID cuts

FIG. 41: Reconstructed Target Distribution (z_{tar}) with Empty Target Background. The outlined histogram in the back contains events which have passed all cuts, but still have the background included. The shaded histogram is the same as the outlined one with the background subtracted. The dark peaks at the bottom are the estimated contributions from the target cell walls.

were applied, this background contributed less than 2% of all accepted events.

Since the liquid hydrogen needed to be contained in an aluminum cell, the electron beam had to pass through the cell windows. There is a probability that some of the electrons in the beam will interact with the nuclei in the cell windows and produce real electron-pion coincidence events. These events need to be removed. In order to estimate the contribution of these events we scattered the same energy electrons off an empty target simulated by the aluminum fins spaced 4.5 cm apart in the “Quintar” (see Fig. 14). Since the empty target fins have a larger effective target length than the cell walls we had to weight the yield of the empty target by 0.071 for the entrance window and 0.104 for the exit window. The weighted yield from the empty target is then subtracted from the yield of the real target. The contribution from the cell windows is estimated to be 0.5%. Fig. 41 shows the z_{tar} distribution with and without background subtractions. The shaded area is the result of subtracting the background after applying all cuts as compared to the outlined histogram, which has no background subtractions. The dark peaks at either end of the spectrum are the estimated contributions from the target cell walls.

4.6 Efficiencies

4.6.1 Tracking

The tracking efficiency is the probability for an event to be associated with a reconstructed trajectory. This efficiency is defined as the ratio of the number of events with associated tracks to the number of events that should have tracks. The events that should have tracks are those that generated a pretrigger (see chapter 3). A tracked event is one that creates a pretrigger, and has passed through the fiducial volume of each detector producing signals that can be correlated with

each other.

The tracking efficiency is highly dependent on the drift chamber efficiency and the tracking algorithm. The majority of problems which affect drift chamber efficiencies come from the electronics. The preamplifiers on the signal wires can fail by either not reporting a signal (dead channel), or becoming noisy (ringing channel). Both dead and ringing channels will reduce the tracking efficiency. In the analysis, the only way to compensate for a ringing channel is to remove it from the tracking algorithm in order to make it a dead channel. Corrections to the criteria for a good track can be made in order to reduce the effect that dead channels have on the efficiency. If the criterion for a good track in the drift chambers is too tight, then a dead channel will reduce the efficiency. The efficiency can be increased by requiring fewer planes per chamber to report hits, but by requiring fewer planes, the likelihood that a track will be produced for random hits in the drift chambers will increase. For this experiment, we required that a particle produce hits on four out of six planes per chamber in the HMS, while in the SOS we required hits on five out of six planes per chamber. Ideally, the more hits the better, but one must remember that each hit must be processed. Therefore, a maximum number of hits per chamber should be selected. A systematic study of the wire chamber efficiency as a function of the maximum number of hit per chamber showed the efficiency starting to level off when the number of hits per chamber equaled 25. This number was used in our analysis to maximize our efficiency while minimizing our processing time.

Fig. 42 shows the tracking efficiency of the HMS and the SOS as a function of the pretrigger rates for runs associated with the $g_{\pi NN}$ portion of this experiment. In general the HMS tracking efficiency of pions was above 98%, while the SOS tracking efficiency of electrons was above 95%. These efficiencies are representative of the entire experiment. We can also see that there is a dependence on the trigger rate. This dependence is due to the electronic dead time (see the next section).

The systematic uncertainty of the tracking efficiency was determined by examining the singles fiducial efficiencies as a function of the run numbers [37]. The correlated uncertainty can be determined by the fluctuation of the efficiencies

about the mean. For both the HMS and SOS, this uncertainty amounts to 0.5%. We also assign a 0.2% uncorrelated uncertainty due to statistical fluctuation.

FIG. 42: Tracking Efficiencies per run vs. Event Rate. Though the efficiency is dependent on the event rate, the HMS efficiency stayed above 98%, while that for the SOS was 95% or better. The event rate dependence is due to the electronic dead time (see section 4.6.2).

In order to ensure better particle identification in the HMS, another tracking efficiency correction was also used. Since we used the particle velocity $\beta = v/c$ in the HMS to separate the pion and proton, we needed to examine the tracking efficiency of the pions as a result of this process. This efficiency was determined in the same way as the efficiency of the total tracking. However, the events in this comparison were required to have a velocity $\beta = \beta_0 \pm 0.5$. β_0 is the normalized velocity produced from the central momentum of the spectrometer (see Eq. 36). A common β tracking efficiency of 98.25% with an uncertainty of 0.25% was used in this analysis.

$$\beta_0 = \frac{p_0}{\sqrt{p_0^2 + m_\pi^2}} \quad (36)$$

4.6.2 Dead Times

During data acquisition, some events are not processed because the data acquisition electronics and computers are unavailable for a short period of time while processing a previous event. This short time interval is either called electronic “dead time” while the trigger electronics are busy passing a trigger to the 8LM, or computer “dead time” while the data acquisition computers record and write data to disk. In determining the cross section, a correction must be made to account for these missing events. This correction is determined from the ratio of the total number of events that were processed to the total number of events that should have been processed. This ratio is equal to the probability that the time interval between events is greater than the “dead time”, and it is called the “live time”. The task is to find the number of events that should have been counted.

In general, if the actual number of events is not directly measured (which is usually the case), then we must fall back on the Poisson distribution. If the average true event rate is R , then the probability of detecting n events in time interval t is given by:

$$P(n) = \frac{(Rt)^n e^{-Rt}}{n!}, \quad (37)$$

and the probability distribution of time between events is given by:

$$P(t) = Re^{-Rt}. \quad (38)$$

For an electronic gate width τ , only those events with a time interval greater than τ will be recorded. The fraction of time intervals greater than τ is given by:

$$P(t > \tau) = R \int_{\tau}^{\infty} e^{-Rt} dt = e^{-R\tau}. \quad (39)$$

Therefore, the measured event rate R is:

$$R_{meas} = Re^{-R\tau}, \quad (40)$$

FIG. 43: Calculation of the Rate Correction. These plots show the typical method of determining the correction to the measured event rate in order to calculate electronic live times. The zero intercept of the line is the correction value. The points represent the ratio of the event rate during a given gate width to the pretrigger rate (measured). The given gate widths are 60 nsec, 90 nsec, and 120 nsec, except in the case of the HMS where the 120 nsec gate was accidentally left at 100 nsec. The limiting gate widths correspond to the hodoscope gate widths.

and the live time L is:

$$\frac{R_{meas}}{R} = e^{-R\tau} \equiv L. \quad (41)$$

To find R , Eq. 40 must be solved numerically, or if L is close enough to unity, then we can approximate Eq. 40 as a linear equation in R .

In order to determine the true event rate R for the electronic live time, event rates were measured using four gate widths ($\tau = 30$ ns, 60 ns, 90 ns, and 120 ns) as shown in Fig. 43. These gate widths were selected because the average gate width in this experiment was 30 ns. They are hard wired into the triggering electronics for each spectrometer and have nothing to do with the coincidence gate width. These gates are measured separately in the trigger logic. One should note that during this experiment, the 120 ns gate was accidentally left at 100 ns for the HMS. The event rate in the 30 nsec gate widths will be small due to the 50 nsec gate width of the hodoscope signals and therefore are not plotted. The line is

FIG. 44: Computer and Electronic Live Time. Left: This shows the computer live time as a function of the total trigger rate. The curve is the Poisson curve for zero events occurring. The $800 \mu\text{sec}$ represents the typical time to write an event's information to disk in the unbuffered mode. The points below the curve are affected by poor prescaling of the singles events. In general, the computer live time was above 60%. Right: This is the electronic live time plotted as a function of the pretrigger rates for the HMS and SOS. The lines are the HMS (60 nsec) and the SOS (73 nsec) logic module input gate widths. The live times should lie on or above these lines depending on the spectrometer. The electronic live times were greater than 98%.

a linear fit whose zero intercept is the correction to R_{meas} to produce R . For the $g_{\pi NN}(t)$ portion of this experiment, The electronic live time was greater than 98.5% for both the HMS and the SOS as shown in right-hand plot of Fig. 44.

The most significant dead time comes from the amount of time the data acquisition computers need to record an event. During this experiment, the data acquisition computers were run in the unbuffered mode to avoid problems with synchronization. In this mode, data transfers usually take $800 \mu\text{sec}$. This is the average amount of time that the Trigger Supervisor (TS) would be busy and not accepting any new triggers. Therefore, if we calculate our computer live time from Eq. 41, then the true event rate would be the event rate of the total number of

FIG. 45: Coincidence Blocking Effects and Corrections. The histogram on the right is a typical distribution of the HMS raw coincidence TDC times. The region in the middle represents properly timed coincidences. The other two regions are events that have improper times assigned to them due to singles events starting or stopping the TDC inappropriately. The plot on the left shows the correction for the mistimed events. The line represents the maximum retiming delay of the stop gate. A detailed discussion of this figure is given in the text.

pretriggers, and the measured event rate would be the total number of triggers seen by the TS. These live times are computed on a run-by-run basis and are shown in the left-hand plot of Fig. 44 as a function of the total trigger rate. The curve represents the live time expected as a function of the total trigger rate for $\tau = 800 \mu\text{sec}$. The live times below the curve are affected by poor prescaling of the singles events. In general the computer live time for this experiment was higher than 60%.

4.6.3 Coincidence Blocking and Retiming

Since this was a coincidence experiment, the signals from the HMS and SOS form a valid coincidence when some portion of their pretrigger gate arrive at

FIG. 46: Coincidence Timing. The schematic shows the typical coincidence time measuring circuit for the HMS Cointime TDC. The timing diagrams show the gate timing of the above circuit. The time line labeled "Region I" is the normal timing. The next two lines show how the coincidence time may be measured too short or too long. The regions refer to the areas shown in the histogram of Fig. 45. A detailed discussion is given in the text.

the 8LM at the same time. Both pretrigger gate widths were adjusted to be 15 nsec. This gate width allows for a 30 nsec coincidence window. As was discussed in section 3.6.2, if the TS is not busy, then the singles and coincidence triggers are sent to the TS from the 8LM to start the reading of the TDCs and ADCs. When these triggers are sent to the TS, the coincidence time is measured by a raw coincidence time TDC for each spectrometer. For example, let's consider the coincidence timing for the HMS. For this example we will assume that the pretriggers arrive at the 8LM at the same time and that the TS is not busy. In order to help make this explanation clearer, please refer to Figs. 45 and 46. When the pretriggers arrive at the 8LM they are passed to the TS as triggers, and since they are in coincidence, a coincidence trigger will also be sent to the TS. The HMS trigger is also sent to a retiming circuit to coordinate the read command timing with the analog signals of the ADCs. Similarly, the SOS trigger is also sent to the coincidence time TDC to start the measurement of the coincidence time. When the coincidence trigger arrives at the TS, a long (on the order of msecs) enable signal is sent to a gate that is activated by the delayed HMS trigger. After the HMS trigger appears at the input to the gate, a stop signal is sent to the coincidence time TDC (see the schematic at the top of Fig. 46). The timing of this process is shown in the timing diagram labeled "Region I" of Fig. 46, and the event's coincidence time will be in Region I of the histogram in Fig. 45. The events in Region I are used in the analysis after the coincidence time cut is applied (see section 4.4.3). However, not all of the good coincidence events are in this peak. Some of the good events have incorrect coincidence times due to singles events starting or stopping the coincidence time TDC inappropriately (Coincidence Blocking). Corrections to the cross sections must be made for these missed events.

There are two ways that a coincidence time can be improperly measured by the circuit in Fig. 46. The first way is when a HMS singles event stops the TDC too soon as shown in the timing diagram labeled "Region III" of Fig. 46. This would result in the event having a coincidence time too small and placing the event in Region III of the histogram in Fig. 45. The second way is when a SOS singles

event starts the timing too early as shown in the diagram labeled “Region II”. This discussion is similar to that given by R. Mohring found in Ref. [26].

The right-hand plot in Fig. 45 shows the coincidence blocking corrections for the runs used in the $g_{\pi NN}(t)$ portion of this experiment. They are plotted as a function of the average pretrigger rate $((\text{HMSpre} + \text{SOSpre})/2)$. The corrections range between 93% and 99.5%, and the line represents the retiming delay.

4.6.4 Pion Absorption

As the pions travel through the material that makeup the enclosures on their way to the detector, there is a probability that they will be scattered by a nucleus in the material. The reaction $\pi^+d \rightarrow pp$ has a very large cross section, which indicates that two-nucleon absorption is important. At this experiment’s kinematics, the center-of-mass energy E_{CM} is approximately 4.0 GeV, which corresponds to a π^+d cross section between 55 and 60 mb. The loss of pions due to absorption was not measured during this experiment. However, proton absorption was studied. The results were used for the pion absorption under the assumption that the total scattering cross section for π^+d is within 20% [32] of that for the proton pd at the HMS central momenta used in this experiment.

Using data from $^1\text{H}(e,e')$ and $^1\text{H}(e,e'p)$ the proton absorption was studied. These reactions were studied simultaneously. The kinematics of the protons were calculated from those of the electrons as measured in the SOS. If the proton was within the angular acceptance of the HMS, then the proton was added to the total possible protons to be detected. If the proton was not detected in the HMS then it was considered absorbed. The ratio of the detected protons to those within the angular acceptance of the HMS is the multiple scattering correction. This correction was 2.8% to 5.5% depending on the proton momentum. The pion absorption correction to the pion electroproduction cross section was taken to be 3.0% with a systematic uncertainty to the cross section of 1.0%.

4.6.5 Target Density

As the electron beam passes through the target, it deposits energy along its path. This causes localized heating and a decrease in the localized density. To reduce this effect the beam is rastered to an amplitude of 1 mm as was discussed in section 3.2. In order to examine this effect, data were taken from the reaction $^1\text{H}(e,e')$ at a beam energy of 4.0 GeV and beam currents from 10 μA to 80 μA . The electrons were collected in the HMS positioned at an angle $\theta_q = 12.5^\circ$. The study showed that the pretrigger count (dead time corrected) was reduced by 5.6%/100 μA . Similarly, the yield on kinematic variables was reduced by 6.2%/100 μA . A value of $6 \pm 1\%$ /100 μA was used as the correction [28].

4.6.6 Charge Measurement

As was mentioned in Sect. 3.2, the beam current and accumulated charge are measured using the BCMs just before the target. These BCMs are calibrated by using the Unser beam current monitor, which has a better signal to noise ratio. The total charge is obtained by integrating the current. In order to avoid integrating unphysical currents due to electronic offsets, the current was not integrated for currents below 1 μA . The random error on the total charge is 0.5%.

Chapter 5

Separating Cross Sections

5.1 Philosophy

With a good understanding of our experimental data, we need to calculate the differential cross sections

$$2\pi \frac{d^2\sigma}{dtd\phi^*} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \frac{d\sigma_{TT}}{dt} \cos(2\phi^*) + \sqrt{2\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{dt} \cos(\phi^*). \quad (42)$$

In order to do this we need to separate them. To remove the ϕ^* dependent terms we can either integrate over ϕ^* , if the ϕ^* coverage is sufficient, or we can plot $d^2\sigma/dtd\phi^*$ as a function of ϕ^* and find the respective terms through fitting analysis. To separate the transverse and longitudinal terms requires that we have cross sections measured at two different virtual photon polarization parameters (ϵ).

The choice of which procedure to use (integrating or fitting) was based on the ϕ^* distribution. Appendix B shows $-t$ versus ϕ^* scatter plots. The grid shown is the binning in the $-t$ and ϕ^* kinematic variables (16 bins in $-t$ and 16 bins in ϕ^*). In order to use the integration method, there needs to be data in all ϕ^* bins for any given $-t$ bin. However, as we go higher in $-t$, we find that there are empty bins. This is especially true for lower ϵ setting. In order to fill more bins (cover more $-t$ - ϕ^* phase space) we took data at HMS angle settings that were

FIG. 47: HMS Angular Settings. In order to acquire more $-t-\phi^*$ phase space, the HMS was moved to more forward and backward angles from the central virtual photon scattering angle θ_q . For the higher ϵ setting we were able to collect additional data at $\theta_q + (2.0^\circ, 4.0^\circ, \text{ and } -2.77^\circ)$. Due to the minimum forward angle of the HMS (see Table II) we were only able to collect additional data for the lower ϵ setting at backward angles of $\theta_q + 2.0^\circ$ and $\theta_q + 4.0^\circ$.

parallel and off parallel to the virtual photon scattering angle θ_q (see Fig. 47). For the higher ϵ setting we were able to collect additional data at $\theta_q + (2.0^\circ, 4.0^\circ, \text{ and } -2.77^\circ)$. Due to the minimum forward angle of the HMS (see Table II) we were only able to collect additional data for the lower ϵ setting at backward angles of $\theta_q + 2.0^\circ$ and $\theta_q + 4.0^\circ$. These additional data gave us a $-t$ range from 0.03 (GeV/c)^2 to 0.065 (GeV/c)^2 which could be integrated over ϕ^* . However, if we use fitting analysis over ϕ^* and combine both the higher and lower ϵ settings in the same fit, we can extend our range in $-t$ from 0.02 (GeV/c)^2 to 0.08 (GeV/c)^2 before low statistics produce unreliable cross sections. Further, doing the fitting analysis in this manner ensures that the separated cross sections in both ϵ settings are consistent.

5.2 Procedure

The differential cross section for this experiment can be defined as:

$$\frac{d^5\sigma}{dE'd\Omega_e d\Omega_\pi^*} = \Gamma_v \frac{d^2\sigma}{d\Omega_\pi^*} = 2\pi\Gamma_v \frac{d^2\sigma}{dt d\phi^*} \frac{dt}{d\cos\theta_{\pi q}^*} \equiv \frac{Yield}{A} \quad (43)$$

where A is the acceptance $\Delta E' \Delta\Omega_e \Delta\Omega_\pi^*$, Γ_v is the virtual photon flux factor, and $dt/d\cos\theta_{\pi q}^*$ is the Jacobian factor [12]. The Yield is corrected for the efficiencies and total charge (see section 5.2.2). The asterisk denotes those quantities in the center-of-mass frame of the pion-nucleon system.

Since this is a two-arm coincidence experiment, finding the acceptance analytically would be very difficult. An alternative method would be to use a Monte-Carlo simulation of the experiment to model the acceptance. If the model's results are comparable to the experimental results, then we can determine the experimental differential cross section through

$$\frac{d^2\sigma_{exp}}{(dtd\phi)_{exp}} = \frac{Yield_{exp}}{Yield_{MC}} \frac{d^2\sigma_{MC}}{(dtd\phi)_{MC}}. \quad (44)$$

Finding the yield is discussed in section 5.2.2.

5.2.1 SIMC

Overview

The standard Hall C simulation package “SIMC” was used in this analysis. This simulator has been used in several previous analyses and a detailed description can be found in Refs. [13] and [22]. SIMC homogeneously generates random events, within the effective target length, along the z_{Lab} axis. This event creation begins with the calculating the incident electron’s momentum ($\mathbf{p}_e$) using the random number generator which is weighted by a Maxwellian distribution about a user provided energy. The event vertex, scattered electron kinematics and the hadron’s scattering angle are then randomly selected. All other necessary quantities, such as the energies and momentum of the hadron in the laboratory reference frame, are calculated through energy and momentum conservation. After all quantities are calculated in the laboratory frame, they are transformed to the center-of-mass frame.

After the event is generated, the scattered particles are followed through their respective spectrometers. As they pass through material, their energy loss is calculated and they may undergo multiple scattering. Internal and external radiative effects are also applied within the target for the generated event. Further, the hadron’s decay probability is calculated throughout the hadron’s spectrometer. The path through the magnetic field and magnet apertures is determined by forward matrix elements generated by the COSY INFINITY program [35], which models the magnetic fields from one aperture to the next. With these matrix elements and the physical characteristics of the collimator, we are able to model the acceptances of the spectrometers. Using realistic wire chamber resolutions and the inverse of the COSY matrix elements, the target quantities are reconstructed. Once these target quantities are reconstructed, then other physical quantities can

be determined and all of these results are written to an output file. These reconstructed values can then be weighted, event by event, with a luminosity factor and a cross section model for comparison to the experimental data. For this experiment, our aim was for the mean of the ratio comparison between the experimental and simulated distributions of the physical quantities to be within 10% of unity. We also desired that the yield ratios should be within 10.0% of unity.

Radiative Processes

Internal and external radiative processes are included for the generated electron and hadron. In SIMC, some fraction of the electrons, and proton or pions are allowed to emit a photon in the direction of motion according to the method described in Ref. [38]. For the pion, the process is calculated as if it were a proton with the mass of the pion. From Fig. 48, we see that the radiative tail is well described for pion electroproduction. The uncertainty in the pion radiation is correlated with ϵ , since the kinematics are the same for the higher and lower ϵ , and taken to be 1.0%, while the electron radiation is uncorrelated and taken to be 0.5%.

Pion Decay

For pion production in SIMC, the pion is allowed to decay as it travels through the hadron spectrometer. The decay probability is calculated at every entrance, exit, and mid point of the major components in the hadron spectrometer. This probability is a function of the distance those points are from the center of the target. A charged pion decays into a muon and neutrino. The charged muon is projected within a forward cone about the pion's trajectory in the laboratory frame. Since the muon has less mass than the pion, the momentum of the muon is smaller, which reduces the likelihood of the muon reaching the detectors if the pion decays in the magnetic field region. If the pion decays after the magnetic

FIG. 48: Missing Energy Comparison. This is a distribution from the higher ϵ setting in parallel kinematics. The shaded histogram is from SIMC and the outlined histogram is from the experimental data. If the SIMC radiative processes are correct, then SIMC's radiative tail to the right should be well matched to the experiment's tail.

FIG. 49: Kinematic Comparison. These four figures show the experimental acceptance (outlined histogram) as compared to the Monte-Carlo modeled acceptance. The average ratio of the experimental to Monte-Carlo acceptance as a function of each kinematic variable was better than 90%. The histograms shown are for the best case.

field region, then it will most likely be detected. As was stated earlier, from the decay length of the charged pion ($c\tau = 7.8045$ m)[32], roughly 20% of the pions will decay within the HMS spectrometer and only 25% of these will pass all cuts. This amounts to 6% contamination due to muons. The uncertainty attributed to pion decay was 1.0%.

Detector Acceptances

A detailed study of the full SOS acceptance was done using data from deep

inelastic electron scattering from deuterium. The SOS target variables were compared between the experimental and simulated data. The results of this study showed that the SOS acceptance model disagreed with the experimental acceptance at larger momentum transfer (Q^2) than were used for this portion of the experiment. For $Q^2 = 0.6 \text{ (GeV/c)}^2$, these discrepancies were negligible (affecting less than 1% of the acceptance), and the Monte-Carlo acceptance agreed with the experimental one to within 10%. Similarly, the HMS agreed to the same level.

The reconstructed coincidence variables were studied using pion electroproduction data. There were no significant discrepancies in the acceptance calculations for Q^2 , W , t , or the target variables (See Fig. 49). The uncertainty attributed to the cross sections from the detection volume is estimated to be 2%.

Modeling the Pion Electroproduction

A critical part of finding the yield is the weighting of the counts with the proper cross section. The experimental data has this weighting implicitly applied, but the Monte-Carlo data needs it explicitly applied. In order to find the correct cross section, one can find the Q^2 , W , t , θ^* , and ϕ^* dependence of the experimental yield and apply it to the Monte-Carlo physics generator. The best way to do this is to start with a reasonable hadron production model in the physics generator that will produce data comparable to the experiment. Then, through binning the above mentioned variables, parameterizing the dependencies, and iteratively fitting the yield ratio until unity of the ratio is achieved, one can determine the correct cross section for the Monte-Carlo simulation.

Both the experimental and simulated data were binned in t , θ^* , and ϕ^* , and in each t bin, the yield ratio $f(\theta^*, \phi^*)$ is calculated for each θ^* and ϕ^* bin by normalizing the counts with the appropriate normalization constant (see section 5.2.2). For the experimental data, this constant was the inverse of the product of the accumulated charge and the total tracking efficiency. For the Monte-Carlo data, the counts were normalized with a factor that took into account the luminosity and the number of completed pion events contributing to the output. The

resulting distributions were fitted to

$$f(\theta^*, \phi^*) = a + b \sin^2(\theta^*) + c \sin^2(\theta^*) \cos(2\phi^*) + d \sin(\theta^*) \cos(\phi^*). \quad (45)$$

This equation is similar to the one used in Appendix C of Amaldi et al. [12] where a is the overall ratio of the yields while the other terms draw out the angular dependencies. Only the higher epsilon data were used in this fit because of their better ϕ^* coverage. Only the central portions of the Q^2 and W distributions were used in order to reduce correlations to these variables. The results of this fit are used to correct the θ^* dependence. Since this fitting is done for each t bin, the t dependence for all terms is determined simultaneously. The newly defined cross sections, as determined from these results, are used in the simulator for another iteration. When this process converges, a will be consistent with unity and the other parameters will be consistent with zero. For the entire pion form factor experiment we took data for four Q^2 settings ($Q^2 = 0.6, 0.75, 1.0$, and 1.6 (GeV/c)²). This procedure was done for each Q^2 setting, and when the results converged in each setting, the Q^2 dependence was determined. The W dependence was not explicitly determined, though it was implicitly determined in the θ^* dependence. The normalization term $(W^2 - M_p^2)^{-2}$ was also used in the cross section model. A detailed description of this procedure can be found in Ref. [28].

The results of the above procedure produces the following model for the pion electroproduction differential cross sections, which are good in the Q^2 range of 0.4 to 1.8 (GeV/c)² and have units of ($\mu\text{b}/\text{GeV}^2$):

$$\frac{d\sigma_L}{dt} = 34.0e^{((-23.5+6.0Q^2)(|t|-m_\pi^2))}, \quad (46)$$

$$\frac{d\sigma_T}{dt} = \frac{10.0}{(Q^2 + (Q^2)^2)}(1 + 4 |t|) - 4 \frac{d\sigma_L}{dt} \sin^2(\theta^*), \quad (47)$$

$$\frac{d\sigma_{LT}}{dt} = (0.94 - 34.4e^{(-2.76Q^2)} + 171.0e^{(-113.9|t|e^{(-0.75Q^2)})}) \sin(\theta^*), \quad (48)$$

$$\frac{d\sigma_{TT}}{dt} = \frac{-2.22}{(Q^2)^2} \frac{|t|}{(|t| + m_\pi^2)^2} \sin^2(\theta^*). \quad (49)$$

These models were used in determining the pion form factor portion of this experiment. However, they do not describe the pole behavior of $d\sigma_L$ and the resulting yield ratios produce an artificial slope in the cross section as a function of $-t$. Therefore, these models were corrected by applying a term to $d\sigma_L$ which ensures proper response at the pole and at $t = 0.0$. $d\sigma_T$ was modified to compensate for the slope and change to $d\sigma_L$. After this correction was applied, the resulting models were iteratively fit until they reproduced, as best as possible, the same model curves as their predecessors. The models used for the $g_{\pi NN}(t)$ analysis were as follows:

$$\frac{d\sigma_L}{dt} = 2.43e^{((-13.9+4.5Q^2)(|t|-m_\pi^2))} \frac{|t|}{(|t| + m_\pi^2)^2}, \quad (50)$$

$$\begin{aligned} \frac{d\sigma_T}{dt} = & \left(\frac{-39.24}{(Q^2 + (Q^2)^2)} (1.0 - .75(|t| - m_\pi^2)) \cos(\theta^*) \right. \\ & - 1.2627 \frac{d\sigma_L}{dt} + 1.2112 \frac{d\sigma_L}{dt} \cos^2(\theta^*) \\ & \left. + \frac{53.9587}{(Q^2 + (Q^2)^2)} \cos^3(\theta^*) \right), \end{aligned} \quad (51)$$

$$\frac{d\sigma_{LT}}{dt} = (0.94 - 34.4e^{(-2.76Q^2)} + 171.0e^{(-113.9|t|e^{(-0.75Q^2)})}) \sin(\theta^*), \quad (52)$$

$$\frac{d\sigma_{TT}}{dt} = \frac{-2.22}{(Q^2)^2} \frac{|t|}{(|t| + m_\pi^2)^2} \sin^2(\theta^*). \quad (53)$$

5.2.2 Calculating Yields and Ratios

For this analysis we chose to calculate the yields from the $\phi_\pi = \phi^*$ distribution in order to be consistent with the calculation of the Monte-Carlo differential cross

section ($d^2\sigma_{MC}/dtd\phi$). Here we binned both the experiment and the Monte-Carlo ϕ_π distributions as is shown in the scatter plots of Appendix B. There are 16 $-t$ bins, and for each $-t$ bin, there are 16 ϕ_π bins. For the experimental data, we corrected the counts in each ϕ_π bin by subtracting the random background and the target windows. Both sets of data (experimental and simulated) were subjected to the same set of data cuts as discussed in section 4.4.

The yields were calculated for each bin by normalizing the counts with the appropriate normalization constant. For the experimental data, this constant was the inverse of the product of the accumulated charge and the total tracking efficiency. The total tracking efficiency was a product of the target density correction, HMS and SOS tracking efficiencies, the HMS β efficiency, pion absorption correction and the electronic and computer live times. For the Monte-Carlo data, the counts were normalized with a factor that took into account the luminosity and the number of completed pion events contributing to the output. After the yields for each experimental run were calculated, the runs for a particular θ_q setting were averaged together, bin for bin, using a weighted average based on the statistical error in the yield. Once the yields for both the experiment and Monte-Carlo were calculated, the yield ratios were then calculated for each ϕ_π bin.

A comparison of the experiment to Monte-Carlo t distributions showed that the yield ratio as a function of t varied depending on the ϵ setting and the HMS spectrometer angle in the lab frame. This variation is due to the Monte-Carlo model did not take into account the different beam energies of the same Q^2 and W values or the different pion three-momenta. Due to this, a correlated systematic uncertainty was added to the cross sections of these settings before they were averaged together (see the following section). These errors are shown in Table IX.

5.2.3 Calculating σ_{exp}

The right-hand side of Eq. 44 requires that the Monte-Carlo cross section be

TABLE IX: Uncertainties in the Yield Ratios.

$\theta_{spect.} - \theta_q$	Value	Uncert. (%)
Lower ϵ		
0.0°		1.0 – (0.9 + 7.5t)
2.0°		1.0 – (0.9 + 4.17t)
4.0°		5.0
Higher ϵ		
-2.77°		10.0
0.0°		0.0
2.0°		5.0
4.0°		5.0

calculated. Part of the output of SIMC was the center of mass cross section calculated according to Eq. 42 and Eq. 50-53. This cross section distribution was binned and cut in the same manner as the yields. For each ϕ_π bin, the mean cross section was extracted. These cross sections were then joined with their associated yield ratios to produce the experimental cross sections according to Eq. 44. In order to find the average cross section in each $(-t, \phi_\pi)$ bin in the upper ϵ setting, we calculated a weighted average of the four HMS θ_q settings. The same was done for the lower ϵ cross sections. These cross sections showed the expected ϕ^* dependence (see Fig. 50). We were now ready to separate the response functions.

5.2.4 Separating σ_T , σ_L , σ_{TT} , and σ_{LT}

In order to separate the response functions, we chose to fit the ϕ^* dependence in each $-t$ bin using MINUIT [39]. In this fit we used both of the ϵ settings and the function

$$2\pi \frac{d^2\sigma_{exp}}{dt d\phi^*} = A_1 + \epsilon A_2 + \epsilon A_3 \cos(2\phi^*) + \sqrt{2\epsilon(\epsilon + 1)} A_4 \cos(\phi^*) \quad (54)$$

to minimize χ^2 . The ϵ and ϕ^* variables were given their average values in each

FIG. 50: Sample Fit to Un-Separated Cross Sections. $2\pi \frac{d^2\sigma_{exp}}{dtd\phi}$ is shown for t bin 7, in both high and low virtual photon polarizations of ϵ . The line is the fit to the data using Minuit and minimizing χ^2 .

$-t$ bin as produced from the SIMC output. An example of the results of these fits is shown in Fig. 50. The final cross sections are shown in Fig. 51 and given in Appendix C. The last two bins are not shown because there were too few statistics in either ϵ setting to produce realistic cross sections.

FIG. 51: Unscaled Differential Cross Sections at $Q^2 = 0.6 \text{ (GeV/c)}^2$ and $W = 1.95 \text{ GeV}$. Fourteen of the sixteen t bins are shown. There were insufficient statistics to produce realistic cross sections for the last two bins.

FIG. 52: Scaled Differential Cross Sections. These cross sections are produced at $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$. Fourteen of the sixteen t bins are shown. There were insufficient statistics to produce realistic cross sections for the last two bins. The only outlying point, at $-t = 0.0725 \text{ (GeV/c)}^2$, is due to the lack of sufficient variation in the fitted data points to provide adequate differentiation of the parameters.

FIG. 53: Scaled Cross Sections versus SIMC Models. The curves are produced from Eq. 50-53 for $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$.

5.2.5 Scaling the Cross Sections

When a theoretical prediction for a cross section is plotted as a function of one kinematic quantity, the plot assumes that all of the other kinematic values are constant. This makes the calculations easier, but does not necessarily reflect what really happens when the cross sections are measured as a function of the same kinematic quantity. For example, if we measure the cross sections for charged pion electroproduction as a function of the momentum transfer $-t$, the associated mean Q^2 and W values will vary over the $-t$ bins within their respective acceptances. This means, if we want a meaningful fit to our cross sections using a theoretical model, then we either have to recalculate the model for different Q^2 and W values at each point in the fit or adjust (scale) the measured cross sections to specific Q^2 and W values and fit these cross sections with the assumption that Q^2 and W are the same for all of the cross sections. Since multiple variable fits are difficult to code into a program and not very economic in processor time, we chose to scale our cross sections. We can do this because our Q^2 and W acceptances are small and we reproduce the experiment to better than 90%. Our scaling philosophy followed the form

$$d^2\sigma_{scaled} = d^2\sigma_{exp} \frac{d^2\sigma_{cal}}{d^2\sigma_{MC}}, \quad (55)$$

where $d^2\sigma_{exp}$ and $d^2\sigma_{MC}$ are from Eq. 44. $d^2\sigma_{cal}$ is the calculated cross section at a selected Q^2 and W using Eqs. 50-53 and Eq. 42 and a given $-t$ value. Since we choose Q^2 , W and $-t$, we must recalculate ϵ and θ^* so that they are consistent with these chosen kinematic values. The value for ϕ^* will remain the average value for each ϕ^* bin. For this experiment we chose to scale our cross sections to $Q^2 = 0.5$ (GeV/c)² and $W = 2.0$ GeV in order to make the minimum momentum transfer $|t_0| = 0.02$ (GeV/c)². These values are within our well understood acceptance.

The calculation of the separated scaled cross sections followed the same procedure as was described in the preceding two sections. The final scaled cross section are shown in Fig. 52 and given in Appendix C. The last two bins are not shown because there were poor statistics in both ϵ settings. The cross section at

TABLE X: Systematic Uncertainties.

Source	Uncorr.	Correl.
Target Density	1.0%	-
Charge	0.5%	-
Tracking Eff.	0.2%	0.5%
Beta Tracking Eff.	-	0.25%
Pion Multiple Scattering	-	1.0%
Pion Decay	-	1.0%
Radiative Processes	0.5%	1.0%
Cut Depend.	0.5%	0.5%
Sum	1.3%	1.9%

$-t = 0.0725 \text{ (GeV/c)}^2$ is deviated due to a large yield ratio in that bin for the lower ϵ setting. As was shown by the scatter plots in Appendix B, the statistics in the lower ϵ setting begin to get smaller beyond a virtual pion momentum of $-t = 0.06 \text{ (GeV/c)}^2$. The result of this fall off is an increase in the yield ratios as we go higher in $-t$. This is also true for very low values of $-t$. The longitudinal and transverse cross sections in this bin were given an additional systematic uncertainty of 300.0% and 25% respectively.

The final scaled cross sections are shown in Fig. 53 with the SIMC models, Eq. 50-53, calculated at the same Q^2 and W . As expected, this procedure reproduces the models well.

5.3 Uncertainty Estimates

The contributions to the uncertainty in the cross sections, both unseparated and separated, came from statistical and systematic errors. The systematic uncertainties are two-fold: correlated and uncorrelated. The correlated errors come from processes that are the same for both the high and low ϵ settings and affect

the separation of the transverse and longitudinal cross sections. Processes that are the same for both spectrometers produce the uncorrelated errors, which are assigned to all cross sections.

The statistical uncertainties of the unseparated cross sections were determined from the calculations of the experimental yields Y_{exp} and the Monte-Carlo yields Y_{MC} . In the Monte-Carlo yields, the uncertainty came from the number of accepted events. The statistical error in the experiment was dominant since the number of Monte-Carlo events was at least one order of magnitude greater than that for the experiment.

The systematic uncertainties from the experiment are described in chapter 4. The significant contributions are shown in Table X. The most significant uncorrelated error came from the target density. The radiative processes, pion decay and multiple scattering dominated the correlated errors.

Another source of systematic error came from the model of the cross section in the Monte-Carlo simulation. To determine this uncertainty we changed the cross sections significantly and recalculated new cross sections. We then compared these new cross sections to the original ones and the percent difference indicated the uncertainty in the cross sections due to the model. Each cross section was increased by 10% individually and the resulting cross sections were compared to the original set. The effects on the models due to modifying each modeled cross

TABLE XI: Average Effects on the Cross Sections Due to Model Changes.

	% Difference on Cross Sections due to:			
	$d\sigma_T/dt+10\%$	$d\sigma_L/dt+10\%$	$d\sigma_{TT}/dt+10\%$	$d\sigma_{LT}/dt+10\%$
Unseparated	0.28	0.24	0.02	0.05
$d\sigma_T/dt$	0.52	0.53	0.07	0.03
$d\sigma_L/dt$	0.30	0.24	0.1	0.12
$d\sigma_{TT}/dt$	1.23	2.22	1.72	1.34
$d\sigma_{LT}/dt$	3.01	1.32	1.78	1.65

FIG. 54: Modification to SIMC Model. These plots show the effect of modifying the SIMC models by 10%.

FIG. 55: Effects on Cross Sections Due to Model Changes. These plots show the effects on the final cross sections as a result of individually modifying each cross section model in SIMC by 10%. The average effect on the cross sections are shown in Table XI. The outlying points occur due to cross sections approaching zero as compared to the overall magnitude of the main group of points and are not used in the averaging.

section is shown in Fig. 54. The resulting percent differences are shown in Fig. 55. We can see in the upper plots in Fig. 55 that the transverse and longitudinal cross sections are more sensitive to changes in the transverse and longitudinal models than changes in the interference terms. Still, the effects are less than 1.0% in the transverse cross section and less than 0.5% in the longitudinal cross section. We can also see that we don't have as good a handle on the interference terms as we do with the transverse and longitudinal terms from their sensitivity to any change in the model. The uncertainty in the interference terms is less than 5.0% and is dependent on t . As the interference cross sections approach zero, the percent difference increased significantly. Table XI shows the average uncertainty applied to the cross section due to the model. These averages do not include the points where the cross sections approached zero.

Chapter 6

Extraction of $g_{\pi NN}(t)$

6.1 Cross Sections

In chapter 4 we demonstrated our understanding of the experimental data which went into the generation of the final cross sections. These data were shown to have little contamination from other particles, less than 1.0%. We showed that the machine's efficiency was high, and that the most significant mechanisms that would cause missed events were examined and corrections were made to the yields to account for these losses.

In chapter 5, the final cross sections were calculated using a comparison between the experimental data and data produced from a Monte-Carlo simulation of the experiment. The Monte-Carlo was configured with information gained from the calibration of the experiment. With this configuration and phenomenological cross section models, the Monte-Carlo simulation reproduced over 95% of the experiment. The next aspect to be investigated is the physics generating the cross sections. The concern here is what contributions are present and how do they affect the separated cross sections. This can be examined by comparing the cross sections to a theoretical model which can have each contribution added in turn.

As described in chapter 2, the Maid2000 Charged Pion Electroproduction model [3] can serve this purpose.

6.1.1 TJNAF Data

In chapter 2, we showed through the MAID2000 model how each reaction channel contributed to the cross section. This examination showed that the baryon resonances dominated the transverse contributions while the Born terms were predominantly in the longitudinal. Fig. 56 shows our cross sections compared to the Maid2000 model. The three curves shown have different resonant contributions. The dashed line shows the Born terms only and indicates that the Born terms have a significant influence on the longitudinal cross section and little contribution to the transverse. The dotted line adds the ρ and ω meson exchanges and shows that these particles have little influence at these kinematics. The solid line has the full Maid model, which add in seven baryon resonances. From this plot we see that the full MAID model is a good approximation to our data, and is appropriate to use for the extraction of $g_{\pi NN}(t)$. This would also indicate that our data includes all the physics.

6.1.2 World Data

The world cross sections we selected were from experiments with kinematic settings close to $W = 2.0$ GeV and $Q^2 = 0.5$ (GeV/c)² which are based on the physical constraints of Hall C at the Thomas Jefferson National Accelerator Facility (TJNAF). Also, we chose experiments that have separated the response functions in the cross sections so that the longitudinal contributions can be examined separately. These requirements left two experiments, which were conducted in the late 1970's at DESY in Hamburg, Germany. The first was from Ackermann *et al.* [40] in which the data were taken at $W = 2.1$ GeV and $Q^2 = 0.35$ (GeV/c)². At these settings, the lowest magnitude of the momentum transfer $-t$ is found to

FIG. 56: Cross Sections Compared to Maid2000. The curves are MAID2000 [3] predictions calculated at $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$. The dotted curve was produced from the Born terms only. The solid line was produce from the Born terms plus the ρ and ω meson exchanges along with seven baryon excitations. The double lines in the top graph show the two different ϵ settings. These cross sections include the systematic errors added in quadrature with the statistical uncertainty.

FIG. 57: World Data. The curves are MAID2000 [3] predictions calculated at $Q^2 = 0.5$ (GeV/c)² and $W = 2.0$ GeV. The dotted curve was produced from the Born terms only. The solid line was produce from the Born terms plus the ρ and ω meson exchanges along with seven baryon excitations. The Brauel et al. [41] cross sections are shown at $Q^2 = 0.7$ (GeV/c)² and $W = 2.19$ GeV. The Ackermann et al. [40] cross sections are shown at $Q^2 = 0.35$ (GeV/c)² and $W = 2.1$ GeV. The double lines in the top graph show two different ϵ settings.

be $|t_0| = 0.01 \text{ (GeV/c)}^2$. The second set came from Brauel *et al.* [41] where the data were taken at $W = 2.19 \text{ GeV}$ and $Q^2 = 0.7 \text{ (GeV/c)}^2$, leading to a minimum momentum transfer $|t_0| = 0.027 \text{ (GeV/c)}^2$. These cross sections are shown in Fig. 57 in comparison with the Maid2000 calculations. The plots show that the cross sections indicate no baryon resonances are present. This is contrary to what we have measured. However, since we are extracting $g_{\pi NN}(t)$ from the longitudinal cross section, which is dominated by the Born terms near the pion pole, this difference may not have an effect on the results for $g_{\pi NN}(t)$. In order to examine this we used two models to extract $g_{\pi NN}(t)$. The first was a parameterization of the MAID model and the second was a simple Born term model. The literature for both of these experiments indicates that the Brauel *et al.* cross sections were reproduced by two models, the first from Actor and Körner [42] and the second from Gutbrod and Kramer [43]. The literature also shows that the model from Gutbrod and Kramer agreed well with the Ackermann *et al.* cross sections. For this analysis we used the Actor and Körner model due to its simplicity to code into a computer program.

6.2 Extracting $g_{\pi NN}(t)$

Two models were selected to extract $g_{\pi NN}(t)$ from our cross sections as well as the World cross sections. The models chosen were the Actor and Körner model and a parameterization of the MAID2000 prediction. The Actor and Körner model is a simple Born term model containing the s- and t-channels and utilizes the Dennery Amplitudes in its calculation. Actor and Körner completed the calculation to leading order in the invariant mass s . The result of this calculation for the longitudinal cross section is

$$Nd\sigma_L \approx -4(eg)^2 F_\pi^2(Q^2) Q^2 \frac{t}{(t - m_\pi^2)^2}. \quad (56)$$

In this model g is the strong coupling form factor $g_{\pi NN}(t)$, $e = 4\pi/137$ is the fine structure constant, $N = 32\pi(s - M^2)^2$ is a normalization factor, and the pion

TABLE XII: The Predicted Values of $\Lambda_{\pi N}$ from the GTR. $g_{\pi NN}(m_\pi^2) = 13.4$ [3].

Model	$\Lambda_{\pi N}$ GeV
Monopole	0.706
Dipole	0.994
Exponential	0.699

form factor is given in the monopole model

$$F_\pi = \frac{1}{1 + \frac{Q^2}{\Lambda_\pi^2}}, \quad (57)$$

where $\Lambda_\pi = 0.483$ GeV/c [28].

The MAID prediction was parameterized by

$$d\sigma_L \approx -\xi g_{\pi NN}^2(t) \frac{t}{(t - m_\pi^2)^2} \quad (58)$$

where ξ is the constant which represents the Q^2 and W dependence. The value of ξ was determined by fitting Eq. 58 to the MAID prediction calculated at $Q^2 = 0.5$ (GeV/c)² and $W = 2.0$ GeV, and fixing the coupling constant $g_{\pi NN}(m_\pi^2) = 13.4$ [3] and the associated cutoff parameter $\Lambda_{\pi N}$ calculated from the Goldberger-Treiman Relation (see Appendix A). The calculated values for $\Lambda_{\pi N}$ for each $g_{\pi NN}(t)$ parameterization are tabulated in Table XII.

In fitting the cross sections we use all three models for $g_{\pi NN}(t)$. In these models, the form of $g_{\pi NN}(t)$ has two free parameters. However, if we use the GTR to constrain $g_{\pi NN}(m_\pi^2)$, then we reduce the number of free parameters to one. The results of these fits are shown in the subsequent subsections.

6.2.1 TJNAF Data

Each model is fitted to six sub groups of our longitudinal cross sections, which included both statistical and systematic uncertainties added in quadrature. The

TABLE XIII: Final Results of the Fits to TJNAF Cross Sections.

Method	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N}$ (GeV)
Monopole Model		
Actor	15.03 ± 0.03	0.367 ± 0.007
Maid $\xi = 0.01202$ (μbc^2)	13.48 ± 0.04	0.633 ± 0.032
Dipole Model		
Actor	14.78 ± 0.02	0.538 ± 0.009
Maid $\xi = 0.01211$ (μbc^2)	13.48 ± 0.03	0.893 ± 0.042
Exponential Model		
Actor	14.58 ± 0.03	0.394 ± 0.006
Maid $\xi = 0.01220$ (μbc^2)	13.48 ± 0.04	0.630 ± 0.028

first group was the entire set of cross sections. The remaining groups were pseudo-randomly selected from the cross sections, and the cross sections in a group were required to be sequential. The smaller groups were used to remove any end-point biasing of the fit. The results of these fits can be found in Appendix D. The first sub group from each set of fits is reproduced in Fig. 58. From these plots and the average χ^2 per degree of freedom (see Appendix D), we see that the Actor and Körner model seems to fit the data better than the MAID prediction. The results of the fits to all six data sets are plotted in Figs. 59, where the lines indicate the weighted average of values 2 thru 6. These averages are shown in Table XIII. Figs. 59 also shows us how the use of two distinctly different models can give us different results. This difference is in the Q^2 and W dependence. If the Actor and Körner model is scaled by $1/\sqrt{2}$, the $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$ would become more consistent with the MAID results. This is a good example of the sensitivity that $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$ have to the cross section model.

FIG. 58: Fits to TJNAF Cross Sections. The cross sections are scaled to $Q^2 = 0.5$ (GeV/c) 2 and $W = 2.0$ GeV and include systematic errors. The fits on the left use the Actor and Körner model [42] and those on the right use the MAID2000 [3] parameterization in Eq. 58.

FIG. 59: Fit Results to TJNAF Cross Sections. The top two plots are for the monopole model of $g_{\pi NN}(t)$. The next two are for the dipole model and the last two are for the exponential. On some points the error is too small to be seen on this scale. The lines indicate the weighted average of the fit results and whose values are given in Table XIII. The data points are from the fits shown in Appendix D.

TABLE XIV: Final Results of the Fits to Brauel *et al.* Cross Sections.

Method	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N}$ (GeV)	χ^2/ndf
Monopole Model			
Actor	13.12 ± 0.07	0.614 ± 0.033	4.820
Maid $\xi = 0.01040$ (μbc^2)	13.28 ± 0.07	0.539 ± 0.024	3.800
Dipole Model			
Actor	13.21 ± 0.06	0.897 ± 0.045	4.400
Maid $\xi = 0.01051$ (μbc^2)	13.36 ± 0.06	0.789 ± 0.032	3.290
Exponential Model			
Actor	13.05 ± 0.07	0.656 ± 0.031	3.960
Maid $\xi = 0.01064$ (μbc^2)	13.16 ± 0.07	0.578 ± 0.022	2.790

TABLE XV: Final Results of the Fits to Ackermann *et al.* Cross Sections.

Method	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N}$ (GeV)	χ^2/ndf
Monopole Model			
Actor	13.31 ± 0.19	0.528 ± 0.090	0.370
Maid $\xi = 0.01298$ (μbc^2)	13.99 ± 0.17	0.391 ± 0.040	0.070
Dipole Model			
Actor	13.42 ± 0.13	0.755 ± 0.125	0.360
Maid $\xi = 0.01305$ (μbc^2)	14.10 ± 0.12	0.561 ± 0.055	0.070
Exponential Model			
Actor	13.24 ± 0.17	0.540 ± 0.087	0.090
Maid $\xi = 0.01313$ (μbc^2)	13.79 ± 0.14	0.402 ± 0.037	0.080

6.2.2 World Data

Both the Actor and Körner model and the MAID2000 parameterization were fit to the Brauel *et al.* and Ackermann *et al.* cross sections. The fits used all three $g_{\pi NN}(t)$ models, one at a time. The MAID parameterization was based on MAID2000 predictions using only the Born terms calculated at the kinematics of the cross sections. A factor of 1.88 was applied to the MAID parameterization in order to reproduce the world cross sections. The fits are shown in Figs. 60 and 61, and the results are tabulated in Tables XIV and XV respectively. From the χ^2 results we see that neither of the models reproduced the Brauel group's cross sections very well. However, if we remove the last two cross sections from the Brauel group's set, the fits improve with $\chi^2/ndf \approx 0.6$. On the other hand, both models reproduce the Ackermann group's cross sections too well. This is due to the lack of statistical variation and the large error bars. From these results, we find that we do not learn anything more about $g_{\pi NN}(t)$ than was predicted by theory.

6.2.3 Discussion

From these results we can see that the values of $g_{\pi NN}(m_\pi^2)$, as determined by all three sets of cross sections, are consistent. $g_{\pi NN}(m_\pi^2)$ remained in the range of 13.0 to 14.0 with the exception of the Actor and Körner model fitted to our cross section. Since $g_{\pi NN}(m_\pi^2)$ is not expected to vary from between the $g_{\pi NN}(t)$ models, we can treat the results for $g_{\pi NN}(m_\pi^2)$ as being obtained from 18 different measurements on three sets of cross sections. With this assumption, we find the weighted average of the $g_{\pi NN}(m_\pi^2)$ for the MAID2000 parameterization results to be $g_{\pi NN}(m_\pi^2) = 13.46 \pm 0.02$, and for the Actor and Körner model the coupling constant is $g_{\pi NN}(m_\pi^2) = 14.55 \pm 0.01$. However, we can not make the same assumption for $\Lambda_{\pi N}$.

The results for $\Lambda_{\pi N}$ varied by more than a factor of 2 depending on the model

FIG. 60: Fits to Brauel *et al.* Cross Sections. The fits on the left use the Actor and Körner model [42] and those on the right use the MAID2000 [3] parameterization in Eq. 58 scaled by $2M$.

FIG. 61: Fits to Ackermann *et al.* Cross Sections. The fits on the left use the Actor and Körner model [42] and those on the right use the MAID2000 [3] parameterization in Eq. 58 scaled by $2M$.

and the cross section fitted. As was indicated in chapter 2, $\Lambda_{\pi N}$ is expected to exhibit more model dependence. However, the important thing to point out here is that $\Lambda_{\pi N}$ remained below 1 GeV regardless of the model.

For both models we assume that the $g_{\pi NN}(t)$ dependence was factorable. This would only be the case if the contribution to the longitudinal cross section was from the pole term (t-channel) alone. This condition would occur if there were no baryon excitations and the other channels were very suppressed. Fig. 56 has shown that our cross sections have significant baryon excitations in them. In chapter 2 we addressed how the excited baryons affected the longitudinal cross section and found that they had little to no affect at our experimental kinematics. However, if the baryon and/or the meson are offshell, then $g_{\pi NN}(t)$ needs to be applied to the excited terms. In the case of the pole term we know that the pion is offshell, so we used the $g_{\pi NN}(t)$ with it. On the other hand, the other terms have excited baryons, which implies that the baryon momenta must be carefully calculated in order to determine t . This process can get complicated and difficult to implement. So we assumed that $g_{\pi NN}(t)$ could be factored out since the pole term dominates the longitudinal cross sections at our experimental kinematics. Because of this assumption we need to determine the systematic uncertainty for our extraction procedure.

6.2.4 Estimates of the Systematic Uncertainty in Fits

In order to determine our systematic uncertainty in using $g_{\pi NN}(t)$ as a factorable piece to $d\sigma_L$, we used a preliminary model provided by Winston Roberts [44]. We refer to this model as the Roberts model. This model consists of the Born terms as well as the $\Delta(1232)$. Each of these terms can be included into the model as needed by the user. The user can also choose to whether or not to factor $g_{\pi NN}(t)$.

The systematic uncertainty was determined by comparing the result from the fit of the Roberts model, using the assumption that $g_{\pi NN}(t)$ is factorable, to

TABLE XVI: Fit Results using the Roberts model. All results are based on a preliminary model from Winston Roberts [44] and are only used to determine the systematic uncertainty of the Actor and Körner model and MAID2000 parameterization fit results.

Method	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N}(GeV)$
s- and t-channel only (Roberts)		
Factored	15.17 ± 0.01	0.358 ± 0.001
Unfactored	15.15 ± 0.01	0.359 ± 0.001
Full Roberts Model		
Factored	13.77 ± 0.01	0.595 ± 0.010
Unfactored	13.56 ± 0.01	0.524 ± 0.012

TABLE XVII: Systematic Uncertainties for $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$. These are calculated as a percent difference between the factored and unfactored results in Table XVI (see text). The first and last lines are applied to the Actor and Körner model fit results. The full model uncertainties are given to the MAID2000 fit results.

Uncertainty for:	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N}(GeV)$
s- and t-channel only (Roberts)	0.1%	0.3%
Full Roberts Model	1.5%	13.5%
Actor and Körner Model	11.7%	31.5%

our cross sections with those result produced by fitting the Roberts model, having $g_{\pi NN}(t)$ applied to the pole term only and multiplying the other terms with $g_{\pi NN}(m_\pi^2)$ (unfactored), to our cross sections. For these models we used the monopole parameterization for $g_{\pi NN}(t)$. In the latter situation we assume that the hadrons in the s-, u-, and KR-channel remain onshell for ease of programming. In all cases, the coupling constant was parameterized through the GTR.

We first examined the Roberts model with just the s- and t-channel applied. This is similar to the Actor and Körner model. The results of these fits are shown in Table XVI as the s- and t-channel only. The percent difference of these results, using the unfactored result as the standard, is shown in Table XVII and represents the systematic uncertainty between the factored and unfactored assumption for the s- and t-channel model. This uncertainty is applied to the fit results presented for the Actor and Körner model.

Next, we investigated the factorability assumption on the full Roberts model. Following the same procedure used for the s- and t-channel version of the Roberts model, the fit results were compared and are shown in Tables XVI and XVII as the full Roberts model. This uncertainty is applied to the MAID2000 fit results.

The last systematic uncertainty for $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$ is that for the use of the Actor and Körner model. for this value we compared the unfactored results from s- and t-channel version of the Roberts model and the unfactored results for the full Roberts model. The percent difference is shown in Table XVII as the Actor and Körner model. This uncertainty is given to the Actor and Körner model fit results.

Chapter 7

Conclusions

Modern theory suggests that the long range force between two baryons is governed by the exchange of mesons. Even in the nucleus, there are exchange currents that arise from the flow of charged mesons between nucleons. From this, one could postulate that the binding of nucleons is governed by the sharing of mesons and that the strength of that binding is proportional to the coupling potential which holds the meson to the nucleon. In the case of the pion, the form factor $g_{\pi NN}(t)$ is the coupling strength between the pion and the nucleon.

Besides the pion mass, the form factor $g_{\pi NN}(t)$ is described by two other parameters, the coupling constant $g_{\pi NN}(m_\pi^2)$ and the cutoff parameter $\Lambda_{\pi N}$. $g_{\pi NN}(m_\pi^2)$ defines the coupling strength while $\Lambda_{\pi N}$ governs the pion momentum range of the form factor. These parameters can only be determined through the fitting of model predictions to experimental measurements of scattering cross section. In this fitting, $g_{\pi NN}(m_\pi^2)$ is constrained through the Goldberger-Treiman relation (GTR) and $\Lambda_{\pi N}$ is allowed to float freely. This makes $\Lambda_{\pi N}$ very model dependent. As was shown in this work, $\Lambda_{\pi N}$ varies significantly depending on the choice of model and how $g_{\pi NN}(t)$ is applied. But overall, $\Lambda_{\pi N}$ took on values which indicated a moderately steep (soft) response in the cross section as a function of the squared virtual pion momentum $-t$.

Traditionally, $g_{\pi NN}(m_\pi^2)$ and $\Lambda_{\pi N}$ are found from cross sections coming from

nucleon-nucleon or pion-nucleon scattering. However, pion photo- and electroproduction have been shown to produce cross sections very near the pion pole (at $t \rightarrow m_\pi^2$) where the cross section response changes dramatically. Using pion electroproduction and measuring the cross sections at very forward angles with low momentum transfer Q^2 , we find the cross sections are dominated by the longitudinal contribution. The purpose of this work was to produce a benchmark set of cross section as close to the pion pole as was physically possible in Hall C of the Thomas Jefferson National Accelerator Facility (TJNAF) in order to examine the pion coupling form factor $g_{\pi NN}(t)$.

In the late fall of 1997, the pion form factor experiment was conducted at TJNAF (TJNAF-93-021). Part of this experiment was dedicated to the extraction of the pion coupling form factor $g_{\pi NN}(t)$. In order to reach more forward angles, the first quadrupole magnet of the High Momentum Spectrometer (HMS) was moved backward from the target 40 cm. Because of this move, an intensive recalibration of the spectrometers was conducted. As a result of this calibration, new magnet setting procedures were established, and the effects of residual fields and core saturation were taken into account when determining the field strengths and calculating the central momentum of each spectrometer.

Since this was a two arm experiment, a Monte-Carlo simulation of the experiment was needed in order to determine the experimental acceptance. This simulation needed a cross section model so that the results were properly weighted. We produced a phenomenological model by examining the experimental dependence on the kinematic variables Q^2 , W , t , θ^* , and ϕ^* (the asterisk refers to the pion-nucleon center-of-mass system).

From the work done on this experiment, two sets of very precise cross sections were generated for the virtual pion momentum transfer range of $0.02 \leq |t| \leq 0.08$ (GeV/c)² at the central virtual photon momentum transfer of $Q^2 = 0.6$ (GeV/c)² and central energy $W = 1.95$ GeV. The difference between the sets was the value of the virtual photon's polarization parameter ϵ . The reason for two sets was separate of the structure functions that make up the cross sections in order to examine the longitudinal contribution alone. The resulting set of cross sections

more than doubled the existing world data and was more precise.

After the longitudinal cross section was isolated, the extraction of $g_{\pi NN}(t)$ was preformed. In this work, $g_{\pi NN}(t)$ was determined using two models. The first was the Actor and Körner model and the second was a parameterization of the MAID2000 prediction calculated at $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$. $g_{\pi NN}(t)$ was assumed to be factorable in both models. This is not a proper assumption because the momentum dependence of $g_{\pi NN}(t)$ differs for each reaction channel. But since the t-channel dominates the longitudinal cross section at our kinematics, this assumption will provide a good first order approximation to $g_{\pi NN}(t)$.

$g_{\pi NN}(t)$ was extracted from two previous measurements and the cross section produce in this experiment. In order to make our cross sections consistent with the way the predictions are calculated, these cross sections were scaled to a common momentum transfer of $Q^2 = 0.5 \text{ (GeV/c)}^2$ and central energy $W = 2.0 \text{ GeV}$ before the separation of the structure functions. The results from the fits to the previous measurements were inconclusive. However, the fits to our cross sections produced results that were more precise than the theoretical predictions. Our final results show the coupling constant to be $g_{\pi NN}(m_\pi^2) = 13.47 \pm 0.20$ and $\Lambda_{\pi N}$ is less than 1 GeV. This result is the weighted average of the Actor and Körner model and MAID2000 parameterization results with the systematic uncertainties, found from the Roberts model, applied. This is consistent with the results from nucleon-nucleon scattering. Further, we find that the ratio $(\Lambda_{\pi N}^2 - m_\pi^2)/\Lambda_{\pi N}^2$ is consistent with the results from S. Coon and M. Sadron [45].

The fact that the best we could do was to show that $\Lambda_{\pi N}$ is less than 1 GeV using essentially six different models would indicate that more work needs to be done on the definition of $g_{\pi NN}(t)$. We encourage the theory community to use this cross section measurement to refine $g_{\pi NN}(t)$.

Appendix A

Chiral Symmetry and The Goldberger-Treiman Relation

The smallness of the pion mass is considered to be a key feature in low energy physics. As an example, the long Compton wavelength of the pion is the length scale for current nuclear physics and the long-range nucleon-nucleon interactions. A significant contribution to QCD is the relationship of the pion mass to an approximate underlying symmetry called chiral symmetry. Chiral symmetry is the basic invariance principle which connects the physics of the strong and weak coupling domains in QCD. For chiral symmetry to be exact, the total Axial current between the quark space inside the nucleon and the hadron space outside the nucleon must be conserved. From the Partially Conserved Axial Current approach (PCAC) we find that the divergence of the axial current produced through charged pion decay is expressed as

$$\partial_\mu A_a^\mu(x) = -f_\pi m_\pi^2 \phi(x), \quad (59)$$

where $f_\pi = 92.42 \pm 0.26$ MeV [32] is the pion decay constant and $\phi(x)$ is the free pion field. For chiral symmetry to be exact from Eq. 59 m_π must be zero. At first glance this would imply that chiral symmetry is broken due to the fact that the pion has mass. However, since the pion's mass is very small, then chiral symmetry

APPENDIX A. CHIRAL SYMMETRY AND THE GOLDBERGER-TREIMAN RELATION

FIG. 62: Feynman Diagram for the Pion Pole Contribution.

is still a valid assumption. A detailed explanation of chiral symmetry and soft pions can be found in many theoretical papers and publications [46], [47].

If we consider $\partial_\mu A_a^\mu(x)$ between two nucleon states we find that the resulting matrix elements can be expressed as

$$\langle N_b | \partial_\mu A_a^\mu(x) | N_a \rangle = -\frac{f_\pi m_\pi^2}{t - m_\pi^2} \bar{u}_b(x) \gamma_5 g_{\pi NN}(t) \tau^a u_a(x), \quad (60)$$

where the kinematics are defined from Fig. 62. The meaning of this result will be shown shortly.

In general, Lorentz invariance requires that the nucleon axial current matrix elements be expressed as:

$$\langle N_b | A_a^\mu | N_a \rangle = \bar{u}_b(x) \frac{\tau^a}{2} [\gamma^\mu G_A(t) + (P_a - P_b) \mu G_P(t)] \gamma_5 u_a(x) \quad (61)$$

Here $G_A(q^2)$ and $G_P(q^2)$ are the axial and pseudoscalar form factors and are determined from

$$G_A(t) = \frac{1}{1 - \frac{t}{\Lambda_A^2}} g_A, \quad (62)$$

$$G_P(t) = \frac{f_\pi}{m_\pi^2 - t} \frac{g_{\pi NN}(t)}{g_{\pi NN}(0)} G_P(0). \quad (63)$$

In Eq. 62, the term $g_A = -1.267 \pm 0.004$ [32] is the axial vector coupling constant. The value of Λ_A is not important in this discussion or in the later analysis, however,

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it has been experimentally determined to be 0.65 ± 0.03 GeV [48]. The small pseudotensor form factor is ignored.

Taking the divergence of Eq. 61 we find

$$\langle N_b | \partial_\mu A_\mu^a | N_a \rangle = \bar{u}_b(x) \frac{\tau^a}{2} [MG_A(t) + tG_P(t)] \gamma_5 u_a(x) \quad (64)$$

where M is the nucleon mass.

We can now relate Eq. 64 to Eq. 60 and find

$$2MG_A(t) + tG_P(t) = -\frac{2f_\pi m_\pi^2}{t - m_\pi^2} g_{\pi NN}(t). \quad (65)$$

If we let $t \rightarrow 0$, we find the form of the Goldberger-Treiman Relation (GTR) [49]

$$\frac{Mg_A}{f_\pi} = g_{\pi NN}(0) \equiv g_{\pi NN}(m_\pi^2) \frac{\Lambda_{\pi N}^2 - m_\pi^2}{\Lambda_{\pi N}^2} \quad (66)$$

for the monopole model of $g_{\pi NN}(t)$. This relation is true for the dipole and exponential models as well. This relation provides us with some constraint to the value of $g_{\pi NN}(m_\pi^2)$ such that

$$g_{\pi NN}(m_\pi^2) = \frac{g_A M}{\eta f_\pi}, \quad (67)$$

where

$$\eta = \frac{\Lambda_{\pi N}^2 - m_\pi^2}{\Lambda_{\pi N}^2}. \quad (68)$$

Appendix B

ϕ^* Scatter Plots

FIG. 63: $|t|$ versus ϕ^* Plots for High ϵ . Angles are in the laboratory frame. The grid represents the binning used in this analysis. The lower plot shows the combined result of the upper plots.

FIG. 64: $|t|$ versus ϕ^* Plots for Low ϵ . Angles are in the laboratory frame. The grid represents the binning used in this analysis. The lower plot shows the combined result of the upper plots.

Appendix C

Final Cross Sections

TABLE XVIII: Scaled, Unseparated Cross Sections ($d\sigma_T + \epsilon d\sigma_L$) for high and low ϵ . Scaled to $Q^2 = 0.5$ (GeV/c) 2 , $W = 2.0$ GeV. Only statistical errors are listed.

$-t$ (GeV/c) 2	θ_π^* (deg)	$d\sigma_{high\ \epsilon}$ $\mu\text{b}/(\text{GeV})^2$	ϵ_{high}	$d\sigma_{low\ \epsilon}$ $\mu\text{b}/(\text{GeV})^2$	ϵ_{low}
0.0225	0.83	29.11 ± 3.66	0.7277	19.20 ± 2.74	0.3522
0.0275	4.77	29.73 ± 2.24	0.7277	20.18 ± 1.65	0.3522
0.0325	6.70	27.95 ± 1.66	0.7277	18.67 ± 1.22	0.3522
0.0375	8.19	26.52 ± 1.64	0.7277	18.71 ± 1.20	0.3522
0.0425	9.44	25.06 ± 1.58	0.7277	17.69 ± 1.15	0.3522
0.0475	10.55	23.36 ± 1.53	0.7277	16.78 ± 1.12	0.3522
0.0525	11.56	21.74 ± 1.56	0.7277	16.54 ± 1.14	0.3522
0.0575	12.48	19.24 ± 1.62	0.7277	15.01 ± 1.18	0.3522
0.0625	13.34	18.99 ± 1.82	0.7277	14.64 ± 1.33	0.3522
0.0675	14.15	15.85 ± 1.94	0.7277	15.00 ± 1.42	0.3522
0.0725	14.92	17.14 ± 2.27	0.7277	14.33 ± 1.66	0.3522
0.0775	15.65	16.78 ± 2.67	0.7277	13.77 ± 1.95	0.3522
0.0825	16.35	15.26 ± 3.11	0.7277	12.32 ± 2.29	0.3522
0.0875	17.02	15.88 ± 3.88	0.7277	11.70 ± 2.87	0.3522
0.0925	17.67	14.38 ± 5.77	0.7277	7.79 ± 4.32	0.3522
0.0975	18.29	12.44 ± 36.54	0.7277	28.10 ± 28.63	0.3522

TABLE XIX: Scaled Separated Cross Sections ($d\sigma_T$ and $d\sigma_L$). Scaled to $Q^2 = 0.5$ $(\text{GeV}/c)^2$, $W = 2.0$ GeV. Only statistical errors are listed.

$-t$ $(\text{GeV}/c)^2$	θ_π^* (deg)	$d\sigma_T$ $\mu\text{b}/(\text{GeV})^2$	$d\sigma_L$ $\mu\text{b}/(\text{GeV})^2$
0.0225	0.83	9.90 ± 2.38	26.40 ± 3.82
0.0275	4.77	11.24 ± 1.42	25.41 ± 2.39
0.0325	6.70	9.97 ± 1.05	24.71 ± 1.78
0.0375	8.19	11.38 ± 1.03	20.81 ± 1.75
0.0425	9.44	10.77 ± 0.99	19.64 ± 1.69
0.0475	10.55	10.60 ± 0.96	17.53 ± 1.65
0.0525	11.56	11.66 ± 0.98	13.86 ± 1.68
0.0575	12.48	11.05 ± 1.01	11.26 ± 1.74
0.0625	13.34	10.55 ± 1.13	11.60 ± 1.95
0.0675	14.15	14.20 ± 1.21	2.27 ± 2.07
0.0725	14.92	11.71 ± 1.42	7.46 ± 2.44
0.0775	15.65	10.95 ± 1.68	8.00 ± 2.86
0.0825	16.35	9.56 ± 1.98	7.83 ± 3.30
0.0875	17.02	7.78 ± 2.48	11.14 ± 4.10
0.0925	17.67	1.62 ± 3.76	17.54 ± 6.03
0.0975	18.29	42.78 ± 25.73	-41.69 ± 35.65

TABLE XX: Scaled Separated Cross Sections ($d\sigma_{TT}$ and $d\sigma_{LT}$). Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$. Only statistical errors are listed.

$-t$ (GeV/c) ²	θ_π^* (deg)	$d\sigma_{TT}$ $\mu\text{b}/(\text{GeV})^2$	$d\sigma_{LT}$ $\mu\text{b}/(\text{GeV})^2$
0.0225	0.83	-0.10 ± 1.87	0.87 ± 0.91
0.0275	4.77	0.98 ± 1.21	1.73 ± 0.62
0.0325	6.70	-0.53 ± 0.90	1.05 ± 0.45
0.0375	8.19	0.74 ± 0.84	0.12 ± 0.41
0.0425	9.44	-2.69 ± 0.79	-0.49 ± 0.36
0.0475	10.55	-2.76 ± 0.79	-0.79 ± 0.35
0.0525	11.56	-3.03 ± 0.80	-0.80 ± 0.35
0.0575	12.48	-3.72 ± 0.84	-1.79 ± 0.34
0.0625	13.34	-3.95 ± 0.93	-2.19 ± 0.38
0.0675	14.15	-3.36 ± 1.04	-1.38 ± 0.40
0.0725	14.92	-6.41 ± 1.25	-1.66 ± 0.44
0.0775	15.65	-5.14 ± 1.53	-1.69 ± 0.51
0.0825	16.35	-6.11 ± 1.90	-1.65 ± 0.54
0.0875	17.02	-9.69 ± 2.47	-0.72 ± 0.62
0.0925	17.67	-3.93 ± 3.45	-1.09 ± 0.85
0.0975	18.29	0.45 ± 4.34	-3.37 ± 1.09

TABLE XXI: Unscaled, Unseparated Cross Sections ($d\sigma_T + \epsilon d\sigma_L$) for high and low ϵ . Only statistical errors are listed.

$-t$ (GeV/c) ²	W (GeV)	Q^2 (GeV/c) ²	θ_π^* (deg)	$d\sigma_{high\ \epsilon}$ $\mu\text{b}/(\text{GeV})^2$	ϵ_{high}	$d\sigma_{low\ \epsilon}$ $\mu\text{b}/(\text{GeV})^2$	ϵ_{low}
0.0225	1.97	0.52	2.76	30.64 $\pm$ 3.82	0.7279	20.12 $\pm$ 2.87	0.3590
0.0275	1.91	0.60	4.16	34.45 $\pm$ 2.61	0.7304	22.69 $\pm$ 1.93	0.3651
0.0325	1.92	0.60	5.21	31.79 $\pm$ 1.95	0.7330	20.80 $\pm$ 1.45	0.3721
0.0375	1.91	0.59	6.09	31.46 $\pm$ 2.06	0.7354	22.02 $\pm$ 1.54	0.3792
0.0425	1.94	0.61	6.89	27.41 $\pm$ 1.86	0.7380	19.17 $\pm$ 1.39	0.3857
0.0475	1.90	0.63	7.83	27.77 $\pm$ 1.96	0.7388	19.63 $\pm$ 1.47	0.3879
0.0525	1.98	0.55	8.59	23.15 $\pm$ 1.86	0.7398	18.04 $\pm$ 1.40	0.3913
0.0575	1.88	0.76	9.50	21.92 $\pm$ 1.94	0.7403	16.01 $\pm$ 1.45	0.3940
0.0625	1.89	0.79	10.40	20.48 $\pm$ 2.04	0.7409	14.59 $\pm$ 1.53	0.3935
0.0675	1.91	0.69	11.77	17.10 $\pm$ 2.18	0.7400	15.34 $\pm$ 1.64	0.3886
0.0725	1.94	0.65	12.83	17.65 $\pm$ 2.43	0.7392	14.25 $\pm$ 1.81	0.3791
0.0775	1.89	0.74	13.93	17.88 $\pm$ 2.67	0.7377	13.35 $\pm$ 1.95	0.3606
0.0825	1.87	0.76	15.06	16.50 $\pm$ 2.97	0.7372	11.84 $\pm$ 2.15	0.3465
0.0875	1.88	0.69	15.88	17.68 $\pm$ 3.82	0.7365	12.02 $\pm$ 2.76	0.3372
0.0925	1.88	0.76	16.56	15.16 $\pm$ 4.96	0.7376	7.41 $\pm$ 3.60	0.3326
0.0975	1.93	0.68	17.34	12.04 $\pm$ 28.86	0.7376	24.07 $\pm$ 22.27	0.3292

TABLE XXII: Unscaled Separated Cross Sections ($d\sigma_T$ and $d\sigma_L$). Only statistical errors are listed.

$-t$ (GeV/c) ²	W (GeV)	Q^2 (GeV/c) ²	θ_π^* (deg)	$d\sigma_T$ $\mu\text{b}/(\text{GeV})^2$	$d\sigma_L$ $\mu\text{b}/(\text{GeV})^2$
0.0225	1.97	0.52	2.76	9.90 ± 2.48	28.49 ± 3.98
0.0275	1.91	0.60	4.16	10.94 ± 1.65	32.20 ± 2.77
0.0325	1.92	0.60	5.21	9.46 ± 1.23	30.46 ± 2.07
0.0375	1.91	0.59	6.09	11.97 ± 1.30	26.49 ± 2.17
0.0425	1.94	0.61	6.89	10.16 ± 1.17	23.37 ± 1.95
0.0475	1.90	0.63	7.83	10.63 ± 1.23	23.21 ± 2.06
0.0525	1.98	0.55	8.59	12.30 ± 1.18	14.67 ± 1.94
0.0575	1.88	0.76	9.50	9.29 ± 1.21	17.06 ± 2.04
0.0625	1.89	0.79	10.40	7.91 ± 1.27	16.97 ± 2.14
0.0675	1.91	0.69	11.77	13.39 ± 1.38	5.01 ± 2.28
0.0725	1.94	0.65	12.83	10.67 ± 1.52	9.44 ± 2.56
0.0775	1.89	0.74	13.93	9.02 ± 1.66	12.00 ± 2.83
0.0825	1.87	0.76	15.06	7.70 ± 1.86	11.93 ± 3.14
0.0875	1.88	0.69	15.88	7.23 ± 2.40	14.20 ± 4.03
0.0925	1.88	0.76	16.56	1.04 ± 3.16	19.14 ± 5.18
0.0975	1.93	0.68	17.34	33.77 ± 20.30	-29.46 ± 27.82

TABLE XXIII: Unscaled Separated Cross Sections ($d\sigma_{TT}$ and $d\sigma_{LT}$). Only statistical errors are listed.

$-t$ (GeV/c) ²	W (GeV)	Q^2 (GeV/c) ²	θ_π^* (deg)	$d\sigma_{TT}$ $\mu\text{b}/(\text{GeV})^2$	$d\sigma_{LT}$ $\mu\text{b}/(\text{GeV})^2$
0.0225	1.97	0.52	2.76	-0.25 ± 1.96	1.63 ± 0.96
0.0275	1.91	0.60	4.16	1.44 ± 1.39	2.38 ± 0.72
0.0325	1.92	0.60	5.21	0.18 ± 1.01	1.58 ± 0.52
0.0375	1.91	0.59	6.09	2.03 ± 0.97	0.57 ± 0.49
0.0425	1.94	0.61	6.89	-1.35 ± 0.84	0.05 ± 0.40
0.0475	1.90	0.63	7.83	-1.23 ± 0.91	-0.10 ± 0.42
0.0525	1.98	0.55	8.59	-1.31 ± 0.83	-0.37 ± 0.38
0.0575	1.88	0.76	9.50	-1.74 ± 0.92	-0.68 ± 0.41
0.0625	1.89	0.79	10.40	-1.61 ± 0.96	-0.93 ± 0.44
0.0675	1.91	0.69	11.77	-1.34 ± 1.08	-0.58 ± 0.46
0.0725	1.94	0.65	12.83	-4.26 ± 1.26	-0.93 ± 0.49
0.0775	1.89	0.74	13.93	-2.67 ± 1.60	-0.60 ± 0.60
0.0825	1.87	0.76	15.06	-4.09 ± 2.00	-0.60 ± 0.65
0.0875	1.88	0.69	15.88	-8.23 ± 2.67	0.15 ± 0.77
0.0925	1.88	0.76	16.56	-1.08 ± 3.55	0.32 ± 0.97
0.0975	1.93	0.68	17.34	2.21 ± 4.42	-2.46 ± 1.09

Appendix D

$g_{\pi NN}(t)$ Results

TABLE XXIV: Fit Results for Actor and Körner Model and Monopole $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	15.15 ± 0.06	0.359 ± 0.012	0.670	15
2	15.08 ± 0.06	0.364 ± 0.013	0.610	9
3	14.81 ± 0.09	0.385 ± 0.021	0.180	5
4	15.19 ± 0.07	0.357 ± 0.015	0.660	6
5	15.07 ± 0.07	0.364 ± 0.014	0.690	8
6	14.92 ± 0.08	0.376 ± 0.017	0.490	5

TABLE XXV: Fit Results for Actor and Körner Model and Dipole $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	14.86 ± 0.04	0.528 ± 0.017	0.590	15
2	14.82 ± 0.04	0.534 ± 0.018	0.520	9
3	14.62 ± 0.06	0.560 ± 0.029	0.180	5
4	14.88 ± 0.05	0.526 ± 0.020	0.570	6
5	14.81 ± 0.04	0.535 ± 0.019	0.580	8
6	14.69 ± 0.05	0.550 ± 0.023	0.430	5

TABLE XXVI: Fit Results for Actor and Körner Model and Exponential $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	14.63 ± 0.05	0.389 ± 0.012	0.520	15
2	14.60 ± 0.05	0.392 ± 0.013	0.440	9
3	14.47 ± 0.07	0.407 ± 0.020	0.190	5
4	14.64 ± 0.06	0.388 ± 0.014	0.490	6
5	14.60 ± 0.05	0.392 ± 0.013	0.490	8
6	14.51 ± 0.06	0.402 ± 0.016	0.370	5

FIG. 65: Actor and Körner Fits with Monopole $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

FIG. 66: Actor and Körner Fits with Dipole $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

FIG. 67: Actor and Körner Fits with Exponential $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

TABLE XXVII: Fit Results for Maid Model and Monopole $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	13.61 ± 0.07	0.597 ± 0.045	1.880	15
2	13.48 ± 0.08	0.651 ± 0.062	2.070	9
3	13.03 ± 0.20	1.236 ± 0.569	0.570	5
4	13.65 ± 0.09	0.582 ± 0.052	1.510	6
5	13.50 ± 0.08	0.644 ± 0.061	2.250	8
6	13.33 ± 0.11	0.747 ± 0.109	1.640	5

TABLE XXVIII: Fit Results for Maid Model and Dipole $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	13.60 ± 0.05	0.843 ± 0.060	1.810	15
2	13.49 ± 0.05	0.913 ± 0.082	1.990	9
3	13.05 ± 0.13	1.627 ± 0.643	0.550	5
4	13.63 ± 0.06	0.824 ± 0.070	1.450	6
5	13.50 ± 0.05	0.905 ± 0.080	2.170	8
6	13.34 ± 0.07	1.040 ± 0.142	1.590	5

TABLE XXIX: Fit Results for Maid Model and Exponential $g_{\pi NN}(t)$. Scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$, $W = 2.0 \text{ GeV}$.

Data Set	$g_{\pi NN}(m_\pi^2)$	$\Lambda_{\pi N} \text{ GeV}$	χ^2/ndf	No. Pts.
1	13.59 ± 0.06	0.596 ± 0.040	1.730	15
2	13.49 ± 0.07	0.642 ± 0.054	1.910	9
3	13.08 ± 0.17	1.082 ± 0.374	0.530	5
4	13.62 ± 0.08	0.585 ± 0.047	1.390	6
5	13.50 ± 0.07	0.637 ± 0.053	2.080	8
6	13.35 ± 0.10	0.725 ± 0.093	1.540	5

FIG. 68: MAID Fits with Monopole $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

FIG. 69: MAID Fits with Dipole $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 \text{ (GeV/c)}^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

FIG. 70: MAID Fits with Exponential $g_{\pi NN}(t)$. The cross sections are scaled to $Q^2 = 0.5 (\text{GeV}/c)^2$ and $W = 2.0 \text{ GeV}$. The data include the systematic errors and the fits take these errors into account.

Appendix E

TJNAF e93-021 Collaboration List

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