

Old Dominion University

ODU Digital Commons

Engineering Management & Systems
Engineering Theses & Dissertations

Engineering Management & Systems
Engineering

Spring 2012

Examining Combat Effectiveness in Asymmetric Engagements with Balanced Forces Using the Information Age Combat Model

Nevan E. N. Shearer
Old Dominion University

Follow this and additional works at: https://digitalcommons.odu.edu/emse_etds



Part of the [Industrial Engineering Commons](#)

Recommended Citation

Shearer, Nevan E.. "Examining Combat Effectiveness in Asymmetric Engagements with Balanced Forces Using the Information Age Combat Model" (2012). Doctor of Philosophy (PhD), Dissertation, Engineering Management & Systems Engineering, Old Dominion University, DOI: 10.25777/938z-yn34
https://digitalcommons.odu.edu/emse_etds/123

This Dissertation is brought to you for free and open access by the Engineering Management & Systems Engineering at ODU Digital Commons. It has been accepted for inclusion in Engineering Management & Systems Engineering Theses & Dissertations by an authorized administrator of ODU Digital Commons. For more information, please contact digitalcommons@odu.edu.

**EXAMINING COMBAT EFFECTIVENESS IN ASYMMETRIC
ENGAGEMENTS WITH BALANCED FORCES USING THE INFORMATION
AGE COMBAT MODEL**

by

Nevan E. N. Shearer
B.S. December 2005, Old Dominion University
M.E.M. May 2007, Old Dominion University

A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY
ENGINEERING MANAGEMENT

OLD DOMINION UNIVERSITY
May 2012

Approved by:

Ghaith Rabadi (Director)

Shannon Bowling (Member)

Resit Unal (Member)

Sean Deller (Member)

ABSTRACT

EXAMINING COMBAT EFFECTIVENESS IN ASYMMETRIC ENGAGEMENTS WITH BALANCED FORCES USING THE INFORMATION AGE COMBAT MODEL

**Nevan E. N. Shearer
Old Dominion University, 2012
Director: Dr. Ghaith Rabadi**

With advances in networked communications, the capabilities of command and control (C2) have come to play an increasingly larger role in battlefield success. Within the past two decades a new military strategy has evolved, known as Network-Centric Operations (NCO), which puts information superiority on the frontline. Moreover, the information advantage that is gained through information superiority is translated into a tactical war-fighting advantage.

A research gap has been identified in the investigation of networked combat force configurations in the realm of asymmetric engagements. Specifically, the research question is, how should an information age combat force be networked in order to increase its combat effectiveness in asymmetric engagements with balanced forces? The objective of this research is to identify which performance metrics are best suited in measuring combat effectiveness in the situations of asymmetric engagements with balanced force sizes. In order to reach conclusions on the research objective, a series of experiments have been conducted using a discrete-event simulation based on the Information Age Combat Model (IACM).

The experiments investigate all of the possible engagements for balanced configurations in the format of X-Y-X, ranging from $3 \leq X \leq 10$, and $3 \leq Y \leq X$, where X represents the number of sensors and influencers, and Y represents the number of

deciders in the network. A total of 1,457,801 unique combat engagement simulations were conducted for data collection. The exact combat network configurations and percentage of wins for both sides were collected for use in the data analysis. Several computer programs were written in order to calculate the various performance metrics associated with each combat configuration. These data, in addition to the win percentages, are used in order to conduct both linear and nonlinear regression models, so that the value of the metrics may be evaluated as combat network performance indicators.

Results indicate that the actual size of the network is a greater predictor for combat performance than any of the metrics calculated from the network configurations. However, it has been determined that network configuration does still play a vital role in combat performance in the case of asymmetric engagements with balanced forces. Moreover, results show that it is possible to configure a network in order to increase its chances of winning in an asymmetric engagement against a larger force size.

ACKNOWLEDGMENTS

I am extremely grateful for the advice and guidance from Dr. Shannon Bowling. Words cannot express how thankful I am for his encouragement and leadership. Without all of his support, advice, and experience I would have never completed this work.

I am also indebted to Dr. Ghaith Rabadi for bringing me into the Ph.D. program and providing me the opportunities of a lifetime, and also for his invaluable advice.

I would also like to thank Dr. Resit Unal for his support and guidance and for providing me with great experience and opportunities.

I am also thankful for Dr. Sean Deller for being a part of my committee, for his commitment to the topic, and for his expert advice.

I also want to thank Mahmoud Khasawneh for his collaboration with me on implementing the number partitioning, combat model logic, and verification. I also want to thank him for his encouragement and inspiration, and for being a great friend.

I especially want to thank my parents, Nick and Martha, for being great, loving parents and supporting me through my education and encouraging me to go so far.

I also want to thank my girlfriend Shannon for her love and support and for putting up with me during this stressful time.

Finally, I want to thank Kim Sibson for helping me tie up all the loose ends, you are a life saver.

Thank you all.

TABLE OF CONTENTS

	Page
LIST OF TABLES.....	viii
LIST OF FIGURES	ix
1. INTRODUCTION	1
INFORMATION ADVANTAGE	2
WARFIGHTING ADVANTAGE	6
EFFECTS-BASED OPERATIONS.....	7
2. BACKGROUND OF THE STUDY	9
EVOLUTION OF MODERN WARFARE	9
NETWORK CENTRIC OPERATIONS.....	15
RESULTS FROM PREVIOUS MODELS.....	27
3. METHODOLOGY	31
OVERVIEW OF METHODOLOGY	31
DETERMINATION OF UNIQUE COMBINATIONS	35
ASYMMETRIC ENGAGEMENTS USING BALANCED FORCES.....	39
INFORMATION AGE COMBAT MODEL USING DISCRETE-EVENT SIMULATION	40
4. RESULTS AND DATA ANALYSIS.....	58
VERIFICATION RESULTS	58
SIMULATION RESULTS SUMMARY.....	62
LINEAR REGRESSION RESULTS.....	65
NON-LINEAR REGRESSION RESULTS.....	71

EXPLORATION OF SURROGATE VARIABLES	75
5. CONCLUSIONS.....	92
DISCUSSION AND CONCLUSIONS	92
FUTURE RESEARCH	93
SUMMARY	95
REFERENCES	96
APPENDIX A: LIST OF UNIQUE COMBAT ENGAGEMENTS.....	100
APPENDIX B: VISUAL BASIC PROGRAM TO FORM MEANINGFUL COMBINATIONS	114
APPENDIX C: VISUAL BASIC PROGRAM USED TO CREATE LIST OF UNIQUE ASSYMMETRIC ENGAGEMENTS.....	118
APPENDIX D: VISUAL BASIC PROGRAM COMBAT MODEL FOR ASYMMETRIC ENGAGEMENTS	120
APPENDIX E: PROGRAM TO CALCULATE DISPARITY	126
APPENDIX F: PROGRAM TO CALCULATE ROBUSTNESS.....	128
APPENDIX G: PROGRAM TO CALCULATE STRENGTH	130
APPENDIX H: PROGRAM TO CALCULATE POWER.....	132
APPENDIX I: PROGRAM TO CALCULATE STABILITY	134
APPENDIX J: PROGRAM TO CALCULATE CONNECTIVITY	136
APPENDIX K: REGRESSION ANALYSIS RESULTS	138
VITA	163

LIST OF TABLES

Table	Page
1. Types of links in the IACM (Deller, 2009).....	25
2. Lower and upper bounds for timely execution of combat model	52

LIST OF FIGURES

Figure	Page
1. Simplest combat network.....	22
2. Simplest two-sided combat network.....	23
3. Simplest complete combat network with two-sides.....	23
4. Adjacency matrix for simplest complete combat network (Cares, 2005, p. 82).....	26
5. General form of the special matrix operation used to determine meaningful combinations (Fidanci, 2010, p. 23).	37
6. Calculation of meaningful combinations for a 5-3-5 network using the special matrix operation (Fidanci, 2010, p. 24).	38
7. Flowchart for combat simulation logic.	42
8. Blue 5-3-5 network vs. Red 4-3-4 network.	44
9. First Combat Cycle Iteration – Red Attacks Blue.	45
10. Second Combat Cycle Iteration – Red Attacks Blue.	46
11. Third Combat Cycle Iteration – Blue Attacks Red.....	47
12. Fourth Combat Cycle Iteration – Blue Attacks Red.	48
13. Fifth Combat Cycle Iteration – Blue Attacks Red.....	49
14. Final Combat Cycle Iteration – Blue Attacks Red.....	50
15. Average ordinal difference versus replications for 7-3-7 configuration.....	59
16. Average ordinal difference versus replications for 8-3-8 configuration.....	60
17. Average ordinal difference versus replications for 9-5-9 configuration.....	61
18. Comparison of asymmetric engagements.	62
19. PDF of percent win for red team for all simulation runs.	63
20. Summary statistics for PDF of percent win for red team.....	64

21. CDF of percent win for red team for all simulation runs.....	65
22. Linear regression model using SDI.....	66
23. Linear regression model using performance metrics.	67
24. Linear regression model with performance metrics minus Disp_Red.....	68
25. Linear regression model with performance metrics + SDI.	69
26. Linear regression model with performance metrics + D.	70
27. Non-linear regression model using SDI.....	71
28. Non-linear regression model using all performance metrics.	72
29. Non-linear regression model using performance metrics with insignificant terms removed.....	73
30. Non-linear regression model using performance metrics + SDI.	74
31. Non-linear regression model using metrics + D with insignificant terms removed. ..	75
32. Stability versus sensors/influencers.	76
33. Correlation between stability and sensors/influencers.....	77
34. Strength versus sensors/influencers.	77
35. Correlation between strength and sensors/influencers.....	78
36. Robustness versus sensors/influencers.	78
37. Correlation between robustness and sensors/influencers.....	79
38. Power versus sensors/influencers.	79
39. Correlation between power and sensors/influencers.....	80
40. Disparity versus sensors/influencers.....	80
41. Correlation between disparity and sensors/influencers.....	81
42. Connectivity versus sensors/influencers.	81
43. Correlation between connectivity and sensors/influencers.....	82

44. Linear regression results using only sensors and influencers.	83
45. Linear regression results using only power.	84
46. Linear regression results using only strength.....	85
47. Linear regression results using only disparity.	86
48. Linear regression results using only connectivity.....	87
49. Linear regression results using only robustness.....	88
50. Linear regression results using only stability.....	89
51. Linear regression results using non-surrogate variables.....	90

CHAPTER 1

INTRODUCTION

The face of war is continually changing. From the line up and shoot tactics predating the American revolution, to the trench warfare and guerrilla tactics of the 20th century (Lind, Nightengale, Schmitt, Sutton, & Wilson, 1989), and now the specialized units of today and beyond, war is forever present and constantly evolving. Conflicts all over the world are potential testing grounds for new tactics and new technologies.

As the world has transitioned from the industrial age to the information age, so have modern militaries. In the past, technology was the catalyst for winning wars, through tactics of using brute force and sheer numbers of troops. However, the shift toward the information age paradigm has placed a high value on information superiority. Although technologically advanced weaponry and large military forces are still important, information superiority acts as a force multiplier by adding to shared situational awareness and communication between entities, allowing for a faster pace of command and control (C2).

With advances in networked communications, the capabilities of C2 have come to play an even larger role in battlefield success. Within the past two decades a new military strategy has evolved, known as Network-Centric Operations (NCO), which puts information superiority on the frontline. Moreover, the information advantage that is gained through information superiority is translated into a tactical warfighting advantage. Perhaps the part of warfare that is most influenced by the information age paradigm shift is command and control. Specifically, as asked by Deller (2009), the question is, “how should an Information Age combat force be organized in order to optimize its

effectiveness?” The purpose of this research is to investigate how combat networks can be organized or configured in order to create a more robust battlefield network and improve combat effectiveness, especially in situations where the forces on either side of the engagements are not of equal size.

INFORMATION ADVANTAGE

One of the major tenets of NCO is the idea of an information advantage. The information advantage is “enabled by the robust networking of well informed geographically dispersed forces” (Department of Defense Office of Force Transformation [DoD OFT], 2005, p. 4). Furthermore, the information advantage is characterized by information sharing, shared situational awareness, and knowledge of the commander’s intent (DoD OFT, 2005). Once an information advantage is gained, the joint forces will be able to obtain a warfighting advantage.

Information Superiority

An information advantage may be achieved through first gaining information superiority. In the second half of the twentieth century, the industrialized world has seen an exponential growth in the use of information technology (IT) in both the civilian and military sectors. In fact, advanced computer and communication technology have become central to all facets of military operations such as command, logistics, and intelligence. Moreover, due to the increasing development of IT systems, the demand for more IT systems integration into the military is likely to continue (Forgues, 2000). These factors, coupled with the relatively low cost of IT systems, means it is plausible for all levels of the military organization to make use of IT systems. However, with this heavy dependence on IT by the military, there has been an emergence of vulnerabilities that can

be exploited in conflict, which has become known as Information Warfare (IW) (Waltz, 1998).

With the emergence of IW a military doctrine has been adopted in order to address the concept of IW and “define how offensive and defensive military operations should be conducted in the new environment of cyberspace” (Forgues, 2000, p. 3). The doctrine for Information Operations (Joint Chiefs of Staff [JCS], 2006, p. 114) published by the United States Joint Chiefs of Staff (JCS) defines information operations and information superiority as follows:

- *Information Operations (IO)*: The integrated employment of the core capabilities of electronic warfare, computer network operations, psychological operations, military deception, and operations security, in concert with specified supporting and related capabilities, to influence, disrupt, corrupt or usurp adversarial human and automated decision making while protecting our own.
- *Information Superiority*: The operational advantage derived from the ability to collect, process, and disseminate an uninterrupted flow of information while exploiting or denying an adversary’s ability to do the same.

Prior to the publication of the aforementioned doctrine, the United States Chairman of the Joint Chiefs of Staff (CJCS) identified information superiority as a factor of *emerging importance* in the Joint Vision 2010 (JCS, 1996). As expected, the concept of information superiority has continued to evolve and has become increasingly important as evident in Joint Vision 2020 (JCS, 2000). In the Joint Vision 2020 doctrine, the CJCS continues to describe his vision of the transformation of the US armed forces needed in order to fulfill the realization of these new capabilities. The embodiment of the CJCS’s

vision is a force “that is dominant across the full spectrum of military operations – persuasive in peace, decisive in war, preeminent in any form of conflict” (JCS, 2000, p. 58).

Moreover, Joint Vision 2020 (JCS, 2000) stresses the important role that IT will have in the transformation of the military. The doctrine emphasizes that “continued development and proliferation of information technologies will substantially change the conduct of military operations,” making “information superiority a key enabler of the transformation of the joint force and the evolution of joint command and control” (JCS, 2000, p. 59). Information superiority will provide the joint forces a competitive advantage, but “only when it is effectively translated into superior knowledge and decisions” (JCS, 2000, p. 62).

According to Cerbrowski and Gartska (1998), the developments in information superiority are central to the shift toward network-centric operations. NCO, which is “characterized by information-intensive interactions between computational nodes on the network,” (Cerbrowski and Gartska, 1998, p. 4) is dependent on the communication of information. Regardless of the domain of these interactions, there is inherent value in information, which is derived from the information’s content, quality, and timeliness. That is to say that the value of information increases “as information moves toward 100% relevant content, 100% accuracy, and zero time delay,” (Cerbrowski and Gartska, 1998, p. 4) which leads toward information superiority.

Shared Situational Awareness

Once information superiority is attained, the joint forces must process the information and share it with other parts of the network in order to increase the shared situational awareness. Information sharing is vital to enhancing the quality of information and shared situational awareness (DoD OFT, 2005). However, information sharing cannot be reliable without the use of a robustly networked force. The most important aspect of shared situational awareness is that it “enables collaboration and self-synchronization, and enhances sustainability and speed of command” (DoD OFT, 2005, p. 7). Consequently, these factors lead to vastly increased mission effectiveness (DoD OFT, 2005).

As a result of shared situational awareness being at the center of NCO, there is an increased emphasis placed upon research in developing it, as well as, developing new organizational approaches to achieving synchronization (DoD OFT, 2005). It is the mission of the DoD to continue to improve upon their ability to “accurately represent NCW-related concepts and capabilities in models and simulations,” (DoD OFT, 2005, p. 11) consequently, helping them to understand and manage the complex combat networks. By having a shared situational awareness, the joint forces can have an enhanced situational understanding of what is going on around them on the battlefield. Furthermore, an information advantage is gained and translated into a cognitive advantage, which is necessary in order to obtain decision superiority and a warfighting advantage.

WARFIGHTING ADVANTAGE

The tenets of NCO work together enabling a distinctive warfighting advantage. By using these tenets including information superiority and shared situational awareness, a warfighting advantage can be gained through self-synchronization and speed of command, and results in overall increased combat power. In the thousands of years of recorded history “the vast majority of innovations that created significant warfighting advantages were concentrated in the physical domain as opposed to the information domain” (DoD OFT, 2005, p. 23) as with NCO. The idea behind NCO’s warfighting advantage is the “ability to develop a higher level of situational awareness, in less time than an adversary, combined with the ability to act on it” (DoD OFT, 2005, p. 24). Although this advantage is not necessarily intuitive, its impact is profound.

Self-Synchronization

The idea of self-synchronization is to enable lower-level forces to operate almost autonomously and “re-task themselves through exploitation of shared awareness and the commander’s intent” (DoD OFT, 2005, p. 9). This can only be achieved by taking advantage of a shared situational awareness obtained through a networked force. Self-synchronization increases the value of subordinate initiative which in turn allows for an increase in operational tempo and responsiveness (DoD OFT, 2005). In essence, a networked force becomes an agile force. By exploiting the agility of a highly trained professional force that is networked, even the low-level forces can rapidly adapt to important developments, becoming self-synchronized, and executing actions that convey the commander’s intent.

Speed of Command

The development of network-centric forces to conduct NCO is a means to achieve greater speed of command. Enabled through shared situational awareness and information superiority provided by the robust combat network, increased speed of command allows the joint forces to obtain a decisive warfighting advantage. A greater speed of command means quicker decision making, and an increased chance of “lock-out” of an adversary’s options, and ultimately the achievement of “option dominance” (DoD, 2003, p. 32).

Rapid speed of command also means the ability to “compress sensor-to-decision-maker-to-shooter timelines” (DoD, 2003, p. 32) to turn the information advantage into a warfighting advantage. By obtaining a rapid speed of command the joint forces are able to increase rates of change on the battlefield, thereby swiftly identifying, adapting to, and changing the opponent’s operating context to the joint force’s advantage. Finally, the fundamental tenets of NCO emphasizing high-quality shared situational awareness, geographically dispersed networked forces, and increased speed of command, create an agile and adaptive force that can conduct “powerful effects-based operations to achieve strategic, operational, and tactical objectives across the full range of military operations” (DoD, 2003, p. 3).

EFFECTS-BASED OPERATIONS

In the emerging way of war, the growing capability of forces to conduct NCO has provided an essential means to conduct effects-based operations (EBO). “EBO is not a new form of warfighting, nor does it replace any of the currently recognized forms of warfare” (DoD, 2003, p. 34), instead it stands as an evolution of the objective-based

planning methodology that has been incorporated into the U.S. military doctrine over the past two decades. EBO is used in all aspects of military operations from peacetime engagement and stability operations to fighting terrorism and other major combat operations (DoD, 2003).

EBO is not necessarily just a mode of warfare, rather, it includes a full range of political, military, and economic actions a nation may take in order to “shape the behavior of an enemy, of a would-be opponent, and even of allies and coalition partners” (DoD, 2003, p. 34). The major idea here is not to win a war through physical attrition, instead the objective is “to induce an opponent or an ally or a neutral to pursue a course of action consistent with [the U.S.’s] security interests” (DoD, 2003, p. 34). This is ultimately achieved by “applying the right force to the right place at the right time” (U.S. Air Force [USAF], 2003, p. 6). Essentially, EBO is about focusing knowledge, precision, speed, and agility on the enemy decision-makers in order to degrade their ability to take coherent action, of which those principles are at the heart of NCO.

CHAPTER 2

BACKGROUND OF THE STUDY

This chapter describes the background of NCO starting with the evolution of warfare and ending with a summary of previous modeling attempts of the information age combat model (IACM). Furthermore, this chapter illustrates the fundamental mechanisms of the IACM and how they are related to the NCO paradigm. Cyberwar and Netwar are also discussed and NCO is described as the latest evolution in modern warfare.

EVOLUTION OF MODERN WARFARE

Throughout history the power of militaries has been derived from the capabilities of the weapons technology of that age. In the information age, however, power is not only derived from weapons and manpower, but perhaps more importantly, power is derived from information. Lind, et al. (1989) describe the evolution of modern warfare by describing different generations of military organization. In fact, Lind, et al. (1989) break down the evolution of warfare into four distinct generations.

The first generation of modern warfare is characterized by the “line-and-column tactics” (Lind, 2004, p. 12) dating back to the seventeenth century. This formal type of warfare was a reinforcement of a culture of order. Around the middle of the nineteenth century the battlefield of order began to break down and the tactics of line-and-column became obsolete, and practically suicidal (Lind, 2004).

Second generation warfare was considered an “answer to the contradiction between the culture of order and the military environment” (Lind, 2004, p. 12). This

evolution of warfare was developed by the French around World War I (WWI) and was characterized by mass firepower, mostly in the form of indirect artillery fire (Lind et al., 1989). In essence this generation represented a shift from a focus on massed manpower to a focus on massed firepower. This transition to second generation warfare was considered a relief to soldiers and their officers because it preserved the top-down discipline associated with the culture of order from first generation warfare. Furthermore, second generation warfare is relevant today because the U.S. Army and U.S. Marine Corps (USMC) learned its tactics from the French during and after WWI, and in some ways it is still the American way of war (Lind, 2004).

Third generation warfare was also a product of WWI, however, it was developed by the German army and is commonly known as blitzkrieg or maneuver warfare (Lind, 2004). Instead of being based on firepower and attrition, third generation warfare is nonlinear and characterized by “speed, surprise, and mental as well as physical dislocation” (Lind, 2004). The main objective is to use the aforementioned tactics to get behind the enemy and collapse them from the rear forward. According to Lind (2004), a “third generation military focuses outward, on the situation, the enemy, and the result the situation requires, not inward on process and method” (p. 13). Unlike previous generations of warfare, initiative was more important than obedience, which depended on self-discipline rather than imposed discipline.

In fourth generation warfare, the characteristics of decentralization and initiative carry over from third generation warfare. In other respects, however, fourth generation warfare marks the most profound change since the seventeenth century, in that “the state loses its monopoly on war” (Lind, 2004, p. 13). This means that state militaries are

finding themselves fighting nonstate opponents such as al-Qaeda, Hamas, Hezbollah, and the Revolutionary Armed Forces of Columbia (Lind, 2004). Moreover, fourth generation warfare is “marked by a return to a world of cultures, not merely states, in conflict” (Lind, 2004, p. 13). The lines drawn between opponents are becoming increasingly blurred and the distinctions between friend and foe are also becoming increasingly difficult.

In addition to identifying different evolutions of warfare, Lind, et al. (1989) identified some catalysts for change. The two major catalysts for changes in generations of warfare are advances in technology and ideas. Lind, et al. (1989) attribute the shift into the first generation warfare to both technology and ideas. The shift toward second generation warfare was predominantly caused by advancements in technology. Finally, the ideas associated with blitzkrieg or maneuver warfare were the primary catalysts for change into the third generation of warfare.

Similar to the ideas of different generations of warfare, Toffler and Toffler (1993) propose an evolution of warfare defined by three waves of societal evolution. They wrote that “the metaphor of history as ‘waves’ of change is more dynamic and revealing” (p. 18) than just talking about transitions to postmodernism. Waves are dynamic and when they crash into one another powerful crosscurrents are unleashed. Similarly, when waves of history collide, conflicts occur and whole civilizations clash. The waves of evolution in society and warfare, are divided into three major types of civilizations (Toffler & Toffler).

The first wave of civilization is characterized by agrarian societies. These societies are “inescapably attached to the land” (Toffler & Toffler, 1993, p. 19). This

wave of civilization was predominant from the early ages of man until around the eighteenth century and was a product of the agricultural revolution. Even today there are multitudes of people who “live and die in premodern, agrarian societies, scrabbling at the unyielding soil as their ancestors did centuries ago” (Toffler & Toffler, 1993, p. 19). Consequently, first wave war bears the “unmistakable stamp of first wave agrarian economies that gave rise to them,” not only in a technological sense, but also with its “organization, communication, logistics, administration, reward structures, leadership styles, and cultural assumptions” (Toffler & Toffler, 1993, p. 37).

Second wave civilization came about with the transformation into a more industrialized society. During this wave of civilization “daring new ideas began to circulate-the idea of progress; the odd doctrine of individual rights; the Rousseauian notion of a social contract; secularism; separation of church and state; and the novel idea that leaders should be chosen by popular will, not divine right” (Toffler & Toffler, 1993, p. 19). These ideas coupled with a new way of making wealth through factory production of goods embodied the second wave civilization into what would be considered a modern system. Inevitably clashes between first and second wave societies resulted in conflicts. Just as mass production was the major principle of industrial nations, “mass destruction became the core principle of industrial-age warfare” (Toffler & Toffler, 1993, p. 38). Not only were there conflicts within industrialized countries, but the whole globe became overrun with conquest, resulting in a “bisected world” (Toffler & Toffler, 1993, p. 20) where first wave civilizations were dominated by second wave civilizations.

Eventually the planet became a “trisected world” with the emergence of a third wave of societal evolution (Toffler & Toffler, 1993, p. 21). Third wave economies

generate their wealth from more intangible goods rather than mass produced industrial goods. This represents a transition into the information age. According to Toffler and Toffler (1993), a third wave civilization will dominate “based on the new ways in which it creates and exploits knowledge” (p. 22). Similarly, third wave warfare taps the awesome power of information through “internal feedback, communication, and self-regulatory adjustment,” making third wave armies “thinking systems” distinguishing them from the “machines” of second wave warfare (Toffler & Toffler, 1993, p. 80).

Each wave of societal evolution is a reflection of the way that society creates wealth. Subsequently, the warfare that is associated with each wave is also a reflection of that society’s economy. Whether it is the first wave “still symbolized by the hoe,” the second wave characterized “by the assembly line,” or the third wave represented “by the computer,” (Toffler & Toffler, 1993, p. 21) it is evident that all three are present in the world today and each one is dominated by the latter.

According to Toffler and Toffler (1993), the Gulf war is considered the first clash between second wave and third wave armies. Once the Iraqi army’s radar and communication equipment were excised they were merely just a second wave military. However, the allied force was continuing to tap their capability to network and share information across the battlespace, and exploit their ability make decisive and devastating precision attacks.

Cyberwar and Netwar

Cyberwar and netwar are byproducts of the information age as seen by Arquilla and Ronfeldt (1993). They too, recognized that the nature of warfare is changing due to

the transformation from the industrial age to the information age. Although the information age is merely beginning, “it is imperative that the growing importance of information strategy be recognized and carefully studied” (Arquilla & Borer, 2007, p. 1).

Arquilla and Ronfeldt (1993, p. 146) define cyberwar as “conducting, and preparing to conduct, military operations according to information-related principles.” Moreover, as stated by Deller (2009), cyberwar “represents high-technology warfare where information is exploited in order to defeat an enemy’s military capabilities” (p. 4). Cyberwar is not only about technology, organization is also an important consideration, such as, “how and where to position what kind of computers and related sensors, networks, databases and so forth” (Arquilla & Ronfeldt, 1993, p. 146).

Contrastingly, netwar reflects “information-related conflict... between nations or societies” (Arquilla & Ronfeldt, 1993, p. 144). Netwar may include non-state entities and is not necessarily violent, instead “it may involve public diplomacy measures, propaganda and psychological campaigns, political and cultural subversion, deception of or interference with local media, infiltration of computer networks and databases, and efforts to promote a dissident or opposition movements across computer networks” (Arquilla & Ronfeldt, 1993, p. 144). Arquilla and Ronfeldt (1993; 1996; 1998; 2000; 2001) continued to develop their concepts of cyberwar and netwar by elaborating on the role of networks and information in the military domain.

After developing their concepts of cyberwar and netwar, Arquilla and Ronfeldt (2000) investigated the concept of swarming and its role in the future of conflict. Four fundamental forms of engagement have emerged throughout history that have evolved military organization and doctrine, starting with melee, then massing, maneuver, and

eventually swarming. Warfare has evolved from “chaotic melees” where every man fights on their own, to fighting in “massed but rigidly shaped formations, and then to the adoption of maneuver” (Arquilla & Ronfeldt, 2000, p. 7) tactics. As these tactics have evolved they gave way to swarming, or the “systematic pulsing of force and /or fire by dispersed, internettted units, so as to strike the adversary from all directions simultaneously” (Arquilla & Ronfeldt, 2000, p. 8). Swarming requires complex organizational innovations and increased information structuring and processing capabilities similar to what would be characteristic of a military force conducting NCO.

Cyberwar and netwar are largely considered a part of fourth generation warfare. Hammes (1994, 2005) synthesizes the statements of Lind, et al. (1989), Toffler and Toffler (1993), and Van Crevald (2000) in order to form his own definition of fourth generation warfare. Hammes also acknowledges the concept of netwar from Arquilla and Ronfeldt (1993) while dismissing cyberwar as third generation warfare that is technologically oriented. His view is that war is evolving in conjunction with the political, economic, and social changes affecting society as a whole. Moreover, it is Hammes’s (1994) position that more intelligence gathering and analytical and dissemination capabilities will be necessary in order to serve a highly flexible, interagency command system that must be utilized in waging fourth generation war.

NETWORK CENTRIC OPERATIONS

Network Centric Warfare (NCW), also known as, NCO, is an emerging paradigm in warfare. Alberts, Garstka, and Stein (1999), emphasized three key concepts of NCO. The first concept is that military forces will be geographically dispersed. The next

concept is that the military forces will be empowered by knowledge. Finally, the last concept is that the military forces will be effectively linked.

Alberts, Garstka, Hayes, and Signori (2001) expand on the definition of NCO by exploring the three domains of warfare and the interactions between them. The three domains of warfare that must be focused on are the physical domain, the information domain, and the cognitive domain. By considering these three domains of warfare and the interactions between them, one can begin to understand the source of increased combat power associated with NCO (DoD, 2001).

The physical domain is the place where the situation exists, in which the military seeks to influence. More specifically, it is the domain where “strike, protect, and maneuver take place across the environments of ground, sea, air and space” (DoD, 2001, p. 44). Moreover, it is the domain where the physical platforms and the communications networks that connect them reside. This is the traditional domain where combat power is measured, and it is “characterized as reality, or ground truth” (DoD, 2001, p. 44).

According to the DoD (2001), the information domain is where information is created, manipulated, and shared. This domain facilitates the communication of information between warfighters. Moreover, it is the domain where “command and control of modern military forces is communicated,” and where the “commander’s intent is conveyed” (DoD, 2001, p. 44). It is the domain where all communications with others takes place, and where information is shared. Consequently, in the new age of warfare, it is increasingly the information domain that must be exploited and protected in order to enable a force to generate combat power.

The cognitive domain is the place where perceptions, awareness, understanding, beliefs, and values reside in the minds of participants, and consequently, where decisions are made (DoD, 2001). The cognitive domain is unique because it is dependent on the individual and their perceptions. This is the domain where intangibles reside, such as, leadership, morale, unit cohesion, training and experience, situational awareness, commander's intent, doctrine, tactics, and so forth. These key attributes have remained relatively constant throughout history since Sun Tzu wrote *The Art of War* (DoD, 2001).

These three domains are interrelated by the flow of information between them. There is one reality, the physical domain, where information is found in its raw form. Next the information is selected and shared through the communications involved in the information domain. Finally, in the cognitive domain, training and experience is used to interpret the information and utilize it in the decision-making process.

According to the DoD (2001, p. 46), NCW involves networking across all three domains, and in its fully mature form, it will possess the following characteristics:

Physical Domain:

- All elements of the force are robustly networked achieving secure and seamless connectivity.

Information Domain:

- The force has the capability to collect, share, access, and protect information.
- The force has the capability to collaborate in the information domain, which enables a force to improve its information position through processes of correlation, fusion, and analysis.

- A force can achieve information advantage over an adversary in the Information Domain.

Cognitive Domain:

- The force has the capability to develop and share high quality situational awareness.
- The force has the capability to develop a shared knowledge of commanders' intent.
- The force has the capability to self-synchronize its operations.

Although there has been a lot of work to explain the contextual role of NCO and define it, there has not been much progress in developing quantifiable metrics to measure the performance of NCO based on its network configurations. Alberts and Hayes (2003) continue to refine the concepts and theory of NCO, however, they do not offer any new techniques or metrics for quantifying network performance. Instead, they endorse a “power to the edge” approach to each of the domains of warfare, which empowers “individuals at the edge of an organization (where the organization interacts with its operating environment to have an impact or effect on that environment)” (Alberts & Hayes, 2003, p. 5) in order to achieve a self-synchronizing capability.

Cerbrowski and Garstka (1998) did identify one quantifiable metric for network-centric computing. This metric is known as Metcalfe’s Law, “which asserts that the ‘power’ of a network is proportional to the square of the number of nodes in the network” (Cerbrowski & Garstka, 1998, p. 3). Alberts, et al. (1999) also discuss Metcalfe’s Law as a way to quantify the power of a network. However, they point out the fact that it is

merely a measurement of potential gains, which will not be attained without “appropriate organizational or doctrinal changes” (Alberts, et al., 1999, p. 103).

Alberts, et al. (2001) propose to measure network performance using the attributes of information richness and information reach. They define information richness as “an aggregate measure of the quality of information” and define information reach as “an aggregate measure of the degree that information is shared” (Alberts, et al., 2001, p. 46). Unfortunately, these metrics are either information technology metrics or traditional platform performance metrics, neither of which “directly quantifies organizational or doctrinal attributes” (Deller, 2009, p. 11).

Alberts, et al. (2001) also introduce a quantifiable metric for measuring the degree of synchronization. This metric is associated with the C2 “processes that arrange and continually adapt the relationships of actions (including moving and tasking forces) in time and space in order to achieve” (Alberts, et al., 2001, p. 206) the mission objectives. In essence, each interaction between every entity is assigned a value from -1 (complete interference) to +1 (complete synchronization) and the values are summed in a combinatorial manner to give a value of overall synchronization of the network. Although Alberts, et al. (2001) admit that this metric likely needs refining, “it is a useful step towards quantifying network performance” (Deller, 2009, p. 11).

Ling, Moon, and Kruzins (2005), also agree that there is a lack of quantifiable metrics to measure network performance. To be specific, Ling, et al. (2005) state that “there is currently no clear means by which one can link the internal metrics of the performance of a network to the external measure of the decision-action cycle rate for a networked force” (p. 5). Ling, et al. (2005) worked on refining the metrics of

connectivity, reach, richness, and tempo, however, the usefulness of these metrics in regards to measuring networked force effectiveness is not yet known.

Fortunately, some work has been done by Deller (2009) and Fidanci (2010) that has investigated some metrics that can potentially be useful in measuring the performance of networked forces. Both Deller (2009) and Fidanci (2010) investigated the use of the Perron-Frobenius Eigenvalues as a possible predictor of network performance. This metric was proposed by Cares (2005) as a possible starting point in quantifying the performance of various combat network configurations. Deller (2009) concluded that the Perron-Frobenius Eigenvalues may be a sufficient indicator of network performance with very small networks, however, as larger networks were studied, the effectiveness of the eigenvalues was not sufficient in measuring network performance. This is due to the fact that the proportion of unique eigenvalues to unique configurations diminishes as the number of unique configurations increases. Deller (2009) decided other metrics would be necessary and proposed two new performance metrics, Disparity and Robustness. Building on the work of Deller (2009), Fidanci (2010) proposed several more metrics including Strength, Power, Stability, and Connectivity. The metrics proposed by Deller (2009) and Fidanci (2010) will be the metrics used in this research and will be explained in Chapter 3.

Information Age Combat Model

Cares (2005) proposes an Information Age Combat Model (IACM) that attempts to describe combat between distributed networked forces. The model proposed by Cares (2005) “explicitly represents interdependencies, appropriately captures fine-scale tactical arrangements and can reproduce tipping point behaviors” (p.75). The premise behind the

IACM is that of a mathematical network, “which at the most basic level is a collection of nodes connected by links” (Cares, 2005, p. 77). Specifically, there are four basic types of nodes in this model represented by sensors, deciders, influencers, and targets. These four types of nodes, in the context of the IACM have the following properties (Cares, 2005):

- *Sensors* receive signals about observable phenomena from other nodes and send them to deciders.
- *Deciders* receive information from sensors and make decisions about the present and future arrangement of other nodes.
- *Influencers* receive direction from deciders and interact with other nodes to affect the state of those nodes.
- *Targets* are nodes that have military value but are not sensors, deciders, or influencers. (p. 77)

Further clarification is needed to completely define the aforementioned node types (Cares, 2005). First, each node can have a characteristic called a “side” which is representative of either friendly or enemy forces. Traditionally those nodes on the “Blue” force would be considered friendly and those on the “Red” side would be considered enemy. Second, in the IACM, targets are anything of military value on either side as long as they are not a sensor, decider, or influencer. Third, sensor logic is not considered a decision-making capability, instead, sensor logic is contained within the sensors. Finally, all sensor information must be processed through a decider. Deciders know the location of all of their side’s nodes given they are within range of that side’s sensors. Also, it is worth noting that influencers can act on any type of node, similarly, sensors can detect any type of node (Deller, 2009).

Furthermore, nodes are connected by various types of directional links. Links may be viewed as “observable phenomenon like radio frequency energy, infrared signals, light signals, communications or acoustic energy that emanate from a node and are detected by a sensor” (Fidanci, 2010, p. 10). Furthermore, links do not necessarily have to be IT connections between nodes, but can represent something more functional such as “tactically driven, operational interactions between nodes” (Cares, 2005, p.78). Through these various links, deciders issue orders to influencers, sensors, and targets. Typically, influencers destroy or render useless the nodes they interact with (Fidanci, 2010).

Figure 1 graphically depicts the combat network as described above in its simplest form. Black nodes represent friendly forces, while light grey nodes represent enemy forces. Moreover, different line styles represent the various types of links between nodes.

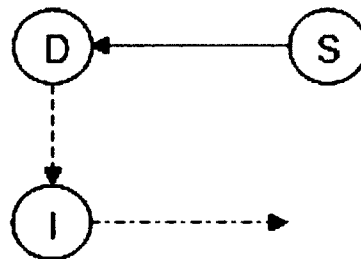


Figure 1: Simplest combat network.

Some links may represent purely physical interactions while other links suggest either physical processes or merely information flows. Figure 2 represents the simplest combat network that involves two opposing forces.

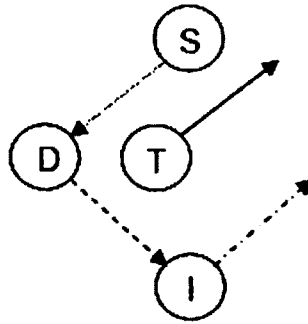


Figure 2: Simplest two-sided combat network.

Cares (2005) goes further and describes the simplest complete two-sided combat network as having 36 possible links. Figure 3 depicts the simplest complete two-sided combat network which represents all of “the ways in which sensors, deciders, influencers, and targets interact meaningfully with each other” (Cares, 2005, p.81).

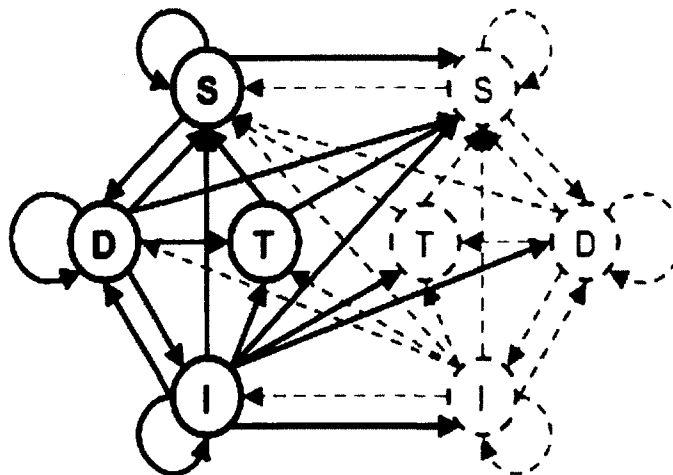


Figure 3: Simplest complete combat network with two-sides.

The complete number and type of links for the simplest complete combat network is actually reduced from the total possible number of links which is 2^8 or 64. Then, the

number is reduced to a total of 36 links by excluding 28 links based on the following assumptions (Deller, 2009, p. 15):

- Targets are passive; their only role is to be sensed and influenced.
Therefore, 12 links from targets to any nodes other than a sensor were excluded.
- Sensors take no action; they provide information to deciders and sensors.
Therefore, 10 links from sensors to any nodes other than a sensor or an own decider were excluded.
- Deciders act only through influencers but can be sensed. Therefore, 6 links from deciders to any adversary nodes except a sensor were excluded.

Furthermore, if one considers the symmetry between the Blue and Red forces, the total number of link types can then be reduced to 18. A complete list of link types is provided in Table 1.

Table 1

Types of links in the IACM (from Deller, 2009)

Link Type	From	To	Interpretation	Link Type	From	To	Interpretation
1	S _{BLUE} S _{RED}	S _{BLUE} S _{RED}	S detecting own S, or S coordinating with own S	10	I _{BLUE} I _{RED}	D _{BLUE} D _{RED}	I attacking own D, or I reporting to own D
2	S _{BLUE} S _{RED}	D _{BLUE} D _{RED}	S reporting to own D	11	I _{BLUE} I _{RED}	I _{BLUE} I _{RED}	I attacking own I, or I coordinating with own I
3	S _{BLUE} S _{RED}	S _{RED} S _{BLUE}	S detecting adversary S	12	I _{BLUE} I _{RED}	T _{BLUE} T _{RED}	I attacking own T
4	D _{BLUE} D _{RED}	S _{BLUE} S _{RED}	S detecting own D, or D commanding own S	13	I _{BLUE} I _{RED}	S _{RED} S _{BLUE}	I attacking adversary S, or S detecting adversary I
5	D _{BLUE} D _{RED}	D _{BLUE} D _{RED}	D commanding own D	14	I _{BLUE} I _{RED}	D _{RED} D _{BLUE}	I attacking adversary D
6	D _{BLUE} D _{RED}	I _{BLUE} I _{RED}	D commanding own I	15	I _{BLUE} I _{RED}	I _{RED} I _{BLUE}	I attacking adversary I
7	D _{BLUE} D _{RED}	T _{BLUE} T _{RED}	D commanding own T	16	T _{RED} T _{BLUE}	T _{RED} T _{BLUE}	I attacking adversary T
8	D _{BLUE} D _{RED}	S _{RED} S _{BLUE}	S detecting adversary D	17	T _{BLUE} T _{RED}	S _{BLUE} S _{RED}	S detecting own T
9	I _{BLUE} I _{RED}	S _{BLUE} S _{RED}	I attacking own S, or S detecting own I	18	T _{BLUE} T _{RED}	S _{RED} S _{BLUE}	S detecting adversary T

The combat networks can also be represented in matrix form rather than graphical depictions. Figure 4 shows the directional links between the nodes of the simplest complete combat network in the form of what is called an adjacency matrix (Cares,

2005). This form of network representation is more valuable in understanding the dimensionality of the different types of networks.

		Friendly				Enemy			
		S	D	I	T	S	D	I	T
Friendly	S	1	1	0	0	1	0	0	0
	D	1	1	1	1	1	0	0	0
	I	1	1	1	1	1	1	1	1
	T	1	0	0	0	1	0	0	0
Enemy	S	1	0	0	0	1	1	0	0
	D	1	0	0	0	1	1	1	1
	I	1	1	1	1	1	1	1	1
	T	1	0	0	0	1	0	0	0

row maps directionally to column = 1, 0 otherwise

Figure 4: Adjacency matrix for simplest complete combat network (Cares, 2005, p. 82).

If all possible links are considered, “combat networks with more than 17 nodes can contain more sub-networks than there are particles of matter in the known universe” (Cares, 2005, p. 82). Consequently, trying to determine “the best arrangement of nodes and links in this huge space of possibilities would be extraordinarily exhaustive” (Cares, 2005, p. 83). Due to the extreme complexity of the combat network configurations, the scope of the problem in this research has been limited to those links and nodes necessary to complete combat cycles, which are link types 2, 3, 6, 13, and 15, which involve only sensors, deciders, and influencers. The same assumptions were present in the previous modeling attempts of Deller (2009) and Fidanci (2010).

Combat cycles, in the context of the IACM and the proposed research, are very important to understanding how the combat model works. Basically, a combat cycle consists of the links and nodes discussed previously, interacting and performing a full rotation of actions of sensing, deciding, and influencing. In order for a sub-network to be able to perform a combat cycle, there must be at least one sensor and one influencer connected to the decider node of that sub-network. The cycle begins with the sensor gathering information about the location and nature of an adversary node, sensor or influencer. Next, the information is communicated to the decider, where it is processed, and the decider passes on their intent to the influencer. In turn, the influencer takes the necessary actions to render the enemy sensor or influencer inactive. In the context of the IACM used in this research, the combat cycle process is repeated iteratively until one of the forces, Red or Blue, is rendered combat ineffective (unable to complete a combat cycle).

RESULTS FROM PREVIOUS MODELS

Although a lot of work has been done on the contextual role of NCW, little research has been done on analyzing the mechanisms involved in the operational strategy. Moreover, modeling NCW has only recently been approached and is still in the early stages (Deller, 2009; Fidanci, 2010). According to Cares (2005), there is still a need to develop a suitable analytical model that appropriately describes distributed networked operations.

Deller (2009) and Fidanci (2010) used agent-based modeling as their modeling paradigm in order to construct the IACM and analyze network performance. Their focus was on recording the Blue or Red win percentage for various network configurations, and

investigating how various metrics could be used as performance indicators for combat networks as a whole. For more information on the agent-based modeling paradigm readers are referred to Bonabeau (2002).

Deller (2009) used Netlogo as the modeling software tool in order to implement the IACM in an agent-based model. Netlogo is an open-source agent-based modeling software developed by the Northwestern University's Center for Connected Learning and Computer-Based Modeling. For more information on Netlogo and its use in modeling and simulation readers are referred to Tisue and Wilensky (2004).

Deller (2009) investigated symmetric engagements using balanced force sizes. In other words, he looked at combat engagements in the format of X-Y-X versus X-Y-X, where X represents the number of sensors and influencers, and Y represents the number of deciders. Deller (2009) was able to experiment with a couple of different sizes of networks, however, the agent-based modeling approach limited him due to the high demand of computational power for simulating large networks. His results indicate that the Perron-Frobenius Eigenvalue was a suitable "indicator for networks with three deciders, however, it was not sufficient for a networked force with five" (Deller, 2009, p. 49). By adding additional performance metrics, disparity and robustness, Deller (2009) was able to show a greater correlation between the metrics and the average probability of a win.

Building on the work of Deller (2009), Fidanci (2010) also developed an ABM to simulate combat between networked forces. Fidanci (2010) used a modeling software called AnyLogic in order to create the IACM, which is a multi-paradigm modeling tool

developed by XJ Technologies. For more information about AnyLogic as used in ABM readers are referred to Oosthuizen, Burke, and Roodt (2010).

Fidanci (2010) added to Deller's work by simulating additional configurations, as well as, proposing additional metrics to measure combat network performance. Fidanci (2010) ran a total of 55 experiments set up in a similar fashion to those of Deller (2009) using symmetric engagements with symmetric force sizes (X-Y-X). Fidanci (2010) confirmed the fact that the Perron-Frobenious Eigenvalues are insufficient in determining combat network performance. He goes on to state that the reason it is not a good predictor is because "the ratio between the number of distinct eigenvalues and the number of different meaningful combinations decreases as the number of Sensor-Influencer and Decider increases" (Fidanci, 2010, p. 87). Consequently, Fidanci (2010) proposed additional metrics of power, strength, connectivity, and stability, in an effort to increase the ability to quantify combat network performance. His results indicate that these performance factors, in addition to those proposed by Deller (2009), do, in fact, improve the ability to measure combat network performance.

Both Deller (2009) and Fidanci (2010) investigated the influence of network factors on symmetric engagements (X-Y-X vs. X-Y-X), using balanced configurations (X-Y-X), through the use of an ABM approach to model the IACM. The research proposed in this document, however, will expand on the idea of using the IACM, in order to model "asymmetric engagements" with "balanced configurations." That is, the combat networks will still be balanced with X-Y-X configurations, however, various network sizes will do battle with networks of different sizes. For example, instead of having a "symmetric engagement" of 5-3-5 vs. 5-3-5, this research will investigate engagements

such as 5-3-5 vs. 7-5-7, which in the context of this research will be identified as “asymmetric engagements with balanced force configurations.” Furthermore, the performance metrics proposed by Deller (2009) and Fidanci (2010) will be evaluated as performance indicators for these asymmetric engagements.

One other evolution this research will take on is the use of the discrete-event simulation (DES) modeling paradigm rather than the ABM paradigm. The reason for this transition deals mainly with the speed of simulation and will be discussed in greater detail in the next chapter. For readers interested in the DES paradigm they are referred to Fishman (2001).

CHAPTER 3

METHODOLOGY

This chapter is dedicated to discussing the methodology utilized in this research. It begins with an overview of the research problem and a high-level articulation of the research methodology. Following the overview, a detailed explanation of the steps involved in the execution of the research methodology is presented.

OVERVIEW OF METHODOLOGY

The scientific method is employed in this research and is summarized below:

Step 1: Identify the Problem. The problem statement is derived from the background information and literature search provided in the previous chapters. A research gap has been identified in the investigation of networked combat force configurations in the realm of asymmetric engagements. Specifically, the research question is, how should an information age combat force be networked in order to increase its combat effectiveness in asymmetric engagements with symmetric force sizes?

Step 2: Literature Review. Chapter 2 provides an in-depth look at the background of the research topic and identifies relevant research pertaining to the simulation of the IACM. This research, in particular, will build off of the previous work of Cares (2005), Deller (2009), and Fidanci (2010).

Step 3: Research Objective. The objective of this research is to identify which performance metrics are best suited in measuring combat effectiveness in the situations of asymmetric engagements with symmetric force sizes.

Step 4: Research Design. In order to reach conclusions on the research objective, a series of experiments will be conducted using a DES combat model based on the IACM. The experiments will investigate all of the possible engagements for symmetric configurations in the format of X-Y-X, ranging from $3 \leq X, Y \leq 10$. A total of 1,457,801 unique combat engagement simulations will be conducted for data collection.

Step 5: Data Collection. Each of the 1,457,801 unique combat engagement simulations will be replicated 30 times in order to obtain an average win percentage for the Red force for each of the unique combat simulation configurations. The exact combat network configurations for both the Blue and Red side will also be collected so they can be used in the data analysis.

Step 6: Data Analysis. Several Visual Basic (VB) programs will be written in order to calculate the various performance metrics associated with each combat configuration. These data, in addition to the win percentages, will be used in order to conduct both linear and nonlinear regression models so that the value of the metrics may be evaluated as combat network performance indicators. The data analysis will be provided in Chapter 4.

Step 7: Conclusions. Conclusions on the value of the performance metrics will be drawn from the data analysis and provided in Chapter 5. Additionally, recommendations for future research will be explored.

Number Partitioning Using Mathematica

Before the combat simulation experiments can be conducted, the various network configurations must be determined. The first part of determining the unique configurations deals with partitioning the number of sensors and influencers with respect to the number of deciders. Since the configurations are symmetric with X-Y-X networks, the partition folders, once generated, can be used for both sensors and influencers. In essence, this is simply an integer-partition problem.

Integer partitioning, in number theory, is a way of writing a positive integer n as a sum of positive integers ("Partition," n.d.). In the context of this research, once the partitions are generated using Mathematica®, they will be combined into what are referred to as unique combinations or meaningful combinations. This means that those configurations that are redundant in respect to the number order are discarded. As an example, the number 8 partitioned 3 at a time would give the following results: 6-1-1, 5-2-1, 4-3-1, 4-2-2, and 3-3-2.

The Mathematica® command that yields the results of the number 8 partitioned 3 at a time is:

```
IntegerPartitions[8, {3}].
```

The code that puts the output in a table format is:

```
TableForm[IntegerPartitions[8, {3}]].
```

The code that counts the number of unique partitions is:

```
Length[IntegerPartitions[8, {3}]],
```

which yields an answer of five (5).

The code that determines the number of permutations for each partition configuration is,

```
TableForm[Permutations[IntegerPartitions[8,{3}][[k]]]
```

where k is the kth item in the list.

For example,

```
TableForm[Permutations[IntegerPartitions[8,{3}][[1]]]
```

yields, all of the permutations for the first partition of the number 8 partitioned 3 at a time. So the output would include all of the permutations of a 6-1-1 configuration which are 6-1-1, 1-6-1, and 1-1-6.

```
TableForm[Permutations[IntegerPartitions[8,{3}][[2]]]
```

yields, all of the permutations for the second partition of the number 8 partitioned 3 at a time. So the output would include all of the permutations of a 5-2-1 configuration which are 5-2-1, 5-1-2, 2-5-1, 2-1-5, 1-5-2, and 1-2-5.

The maximum value for k for an 8-3 partition is 5 because that is the total number of unique partitions for the problem.

In order to determine all of the unique combat configurations for all Sensors, Deciders, and Influencers as described in Appendix A, all of the permutations for the Sensors and Deciders; and Deciders and Influencers must first be determined. Because the number of deciders for each combat cycle is always one, essentially the problem reduces to simply the permutations for the Sensors. The same technique used to determine the permutations for the Sensors can be used to determine the permutations for the Influencers.

The following Mathematica® code produces a number of data files that contain the number of permutations for each X-Y partition. These data files are then used as the input for the Visual Basic® program that uses the technique described by Fidanci (2010) to determine the number of unique combat configurations which is used in the actual

combat simulation model. The Mathematica® code determines the total number of permutations for $3 \leq x \leq 20$ and $3 \leq y \leq x$ is provided below:

```
For[i=3,i<=20,i++,
For[j=3,j<=i,j++,
Export["D:/Partitions/Partitions_"<>IntegerString[i]<>
_"<>IntegerString[j]<>".dat",TableForm[Flatten[Table[
Permutations[IntegerPartitions[i,{j}][[k]]],{k,1,Length[
IntegerPartitions[i,{j}]]}],1]]]]
```

Although the code was used to create partitions for $3 \leq x \leq 20$ and $3 \leq y \leq x$, only the partitions for $3 \leq x \leq 10$ and $3 \leq y \leq x$ will be used in this research. The scope was limited to this because the complexity of the networks exponentially increases for every increase in x and y .

DETERMINATION OF UNIQUE COMBINATIONS

Once the partitions and all permutations of the partitions are created using Mathematica®, a Visual Basic code is used to create the meaningful combinations. In other words, the partition permutations are combined with one another in order to make the X-Y-X or sensor-decider-influencer network configurations. Furthermore, the X-Y-X configurations will be “sorted” using a methodology developed by Fidanci (2010) in order to eliminate redundant configurations, leaving only the meaningful combinations to be examined in the combat simulation.

There is a finite number of ways to link the sensors deciders and influencers to each other. Deller (2009) made two important scoping decisions for the rules of the

combat simulation. This research also uses the same scoping decisions. First, each Sensor and Influencer would only be linked to one Decider. However, deciders do not have the same limitation; they can be linked to multiple sensors and influencers, with a minimum requirement of at least one sensor and one influencer (so that they can perform a combat cycle). Second, the connectivity within any X-Y-X arrangements is subject to only those hierarchical links in the chain of command necessary to create combat cycles (link types 2, 3, 6, 13, and 15).

The number of possible configurations for an X-Y-X force grows exponentially as X increases. Also, the number of different meaningful combinations for any X-Y-X number template is simply a combinatorial coupling relation of X and Y (Fidanci, 2010).

For example, there are a total of thirty-six possible ways to distribute five sensors and five influencers across three deciders, essentially a 5-3-5 configuration. When the number 5 is partitioned by 3 and all permutations are given, there are a total of six possible arrangements. This represents the sensor to influencer connections, and will be called sub matrix, A, m by three in dimension. Similarly, since we have the same number of influencers, there will be the same six possible configurations for the influencer to decider connections; the same sub matrix, A, m by three in dimension. Consequently, the total number of sensor-decider-influencer combinations is six times six, which is equal to a total of thirty-six combinations, as mentioned earlier. However, some combinations may actually be repeat combinations and are considered redundant.

In order to distinguish the different meaningful combinations from the total possible thirty-six configurations, a special matrix operation is applied to the two identical matrices, one representing the sensor-decider connections and the other

representing decider-influencer connections. The special matrix operation yields thirty six real numbers with fractions; some are repeated, but some are distinct. These numbers work as an index. In essence, the fractional numbers detect the difference between all of the possible combinations. Each distinct number in the resulting matrix represents a meaningful combination and thusly the number of meaningful combinations for each X-Y-X network template is determined. The general form of the special matrix operation is defined in Figure 5:

$$A \circ A' = \begin{bmatrix} a_{11} & \cdots & a_{1y} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{my} \end{bmatrix} \circ \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{y1} & \cdots & a_{ym} \end{bmatrix} = \sum_{j=1}^y \frac{a_{ij}^y}{a_{ji}^{1/y}}$$

Where,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1y} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{my} \end{bmatrix}$$

$$A' = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{y1} & \cdots & a_{ym} \end{bmatrix}$$

$$1 \leq i \leq m \text{ and } y \text{ as } n\text{Deciders}$$

Figure 5: General form of the special matrix operation used to determine meaningful combinations (Fidanci, 2010, p. 23).

Furthermore, the example of the 5-3-5 template and the operations to determine the number of meaningful combinations is given in Figure 6.

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix} \circ \begin{bmatrix} 3 & 2 & 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 & 2 & 1 \\ 1 & 1 & 2 & 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3^2}{3^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \\ \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \\ \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \\ \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \\ \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \\ \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} & \frac{3^2}{2^2} + \frac{1^2}{2^2} + \frac{1^2}{1^2} \end{bmatrix}$$

$$= \begin{bmatrix} 20.7208 & 23.2236 & 23.2236 & 28.6934 & 28.5874 & 28.6934 \\ 14.5469 & 13.6992 & 15.1433 & 14.5469 & 15.1433 & 16.6934 \\ 14.5469 & 15.1433 & 13.6992 & 16.6934 & 15.1433 & 14.5469 \\ 28.6934 & 23.2236 & 28.5874 & 20.7208 & 23.2236 & 28.6934 \\ 16.6934 & 15.1433 & 15.1433 & 14.5469 & 13.6992 & 14.5469 \\ 28.6934 & 28.5874 & 23.2236 & 28.6934 & 23.2236 & 20.7208 \end{bmatrix}$$

Figure 6: Calculation of meaningful combinations for a 5-3-5 network using the special matrix operation (Fidanci, 2010, p. 24).

The 5-3-5 network has a total of eight meaningful combinations, as can be determined from counting how many unique numbers there are in the last matrix in Figure 6.

The methodology developed by Fidanci (2010) to determine the number of unique configurations was employed through a Visual Basic program that is included in Appendix B. The Visual Basic program not only determines the number of meaningful combinations for each X-Y-X template examined, but it also combines the partitions from Mathematica® to form the actual unique combinations. This Visual Basic program outputs several folders labeled “PartitionData_X_Y,” where $3 \leq X, Y \leq 10$, that actually

contain the various unique combinations associated with each X-Y-X template, which in turn will be used as one of the inputs into the combat model.

ASYMMETRIC ENGAGEMENTS USING BALANCED FORCES

The contribution of this research is to investigate the value of the proposed performance metrics with respect to asymmetric engagements. Previous research taken on by Deller (2009) and Fidanci (2010) explored symmetric engagements with symmetric network combinations. This research takes the investigation a step further by looking at asymmetric engagements, or the ability for smaller, balanced networks, to combat larger balanced networks. For example, instead of having a 7-3-7 network combat a 7-3-7 network, this research will examine instances where a 7-3-7 network will combat an 8-3-8 network and a 9-3-9 network and so on. In fact this research proposes to run experiments that will investigate all of the possible engagements for symmetric configurations in the format of X-Y-X, ranging from $3 \leq X, Y \leq 10$. This is a total of 540 combat engagement simulations that will be conducted for data collection. Furthermore, each of the 540 combat engagements has a number of unique engagements that is associated with it. A complete list of all 540 combat engagements and the number of unique engagements associated with them is provided in Appendix A.

A Visual Basic code was written to calculate each engagement configuration, as well as, count the number of unique engagements. The output file created by this code, named XYXconfig.dat, will be used as one of the inputs into the combat model. This is a critical step in scoping the experiments as it allows for the segregation of runs to be carried out on multiple computers simultaneously, drastically reducing the time needed to run all 1,457,801 unique combat engagement simulations, thereby shortening the time

needed for data collection. The Visual Basic code used for creating the `XYXconfig.dat` file can be found in Appendix C.

INFORMATION AGE COMBAT MODEL USING DISCRETE-EVENT SIMULATION

The previous modeling attempts of the IACM, as conducted by Deller (2009) and Fidanci (2010), used an ABM paradigm. Although the simulations they developed worked well in representing the mechanics of the IACM, they were slow and computationally expensive, relative to the proposed methodology in modeling the IACM. This research proposes to build the IACM using the DES modeling paradigm. The reason behind this is to increase speed and efficiency of the experiments. After much thought, it was decided that the essence of the IACM and previous attempts could be captured through using the DES paradigm. The combat networks can still be represented in the same way as the previous models, however, the combat engagements are direct rather than dispersed in the simulation space.

Discrete-event simulation depicts the points in which the entities in the system, or the system itself, changes values or states (Fishman, 2001). So by definition, DES is a representation of a system's operation through a chronological succession of events. In essence, the IACM is a chronological succession of events where the various nodes change state throughout the combat engagements. Unlike the ABM approach where the nodes (sensors, deciders, and influencers) move around between interactions (events), this research approach eliminates the moving of the agents and in turn allows for "direct combat" of the various networks being studied. Because the nodes in the ABM approach

are placed randomly and move randomly, essentially there is no value added by modeling the IACM this way compared to modeling it with the DES paradigm. In fact, the direct combat approach proposed in this research should drastically improve simulation speed and efficiency, while still capturing the essence and mechanics of the previous models of the IACM.

Visual Basic was chosen as the programming language for building the DES combat model. The reasoning for this is because of not only the general familiarity with the programming language, but also because of its ability to quickly manipulate data files for both input and output purposes. Moreover, the integration of various Visual Basic programs will allow for streamlined approach to the calculation of various metrics that will be used in the data analysis. By adopting the DES paradigm and coding the IACM in Visual Basic, the speed and efficiency gained should allow for a drastically larger number of experiments to be studied. Whereas Deller (2009) was able to investigate 55 combat engagement configurations, this methodology allows for the examination of the 540 combat engagement configurations proposed in this research.

The underlying logic of the DES model used in this research is illustrated below in Figure 7. Before the simulation actually begins, there are several inputs given to the model for the initial setup. These inputs include the configuration details for each side (red and blue) and the number of replications desired for the engagement. The iterative process described in Figure 7 is for one replication of combat engagement. The engagement begins with a simple coin toss. However, instead of each side having a 50/50 chance of sensing a target, a weighted average is utilized using the number of sensors as weights. That is to say, the side with more sensors is more likely to sense a target, and is

therefore more likely to have an opportunity to influence a target. Once the decision is made on which side has sensed a target, the target selection process takes place.

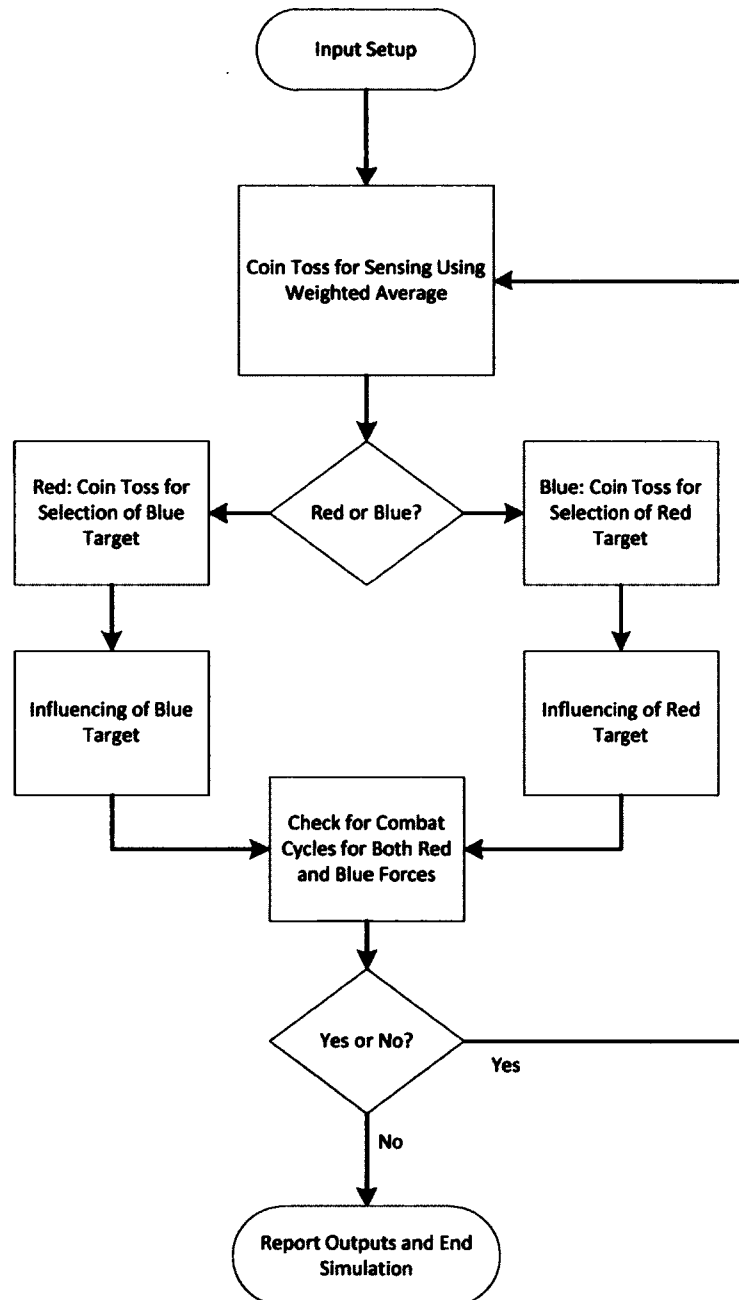


Figure 7: Flowchart for combat simulation logic.

If red senses a target, then another number is randomly generated to choose which blue target (sensor or influencer) will be eliminated or “influenced.” Conversely, if blue senses a target, then another number is randomly generated to choose which red target (sensor or influencer) will be eliminated or “influenced.” After the target has been influenced, it is removed from the simulation. Next there is a process that takes place in order to determine if there are any remaining combat cycles for either side. The purpose of this is to determine if one side is defeated, and if the simulation is over. If it is determined that both sides still have functioning combat cycles then the next iteration begins, going back to the sensing process using the weighted average. If there are no remaining combat cycles for one side then the simulation is over and the outputs are recorded.

The logic of the proposed combat model will be illustrated using an example. Figure 8 shows a 5-3-5 combat network and a 4-3-4 combat network, Blue and Red respectively. This is an example of an asymmetric engagement using balanced forces. Specifically, in the Blue Force, the way sensors are distributed across deciders is 3-1-1, similarly the influencers follow a 3-1-1 format. In the Red Force, the sensors have a 2-1-1 format, while the influencers have a 1-1-2 format.

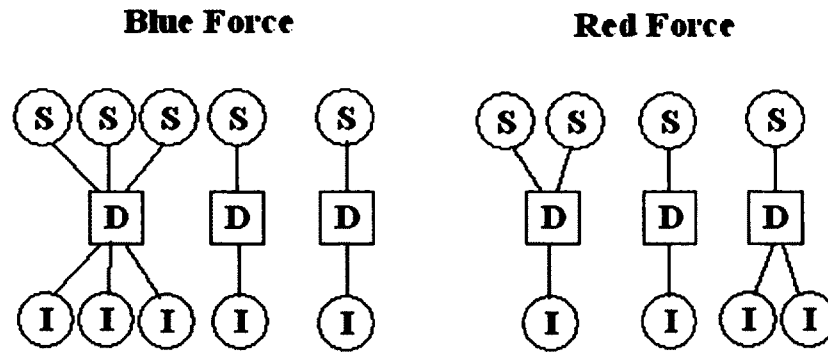


Figure 8: Blue 5-3-5 network vs. Red 4-3-4 network.

The manner in which the proposed combat model decides who takes the first shot is by random number generation; a coin toss. Specifically, at each round or iteration, the number of sensors is counted and a random number is generated based on the sensors count. For instance, the total sensors count, Blue and Red, in Figure 8 is 9. Therefore, the model generates a random number between 1 and 9. Say the generated number was 4. Since the 4th sensor is in the Blue force, the Blue force takes the shot. Once the model determines who takes the first shot, another random number generation takes place to decide which adversary sensor or influencer is destroyed. Since there are a total of 4 sensors and 4 influencers on the Red Force (a total of 8 potential targets), the model generates a number between 1 and 8 and the selected target is then eliminated from that combat force.

As we go through the iterations in this battle, it will be noticed that the model will only select a sensor that is a part of a combat cycle. Specifically, the coin toss will never pick a sensor that has no corresponding influencer(s) that can take the shot and destroy an adversary sensor or influencer. This is an important function of the model and will start happening as forces get depleted. This “one shot, one kill” approach is also important

because as we iterate, the coin toss acts like a weighted average, where an advantage is given to the force that has more sensors. The logic behind this is that the force with more sensors is more likely to find a target.

We will now do the first iteration for the example in Figure 8. We will assume the first random number generated is a 7. Since the 7th sensor is in the Red force, the Red force will take the first shot. The next step is to generate another random number which will determine the adversary target. Since there are a total of 10 potential targets on the Blue Force, a random number is generated between 1 and 10. We will assume that number is 3. This means that the third sensor which is connected to the first decider in the Blue force is the acquired target. The Red sensor transmits this information to the decider to which it is connected. The Red decider instructs the influencer attached to it to attack the acquired Blue target. At that point, the Blue target is destroyed. Figure 9 shows this engagement and the highlights Red combat cycle which destroyed the Blue target.

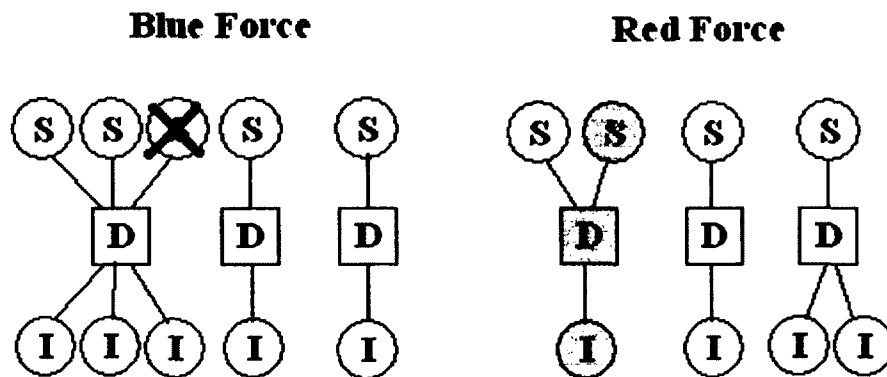


Figure 9: First Combat Cycle Iteration – Red Attacks Blue.

Another random number is generated to decide who takes the next shot. Since there are eight remaining sensors, that random number is going to be between one and

eight. You can see that since the Red force has the same number of sensors as the Blue force, they are both equally likely to acquire the next target. We will assume that number is five. The Red force takes the shot. A random number is generated to decide which target is eliminated. Say that number is six. The target acquired is therefore the second influencer connected to the first decider of the Blue force. The Red sensor sends this information to its decider, and the decider instructs its influencer to eliminate the Blue target. This Red combat cycle and the combat engagement are shown in Figure 10. A grey-colored “X” is used to illustrate the current target being destroyed.

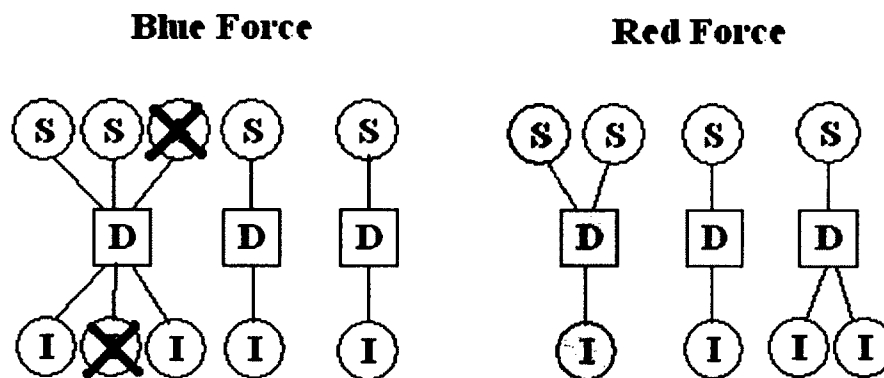


Figure 10: Second Combat Cycle Iteration – Red Attacks Blue.

Another random number is generated; let us say it is one. The Blue force now takes a shot for the first time. A random number between one and eight is generated to decide which Red target is destroyed, let us say it is seven. The Red target acquired is the first influencer connected to the third decider. The Blue sensor informs its decider of the acquired target, and the decider orders one of its remaining influencers to attack and destroy the Red target. Figure 11 depicts this engagement and shows the Blue combat cycle that acquired and destroyed the Red target.

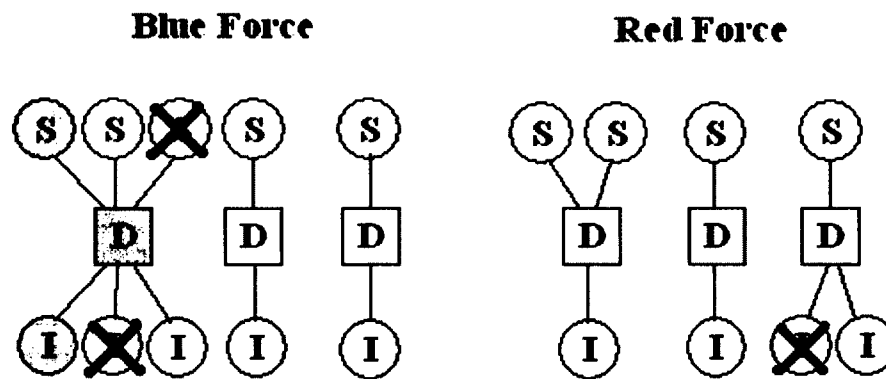


Figure 11: Third Combat Cycle Iteration – Blue Attacks Red.

Another round is initiated now and we will assume the random number is four.

The Blue force takes the next shot which means detection occurs through its fourth sensor, which is connected to the third decider. A random number generation takes place to decide which Red target is acquired. Let us assume it yields a five. This means that the influencer attached to the first Red decider is the next target. Accordingly, the Blue sensor informs its decider of the acquired target, and the decider orders its influencer to take it out. Figure 11 depicts this engagement. An important observation in Figure 12 is how the first and second Red sensors are no longer a part of an effective combat cycle and have an NCC (No Combat Cycle) label on top of them. The same applies to their decider. This is all due to the fact that their decider lost its firepower when its influencer was eliminated. Therefore, those two Red sensors and their corresponding decider are rendered combat ineffective. However, the sensors can still be targeted by the Blue force.

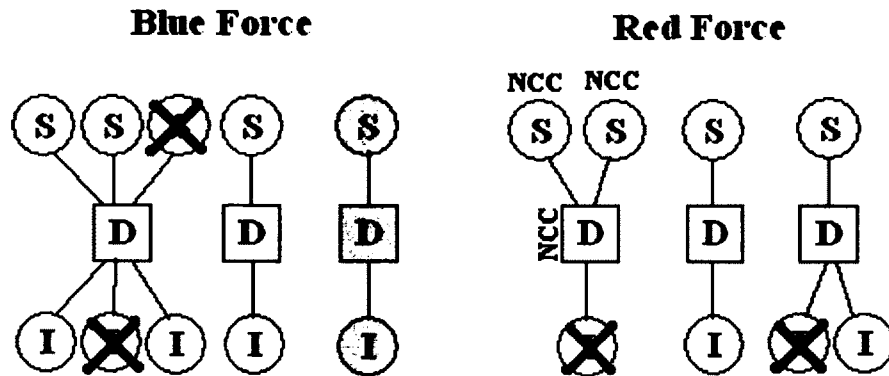


Figure 12: Fourth Combat Cycle Iteration – Blue Attacks Red.

Another engagement takes place. Since there are still six operational sensors left in battle, the random number generated will be between one and six. Let the random number this time be three. This means the third Blue sensor gets selected and therefore the Blue force takes the next shot. Another random number is generated and this time it will be between one and six since there are a total of six Red targets left in Battle, two of which are combat ineffective. Let that number be four. The fourth remaining Red sensor is acquired as a target. Consequently, the Blue sensor sends the information to its decider who will give orders to the sole influencer it has to take that target out. Figure 13 shows this engagement. Notice that this engagement results in taking the third Red decider and its influencer out of battle as they are no longer part of a combat cycle.

The model keeps running as both forces still have combat cycles left in their network. Notice that the Blue force is more likely to take the next shot because it possesses four operational sensors, compared to one sensor the Red force still has operational as part of a combat cycle.

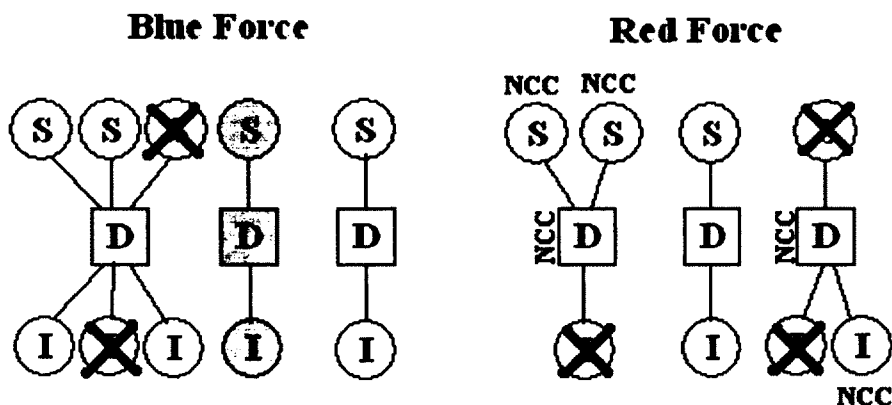


Figure 13: Fifth Combat Cycle Iteration – Blue Attacks Red.

A random number between one and five is generated and let us say it is two. This means the second Blue sensor is selected and therefore the Blue force takes the next shot. Another random number between one and five is generated to decide the acquired target, since there are actually five potential targets left on the Red Force, even though only two of the five targets are still operational as part of a combat cycle. Let that number be four. This means the first remaining Red influencer is acquired as a target. The Blue sensor communicates this information to its decider, and the decider sends the influencer to take out the Red target. This engagement effectively ends the battle as the Red force is no longer combat effective because the engagement eliminated its last remaining combat cycle. The Blue force is victorious. Figure 14 illustrates the final engagement.

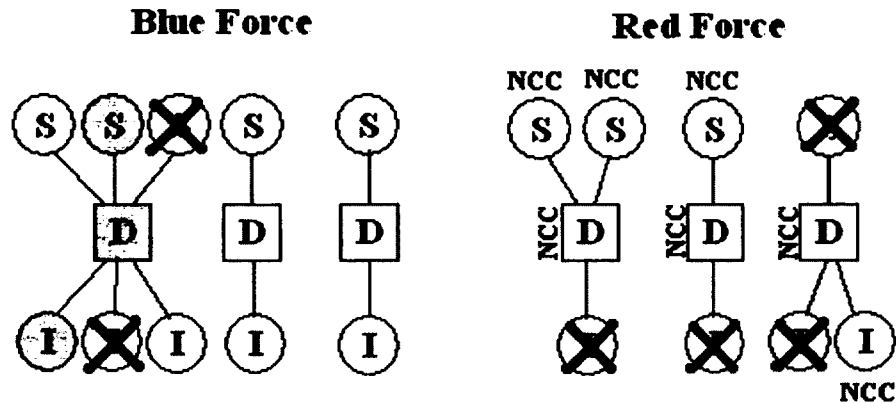


Figure 14: Final Combat Cycle Iteration – Blue Attacks Red.

One can notice that the logic behind the Combat model is not complex. Moreover, the importance of the combat cycle becomes evident with the example. Take the hypothetical case of having the Blue force losing its first three influencers at the beginning of battle. This will render their decider, and the three sensors connected to it, combat ineffective. They lost their firepower and can no longer form a functional combat cycle. It is clear how combat cycles play an important role in keeping assets operational on the battlefield. This issue will be examined in depth in this research as we start looking at the performance metrics of each combat configuration in battle. Those performance metrics should enable us to quantitatively assess the strengths and weaknesses of each unique configuration. The full combat model code can be found in Appendix D.

Dividing Simulation Runs

Due to the enormous amount of simulation experiments required to analyze configurations from $3 \leq X, Y \leq 10$, it was decided to make the combat model flexible enough to run various configurations separately, rather than have all of them run on one

execution on one machine. The main reason for designing the Visual Basic combat model with this in mind was to significantly reduce the time needed for data collection. By separating the simulation runs into various combat model executions, several processors can be used at one time. In other words, several computers will be utilized in conducting the experiments, rather than using one single computer. Also, setting up the simulations in this fashion allows for better detection of errors within configuration settings, and added reliability in the case of a computer malfunction or power outage.

Once the combat model design was complete, the next step was to segregate the runs using an Excel spreadsheet. In order to do this, the 540 combat configurations were grouped together to create a total number of engagements for each segment to be around two hundred thousand unique combat engagements. There were a couple of exceptions where certain configurations yielded around six to eight million unique combat engagements that could not be separated further. The resulting ranges are then hard-coded into the combat model as “lowerbound” and “upperbound” variables, in order to set up the experiments. Table 2 shows a summary of the ranges used in executing the experiments, as well as the number of unique combat engagements associated with those ranges.

Table 2

Lower and upper bounds for timely execution of combat model.

Processor	Lower Bound	Upper Bound	# of Engagements
1	1	366	194,287
2	367	408	204,178
3	409	451	191,759
4	452	470	208,517
5	471	506	179,643
6	507	513	220,473
7	514	520	205,252
8	521	540	53,692
Total			1,457,801

Model Verification Overview

Although previous attempts at modeling the IACM by Deller (2009) and Fidanci (2010) used ABM, the model developed using the DES paradigm can be verified using their results. Despite the different modeling paradigms, the new combat model should yield similar results to those of Deller (2009) and Fidanci (2010). The verification of the proposed combat model will come from comparing the results from symmetric engagements in the new model, to the previous models' results reported by Deller (2009). The reason why symmetric engagements must be used for the verification is because those are the type of engagements analyzed in the research of Deller (2009) and Fidanci (2010). Specifically, the ordinal ranking of various networks that resulted in the research of Deller (2009) will be compared to the ordinal ranking of those same networks using the results of this research. By making this comparison it can be verified that the model is performing as intended and the networks give similar performance levels as found in previous research.

Performance Metrics

The performance metrics used in this research are adapted from the work of Deller (2009) and Fidanci (2010). The Perron-Frobenius Eigenvalue (PFE) was not included in the list of factors evaluated because it was determined in previous research that the added value of the PFE decreased in larger configuration sizes. That is to say that the proportion of unique eigenvalues to the number of unique configurations decreases as the configuration sizes increase. It is for this reason that it was decided to focus on the other performance metrics as described in the dissertations of Deller (2009) and Fidanci (2010). The performance metrics that will be investigated, however, include disparity, strength, power, connectivity, robustness, and stability.

Disparity

Disparity is defined as the sum of the difference of the maximum and minimum number of sensors and influencers across the deciders (Deller, 2009). This metric represents the imbalance of the distribution of sensors and influencers across all of the deciders. This can be formulated as:

$$Disparity = [\max(S_n) - \min(S_n)] + [\max(I_n) - \min(I_n)]$$

Where, S_n : the number of Sensors assigned to each of n Deciders

I_n : the number of Influencers assigned to each of n Deciders

According to Deller (2009), the greater disparity of a configuration, the more likely that configuration will have either an extremely high or extremely low value for its probability to win. Consequently, disparity can have a positive or negative impact on the win probability.

Robustness

Robustness is the minimum number of either sensors or influencers lost that would render the entire configuration of the nodes nonfunctional (Deller, 2009).

Robustness can be mathematically expressed as:

$$Robustness = \sum_{i=1}^n \min (S_i, I_i) \quad (1)$$

where, S_i : the number of Sensors assigned to Decider i

I_i : the number of Influencers assigned to Deciders i

The greater the robustness value of a given configuration, the longer that configuration will be combat effective. Configurations with a higher robustness value have a greater probability of win, while less robust configurations have a lower probability of win.

Strength

Strength of connectivity, or simply strength, is a metric proposed by Fidanci (2010). Strength is the sum of the weighted average of sensors and influencers linked to each decider according to the logarithmic function. The combinations with greater strength reflect the number of nodes of sensors and influencers linked to each decider so that the entire configuration maintains combat effectiveness. Strength can be formulated as:

$$Strength = \sum_{i=1}^n \{\log_{10}(\# \text{ of Sensor}_i + 1) * \log_{10}(\# \text{ of Influencer}_i + 1)\} \quad (2)$$

According to Fidanci (2010), the greater the strength value, the more likely that configuration will be to win.

Power

Another metric proposed by Fidanci (2010) is the power of the deciders, or simply Power. Power is a sum of the weighted average of sensors and influencers linked to each decider according to the square-root function. Power can be mathematically expressed as:

$$\text{Power} = \sum_{i=1}^n \{\text{Sqrt}(\# \text{ of Sensor}_i) * \text{Sqrt}(\# \text{ of Influencer}_i)\} \quad (3)$$

According to Fidanci (2010), the larger the power value, the more reliable and readily available fighting units maintains the combat effectiveness.

Stability

Another metric proposed by Fidanci (2010) is stability of deciders, simply referred to as stability. Stability is the sum of the quotient of sensors and influencers connected to each decider. Stability can be formulated as:

$$\text{Stability} = \sum_{i=1}^n \{\text{Quotient}(\# \text{ of Sensor}_i, \# \text{ of Influencer}_i)\} \quad (4)$$

According to Fidanci (2010), there is a negative correlation between the combat performance and the stability value. In essence, stability shows the number of ineffectively used decider nodes.

Connectivity

The last metric proposed by Fidanci (2010) is connectivity of sensors and influencers, referred to as connectivity. Similar to disparity, connectivity is the sum of the absolute value of the unbalanced number of sensors and influencers connected to each decider. Connectivity is mathematically represented as:

$$Connectivity = \sum_{i=1}^n \{ABS(\# \text{ of Sensor}_i) - (\# \text{ of Influencer}_i)\} \quad (5)$$

According to Fidanci (2010), connectivity represents the number of unproductive sensors and influencers.

Data Analysis

The data analysis proposed in this research will utilize the performance metrics previously discussed. They will be evaluated as performance indicators for the combat effectiveness of various combat networks. Specifically, their utility with respect to predicting the combat effectiveness of balanced forces in asymmetric engagements will be analyzed. In addition to gathering descriptive statistics about the percentage of wins for various engagements, regression analysis will be used as a basis for evaluating the various metrics and their ability to quantify combat network performance.

Descriptive Statistics

The descriptive statistics that are to be gathered deal with the average percentage of wins associated with each combat configuration with respect to their performance against configurations of the same size, as well as, their performance versus larger configurations. The results will be presented as histograms illustrating the average percentage of wins for each of the compared engagements. Consequently, as mentioned

in the section on model verification, symmetric engagements of balanced forces should have a total win percentage around 50 percent on average. Furthermore, as configurations battle larger combat forces, their win percentage will be less than 50 percent on average, with an observable trend emerging.

Linear Regression

The first type of regression analysis to be performed is a linear regression. Each of the performance metrics discussed earlier will be evaluated for their utility as performance indicators. Several multiple regressions will be performed in order to determine which combination of metrics works best in predicting combat network performance with respect to the asymmetric engagements analyzed in this research. In addition to the performance metrics previously discussed, the network sizes in the form of number of sensors, deciders, and influencers will also be included in the analysis.

Nonlinear Regression

A second regression will be performed to investigate the possible nonlinear behavior of the performance metrics. Once again each of the performance metrics discussed earlier will be evaluated for their utility as performance indicators, this time using a nonlinear regression. Finally, several multiple nonlinear regressions will be performed in order to gain insight on the utility of the performance metrics with respect to the asymmetric engagements studied in this research. Also, in addition to the performance metrics previously discussed, the network sizes in the form of number of sensors, deciders, and influencers will also be included in the analysis.

CHAPTER 4

RESULTS AND DATA ANALYSIS

VERIFICATION RESULTS

One of the most important steps in any modeling and simulation research is the verification of the model. Verification is done to ensure that the model is running in the way it was envisioned and implemented to run and that the algorithms incorporated in it are working properly.

The way the verification of the model developed in this research was approached is by comparing the results with those of Deller (2009). Originally the idea was to compare the regression analysis between the current model's results and the previous results of Deller (2009). However, it was quickly realized that this was not a good approach to the verification because the models were developed using two different modeling paradigms. Thus, an alternative approach to the model verification was conceived using the ordinal ranking of the various configurations. Specifically, the ordinal rankings of Deller's results were compared to the ordinal rankings of the current model's results. The verification was done using the three configurations that were analyzed by Deller (2009) which are the 7-3-7, 8-3-8, and 9-5-9 configurations. Additionally, the ordinal rankings were compared using different numbers of replications in order to examine the impact of replications on the modeling results. The comparison was done by ordering the unique combinations in terms of their average win percentage. Then the average difference between the current model's ordinal rankings and Deller's ordinal rankings was calculated.

The verification results for the 7-3-7 configuration show that as the number of replications increases, the average ordinal difference decreases. For instance, at 30 replications, the average difference in ordinal rank between the two models was 8.5. This means that on average the rankings differed by 8.5 ranks. At 100 replications the average ordinal difference decreases to 6.3. At 1000 replications the average ordinal difference decreases to 3.1. Consequently, as the number of replications was increased the current model more closely reflected the results of Deller (2009). This shows that the current model is working in the way it was intended, modeling the logic of the Information Age Combat Model.

Figure 15 depicts a graphical representation of the relationship between the number of replications and the average difference between the ordinal rankings of the results from the two different models, for the 7-3-7 configuration.

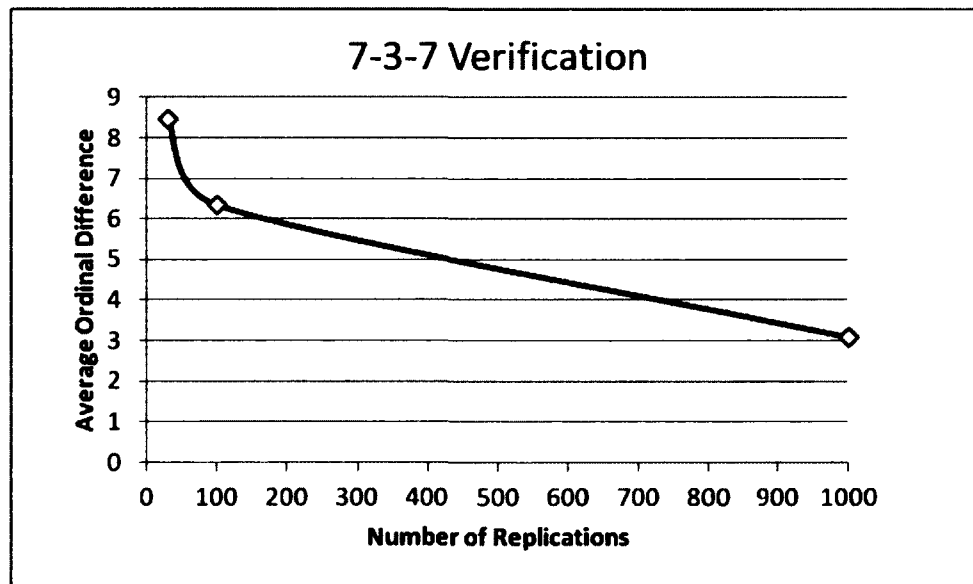


Figure 15: Average ordinal difference versus replications for 7-3-7 configuration.

The verification for the 8-3-8 configuration shows similar results. For instance, at 30 replications, the average difference in ordinal rank between the two models was 11.0. This means that on average the rankings differed by 11.0 ranks. At 100 replications the average ordinal difference decreases to 7.6. At 1000 replications the average ordinal difference decreases to 4.3. Subsequently, as the number of replications was increased the current model more closely reflected the results of Deller (2009). Once again, this shows that the current model is working in the way it was intended, modeling the logic of the Information Age Combat Model.

Figure 16 displays a graphical representation for the 8-3-8 configuration of the relationship between the number of replications and the average difference between the ordinal rankings of the results from the two different models.

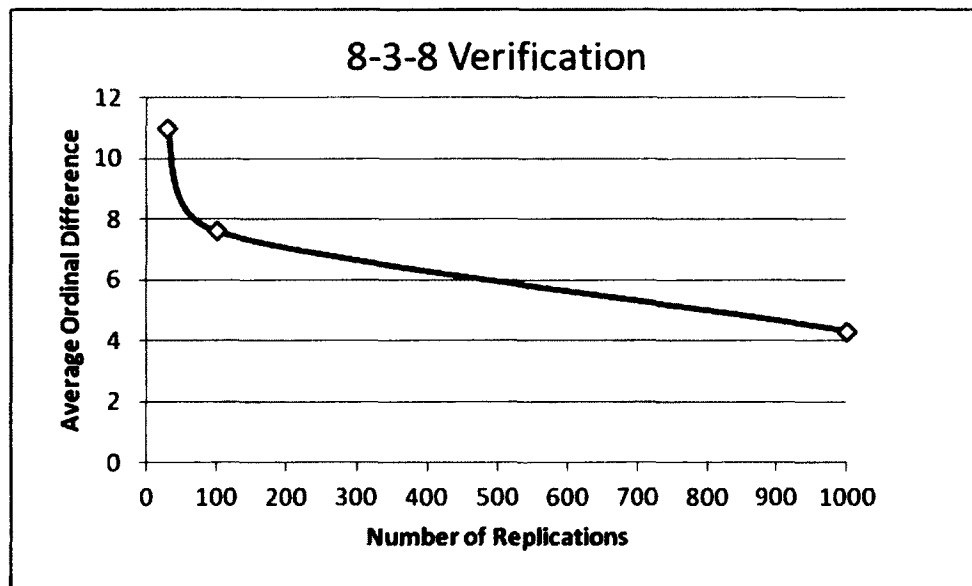


Figure 16: Average ordinal difference versus replications for 8-3-8 configuration.

The verification for the 9-5-9 configuration also showed similar results. For example, at 30 replications, the average difference in ordinal rank between the two models was 17.7. This means that on average the rankings differed by 17.7 ranks. At 100 replications the average ordinal difference decreases to 14.3. At 1000 replications the average ordinal difference decreases to 10.0. Therefore, as the number of replications was increased the current model more closely reflected the results of Deller (2009). Once more, this verifies that the current model is working in the way it was intended, modeling the logic of the Information Age Combat Model.

Figure 17 graphically depicts the relationship between the number of replications and the average difference between the ordinal rankings of the results from the two different models, for the 9-5-9 configuration.

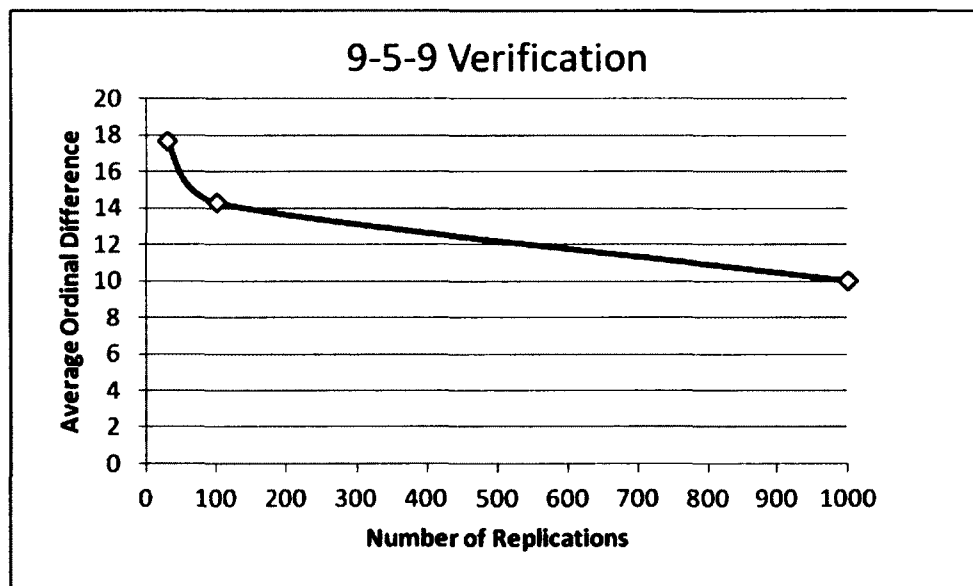


Figure 17: Average ordinal difference versus replications for 9-5-9 configuration.

SIMULATION RESULTS SUMMARY

The 540 experiments performed in this research consisted of a total of 1,457,801 unique combat engagements. The purpose of this is to examine the possibility for a smaller combat configuration to win against a larger configuration. Figure 18 illustrates a comparison of the asymmetric engagements for X-Y-X, where X ranges from X to X+3 and $3 \leq X \leq 7$ for all $Y \leq X$.

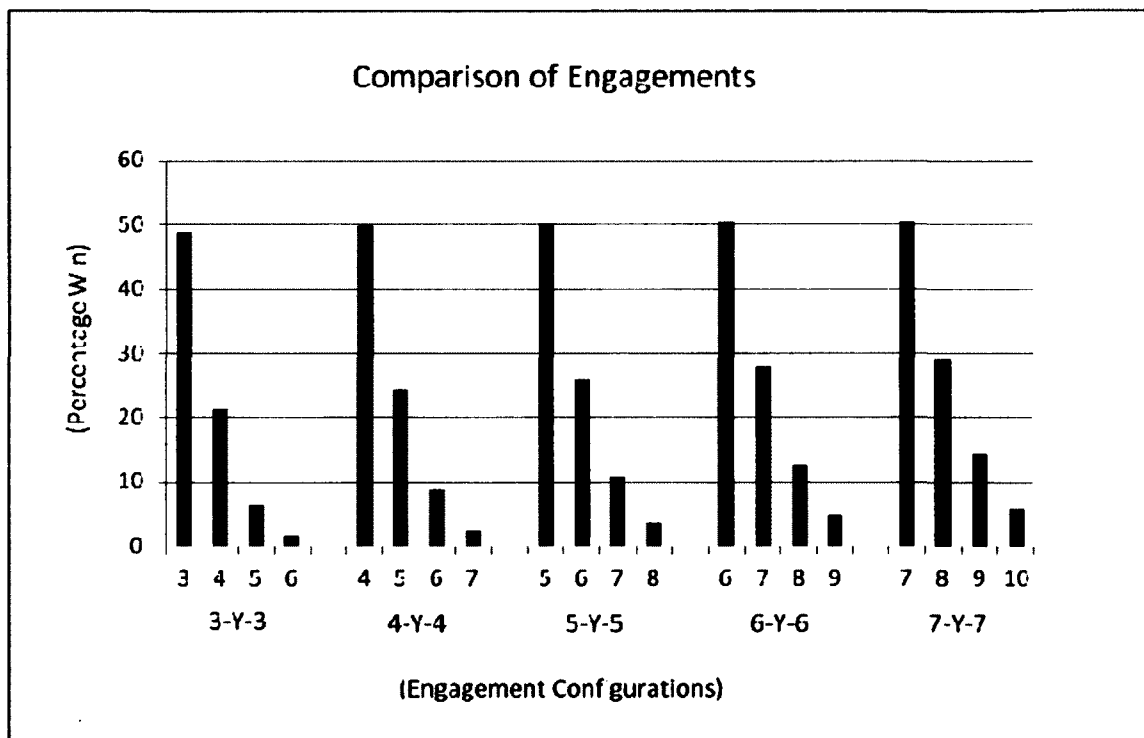


Figure 18: Comparison of asymmetric engagements.

The comparisons in Figure 18 show that on average when a configuration battles an identical configuration it wins about fifty percent of the time. Furthermore, when one configuration does battle with another configuration that is larger by one sensor and one influencer, its chance at winning decreases to about twenty-five percent on average. When one configuration combats another configuration that is larger by two sensors and

two influencers, its chance at winning decreases to about ten percent on average. Finally, when one configuration battles another configuration that is larger by three sensors and three influencers, its chance at winning decreases to about five percent or less on average. One other observation that can be taken from this is that it is possible, in fact, for a smaller combat network to defeat a larger combat network. This points to the idea that the combat network configuration can and does play a vital role in determining its success.

Figure 19 is a PDF displaying all of the win percentages collected from the 1,457,801 combat engagements. This confirms that the simulation model is working as intended, as it illustrates the multi-modal nature of the asymmetric engagements. In other words, each of the “mini normal distributions” represents a certain type of engagement. For instance, the distribution centered around fifty percent represents symmetric engagements, whereas the distributions to the left represent the asymmetric engagements. Furthermore, the farther left on the distribution, the greater the asymmetry in the engagements.

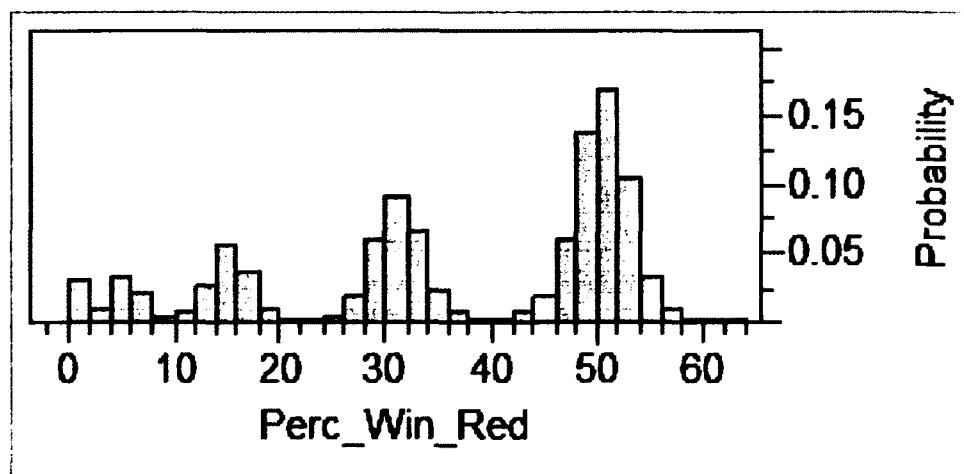


Figure 19: PDF of percent win for red team for all simulation runs.

Quantiles		
100.0%	maximum	62.8
99.5%		56.3
97.5%		54.5
90.0%		52.6
75.0%	quartile	50.7
50.0%	median	46.3
25.0%	quartile	28.8
10.0%		13.1
2.5%		1.6
0.5%		0.1
0.0%	minimum	0
Moments		
Mean		37.013952
Std Dev		16.217794
Std Err Mean		0.0134321
Upper 95% Mean		37.040278
Lower 95% Mean		36.987626
N		1457801

Figure 20: Summary statistics for PDF of percent win for red team.

Figure 20 shows all of the summary statistics associated with the percent win for the red team for all simulation runs. It should be noted that the win percentages range from 0.0 percent to 62.8 percent. To further illustrate the multi-modal nature of the simulation results, *Figure 21* displays the CDF of percent win for the red team for all simulation runs. Each S-curve in the CDF represents a normal distribution within the aggregated results.

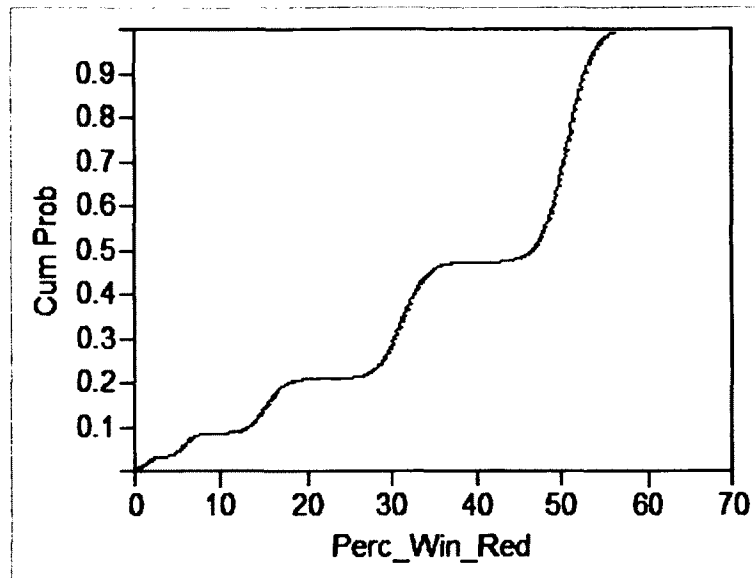


Figure 21: CDF of percent win for red team for all simulation runs.

LINEAR REGRESSION RESULTS

The results from the 1,457,801 combat engagements were used to help resolve what the major factors are that determine the outcome of the engagements. JMP 9 was used to perform all of the regression analyses found in this section. The initial analysis was performed using a linear regression with the percent win for red as the dependent variable and the number of sensors, deciders, and influencers for both red and blue as the independent variables. Figure 22 shows a summary of the results for the linear regression model using only sensors, deciders, and influencers as the independent variables. In this regression analysis the RSquare was found to be about 0.91. Also, although all of the variables were found to be statistically significant, it is the number of sensors and influencers that have the greatest impact on the engagement results; whereas the number of deciders has little impact on the engagement results.

Summary of Fit				
RSquare			0.912051	
RSquare Adj			0.91205	
Root Mean Square Error			4.809598	
Mean of Response			37.01395	
Observations (or Sum Wgts)			1457801	

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	349703887	87425972	3779401
Error	1.5e+6	33722070	23.132229	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	535	26556285	49637.9	10094.55
Pure Error	1.5e+6	7165786	4.9	Prob > F
Total Error	1.5e+6	33722070		<.0001*
				Max RSq
				0.9813

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.784843	0.073699	580.54	<.0001*
Red S,I	13.992586	0.003664	3819.1	<.0001*
Red D	-0.363777	0.005902	-61.63	<.0001*
Blue S,I	-13.46769	0.008207	-1641	<.0001*
Blue D	0.3961034	0.003958	100.08	<.0001*

Figure 22: Linear regression model using SDI.

It should be also noted that the number of red deciders has a negative correlation to the average percent win for the red side. This means that it is better to have more sensors and influencers and less deciders in order to increase the chances of winning an engagement.

The next regression model that was constructed examines the performance metrics defined earlier in the research, and their predictive capability on the combat network performance. Figure 23 displays the results from the regression model using all of the performance metrics defined earlier as independent variables. The RSquare in this regression model was calculated to be around 0.92, which is a slight increase from the previous regression model. All of the variables were found to be statistically significant except for red disparity.

Summary of Fit				
RSquare				0.921313
RSquare Adj				0.921312
Root Mean Square Error				4.549317
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	353255160	29437930	1422377
Error	1.5e+6	30170797	20.696286	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	30091728	21.1579	9.5101
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	30170797		<.0001*
				Max RSq
				0.9998

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.822792	0.069884	612.77	<.0001*
Conn_Red	6.4651261	0.009793	660.15	<.0001*
Conn_Blue	-5.587557	0.01102	-507.0	<.0001*
Disp_Red	0.0014623	0.002423	0.60	0.5462
Disp_Blue	-0.149656	0.002598	-57.60	<.0001*
Pow_Red	1.8137686	0.032569	55.69	<.0001*
Pow_Blue	-1.045457	0.031649	-33.03	<.0001*
Rob_Red	13.043787	0.026716	488.23	<.0001*
Rob_Blue	-10.92455	0.0289	-378.0	<.0001*
Stab_Red	-0.358176	0.005404	-66.28	<.0001*
Stab_Blue	-0.093718	0.005754	-16.29	<.0001*
Stre_Red	-4.799564	0.191753	-25.03	<.0001*
Stre_Blue	-12.76768	0.165811	-77.00	<.0001*

Figure 23: Linear regression model using performance metrics.

The same regression model was recalculated with red disparity removed, and the results can be found in Figure 24. After recalculating this regression model the RSquare remained at 0.92, and all other variables remained significant.

Summary of Fit				
RSquare		0.921313		
RSquare Adj		0.921312		
Root Mean Square Error		4.549316		
Mean of Response		37.01395		
Observations (or Sum Wgts)		1457801		

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	353255153	32114105	1551685
Error	1.5e+6	30170805	20.696277	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	30033119	21.5532	10.0733
Pure Error	64350	137686	2.1396	Prob > F
Total Error	1.5e+6	30170805		<.0001*
				Max RSq
				0.9996

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.824607	0.06982	613.36	<.0001*
Conn_Red	6.4684987	0.008042	804.33	<.0001*
Conn_Blue	-5.587493	0.01102	-507.0	<.0001*
Disp_Blue	-0.14962	0.002597	-57.60	<.0001*
Pow_Red	1.8110036	0.032245	56.16	<.0001*
Pow_Blue	-1.046298	0.031618	-33.09	<.0001*
Rob_Red	13.051465	0.023492	555.58	<.0001*
Rob_Blue	-10.9244	0.028899	-378.0	<.0001*
Stab_Red	-0.359554	0.004898	-73.41	<.0001*
Stab_Blue	-0.093764	0.005753	-16.30	<.0001*
Stre_Red	-4.829129	0.185388	-26.05	<.0001*
Stre_Blue	-12.76304	0.165632	-77.06	<.0001*

Figure 24: Linear regression model with performance metrics minus Disp_Red.

The next regression model examines the ability to combine all of the performance metrics as well as the number of sensors, deciders, and influencers in order to increase the predictive capability of the regression model. It turns out that there is a multicollinearity problem between the number of sensors and influencers and several of the performance metrics. Figure 25 summarizes the results from this regression model using all of the performance metrics and the number of sensors, deciders, and influencers for both red and blue.

Singularity Details

Conn_Red = - 2*Rob_Red + 2*Red S,I
Conn_Blue = - 2*Rob_Blue + 2*Blue S,I

Summary of Fit

RSquare0.923779

RSquare Adj0.923778

Root Mean Square Error4.477449

Mean of Response37.01395

Observations (or Sum Wgts)1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	354200918	25300065	1262003
Error	1.5e+6	29225042	20.047553	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	29145973	20.4929	9.2112
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	29225042		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob> t
Intercept	Biased	45.279405	0.070593	641.42	<.0001*
Conn_Red	Biased	5.8603869	0.010038	583.80	<.0001*
Conn_Blue	Biased	-5.572395	0.011263	-494.7	<.0001*
Disp_Red	Biased	-0.606284	0.003688	-164.4	<.0001*
Disp_Blue	Biased	-0.083937	0.003835	-21.89	<.0001*
Pow_Red	Biased	14.99157	0.068908	217.56	<.0001*
Pow_Blue	Biased	-2.588073	0.073481	-35.22	<.0001*
Rob_Red	Biased	11.94984	0.02678	446.21	<.0001*
Rob_Blue	Biased	-10.95054	0.028803	-380.2	<.0001*
Stab_Red	Biased	-0.13899	0.005416	-25.66	<.0001*
Stab_Blue		-0.092874	0.00573	-16.21	<.0001*
Stre_Red	Biased	-93.70946	0.452658	-207.0	<.0001*
Stre_Blue	Biased	-1.662565	0.483609	-3.44	0.0006*
Red S,I	Zeroed	0	0	.	.
Red D	Biased	-4.449238	0.020538	-216.6	<.0001*
Blue S,I	Zeroed	0	0	.	.
Blue D		0.5855079	0.022418	26.12	<.0001*

Figure 25: Linear regression model with performance metrics + SDI.

There is only a negligible improvement in the RSquare when the performance metrics and number of sensors, deciders, and influencers are included as independent variables in the regression analysis.

Summary of Fit				
RSquare			0.923779	
RSquare Adj			0.923778	
Root Mean Square Error			4.477449	
Mean of Response			37.01395	
Observations (or Sum Wgts)			1457801	

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	354200916	25300065	1262003
Error	1.5e+6	29225042	20.047553	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	29145973	20.4929	9.2112
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	29225042		<.0001*
				Max RSq
				0.9998

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	45.279405	0.070593	641.42	<.0001*
Conn_Red	5.8603869	0.010038	583.80	<.0001*
Conn_Blue	-5.572395	0.011263	-494.7	<.0001*
Disp_Red	-0.606284	0.003688	-164.4	<.0001*
Disp_Blue	-0.083937	0.003835	-21.89	<.0001*
Pow_Red	14.99157	0.068908	217.56	<.0001*
Pow_Blue	-2.588073	0.073481	-35.22	<.0001*
Rob_Red	11.94964	0.02678	446.21	<.0001*
Rob_Blue	-10.95054	0.028803	-380.2	<.0001*
Stab_Red	-0.13899	0.005416	-25.66	<.0001*
Stab_Blue	-0.092874	0.00573	-16.21	<.0001*
Stre_Red	-93.70946	0.452658	-207.0	<.0001*
Stre_Blue	-1.662565	0.483609	-3.44	0.0006*
Red D	-4.449238	0.020538	-216.6	<.0001*
Blue D	0.5855079	0.022418	26.12	<.0001*

Figure 26: Linear regression model with performance metrics + D.

Figure 26 illustrates the results from the regression analysis using the performance metrics and number of deciders for red and blue (number of sensors and influencers were removed because of multicollinearity with other variables). The multicollinearity that was discovered during the regression analysis is explored further in a later section in this chapter.

NON-LINEAR REGRESSION RESULTS

The next analysis was performed using a non-linear regression with the percent win for red as the dependent variable and the number of sensors, deciders, and influencers for both red and blue as the independent variables. Non-linear terms were also included in the form of quadratic terms and all two-way interaction terms.

Summary of Fit				
RSquare				0.980288
RSquare Adj				0.980287
Root Mean Square Error				2.277007
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	375867687	26847692	5178194
Error	1.5e+6	7558270	5.1847598	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	525	392484.7	747.590	152.0327
Pure Error	1.5e+6	7165785.5	4.917	Prob > F
Total Error	1.5e+6	7558270.3		<.0001*
				Max RSq
				0.9813

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	44.213009	0.081364	720.50	<.0001*
Red S,I	18.146477	0.002542	7139.5	<.0001*
Red D	-0.466036	0.003453	-135.0	<.0001*
Blue S,I	-17.67417	0.006649	-2658	<.0001*
Blue D	0.4011586	0.002412	166.35	<.0001*
(Red S,I-8.94429)*(Red D-3.69742)	-0.101324	0.002905	-34.88	<.0001*
(Red S,I-8.94429)*(Blue S,I-9.759)	-5.039053	0.004035	-1249	<.0001*
(Red S,I-8.94429)*(Blue D-4.67447)	0.0975055	0.001684	58.58	<.0001*
(Red D-3.69742)*(Blue S,I-9.759)	0.1236491	0.006788	18.22	<.0001*
(Red D-3.69742)*(Blue D-4.67447)	-0.009678	0.003259	-2.97	0.0030*
(Blue S,I-9.759)*(Blue D-4.67447)	-0.135173	0.004138	-32.87	<.0001*
(Red S,I-8.94429)*(Red S,I-8.94429)	2.3021673	0.001075	2142.4	<.0001*
(Red D-3.69742)*(Red D-3.69742)	0.037035	0.003436	10.78	<.0001*
(Blue S,I-9.759)*(Blue S,I-9.759)	2.7065405	0.005083	532.50	<.0001*
(Blue D-4.67447)*(Blue D-4.67447)	-0.011779	0.001193	-9.87	<.0001*

Figure 27: Non-linear regression model using SDI.

Figure 27 shows a summary of the results for the non-linear regression model using only sensors, deciders, and influencers as the independent variables. In this regression analysis the RSquare was found to be about 0.98, which is a significant increase over all of the linear regression models. Also, as in the linear regression models, all of the variables were found to be statistically significant. Once again it is the number of sensors and influencers that have the greatest impact on the engagement results; whereas the number of deciders has little impact on the engagement results. Additionally, it should be again noted that the number of red deciders has a negative correlation to the average percent win for the red side. This means that it is better to have more sensors and influencers and less deciders in order to increase the chances of winning an engagement.

Summary of Fit				
RSquare				0.990684
RSquare Adj				0.990684
Root Mean Square Error				1.56536
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	90	379854054	4220601	1722446
Error	1.5e+6	3571904	2.450353	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3492834.9	2.45599	Prob > F
Pure Error	35540	79068.8	2.22478	<.0001*
Total Error	1.5e+6	3571903.7		Max RSq 0.9998

Figure 28: Non-linear regression model using all performance metrics.

Figure 28 shows a summary of the results from a non-linear regression analysis using all of the performance metrics, their quadratic terms, and all two-way interactions.

This regression analysis yielded an RSquare of 0.99, which is a slight increase from the non-linear regression using the number of sensors, deciders, and influencers as predictors. In this instance all of the main effects were found to be significant, however, there were several interaction and quadratic terms that were found to be insignificant. The parameter estimates for this model and all succeeding models, as well as, the complete regression results from all of the regression models discussed in this chapter, can be found in Appendix K.

Summary of Fit				
RSquare			0.990882	
RSquare Adj			0.990881	
Root Mean Square Error			1.56557	
Mean of Response			37.01395	
Observations (or Sum Wgts)			1457801	

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	67	379853038	5669448	2313106
Error	1.5e+6	3572920	2.451011	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3493850.7	2.45666	Prob > F
Pure Error	35540	79068.8	2.22478	<.0001*
Total Error	1.5e+6	3572919.5		Max RSq 0.9998

Figure 29: Non-linear regression model using performance metrics with insignificant terms removed.

Figure 29 summarizes the recalculation of the previous model with the insignificant terms excluded. The RSquare of the recalculated model remains at 0.99.

Summary of Fit				
RSquare				0.990814
RSquare Adj				0.990813
Root Mean Square Error				1.554449
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	129	379903769	2944990	1218796
Error	1.5e+6	3522188	2.416312	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3443118.9	2.42110	Prob > F
Pure Error	35540	79068.8	2.22478	<.0001*
Total Error	1.5e+6	3522187.7		Max RSq 0.9998

Figure 30: Non-linear regression model using performance metrics + SDI.

summarizes the results from a non-linear regression analysis that included all of the performance metrics and the number of sensors, deciders, and influencers for red and blue, as well as, all quadratic terms and two-way interactions. As in the linear regression, when the metrics and number of SDI were used in the regression analysis, there is a problem with multicollinearity between variables. The singularity details (correlation details) and parameter estimates can be found in Appendix K. JMP 9 automatically zeroes the variables causing the multicollinearity and yields an RSquare of 0.99. Once again, it is the number of sensors and influencers for both red and blue that are causing the multicollinearity problems. Those variables as well as all of their quadratic and interaction terms and all other insignificant terms were then removed from the model and the regression analysis is recalculated. These final results of this regression model are summarized in Figure 31.

Summary of Fit				
RSquare				0.99081
RSquare Adj				0.990809
Root Mean Square Error				1.554769
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801
Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	82	379902208	4632954	1916578
Error	1.5e+6	3523749	2.417305	Prob > F
C. Total	1.5e+6	383425957		<.0001*
Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3444680.7	2.42212	1.0887
Pure Error	35540	79068.8	2.22478	Prob > F
Total Error	1.5e+6	3523749.5		<.0001*
				Max RSq
				0.9998

Figure 31: Non-linear regression model using metrics + D with insignificant terms removed.

As evident from all of the regression models analyzed in this research, the added benefit of using a non-linear regression model may be outweighed by the simplicity of explanation in using a linear model. That is to say that even though the non-linear model yields higher RSquare values, the interpretation of quadratic terms and interaction terms may not be simple or even possible. In addition, using the performance metrics as defined by Deller (2009) and Fidanci (2010) in combination with the number of sensors and influencers has been found to be infeasible and will be explored in the next section of this chapter.

EXPLORATION OF SURROGATE VARIABLES

The multicollinearity problem encountered during the regression analysis has sparked an interest to examine the possibility of the performance metrics acting as surrogates for network size. In other words, are certain performance metrics simply

mimicking the number of sensors and influencers. If this is the case then the values of the metrics acting as surrogates will closely follow the number of sensors and influencers associated with each configuration. Moreover, this may help explain the multicollinearity problem that was encountered when the performance metrics were used in combination with the number of sensors and influencers.

In order to examine the surrogate nature of each variable, each of the metrics was plotted against the number of sensors/influencers for each configuration of the red side (the blue side has all identical configurations). If the metric closely follows the number of sensors/influencers (little variability across the number of sensors/influencers) then that variable will be considered a surrogate for network size. If the variable has a large range across each number of sensors/influencers then it will not be considered a surrogate. In addition, the correlation estimates between each variable and the number of sensors/influencers is calculated to give further confirmation of the surrogate nature of the variables.

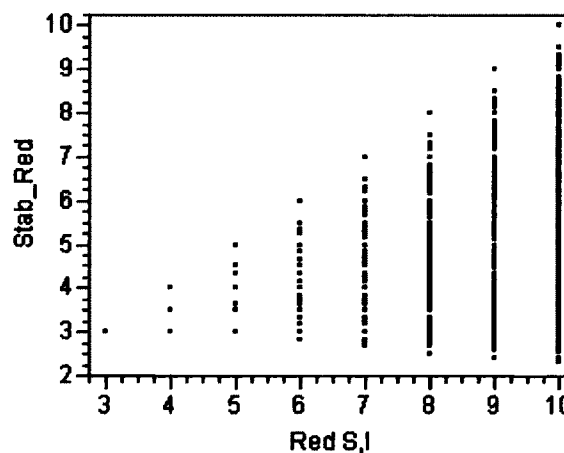


Figure 32: Stability versus sensors/influencers.

Figure 32 shows the range of stability values across each number of sensors/influencers. Since the range varies greatly across the number of sensors/influencers, it is not considered a surrogate for network size. Moreover, Figure 33 confirms that there is little correlation between the metric stability and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Stab_Red	Stab_Blue
Intercept	1.0000	0.0171	-0.8965	-0.0521	-0.0354
Red S,I	0.0171	1.0000	-0.3833	-0.2439	-0.0163
Blue S,I	-0.8965	-0.3833	1.0000	-0.0008	-0.1396
Stab_Red	-0.0521	-0.2439	-0.0008	1.0000	-0.1124
Stab_Blue	-0.0354	-0.0163	-0.1396	-0.1124	1.0000

Figure 33: Correlation between stability and sensors/influencers.

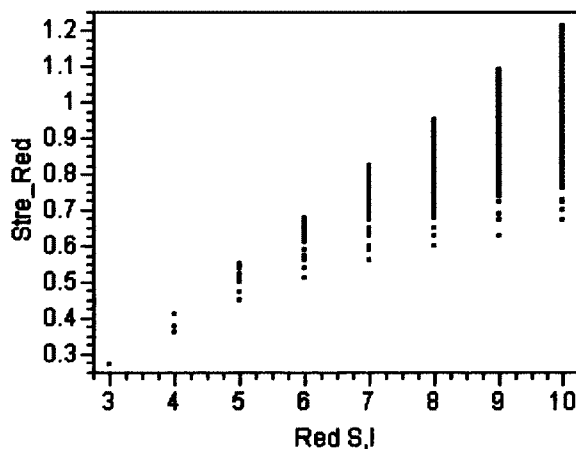


Figure 34: Strength versus sensors/influencers.

Figure 34 shows the range of strength values across each number of sensors/influencers. Since there is little variability in the metric for each number of sensors/influencers, it is

considered a surrogate for network size. This is confirmed in Figure 35 with the high correlations between the strength metric and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Stre_Red	Stre_Blue
Intercept	1.0000	0.0036	-0.7860	-0.0025	-0.0035
Red S,I	0.0036	1.0000	-0.2330	-0.7930	0.0239
Blue S,I	-0.7860	-0.2330	1.0000	0.0178	-0.5063
Stre_Red	-0.0025	-0.7930	0.0178	1.0000	-0.0165
Stre_Blue	-0.0035	0.0239	-0.5063	-0.0165	1.0000

Figure 35: Correlation between strength and sensors/influencers.

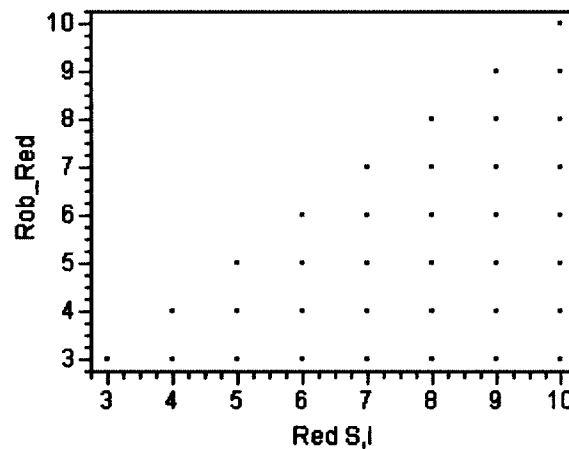


Figure 36: Robustness versus sensors/influencers.

Figure 36 illustrates the range of robustness values across each number of sensors/influencers. Since the range varies greatly across the number of sensors/influencers, it is not considered a surrogate for network size. Figure 37 confirms that there is little correlation between the metric robustness and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Rob_Red	Rob_Blue
Intercept	1.0000	0.0188	-0.8840	-0.0379	-0.0418
Red S,I	0.0188	1.0000	-0.3571	-0.4149	-0.0093
Blue S,I	-0.8840	-0.3571	1.0000	-0.0039	-0.2039
Rob_Red	-0.0379	-0.4149	-0.0039	1.0000	-0.0145
Rob_Blue	-0.0418	-0.0093	-0.2039	-0.0145	1.0000

Figure 37: Correlation between robustness and sensors/influencers.

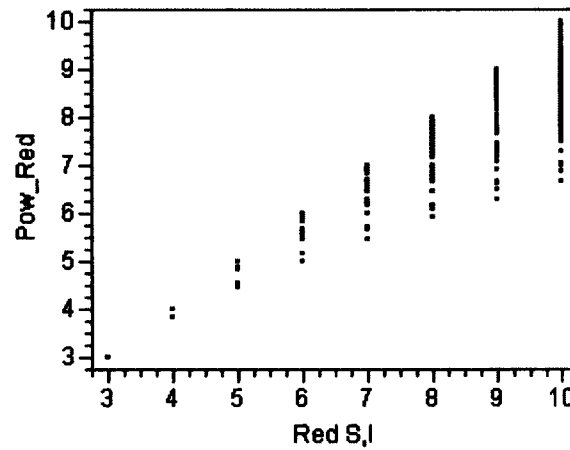


Figure 38: Power versus sensors/influencers.

Figure 38 shows the range of power values across each number of sensors/influencers.

Since the ranges closely mimic the number of sensors/influencers, it is considered a surrogate for network size. This is confirmed in Figure 39 with the high correlations between the power metric and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Pow_Red	Pow_Blue
Intercept	1.0000	0.0356	-0.6832	-0.0391	-0.0418
Red S,I	0.0356	1.0000	-0.1575	-0.8650	0.0028
Blue S,I	-0.6832	-0.1575	1.0000	0.0047	-0.6281
Pow_Red	-0.0391	-0.8650	0.0047	1.0000	-0.0128
Pow_Blue	-0.0418	0.0028	-0.6281	-0.0128	1.0000

Figure 39: Correlation between power and sensors/influencers.

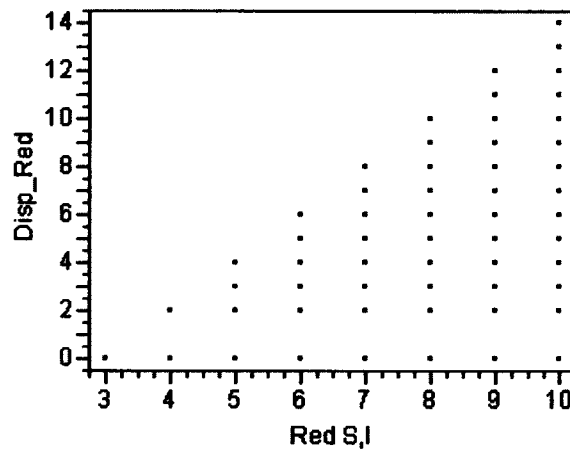


Figure 40: Disparity versus sensors/influencers.

Figure 40 illustrates the range of disparity values across each number of sensors/influencers. Since the range varies greatly across the number of sensors/influencers, it is not considered a surrogate for network size. Moreover, *Figure 41* confirms that there is little correlation between the metric disparity and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Disp_Red	Disp_Blue
Intercept	1.0000	-0.0055	-0.9001	0.0182	-0.0012
Red S,I	-0.0055	1.0000	-0.3832	-0.4348	0.0865
Blue S,I	-0.9001	-0.3832	1.0000	0.0374	-0.1648
Disp_Red	0.0182	-0.4348	0.0374	1.0000	-0.1064
Disp_Blue	-0.0012	0.0865	-0.1648	-0.1064	1.0000

Figure 41: Correlation between disparity and sensors/influencers.

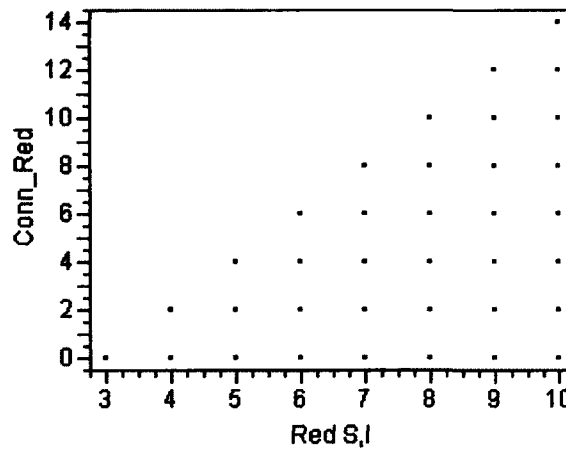


Figure 42: Connectivity versus sensors/influencers.

Figure 42 illustrates the range of connectivity values across each number of sensors/influencers. Since the range varies greatly across the number of sensors/influencers, it is not considered a surrogate for network size. Figure 43 confirms that there is little correlation between the metric connectivity and the number of sensors/influencers.

Correlation of Estimates

Corr	Intercept	Red S,I	Blue S,I	Conn_Red	Conn_Blue
Intercept	1.0000	-0.0092	-0.9080	0.0379	0.0418
Red S,I	-0.0092	1.0000	-0.3848	-0.3288	0.0207
Blue S,I	-0.9080	-0.3848	1.0000	0.0091	-0.1463
Conn_Red	0.0379	-0.3288	0.0091	1.0000	-0.0145
Conn_Blue	0.0418	0.0207	-0.1463	-0.0145	1.0000

Figure 43: Correlation between connectivity and sensors/influencers.

This gives definitive proof of the correlation between the number of sensors and influencers and the metrics. The highest amount of correlation between the number of sensors and influencers and the non-surrogate metrics is with disparity, at about 0.43. However, when the surrogate variables, strength and power, are examined, the correlation is as high as 0.87. This confirms that there is, in fact, a multicollinearity issue between the surrogate variables of strength and power and the network size. This is also shown with the fact that there is less variability in those two metrics, strength and power, across the number of sensors and influencers.

After determining that strength and power are indeed acting as surrogates for network size it should be expected that they would have the greatest impact on predicting the percentage of red wins when analyzed separately. As a starting point, Figure 44 shows a linear regression that only uses the number of sensors and influencers for both red and blue as independent variables. The results show an RSquare of 0.91, which means that network size in terms of the number of sensors and influencers is a formidable predictor in the case of asymmetric engagements.

Summary of Fit

RSquare	0.91141
RSquare Adj	0.911409
Root Mean Square Error	4.827092
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	349458067	174729033	7498836
Error	1.5e+6	33967890	23.300821	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	33	26520691	803657	157313.3
Pure Error	1.5e+6	7447199	5.108642	Prob > F
Total Error	1.5e+6	33967890		<.0001*
				Max RSq
				0.9806

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.762678	0.073153	584.56	<.0001*
Red S,I	13.973365	0.003609	3872.1	<.0001*
Blue S,I	-13.3959	0.008191	-1635	<.0001*

Figure 44: Linear regression results using only sensors and influencers.

It is logical to assume then that power and strength would yield similar results if they are truly acting as surrogates for network size. Figure 45 shows the results of a linear regression using only power as a predictor. The results yielded an RSquare of 0.70 which is substantially high and confirms the notion that power is acting as a surrogate for network size.

Summary of Fit

RSquare	0.699256
RSquare Adj	0.699256
Root Mean Square Error	8.893864
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	268112953	134056476	1694755
Error	1.5e+6	115313005	79.100811	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	41897	57084044	1362.49	33.1303
Pure Error	1.4e+6	58228961	41.13	Prob > F
Total Error	1.5e+6	115313005		<.0001*
				Max RSq
				0.8481

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	-5.346655	0.098095	-54.50	<.0001*
Pow_Red	11.68942	0.006358	1838.7	<.0001*
Pow_Blue	-5.976894	0.010697	-558.8	<.0001*

Figure 45: Linear regression results using only power.

The next regression is used to confirm the surrogate nature of strength. Figure 46 displays the results for a linear regression model using only strength as the independent variables. The results show a calculated RSquare of 0.60, which is also substantially high when compared to the regression results using the number of sensors and influencers as predictors. This also confirms that strength is, in fact, acting as a surrogate for network size.

Summary of Fit

RSquare	0.6048
RSquare Adj	0.604799
Root Mean Square Error	10.19531
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	231895964	115947982	1115480
Error	1.5e+6	151529994	103.94444	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	4518	17921476	3966.68	43.1462
Pure Error	1.5e+6	133608518	91.94	Prob > F
Total Error	1.5e+6	151529994		<.0001*
				Max RSq
				0.6515

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	-4.45096	0.091172	-48.82	<.0001*
Stre_Red	82.090368	0.055256	1485.6	<.0001*
Stre_Blue	-34.64119	0.085112	-407.0	<.0001*

Figure 46: Linear regression results using only strength.

As a confirmation of the other metrics not being surrogates their linear regression models will also be examined. It is predicted that those linear regression models using the other metrics on their own will yield much lower RSquare values. The results for disparity as a predictor are listed below in Figure 47. With an RSquare value of only 0.15 this confirms that disparity does not act as a surrogate for network size, rather it would be considered a metric that deals with the actual network configuration.

Summary of Fit

RSquare	0.146202
RSquare Adj	0.146201
Root Mean Square Error	14.98544
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	56057815	28028908	124815.1
Error	1.5e+6	327368142	224.56345	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	193	6564647	34013.7	154.5450
Pure Error	1.5e+6	320803495	220.1	Prob > F
Total Error	1.5e+6	327368142		<.0001*
				Max RSq
				0.1633

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	23.393092	0.048653	480.81	<.0001*
Disp_Red	2.7267329	0.005483	497.30	<.0001*
Disp_Blue	-0.590745	0.006013	-98.24	<.0001*

Figure 47: Linear regression results using only disparity.

The results for connectivity as a predictor are listed below in Figure 48. With an RSquare value of only 0.06 this confirms that disparity does not act as a surrogate for network size, rather it would be considered a metric that deals with the actual network configuration.

Summary of Fit				
RSquare				0.0608
RSquare Adj				0.060799
Root Mean Square Error				15.71705
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	23312455	11656228	47186.30
Error	1.5e+6	360113502	247.02565	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	61	1040568	17058.5	69.2528
Pure Error	1.5e+6	359072934	246.3	Prob > F
Total Error	1.5e+6	360113502		<.0001*
				Max RSq
				0.0635

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	28.904051	0.037362	773.63	<.0001*
Conn_Red	1.346676	0.004422	304.55	<.0001*
Conn_Blue	0.1474085	0.004695	31.40	<.0001*

Figure 48: Linear regression results using only connectivity.

The results for robustness as a predictor are listed below in Figure 49. With an RSquare value of only 0.20 this confirms that disparity does not act as a surrogate for network size, rather it would be considered a metric that deals with the actual network configuration.

Summary of Fit

RSquare	0.203519
RSquare Adj	0.203518
Root Mean Square Error	14.47371
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	78034480	39017240	186250.3
Error	1.5e+6	305391477	209.4882	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	61	703714	11536.3	55.1938
Pure Error	1.5e+6	304687763	209.0	Prob > F
Total Error	1.5e+6	305391477		<.0001*
				Max RSq
				0.2054

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	12.767266	0.0753	169.55	<.0001*
Rob_Red	4.7378057	0.007791	608.14	<.0001*
Rob_Blue	-0.763704	0.008529	-89.54	<.0001*

Figure 49: Linear regression results using only robustness.

The results for stability as a predictor are listed below in Figure 50. With an RSquare value of only 0.04 this confirms that disparity does not act as a surrogate for network size, rather it would be considered a metric that deals with the actual network configuration.

Summary of Fit				
RSquare				0.03648
RSquare Adj				0.036479
Root Mean Square Error				15.91924
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	13987488	6993744	27597.20
Error	1.5e+6	369438470	253	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	45497	49295452	1083.49	Prob > F
Pure Error	1.4e+6	320143018	226.68	<.0001*
Total Error	1.5e+6	369438470		Max RSq 0.1650

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	23.557538	0.064559	364.90	<.0001*
Stab_Red	1.7678883	0.00865	204.39	<.0001*
Stab_Blue	0.7305861	0.008468	86.28	<.0001*

Figure 50: Linear regression results using only stability.

The regression results presented in this section confirm that strength and power do act as surrogates for the network size in terms of the number of sensors and influencers. Moreover, the other metrics of disparity, connectivity, robustness, and stability are not surrogates for network size. Rather, they are measures of the actual various connections possible for each network configuration.

After determining which factors examine the actual network configurations, rather than act as a surrogate for network size, it was decided to run a final linear regression model using only the non-surrogate metrics of disparity, connectivity, robustness, and stability. The results in Figure 51 show an RSquare value of 0.92. This is perhaps the

most meaningful model presented in this research with regards to predicting combat performance in asymmetric engagements in the context of the information age combat model.

Summary of Fit				
RSquare				0.920321
RSquare Adj				0.92032
Root Mean Square Error				4.577898
Mean of Response				37.01395
Observations (or Sum Wgts)				1457801

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	8	352874791	44109349	2104740
Error	1.5e+6	30551166	20.95715	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit				
Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.1e+6	29735217	26.9514	11.7094
Pure Error	354500	815948	2.3017	Prob > F
Total Error	1.5e+6	30551166		<.0001*
				Max RSq
				0.9979

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.645228	0.070071	608.60	<.0001*
Conn_Red	6.9001885	0.002756	2503.9	<.0001*
Conn_Blue	-6.673687	0.004434	-1505	<.0001*
Disp_Red	-0.036673	0.002136	-17.17	<.0001*
Disp_Blue	-0.017356	0.002285	-7.60	<.0001*
Rob_Red	14.328047	0.003876	3696.1	<.0001*
Rob_Blue	-13.74079	0.007983	-1721	<.0001*
Stab_Red	-0.338899	0.003523	-96.20	<.0001*
Stab_Blue	0.4382079	0.003074	142.55	<.0001*

Figure 51: Linear regression results using non-surrogate variables.

The reason this final regression model should be considered the most meaningful is because the bias of network size has been limited through the choice of non-surrogate

variables. Moreover, when looking at the parameter estimates, it can be concluded that connectivity and robustness are the most important network configuration factors in determining combat network performance in the case of asymmetric engagements, from the metrics examined in this research.

CHAPTER 5

CONCLUSIONS

DISCUSSION AND CONCLUSIONS

There were two primary objectives in the research effort presented. The first objective was to successfully build a computationally fast and versatile simulation model of the IACM using a discrete-event simulation modeling paradigm. The second objective was to use the simulation model to examine the effectiveness of previously defined performance metrics in the realm of engagements with balanced forces of unequal assets, also called asymmetric engagements.

Both research objectives have been accomplished. Visual Basic turned out to be an adequate medium for the programming of the simulation model using the DES approach. Moreover, the use of VB also allowed for an efficient means of data manipulation. After the simulation was verified to represent characteristics taken from the IACM, the analysis of asymmetric engagements using balanced forces proceeded.

The results of the analysis of asymmetric engagements using balanced forces provided the information needed to draw several conclusions. The first conclusion that can be drawn is that, in fact, it is possible for a smaller networked force to defeat a larger networked force. This points to the idea that the combat network configuration can and does play a vital role in determining its success.

The second conclusion that can be drawn from this research is that the network size will, eventually, always trump organization (i.e., make the number disparity of assets great enough and the larger force will always win), organization can dominate when the number of assets are within a certain range of each other. The addition of a single sensor

or influencer may seem like a small advantage, but proportionally this increase of assets to one side of the engagement can be quite large. Moreover, the fact that better organized forces can overcome this disparity in assets is significant.

The third and final conclusion that should be taken from this research is that robustness and connectivity of the network configurations are the most important factors, from those examined in this research, in determining the outcome of asymmetric engagements with balanced forces. These two network factors should be the primary focus when configuring a network with balanced forces for asymmetric engagements.

FUTURE RESEARCH

The research presented in this paper investigates NCO engagements of configurations with an unequal number of assets, or asymmetric engagements. However, there is still a need for further research on the investigation of networked effects using the IACM with several other areas of focus. There are at least four major focuses that should be considered in future research with regards to this topic.

- One focus area would be to include several other links discussed in Section 2.4 of this document. This would include horizontal links between sensors, direct sensor to influencer links, and links between sensors, influencers, and multiple deciders.
- Another direction for future research would be to include a stochastic element representing probabilities of the various functions being carried out. This could also be implemented in communication links between nodes. This would allow for the investigation of how well the performance metrics predict in a stochastic network environment.

- A third addition to this research could be to go back and examine the effectiveness of Perron-Frobenius Eigenvalues as predictors of network performance with respect to asymmetric engagements. Is there a significant correlation with the PFE value with respect to winning? Can a normalized PFE (i.e., coefficient of networked effects (Cares, 2005)) be included to increase the predictive capability of the PFE?
- A fourth future research focus should be to incorporate asymmetric engagements with unbalanced forces. The number of configurations needed to investigate increases exponentially when compared to only investigating asymmetric engagements with balanced forces (X-Y-X), or symmetric engagements with unbalanced forces (X-Y-Z). This could lead to a conclusion of whether sensors or influencers are more important than one another. Is the value of a sensor in the network the same as the value of an influencer? Are sensor-heavy configurations better networked than influencer-heavy configurations?

There could also be other contributions to future research including the addition of several variables into the model. These variables could include things such as capabilities for the sensors, deciders, and influencers in the form of movement logic, survivability, sensing and influencing ranges, and other characteristics (Deller, 2009). Another addition to the simulation model could be the addition of terrain data rather than agents moving around in a flat two-dimensional space. All of these additions, however, are far off in the horizon and will take a substantial amount of thought and consideration to include in future evolutions of the IACM.

SUMMARY

The need for smaller, geographically dispersed, networked forces on the battlefield has become evident with the evolutionary shifts in the way society functions and the way war is waged. Although technology still plays a large role in dominant forces all over the world, it is the sharing of information and the networks used to communicate that make the technology effective. This is why the configuration of these distributed networks is vital to the effectiveness of networked operations. The understanding of how these networks function under different configurations will allow for more effective networked operations. This applies to not only the military application of Network-Centric Operations, but also to its civilian counterpart of Distributed Network Operations. The abstract functions of sensing information, deciding, and influencing based on the information shared, allows the IACM “to model almost any activity involving planning and decision-making” (Deller, 2009, p. 51). Consequently, this research furthers the understanding on how to configure these networks for more effective planning and decision-making with respect to networked operations.

REFERENCES

- Alberts, D. S., Garstka, J. H., & Stein, F. P. (1999). *Network centric warfare*. Washington, DC: CCRP Press.
- Alberts, D. S., Garstka, J. H., Hayes, R. E., & Signori, D. A. (2001). *Understanding information age warfare*. Washington, DC: CCRP Press.
- Alberts, D. S., & Hayes, R. E. (2003). *Power to the edge*. Washington, DC: CCRP Press.
- Arquilla, J., & Borer, D. A. (Eds.). (2007). *Information strategy and warfare: A guide to theory and practice*. New York, NY: Routledge.
- Arquilla, J., & Ronfeldt, D. (1993). Cyberwar is coming. *Comparative Strategy*, 12(2), 141–165.
- Arquilla, J., & Ronfeldt, D. (1996). *The advent of netwar*. Santa Monica, CA: RAND. Retrieved from http://www.rand.org/content/dam/rand/pubs/monograph_reports/MR880/MR880.ch12.pdf
- Arquilla, J., & Ronfeldt, D. (2000). *Swarming & the future of conflict*. Santa Monica, CA: RAND.
- Arquilla, J., & Ronfeldt, D. (Eds.). (2001). *Networks and netwars: The future of terror, crime, and militancy*. Santa Monica, CA: RAND.
- Arquilla, J., & Ronfeldt, D., Fuller, G., Fuller, M. (1998). *The Zapatista social netwar in Mexico*. Santa Monica, CA: RAND. Retrieved from http://www.rand.org/pubs/monograph_reports/MR994.html
- Bonabeau, E. (2002). Agent-based modeling: Methods and techniques for simulating human systems. Proceedings from the National Academy of Sciences: *Adaptive Agents, Intelligence, and Emergent Human Organization: Capturing Complexity*

- through Agent-Based Modeling*. Irvine, CA. Retrieved from <http://www.pnas.org/content/99/suppl.3/7280.full.pdf>
- Cebrowski, A. K., & Garstka, J. H. (1998). Network centric warfare: Its origin and future. Proceedings from *U.S. Naval Institute*, 28-35. Retrieved from http://www.kinection.com/ncoic/ncw_origin_future.pdf
- Deller, S. (2009). Towards the quantitative analysis of the connectivity value of networked operations. (Doctoral dissertation) Retrieved from ProQuest (Accession No. AAT 3377889).
- Department of Defense (2001). *Network centric warfare: Report to Congress*. Retrieved from http://cio-nii.defense.gov/docs/pt2_ncw_main.pdf
- Department of Defense. (2003). *Military transformation: A strategic approach*. Retrieved from <http://www.iwar.org.uk/rma/resources/transformation/military-transformation-a-strategic-approach.pdf>
- Department of Defense Office of Force Transformation. (2005). *The implementation of network-centric warfare*. Retrieved from http://www.au.af.mil/au/awc/awcgate/transformation/oft_implementation_ncw.pdf
- Fidanci, M. (2010). The influence of network factors on network centric operations. (Doctoral dissertation) Retrieved from ProQuest (Accession No. AAT 3429551).
- Fishman, G. S. (2001). *Discrete-event simulation: Modeling, programming, and analysis*. New York, NY: Springer-Verlag.
- Hammes, T. X. (1994). The evolution of war: The fourth generation. *Marine Corps Gazette*, 78(9), 35-44.

Joint Chiefs of Staff. (1996). *Joint Vision 2010*. Retrieved from <http://www.dtic.mil/jv2010/jv2010.pdf>

Joint Chiefs of Staff. (2000). *Joint Vision 2020*. Retrieved from http://www.dtic.mil/doctrine/jel/jfq_pubs/1225.pdf

Joint Chiefs of Staff. (2006). Information operations. Retrieved from http://www.dtic.mil/doctrine/new_pubs/jp3_13.pdf

Lind, W. S. (2004). Understanding fourth generation warfare. *Military Review*, 84, 12-16. Retrieved from <http://www.au.af.mil/au/awc/awcgate/milreview/lind.pdf>

Lind, W. S., Nightengale, K., Schmitt, J. F., Sutton, J. W., & Wilson, G. I. (1989). The changing face of war: Into the fourth generation. *Marine Corps Gazette*, 73(10), 22-26.

Ling, M. F., Moon, T., & Kruzins, E. (2005). Proposed network centric warfare metrics: From connectivity to the OODA cycle. *Military Operations Research*, 10(1), 5-13.

Oosthuizen, R., Burke, I., & Roodt, J. (2010). Initial investigation of concepts of Command and Control using an agent based approach. Proceedings from *The Workshop on ICT Uses in Warfare and the Safeguarding of Peace*. Forever Resort, Bela Bela. Retrieved from http://researchspace.csir.co.za/dspace/bitstream/10204/4691/1/Burke3_2010.pdf

Partition Number Theory. (n.d.). Retrieved June 28, 2011 from the Wikipedia: [http://en.wikipedia.org/wiki/Partition_\(number_theory\)](http://en.wikipedia.org/wiki/Partition_(number_theory))

- Tisue, S., & Wilensky, U. (2004). Netlogo: A simple environment for modeling complexity. Proceedings from the ICCS: *International Conference on Complex Systems*. Boston, MA. Retrieved from <http://ccl.northwestern.edu/papers/netlogo-iccs2004.pdf>
- Toffler, A., & Toffler, H. (1993). *War and anti-war: Survival at the dawn of the 21st century*. Boston: Little, Brown and Company.
- U.S. Air Force. (2003). *The Air Force transformation flight plan*. Retrieved from http://www.au.af.mil/au/awc/awcgate/af/af_trans_flightplan_nov03.pdf
- Van Creveld, M. (2000). *Through a glass darkly: Some reflections on the future of war*. Retrieved from <http://dnipogo.org/creveld/throughaglassdarkly.html>
- Waltz, E. (1998). The US transition to information warfare. *Journal of Electronic Defense*, 21(12), 35-42.

APPENDICES

APPENDIX A: LIST OF UNIQUE COMBAT ENGAGEMENTS

Simulation Configuration No.	Red Sensors	Red Deciders	Red Influencers		Blue Sensors	Blue Deciders	Blue Influencers	No. of Unique Engagements
1	3	3	3	vs	3	3	3	1
2	3	3	3	vs	4	3	4	2
3	3	3	3	vs	4	4	4	1
4	3	3	3	vs	5	3	5	8
5	3	3	3	vs	5	4	5	2
6	3	3	3	vs	5	5	5	1
7	3	3	3	vs	6	3	6	19
8	3	3	3	vs	6	4	6	9
9	3	3	3	vs	6	5	6	2
10	3	3	3	vs	6	6	6	1
11	3	3	3	vs	7	3	7	42
12	3	3	3	vs	7	4	7	27
13	3	3	3	vs	7	5	7	9
14	3	3	3	vs	7	6	7	2
15	3	3	3	vs	7	7	7	1
16	3	3	3	vs	8	3	8	78
17	3	3	3	vs	8	4	8	74
18	3	3	3	vs	8	5	8	30
19	3	3	3	vs	8	6	8	9
20	3	3	3	vs	8	7	8	2
21	3	3	3	vs	8	8	8	1
22	3	3	3	vs	9	3	9	139
23	3	3	3	vs	9	4	9	168
24	3	3	3	vs	9	5	9	95
25	3	3	3	vs	9	6	9	31
26	3	3	3	vs	9	7	9	9
27	3	3	3	vs	9	8	9	2
28	3	3	3	vs	9	9	9	1
29	3	3	3	vs	10	3	10	224
30	3	3	3	vs	10	4	10	363

31	3	3	3	vs	10	5	10	248
32	3	3	3	vs	10	6	10	105
33	3	3	3	vs	10	7	10	31
34	3	3	3	vs	10	8	10	9
35	3	3	3	vs	10	9	10	2
36	3	3	3	vs	10	10	10	1
37	4	3	4	vs	4	3	4	4
38	4	3	4	vs	4	4	4	2
39	4	3	4	vs	5	3	5	16
40	4	3	4	vs	5	4	5	4
41	4	3	4	vs	5	5	5	2
42	4	3	4	vs	6	3	6	38
43	4	3	4	vs	6	4	6	18
44	4	3	4	vs	6	5	6	4
45	4	3	4	vs	6	6	6	2
46	4	3	4	vs	7	3	7	84
47	4	3	4	vs	7	4	7	54
48	4	3	4	vs	7	5	7	18
49	4	3	4	vs	7	6	7	4
50	4	3	4	vs	7	7	7	2
51	4	3	4	vs	8	3	8	156
52	4	3	4	vs	8	4	8	148
53	4	3	4	vs	8	5	8	60
54	4	3	4	vs	8	6	8	18
55	4	3	4	vs	8	7	8	4
56	4	3	4	vs	8	8	8	2
57	4	3	4	vs	9	3	9	278
58	4	3	4	vs	9	4	9	336
59	4	3	4	vs	9	5	9	190
60	4	3	4	vs	9	6	9	62
61	4	3	4	vs	9	7	9	18
62	4	3	4	vs	9	8	9	4
63	4	3	4	vs	9	9	9	2
64	4	3	4	vs	10	3	10	448
65	4	3	4	vs	10	4	10	726
66	4	3	4	vs	10	5	10	496
67	4	3	4	vs	10	6	10	210
68	4	3	4	vs	10	7	10	62
69	4	3	4	vs	10	8	10	18
70	4	3	4	vs	10	9	10	4

71	4	3	4	vs	10	10	10	2
72	4	4	4	vs	4	4	4	1
73	4	4	4	vs	5	4	5	2
74	4	4	4	vs	5	5	5	1
75	4	4	4	vs	6	4	6	9
76	4	4	4	vs	6	5	6	2
77	4	4	4	vs	6	6	6	1
78	4	4	4	vs	7	4	7	27
79	4	4	4	vs	7	5	7	9
80	4	4	4	vs	7	6	7	2
81	4	4	4	vs	7	7	7	1
82	4	4	4	vs	8	4	8	74
83	4	4	4	vs	8	5	8	30
84	4	4	4	vs	8	6	8	9
85	4	4	4	vs	8	7	8	2
86	4	4	4	vs	8	8	8	1
87	4	4	4	vs	9	4	9	168
88	4	4	4	vs	9	5	9	95
89	4	4	4	vs	9	6	9	31
90	4	4	4	vs	9	7	9	9
91	4	4	4	vs	9	8	9	2
92	4	4	4	vs	9	9	9	1
93	4	4	4	vs	10	4	10	363
94	4	4	4	vs	10	5	10	248
95	4	4	4	vs	10	6	10	105
96	4	4	4	vs	10	7	10	31
97	4	4	4	vs	10	8	10	9
98	4	4	4	vs	10	9	10	2
99	4	4	4	vs	10	10	10	1
100	5	3	5	vs	5	3	5	64
101	5	3	5	vs	5	4	5	16
102	5	3	5	vs	5	5	5	8
103	5	3	5	vs	6	3	6	152
104	5	3	5	vs	6	4	6	72
105	5	3	5	vs	6	5	6	16
106	5	3	5	vs	6	6	6	8
107	5	3	5	vs	7	3	7	336
108	5	3	5	vs	7	4	7	216
109	5	3	5	vs	7	5	7	72
110	5	3	5	vs	7	6	7	16

111	5	3	5	vs	7	7	7	8
112	5	3	5	vs	8	3	8	624
113	5	3	5	vs	8	4	8	592
114	5	3	5	vs	8	5	8	240
115	5	3	5	vs	8	6	8	72
116	5	3	5	vs	8	7	8	16
117	5	3	5	vs	8	8	8	8
118	5	3	5	vs	9	3	9	1112
119	5	3	5	vs	9	4	9	1344
120	5	3	5	vs	9	5	9	760
121	5	3	5	vs	9	6	9	248
122	5	3	5	vs	9	7	9	72
123	5	3	5	vs	9	8	9	16
124	5	3	5	vs	9	9	9	8
125	5	3	5	vs	10	3	10	1792
126	5	3	5	vs	10	4	10	2904
127	5	3	5	vs	10	5	10	1984
128	5	3	5	vs	10	6	10	840
129	5	3	5	vs	10	7	10	248
130	5	3	5	vs	10	8	10	72
131	5	3	5	vs	10	9	10	16
132	5	3	5	vs	10	10	10	8
133	5	4	5	vs	5	4	5	4
134	5	4	5	vs	5	5	5	2
135	5	4	5	vs	6	4	6	18
136	5	4	5	vs	6	5	6	4
137	5	4	5	vs	6	6	6	2
138	5	4	5	vs	7	4	7	54
139	5	4	5	vs	7	5	7	18
140	5	4	5	vs	7	6	7	4
141	5	4	5	vs	7	7	7	2
142	5	4	5	vs	8	4	8	148
143	5	4	5	vs	8	5	8	60
144	5	4	5	vs	8	6	8	18
145	5	4	5	vs	8	7	8	4
146	5	4	5	vs	8	8	8	2
147	5	4	5	vs	9	4	9	336
148	5	4	5	vs	9	5	9	190
149	5	4	5	vs	9	6	9	62
150	5	4	5	vs	9	7	9	18

151	5	4	5	vs	9	8	9	4
152	5	4	5	vs	9	9	9	2
153	5	4	5	vs	10	4	10	726
154	5	4	5	vs	10	5	10	496
155	5	4	5	vs	10	6	10	210
156	5	4	5	vs	10	7	10	62
157	5	4	5	vs	10	8	10	18
158	5	4	5	vs	10	9	10	4
159	5	4	5	vs	10	10	10	2
160	5	5	5	vs	5	5	5	1
161	5	5	5	vs	6	5	6	2
162	5	5	5	vs	6	6	6	1
163	5	5	5	vs	7	5	7	9
164	5	5	5	vs	7	6	7	2
165	5	5	5	vs	7	7	7	1
166	5	5	5	vs	8	5	8	30
167	5	5	5	vs	8	6	8	9
168	5	5	5	vs	8	7	8	2
169	5	5	5	vs	8	8	8	1
170	5	5	5	vs	9	5	9	95
171	5	5	5	vs	9	6	9	31
172	5	5	5	vs	9	7	9	9
173	5	5	5	vs	9	8	9	2
174	5	5	5	vs	9	9	9	1
175	5	5	5	vs	10	5	10	248
176	5	5	5	vs	10	6	10	105
177	5	5	5	vs	10	7	10	31
178	5	5	5	vs	10	8	10	9
179	5	5	5	vs	10	9	10	2
180	5	5	5	vs	10	10	10	1
181	6	3	6	vs	6	3	6	361
182	6	3	6	vs	6	4	6	171
183	6	3	6	vs	6	5	6	38
184	6	3	6	vs	6	6	6	19
185	6	3	6	vs	7	3	7	798
186	6	3	6	vs	7	4	7	513
187	6	3	6	vs	7	5	7	171
188	6	3	6	vs	7	6	7	38
189	6	3	6	vs	7	7	7	19
190	6	3	6	vs	8	3	8	1482

191	6	3	6	vs	8	4	8	1406
192	6	3	6	vs	8	5	8	570
193	6	3	6	vs	8	6	8	171
194	6	3	6	vs	8	7	8	38
195	6	3	6	vs	8	8	8	19
196	6	3	6	vs	9	3	9	2641
197	6	3	6	vs	9	4	9	3192
198	6	3	6	vs	9	5	9	1805
199	6	3	6	vs	9	6	9	589
200	6	3	6	vs	9	7	9	171
201	6	3	6	vs	9	8	9	38
202	6	3	6	vs	9	9	9	19
203	6	3	6	vs	10	3	10	4256
204	6	3	6	vs	10	4	10	6897
205	6	3	6	vs	10	5	10	4712
206	6	3	6	vs	10	6	10	1995
207	6	3	6	vs	10	7	10	589
208	6	3	6	vs	10	8	10	171
209	6	3	6	vs	10	9	10	38
210	6	3	6	vs	10	10	10	19
211	6	4	6	vs	6	4	6	81
212	6	4	6	vs	6	5	6	18
213	6	4	6	vs	6	6	6	9
214	6	4	6	vs	7	4	7	243
215	6	4	6	vs	7	5	7	81
216	6	4	6	vs	7	6	7	18
217	6	4	6	vs	7	7	7	9
218	6	4	6	vs	8	4	8	666
219	6	4	6	vs	8	5	8	270
220	6	4	6	vs	8	6	8	81
221	6	4	6	vs	8	7	8	18
222	6	4	6	vs	8	8	8	9
223	6	4	6	vs	9	4	9	1512
224	6	4	6	vs	9	5	9	855
225	6	4	6	vs	9	6	9	279
226	6	4	6	vs	9	7	9	81
227	6	4	6	vs	9	8	9	18
228	6	4	6	vs	9	9	9	9
229	6	4	6	vs	10	4	10	3267
230	6	4	6	vs	10	5	10	2232

231	6	4	6	vs	10	6	10	945
232	6	4	6	vs	10	7	10	279
233	6	4	6	vs	10	8	10	81
234	6	4	6	vs	10	9	10	18
235	6	4	6	vs	10	10	10	9
236	6	5	6	vs	6	5	6	4
237	6	5	6	vs	6	6	6	2
238	6	5	6	vs	7	5	7	18
239	6	5	6	vs	7	6	7	4
240	6	5	6	vs	7	7	7	2
241	6	5	6	vs	8	5	8	60
242	6	5	6	vs	8	6	8	18
243	6	5	6	vs	8	7	8	4
244	6	5	6	vs	8	8	8	2
245	6	5	6	vs	9	5	9	190
246	6	5	6	vs	9	6	9	62
247	6	5	6	vs	9	7	9	18
248	6	5	6	vs	9	8	9	4
249	6	5	6	vs	9	9	9	2
250	6	5	6	vs	10	5	10	496
251	6	5	6	vs	10	6	10	210
252	6	5	6	vs	10	7	10	62
253	6	5	6	vs	10	8	10	18
254	6	5	6	vs	10	9	10	4
255	6	5	6	vs	10	10	10	2
256	6	6	6	vs	6	6	6	1
257	6	6	6	vs	7	6	7	2
258	6	6	6	vs	7	7	7	1
259	6	6	6	vs	8	6	8	9
260	6	6	6	vs	8	7	8	2
261	6	6	6	vs	8	8	8	1
262	6	6	6	vs	9	6	9	31
263	6	6	6	vs	9	7	9	9
264	6	6	6	vs	9	8	9	2
265	6	6	6	vs	9	9	9	1
266	6	6	6	vs	10	6	10	105
267	6	6	6	vs	10	7	10	31
268	6	6	6	vs	10	8	10	9
269	6	6	6	vs	10	9	10	2
270	6	6	6	vs	10	10	10	1

271	7	3	7	vs	7	3	7	1764
272	7	3	7	vs	7	4	7	1134
273	7	3	7	vs	7	5	7	378
274	7	3	7	vs	7	6	7	84
275	7	3	7	vs	7	7	7	42
276	7	3	7	vs	8	3	8	3276
277	7	3	7	vs	8	4	8	3108
278	7	3	7	vs	8	5	8	1260
279	7	3	7	vs	8	6	8	378
280	7	3	7	vs	8	7	8	84
281	7	3	7	vs	8	8	8	42
282	7	3	7	vs	9	3	9	5838
283	7	3	7	vs	9	4	9	7056
284	7	3	7	vs	9	5	9	3990
285	7	3	7	vs	9	6	9	1302
286	7	3	7	vs	9	7	9	378
287	7	3	7	vs	9	8	9	84
288	7	3	7	vs	9	9	9	42
289	7	3	7	vs	10	3	10	9408
290	7	3	7	vs	10	4	10	15246
291	7	3	7	vs	10	5	10	10416
292	7	3	7	vs	10	6	10	4410
293	7	3	7	vs	10	7	10	1302
294	7	3	7	vs	10	8	10	378
295	7	3	7	vs	10	9	10	84
296	7	3	7	vs	10	10	10	42
297	7	4	7	vs	7	4	7	729
298	7	4	7	vs	7	5	7	243
299	7	4	7	vs	7	6	7	54
300	7	4	7	vs	7	7	7	27
301	7	4	7	vs	8	4	8	1998
302	7	4	7	vs	8	5	8	810
303	7	4	7	vs	8	6	8	243
304	7	4	7	vs	8	7	8	54
305	7	4	7	vs	8	8	8	27
306	7	4	7	vs	9	4	9	4536
307	7	4	7	vs	9	5	9	2565
308	7	4	7	vs	9	6	9	837
309	7	4	7	vs	9	7	9	243
310	7	4	7	vs	9	8	9	54

311	7	4	7	vs	9	9	9	27
312	7	4	7	vs	10	4	10	9801
313	7	4	7	vs	10	5	10	6696
314	7	4	7	vs	10	6	10	2835
315	7	4	7	vs	10	7	10	837
316	7	4	7	vs	10	8	10	243
317	7	4	7	vs	10	9	10	54
318	7	4	7	vs	10	10	10	27
319	7	5	7	vs	7	5	7	81
320	7	5	7	vs	7	6	7	18
321	7	5	7	vs	7	7	7	9
322	7	5	7	vs	8	5	8	270
323	7	5	7	vs	8	6	8	81
324	7	5	7	vs	8	7	8	18
325	7	5	7	vs	8	8	8	9
326	7	5	7	vs	9	5	9	855
327	7	5	7	vs	9	6	9	279
328	7	5	7	vs	9	7	9	81
329	7	5	7	vs	9	8	9	18
330	7	5	7	vs	9	9	9	9
331	7	5	7	vs	10	5	10	2232
332	7	5	7	vs	10	6	10	945
333	7	5	7	vs	10	7	10	279
334	7	5	7	vs	10	8	10	81
335	7	5	7	vs	10	9	10	18
336	7	5	7	vs	10	10	10	9
337	7	6	7	vs	7	6	7	4
338	7	6	7	vs	7	7	7	2
339	7	6	7	vs	8	6	8	18
340	7	6	7	vs	8	7	8	4
341	7	6	7	vs	8	8	8	2
342	7	6	7	vs	9	6	9	62
343	7	6	7	vs	9	7	9	18
344	7	6	7	vs	9	8	9	4
345	7	6	7	vs	9	9	9	2
346	7	6	7	vs	10	6	10	210
347	7	6	7	vs	10	7	10	62
348	7	6	7	vs	10	8	10	18
349	7	6	7	vs	10	9	10	4
350	7	6	7	vs	10	10	10	2

351	7	7	7	vs	7	7	7	1
352	7	7	7	vs	8	7	8	2
353	7	7	7	vs	8	8	8	1
354	7	7	7	vs	9	7	9	9
355	7	7	7	vs	9	8	9	2
356	7	7	7	vs	9	9	9	1
357	7	7	7	vs	10	7	10	31
358	7	7	7	vs	10	8	10	9
359	7	7	7	vs	10	9	10	2
360	7	7	7	vs	10	10	10	1
361	8	3	8	vs	8	3	8	6084
362	8	3	8	vs	8	4	8	5772
363	8	3	8	vs	8	5	8	2340
364	8	3	8	vs	8	6	8	702
365	8	3	8	vs	8	7	8	156
366	8	3	8	vs	8	8	8	78
367	8	3	8	vs	9	3	9	10842
368	8	3	8	vs	9	4	9	13104
369	8	3	8	vs	9	5	9	7410
370	8	3	8	vs	9	6	9	2418
371	8	3	8	vs	9	7	9	702
372	8	3	8	vs	9	8	9	156
373	8	3	8	vs	9	9	9	78
374	8	3	8	vs	10	3	10	17472
375	8	3	8	vs	10	4	10	28314
376	8	3	8	vs	10	5	10	19344
377	8	3	8	vs	10	6	10	8190
378	8	3	8	vs	10	7	10	2418
379	8	3	8	vs	10	8	10	702
380	8	3	8	vs	10	9	10	156
381	8	3	8	vs	10	10	10	78
382	8	4	8	vs	8	4	8	5476
383	8	4	8	vs	8	5	8	2220
384	8	4	8	vs	8	6	8	666
385	8	4	8	vs	8	7	8	148
386	8	4	8	vs	8	8	8	74
387	8	4	8	vs	9	4	9	12432
388	8	4	8	vs	9	5	9	7030
389	8	4	8	vs	9	6	9	2294
390	8	4	8	vs	9	7	9	666

391	8	4	8	vs	9	8	9	148
392	8	4	8	vs	9	9	9	74
393	8	4	8	vs	10	4	10	26862
394	8	4	8	vs	10	5	10	18352
395	8	4	8	vs	10	6	10	7770
396	8	4	8	vs	10	7	10	2294
397	8	4	8	vs	10	8	10	666
398	8	4	8	vs	10	9	10	148
399	8	4	8	vs	10	10	10	74
400	8	5	8	vs	8	5	8	900
401	8	5	8	vs	8	6	8	270
402	8	5	8	vs	8	7	8	60
403	8	5	8	vs	8	8	8	30
404	8	5	8	vs	9	5	9	2850
405	8	5	8	vs	9	6	9	930
406	8	5	8	vs	9	7	9	270
407	8	5	8	vs	9	8	9	60
408	8	5	8	vs	9	9	9	30
409	8	5	8	vs	10	5	10	7440
410	8	5	8	vs	10	6	10	3150
411	8	5	8	vs	10	7	10	930
412	8	5	8	vs	10	8	10	270
413	8	5	8	vs	10	9	10	60
414	8	5	8	vs	10	10	10	30
415	8	6	8	vs	8	6	8	81
416	8	6	8	vs	8	7	8	18
417	8	6	8	vs	8	8	8	9
418	8	6	8	vs	9	6	9	279
419	8	6	8	vs	9	7	9	81
420	8	6	8	vs	9	8	9	18
421	8	6	8	vs	9	9	9	9
422	8	6	8	vs	10	6	10	945
423	8	6	8	vs	10	7	10	279
424	8	6	8	vs	10	8	10	81
425	8	6	8	vs	10	9	10	18
426	8	6	8	vs	10	10	10	9
427	8	7	8	vs	8	7	8	4
428	8	7	8	vs	8	8	8	2
429	8	7	8	vs	9	7	9	18
430	8	7	8	vs	9	8	9	4

431	8	7	8	vs	9	9	9	2
432	8	7	8	vs	10	7	10	62
433	8	7	8	vs	10	8	10	18
434	8	7	8	vs	10	9	10	4
435	8	7	8	vs	10	10	10	2
436	8	8	8	vs	8	8	8	1
437	8	8	8	vs	9	8	9	2
438	8	8	8	vs	9	9	9	1
439	8	8	8	vs	10	8	10	9
440	8	8	8	vs	10	9	10	2
441	8	8	8	vs	10	10	10	1
442	9	3	9	vs	9	3	9	19321
443	9	3	9	vs	9	4	9	23352
444	9	3	9	vs	9	5	9	13205
445	9	3	9	vs	9	6	9	4309
446	9	3	9	vs	9	7	9	1251
447	9	3	9	vs	9	8	9	278
448	9	3	9	vs	9	9	9	139
449	9	3	9	vs	10	3	10	31136
450	9	3	9	vs	10	4	10	50457
451	9	3	9	vs	10	5	10	34472
452	9	3	9	vs	10	6	10	14595
453	9	3	9	vs	10	7	10	4309
454	9	3	9	vs	10	8	10	1251
455	9	3	9	vs	10	9	10	278
456	9	3	9	vs	10	10	10	139
457	9	4	9	vs	9	4	9	28224
458	9	4	9	vs	9	5	9	15960
459	9	4	9	vs	9	6	9	5208
460	9	4	9	vs	9	7	9	1512
461	9	4	9	vs	9	8	9	336
462	9	4	9	vs	9	9	9	168
463	9	4	9	vs	10	4	10	60984
464	9	4	9	vs	10	5	10	41664
465	9	4	9	vs	10	6	10	17640
466	9	4	9	vs	10	7	10	5208
467	9	4	9	vs	10	8	10	1512
468	9	4	9	vs	10	9	10	336
469	9	4	9	vs	10	10	10	168
470	9	5	9	vs	9	5	9	9025

471	9	5	9	vs	9	6	9	2945
472	9	5	9	vs	9	7	9	855
473	9	5	9	vs	9	8	9	190
474	9	5	9	vs	9	9	9	95
475	9	5	9	vs	10	5	10	23560
476	9	5	9	vs	10	6	10	9975
477	9	5	9	vs	10	7	10	2945
478	9	5	9	vs	10	8	10	855
479	9	5	9	vs	10	9	10	190
480	9	5	9	vs	10	10	10	95
481	9	6	9	vs	9	6	9	961
482	9	6	9	vs	9	7	9	279
483	9	6	9	vs	9	8	9	62
484	9	6	9	vs	9	9	9	31
485	9	6	9	vs	10	6	10	3255
486	9	6	9	vs	10	7	10	961
487	9	6	9	vs	10	8	10	279
488	9	6	9	vs	10	9	10	62
489	9	6	9	vs	10	10	10	31
490	9	7	9	vs	9	7	9	81
491	9	7	9	vs	9	8	9	18
492	9	7	9	vs	9	9	9	9
493	9	7	9	vs	10	7	10	279
494	9	7	9	vs	10	8	10	81
495	9	7	9	vs	10	9	10	18
496	9	7	9	vs	10	10	10	9
497	9	8	9	vs	9	8	9	4
498	9	8	9	vs	9	9	9	2
499	9	8	9	vs	10	8	10	18
500	9	8	9	vs	10	9	10	4
501	9	8	9	vs	10	10	10	2
502	9	9	9	vs	9	9	9	1
503	9	9	9	vs	10	9	10	2
504	9	9	9	vs	10	10	10	1
505	10	3	10	vs	10	3	10	50176
506	10	3	10	vs	10	4	10	81312
507	10	3	10	vs	10	5	10	55552
508	10	3	10	vs	10	6	10	23520
509	10	3	10	vs	10	7	10	6944
510	10	3	10	vs	10	8	10	2016

511	10	3	10	vs	10	9	10	448
512	10	3	10	vs	10	10	10	224
513	10	4	10	vs	10	4	10	131769
514	10	4	10	vs	10	5	10	90024
515	10	4	10	vs	10	6	10	38115
516	10	4	10	vs	10	7	10	11253
517	10	4	10	vs	10	8	10	3267
518	10	4	10	vs	10	9	10	726
519	10	4	10	vs	10	10	10	363
520	10	5	10	vs	10	5	10	61504
521	10	5	10	vs	10	6	10	26040
522	10	5	10	vs	10	7	10	7688
523	10	5	10	vs	10	8	10	2232
524	10	5	10	vs	10	9	10	496
525	10	5	10	vs	10	10	10	248
526	10	6	10	vs	10	6	10	11025
527	10	6	10	vs	10	7	10	3255
528	10	6	10	vs	10	8	10	945
529	10	6	10	vs	10	9	10	210
530	10	6	10	vs	10	10	10	105
531	10	7	10	vs	10	7	10	961
532	10	7	10	vs	10	8	10	279
533	10	7	10	vs	10	9	10	62
534	10	7	10	vs	10	10	10	31
535	10	8	10	vs	10	8	10	81
536	10	8	10	vs	10	9	10	18
537	10	8	10	vs	10	10	10	9
538	10	9	10	vs	10	9	10	4
539	10	9	10	vs	10	10	10	2
540	10	10	10	vs	10	10	10	1

APPENDIX B: VISUAL BASIC PROGRAM TO FORM MEANINGFUL COMBINATIONS

```

Private Sub Partition_Click()

Open "D:\output.dat" For Output As #2

For i = 3 To 15
    For j = 3 To i

'Count number of rows in file to determine array size
    x = 0
    FileName = "D:\Partitions\Partitions_" & i & "_" & j & ".dat"

        Open FileName For Input As #1    ' Open file for input.
        Do While Not EOF(1)
            Input #1, Test
            x = x + 1
        Loop

    Rows = x / j
    Close #1

'Dimension Array to proper size and load data from file
ReDim Partition(Rows, j)

        x = 0
        Open FileName For Input As #1    ' Open file for input.
        Do While Not EOF(1)

            Input #1, Temp
            k = Int(x / j) + 1
            m = x Mod j + 1
            Partition(k, m) = Temp

            x = x + 1
        Loop
        Close #1

'Load new array with evaluation function to determine uniqueness
TotalRows = Rows ^ 2
ReDim Unique(TotalRows)

'Create unique column from matrix

    k = 1
    Unique(k) = 0
    For h = 1 To Rows
        For g = 1 To Rows

```

```

        Unique(k) = 0
        For f = 1 To j

            'Unique(k) = Round(Unique(k) + Log(Partition(h, f) + 1) *
            Log(Partition(g, f) + 1), 6)
            Unique(k) = Round(Unique(k) + Partition(h, f) ^ 3 /
            Partition(g, f) ^ (1 / 3), 6)

        Next f
        k = k + 1

    Next g
Next h

```

```

ReDim UniqueTemp(1)
Row1 = 2 * j
ReDim UniqueData(Row1, 1)
Flag = 0
x = 1
UniqueTemp(1) = Unique(1)

'Write first config to array in first column
For a = 1 To j

    UniqueData(2 * (a - 1) + 1, 1) = Partition(1, a)
    UniqueData(2 * a, 1) = Partition(1, a)

Next a

```

```

For k = 2 To TotalRows
For w = 1 To x

    If Unique(k) = UniqueTemp(w) Then
        Flag = 1
    End If

Next w

    If Flag = 0 Then

        x = x + 1
        ReDim Preserve UniqueTemp(x)
        ReDim Preserve UniqueData(Row1, x)
        UniqueTemp(x) = Unique(k)

        place1 = Int((k - 1) / Rows) + 1

```

```

        place2 = k Mod Rows
        If place2 = 0 Then place2 = Rows

'For-Next Loop needed here to iterate writing of row data

        For b = 1 To j
            UniqueData(2 * (b - 1) + 1, x) = Partition(place1, b)
            UniqueData(2 * b, x) = Partition(place2, b)

'Need something to write second half of config

                Next b

        End If

Flag = 0

Next k

Print i, j, x
Write #2, i, j, x

'Write Files
FileName2 = "D:\PartitionData\PartitionD_" & i & "_" & j & ".dat"

Open FileName2 For Output As #3

'Write Raw Data to files
Config = ""
For u = 1 To x

        For r = 1 To j
            Config = Config & UniqueData(2 * (r - 1) + 1, u) & " " &
UniqueData(2 * r, u) & " "
        Next r

Print #3, Config

Config = ""
Next u

Close #3

        Next j
Next i

Print "Done"

```

Close #2

End Sub

APPENDIX C: VISUAL BASIC PROGRAM USED TO CREATE LIST OF UNIQUE ASSYMMETRIC ENGAGEMENTS

```

Private Sub Combinations_Click()

lowerbound = 3
upperbound = 13

Open "D:\XYXConfig.dat" For Output As #1
x = 0

For RedPX = lowerbound To upperbound
    For RedPY = lowerbound To RedPX

        For BluePX = RedPX To upperbound
            For BluePY = RedPY To BluePX

                y = 0
                FileName1 = "D:\PartitionDataR\PartitionD_" & RedPX & "_" &
RedPY & ".dat"

                Open FileName1 For Input As #2    ' Open file for input.
                Do While Not EOF(2)
                    Input #2, Test
                    y = y + 1
                Loop

                RowsR = y / (2 * BluePY)
                Close #2

                y = 0
                FileName1 = "D:\PartitionDataB\PartitionD_" & BluePX & "_" &
BluePY & ".dat"

                Open FileName1 For Input As #2    ' Open file for input.
                Do While Not EOF(2)
                    Input #2, Test
                    y = y + 1
                Loop

                RowsB = y / (2 * RedPY)
                Close #2

                Rows = RowsR * RowsB
                x = x + 1
                Print #1, x & " " & RedPX & " " & RedPY & " " & BluePX &
" " & BluePY & " " & Rows
            
```

```
        Next BluePY
      Next BluePX
    Next RedPY
  Next RedPX

Close #1

Print "Done"

End Sub
```

APPENDIX D: VISUAL BASIC PROGRAM COMBAT MODEL FOR ASYMMETRIC ENGAGEMENTS

```

Dim TempNodes, RedX, BlueX, TempSensors, TempDeciders
Dim RedPX, RedPY, BluePX, BluePY

Public Sub Combat_Click()
Randomize
Cls

Open "D:\output.dat" For Output As #2
'*****
Counttotal = 0
Replications = 1000
lowerbound = 3
upperbound = 6

For RedPX = lowerbound To upperbound
    For RedPY = lowerbound To RedPX
        For BluePX = RedPX To upperbound
            For BluePY = RedPY To BluePX

UpdateRX = "Red " & RedPX & "-" & RedPY & " vs Blue " & BluePX &
 "-" & BluePY
UpdateRX.Refresh
Form1.Show

'Load Red Source Matrix to Array from file

a = RedPX
b = RedPY

'Count number of rows in file to determine array size
x = 0
FileName1 = "D:\PartitionData\PartitionD_" & a & "_" & b &
".dat"

        Open FileName1 For Input As #1    ' Open file for input.
        Do While Not EOF(1)
            Input #1, Test
            x = x + 1
        Loop

RowsRed = x / (2 * b)

```

```

        Close #1

TempNodes = 0
RedY = b
TotalRed = RowsRed
i = 1

'Print RowsRed

ReDim Red(TotalRed, RedY, 2)
ReDim RedTemp(TotalRed, RedY, 2)

Open FileName1 For Input As #1    ' Open file for input.
Do While Not EOF(1)
    TempNodes = TempNodes + 1
    Input #1, TempSensors, TempDeciders
    Red(i, TempNodes, 1) = TempSensors
    RedTemp(i, TempNodes, 1) = TempSensors
    Red(i, TempNodes, 2) = TempDeciders
    RedTemp(i, TempNodes, 2) = TempDeciders
    If TempNodes = RedY Then
        TempNodes = 0
        i = i + 1
    End If
Loop
Close #1

'*****
'Load Blue Source Matrix to Array from file

c = BluePX
d = BluePY

'Count number of rows in file to determine array size
x = 0
FileName2 = "D:\PartitionData\PartitionD_" & c & "_" & d &
".dat"

        Open FileName2 For Input As #1    ' Open file for input.
        Do While Not EOF(1)
            Input #1, Test
            x = x + 1
        Loop

        RowsBlue = x / (2 * d)
        Close #1

TempNodes = 0
BlueY = d

```

```
TotalBlue = RowsBlue
i = 1
```

```
ReDim Blue(TotalBlue, BlueY, 2)
ReDim BlueTemp(TotalBlue, BlueY, 2)
```

```
Open FileName2 For Input As #1    ' Open file for input.
Do While Not EOF(1)
TempNodes = TempNodes + 1
Input #1, TempSensors, TempDeciders
Blue(i, TempNodes, 1) = TempSensors
BlueTemp(i, TempNodes, 1) = TempSensors
Blue(i, TempNodes, 2) = TempDeciders
BlueTemp(i, TempNodes, 2) = TempDeciders
    If TempNodes = BlueY Then
        TempNodes = 0
        i = i + 1
    End If

Loop
Close #1
```

```
'*****
'Do Battle
```

```
CountRep = 0
RedCount = 0
Do While RedCount < TotalRed
RedCount = RedCount + 1
BlueCount = 0
Do While BlueCount < TotalBlue
BlueCount = BlueCount + 1
CountRep = CountRep + 1
Counttotal = Counttotal + 1
```

```
'Determine Number of Replications (e.g. 30)
RedWins = 0
BlueWins = 0
Do While RedWins + BlueWins < Replications
```

```
'Need to reinitialize the matrix each time
```

```
'*****
'Load Red Source Matrix from Initial Temp Matrix
```

```
For i = 1 To TotalRed
For j = 1 To RedY
```

```

Red(i, j, 1) = RedTemp(i, j, 1)
Red(i, j, 2) = RedTemp(i, j, 2)
Next j
Next i

```

```

'*****

```

```

'Load Blue Source Matrix from Initial Temp Matrix
For i = 1 To TotalBlue
For j = 1 To BlueY
Blue(i, j, 1) = BlueTemp(i, j, 1)
Blue(i, j, 2) = BlueTemp(i, j, 2)
Next j
Next i

```

```

'Determine winner of each replication
Winner = ""
Do While Winner = ""

```

```

'Count Red Sensors and Influencers and Combat Cycles
TotalActiveRedSensors = 0
TotalActiveRedInfluencers = 0
TotalActiveRedCombatCycles = 0
For i = 1 To RedY
    RedFlagS = 0
    RedFlagI = 0
    TotalActiveRedSensors = TotalActiveRedSensors + Red(RedCount,
i, 1)
    TotalActiveRedInfluencers = TotalActiveRedInfluencers +
Red(RedCount, i, 2)
    If Red(RedCount, i, 1) > 0 Then RedFlagS = 1
    If Red(RedCount, i, 2) > 0 Then RedFlagI = 1
    TotalActiveRedCombatCycles = TotalActiveRedCombatCycles +
RedFlagS * RedFlagI
Next i

```

```

If TotalActiveRedCombatCycles = 0 Then
Bluewins = Bluewins + 1
'Print "Blue Wins"
Winner = "Blue"

```

```

GoTo 10
End If

```

```

'Count Blue Sensors and Influencers and Combat Cycles
TotalActiveBlueSensors = 0
TotalActiveBlueInfluencers = 0
TotalActiveBlueCombatCycles = 0
For i = 1 To BlueY
    BlueFlagS = 0
    BlueFlagI = 0

```

```

    TotalActiveBlueSensors = TotalActiveBlueSensors +
Blue(BlueCount, i, 1)
    TotalActiveBlueInfluencers = TotalActiveBlueInfluencers +
Blue(BlueCount, i, 2)
    If Blue(BlueCount, i, 1) > 0 Then BlueFlagS = 1
    If Blue(BlueCount, i, 2) > 0 Then BlueFlagI = 1
    TotalActiveBlueCombatCycles = TotalActiveBlueCombatCycles +
BlueFlagS * BlueFlagI
Next i

If TotalActiveBlueCombatCycles = 0 Then
RedWins = RedWins + 1
'Print "Red Wins"
Winner = "Red"

GoTo 10
End If

```

'Pick Side to Shoot and Destroy Sensor or Influencer on Opposing Side

```

TotalActiveEverything = TotalActiveRedSensors +
TotalActiveBlueSensors

```

```

'ShootSide = Int(Rnd() * TotalActiveEverything) + 1
ShootSide = Rnd() * TotalActiveEverything

```

```

If ShootSide <= TotalActiveRedSensors Then
'Red won toss so destroy Blue target (sensor or influencer)
    BlueDestroy = Int(Rnd * (TotalActiveBlueSensors +
TotalActiveBlueInfluencers)) + 1
    BlueTrack = 0
    For j = 1 To 2
    For i = 1 To BlueY
        BlueTrack = BlueTrack + Blue(BlueCount, i, j)
        If BlueTrack >= BlueDestroy Then
            Blue(BlueCount, i, j) = Blue(BlueCount, i, j) - 1
            GoTo 20
        End If
    Next i
    Next j

```

Else

```

'Blue won toss so destroy Red target (sensor or influencer)
    RedDestroy = Int(Rnd * (TotalActiveRedSensors +
TotalActiveRedInfluencers)) + 1
    RedTrack = 0

```

```

    For j = 1 To 2
    For i = 1 To RedY
        RedTrack = RedTrack + Red(RedCount, i, j)
        If RedTrack >= RedDestroy Then
            Red(RedCount, i, j) = Red(RedCount, i, j) - 1
            GoTo 20
        End If
    Next i
    Next j

End If

20
Loop
10
Loop

HolderRed = ""
HolderBlue = ""
For p = 1 To b

HolderRed = HolderRed & " " & RedTemp(RedCount, p, 1) & " " &
RedTemp(RedCount, p, 2)

Next p

For p = 1 To d

HolderBlue = HolderBlue & " " & BlueTemp(BlueCount, p, 1) & " " &
BlueTemp(BlueCount, p, 2)

Next p

Print #2, Counttotal & " " & CountRep & " " & RedPX & " " & RedPY
& " " & BluePX & " " & BluePY & " " & HolderRed & " , " &
HolderBlue & " " & Round((100 * RedWins / (RedWins + BlueWins)),
2)

Loop
Loop

        Next BluePY
    Next BluePX
Next RedPY
Next RedPX

Close #2

Print CountRep, "Done"
End Sub

```

APPENDIX E: PROGRAM TO CALCULATE DISPARITY

```

Private Sub Combinations_Click()

CounterIndex = 0
Open "D:\Disparity_Output1000.dat" For Output As #2

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1      ' Open file for input.

        Do While Not EOF(1)
            CounterIndex = CounterIndex + 1

            Input #1, IndexCount
            Input #1, TotalCount
            Input #1, ConfigCount
            Input #1, RedS, RedI, BlueS, BlueI

            ReDim RedConfig(2 * RedI)
            ReDim BlueConfig(2 * BlueI)

            For i = 1 To 2 * RedI
                Input #1, RedConfig(i)
            Next i

            For i = 1 To 2 * BlueI
                Input #1, BlueConfig(i)
            Next i

            Input #1, PercentWin

'Calculate Metrics

            MaxSensor = RedConfig(1)
            MinSensor = RedConfig(1)
            MaxInfluencer = RedConfig(2)
            MinInfluencer = RedConfig(2)

            For i = 1 To (2 * RedI - 1)

                If RedConfig(i) > MaxSensor Then MaxSensor = RedConfig(i)
                If RedConfig(i) < MinSensor Then MinSensor = RedConfig(i)

                If RedConfig(i + 1) > MaxInfluencer Then MaxInfluencer =
RedConfig(i + 1)

```

```

        If RedConfig(i + 1) < MinInfluencer Then MinInfluencer =
RedConfig(i + 1)

        RedDisparity = (MaxSensor - MinSensor) + (MaxInfluencer -
MinInfluencer)

        i = i + 1
    Next i

    MaxSensor = BlueConfig(1)
    MinSensor = BlueConfig(1)
    MaxInfluencer = BlueConfig(2)
    MinInfluencer = BlueConfig(2)

    For i = 1 To (2 * BlueI - 1)

        If BlueConfig(i) > MaxSensor Then MaxSensor =
BlueConfig(i)
        If BlueConfig(i) < MinSensor Then MinSensor =
BlueConfig(i)

        If BlueConfig(i + 1) > MaxInfluencer Then MaxInfluencer =
BlueConfig(i + 1)
        If BlueConfig(i + 1) < MinInfluencer Then MinInfluencer =
BlueConfig(i + 1)

        BlueDisparity = (MaxSensor - MinSensor) + (MaxInfluencer
- MinInfluencer)

        i = i + 1
    Next i

    Print #2, CounterIndex, RedDisparity, BlueDisparity,
Round(PercentWin, 2)

    Loop

    Print "Done"

    Close #1
    Close #2

End Sub

```

APPENDIX F: PROGRAM TO CALCULATE ROBUSTNESS

```

Private Sub Combinations_Click()

CounterIndex = 0
Open "D:\Robustness_Output1000.dat" For Output As #2

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1      ' Open file for input.

        Do While Not EOF(1)
            CounterIndex = CounterIndex + 1
            Input #1, IndexCount
            Input #1, TotalCount
            Input #1, ConfigCount
            Input #1, RedS, RedI, BlueS, BlueI

            ReDim RedConfig(2 * RedI)
            ReDim BlueConfig(2 * BlueI)

            For i = 1 To 2 * RedI
                Input #1, RedConfig(i)
            Next i

            For i = 1 To 2 * BlueI
                Input #1, BlueConfig(i)
            Next i

            Input #1, PercentWin

'Calculate Metrics
            RedRobustness = 0
            BlueRobustness = 0

            For i = 1 To (2 * RedI - 1)

                RedRobustnessTemp = RedConfig(i)

                If RedConfig(i + 1) < RedConfig(i) Then RedRobustnessTemp
= RedConfig(i + 1)

                RedRobustness = RedRobustness + RedRobustnessTemp

                i = i + 1
            Next i
        End Do
    End Do
End Sub

```

```
    For i = 1 To (2 * BlueI - 1)

        BlueRobustnessTemp = BlueConfig(i)

        If BlueConfig(i + 1) < BlueConfig(i) Then
BlueRobustnessTemp = BlueConfig(i + 1)

        BlueRobustness = BlueRobustness + BlueRobustnessTemp

        i = i + 1
    Next i

    Print #2, CounterIndex, RedRobustness, BlueRobustness,
Round(PercentWin, 2)

    Loop

    Print "Done"

    Close #1
    Close #2

End Sub
```

APPENDIX G: PROGRAM TO CALCULATE STRENGTH

```

Private Sub Combinations_Click()

Open "D:\Strength_Output1000.dat" For Output As #2
CounterIndex = 0

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1    ' Open file for input.

        Do While Not EOF(1)
        CounterIndex = CounterIndex + 1
        Input #1, IndexCount
        Input #1, TotalCount
        Input #1, ConfigCount
        Input #1, RedS, RedI, BlueS, BlueI

        ReDim RedConfig(2 * RedI)
        ReDim BlueConfig(2 * BlueI)

        For i = 1 To 2 * RedI
        Input #1, RedConfig(i)
        Next i

        For i = 1 To 2 * BlueI
        Input #1, BlueConfig(i)
        Next i

        Input #1, PercentWin

'Calculate Metrics

        RedStrength = 0
        For i = 1 To (2 * RedI - 1)
            RedStrength = RedStrength + (Log(RedConfig(i) + 1) /
Log(10)) * (Log(RedConfig(i + 1) + 1) / Log(10))
            i = i + 1
        Next i

        BlueStrength = 0
        For i = 1 To (2 * BlueI - 1)
            BlueStrength = BlueStrength + (Log(BlueConfig(i) + 1)
/ Log(10)) * (Log(BlueConfig(i + 1) + 1) / Log(10))
            i = i + 1
        Next i

```

```
Print #2, CounterIndex, Round(RedStrength, 2),  
Round(BlueStrength, 2), Round(PercentWin, 2)
```

```
Loop
```

```
Print "Done"
```

```
Close #1
```

```
Close #2
```

```
End Sub
```

APPENDIX H: PROGRAM TO CALCULATE POWER

```

Private Sub Combinations_Click()

CounterIndex = 0
Open "D:\Power_Output1000.dat" For Output As #2

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1      ' Open file for input.

        Do While Not EOF(1)
            CounterIndex = CounterIndex + 1
            Input #1, IndexCount
            Input #1, TotalCount
            Input #1, ConfigCount
            Input #1, RedS, RedI, BlueS, BlueI

            ReDim RedConfig(2 * RedI)
            ReDim BlueConfig(2 * BlueI)

            For i = 1 To 2 * RedI
                Input #1, RedConfig(i)
            Next i

            For i = 1 To 2 * BlueI
                Input #1, BlueConfig(i)
            Next i

            Input #1, PercentWin

'Calculate Metrics

            RedPower = 0
            For i = 1 To (2 * RedI - 1)
                RedPower = RedPower + RedConfig(i) ^ 0.5 *
RedConfig(i + 1) ^ 0.5
                i = i + 1
            Next i

            BluePower = 0
            For i = 1 To (2 * BlueI - 1)
                BluePower = BluePower + BlueConfig(i) ^ 0.5 *
BlueConfig(i + 1) ^ 0.5
                i = i + 1
            Next i

```

```
Print #2, CounterIndex, Round(RedPower, 2),  
Round(BluePower, 2), Round(PercentWin, 2)
```

```
Loop
```

```
Print "Done"
```

```
Close #1
```

```
Close #2
```

```
End Sub
```

APPENDIX I: PROGRAM TO CALCULATE STABILITY

```

Private Sub Combinations_Click()

CounterIndex = 0
Open "D:\Stability_Output1000.dat" For Output As #2

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1    ' Open file for input.

        Do While Not EOF(1)
            CounterIndex = CounterIndex + 1
            Input #1, IndexCount
            Input #1, TotalCount
            Input #1, ConfigCount
            Input #1, RedS, RedI, BlueS, BlueI

            ReDim RedConfig(2 * RedI)
            ReDim BlueConfig(2 * BlueI)

            For i = 1 To 2 * RedI
                Input #1, RedConfig(i)
            Next i

            For i = 1 To 2 * BlueI
                Input #1, BlueConfig(i)
            Next i

            Input #1, PercentWin

'Calculate Metrics

            RedStability = 0
            For i = 1 To (2 * RedI - 1)
                RedStability = RedStability + (RedConfig(i) /
RedConfig(i + 1))
                i = i + 1
            Next i

            BlueStability = 0
            For i = 1 To (2 * BlueI - 1)
                BlueStability = BlueStability + (BlueConfig(i) /
BlueConfig(i + 1))
                i = i + 1
            Next i

```

```
Print #2, CounterIndex, Round(RedStability, 2),  
Round(BlueStability, 2), Round(PercentWin, 2)
```

```
Loop
```

```
Print "Done"
```

```
Close #1
```

```
Close #2
```

```
End Sub
```

APPENDIX J: PROGRAM TO CALCULATE CONNECTIVITY

```

Private Sub Combinations_Click()

CounterIndex = 0
Open "D:\Connectivity_Output1000.dat" For Output As #2

    FileName1 = "D:\nevan1000.dat"

        Open FileName1 For Input As #1      ' Open file for input.

        Do While Not EOF(1)
            CounterIndex = CounterIndex + 1
            Input #1, IndexCount
            Input #1, TotalCount
            Input #1, ConfigCount
            Input #1, RedS, RedI, BlueS, BlueI

            ReDim RedConfig(2 * RedI)
            ReDim BlueConfig(2 * BlueI)

            For i = 1 To 2 * RedI
                Input #1, RedConfig(i)
            Next i

            For i = 1 To 2 * BlueI
                Input #1, BlueConfig(i)
            Next i

            Input #1, PercentWin

'Calculate Metrics

            RedConnectivity = 0
            For i = 1 To (2 * RedI - 1)
                RedConnectivity = RedConnectivity + Abs(RedConfig(i)
- RedConfig(i + 1))
                i = i + 1
            Next i

            BlueConnectivity = 0
            For i = 1 To (2 * BlueI - 1)
                BlueConnectivity = BlueConnectivity +
Abs(BlueConfig(i) - BlueConfig(i + 1))
                i = i + 1
            Next i

```

```
Print #2, CounterIndex, Round(RedConnectivity, 2),  
Round(BlueConnectivity, 2), Round(PercentWin, 2)
```

```
Loop
```

```
Print "Done"
```

```
Close #1
```

```
Close #2
```

```
End Sub
```

APPENDIX K: REGRESSION ANALYSIS RESULTS

Linear Model with only SDI

Summary of Fit

RSquare	0.912051
RSquare Adj	0.91205
Root Mean Square Error	4.809598
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	349703887	87425972	3779401
Error	1.5e+6	33722070	23.132229	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	535	26556285	49637.9	10094.55
Pure Error	1.5e+6	7165786	4.9	Prob > F
Total Error	1.5e+6	33722070		<.0001*
				Max RSq
				0.9813

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.784843	0.073699	580.54	<.0001*
Red S,I	13.992586	0.003664	3819.1	<.0001*
Red D	-0.363777	0.005902	-61.63	<.0001*
Blue S,I	-13.46769	0.008207	-1641	<.0001*
Blue D	0.3961034	0.003958	100.08	<.0001*

Linear Model with only Metrics

Summary of Fit

RSquare	0.921313
RSquare Adj	0.921312
Root Mean Square Error	4.549317
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	353255160	29437930	1422377
Error	1.5e+6	30170797	20.696286	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	30091728	21.1579	9.5101
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	30170797		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.822792	0.069884	612.77	<.0001*
Conn_Red	6.4851261	0.009793	660.15	<.0001*
Conn_Blue	-5.587557	0.01102	-507.0	<.0001*
Disp_Red	0.0014623	0.002423	0.60	0.5462
Disp_Blue	-0.149856	0.002598	-57.60	<.0001*
Pow_Red	1.8137686	0.032569	55.69	<.0001*
Pow_Blue	-1.045457	0.031649	-33.03	<.0001*
Rob_Red	13.043787	0.026716	488.23	<.0001*
Rob_Blue	-10.92455	0.0289	-378.0	<.0001*
Stab_Red	-0.358176	0.005404	-66.28	<.0001*
Stab_Blue	-0.093718	0.005754	-16.29	<.0001*
Stre_Red	-4.799564	0.191753	-25.03	<.0001*
Stre_Blue	-12.76768	0.165811	-77.00	<.0001*

Linear Model with only Metrics minus Disp_Red

Summary of Fit

RSquare	0.921313
RSquare Adj	0.921312
Root Mean Square Error	4.549316
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	353255153	32114105	1551685
Error	1.5e+6	30170805	20.696277	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	30033119	21.5532	10.0733
Pure Error	64350	137886	2.1396	Prob > F
Total Error	1.5e+6	30170805		<.0001*
				Max RSq
				0.9996

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	42.824607	0.06982	613.36	<.0001*
Conn_Red	6.4684987	0.008042	804.33	<.0001*
Conn_Blue	-5.587493	0.01102	-507.0	<.0001*
Disp_Blue	-0.14962	0.002597	-57.60	<.0001*
Pow_Red	1.8110036	0.032245	56.16	<.0001*
Pow_Blue	-1.046298	0.031618	-33.09	<.0001*
Rob_Red	13.051465	0.023492	555.58	<.0001*
Rob_Blue	-10.9244	0.028899	-378.0	<.0001*
Stab_Red	-0.359554	0.004898	-73.41	<.0001*
Stab_Blue	-0.093764	0.005753	-16.30	<.0001*
Stre_Red	-4.829129	0.185388	-26.05	<.0001*
Stre_Blue	-12.76304	0.165632	-77.06	<.0001*

Linear Model with Metrics + SDI

Singularity Details

Conn_Red = - 2*Rob_Red + 2*Red S,I

Conn_Blue = - 2*Rob_Blue + 2*Blue S,I

Summary of Fit

RSquare	0.923779
RSquare Adj	0.923778
Root Mean Square Error	4.477449
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	354200916	25300065	1262003
Error	1.5e+6	29225042	20.047553	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	29145973	20.4929	9.2112
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	29225042		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob> t
Intercept	Biased	45.279405	0.070593	641.42	<.0001*
Conn_Red	Biased	5.8603869	0.010038	583.80	<.0001*
Conn_Blue	Biased	-5.572395	0.011263	-494.7	<.0001*
Disp_Red	Biased	-0.606284	0.003688	-164.4	<.0001*
Disp_Blue	Biased	-0.083937	0.003835	-21.89	<.0001*
Pow_Red	Biased	14.99157	0.068908	217.56	<.0001*
Pow_Blue	Biased	-2.588073	0.073481	-35.22	<.0001*
Rob_Red	Biased	11.94964	0.02678	446.21	<.0001*
Rob_Blue	Biased	-10.95054	0.028803	-380.2	<.0001*
Stab_Red	Biased	-0.13899	0.005416	-25.66	<.0001*
Stab_Blue		-0.092874	0.00573	-16.21	<.0001*
Stre_Red	Biased	-93.70946	0.452658	-207.0	<.0001*
Stre_Blue	Biased	-1.662565	0.483609	-3.44	0.0006*
Red S,I	Zeroed	0	0	.	.
Red D	Biased	-4.449238	0.020538	-216.6	<.0001*
Blue S,I	Zeroed	0	0	.	.
Blue D		0.5855079	0.022418	26.12	<.0001*

Linear Model with Metrics + D

Summary of Fit

RSquare	0.923779
RSquare Adj	0.923778
Root Mean Square Error	4.477449
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	354200918	25300065	1262003
Error	1.5e+6	29225042	20.047553	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	29145973	20.4929	9.2112
Pure Error	35540	79069	2.2248	Prob > F
Total Error	1.5e+6	29225042		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	45.279405	0.070593	641.42	<.0001*
Conn_Red	5.8603889	0.010038	583.80	<.0001*
Conn_Blue	-5.572395	0.011263	-494.7	<.0001*
Disp_Red	-0.606284	0.003688	-164.4	<.0001*
Disp_Blue	-0.083937	0.003835	-21.89	<.0001*
Pow_Red	14.99157	0.068908	217.56	<.0001*
Pow_Blue	-2.588073	0.073481	-35.22	<.0001*
Rob_Red	11.94964	0.02678	446.21	<.0001*
Rob_Blue	-10.95054	0.028803	-380.2	<.0001*
Stab_Red	-0.13899	0.005416	-25.66	<.0001*
Stab_Blue	-0.092874	0.00573	-16.21	<.0001*
Stre_Red	-93.70946	0.452658	-207.0	<.0001*
Stre_Blue	-1.662565	0.483609	-3.44	0.0006*
Red D	-4.449238	0.020538	-216.6	<.0001*
Blue D	0.5855079	0.022418	26.12	<.0001*

Non-Linear Model with SDI (2-way Interactions and Quadratic terms)

Summary of Fit

RSquare	0.980288
RSquare Adj	0.980287
Root Mean Square Error	2.277007
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	375867687	26847692	5178194
Error	1.5e+6	7558270	5.1847598	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	525	392484.7	747.590	152.0327
Pure Error	1.5e+6	7165785.5	4.917	Prob > F
Total Error	1.5e+6	7558270.3		<.0001*
				Max RSq
				0.9813

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	44.213009	0.061364	720.50	<.0001*
Red S,I	18.146477	0.002542	7139.5	<.0001*
Red D	-0.466036	0.003453	-135.0	<.0001*
Blue S,I	-17.67417	0.006649	-2658	<.0001*
Blue D	0.4011586	0.002412	166.35	<.0001*
(Red S,I-8.94429)*(Red D-3.69742)	-0.101324	0.002905	-34.88	<.0001*
(Red S,I-8.94429)*(Blue S,I-9.759)	-5.039053	0.004035	-1249	<.0001*
(Red S,I-8.94429)*(Blue D-4.67447)	0.0975055	0.001664	58.58	<.0001*
(Red D-3.69742)*(Blue S,I-9.759)	0.1236491	0.006788	18.22	<.0001*
(Red D-3.69742)*(Blue D-4.67447)	-0.009678	0.003259	-2.97	0.0030*
(Blue S,I-9.759)*(Blue D-4.67447)	-0.135173	0.004138	-32.67	<.0001*
(Red S,I-8.94429)*(Red S,I-8.94429)	2.3021673	0.001075	2142.4	<.0001*
(Red D-3.69742)*(Red D-3.69742)	0.037035	0.003436	10.78	<.0001*
(Blue S,I-9.759)*(Blue S,I-9.759)	2.7065405	0.005083	532.50	<.0001*
(Blue D-4.67447)*(Blue D-4.67447)	-0.011779	0.001193	-9.87	<.0001*

Non-Linear Model with Metrics (2-way Interactions and Quadratic terms)

Summary of Fit

RSquare	0.990684
RSquare Adj	0.990684
Root Mean Square Error	1.56536
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	90	379854054	4220601	1722448
Error	1.5e+6	3571904	2.450353	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3492834.9	2.45599	1.1039
Pure Error	35540	79068.8	2.22478	Prob > F
Total Error	1.5e+6	3571903.7		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	43.993504	0.042622	1032.2	<.0001*
Conn_Red	8.3513251	0.005837	1430.8	<.0001*
Conn_Blue	-8.156417	0.006319	-1291	<.0001*
Disp_Red	0.1481417	0.001139	130.03	<.0001*
Disp_Blue	-0.127242	0.001224	-103.9	<.0001*
Pow_Red	-0.602685	0.019297	-31.23	<.0001*
Pow_Blue	0.4723847	0.018207	25.95	<.0001*
Rob_Red	16.781052	0.016149	1039.2	<.0001*
Rob_Blue	-16.38136	0.016711	-980.2	<.0001*
Stab_Red	0.1333046	0.002748	48.50	<.0001*
Stab_Blue	-0.069186	0.002875	-24.06	<.0001*
Stre_Red	16.773184	0.09398	178.48	<.0001*
Stre_Blue	-15.38568	0.083092	-185.2	<.0001*
(Conn_Red-5.40938)*(Conn_Blue-5.59815)	-1.096842	0.009933	-110.4	<.0001*
(Conn_Red-5.40938)*(Disp_Red-6.28209)	-0.131034	0.002956	-44.33	<.0001*
(Conn_Red-5.40938)*(Disp_Blue-5.93948)	-0.018088	0.002261	-8.00	<.0001*
(Conn_Red-5.40938)*(Pow_Red-8.25877)	-1.036628	0.038797	-26.72	<.0001*
(Conn_Red-5.40938)*(Pow_Blue-9.06484)	0.0706146	0.027527	2.57	0.0103*
(Conn_Red-5.40938)*(Rob_Red-6.2396)	2.9824052	0.038712	77.04	<.0001*
(Conn_Red-5.40938)*(Rob_Blue-6.95993)	-2.188461	0.025756	-84.97	<.0001*
(Conn_Red-5.40938)*(Stab_Red-5.07463)	-0.156882	0.006668	-23.53	<.0001*
(Conn_Red-5.40938)*(Stab_Blue-6.13897)	-0.009366	0.004994	-1.88	0.0608
(Conn_Red-5.40938)*(Stre_Red-0.92333)	4.410406	0.228779	19.28	<.0001*
(Conn_Red-5.40938)*(Stre_Blue-0.99106)	-1.709078	0.145405	-11.75	<.0001*
(Conn_Blue-5.59815)*(Disp_Red-6.28209)	0.0032861	0.002475	1.33	0.1843
(Conn_Blue-5.59815)*(Disp_Blue-5.93948)	0.1424568	0.003293	43.25	<.0001*
(Conn_Blue-5.59815)*(Pow_Red-8.25877)	-0.078008	0.034698	-2.25	0.0246*
(Conn_Blue-5.59815)*(Pow_Blue-9.06484)	0.7804666	0.042662	18.29	<.0001*
(Conn_Blue-5.59815)*(Rob_Red-6.2396)	-2.241939	0.027068	-82.83	<.0001*
(Conn_Blue-5.59815)*(Rob_Blue-6.95993)	1.2478186	0.045491	27.43	<.0001*
(Conn_Blue-5.59815)*(Stab_Red-5.07463)	0.0513736	0.005494	9.35	<.0001*
(Conn_Blue-5.59815)*(Stab_Blue-6.13897)	0.1419956	0.007446	19.07	<.0001*
(Conn_Blue-5.59815)*(Stre_Red-0.92333)	-0.358211	0.208494	-1.72	0.0858
(Conn_Blue-5.59815)*(Stre_Blue-0.99106)	0.7745981	0.218554	3.54	0.0004*
(Disp_Red-6.28209)*(Disp_Blue-5.93948)	-0.000549	0.000572	-0.96	0.3364
(Disp_Red-6.28209)*(Pow_Red-8.25877)	0.2761384	0.007928	34.83	<.0001*

Term	Estimate	Std Error	t Ratio	Prob> t
(Disp_Red-6.28209)*(Pow_Blue-9.06484)	-0.000297	0.006984	-0.04	0.9661
(Disp_Red-6.28209)*(Rob_Red-6.2396)	-0.30511	0.00754	-40.46	<.0001*
(Disp_Red-6.28209)*(Rob_Blue-6.95993)	0.0076405	0.006443	1.19	0.2357
(Disp_Red-6.28209)*(Stab_Red-5.07463)	0.0418397	0.00151	27.71	<.0001*
(Disp_Red-6.28209)*(Stab_Blue-6.13897)	-0.001109	0.00127	-0.87	0.3825
(Disp_Red-6.28209)*(Stre_Red-0.92333)	0.1214346	0.05153	2.36	0.0184*
(Disp_Red-6.28209)*(Stre_Blue-0.99106)	-0.07309	0.036635	-2.00	0.0460*
(Disp_Blue-5.93948)*(Pow_Red-8.25877)	0.001421	0.007934	0.18	0.8579
(Disp_Blue-5.93948)*(Pow_Blue-9.06484)	-0.20588	0.008371	-24.59	<.0001*
(Disp_Blue-5.93948)*(Rob_Red-6.2396)	-0.034086	0.006198	-5.50	<.0001*
(Disp_Blue-5.93948)*(Rob_Blue-6.95993)	0.3149481	0.008221	38.31	<.0001*
(Disp_Blue-5.93948)*(Stab_Red-5.07463)	0.0003366	0.00125	0.27	0.7877
(Disp_Blue-5.93948)*(Stab_Blue-6.13897)	-0.036683	0.001593	-23.03	<.0001*
(Disp_Blue-5.93948)*(Stre_Red-0.92333)	-0.058352	0.047931	-1.22	0.2234
(Disp_Blue-5.93948)*(Stre_Blue-0.99106)	-0.390652	0.04682	-8.34	<.0001*
(Pow_Red-8.25877)*(Pow_Blue-9.06484)	-0.314697	0.097205	-3.24	0.0012*
(Pow_Red-8.25877)*(Rob_Red-6.2396)	-2.371218	0.106555	-22.25	<.0001*
(Pow_Red-8.25877)*(Rob_Blue-6.95993)	-0.158267	0.089601	-1.77	0.0773
(Pow_Red-8.25877)*(Stab_Red-5.07463)	-0.127939	0.018936	-6.76	<.0001*
(Pow_Red-8.25877)*(Stab_Blue-6.13897)	-0.028989	0.017697	-1.64	0.1014
(Pow_Red-8.25877)*(Stre_Red-0.92333)	-24.77338	0.793689	-31.21	<.0001*
(Pow_Red-8.25877)*(Stre_Blue-0.99106)	1.8172002	0.504997	3.60	0.0003*
(Pow_Blue-9.06484)*(Rob_Red-6.2396)	0.2282333	0.075443	3.03	0.0025*
(Pow_Blue-9.06484)*(Rob_Blue-6.95993)	1.691063	0.115009	14.70	<.0001*
(Pow_Blue-9.06484)*(Stab_Red-5.07463)	-0.023584	0.015212	-1.55	0.1211
(Pow_Blue-9.06484)*(Stab_Blue-6.13897)	0.0249953	0.019814	1.26	0.2071
(Pow_Blue-9.06484)*(Stre_Red-0.92333)	0.3776552	0.588507	0.64	0.5211
(Pow_Blue-9.06484)*(Stre_Blue-0.99106)	10.497887	0.674826	15.56	<.0001*
(Rob_Red-6.2396)*(Rob_Blue-6.95993)	-4.484521	0.070304	-63.79	<.0001*
(Rob_Red-6.2396)*(Stab_Red-5.07463)	-0.355475	0.018409	-19.31	<.0001*
(Rob_Red-6.2396)*(Stab_Blue-6.13897)	-0.01997	0.013693	-1.46	0.1447
(Rob_Red-6.2396)*(Stre_Red-0.92333)	10.350104	0.616598	16.79	<.0001*
(Rob_Red-6.2396)*(Stre_Blue-0.99106)	-3.633275	0.397968	-9.13	<.0001*
(Rob_Blue-6.95993)*(Stab_Red-5.07463)	0.1051439	0.014238	7.38	<.0001*
(Rob_Blue-6.95993)*(Stab_Blue-6.13897)	0.3321768	0.01983	16.75	<.0001*
(Rob_Blue-6.95993)*(Stre_Red-0.92333)	-0.483976	0.537744	-0.90	0.3681
(Rob_Blue-6.95993)*(Stre_Blue-0.99106)	1.4527134	0.570436	2.55	0.0109*
(Stab_Red-5.07463)*(Stab_Blue-6.13897)	-0.000819	0.00276	-0.30	0.7666
(Stab_Red-5.07463)*(Stre_Red-0.92333)	3.0801015	0.136776	22.52	<.0001*
(Stab_Red-5.07463)*(Stre_Blue-0.99106)	0.0233139	0.080308	0.29	0.7716
(Stab_Blue-6.13897)*(Stre_Red-0.92333)	0.1668256	0.107357	1.55	0.1202
(Stab_Blue-6.13897)*(Stre_Blue-0.99106)	-2.242344	0.133503	-16.80	<.0001*
(Stre_Red-0.92333)*(Stre_Blue-0.99106)	-10.5634	3.036763	-3.48	0.0005*
(Conn_Red-5.40938)*(Conn_Red-5.40938)	0.7109352	0.007242	98.17	<.0001*
(Conn_Blue-5.59815)*(Conn_Blue-5.59815)	0.3456967	0.008775	39.40	<.0001*
(Disp_Red-6.28209)*(Disp_Red-6.28209)	0.0146155	0.000409	35.76	<.0001*
(Disp_Blue-5.93948)*(Disp_Blue-5.93948)	-0.015004	0.000416	-36.07	<.0001*
(Pow_Red-8.25877)*(Pow_Red-8.25877)	2.4590635	0.079728	30.84	<.0001*
(Pow_Blue-9.06484)*(Pow_Blue-9.06484)	-1.194085	0.075769	-15.76	<.0001*
(Rob_Red-6.2396)*(Rob_Red-6.2396)	3.0915075	0.052618	58.75	<.0001*
(Rob_Blue-6.95993)*(Rob_Blue-6.95993)	1.1241966	0.059996	18.74	<.0001*
(Stab_Red-5.07463)*(Stab_Red-5.07463)	0.0670464	0.002291	29.26	<.0001*
(Stab_Blue-6.13897)*(Stab_Blue-6.13897)	-0.047931	0.002471	-19.40	<.0001*
(Stre_Red-0.92333)*(Stre_Red-0.92333)	62.86346	2.737686	22.96	<.0001*
(Stre_Blue-0.99106)*(Stre_Blue-0.99106)	-24.55154	2.159308	-11.37	<.0001*

Non-Linear Model with Metrics (2-way Interactions and Quadratic terms) minus Insignificant Terms

Summary of Fit

RSquare	0.990682
RSquare Adj	0.990681
Root Mean Square Error	1.56557
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	67	379853038	5669448	2313106
Error	1.5e+6	3572920	2.451011	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3493850.7	2.45666	1.1042
Pure Error	35540	79068.8	2.22478	Prob > F
Total Error	1.5e+6	3572919.5		<.0001*
				Max RSq
				0.9998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	43.994085	0.042492	1035.3	<.0001*
Conn_Red	8.3494802	0.005643	1479.5	<.0001*
Conn_Blue	-8.156343	0.006024	-1354	<.0001*
Disp_Red	0.1488632	0.001111	134.00	<.0001*
Disp_Blue	-0.127904	0.001147	-111.6	<.0001*
Pow_Red	-0.608373	0.018732	-32.48	<.0001*
Pow_Blue	0.4803467	0.01755	27.37	<.0001*
Rob_Red	16.774446	0.015576	1077.0	<.0001*
Rob_Blue	-16.37873	0.015948	-1027	<.0001*
Stab_Red	0.134003	0.002718	49.31	<.0001*
Stab_Blue	-0.068002	0.002526	-26.92	<.0001*
Stre_Red	16.860143	0.093201	180.90	<.0001*
Stre_Blue	-15.46885	0.075994	-203.6	<.0001*
(Conn_Red-5.40938)*(Conn_Blue-5.59815)	-1.129048	0.00233	-484.5	<.0001*
(Conn_Red-5.40938)*(Disp_Red-6.28209)	-0.129443	0.002716	-47.66	<.0001*
(Conn_Red-5.40938)*(Disp_Blue-5.93948)	-0.01739	0.000392	-44.41	<.0001*
(Conn_Red-5.40938)*(Pow_Red-8.25877)	-1.055863	0.036402	-29.01	<.0001*
(Conn_Red-5.40938)*(Pow_Blue-9.06484)	0.103461	0.010623	9.74	<.0001*
(Conn_Red-5.40938)*(Rob_Red-6.2396)	2.9819314	0.037814	78.86	<.0001*
(Conn_Red-5.40938)*(Rob_Blue-6.95993)	-2.274801	0.00473	-480.9	<.0001*
(Conn_Red-5.40938)*(Stab_Red-5.07463)	-0.152005	0.005528	-27.50	<.0001*
(Conn_Red-5.40938)*(Stre_Red-0.92333)	4.588683	0.162762	28.19	<.0001*
(Conn_Red-5.40938)*(Stre_Blue-0.99106)	-1.419584	0.061523	-23.07	<.0001*
(Conn_Blue-5.59815)*(Disp_Blue-5.93948)	0.1363065	0.002959	46.06	<.0001*
(Conn_Blue-5.59815)*(Pow_Red-8.25877)	-0.051045	0.003985	-12.81	<.0001*
(Conn_Blue-5.59815)*(Pow_Blue-9.06484)	0.8432741	0.034608	24.37	<.0001*
(Conn_Blue-5.59815)*(Rob_Red-6.2396)	-2.341794	0.005184	-451.7	<.0001*
(Conn_Blue-5.59815)*(Rob_Blue-6.95993)	1.3126379	0.04012	32.72	<.0001*
(Conn_Blue-5.59815)*(Stab_Red-5.07463)	0.0498686	0.001725	28.91	<.0001*
(Conn_Blue-5.59815)*(Stab_Blue-6.13897)	0.119806	0.00362	33.10	<.0001*
(Disp_Red-6.28209)*(Pow_Red-8.25877)	0.2891276	0.006613	43.72	<.0001*
(Disp_Red-6.28209)*(Rob_Red-6.2396)	-0.301483	0.007123	-42.32	<.0001*
(Disp_Red-6.28209)*(Stab_Red-5.07463)	0.0398945	0.000936	42.61	<.0001*
(Disp_Red-6.28209)*(Stre_Blue-0.99106)	-0.04251	0.008368	-5.08	<.0001*
(Disp_Blue-5.93948)*(Pow_Blue-9.06484)	-0.211906	0.007319	-28.95	<.0001*

Term	Estimate	Std Error	t Ratio	Prob> t
(Disp_Blue-5.93948)*(Rob_Red-6.2396)	-0.035218	0.000756	-46.58	<.0001*
(Disp_Blue-5.93948)*(Rob_Blue-6.95993)	0.3027231	0.007415	40.82	<.0001*
(Disp_Blue-5.93948)*(Stab_Blue-6.13897)	-0.032993	0.001234	-26.73	<.0001*
(Disp_Blue-5.93948)*(Stre_Blue-0.99106)	-0.285358	0.031114	-9.17	<.0001*
(Pow_Red-8.25877)*(Pow_Blue-9.06484)	-0.452502	0.028716	-15.76	<.0001*
(Pow_Red-8.25877)*(Rob_Red-6.2396)	-2.430333	0.101089	-24.04	<.0001*
(Pow_Red-8.25877)*(Stab_Red-5.07463)	-0.126061	0.018671	-6.75	<.0001*
(Pow_Red-8.25877)*(Stre_Red-0.92333)	-23.70782	0.756425	-31.34	<.0001*
(Pow_Red-8.25877)*(Stre_Blue-0.99106)	1.7395036	0.20513	8.48	<.0001*
(Pow_Blue-9.06484)*(Rob_Red-6.2396)	0.3856924	0.031417	12.28	<.0001*
(Pow_Blue-9.06484)*(Rob_Blue-6.95993)	1.8041047	0.095309	18.93	<.0001*
(Pow_Blue-9.06484)*(Stre_Blue-0.99106)	10.188595	0.33504	30.41	<.0001*
(Rob_Red-6.2396)*(Rob_Blue-6.95993)	-4.763405	0.008714	-546.7	<.0001*
(Rob_Red-6.2396)*(Stab_Red-5.07463)	-0.343639	0.015996	-21.48	<.0001*
(Rob_Red-6.2396)*(Stre_Red-0.92333)	10.854235	0.48152	22.54	<.0001*
(Rob_Red-6.2396)*(Stre_Blue-0.99106)	-3.067409	0.188537	-16.27	<.0001*
(Rob_Blue-6.95993)*(Stab_Red-5.07463)	0.0913649	0.003677	24.85	<.0001*
(Rob_Blue-6.95993)*(Stab_Blue-6.13897)	0.2910897	0.009109	31.96	<.0001*
(Stab_Red-5.07463)*(Stre_Red-0.92333)	2.9661504	0.108499	27.34	<.0001*
(Stab_Blue-6.13897)*(Stre_Blue-0.99106)	-1.977443	0.069542	-28.44	<.0001*
(Stre_Red-0.92333)*(Stre_Blue-0.99106)	-9.257045	0.659028	-14.05	<.0001*
(Conn_Red-5.40938)*(Conn_Red-5.40938)	0.7105084	0.007011	101.35	<.0001*
(Conn_Blue-5.59815)*(Conn_Blue-5.59815)	0.3620063	0.007599	47.64	<.0001*
(Disp_Red-6.28209)*(Disp_Red-6.28209)	0.0143599	0.000347	41.42	<.0001*
(Disp_Blue-5.93948)*(Disp_Blue-5.93948)	-0.014621	0.000382	-38.24	<.0001*
(Pow_Red-8.25877)*(Pow_Red-8.25877)	2.4123375	0.075759	31.84	<.0001*
(Pow_Blue-9.06484)*(Pow_Blue-9.06484)	-1.201924	0.05586	-21.52	<.0001*
(Rob_Red-6.2396)*(Rob_Red-6.2396)	3.0931046	0.051834	59.67	<.0001*
(Rob_Blue-6.95993)*(Rob_Blue-6.95993)	1.1895097	0.054202	21.95	<.0001*
(Stab_Red-5.07463)*(Stab_Red-5.07463)	0.06496	0.001909	34.03	<.0001*
(Stab_Blue-6.13897)*(Stab_Blue-6.13897)	-0.045284	0.001752	-25.85	<.0001*
(Stre_Red-0.92333)*(Stre_Red-0.92333)	56.942345	2.404866	23.68	<.0001*
(Stre_Blue-0.99106)*(Stre_Blue-0.99106)	-20.97396	1.404039	-14.94	<.0001*

Non-Linear Model with Metrics + SDI (2-way Interactions and Quadratic terms)

Singularity Details

Conn_Red = - 2*Rob_Red + 2*Red S,I

Conn_Blue = - 2*Rob_Blue - 2.04e-6*Stre_Red + 0.00077*Red S,I + 2*Blue S,I = - 2*Rob_Blue - 2.04e-6*Stre_Red + 0.00077*Red S,I + 0.00099*Blue S,I + 2*Blue D

Red S,I = - 1.94213*Blue S,I - 1.94213*Blue D + 11772.1*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 4356.89*(Conn_Red-5.40938)*(Disp_Red-6.28209) + 26382.1*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 16633*(Conn_Red-5.40938)*(Rob_Blue-6.95993) - 124666*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 16633*(Conn_Red-5.40938)*(Blue S,I-9.759) = - 1.94213*Blue S,I - 1.94213*Blue D + 11772.1*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 4356.89*(Conn_Red-5.40938)*(Disp_Red-6.28209) + 26382.1*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 16633*(Conn_Red-5.40938)*(Rob_Blue-6.95993) - 124666*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 5.85337*(Conn_Red-5.40938)*(Blue S,I-9.759) - 16633*(Conn_Red-5.40938)*(Blue D-9.759) = - 0.88082*Blue S,I - 0.88082*Blue D + 476.972*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 441.642*(Conn_Red-5.40938)*(Disp_Red-6.28209) + 1884.08*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 11678.3*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 0.37191*(Conn_Red-5.40938)*(Blue S,I-9.759) - 0.37191*(Conn_Red-5.40938)*(Blue D-9.759) - 90.6219*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - 6458.82*(Conn_Blue-5.59815)*(Disp_Blue-5.93948) - 429.957*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 1325.71*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 1121.25*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) - 953.944*(Conn_Blue-5.59815)*(Red S,I-8.94429) + 883.283*(Disp_Red-6.28209)*(Rob_Red-6.2396) - 181.244*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 883.283*(Disp_Red-6.28209)*(Red S,I-8.94429) + 181.244*(Disp_Red-6.28209)*(Blue S,I-9.759) - 12917.6*(Disp_Blue-5.93948)*(Rob_Blue-6.95993) - 0.33837*(Disp_Blue-5.93948)*(Red S,I-8.94429) + 12917.6*(Disp_Blue-5.93948)*(Blue S,I-9.759) = - 0.88082*Blue S,I - 0.88082*Blue D + 476.972*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 441.642*(Conn_Red-5.40938)*(Disp_Red-6.28209) + 1884.08*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 11678.3*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 0.37191*(Conn_Red-5.40938)*(Blue S,I-9.759) - 0.37191*(Conn_Red-5.40938)*(Blue D-9.759) - 90.6219*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - 6458.82*(Conn_Blue-5.59815)*(Disp_Blue-5.93948) - 429.957*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 1325.71*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 1121.25*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) - 953.944*(Conn_Blue-5.59815)*(Red S,I-8.94429) + 883.283*(Disp_Red-6.28209)*(Rob_Red-6.2396) - 181.244*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 883.283*(Disp_Red-6.28209)*(Red S,I-8.94429) + 181.244*(Disp_Red-6.28209)*(Blue S,I-9.759) - 12917.6*(Disp_Blue-5.93948)*(Rob_Blue-6.95993) - 0.33837*(Disp_Blue-5.93948)*(Red S,I-8.94429) + 2.2331*(Disp_Blue-5.93948)*(Blue S,I-9.759) + 12917.6*(Disp_Blue-5.93948)*(Blue D-9.759) = 0.65672*Blue S,I + 0.65672*Blue D - 506.452*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 11616.9*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 2709.09*(Conn_Red-5.40938)*(Pow_Red-8.25877) - 2107.32*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 0.07599*(Conn_Red-5.40938)*(Stre_Red-0.92333) - 8346.61*(Conn_Red-5.40938)*(Stre_Blue-0.99106) + 0.34165*(Conn_Red-5.40938)*(Blue S,I-9.759) + 0.34165*(Conn_Red-5.40938)*(Blue D-9.759) + 640.529*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 633.359*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + 1474.94*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 437.748*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 1012.9*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 23233.7*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 1281.06*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 0.04052*(Disp_Red-6.28209)*(Stre_Red-0.92333) + 23233.7*(Disp_Red-6.28209)*(Red S,I-8.94429) - 1281.06*(Disp_Red-6.28209)*(Blue S,I-9.759) - 0.04272*(Disp_Blue-5.93948)*(Blue S,I-9.759) - 0.04272*(Disp_Blue-5.93948)*(Blue D-9.759) - 5418.18*(Pow_Red-8.25877)*(Rob_Red-6.2396) + 1266.72*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 1.00091*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 5418.15*(Pow_Red-8.25877)*(Red S,I-8.94429) = - 0.8867*Blue S,I - 0.8867*Blue D - 353.778*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 1476.14*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 219.088*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 34126.9*(Conn_Red-5.40938)*(Stre_Blue-0.99106) + 0.20501*(Conn_Red-5.40938)*(Blue S,I-9.759) + 0.20501*(Conn_Red-5.40938)*(Blue D-9.759) + 252.617*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - 1022.06*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 20175.6*(Conn_Blue-5.59815)*(Pow_Blue-9.06484) + 1036.13*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 2066.76*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 707.556*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 2952.27*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 505.235*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 2952.27*(Disp_Red-6.28209)*(Red S,I-8.94429) - 505.235*(Disp_Red-6.28209)*(Blue S,I-9.759) + 0.83562*(Disp_Blue-5.93948)*(Blue D-9.759) - 2044.13*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 0.11023*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 0.36614*(Pow_Red-8.25877)*(Red S,I-8.94429) + 2044.13*(Pow_Red-8.25877)*(Blue S,I-9.759) - 438.177*(Pow_Blue-

9.06484)*(Rob_Red-6.2396) - 40351.2*(Pow_Blue-9.06484)*(Rob_Blue-6.95993) + 438.177*(Pow_Blue-9.06484)*(Red S,I-8.94429) + 40351.2*(Pow_Blue-9.06484)*(Blue S,I-9.759) = - 0.8867*Blue S,I - 0.8867*Blue D - 353.778*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 1476.14*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 219.088*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 34126.9*(Conn_Red-5.40938)*(Stre_Blue-0.99106) + 0.20501*(Conn_Red-5.40938)*(Blue S,I-9.759) + 0.20501*(Conn_Red-5.40938)*(Blue D-9.759) + 252.617*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - 1022.06*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 20175.6*(Conn_Blue-5.59815)*(Pow_Blue-9.06484) + 1036.13*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 2066.76*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 707.556*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 2952.27*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 505.235*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 2952.27*(Disp_Red-6.28209)*(Red S,I-8.94429) - 505.235*(Disp_Red-6.28209)*(Blue S,I-9.759) + 0.83562*(Disp_Blue-5.93948)*(Blue S,I-9.759) + 0.83562*(Disp_Blue-5.93948)*(Blue D-9.759) - 2044.13*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 0.11023*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 0.36614*(Pow_Red-8.25877)*(Red S,I-8.94429) + 2044.13*(Pow_Red-8.25877)*(Blue S,I-9.759) - 438.177*(Pow_Blue-9.06484)*(Rob_Red-6.2396) - 40351.2*(Pow_Blue-9.06484)*(Rob_Blue-6.95993) + 438.177*(Pow_Blue-9.06484)*(Red S,I-8.94429) + 0.5051*(Pow_Blue-9.06484)*(Blue S,I-9.759) + 40351.2*(Pow_Blue-9.06484)*(Blue D-9.759) = - 2.43421*Blue S,I - 2.43421*Blue D + 11661.7*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 11857.7*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 13518.7*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 32104.8*(Conn_Red-5.40938)*(Rob_Blue-6.95993) + 135999*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 2.79894*(Conn_Red-5.40938)*(Blue S,I-9.759) - 2.79894*(Conn_Red-5.40938)*(Blue D-9.759) - 3397.24*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 4421.74*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 21612*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 1480.58*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) - 23323.5*(Conn_Blue-5.59815)*(Red S,I-8.94429) + 23715.4*(Disp_Red-6.28209)*(Rob_Red-6.2396) - 6794.47*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 23715.4*(Disp_Red-6.28209)*(Red S,I-8.94429) + 6794.47*(Disp_Red-6.28209)*(Blue S,I-9.759) + 3.06204*(Disp_Blue-5.93948)*(Red S,I-8.94429) + 0.65538*(Disp_Blue-5.93948)*(Blue S,I-9.759) + 0.65538*(Disp_Blue-5.93948)*(Blue D-9.759) + 8843.48*(Pow_Red-8.25877)*(Rob_Blue-6.95993) + 0.5621*(Pow_Red-8.25877)*(Stre_Red-0.92333) - 2.77156*(Pow_Red-8.25877)*(Red S,I-8.94429) - 8843.47*(Pow_Red-8.25877)*(Blue S,I-9.759) - 27037.5*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + 27037.5*(Pow_Blue-9.06484)*(Red S,I-8.94429) - 0.39689*(Pow_Blue-9.06484)*(Blue S,I-9.759) - 0.39689*(Pow_Blue-9.06484)*(Blue D-9.759) - 25661.2*(Rob_Red-6.2396)*(Rob_Blue-6.95993) + 271998*(Rob_Red-6.2396)*(Stre_Blue-0.99106) + 89870.9*(Rob_Red-6.2396)*(Blue S,I-9.759) - 64209.7*(Rob_Blue-6.95993)*(Red S,I-8.94429) = 0.7441*Blue S,I + 0.7441*Blue D - 553.643*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 11687.1*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 2191.64*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 7725.7*(Conn_Red-5.40938)*(Stab_Red-5.07463) + 0.05743*(Conn_Red-5.40938)*(Stre_Red-0.92333) - 1137.75*(Conn_Red-5.40938)*(Stre_Blue-0.99106) + 0.38871*(Conn_Red-5.40938)*(Blue S,I-9.759) + 0.38871*(Conn_Red-5.40938)*(Blue D-9.759) + 607.55*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 506.232*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + 1483.75*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 413.55*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 1107.29*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 23374.1*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 1215.1*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 0.03757*(Disp_Red-6.28209)*(Stre_Red-0.92333) + 23374.1*(Disp_Red-6.28209)*(Red S,I-8.94429) - 1215.1*(Disp_Red-6.28209)*(Blue S,I-9.759) - 0.04242*(Disp_Blue-5.93948)*(Red S,I-8.94429) - 0.03492*(Disp_Blue-5.93948)*(Blue S,I-9.759) - 0.03492*(Disp_Blue-5.93948)*(Blue D-9.759) + 1012.46*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 0.88503*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 3.30772*(Pow_Red-8.25877)*(Red S,I-8.94429) - 0.0361*(Pow_Red-8.25877)*(Red D-3.69742) - 1012.46*(Pow_Red-8.25877)*(Blue S,I-9.759) - 4383.27*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + 4383.27*(Pow_Blue-9.06484)*(Red S,I-8.94429) + 0.04806*(Pow_Blue-9.06484)*(Blue S,I-9.759) + 0.04806*(Pow_Blue-9.06484)*(Blue D-9.759) + 5182.07*(Rob_Red-6.2396)*(Rob_Blue-6.95993) + 15451.4*(Rob_Red-6.2396)*(Stab_Red-5.07463) + 0.13513*(Rob_Red-6.2396)*(Stre_Red-0.92333) - 2275.5*(Rob_Red-6.2396)*(Stre_Blue-0.99106) - 5182.07*(Rob_Red-6.2396)*(Blue S,I-9.759) + 0.27007*(Rob_Blue-6.95993)*(Red S,I-8.94429) - 827.099*(Rob_Blue-6.95993)*(Blue S,I-9.759) - 15451.4*(Stab_Red-5.07463)*(Red S,I-8.94429) = - 1986.28*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 10436.5*(Conn_Red-5.40938)*(Disp_Red-6.28209) + 26395*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 57494.3*(Conn_Red-5.40938)*(Stab_Blue-6.13897) - 339756*(Conn_Red-5.40938)*(Stre_Blue-0.99106) - 1.52302*(Conn_Red-5.40938)*(Blue S,I-9.759) - 1.52302*(Conn_Red-5.40938)*(Blue D-9.759) + 1725.1*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - 3772.27*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 4406.13*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + 1217.1*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 3972.57*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 20873*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 3450.21*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 20873*(Disp_Red-6.28209)*(Red S,I-8.94429) - 3450.21*(Disp_Red-6.28209)*(Blue S,I-9.759) - 7544.54*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 0.68473*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 3.39974*(Pow_Red-8.25877)*(Red S,I-8.94429) + 7544.54*(Pow_Red-8.25877)*(Blue S,I-9.759) + 52790.1*(Pow_Blue-9.06484)*(Rob_Red-6.2396) -

$$\begin{aligned}
& 52790.1 * (\text{Pow_Blue}-9.06484) * (\text{Red S,I}-8.94429) - 867.115 * (\text{Rob_Red}-6.2396) * (\text{Rob_Blue}-6.95993) + \\
& 114989 * (\text{Rob_Red}-6.2396) * (\text{Stab_Blue}-6.13897) - 679512 * (\text{Rob_Red}-6.2396) * (\text{Stre_Blue}-0.99106) + \\
& 867.115 * (\text{Rob_Red}-6.2396) * (\text{Blue S,I}-9.759) + 1.18397 * (\text{Rob_Blue}-6.95993) * (\text{Red S,I}-8.94429) - \\
& 2434.2 * (\text{Rob_Blue}-6.95993) * (\text{Blue S,I}-9.759) - 1.05619 * (\text{Stab_Red}-5.07463) * (\text{Red S,I}-8.94429) - \\
& 114989 * (\text{Stab_Blue}-6.13897) * (\text{Red S,I}-8.94429) = 0.53817 * \text{Blue S,I} + 0.53817 * \text{Blue D} - \\
& 344.033 * (\text{Conn_Red}-5.40938) * (\text{Conn_Blue}-5.59815) - 19469.5 * (\text{Conn_Red}-5.40938) * (\text{Disp_Red}-6.28209) - \\
& 1375.36 * (\text{Conn_Red}-5.40938) * (\text{Pow_Blue}-9.06484) + 0.10901 * (\text{Conn_Red}-5.40938) * (\text{Stre_Red}-0.92333) + \\
& 6291 * (\text{Conn_Red}-5.40938) * (\text{Stre_Blue}-0.99106) - 46000.7 * (\text{Conn_Red}-5.40938) * (\text{Red D}-3.69742) + \\
& 0.24098 * (\text{Conn_Red}-5.40938) * (\text{Blue S,I}-9.759) + 0.24098 * (\text{Conn_Red}-5.40938) * (\text{Blue D}-9.759) + \\
& 1222.32 * (\text{Conn_Blue}-5.59815) * (\text{Disp_Red}-6.28209) + 517.915 * (\text{Conn_Blue}-5.59815) * (\text{Pow_Red}-8.25877) + \\
& 715.839 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Red}-6.2396) + 1084.43 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Blue}-6.95993) + \\
& 688.066 * (\text{Conn_Blue}-5.59815) * (\text{Red S,I}-8.94429) - 38939 * (\text{Disp_Red}-6.28209) * (\text{Rob_Red}-6.2396) + \\
& 2444.64 * (\text{Disp_Red}-6.28209) * (\text{Rob_Blue}-6.95993) + 38939 * (\text{Disp_Red}-6.28209) * (\text{Red S,I}-8.94429) - \\
& 2444.64 * (\text{Disp_Red}-6.28209) * (\text{Blue S,I}-9.759) - 0.33182 * (\text{Disp_Blue}-5.93948) * (\text{Red S,I}-8.94429) + \\
& 1035.83 * (\text{Pow_Red}-8.25877) * (\text{Rob_Blue}-6.95993) - 1.07268 * (\text{Pow_Red}-8.25877) * (\text{Stre_Red}-0.92333) + \\
& 5.86796 * (\text{Pow_Red}-8.25877) * (\text{Red S,I}-8.94429) - 1035.83 * (\text{Pow_Red}-8.25877) * (\text{Blue S,I}-9.759) - \\
& 2750.71 * (\text{Pow_Blue}-9.06484) * (\text{Rob_Red}-6.2396) + 2750.71 * (\text{Pow_Blue}-9.06484) * (\text{Red S,I}-8.94429) + \\
& 0.16219 * (\text{Pow_Blue}-9.06484) * (\text{Blue S,I}-9.759) + 0.16219 * (\text{Pow_Blue}-9.06484) * (\text{Blue D}-9.759) + \\
& 2807.81 * (\text{Rob_Red}-6.2396) * (\text{Rob_Blue}-6.95993) + 0.2698 * (\text{Rob_Red}-6.2396) * (\text{Stre_Red}-0.92333) + \\
& 12582 * (\text{Rob_Red}-6.2396) * (\text{Stre_Blue}-0.99106) - 92001.5 * (\text{Rob_Red}-6.2396) * (\text{Red D}-3.69742) - \\
& 2807.81 * (\text{Rob_Red}-6.2396) * (\text{Blue S,I}-9.759) + 0.25297 * (\text{Rob_Blue}-6.95993) * (\text{Red S,I}-8.94429) - \\
& 2168.87 * (\text{Rob_Blue}-6.95993) * (\text{Blue S,I}-9.759) - 2.19801 * (\text{Stab_Red}-5.07463) * (\text{Red S,I}-8.94429) - \\
& 0.19839 * (\text{Stab_Blue}-6.13897) * (\text{Red S,I}-8.94429) + 0.2524 * (\text{Stre_Red}-0.92333) * (\text{Red D}-3.69742) - \\
& 12582 * (\text{Stre_Blue}-0.99106) * (\text{Red S,I}-8.94429) + 92001.5 * (\text{Red S,I}-8.94429) * (\text{Red D}-3.69742) = - \\
& 10.6669 * \text{Blue S,I} - 10.6669 * \text{Blue D} + 28144.1 * (\text{Conn_Red}-5.40938) * (\text{Conn_Blue}-5.59815) + \\
& 72160.1 * (\text{Conn_Red}-5.40938) * (\text{Disp_Red}-6.28209) + 32207.8 * (\text{Conn_Red}-5.40938) * (\text{Pow_Blue}-9.06484) + \\
& 36146.7 * (\text{Conn_Red}-5.40938) * (\text{Rob_Blue}-6.95993) - 307726 * (\text{Conn_Red}-5.40938) * (\text{Stre_Blue}-0.99106) - \\
& 10.7517 * (\text{Conn_Red}-5.40938) * (\text{Blue S,I}-9.759) - 10.7517 * (\text{Conn_Red}-5.40938) * (\text{Blue D}-9.759) - \\
& 7892.44 * (\text{Conn_Blue}-5.59815) * (\text{Disp_Red}-6.28209) - 5319.25 * (\text{Conn_Blue}-5.59815) * (\text{Pow_Red}-8.25877) - \\
& 33566.5 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Red}-6.2396) - 1188.59 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Blue}-6.95993) - \\
& 20141.6 * (\text{Conn_Blue}-5.59815) * (\text{Red S,I}-8.94429) + 144320 * (\text{Disp_Red}-6.28209) * (\text{Rob_Red}-6.2396) - \\
& 15784.9 * (\text{Disp_Red}-6.28209) * (\text{Rob_Blue}-6.95993) - 144320 * (\text{Disp_Red}-6.28209) * (\text{Red S,I}-8.94429) + \\
& 15784.9 * (\text{Disp_Red}-6.28209) * (\text{Blue S,I}-9.759) + 2.26869 * (\text{Disp_Blue}-5.93948) * (\text{Red S,I}-8.94429) + \\
& 1.678 * (\text{Disp_Blue}-5.93948) * (\text{Blue S,I}-9.759) + 1.678 * (\text{Disp_Blue}-5.93948) * (\text{Blue D}-9.759) - \\
& 10638.5 * (\text{Pow_Red}-8.25877) * (\text{Rob_Blue}-6.95993) + 5.81693 * (\text{Pow_Red}-8.25877) * (\text{Stre_Red}-0.92333) - \\
& 19.8044 * (\text{Pow_Red}-8.25877) * (\text{Red S,I}-8.94429) + 10638.5 * (\text{Pow_Red}-8.25877) * (\text{Blue S,I}-9.759) + \\
& 64415.7 * (\text{Pow_Blue}-9.06484) * (\text{Rob_Red}-6.2396) - 64415.7 * (\text{Pow_Blue}-9.06484) * (\text{Red S,I}-8.94429) - \\
& 0.81695 * (\text{Pow_Blue}-9.06484) * (\text{Blue S,I}-9.759) - 0.81695 * (\text{Pow_Blue}-9.06484) * (\text{Blue D}-9.759) - \\
& 107416 * (\text{Rob_Red}-6.2396) * (\text{Rob_Blue}-6.95993) - 1.02762 * (\text{Rob_Red}-6.2396) * (\text{Stre_Red}-0.92333) - \\
& 615451 * (\text{Rob_Red}-6.2396) * (\text{Stre_Blue}-0.99106) + 179710 * (\text{Rob_Red}-6.2396) * (\text{Blue S,I}-9.759) - \\
& 5.47532 * (\text{Rob_Blue}-6.95993) * (\text{Red S,I}-8.94429) + 2377.18 * (\text{Rob_Blue}-6.95993) * (\text{Blue S,I}-9.759) + \\
& 6.01654 * (\text{Stab_Red}-5.07463) * (\text{Red S,I}-8.94429) + 0.87166 * (\text{Stab_Red}-5.07463) * (\text{Blue S,I}-9.759) + \\
& 0.87166 * (\text{Stab_Red}-5.07463) * (\text{Blue D}-9.759) + 1.23857 * (\text{Stab_Blue}-6.13897) * (\text{Red S,I}-8.94429) - \\
& 1.50472 * (\text{Stre_Red}-0.92333) * (\text{Red D}-3.69742) + 615451 * (\text{Stre_Blue}-0.99106) * (\text{Red S,I}-8.94429) - \\
& 1.95364 * (\text{Red S,I}-8.94429) * (\text{Red D}-3.69742) - 72293.4 * (\text{Red S,I}-8.94429) * (\text{Blue S,I}-9.759) = - \\
& 10.6669 * \text{Blue S,I} - 10.6669 * \text{Blue D} + 28144.1 * (\text{Conn_Red}-5.40938) * (\text{Conn_Blue}-5.59815) + \\
& 72160.1 * (\text{Conn_Red}-5.40938) * (\text{Disp_Red}-6.28209) + 32207.8 * (\text{Conn_Red}-5.40938) * (\text{Pow_Blue}-9.06484) + \\
& 36146.7 * (\text{Conn_Red}-5.40938) * (\text{Rob_Blue}-6.95993) - 307726 * (\text{Conn_Red}-5.40938) * (\text{Stre_Blue}-0.99106) - \\
& 10.7517 * (\text{Conn_Red}-5.40938) * (\text{Blue S,I}-9.759) - 10.7517 * (\text{Conn_Red}-5.40938) * (\text{Blue D}-9.759) - \\
& 7892.44 * (\text{Conn_Blue}-5.59815) * (\text{Disp_Red}-6.28209) - 5319.25 * (\text{Conn_Blue}-5.59815) * (\text{Pow_Red}-8.25877) - \\
& 33566.5 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Red}-6.2396) - 1188.59 * (\text{Conn_Blue}-5.59815) * (\text{Rob_Blue}-6.95993) - \\
& 20141.6 * (\text{Conn_Blue}-5.59815) * (\text{Red S,I}-8.94429) + 144320 * (\text{Disp_Red}-6.28209) * (\text{Rob_Red}-6.2396) - \\
& 15784.9 * (\text{Disp_Red}-6.28209) * (\text{Rob_Blue}-6.95993) - 144320 * (\text{Disp_Red}-6.28209) * (\text{Red S,I}-8.94429) + \\
& 15784.9 * (\text{Disp_Red}-6.28209) * (\text{Blue S,I}-9.759) + 2.26869 * (\text{Disp_Blue}-5.93948) * (\text{Red S,I}-8.94429) + \\
& 1.678 * (\text{Disp_Blue}-5.93948) * (\text{Blue S,I}-9.759) + 1.678 * (\text{Disp_Blue}-5.93948) * (\text{Blue D}-9.759) - \\
& 10638.5 * (\text{Pow_Red}-8.25877) * (\text{Rob_Blue}-6.95993) + 5.81693 * (\text{Pow_Red}-8.25877) * (\text{Stre_Red}-0.92333) - \\
& 19.8044 * (\text{Pow_Red}-8.25877) * (\text{Red S,I}-8.94429) + 10638.5 * (\text{Pow_Red}-8.25877) * (\text{Blue S,I}-9.759) + \\
& 64415.7 * (\text{Pow_Blue}-9.06484) * (\text{Rob_Red}-6.2396) - 64415.7 * (\text{Pow_Blue}-9.06484) * (\text{Red S,I}-8.94429) - \\
& 0.81695 * (\text{Pow_Blue}-9.06484) * (\text{Blue S,I}-9.759) - 0.81695 * (\text{Pow_Blue}-9.06484) * (\text{Blue D}-9.759) - \\
& 107416 * (\text{Rob_Red}-6.2396) * (\text{Rob_Blue}-6.95993) - 1.02762 * (\text{Rob_Red}-6.2396) * (\text{Stre_Red}-0.92333) - \\
& 615451 * (\text{Rob_Red}-6.2396) * (\text{Stre_Blue}-0.99106) + 179710 * (\text{Rob_Red}-6.2396) * (\text{Blue S,I}-9.759) - \\
& 5.47532 * (\text{Rob_Blue}-6.95993) * (\text{Red S,I}-8.94429) + 2377.18 * (\text{Rob_Blue}-6.95993) * (\text{Blue S,I}-9.759) +
\end{aligned}$$

$$\begin{aligned}
& 6.01654*(\text{Stab_Red-5.07463})*(\text{Red S,I-8.94429}) + 0.87166*(\text{Stab_Red-5.07463})*(\text{Blue S,I-9.759}) + \\
& 0.87166*(\text{Stab_Red-5.07463})*(\text{Blue D-9.759}) + 1.23857*(\text{Stab_Blue-6.13897})*(\text{Red S,I-8.94429}) - \\
& 1.50472*(\text{Stre_Red-0.92333})*(\text{Red D-3.69742}) + 615451*(\text{Stre_Blue-0.99106})*(\text{Red S,I-8.94429}) - \\
& 1.95364*(\text{Red S,I-8.94429})*(\text{Red D-3.69742}) - 11.6016*(\text{Red S,I-8.94429})*(\text{Blue S,I-9.759}) - 72293.4*(\text{Red} \\
& \text{S,I-8.94429})*(\text{Blue D-9.759}) = - 1.21428*\text{Blue S,I} - 1.21428*\text{Blue D} - 259.151*(\text{Conn_Red-} \\
& 5.40938)*(\text{Conn_Blue-5.59815}) - 747.619*(\text{Conn_Red-5.40938})*(\text{Disp_Red-6.28209}) - 1196.42*(\text{Conn_Red-} \\
& 5.40938)*(\text{Pow_Blue-9.06484}) - 17243.6*(\text{Conn_Red-5.40938})*(\text{Stre_Blue-0.99106}) + 0.10337*(\text{Conn_Red-} \\
& 5.40938)*(\text{Blue S,I-9.759}) + 0.10337*(\text{Conn_Red-5.40938})*(\text{Blue D-9.759}) + 1889.72*(\text{Conn_Blue-} \\
& 5.59815)*(\text{Disp_Red-6.28209}) + 224.679*(\text{Conn_Blue-5.59815})*(\text{Pow_Red-8.25877}) - 208.967*(\text{Conn_Blue-} \\
& 5.59815)*(\text{Rob_Red-6.2396}) + 1217.58*(\text{Conn_Blue-5.59815})*(\text{Rob_Blue-6.95993}) + 518.301*(\text{Conn_Blue-} \\
& 5.59815)*(\text{Red S,I-8.94429}) - 23352.5*(\text{Conn_Blue-5.59815})*(\text{Red D-3.69742}) - 1495.24*(\text{Disp_Red-} \\
& 6.28209)*(\text{Rob_Red-6.2396}) + 3779.43*(\text{Disp_Red-6.28209})*(\text{Rob_Blue-6.95993}) + 1495.24*(\text{Disp_Red-} \\
& 6.28209)*(\text{Red S,I-8.94429}) - 3779.43*(\text{Disp_Red-6.28209})*(\text{Blue S,I-9.759}) - 0.22897*(\text{Disp_Blue-} \\
& 5.93948)*(\text{Blue S,I-9.759}) - 0.22897*(\text{Disp_Blue-5.93948})*(\text{Blue D-9.759}) + 449.359*(\text{Pow_Red-} \\
& 8.25877)*(\text{Rob_Blue-6.95993}) + 0.18712*(\text{Pow_Red-8.25877})*(\text{Red S,I-8.94429}) - 449.358*(\text{Pow_Red-} \\
& 8.25877)*(\text{Blue S,I-9.759}) - 2392.84*(\text{Pow_Blue-9.06484})*(\text{Rob_Red-6.2396}) + 2392.84*(\text{Pow_Blue-} \\
& 9.06484)*(\text{Red S,I-8.94429}) + 0.07924*(\text{Pow_Blue-9.06484})*(\text{Blue S,I-9.759}) + 0.07924*(\text{Pow_Blue-} \\
& 9.06484)*(\text{Blue D-9.759}) + 618.668*(\text{Rob_Red-6.2396})*(\text{Rob_Blue-6.95993}) - 34487.1*(\text{Rob_Red-} \\
& 6.2396)*(\text{Stre_Blue-0.99106}) - 618.668*(\text{Rob_Red-6.2396})*(\text{Blue S,I-9.759}) + 0.07674*(\text{Rob_Blue-} \\
& 6.95993)*(\text{Red S,I-8.94429}) - 46705.1*(\text{Rob_Blue-6.95993})*(\text{Red D-3.69742}) - 2435.16*(\text{Rob_Blue-} \\
& 6.95993)*(\text{Blue S,I-9.759}) + 0.19669*(\text{Stab_Red-5.07463})*(\text{Blue S,I-9.759}) + 0.19669*(\text{Stab_Red-} \\
& 5.07463)*(\text{Blue D-9.759}) + 34487.1*(\text{Stre_Blue-0.99106})*(\text{Red S,I-8.94429}) - 0.07769*(\text{Red S,I-} \\
& 8.94429)*(\text{Red D-3.69742}) + 0.12267*(\text{Red S,I-8.94429})*(\text{Blue S,I-9.759}) + 0.12267*(\text{Red S,I-} \\
& 8.94429)*(\text{Blue D-9.759}) + 46705.1*(\text{Red D-3.69742})*(\text{Blue S,I-9.759}) = - 1.21428*\text{Blue S,I} - 1.21428*\text{Blue} \\
& \text{D} - 259.151*(\text{Conn_Red-5.40938})*(\text{Conn_Blue-5.59815}) - 747.619*(\text{Conn_Red-5.40938})*(\text{Disp_Red-} \\
& 6.28209) - 1196.42*(\text{Conn_Red-5.40938})*(\text{Pow_Blue-9.06484}) - 17243.6*(\text{Conn_Red-5.40938})*(\text{Stre_Blue-} \\
& 0.99106) + 0.10337*(\text{Conn_Red-5.40938})*(\text{Blue S,I-9.759}) + 0.10337*(\text{Conn_Red-5.40938})*(\text{Blue D-9.759}) \\
& + 1889.72*(\text{Conn_Blue-5.59815})*(\text{Disp_Red-6.28209}) + 224.679*(\text{Conn_Blue-5.59815})*(\text{Pow_Red-8.25877}) \\
& - 208.967*(\text{Conn_Blue-5.59815})*(\text{Rob_Red-6.2396}) + 1217.58*(\text{Conn_Blue-5.59815})*(\text{Rob_Blue-6.95993}) + \\
& 518.301*(\text{Conn_Blue-5.59815})*(\text{Red S,I-8.94429}) - 23352.5*(\text{Conn_Blue-5.59815})*(\text{Red D-3.69742}) - \\
& 1495.24*(\text{Disp_Red-6.28209})*(\text{Rob_Red-6.2396}) + 3779.43*(\text{Disp_Red-6.28209})*(\text{Rob_Blue-6.95993}) + \\
& 1495.24*(\text{Disp_Red-6.28209})*(\text{Red S,I-8.94429}) - 3779.43*(\text{Disp_Red-6.28209})*(\text{Blue S,I-9.759}) - \\
& 0.22897*(\text{Disp_Blue-5.93948})*(\text{Blue S,I-9.759}) - 0.22897*(\text{Disp_Blue-5.93948})*(\text{Blue D-9.759}) + \\
& 449.359*(\text{Pow_Red-8.25877})*(\text{Rob_Blue-6.95993}) + 0.18712*(\text{Pow_Red-8.25877})*(\text{Red S,I-8.94429}) - \\
& 449.358*(\text{Pow_Red-8.25877})*(\text{Blue S,I-9.759}) - 2392.84*(\text{Pow_Blue-9.06484})*(\text{Rob_Red-6.2396}) + \\
& 2392.84*(\text{Pow_Blue-9.06484})*(\text{Red S,I-8.94429}) + 0.07924*(\text{Pow_Blue-9.06484})*(\text{Blue S,I-9.759}) + \\
& 0.07924*(\text{Pow_Blue-9.06484})*(\text{Blue D-9.759}) + 618.668*(\text{Rob_Red-6.2396})*(\text{Rob_Blue-6.95993}) - \\
& 34487.1*(\text{Rob_Red-6.2396})*(\text{Stre_Blue-0.99106}) - 618.668*(\text{Rob_Red-6.2396})*(\text{Blue S,I-9.759}) + \\
& 0.07674*(\text{Rob_Blue-6.95993})*(\text{Red S,I-8.94429}) - 46705.1*(\text{Rob_Blue-6.95993})*(\text{Red D-3.69742}) - \\
& 2435.16*(\text{Rob_Blue-6.95993})*(\text{Blue S,I-9.759}) + 0.19669*(\text{Stab_Red-5.07463})*(\text{Blue S,I-9.759}) + \\
& 0.19669*(\text{Stab_Red-5.07463})*(\text{Blue D-9.759}) + 34487.1*(\text{Stre_Blue-0.99106})*(\text{Red S,I-8.94429}) - \\
& 0.07769*(\text{Red S,I-8.94429})*(\text{Red D-3.69742}) + 0.12267*(\text{Red S,I-8.94429})*(\text{Blue S,I-9.759}) + 0.12267*(\text{Red} \\
& \text{S,I-8.94429})*(\text{Blue D-9.759}) + 0.54474*(\text{Red D-3.69742})*(\text{Blue S,I-9.759}) + 46705.1*(\text{Red D-3.69742})*(\text{Blue} \\
& \text{D-9.759}) = - 1.43269*\text{Blue S,I} - 1.43269*\text{Blue D} - 1969.27*(\text{Conn_Red-5.40938})*(\text{Conn_Blue-5.59815}) - \\
& 1306.94*(\text{Conn_Red-5.40938})*(\text{Disp_Red-6.28209}) - 5075.57*(\text{Conn_Red-5.40938})*(\text{Pow_Blue-9.06484}) - \\
& 132972*(\text{Conn_Red-5.40938})*(\text{Stre_Blue-0.99106}) + 0.48612*(\text{Conn_Red-5.40938})*(\text{Blue S,I-9.759}) + \\
& 0.48612*(\text{Conn_Red-5.40938})*(\text{Blue D-9.759}) + 138.7*(\text{Conn_Blue-5.59815})*(\text{Disp_Red-6.28209}) + \\
& 5322.69*(\text{Conn_Blue-5.59815})*(\text{Pow_Red-8.25877}) + 3874.49*(\text{Conn_Blue-5.59815})*(\text{Rob_Red-6.2396}) - \\
& 1641.6*(\text{Conn_Blue-5.59815})*(\text{Rob_Blue-6.95993}) + 3938.53*(\text{Conn_Blue-5.59815})*(\text{Red S,I-8.94429}) - \\
& 14734.3*(\text{Conn_Blue-5.59815})*(\text{Blue S,I-9.759}) - 2613.88*(\text{Disp_Red-6.28209})*(\text{Rob_Red-6.2396}) + \\
& 277.4*(\text{Disp_Red-6.28209})*(\text{Rob_Blue-6.95993}) + 2613.87*(\text{Disp_Red-6.28209})*(\text{Red S,I-8.94429}) - \\
& 277.4*(\text{Disp_Red-6.28209})*(\text{Blue S,I-9.759}) - 0.31482*(\text{Disp_Blue-5.93948})*(\text{Red S,I-8.94429}) - \\
& 0.41507*(\text{Disp_Blue-5.93948})*(\text{Blue S,I-9.759}) - 0.41507*(\text{Disp_Blue-5.93948})*(\text{Blue D-9.759}) + \\
& 10645.4*(\text{Pow_Red-8.25877})*(\text{Rob_Blue-6.95993}) + 0.58088*(\text{Pow_Red-8.25877})*(\text{Red S,I-8.94429}) - \\
& 10645.4*(\text{Pow_Red-8.25877})*(\text{Blue S,I-9.759}) - 10151.1*(\text{Pow_Blue-9.06484})*(\text{Rob_Red-6.2396}) + \\
& 10151.1*(\text{Pow_Blue-9.06484})*(\text{Red S,I-8.94429}) - 0.38184*(\text{Pow_Blue-9.06484})*(\text{Blue S,I-9.759}) - \\
& 0.38184*(\text{Pow_Blue-9.06484})*(\text{Blue D-9.759}) + 15626*(\text{Rob_Red-6.2396})*(\text{Rob_Blue-6.95993}) - \\
& 265943*(\text{Rob_Red-6.2396})*(\text{Stre_Blue-0.99106}) - 15626*(\text{Rob_Red-6.2396})*(\text{Blue S,I-9.759}) + \\
& 0.7786*(\text{Rob_Blue-6.95993})*(\text{Red S,I-8.94429}) - 26185.3*(\text{Rob_Blue-6.95993})*(\text{Blue S,I-9.759}) - \\
& 0.29813*(\text{Stab_Blue-6.13897})*(\text{Red S,I-8.94429}) + 265943*(\text{Stre_Blue-0.99106})*(\text{Red S,I-8.94429}) + \\
& 0.2909*(\text{Red S,I-8.94429})*(\text{Blue S,I-9.759}) + 0.2909*(\text{Red S,I-8.94429})*(\text{Blue D-9.759}) + 29468.5*(\text{Blue S,I-} \\
& 9.759)*(\text{Blue D-9.759}) = - 1.76632*\text{Blue S,I} - 1.76632*\text{Blue D} + 388.407*(\text{Conn_Red-5.40938})*(\text{Conn_Blue-}
\end{aligned}$$

$$\begin{aligned}
& 5.59815) + 3150.94 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) + 2270.1 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484}) - 5783.97 * (\text{Conn_Red-5.40938}) * (\text{Rob_Red-6.2396}) - 24020.3 * (\text{Conn_Red-5.40938}) * (\text{Stre_Blue-0.99106}) + 5783.97 * (\text{Conn_Red-5.40938}) * (\text{Red S,I-8.94429}) + 5.69209 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) + 508.845 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) - 112.936 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) + 504.112 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) - 776.815 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) + 6301.88 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) + 11.3842 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) - 6301.88 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) - 11.3842 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) - 0.16116 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) - 0.20697 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) - 0.20697 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) + 1017.69 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) - 1.19037 * (\text{Pow_Red-8.25877}) * (\text{Red S,I-8.94429}) - 1017.69 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) + 4540.2 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) - 4540.2 * (\text{Pow_Blue-9.06484}) * (\text{Red S,I-8.94429}) + 0.05268 * (\text{Pow_Blue-9.06484}) * (\text{Blue S,I-9.759}) + 0.05268 * (\text{Pow_Blue-9.06484}) * (\text{Blue D-9.759}) - 1779.5 * (\text{Rob_Red-6.2396}) * (\text{Rob_Blue-6.95993}) - 48040.5 * (\text{Rob_Red-6.2396}) * (\text{Stre_Blue-0.99106}) + 1779.5 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) - 1008.22 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) + 0.36187 * (\text{Stab_Red-5.07463}) * (\text{Red S,I-8.94429}) + 48040.5 * (\text{Stre_Blue-0.99106}) * (\text{Red S,I-8.94429}) - 0.06197 * (\text{Red S,I-8.94429}) * (\text{Red D-3.69742}) - 0.95189 * (\text{Red S,I-8.94429}) * (\text{Blue S,I-9.759}) - 0.95189 * (\text{Red S,I-8.94429}) * (\text{Blue D-9.759}) - 0.11962 * (\text{Blue S,I-9.759}) * (\text{Blue D-9.759}) - 2891.98 * (\text{Conn_Red-5.40938}) * (\text{Conn_Red-5.40938}) = - 2.07844 * \text{Blue S,I} - 2.07844 * \text{Blue D} + 975.192 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815}) + 80.1604 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) + 3192.25 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484}) - 10662.8 * (\text{Conn_Red-5.40938}) * (\text{Stre_Blue-0.99106}) - 0.72818 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) - 0.72818 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) - 139.769 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) - 1560.16 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) - 1505.53 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) - 2313.52 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) - 1950.38 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) + 4483.93 * (\text{Conn_Blue-5.59815}) * (\text{Blue S,I-9.759}) + 160.321 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) - 279.537 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) - 160.321 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) + 279.537 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) + 0.02342 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) - 0.85499 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) - 0.85499 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) - 3120.31 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) + 0.04677 * (\text{Pow_Red-8.25877}) * (\text{Stre_Red-0.92333}) + 3120.31 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) + 6384.5 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) - 6384.5 * (\text{Pow_Blue-9.06484}) * (\text{Red S,I-8.94429}) - 0.44728 * (\text{Pow_Blue-9.06484}) * (\text{Blue S,I-9.759}) - 0.44728 * (\text{Pow_Blue-9.06484}) * (\text{Blue D-9.759}) - 6911.83 * (\text{Rob_Red-6.2396}) * (\text{Rob_Blue-6.95993}) - 21325.5 * (\text{Rob_Red-6.2396}) * (\text{Stre_Blue-0.99106}) + 6911.83 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) - 0.26893 * (\text{Rob_Blue-6.95993}) * (\text{Red S,I-8.94429}) - 4340.83 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) + 0.0267 * (\text{Stab_Red-5.07463}) * (\text{Blue S,I-9.759}) + 0.0267 * (\text{Stab_Red-5.07463}) * (\text{Blue D-9.759}) - 0.15909 * (\text{Stab_Blue-6.13897}) * (\text{Blue S,I-9.759}) - 0.15909 * (\text{Stab_Blue-6.13897}) * (\text{Blue D-9.759}) + 21325.5 * (\text{Stre_Blue-0.99106}) * (\text{Red S,I-8.94429}) + 0.03416 * (\text{Stre_Blue-0.99106}) * (\text{Blue S,I-9.759}) + 0.03416 * (\text{Stre_Blue-0.99106}) * (\text{Blue D-9.759}) + 0.03354 * (\text{Red S,I-8.94429}) * (\text{Red D-3.69742}) - 0.2439 * (\text{Red S,I-8.94429}) * (\text{Blue S,I-9.759}) - 0.2439 * (\text{Red S,I-8.94429}) * (\text{Blue D-9.759}) + 0.17115 * (\text{Red D-3.69742}) * (\text{Blue S,I-9.759}) + 0.17115 * (\text{Red D-3.69742}) * (\text{Blue D-9.759}) - 0.52821 * (\text{Blue S,I-9.759}) * (\text{Blue D-9.759}) + 0.46277 * (\text{Conn_Red-5.40938}) * (\text{Conn_Red-5.40938}) - 2241.97 * (\text{Conn_Blue-5.59815}) * (\text{Conn_Blue-5.59815}) = - 0.51676 * \text{Blue S,I} - 0.51676 * \text{Blue D} + 399.793 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815}) - 3070.98 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) + 791.677 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484}) - 1890.31 * (\text{Conn_Red-5.40938}) * (\text{Rob_Red-6.2396}) + 15538.5 * (\text{Conn_Red-5.40938}) * (\text{Stre_Blue-0.99106}) - 0.32944 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) - 0.32944 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) + 254.508 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) - 1127.98 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) - 1113.15 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) + 504.658 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) - 799.586 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) - 6141.95 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) + 509.016 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) + 6141.95 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) - 509.016 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) + 0.03617 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) + 0.08123 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) + 0.08123 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) - 2255.96 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) - 0.24394 * (\text{Pow_Red-8.25877}) * (\text{Stre_Red-0.92333}) + 0.99787 * (\text{Pow_Red-8.25877}) * (\text{Red S,I-8.94429}) + 2255.96 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) + 1583.35 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) - 1583.35 * (\text{Pow_Blue-9.06484}) * (\text{Red S,I-8.94429}) - 3825.47 * (\text{Rob_Red-6.2396}) * (\text{Rob_Blue-6.95993}) + 0.04614 * (\text{Rob_Red-6.2396}) * (\text{Stre_Red-0.92333}) + 31077 * (\text{Rob_Red-6.2396}) * (\text{Stre_Blue-0.99106}) + 3780.62 * (\text{Rob_Red-6.2396}) * (\text{Red S,I-8.94429}) + 3825.47 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) - 0.1628 * (\text{Rob_Blue-6.95993}) * (\text{Red S,I-8.94429}) - 1009.32 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) - 0.37368 * (\text{Stab_Red-5.07463}) * (\text{Red S,I-8.94429}) + 0.0625 * (\text{Stre_Red-0.92333}) * (\text{Red D-3.69742}) - 31077 * (\text{Stre_Blue-0.99106}) * (\text{Red S,I-8.94429}) + 0.15689 * (\text{Red S,I-8.94429}) * (\text{Red D-3.69742}) - 0.09354 * (\text{Blue S,I-9.759}) * (\text{Blue D-9.759}) - 8.84891 * (\text{Conn_Red-5.40938}) * (\text{Conn_Red-5.40938}) + 0.14818 * (\text{Conn_Blue-5.59815}) * (\text{Conn_Blue-5.59815}) - 3780.62 * (\text{Rob_Red-6.2396}) * (\text{Rob_Red-6.2396}) = 2.87831 * \text{Blue S,I} + 2.87831 * \text{Blue D} - 1381.78 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815}) -
\end{aligned}$$

22460.4*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 5053.35*(Conn_Red-5.40938)*(Pow_Blue-9.06484) +
 0.14484*(Conn_Red-5.40938)*(Stre_Red-0.92333) - 8282.41*(Conn_Red-5.40938)*(Stre_Blue-0.99106) -
 1492.4*(Conn_Red-5.40938)*(Red S,I-8.94429) + 0.68189*(Conn_Red-5.40938)*(Blue S,I-9.759) +
 0.68189*(Conn_Red-5.40938)*(Blue D-9.759) + 855.928*(Conn_Blue-5.59815)*(Disp_Red-6.28209) +
 568.595*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + 3554.72*(Conn_Blue-5.59815)*(Rob_Red-6.2396) +
 930.536*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 2763.56*(Conn_Blue-5.59815)*(Red S,I-8.94429) -
 44920.8*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 1711.86*(Disp_Red-6.28209)*(Rob_Blue-6.95993) +
 0.0727*(Disp_Red-6.28209)*(Stre_Red-0.92333) + 44920.8*(Disp_Red-6.28209)*(Red S,I-8.94429) -
 1711.86*(Disp_Red-6.28209)*(Blue S,I-9.759) - 0.06454*(Disp_Blue-5.93948)*(Red S,I-8.94429) -
 0.06525*(Disp_Blue-5.93948)*(Blue S,I-9.759) - 0.06525*(Disp_Blue-5.93948)*(Blue D-9.759) -
 0.05055*(Pow_Red-8.25877)*(Rob_Red-6.2396) + 1137.19*(Pow_Red-8.25877)*(Rob_Blue-6.95993) -
 1.83707*(Pow_Red-8.25877)*(Stre_Red-0.92333) + 6.61492*(Pow_Red-8.25877)*(Red S,I-8.94429) -
 0.07523*(Pow_Red-8.25877)*(Red D-3.69742) - 1137.19*(Pow_Red-8.25877)*(Blue S,I-9.759) -
 10106.7*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + 10106.7*(Pow_Blue-9.06484)*(Red S,I-8.94429) +
 0.10815*(Pow_Blue-9.06484)*(Blue S,I-9.759) + 0.10815*(Pow_Blue-9.06484)*(Blue D-9.759) +
 12636.6*(Rob_Red-6.2396)*(Rob_Blue-6.95993) + 0.34714*(Rob_Red-6.2396)*(Stre_Red-0.92333) -
 16564.8*(Rob_Red-6.2396)*(Stre_Blue-0.99106) - 2984.81*(Rob_Red-6.2396)*(Red S,I-8.94429) -
 12636.6*(Rob_Red-6.2396)*(Blue S,I-9.759) + 0.55612*(Rob_Blue-6.95993)*(Red S,I-8.94429) -
 1861.07*(Rob_Blue-6.95993)*(Blue S,I-9.759) - 2.32438*(Stab_Red-5.07463)*(Red S,I-8.94429) -
 0.34131*(Stab_Blue-6.13897)*(Red S,I-8.94429) - 0.06036*(Stre_Red-0.92333)*(Stre_Blue-0.99106) +
 0.46698*(Stre_Red-0.92333)*(Red D-3.69742) + 16564.8*(Stre_Blue-0.99106)*(Red S,I-8.94429) +
 0.77281*(Red S,I-8.94429)*(Red D-3.69742) + 3.02576*(Red S,I-8.94429)*(Blue S,I-9.759) + 3.02576*(Red
 S,I-8.94429)*(Blue D-9.759) + 0.04653*(Red D-3.69742)*(Blue S,I-9.759) + 0.04653*(Red D-3.69742)*(Blue
 D-9.759) + 2.376*(Conn_Red-5.40938)*(Conn_Red-5.40938) - 0.26439*(Conn_Blue-5.59815)*(Conn_Blue-
 5.59815) + 0.14682*(Pow_Red-8.25877)*(Pow_Red-8.25877) - 3.13084*(Rob_Red-6.2396)*(Rob_Red-
 6.2396) + 1861.07*(Rob_Blue-6.95993)*(Rob_Blue-6.95993) + 5.62781*(Stre_Red-0.92333)*(Stre_Red-
 0.92333) + 2984.81*(Red S,I-8.94429)*(Red S,I-8.94429) = - 1.43269*Blue S,I - 1.43269*Blue D -
 1969.25*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - 1306.93*(Conn_Red-5.40938)*(Disp_Red-6.28209) -
 5075.51*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 132970*(Conn_Red-5.40938)*(Stre_Blue-0.99106) +
 0.48612*(Conn_Red-5.40938)*(Blue S,I-9.759) + 0.48612*(Conn_Red-5.40938)*(Blue D-9.759) +
 138.7*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 5322.63*(Conn_Blue-5.59815)*(Pow_Red-8.25877) +
 3874.45*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 1641.57*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) +
 3938.49*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 14734.1*(Conn_Blue-5.59815)*(Blue S,I-9.759) -
 2613.86*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 277.4*(Disp_Red-6.28209)*(Rob_Blue-6.95993) +
 2613.86*(Disp_Red-6.28209)*(Red S,I-8.94429) - 277.4*(Disp_Red-6.28209)*(Blue S,I-9.759) -
 0.31482*(Disp_Blue-5.93948)*(Red S,I-8.94429) - 0.41507*(Disp_Blue-5.93948)*(Blue S,I-9.759) -
 0.41507*(Disp_Blue-5.93948)*(Blue D-9.759) + 10645.3*(Pow_Red-8.25877)*(Rob_Blue-6.95993) +
 0.58087*(Pow_Red-8.25877)*(Red S,I-8.94429) - 10645.3*(Pow_Red-8.25877)*(Blue S,I-9.759) -
 10151*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + 10151*(Pow_Blue-9.06484)*(Red S,I-8.94429) -
 0.38183*(Pow_Blue-9.06484)*(Blue S,I-9.759) - 0.38183*(Pow_Blue-9.06484)*(Blue D-9.759) +
 15625.9*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 265940*(Rob_Red-6.2396)*(Stre_Blue-0.99106) -
 15625.9*(Rob_Red-6.2396)*(Blue S,I-9.759) + 0.7786*(Rob_Blue-6.95993)*(Red S,I-8.94429) -
 26185.1*(Rob_Blue-6.95993)*(Blue S,I-9.759) - 0.29812*(Stab_Blue-6.13897)*(Red S,I-8.94429) +
 265940*(Stre_Blue-0.99106)*(Red S,I-8.94429) + 0.2909*(Red S,I-8.94429)*(Blue S,I-9.759) + 0.2909*(Red
 S,I-8.94429)*(Blue D-9.759) + 0.6013*(Blue S,I-9.759)*(Blue D-9.759) + 1.21885*(Conn_Red-
 5.40938)*(Conn_Red-5.40938) + 6.94278*(Conn_Blue-5.59815)*(Conn_Blue-5.59815) +
 0.72914*(Rob_Red-6.2396)*(Rob_Red-6.2396) - 3283.14*(Rob_Blue-6.95993)*(Rob_Blue-6.95993) +
 29468.2*(Blue S,I-9.759)*(Blue S,I-9.759) = - 1.43269*Blue S,I - 1.43269*Blue D - 1969.25*(Conn_Red-
 5.40938)*(Conn_Blue-5.59815) - 1306.93*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 5075.51*(Conn_Red-
 5.40938)*(Pow_Blue-9.06484) - 132970*(Conn_Red-5.40938)*(Stre_Blue-0.99106) + 0.48612*(Conn_Red-
 5.40938)*(Blue S,I-9.759) + 0.48612*(Conn_Red-5.40938)*(Blue D-9.759) + 138.7*(Conn_Blue-
 5.59815)*(Disp_Red-6.28209) + 5322.63*(Conn_Blue-5.59815)*(Pow_Red-8.25877) +
 3874.45*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 1641.57*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) +
 3938.49*(Conn_Blue-5.59815)*(Red S,I-8.94429) - 14734.1*(Conn_Blue-5.59815)*(Blue S,I-9.759) -
 2613.86*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 277.4*(Disp_Red-6.28209)*(Rob_Blue-6.95993) +
 2613.86*(Disp_Red-6.28209)*(Red S,I-8.94429) - 277.4*(Disp_Red-6.28209)*(Blue S,I-9.759) -
 0.31482*(Disp_Blue-5.93948)*(Red S,I-8.94429) - 0.41507*(Disp_Blue-5.93948)*(Blue S,I-9.759) -
 0.41507*(Disp_Blue-5.93948)*(Blue D-9.759) + 10645.3*(Pow_Red-8.25877)*(Rob_Blue-6.95993) +
 0.58087*(Pow_Red-8.25877)*(Red S,I-8.94429) - 10645.3*(Pow_Red-8.25877)*(Blue S,I-9.759) -
 10151*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + 10151*(Pow_Blue-9.06484)*(Red S,I-8.94429) -
 0.38183*(Pow_Blue-9.06484)*(Blue S,I-9.759) - 0.38183*(Pow_Blue-9.06484)*(Blue D-9.759) +
 15625.9*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 265940*(Rob_Red-6.2396)*(Stre_Blue-0.99106) -

$15625.9 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) + 0.7786 * (\text{Rob_Blue-6.95993}) * (\text{Red S,I-8.94429}) -$
 $26185.1 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) - 0.29812 * (\text{Stab_Blue-6.13897}) * (\text{Red S,I-8.94429}) +$
 $265940 * (\text{Stre_Blue-0.99106}) * (\text{Red S,I-8.94429}) + 0.2909 * (\text{Red S,I-8.94429}) * (\text{Blue S,I-9.759}) + 0.2909 * (\text{Red}$
 $\text{S,I-8.94429}) * (\text{Blue D-9.759}) + 0.6013 * (\text{Blue S,I-9.759}) * (\text{Blue D-9.759}) + 1.21885 * (\text{Conn_Red-}$
 $\text{5.40938}) * (\text{Conn_Red-5.40938}) + 6.94278 * (\text{Conn_Blue-5.59815}) * (\text{Conn_Blue-5.59815}) +$
 $0.72914 * (\text{Rob_Red-6.2396}) * (\text{Rob_Red-6.2396}) - 3283.14 * (\text{Rob_Blue-6.95993}) * (\text{Rob_Blue-6.95993}) +$
 $0.6013 * (\text{Blue S,I-9.759}) * (\text{Blue S,I-9.759}) + 29468.2 * (\text{Blue D-9.759}) * (\text{Blue D-9.759})$

$(\text{Conn_Red-5.40938}) * (\text{Stre_Blue-0.99106}) = 509589 * (\text{Conn_Blue-5.59815}) * (\text{Blue S,I-9.759}) -$
 $509589 * (\text{Conn_Blue-5.59815}) * (\text{Blue D-9.759}) = -0.03551 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815}) -$
 $0.02292 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) + 0.93343 * (\text{Conn_Red-5.40938}) * (\text{Disp_Blue-5.93948}) +$
 $0.13978 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484}) + 0.00011 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) +$
 $0.00011 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) - 0.00835 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) +$
 $0.20191 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) + 0.06886 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) +$
 $0.00228 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) + 0.07102 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) -$
 $0.04584 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) - 0.01669 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) +$
 $0.04584 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) + 0.01669 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) +$
 $1.86686 * (\text{Disp_Blue-5.93948}) * (\text{Rob_Red-6.2396}) - 1.86686 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) =$
 $0.0002 * \text{Red S,I} + 0.00028 * \text{Blue S,I} + 0.00028 * \text{Blue D} - 1.02923 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815})$
 $- 1.19181 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) - 1.36359 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484})$
 $+ 0.00101 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) + 0.00101 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) -$
 $0.06017 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) + 1.23961 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) +$
 $0.52419 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) + 0.08091 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) +$
 $32.5973 * (\text{Conn_Blue-5.59815}) * (\text{Stab_Red-5.07463}) + 2.05847 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) -$
 $2.38362 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) - 0.12033 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) +$
 $2.38362 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) + 0.12033 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) + 7.64e-$
 $5 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) - 0.00024 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) -$
 $0.00024 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) + 2.47921 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) - 7.54e-$
 $5 * (\text{Pow_Red-8.25877}) * (\text{Stre_Red-0.92333}) + 0.00032 * (\text{Pow_Red-8.25877}) * (\text{Red S,I-8.94429}) -$
 $2.47921 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) - 2.72717 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) +$
 $2.72717 * (\text{Pow_Blue-9.06484}) * (\text{Red S,I-8.94429}) + 5.16531 * (\text{Rob_Red-6.2396}) * (\text{Rob_Blue-6.95993}) -$
 $2 * (\text{Rob_Red-6.2396}) * (\text{Stre_Blue-0.99106}) - 5.16531 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) +$
 $65.1946 * (\text{Rob_Blue-6.95993}) * (\text{Stab_Red-5.07463}) + 0.00051 * (\text{Rob_Blue-6.95993}) * (\text{Red S,I-8.94429}) -$
 $0.16182 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) - 65.1946 * (\text{Stab_Red-5.07463}) * (\text{Blue S,I-9.759}) =$
 $0.0002 * \text{Red S,I} + 0.00028 * \text{Blue S,I} + 0.00028 * \text{Blue D} - 1.02923 * (\text{Conn_Red-5.40938}) * (\text{Conn_Blue-5.59815})$
 $- 1.19181 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) - 1.36359 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484})$
 $+ 0.00101 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) + 0.00101 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) -$
 $0.06017 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) + 1.23961 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) +$
 $0.52419 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) + 0.08091 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) +$
 $32.5973 * (\text{Conn_Blue-5.59815}) * (\text{Stab_Red-5.07463}) + 2.05847 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) -$
 $2.38362 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) - 0.12033 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) +$
 $2.38362 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) + 0.12033 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) + 7.64e-$
 $5 * (\text{Disp_Blue-5.93948}) * (\text{Red S,I-8.94429}) - 0.00024 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) -$
 $0.00024 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) + 2.47921 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) - 7.54e-$
 $5 * (\text{Pow_Red-8.25877}) * (\text{Stre_Red-0.92333}) + 0.00032 * (\text{Pow_Red-8.25877}) * (\text{Red S,I-8.94429}) -$
 $2.47921 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) - 2.72717 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) +$
 $2.72717 * (\text{Pow_Blue-9.06484}) * (\text{Red S,I-8.94429}) + 5.16531 * (\text{Rob_Red-6.2396}) * (\text{Rob_Blue-6.95993}) -$
 $2 * (\text{Rob_Red-6.2396}) * (\text{Stre_Blue-0.99106}) - 5.16531 * (\text{Rob_Red-6.2396}) * (\text{Blue S,I-9.759}) +$
 $65.1946 * (\text{Rob_Blue-6.95993}) * (\text{Stab_Red-5.07463}) + 0.00051 * (\text{Rob_Blue-6.95993}) * (\text{Red S,I-8.94429}) -$
 $0.16182 * (\text{Rob_Blue-6.95993}) * (\text{Blue S,I-9.759}) - 0.00079 * (\text{Stab_Red-5.07463}) * (\text{Blue S,I-9.759}) -$
 $65.1946 * (\text{Stab_Red-5.07463}) * (\text{Blue D-9.759}) = 5.37e-6 * \text{Blue S,I} + 5.37e-6 * \text{Blue D} - 0.00261 * (\text{Conn_Red-}$
 $\text{5.40938}) * (\text{Conn_Blue-5.59815}) + 0.01738 * (\text{Conn_Red-5.40938}) * (\text{Disp_Red-6.28209}) +$
 $0.08983 * (\text{Conn_Red-5.40938}) * (\text{Pow_Blue-9.06484}) + 5.58e-6 * (\text{Conn_Red-5.40938}) * (\text{Blue S,I-9.759}) +$
 $5.58e-6 * (\text{Conn_Red-5.40938}) * (\text{Blue D-9.759}) + 0.00221 * (\text{Conn_Blue-5.59815}) * (\text{Disp_Red-6.28209}) +$
 $0.00639 * (\text{Conn_Blue-5.59815}) * (\text{Pow_Red-8.25877}) + 0.02643 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Red-6.2396}) -$
 $0.00167 * (\text{Conn_Blue-5.59815}) * (\text{Rob_Blue-6.95993}) + 0.99395 * (\text{Conn_Blue-5.59815}) * (\text{Stab_Blue-6.13897})$
 $+ 0.00522 * (\text{Conn_Blue-5.59815}) * (\text{Red S,I-8.94429}) + 0.03476 * (\text{Disp_Red-6.28209}) * (\text{Rob_Red-6.2396}) +$
 $0.00442 * (\text{Disp_Red-6.28209}) * (\text{Rob_Blue-6.95993}) - 0.03476 * (\text{Disp_Red-6.28209}) * (\text{Red S,I-8.94429}) -$
 $0.00442 * (\text{Disp_Red-6.28209}) * (\text{Blue S,I-9.759}) - 8.21e-6 * (\text{Disp_Blue-5.93948}) * (\text{Blue S,I-9.759}) - 8.21e-$
 $6 * (\text{Disp_Blue-5.93948}) * (\text{Blue D-9.759}) + 0.01277 * (\text{Pow_Red-8.25877}) * (\text{Rob_Blue-6.95993}) + 2.52e-$
 $6 * (\text{Pow_Red-8.25877}) * (\text{Stre_Red-0.92333}) - 2.89e-6 * (\text{Pow_Red-8.25877}) * (\text{Red S,I-8.94429}) -$
 $0.01277 * (\text{Pow_Red-8.25877}) * (\text{Blue S,I-9.759}) + 0.17966 * (\text{Pow_Blue-9.06484}) * (\text{Rob_Red-6.2396}) -$

$$\begin{aligned}
& 0.17966*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) + 2.72e-5*(Pow_Blue-9.06484)*(Blue\ S,I-9.759) + 2.72e- \\
& 5*(Pow_Blue-9.06484)*(Blue\ D-9.759) + 0.0633*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 2*(Rob_Red- \\
& 6.2396)*(Stre_Blue-0.99106) - 0.0633*(Rob_Red-6.2396)*(Blue\ S,I-9.759) + 1.9879*(Rob_Blue- \\
& 6.95993)*(Stab_Blue-6.13897) + 4.85e-6*(Rob_Blue-6.95993)*(Red\ S,I-8.94429) + 0.00333*(Rob_Blue- \\
& 6.95993)*(Blue\ S,I-9.759) - 1.9879*(Stab_Blue-6.13897)*(Blue\ S,I-9.759) = 5.37e-6*Blue\ S,I + 5.37e-6*Blue \\
& D - 0.00261*(Conn_Red-5.40938)*(Conn_Blue-5.59815) + 0.01738*(Conn_Red-5.40938)*(Disp_Red- \\
& 6.28209) + 0.08983*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + 5.58e-6*(Conn_Red-5.40938)*(Blue\ S,I- \\
& 9.759) + 5.58e-6*(Conn_Red-5.40938)*(Blue\ D-9.759) + 0.00221*(Conn_Blue-5.59815)*(Disp_Red- \\
& 6.28209) + 0.00639*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + 0.02643*(Conn_Blue-5.59815)*(Rob_Red- \\
& 6.2396) - 0.00167*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + 0.99395*(Conn_Blue-5.59815)*(Stab_Blue- \\
& 6.13897) + 0.00522*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) + 0.03476*(Disp_Red-6.28209)*(Rob_Red- \\
& 6.2396) + 0.00442*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 0.03476*(Disp_Red-6.28209)*(Red\ S,I- \\
& 8.94429) - 0.00442*(Disp_Red-6.28209)*(Blue\ S,I-9.759) - 8.21e-6*(Disp_Blue-5.93948)*(Blue\ S,I-9.759) - \\
& 8.21e-6*(Disp_Blue-5.93948)*(Blue\ D-9.759) + 0.01277*(Pow_Red-8.25877)*(Rob_Blue-6.95993) + 2.52e- \\
& 6*(Pow_Red-8.25877)*(Stre_Red-0.92333) - 2.89e-6*(Pow_Red-8.25877)*(Red\ S,I-8.94429) - \\
& 0.01277*(Pow_Red-8.25877)*(Blue\ S,I-9.759) + 0.17966*(Pow_Blue-9.06484)*(Rob_Red-6.2396) - \\
& 0.17966*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) + 2.72e-5*(Pow_Blue-9.06484)*(Blue\ S,I-9.759) + 2.72e- \\
& 5*(Pow_Blue-9.06484)*(Blue\ D-9.759) + 0.0633*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 2*(Rob_Red- \\
& 6.2396)*(Stre_Blue-0.99106) - 0.0633*(Rob_Red-6.2396)*(Blue\ S,I-9.759) + 1.9879*(Rob_Blue- \\
& 6.95993)*(Stab_Blue-6.13897) + 4.85e-6*(Rob_Blue-6.95993)*(Red\ S,I-8.94429) + 0.00333*(Rob_Blue- \\
& 6.95993)*(Blue\ S,I-9.759) - 0.00011*(Stab_Blue-6.13897)*(Blue\ S,I-9.759) - 1.9879*(Stab_Blue- \\
& 6.13897)*(Blue\ D-9.759) = 2.76e-5*Red\ S,I + 2.77e-5*Blue\ S,I + 2.77e-5*Blue\ D - 0.00512*(Conn_Red- \\
& 5.40938)*(Conn_Blue-5.59815) - 0.01617*(Conn_Red-5.40938)*(Disp_Red-6.28209) + \\
& 0.06534*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 3.2734*(Conn_Red-5.40938)*(Stre_Red-0.92333) + \\
& 7.17e-6*(Conn_Red-5.40938)*(Blue\ S,I-9.759) + 7.17e-6*(Conn_Red-5.40938)*(Blue\ D-9.759) - \\
& 0.00399*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 0.09912*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + \\
& 0.03142*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 0.02607*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + \\
& 0.01023*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) - 0.03235*(Disp_Red-6.28209)*(Rob_Red-6.2396) - \\
& 0.00797*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + 0.03235*(Disp_Red-6.28209)*(Red\ S,I-8.94429) + \\
& 0.00797*(Disp_Red-6.28209)*(Blue\ S,I-9.759) + 0.19823*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 4.86e- \\
& 5*(Pow_Red-8.25877)*(Stre_Red-0.92333) - 0.00001*(Pow_Red-8.25877)*(Red\ S,I-8.94429) - \\
& 0.19823*(Pow_Red-8.25877)*(Blue\ S,I-9.759) + 0.13067*(Pow_Blue-9.06484)*(Rob_Red-6.2396) - \\
& 0.13067*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) + 0.0833*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - \\
& 6.5468*(Rob_Red-6.2396)*(Stre_Red-0.92333) - 2*(Rob_Red-6.2396)*(Stre_Blue-0.99106) - \\
& 0.0833*(Rob_Red-6.2396)*(Blue\ S,I-9.759) + 0.00001*(Rob_Blue-6.95993)*(Red\ S,I-8.94429) + \\
& 0.05215*(Rob_Blue-6.95993)*(Blue\ S,I-9.759) - 2.67e-5*(Stab_Red-5.07463)*(Red\ S,I-8.94429) + \\
& 6.5468*(Stre_Red-0.92333)*(Red\ S,I-8.94429) = - 0.06594*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - \\
& 0.04927*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 0.37761*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + \\
& 0.00012*(Conn_Red-5.40938)*(Blue\ S,I-9.759) + 0.00012*(Conn_Red-5.40938)*(Blue\ D-9.759) + \\
& 0.06066*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 0.51879*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + \\
& 0.41016*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 0.01725*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + \\
& 9.46041*(Conn_Blue-5.59815)*(Stre_Red-0.92333) + 0.13187*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) - \\
& 0.09853*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 0.12132*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + \\
& 0.09853*(Disp_Red-6.28209)*(Red\ S,I-8.94429) - 0.12132*(Disp_Red-6.28209)*(Blue\ S,I-9.759) + \\
& 1.03758*(Pow_Red-8.25877)*(Rob_Blue-6.95993) + 2.15e-5*(Pow_Red-8.25877)*(Red\ S,I-8.94429) - \\
& 1.03758*(Pow_Red-8.25877)*(Blue\ S,I-9.759) - 0.75522*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + \\
& 0.75522*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) + 1.08407*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - \\
& 2*(Rob_Red-6.2396)*(Stre_Blue-0.99106) - 1.08407*(Rob_Red-6.2396)*(Blue\ S,I-9.759) + \\
& 18.9208*(Rob_Blue-6.95993)*(Stre_Red-0.92333) + 0.0345*(Rob_Blue-6.95993)*(Blue\ S,I-9.759) - \\
& 18.9208*(Stre_Red-0.92333)*(Blue\ S,I-9.759) = - 0.06594*(Conn_Red-5.40938)*(Conn_Blue-5.59815) - \\
& 0.04927*(Conn_Red-5.40938)*(Disp_Red-6.28209) - 0.37761*(Conn_Red-5.40938)*(Pow_Blue-9.06484) + \\
& 0.00012*(Conn_Red-5.40938)*(Blue\ S,I-9.759) + 0.00012*(Conn_Red-5.40938)*(Blue\ D-9.759) + \\
& 0.06066*(Conn_Blue-5.59815)*(Disp_Red-6.28209) + 0.51879*(Conn_Blue-5.59815)*(Pow_Red-8.25877) + \\
& 0.41016*(Conn_Blue-5.59815)*(Rob_Red-6.2396) - 0.01725*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) + \\
& 9.46041*(Conn_Blue-5.59815)*(Stre_Red-0.92333) + 0.13187*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) - \\
& 0.09853*(Disp_Red-6.28209)*(Rob_Red-6.2396) + 0.12132*(Disp_Red-6.28209)*(Rob_Blue-6.95993) + \\
& 0.09853*(Disp_Red-6.28209)*(Red\ S,I-8.94429) - 0.12132*(Disp_Red-6.28209)*(Blue\ S,I-9.759) + \\
& 1.03758*(Pow_Red-8.25877)*(Rob_Blue-6.95993) + 2.15e-5*(Pow_Red-8.25877)*(Red\ S,I-8.94429) - \\
& 1.03758*(Pow_Red-8.25877)*(Blue\ S,I-9.759) - 0.75522*(Pow_Blue-9.06484)*(Rob_Red-6.2396) + \\
& 0.75522*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) + 1.08407*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - \\
& 2*(Rob_Red-6.2396)*(Stre_Blue-0.99106) - 1.08407*(Rob_Red-6.2396)*(Blue\ S,I-9.759) + \\
& 18.9208*(Rob_Blue-6.95993)*(Stre_Red-0.92333) + 0.0345*(Rob_Blue-6.95993)*(Blue\ S,I-9.759) -
\end{aligned}$$

$$\begin{aligned}
&18.9208*(Stre_Red-0.92333)*(Blue\ D-9.759) = 1.66e-5*Red\ S,I + 0.00749*(Conn_Red- \\
&5.40938)*(Conn_Blue-5.59815) + 0.03005*(Conn_Red-5.40938)*(Disp_Red-6.28209) + \\
&0.15359*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 7.41e-6*(Conn_Red-5.40938)*(Blue\ S,I-9.759) - \\
&7.41e-6*(Conn_Red-5.40938)*(Blue\ D-9.759) + 0.00024*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - \\
&0.02446*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 0.01836*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + \\
&0.03925*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) - 3.14155*(Conn_Blue-5.59815)*(Stre_Blue-0.99106) - \\
&0.01498*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) + 0.0601*(Disp_Red-6.28209)*(Rob_Red-6.2396) + \\
&0.00047*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 0.0601*(Disp_Red-6.28209)*(Red\ S,I-8.94429) - \\
&0.00047*(Disp_Red-6.28209)*(Blue\ S,I-9.759) - 6.3e-6*(Disp_Blue-5.93948)*(Blue\ S,I-9.759) - 6.3e- \\
&6*(Disp_Blue-5.93948)*(Blue\ D-9.759) - 0.04892*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 8.44e- \\
&6*(Pow_Red-8.25877)*(Red\ S,I-8.94429) + 0.04892*(Pow_Red-8.25877)*(Blue\ S,I-9.759) + \\
&0.30717*(Pow_Blue-9.06484)*(Rob_Red-6.2396) - 0.30717*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) - \\
&0.06668*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 2*(Rob_Red-6.2396)*(Stre_Blue-0.99106) + \\
&0.06668*(Rob_Red-6.2396)*(Blue\ S,I-9.759) - 6.28311*(Rob_Blue-6.95993)*(Stre_Blue-0.99106) - \\
&0.07849*(Rob_Blue-6.95993)*(Blue\ S,I-9.759) + 2*(Stre_Blue-0.99106)*(Red\ S,I-8.94429) + \\
&6.28311*(Stre_Blue-0.99106)*(Blue\ S,I-9.759) = 1.66e-5*Red\ S,I + 0.00749*(Conn_Red- \\
&5.40938)*(Conn_Blue-5.59815) + 0.03005*(Conn_Red-5.40938)*(Disp_Red-6.28209) + \\
&0.15359*(Conn_Red-5.40938)*(Pow_Blue-9.06484) - 7.41e-6*(Conn_Red-5.40938)*(Blue\ S,I-9.759) - \\
&7.41e-6*(Conn_Red-5.40938)*(Blue\ D-9.759) + 0.00024*(Conn_Blue-5.59815)*(Disp_Red-6.28209) - \\
&0.02446*(Conn_Blue-5.59815)*(Pow_Red-8.25877) - 0.01836*(Conn_Blue-5.59815)*(Rob_Red-6.2396) + \\
&0.03925*(Conn_Blue-5.59815)*(Rob_Blue-6.95993) - 3.14155*(Conn_Blue-5.59815)*(Stre_Blue-0.99106) - \\
&0.01498*(Conn_Blue-5.59815)*(Red\ S,I-8.94429) + 0.0601*(Disp_Red-6.28209)*(Rob_Red-6.2396) + \\
&0.00047*(Disp_Red-6.28209)*(Rob_Blue-6.95993) - 0.0601*(Disp_Red-6.28209)*(Red\ S,I-8.94429) - \\
&0.00047*(Disp_Red-6.28209)*(Blue\ S,I-9.759) - 6.3e-6*(Disp_Blue-5.93948)*(Blue\ S,I-9.759) - 6.3e- \\
&6*(Disp_Blue-5.93948)*(Blue\ D-9.759) - 0.04892*(Pow_Red-8.25877)*(Rob_Blue-6.95993) - 8.44e- \\
&6*(Pow_Red-8.25877)*(Red\ S,I-8.94429) + 0.04892*(Pow_Red-8.25877)*(Blue\ S,I-9.759) + \\
&0.30717*(Pow_Blue-9.06484)*(Rob_Red-6.2396) - 0.30717*(Pow_Blue-9.06484)*(Red\ S,I-8.94429) - \\
&0.06668*(Rob_Red-6.2396)*(Rob_Blue-6.95993) - 2*(Rob_Red-6.2396)*(Stre_Blue-0.99106) + \\
&0.06668*(Rob_Red-6.2396)*(Blue\ S,I-9.759) - 6.28311*(Rob_Blue-6.95993)*(Stre_Blue-0.99106) - \\
&0.07849*(Rob_Blue-6.95993)*(Blue\ S,I-9.759) + 2*(Stre_Blue-0.99106)*(Red\ S,I-8.94429) + \\
&6.28311*(Stre_Blue-0.99106)*(Blue\ D-9.759)
\end{aligned}$$

$$(Disp_Red-6.28209)*(Blue\ S,I-9.759) = (Disp_Red-6.28209)*(Blue\ D-9.759)$$

$$(Pow_Red-8.25877)*(Blue\ S,I-9.759) = (Pow_Red-8.25877)*(Blue\ D-9.759)$$

$$(Rob_Red-6.2396)*(Blue\ S,I-9.759) = (Rob_Red-6.2396)*(Blue\ D-9.759)$$

$$(Rob_Blue-6.95993)*(Blue\ S,I-9.759) = (Rob_Blue-6.95993)*(Blue\ D-9.759)$$

Summary of Fit

RSquare	0.990814
RSquare Adj	0.990813
Root Mean Square Error	1.554449
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	129	379903789	2944990	1218796
Error	1.5e+6	3522188	2.416312	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3443118.9	2.42110	Prob > F
Pure Error	35540	79068.8	2.22478	<.0001*
Total Error	1.5e+6	3522187.7		Max RSq 0.9998

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob> t
Intercept	Biased	44.367612	0.043522	1019.4	<.0001*
Conn_Red	Biased	8.1367168	0.006774	1201.2	<.0001*
Conn_Blue	Biased	-7.956626	0.007261	-1096	<.0001*
Disp_Red	Biased	0.0465067	0.001589	29.28	<.0001*
Disp_Blue	Biased	-0.0554	0.001593	-34.77	<.0001*
Pow_Red	Biased	3.1092805	0.04811	64.63	<.0001*
Pow_Blue	Biased	-2.812379	0.050538	-55.65	<.0001*
Rob_Red	Biased	16.349445	0.017665	925.53	<.0001*
Rob_Blue	Biased	-16.01418	0.01808	-885.7	<.0001*
Stab_Red	Biased	0.137732	0.002877	47.88	<.0001*
Stab_Blue	Biased	-0.075154	0.003011	-24.96	<.0001*
Stre_Red	Biased	-7.560553	0.299977	-25.20	<.0001*
Stre_Blue	Biased	5.9235653	0.314379	18.84	<.0001*
Red S,I	Zeroed	0	0	.	.
Red D	Biased	-1.100112	0.012907	-85.24	<.0001*
Blue S,I	Zeroed	0	0	.	.
Blue D	Biased	0.9852416	0.013986	70.44	<.0001*
(Conn_Red-5.40938)*(Conn_Blue-5.59815)	Biased	-0.827799	45.07537	-0.02	0.9853
(Conn_Red-5.40938)*(Disp_Red-6.28209)	Biased	1.9913778	137.7132	0.01	0.9885
(Conn_Red-5.40938)*(Disp_Blue-5.93948)	Biased	-0.006232	0.003559	-1.75	0.0800
(Conn_Red-5.40938)*(Pow_Red-8.25877)	Biased	0.2752861	0.084595	3.25	0.0011*
(Conn_Red-5.40938)*(Pow_Blue-9.06484)	Biased	-0.340415	280.3389	-0.00	0.9990
(Conn_Red-5.40938)*(Rob_Red-6.2396)	Biased	0.0162354	0.002684	6.05	<.0001*
(Conn_Red-5.40938)*(Rob_Blue-6.95993)	Biased	-1.953389	0.027001	-72.34	<.0001*
(Conn_Red-5.40938)*(Stab_Red-5.07463)	Biased	0.0461908	0.007185	6.43	<.0001*
(Conn_Red-5.40938)*(Stab_Blue-6.13897)	Biased	-0.015295	0.005222	-2.93	0.0034*
(Conn_Red-5.40938)*(Stre_Red-0.92333)	Biased	1.7104996	0.519405	3.29	0.0010*
(Conn_Red-5.40938)*(Stre_Blue-0.99106)	Biased	-5.970419	2284.195	-0.00	0.9979
(Conn_Red-5.40938)*(Red S,I-8.94429)	Biased	0.9448042	0.016654	56.73	<.0001*
(Conn_Red-5.40938)*(Red D-3.69742)	Biased	-0.288378	0.02235	-12.90	<.0001*
(Conn_Red-5.40938)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Conn_Red-5.40938)*(Blue D-4.67447)	Biased	0.4721635	87.99787	0.01	0.9957
(Conn_Blue-5.59815)*(Disp_Red-6.28209)	Biased	0.003749	57.14276	0.00	0.9999
(Conn_Blue-5.59815)*(Disp_Blue-5.93948)	Biased	0.0475614	0.004666	10.19	<.0001*
(Conn_Blue-5.59815)*(Pow_Red-8.25877)	Biased	-1.693887	212.1807	-0.01	0.9936
(Conn_Blue-5.59815)*(Pow_Blue-9.06484)	Biased	-0.000536	0.100789	-0.01	0.9958
(Conn_Blue-5.59815)*(Rob_Red-6.2396)	Biased	0.8684825	103.8361	0.01	0.9933
(Conn_Blue-5.59815)*(Rob_Blue-6.95993)	Biased	1.5332439	45.4798	0.03	0.9731
(Conn_Blue-5.59815)*(Stab_Red-5.07463)	Biased	0.0175595	0.005743	3.06	0.0022*
(Conn_Blue-5.59815)*(Stab_Blue-6.13897)	Biased	-0.007084	0.007842	-0.90	0.3664

Term		Estimate	Std Error	t Ratio	Prob> t
(Conn_Blue-5.59815)*(Stre_Red-0.92333)	Biased	10.802891	0.486775	22.19	<.0001*
(Conn_Blue-5.59815)*(Stre_Blue-0.99106)	Biased	0.6061567	0.613517	0.99	0.3232
(Conn_Blue-5.59815)*(Red S,I-8.94429)	Biased	-0.307975	90.15074	-0.00	0.9973
(Conn_Blue-5.59815)*(Red D-3.69742)	Biased	0.5790593	0.022361	25.90	<.0001*
(Conn_Blue-5.59815)*(Blue S,I-9.759)	Biased	1.056259	0.0198	53.35	<.0001*
(Conn_Blue-5.59815)*(Blue D-4.67447)	Biased	0.1816608	85.51397	0.00	0.9983
(Disp_Red-6.28209)*(Disp_Blue-5.93948)	Biased	-0.003134	0.001321	-2.37	0.0176*
(Disp_Red-6.28209)*(Pow_Red-8.25877)	Biased	0.6517767	0.032542	20.03	<.0001*
(Disp_Red-6.28209)*(Pow_Blue-9.06484)	Biased	-0.034828	0.025224	-1.38	0.1674
(Disp_Red-6.28209)*(Rob_Red-6.2396)	Biased	3.9979331	275.4265	0.01	0.9884
(Disp_Red-6.28209)*(Rob_Blue-6.95993)	Biased	0.0189285	114.2855	0.00	0.9999
(Disp_Red-6.28209)*(Stab_Red-5.07463)	Biased	0.003613	0.002193	1.65	0.0995
(Disp_Red-6.28209)*(Stab_Blue-6.13897)	Biased	-0.000907	0.001974	-0.46	0.6458
(Disp_Red-6.28209)*(Stre_Red-0.92333)	Biased	-4.581194	0.220716	-20.76	<.0001*
(Disp_Red-6.28209)*(Stre_Blue-0.99106)	Biased	-0.191738	0.166007	-1.16	0.2481
(Disp_Red-6.28209)*(Red S,I-8.94429)	Biased	-4.068337	275.4265	-0.01	0.9882
(Disp_Red-6.28209)*(Red D-3.69742)	Biased	-0.16828	0.010018	-16.80	<.0001*
(Disp_Red-6.28209)*(Blue S,I-9.759)	Biased	0.120059	114.2855	0.00	0.9992
(Disp_Red-6.28209)*(Blue D-4.67447)	Biased	-0.004568	0.00769	-0.59	0.5525
(Disp_Blue-5.93948)*(Pow_Red-8.25877)	Biased	0.0320313	0.024503	1.31	0.1911
(Disp_Blue-5.93948)*(Pow_Blue-9.06484)	Biased	-0.052503	0.035442	-1.48	0.1385
(Disp_Blue-5.93948)*(Rob_Red-6.2396)	Biased	-0.00869	0.009518	-0.91	0.3613
(Disp_Blue-5.93948)*(Rob_Blue-6.95993)	Biased	0.0884217	0.011303	7.82	<.0001*
(Disp_Blue-5.93948)*(Stab_Red-5.07463)	Biased	-0.000157	0.001925	-0.08	0.9351
(Disp_Blue-5.93948)*(Stab_Blue-6.13897)	Biased	-0.015653	0.002311	-6.77	<.0001*
(Disp_Blue-5.93948)*(Stre_Red-0.92333)	Biased	-0.319411	0.161123	-1.98	0.0474*
(Disp_Blue-5.93948)*(Stre_Blue-0.99106)	Biased	-0.00355	0.243827	-0.01	0.9884
(Disp_Blue-5.93948)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Disp_Blue-5.93948)*(Red D-3.69742)	Biased	-0.015451	0.007368	-2.10	0.0360*
(Disp_Blue-5.93948)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Disp_Blue-5.93948)*(Blue D-4.67447)	Biased	-0.012343	0.011116	-1.11	0.2668
(Pow_Red-8.25877)*(Pow_Blue-9.06484)	Biased	0.8727589	0.459596	1.90	0.0576
(Pow_Red-8.25877)*(Rob_Red-6.2396)	Biased	-0.220472	0.216231	-1.02	0.3079
(Pow_Red-8.25877)*(Rob_Blue-6.95993)	Biased	-3.598335	424.3615	-0.01	0.9932
(Pow_Red-8.25877)*(Stab_Red-5.07463)	Biased	-0.075179	0.034763	-2.16	0.0306*
(Pow_Red-8.25877)*(Stab_Blue-	Biased	-0.022454	0.036294	-0.62	0.5361

Term		Estimate	Std Error	t Ratio	Prob> t
6.13897)					
(Pow_Red-8.25877)*(Stre_Red-0.92333)	Biased	59.375168	3.928465	15.11	<.0001*
(Pow_Red-8.25877)*(Stre_Blue-0.99106)	Biased	1.1523922	3.029415	0.38	0.7036
(Pow_Red-8.25877)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Pow_Red-8.25877)*(Red D-3.69742)	Biased	3.4706517	0.174476	19.89	<.0001*
(Pow_Red-8.25877)*(Blue S,I-9.759)	Biased	-0.226345	424.3615	-0.00	0.9996
(Pow_Red-8.25877)*(Blue D-4.67447)	Biased	-0.006162	0.139913	-0.04	0.9649
(Pow_Blue-9.06484)*(Rob_Red-6.2396)	Biased	-0.619368	560.6777	-0.00	0.9991
(Pow_Blue-9.06484)*(Rob_Blue-6.95993)	Biased	0.2457039	0.24809	0.99	0.3220
(Pow_Blue-9.06484)*(Stab_Red-5.07463)	Biased	0.0087044	0.035938	0.24	0.8086
(Pow_Blue-9.06484)*(Stab_Blue-6.13897)	Biased	0.1960488	0.038764	5.06	<.0001*
(Pow_Blue-9.06484)*(Stre_Red-0.92333)	Biased	-6.671514	3.030752	-2.20	0.0277*
(Pow_Blue-9.06484)*(Stre_Blue-0.99106)	Biased	-8.829369	4.424318	-2.00	0.0460*
(Pow_Blue-9.06484)*(Red S,I-8.94429)	Biased	0.1537168	560.6777	0.00	0.9998
(Pow_Blue-9.06484)*(Red D-3.69742)	Biased	-0.361552	0.141207	-2.56	0.0105*
(Pow_Blue-9.06484)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Pow_Blue-9.06484)*(Blue D-4.67447)	Biased	-0.848988	0.200167	-4.24	<.0001*
(Rob_Red-6.2396)*(Rob_Blue-6.95993)	Biased	1.1316551	281.5243	0.00	0.9968
(Rob_Red-6.2396)*(Stab_Red-5.07463)	Biased	0.0866669	0.019287	4.49	<.0001*
(Rob_Red-6.2396)*(Stab_Blue-6.13897)	Biased	-0.03477	0.014028	-2.48	0.0132*
(Rob_Red-6.2396)*(Stre_Red-0.92333)	Biased	9.5150179	1.333589	7.13	<.0001*
(Rob_Red-6.2396)*(Stre_Blue-0.99106)	Biased	-12.01449	4568.391	-0.00	0.9979
(Rob_Red-6.2396)*(Red S,I-8.94429)	Biased	1.9777811	0.056858	34.78	<.0001*
(Rob_Red-6.2396)*(Red D-3.69742)	Biased	-0.34776	0.056806	-6.12	<.0001*
(Rob_Red-6.2396)*(Blue S,I-9.759)	Biased	-5.158963	281.5243	-0.02	0.9854
(Rob_Red-6.2396)*(Blue D-4.67447)	Biased	0.9441113	175.9958	0.01	0.9957
(Rob_Blue-6.95993)*(Stab_Red-5.07463)	Biased	0.0335018	0.014566	2.30	0.0214*
(Rob_Blue-6.95993)*(Stab_Blue-6.13897)	Biased	0.015798	0.020448	0.77	0.4398
(Rob_Blue-6.95993)*(Stre_Red-0.92333)	Biased	23.215116	1.230579	18.87	<.0001*
(Rob_Blue-6.95993)*(Stre_Blue-0.99106)	Biased	-1.165374	1.525652	-0.76	0.4450
(Rob_Blue-6.95993)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Rob_Blue-6.95993)*(Red D-3.69742)	Biased	1.2336429	0.056305	21.91	<.0001*
(Rob_Blue-6.95993)*(Blue S,I-9.759)	Biased	-1.046973	90.95961	-0.01	0.9908
(Rob_Blue-6.95993)*(Blue D-4.67447)	Biased	0.2398348	171.0279	0.00	0.9989
(Stab_Red-5.07463)*(Stab_Blue-6.13897)	Biased	0.0005889	0.002825	0.21	0.8349
(Stab_Red-5.07463)*(Stre_Red-0.92333)	Biased	-0.344438	0.251322	-1.37	0.1705
(Stab_Red-5.07463)*(Stre_Blue-0.99106)	Biased	-0.059639	0.237108	-0.25	0.8014
(Stab_Red-5.07463)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Stab_Red-5.07463)*(Red D-3.69742)	Biased	0.0485239	0.01206	4.02	<.0001*
(Stab_Red-5.07463)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Stab_Red-5.07463)*(Blue D-4.67447)	Biased	-0.000689	0.010929	-0.06	0.9497
(Stab_Blue-6.13897)*(Stre_Red-	Biased	0.1902797	0.239128	0.80	0.4262

Term		Estimate	Std Error	t Ratio	Prob> t
0.92333)					
(Stab_Blue-6.13897)*(Stre_Blue-0.99106)	Biased	-1.199448	0.275495	-4.35	<.0001*
(Stab_Blue-6.13897)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Stab_Blue-6.13897)*(Red D-3.69742)	Biased	0.0070087	0.010869	0.64	0.5190
(Stab_Blue-6.13897)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Stab_Blue-6.13897)*(Blue D-4.67447)	Biased	-0.101046	0.013237	-7.63	<.0001*
(Stre_Red-0.92333)*(Stre_Blue-0.99106)	Biased	-11.57776	19.96643	-0.58	0.5620
(Stre_Red-0.92333)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Stre_Red-0.92333)*(Red D-3.69742)	Biased	-24.80692	1.166092	-21.27	<.0001*
(Stre_Red-0.92333)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Stre_Red-0.92333)*(Blue D-4.67447)	Biased	-0.057139	0.922795	-0.06	0.9506
(Stre_Blue-0.99106)*(Red S,I-8.94429)	Biased	12.369056	4568.391	0.00	0.9978
(Stre_Blue-0.99106)*(Red D-3.69742)	Biased	-0.113736	0.931063	-0.12	0.9028
(Stre_Blue-0.99106)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Stre_Blue-0.99106)*(Blue D-4.67447)	Biased	3.9784564	1.330886	2.99	0.0028*
(Red S,I-8.94429)*(Red D-3.69742)	Zeroed	0	0	.	.
(Red S,I-8.94429)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Red S,I-8.94429)*(Blue D-4.67447)	Biased	-0.773108	175.9958	-0.00	0.9965
(Red D-3.69742)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Red D-3.69742)*(Blue D-4.67447)	Biased	0.0071607	0.043086	0.17	0.8680
(Blue S,I-9.759)*(Blue D-4.67447)	Biased	-0.163236	171.0279	-0.00	0.9992
(Conn_Red-5.40938)*(Conn_Red-5.40938)	Zeroed	0	0	.	.
(Conn_Blue-5.59815)*(Conn_Blue-5.59815)	Zeroed	0	0	.	.
(Disp_Red-6.28209)*(Disp_Red-6.28209)	Biased	-0.01476	0.000997	-14.81	<.0001*
(Disp_Blue-5.93948)*(Disp_Blue-5.93948)	Biased	-0.002845	0.001074	-2.65	0.0080*
(Pow_Red-8.25877)*(Pow_Red-8.25877)	Biased	-3.869132	0.313967	-12.32	<.0001*
(Pow_Blue-9.06484)*(Pow_Blue-9.06484)	Biased	1.1188826	0.356139	3.14	0.0017*
(Rob_Red-6.2396)*(Rob_Red-6.2396)	Zeroed	0	0	.	.
(Rob_Blue-6.95993)*(Rob_Blue-6.95993)	Biased	3.1057226	90.9596	0.03	0.9728
(Stab_Red-5.07463)*(Stab_Red-5.07463)	Biased	-0.037045	0.002884	-12.84	<.0001*
(Stab_Blue-6.13897)*(Stab_Blue-6.13897)	Biased	0.0274793	0.002882	9.53	<.0001*
(Stre_Red-0.92333)*(Stre_Red-0.92333)	Biased	-264.0635	13.2649	-19.91	<.0001*
(Stre_Blue-0.99106)*(Stre_Blue-0.99106)	Biased	40.436364	14.93415	2.71	0.0068*
(Red S,I-8.94429)*(Red S,I-8.94429)	Zeroed	0	0	.	.
(Red D-3.69742)*(Red D-3.69742)	Biased	-0.588441	0.028657	-20.53	<.0001*
(Blue S,I-9.759)*(Blue S,I-9.759)	Zeroed	0	0	.	.
(Blue D-4.67447)*(Blue D-4.67447)		0.1253343	0.032136	3.90	<.0001*

Non-Linear Model with Metrics + D (2-way Interactions and Quadratic terms) minus Correlated and Insignificant Terms

Summary of Fit

RSquare	0.99081
RSquare Adj	0.990809
Root Mean Square Error	1.554769
Mean of Response	37.01395
Observations (or Sum Wgts)	1457801

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	82	379902208	4632954	1916578
Error	1.5e+6	3523749	2.417305	Prob > F
C. Total	1.5e+6	383425957		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	1.4e+6	3444880.7	2.42212	Prob > F
Pure Error	35540	79068.8	2.22478	<.0001*
Total Error	1.5e+6	3523749.5		Max RSq 0.9998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	44.336628	0.043375	1022.2	<.0001*
Conn_Red	8.1319231	0.006637	1225.3	<.0001*
Conn_Blue	-7.947685	0.00647	-1228	<.0001*
Disp_Red	0.0463191	0.001573	29.45	<.0001*
Disp_Blue	-0.049504	0.001524	-32.47	<.0001*
Pow_Red	3.128375	0.046716	66.97	<.0001*
Pow_Blue	-2.936691	0.044749	-65.63	<.0001*
Rob_Red	16.337918	0.017445	936.52	<.0001*
Rob_Blue	-16.00884	0.016338	-979.8	<.0001*
Stab_Red	0.1405397	0.002761	50.90	<.0001*
Stab_Blue	-0.083348	0.00235	-35.47	<.0001*
Stre_Red	-7.619248	0.290284	-26.25	<.0001*
Stre_Blue	6.7768917	0.283127	23.94	<.0001*
Red D	-1.105726	0.012321	-89.75	<.0001*
Blue D	1.0365464	0.012546	82.62	<.0001*
(Conn_Red-5.40938)*(Conn_Blue-5.59815)	-1.002073	0.005691	-176.1	<.0001*
(Conn_Red-5.40938)*(Disp_Red-6.28209)	-0.039216	0.003231	-12.14	<.0001*
(Conn_Red-5.40938)*(Pow_Red-8.25877)	0.3790522	0.014952	25.35	<.0001*
(Conn_Red-5.40938)*(Pow_Blue-9.06484)	-0.187893	0.009604	-19.56	<.0001*
(Conn_Red-5.40938)*(Rob_Red-6.2396)	1.8907719	0.030214	62.58	<.0001*
(Conn_Red-5.40938)*(Rob_Blue-6.95993)	-2.00649	0.013584	-147.7	<.0001*
(Conn_Red-5.40938)*(Stab_Red-5.07463)	0.0604311	0.006323	9.56	<.0001*
(Conn_Red-5.40938)*(Stab_Blue-6.13897)	-0.015702	0.000945	-16.62	<.0001*
(Conn_Red-5.40938)*(Stre_Red-0.92333)	1.330492	0.229235	5.80	<.0001*
(Conn_Red-5.40938)*(Red D-3.69742)	-0.309947	0.011736	-26.41	<.0001*
(Conn_Red-5.40938)*(Blue D-4.67447)	0.0811908	0.001243	65.30	<.0001*
(Conn_Blue-5.59815)*(Disp_Red-6.28209)	0.0444395	0.0014	31.75	<.0001*
(Conn_Blue-5.59815)*(Disp_Blue-5.93948)	0.0125159	0.001142	10.96	<.0001*
(Conn_Blue-5.59815)*(Pow_Red-8.25877)	-1.466761	0.032704	-44.85	<.0001*
(Conn_Blue-5.59815)*(Rob_Red-6.2396)	-2.053441	0.016032	-128.1	<.0001*
(Conn_Blue-5.59815)*(Rob_Blue-6.95993)	2.1848416	0.00908	240.61	<.0001*
(Conn_Blue-5.59815)*(Stab_Red-5.07463)	0.0184137	0.002461	7.48	<.0001*
(Conn_Blue-5.59815)*(Stre_Red-0.92333)	8.5334593	0.222316	38.38	<.0001*
(Conn_Blue-5.59815)*(Red D-3.69742)	0.4845086	0.011028	43.94	<.0001*
(Conn_Blue-5.59815)*(Blue D-4.67447)	0.0092757	0.001229	7.55	<.0001*

Term	Estimate	Std Error	t Ratio	Prob> t
(Disp_Red-6.28209)*(Disp_Blue-5.93948)	-0.002837	0.000358	-7.92	<.0001*
(Disp_Red-6.28209)*(Pow_Red-8.25877)	0.6520739	0.032524	20.05	<.0001*
(Disp_Red-6.28209)*(Rob_Red-6.2396)	-0.063031	0.008467	-7.44	<.0001*
(Disp_Red-6.28209)*(Rob_Blue-6.95993)	0.0776475	0.002754	28.20	<.0001*
(Disp_Red-6.28209)*(Stre_Red-0.92333)	-4.638685	0.217479	-21.33	<.0001*
(Disp_Red-6.28209)*(Red D-3.69742)	-0.165432	0.009926	-16.67	<.0001*
(Disp_Blue-5.93948)*(Rob_Blue-6.95993)	0.0147934	0.002303	6.42	<.0001*
(Disp_Blue-5.93948)*(Stab_Blue-6.13897)	-0.010605	0.000974	-10.89	<.0001*
(Disp_Blue-5.93948)*(Stre_Red-0.92333)	-0.120488	0.005957	-20.22	<.0001*
(Disp_Blue-5.93948)*(Red D-3.69742)	-0.011466	0.001346	-8.52	<.0001*
(Pow_Red-8.25877)*(Rob_Blue-6.95993)	-2.773756	0.065089	-42.61	<.0001*
(Pow_Red-8.25877)*(Stab_Red-5.07463)	-0.147951	0.016303	-9.08	<.0001*
(Pow_Red-8.25877)*(Stre_Red-0.92333)	60.915222	3.686764	16.52	<.0001*
(Pow_Red-8.25877)*(Red D-3.69742)	3.5346213	0.162585	21.74	<.0001*
(Pow_Blue-9.06484)*(Rob_Red-6.2396)	-0.337291	0.032051	-10.52	<.0001*
(Pow_Blue-9.06484)*(Stab_Blue-6.13897)	0.0934923	0.021046	4.44	<.0001*
(Pow_Blue-9.06484)*(Stre_Red-0.92333)	-1.22269	0.229803	-5.32	<.0001*
(Pow_Blue-9.06484)*(Stre_Blue-0.99106)	-6.968522	1.423358	-4.90	<.0001*
(Pow_Blue-9.06484)*(Red D-3.69742)	-0.075666	0.0147	-5.15	<.0001*
(Pow_Blue-9.06484)*(Blue D-4.67447)	-0.638281	0.055391	-11.52	<.0001*
(Rob_Red-6.2396)*(Rob_Blue-6.95993)	-4.117799	0.0407	-101.2	<.0001*
(Rob_Red-6.2396)*(Stab_Red-5.07463)	0.1197663	0.016862	7.10	<.0001*
(Rob_Red-6.2396)*(Stab_Blue-6.13897)	-0.03386	0.001829	-18.51	<.0001*
(Rob_Red-6.2396)*(Stre_Red-0.92333)	8.736577	0.589606	14.82	<.0001*
(Rob_Red-6.2396)*(Red D-3.69742)	-0.397646	0.027889	-14.26	<.0001*
(Rob_Red-6.2396)*(Blue D-4.67447)	0.1625646	0.002438	66.68	<.0001*
(Rob_Blue-6.95993)*(Stab_Red-5.07463)	0.0350672	0.004892	7.17	<.0001*
(Rob_Blue-6.95993)*(Red D-3.69742)	0.931961	0.024724	37.69	<.0001*
(Stab_Red-5.07463)*(Red D-3.69742)	0.0599182	0.006885	8.70	<.0001*
(Stab_Blue-6.13897)*(Stre_Blue-0.99106)	-0.307834	0.153723	-2.00	0.0452*
(Stab_Blue-6.13897)*(Blue D-4.67447)	-0.032724	0.006111	-5.35	<.0001*
(Stre_Red-0.92333)*(Red D-3.69742)	-24.94967	1.152139	-21.66	<.0001*
(Stre_Blue-0.99106)*(Blue D-4.67447)	2.3524179	0.384328	6.12	<.0001*
(Conn_Red-5.40938)*(Conn_Red-5.40938)	0.4590018	0.005666	81.01	<.0001*
(Conn_Blue-5.59815)*(Conn_Blue-5.59815)	0.5536526	0.002028	272.99	<.0001*
(Disp_Red-6.28209)*(Disp_Red-6.28209)	-0.015045	0.000972	-15.49	<.0001*
(Disp_Blue-5.93948)*(Disp_Blue-5.93948)	0.0033687	0.000412	8.17	<.0001*
(Pow_Red-8.25877)*(Pow_Red-8.25877)	-4.077721	0.250089	-16.31	<.0001*
(Pow_Blue-9.06484)*(Pow_Blue-9.06484)	1.1563473	0.102441	11.29	<.0001*
(Rob_Red-6.2396)*(Rob_Red-6.2396)	1.9132598	0.042083	45.46	<.0001*
(Rob_Blue-6.95993)*(Rob_Blue-6.95993)	2.1542114	0.010734	200.68	<.0001*
(Stab_Red-5.07463)*(Stab_Red-5.07463)	-0.03848	0.002869	-13.41	<.0001*
(Stab_Blue-6.13897)*(Stab_Blue-6.13897)	0.0153711	0.001702	9.03	<.0001*
(Stre_Red-0.92333)*(Stre_Red-0.92333)	-266.8062	13.09057	-20.38	<.0001*
(Stre_Blue-0.99106)*(Stre_Blue-0.99106)	25.030033	5.240361	4.78	<.0001*
(Red D-3.69742)*(Red D-3.69742)	-0.590705	0.027275	-21.66	<.0001*
(Blue D-4.67447)*(Blue D-4.67447)	0.0794042	0.009507	8.35	<.0001*
(Rob_Blue-6.95993)*(Stre_Red-0.92333)	16.096905	0.478954	33.61	<.0001*

VITA

Nevan Edward Norris Shearer was born in Portsmouth, Virginia on March 14, 1983. He graduated from Old Dominion University in 2005 with a B.S. in Mechanical Engineering Technology. Continuing his education, he earned his M.E.M from Old Dominion University in 2007. After graduating with his M.E.M, Nevan was given an opportunity to earn his Ph.D. in Engineering Management while working as a graduate research assistant and graduate teaching assistant at Old Dominion University. He conducted this research during his time as a graduate assistant in the Department of Engineering Management and Systems Engineering, Old Dominion University, Kaufman Hall, Norfolk, VA, 23529.