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Coherence Between Wind and Current Events in a Nearshore, Shallow Water, Wave Dominated Environment

George Milton Hecker Old Dominion University

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COHERENCE BETWEEN WIND AND CURRENT EVENTS IN A NEARSHORE,

SHALLOW WATER, WAVE DOMINATED ENVIRONMENT

by

George Milton Hecker B.S. June 1944, U.S. Naval Academy M.S. August 1968, Rensselaer Polytechnic Institute M.S. August 1970, Old Dominion University

A Dissertation Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirements for the Degree of $\ddot{}$

DOCTOR OF PHILOSOPHY

OCEANOGRAPHY

OLD DOMINION UNIVERSITY August 1980

ABSTRACT

COHERENCE BETWEEN WIND AND CURRENT EVENTS IN A NEARSHORE, SHALLOW WATER, WAVE DOMINATED ENVIRONMENT

George Milton Hecker Old Dominion University, 1980 Director: Dr. Ronald E. Johnson

Five days of observation were made of the current at four depths in a water depth of eight meters, and accompanying record of local wind one-half Kilometer from shore at the U.S. Army Corps of Engineers Coastal Engineering Research Center field facility at Duck, North Carolina beginning five May 1978. The current was measured with a state-of-the-art electromagnetic current meter having a 0.2-second response time and an accuracy of \pm 2 centimeters per second. At each depth a ten-minute record was made each hour with a sampling interval of one-eighth second. The frequency components of the record were analyzed and found to be essentially free from The record of one hundred and twenty (less five lost noise and bias. due to loss of amplifier power supply) ten-minute averages was also analyzed for frequency components. Semidiurnal and diurnal tidal components and quadratic curve trends were identified. Analyses by fast fourier transform revealed that the component of the wind perpendicular to the coast had significant coherence with the alongshore component of the current.

DEDICATION

To my wife Janice Mori Hecker

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ACKNOWLEDGEMENTS

I wish to express my gratitude to the many people who provided assistance and especially encouragement to carry on throughout the course of this study.

First those at Old Dominion University. My dissertation director, Dr. Ronald Johnson, skillfully maneuvered me into a position in which I had to complete my research, when a personal tragedy intervened causing a two year slowdown. He also assisted me in the field calibration of the primary instrument. Dr. John Ludwick, in addition to continuous direct support, always made me feel that my work was critical to the advancement of science. Dr. Chester Grosch saw to it that I became an adequate programmer by the "toss him in" technique. He also dragged me (often screaming) through filter operations, Cooley-Tukey autocorrelations and fast Fourier transform spectrum analyses, quadratic curve-fitting, etc. . . . I came to dread his "why don't you try . . . " because it usually meant a whole weekend of writing programs for analyzing and plotting data. His spirited response to questions about data analysis and graphic output at all hours are memories to be treasured. Dr. George Ofelt provided an effective sounding board during meetings with my committee. Dr. Chin Kuo, a member of my committee until V.P.I. was fortunate enough to obtain his services, provided the engineer's approach so much needed and so greatly missed this past year. Dr. Earl Thornton,

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professor of mechanical engineering mechanics, became interested in my study because of the unique requirements of the instrument support structure. He, along with some of his graduate students, conducted a rather complete study of its vibrational modes. In addition, he joined me in the field to supervise the installation of accelerometers and to observe their response.

Many graduate students assisted in the transportation, installation and continuous operation of the equipment and portable laboratory. They include: Paul Bowen, Jack Frye, Fred Hilder (now Ph.D.), Greg Kopanski, Dennis Lundgren, Jack McCambridge, Al Moore, Ray Stewart, Dave Timpy and Kim Zauderer. A special thanks is due to two graduate student friends from Korea. Jong Yul Chung (now Ph.D.) who took the data acquisition system over after this study, paid me back handsomely by getting me off the ground with energy spectra. His associate Im Sang Oh continued to help me debug plotting and filtering programs. Help also came from Argentina in the persons of Gerardo Perillo and his vivacious wife Cintia both of whom were ever-present and helped me converse with the DEC-10 (University computer). Thurman Gardner, supervisor of the school of sciences instrument shop, welded the instrument support structure. Not one weld failed. Bobby Powell, of the same shop, constructed the instrumentation amplifier and provided certain of the electronic calibration and monitoring equipment in the trailer. The diagram of the amplifier was provided by Prof. Bill Thornton of the Electrical Engineering Technology department. The trailer, which served as a portable laboratory and bunk room, was provided by Dr. Gary Copeland of the Department of Geophysical Sciences for the

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price of ten days of ozone measurements. Other equipment from the same department included an anemometer and strip chart recorder, a barograph and a thermograph.

The staff of the computer center were most supportive. In particular Tim Hendrickson met every reasonable (and some unreasonable) request for facilities and services; Nancy Salyers worked out the "change" utility that was necessary for the final phase of data file creation within the University computer. Programming consultants that assisted include: Tom Brown, Bob Dawson, Juny Samson and Joy Woo.

Equipment from the Department of Oceanography included a mini computer with high speed paper tape punch, a tide gauge, salinometer, electric hoist and the use of the ship's truck to tow the portable laboratory to Duck.

This study could not have been conducted without the generosity of Dick McBirney and Larry Marsh of 8595 Grovemont Circle, Gaithersberg, Maryland 20760, with whom I had become acquainted while evaluating an earlier prototype electromagnetic current meter of their design (Hecker, 1970). They provided me the current meter employed as well as the advice of Jim Darbey of the Marsh McBirney company.

A large measure of thanks must also be given to the United States Army Corps of Engineers which allowed me to lash the portable laboratory to the end of the pier at the field research station at Duck, North Carolina. This permission was granted through contact with Dr. D. Lee Harris of the Coastal Engineering Research Center, Ft. Belvoir, Virginia. Dr. Harris also provided invaluable insight into the editing and analysis of time records. Contact at C.E.R.C.

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was shifted to Dr. Curt Mason (now director of the field station at Duck). At Duck, Charles Judge and his able assistant Gene Bichtner, could not have been more helpful during the course of the field work They furnished copies of data that was routinely being taken there. there and in addition provided an electronic power supply to replace the battery supply to the instrumentation amplifier.

I also appreciate the advice and offer of assistance by Jerry Appel and Dick Ribe of NOAA Instrument Test and Evaluation department in Washington, D.C.

Jim Davenport, a former graduate student with previous knowledge of the data acquisition system and now at NASA Langley, did an outstanding job on a consulting basis in repairing the PDP-8 and debugging the software. He also arranged for use of a computer at NASA and wrote the program to read the paper tape and write the data onto magnetic tape which could be read by the University computer. Milton Skolaut, the real master of the HP 2100, came to my rescue on numerous occasions when the going was rough trying to make the system work most efficiently. I am afraid these gentlemen, and their boss, Steve Katzberg, had cause to regret their offer. We were all surprised at the extent of the time and effort needed to do the job.

Financial support was provided as follows: Department of Oceanography, for payment of consulting fee and for the purchase of paper tape: \$273.32; School of Graduate Studies, for materials to manufacture some of the needed structure: \$200.00; Southern Regional Education Board, Atlanta, Georgia, for travel costs required in the use of facilities not available at the home institution (The Alfred P. Sloan Foundation): \$427.30.

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A special word of thanks to my close personal friends Carvel and "AD" Blair, whose constant encouragement and weekly care and feeding sustained me during the last eight months. Finally, the most important, my family. Words cannot express all that I owe to my marvellous and completely unique wife. The month before the field work, she began a losing battle against cancer. She would not let me abandon my study. My daughters Carla and Sandy have hovered over me constantly and have (as a birthday gift) paid for the typing of this paper. My son Mark procured the material used in the construction of the entire support structure. As a matter of principle, except for stainless steel bolts, paint, and two replacement skate-board ball bearings, all of the material is recycled (i.e. junk). In addition, Mark put in two days on watch in the portable laboratory.

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Chapter 1

INTRODUCTION

One of the most difficult tasks in the investigation of natural phenomena is the reduction of number of variables. As our instrumentation and our data processing techniques have become more sophisticated, our models of these phenomena have improved. Analysis of data that more closely represents the real world, allows us to identify and separate some of the variable forcing functions. It is axiomatic therefore, that existing models must be reexamined and improved whenever significant advances are made in our ability to measure and analyze. Such an advance is the electromagnetic current meter: a meter without moving parts in which an electrical potential proportional to the current is produced by the flow of water through a magnetic field. Although its origin can be traced to Smith and Slepian (1917) it has only been during the last decade that these meters have become available to all investigators. Many excellent studies made during the past ten years with the new meters have been reported. Some of these are on a fairly large scale which is necessary to model circulation, while others are in the process of planning, execution or analysis.

This study focuses attention on a region that has not been recently reported; this region is just beyond the surf zone, beyond the "radiation stress"-created "longshore" current, but where the

waves still produce the dominant instantaneous motion, which heretofore has caused errors in data taken by impeller-type instruments. A region Murray (1975) referred to as follows:

... In contrast to recent advances in our knowledge of both wave-driven currents inside the breaker zone (Longuet-Higgins, 1972) and currents on the continental shelf (Smith, 1974), wind-driven currents in the intermediate zone [underlining added] immediately seaward of the breaking point, where the effect of the coast is still of great importance, remains poorly understood.

The nearshore region is a subset of the coastal boundary layer which is in turn a subset (approximately 10 Km wide) of the continental shelf. Hails (1974) divides "nearshore" into swash, surf, breaker and offshore zones. As mentioned above, Murray has used the term "intermediate zone" to designate the region between the surf and breaker area and the rest of the coastal boundary layer.

Previous Work

As stated above reports of work done in this water depth with electromagnetic current meters were not found although it is known that such is currently in progress or was simply not accessible at the time of planning of this investigation (1977).

Presented here is a summary of some of the literature which deals with the area and results of investigations some of which are based on measuring devices other than the electromagnetic current meter.

First consider the overall shelf circulation which will make a contribution to the motion in the intermediate zone. This circulation has in the past decade been under intensive study. Niiler (1975) identifies the forcing functions of shelf circulation as local

variations in atmospheric wind stress and thermohaline forcing on both synoptic and seasonal time scales, eddies and long waves which impinge from the deep sea and tidal phenomenon. He cites reported high coherence between alongshore variable currents and alongshore variable winds and notes that "precious little is known on parameterization of turbulent diffusion near shore." Csanady (1976) identifies the coastal boundary layer of about 10 Km width and the mid-shelf region beyond. He identifies forcing functions as winds, tides, river inflow, longshore pressure gradients and density differences. He reports that while the general flow within the CBL (coastal boundary layer) within the New York bight is west and south, "complex average effects of variables probably predominate." Csanady (1976b) reporting at the Continental Shelf Conference at Brookhaven National Laboratory in November 1975 further grouped the forcing functions into wind stress and horizontal pressure gradients; the latter being further subdivided into density differences and tidal waves propagating toward shore from deep water. With respect to wind-driven flow he identifies depth as the dominant influence. He explains a model in which the slope caused by wind stress exactly balances the wind stress: wind stress/wind set-up (in geostrophic balance) where

$$
g \frac{\partial \zeta}{\partial x} = \frac{F}{h}
$$

 $x =$ the displacement in the direction of the wind $F =$ kinematic wind stress = T/ρ = wind stress : density $h = depth$

 $g = acceleration of gravity$ $\partial \zeta / \partial x$ = slope of sea surface when in balance.

If one examines the difference in slope between 100m and 10m depths (other factors constant) the slope in 10m is ten times that in 100m (e.g. 10^{-6} vs 10^{-7}) for the same wind stress. Csanady goes on to define topographic waves, coastal jets and Kelvin waves as forcing functions having horizontal pressure gradients. He also discusses "frontal adjustments" arising from the presence of less dense water near the coast (as from solar heating) creating a lens of low density and thereby uncoupled. Concerning studies of shelf circulation he specifically warns that random survey work over such a vast region which is subject to such complex physical influences is likely to be wasteful. Csanady (1978) adds a discussion of the "arrested topographic wave" (wave motion which is modified by topographic features) and discusses causes of south-going currents in the mid-Atlantic bight. Csanady (1980) discusses longshore pressure gradients caused by offshore winds. Here he refers to a region just outside the CBL in 50m depth, and observes that an offshore wind can produce an acceleration to the right (alongshore). This was one of the purposes of the Coastal Boundary layer transect experiment (COBOLT) to "elucidate the structure of flow along a straight and relatively uncomplicated open oceanic coast."

In this experiment four vertical arrays of four electromagnetic current meters each were moored 3, 6, 9, and 12 Km from shore. Allen (1980) in summarizing models of wind-driven currents on the continental shelf reports that "the important forcing function is the alongshore

component of the wind stress which drives the cross-shelf transport in the surface Ekman layer." Winant (1980) reviewing coastal circulation and wind-induced currents cites the large number of studies precipitated by the availability of better instrumentation. He also mentions the modifying effects of climate, geomorphology and stratification on the standard forcing functions. Winant identifies edge waves (Munk, 1956, 1964) as being of atmospheric origin and also speaks of a number of events that occur at tidal frequencies. He points out that forcing functions of periods between one and thirty days are generally related to atmospheric events, and referring to Beardsley and Winant (1979), concludes that coastal sea surface slope of approximately 10^{-7} is caused by large scale oceanic circulation. Winant also points out that high coherence between coastwise currents and wind stress decays with distance offshore and that currents that propagate along shore are trapped along the boundary either by refraction (edge waves) or coriolis (Kelvin waves). The determination of coherence by the utilization of empirical orthogonal functions, "EOF" is also discussed. Some 26 reports of local wind forcing (almost exclusively in water of 30m or greater depth) are cited and summarized:

... The number of studies involved on the shelf of North America alone underlines the differences in observation. To say that the only common denominator between these reports is that when the wind blows alongshore, the water moves in the same direction is an oversimplification, yet clearly the wide variety of climatological and geomorphological setting results in different shelf responses to wind forcing.

Under future work Winant states that we need to know how the inshore frictional dynamics merge with the offshore Ekman dynamics: "The resolution of boundary layer and internal shear-layer dynamics is one of the most pressing areas where observations are required."

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in the Gulf of Mexico. He reports that nearshore currents respond with a lag of 3 to 5 hours to the sea breeze cycle with amplitude up to 25 cm/sec, essentially parallel to the coastline. Current reversal in both a.m. and p.m. was counter clockwise taking about one hour to reverse. Measurements were made with drogues and ducted, vane-type current meters. Observation and analysis with a depthdependent eddy viscosity show a three-tiered velocity profile with near surface and near bottom flow predominantly parallel to shore with a small offshore component. At mid-depth flow was still alongshore with a small offshore component. Murray (1975) provided a more complete report of the seabreeze experiment, and commented on the inability of the impeller current meters to cope with shallow-water, wave dominated flow. (This was underscored by personal communication two years later that the current meter data was not used.) He concluded that the direction of currents driven by local winds is predominantly alongshore; there is little dependence on wind speed or direction. Under moderate conditions, current speed is strongly controlled by the wind angle to the shoreline and to a lesser degree by the wind speed. Murray reaffirmed that solutions to the model of Jeffreys gave "good" results using a constant eddy viscosity and a coriolis term. Winant and Olson (1976) made observation in 18m depth employing a vertical array of four electromagnetic current meters for 33 days 1.5 Km off coast of La Jolla, California at the U.S. Naval Undersea Center during late summer. The current meters signal processing unit time constant was set to give four 15-minute averages per They concluded that there is no significant coherence between hour. wind and current (0.43 at 95% confidence) over the study period.

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Much work has been done which is in or near the "intermediate zone." However, there are two important similarities of these investigations; most were done in a water depth greater than nine meters and most were done with drogues or impeller-type meters. Herewith is a summary of some of these studies and comments.

Harrison et al. (1967) reported a major study of shelf waters off the Chesapeake Bight using primarily Lagrangian methods which shows south-going alongshore currents in April and June but northgoing bottom current in April in this study area.

Hails (1974) points out that there is a need to determine the budget of water flowing in the nearshore (speaking of inside the breakers) circulation system. Murray (1972), using a vane-directed impeller instrument, studied wind-driven currents in a water depth of 15m. After removal of the K1 and M2 tidal components, he analyzed the nontidal currents as wind-drift or induced, slope currents caused by wind set-up against a coast or as density currents. Murray concluded that light onshore winds resulted in alongshore currents; strong onshore winds produced seaward bottom flow. He also reported that he observed a five to ten degree Ekman flow in an open area and suggests that the model by Jeffreys (1923) was still the best model available. Swift et al. (1973) employing a meter with orthogonally mounted impellers in a water depth of ten meters, found southeast bottom currents of 15 to 20 cm/sec "several hours" after a northeasterly wind of 25 m/sec (the work was off False Cape, N.C.). Sonu et al. (1973) made a preliminary report of an extensively instrumented experiment to study the sea breeze effect. Observations were made between the shore and a water depth of 12m for a period of ten days

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Scott and Csanady (1976) refer to the coastal boundary layer transect experiment (COBOLT). The measurements were made 11 Km south of Long Island, N.Y., in water of 32m depth for 23 days in September 1975. They compared alongshore wind stress to onshore bottom velocity as well as alongshore wind stress and alongshore current and found a better linear regression model of the former rather than the latter. They also conclude that the averaged current meter data must be accurate to a high degree to get such agreement. They report an accuracy of \pm 0.2 cm/sec at the bottom and 1 to 2 cm/sec at the surface. They also state:

... Tidal motions and wind-induced transients are independently driven and are not significantly influenced by the residual circulation. Thus they act much as molecular agitation in a fluid.

Barwick, Evans and Weber (1977) refer to the extensive literature about the shortcomings of surface current meter data and report on a unique radar system to determine a complete, wide-area, synoptic, surface circulation picture. Their "preliminary estimate of the precision of current velocity measurements show it to be better than 30 cm/sec" (effort being made to check this figure for possible typographic error). Hunter, Boicourt and Hacker (1977) recorded velocities off the entrance to Chesapeake Bay in 38m depth employing impellertype meter for 27 days. They expressed a need for better local atmospheric data, and measurement uncertainties in the cross-shelf They report that current response to wind stress seemed direction. to be instantaneous. Ludwick (1978) evaluated near surface and bottom data from 3.5 to 7 Km offshore in depths from 8.2 to 32.8m for two one-month periods (July-August and September-October) made by moored

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Savonious-rotor current meters. He also cited extensive literature about current meters in an oscillating flow and stated:

. . . . These known defects are nearly enough to discourage any analysis of the data set whatsoever: however when an assessment is made of the present state of knowledge of shallow-water coastal flow fields, it appears that there could be some profit in the study of the subject measurements.

He then proceeeds to do a masterful job of identifying seven tidal components of significant energy and discusses the competency of these currents coupled with net north and south (depending on time of year) bottom currents. He reported that current response time to wind stress appeared to be less than three hours. Finally, Winant and Beardsley (1979) compared some wind/current data results and reported that the large spread in results may be explained by any combination of: experiment differences in measuring currents, geomorphological differences in sites, similarity between tide and wind events (in time and amplitude) and in "additional dynamics." One must take note that these comments referred to conditions in water depths of 25 to 40m deep.

Purposes of the Study

Determine the coherence between wind events and current events in a nearshore, shallow water, wave dominated environment.

Determine the suitability of a state-of-the-art electromagnetic current meter for such an environment.

Chapter 2

FIELD PROGRAM AND METHODS

Data Requirements

In order to study the interaction between wind and water motion it is necessary to obtain vertical profiles of horizontal water velocity (from which components other than wind-driven have been removed) and wind velocity at the interface. Ideally these should be determined continuously and synoptically over as large a horizontal area as possible.

Other data to allow isolation of the required data could include: data to provide a complete description of wave climate for use in boundary roughness and wave-drift consideration, data to provide description of sea surface still water level elevation over a wide area to provide vertical distance from surface to measured water velocity (a function of the tide) and to determine local and area sea level slopes caused by many different forcing functions (such as horizontal barometric pressure distribution and various large-scale inertial water mass movements), data on possible coastal fresh water run-off or if near an inlet data to determine effect of such flow, data to describe the vertical density distribution in the water column again over a wide area to provide analysis of stratification (and resulting momentum transfer loss) as well as to detect

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large scale water mass movements such as ocean fronts, and finally visual observation (such as remote sensing or aerial photography) to assist in the identification of some of the above features (and including local rip currents).

Under the constraints imposed by a single, moderately funded, observer the following primary equipment was assembled to obtain adequate data to achieve the purpose of the study:

1. One model 511 Marsh-McBirney electromagnetic current meter

> One Climet model 026-1 recording anemometer $2.$

 $3.$ One General Dynamics model 3040 Tide guage

4. One Bekman model RS5-3 Induction Salinometer

One Belfort model 5-800A Microbarograph $5.$

One DEC PDP-8 computer with Facit-Addo model 4070 high 6. speed paper tape punch

> $7.$ One manufactured instrumentation amplifier

8. One manufactured instrument support structure.

Secondary equipment included a precision two channel strip chart recorder, precision digital voltmeter, sensitive CRT for noise analysis, accelerometer system to analyze motion of current meter, a rain gauge, a thermograph and a camera. Details of construction, calibration and performance of important equipment are included in Appendix A. Figures 1 and 2 show general equipment arrangement.

Site Selection

A study of any natural process is so complex that the investigator who cannot instrument a large area must try to minimize effects

Fig. 1. Interior of the portable **laboratory**

Fig. 2. External view of portable **laboratory**

that cannot be measured. The topic was chosen to test the validity of data taken in a difficult environment. In an effort to minimize some of the potential problems a coastal site was chosen along a long straight beach as far as possible from inlets, a beach also with minimal run-off effects during heavy rain, etc. Such a location is Duck, North Carolina, chosen by the Army Corps of Engineers for the site of their coastal engineering field research facility (Mason, 1979). A water depth of less than nine meters but clear of the breaker zone and the region of wave-created longshore current was desired. This is easily achieved on the pier at Duck. The depth at a location of 29.6m (97 ft.) from the seaward end of the pier (519.1m [1703 ft.] from shore end) was found to be 8.72 ± 0.61 m (28.6 \pm 2.0 ft.). At this depth one is led to expect that wind-driven currents will be in the same direction as the wind, modified of course by the presence of the shore. It was planned to conduct the experiment during the latter part of winter to minimize the possibility of not having a homogeneous water column. The schedule was delayed, and as expected, some stratification was found (see Figure 3).

Field Program

As previously stated, a prime requirement was to provide data for a vertical profile of horizontal current. This was accomplished with one current meter by recording data for ten minutes out of each hour at four different depths (measured from the lowest position, one meter above the bed). Data acquisition began at 1800 EST on May 5, 1978 and continued for 120 hours.

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Fig. 3. Site location chart showing CERC field
research facility at Duck, N.C.

The sampling rate during the ten minutes was eight times per second or 4800 digital data points in each record for both the x component (parallel to shore) and the y component (perpendicular to the shore).

The Δt of $1/8$ second was selected as the shortest practical interval with the available equipment. A higher rate was attempted during the rehearsal data acquisition period 8 and 9 April but inadequate time was provided to change the depth of the sensor and to replace the paper tape. Forty minutes of recording essentially required one full roll of paper tape.

The sampling rate should, over a ten minute period, adequately represent phenomenon whose periods are between the Nyquist frequency, $f_n = 1/2 \Delta t$, four cycles per second and the resolution frequency. The resolution frequency depends on parameters chosen during data processing but it will approach that which has a period of about twice the length of the record. Therefore, each ten minute record will give an acceptable representation of frequencies between 4 and 8×10^{-4} Hz (or periods between 0.25 sec and 20 minutes). This however could be modified by the response time of the instrument which in this case is 0.2 sec ("response time" is the time in seconds from impressment of a voltage on a circuit and the time when the voltage in the circuit builds up to 63% of the amplitude of the impressed voltage). The spectral window created by $t_0 = 0.2$ sec is calculated as follows:

The spectral power function of an exponential filter is

$$
H(f) = \frac{1}{1 + 4\pi^2 f^2 t_0^2}
$$

Assuming that a spectral power of 100^{-2} is sufficient, the spectral window can be determined by

$$
100^{-2} = H(f_w) = \frac{1}{1 + 4^{2} f_w^{2} t_0^{2}}
$$

from which $f_w = 7.92$ Hz, hence the frequency range of the electromagnetic current meter itself (with time constant set at 0.2 sec) is 0.00 to 7.92 Hz, and we are not limited by the time constant but by the digitization interval (0.125 sec).

We see therefore that the ten minute sampling period will provide adequate representation of all water movement between 4.0 and 8.0×10^{-4} Hz (periods between 0.25 sec and 20.00 min). This band includes motion caused by wind and gravity waves and turbulence (Chung, 1979). Vortex shedding was not considered to be a problem unless it created motion of frequencies above 4.0 Hz, and care had been taken to see that this was not occurring.

The wind direction and velocity ten m above the sea surface was measured continuously and recorded on a strip chart recorder.

The barometric pressure was recorded continuously.

Temperature and salinity measurements of the water column were taken three times each day.

Air temperature was continuously recorded by a thermograph placed on the pier under the trailer.

The elevation of the S.W.L. (still water level) was recorded automatically eight times an hour.

Visual observations of wave heights, periods and directions were made and recorded as well as photographed three times per day.

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Precipitation was measured and recorded after each precipitation event.

Any other features seen during daylight hours, such as rip cells, reflected waves, turbidity and turbidity interfaces and marine life activity (large runs of bluefish) were noted.

The bottom was visited two times (at the beginning and end of the sampling period) to get a sediment sample and description of bed forms.

Current Meter Calibration

Because of the importance attached to the care with which one must take data in such a high energy environment, a detailed description has been provided in Appendix A. The following is a brief summary of the various steps in the process.

1. Calibration by manufacturer.

 $2.$ Check operation and interfacing with computer (PDP-8) in university flume.

- $3.$ Open-water calibration against known mechanical meter.
- $4.$ On site pre-installation operational calibration.
- $5.$ Calibration during sampling period.
- 6. On site post-installation operational calibration.
- $7.$ Calibration recheck by manufacturer.

Chapter 3

THE DATA

Current Data

Orthogonal vectors of current at four depths were measured: parallel to the shore x, and perpendicular to the shore y. Positive x is southward and positive y is shoreward. The coastwise component of the current was predominantly north-going at all depths (95 out of 120 hours). The vertical structure of the on-offshore component of the current exhibited a complex pattern, but during the 120 hours of this study the surface current was predominantly offshore, the bottom onshore and the mid depth currents approximately equally distributed. Each vector was recorded eight time per second for ten minutes of each hour. One hundred and twenty such records were obtained.

Wind Data

The wind speed and direction ten meters above the sea surface recorded continuously. The coastwise component was predominantly toward the north. The on-offshore component was more evenly distributed, but more offshore (75:45).

Processing of Current Data

The sampling program was controlled and recorded by the data acquisition system planned for a previous study at Old Dominion

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University (Ludwick, 1977). The heart of the system is a DEC PDP-8 computer which controlled the sampling and the paper tape punch. A computer (HP 2100) at NASA Langley was employed to read the paper tape, translate it and record the data on magnetic tape. This magnetic tape was then read into the University DEC-10 disc storage by another translation utility program ("CHANGE") and again from disc back to magnetic tape. Fortunately this procedure is no longer necessary at Old Dominion University; a system now exists which permits recording directly on magnetic tape.

Selected ten minute records were first edited for any spurious In general, there were no such signals. Problems were signals. discovered which required some editing of data identification code to permit the records to be read by analyzing programs. It was also discovered that some five hours of records were lost because the amplifier batteries lasted only six hours instead of the estimated two to three days. The records required less than 2.5 percent editing whenever the record was made.

After editing, these records were subjected to spectrum analysis to determine if they were coherent, unbiased representation of the real phenomenon, free of noise and instrument motion (see Appendix B for computer programs). Analysis of the ten minute records was accomplished by filtering and autocorrelation techniques (Grosch, 1978) while that of the 120 ten-minute averages was done by least squares, curve-fitting, and fast fourier transform techniques (Hinich and Clay, 1968). A brief summary of the spectral analysis methods employed is included as Appendix C. Figure 25 (p. 46) indicates that the frequencies with the greatest energy are the wind and gravity waves observed.

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During the course of this analysis a surprising discovery was made that should be further studied. One of the records which was passed through various filtering operations, revealed a clear component with a period of about ten minutes (see Figures 4 and 5). One possible forcing function suggested (Dr. Grosch, private correspondence) is a precursor wave in the atmospheric frontal system (see Figure 6 for the barometric pressure record).

Once it was assured that the data are valid, each ten minute record was then averaged, converted to units of cm/sec, corrected for zero offset and amplification factors and stored in a final data file with the time and distance from the bed for each record. Figure 7 (p. 24) gives a summary of these data for the purpose of illustrating the extent of editing.

The wind data were graphically integrated, filed and converted to hourly average orthogonal components in units of meters per second. See Figures 8 and 9 (pp. 25 and 26) for the vectors of wind speed ten meters above the sea surface.

Final Analysis of Data

Figures 10 through 17 (pp. 27 through 34) show the complete 120 hour record of the orthogonal vectors of current at each of the four levels. It must be noted that a linear function has been substituted for the missing hours of data (hours seven through eleven).

One notices that the data appears to present almost three quarters of a complete frontal passage cycle. This is substantiated by Figure 6 the barometric pressure record.

20

1MIN1A, AVG : FLUCT(U)

Fig. 4. Alongshore component of a component having a period of about one minute.

Onshore component of a component Fig. $5.$ having a period of about one minute.

Fig. 6. The record of the barometric pressure.

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Fig. 7. The low-passed data. Blocked out numbers indicates lost data.

Fig. 8. The alongshore component of the low-passed wind record.

Fig. 9. The onshore component of the low-passed wind record.

Fig. 10. The alongshore surface component of the low-passed current record.

Fig. 11. The onshore component of the surface, low passed current.

Fig. 12. Alongshore component of the second level.

Fig. 13. The onshore component second level.

Fig. 14. The alongshore component of the current at the third level.

Fig. 15. The onshore component third level down.

Alongshore component of current at the Fig. 16. bottom level (1.0 m above the bed).

The bottom component of the onshore component Fig. 17. of current (one meter above the bed).

The first step taken was to fit a least squares quadratic function to the data. See Figures 18 through 19 (pp. 36 and 37) for examples of the quadratic functions obtained for the components of the surface current and for wind ten meters above the surface.

This function was then subtracted from the data and the resulting record examined for tidal components. The data was bandpassed through Doodson's semi diurnal tide (see record of sea surface S.W.L., Figure 20 [p. 38]) filter and then through Doodson diurnal band pass filter (Godin, 1972). See Figures 21 and 22 (pp. 39 and 40) for these tidal components of the surface record. One normally would then have subtracted these components from the record from which the quadratic function had been removed. To do so, however, would have, owing to the truncating effect of filtering operating, reduced the record length to 56 hours. Analysis showed that the energies of the semidiurnal and diurnal tidal components to be 2.2 and 8.0 percent respectively of the total energy. It was considered the lesser of the evils not to remove these components prior to examining wind and current coherence.

Finally then, the orthogonal components of the wind and current records were normalized by dividing by the square root of the total energy, subjected to fast fourier transforms and examined for spectral properties and coherence.

Fluctuating component of the low-passed
record of the sea surface (tide guage). Fig. 20.

The semidiurnal component of the surface Fig. 21. alongshore current.

Fig. 22. The diurnal tidal component of the alongshore component of the surface current (please note the change in scale).

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Chapter 4

EXPERIMENTAL RESULTS

A preliminary result of this study is the production of data having a demonstrated reliability. Approximately 115 ten-minute records of orthogonal components of water motion at each of four depths have been produced which can be used to study events having periods between one fourth of a second and twenty minutes. This would include phenomenon such as wind and gravity waves, edge waves and turbulence. One hundred and twenty-hour records (less one five hour period) of the low-passed (one ten minute average each hours), orthogonal components of water motion at each of four depths was produced. Further study with this output include such phenomena as atmospheric interaction, turbulent transfer of momentum, bottom friction and sediment transport, tidal component and stratification effects.

The specific result of interest is that the greatest coherence between wind and current events was found to exist between the component of the wind perpendicular to the shore and the alongshore component of the current.

The uniqueness of this study is twofold: employment of a state-of-the-art electromagnetic current meter and measurement in a nearshore, shallow-water, wave dominated coastal environment.

Data recorded to accompany that of the wind and current includes temperature/salinity distribution, barometric pressure,

precipitation, wave observations (visual), tide record, bottom profile, sediment samples, air temperature. Detailed wave records for the same period are on record at the Coastal Engineering Research Center, Ft. Belvoir, Virginia.

Chapter 5

DISCUSSION

Validity of Data

More than one hundred operations and resulting graphs were performed on the data to establish its validity. Of the ten-minute records, the study concentrated on the first hour, 1800 E.S.T. 5 May. For purposes of simplicity we will discuss and illustrate treatment of the onshore component of the current one meter below the surface. In analyzing this record, amplitude was uncorrected and in units of 0.0195 volts. The reader may view them as arbitrary units. Analysis of these records was accomplished by use of the Cooley-Tukey method of autocorrelation (Blackman and Tukey, 1958). The selection of this method was based on need for practical experience. Alternatively, and again primarily for experience in methodology, the fast fourier transform method was used on the low-pass 120-hour record.

Attention is now invited to Figures 23 through 28 (pp. 44 through 50) which represent a series of analysis operations performed on the ten-minute records; the onshore surface current record is used for illustration. In viewing these graphs note that positive is onshore. The first three figures show a portion of the high-passed (mean subtracted) record, the autocorrection function and the spectral energy. The autocorrelation function by failing to decay uniformly

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Fig. 23. An example of the appearance of record
with only the average subtracted.

t,

Fig. 24. The autocorrelation function of the fluctuating component of onshore component of surface current.

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surface current.

to a value below 0.2 but instead increases and decreases reveals the presence of a relatively low "beat" frequency component similar to surf beats (see Bendat and Piersol, 1971, pp. 17-21). This is also evident in viewing the entire unfiltered record (not illustrated herein). The spectral energy distribution clearly identifies the presence of several wave systems with periods between six and twelve seconds which agrees with visual observations. Figure 26 shows the effect of a two hertz low-pass filter on the record of Figure 23 (p. 44). The next two figures (27 and 28) illustrate the same data, first high-passed and then treated with a sliding average-type filter operation in an effort to isolate the periodic signal mentioned above. This process was repeated with seven different averaging periods. Those analyses show no evidence of white noise and present a spectrum which seemes to represent the natural environment observed. During this analysis the number of lags selected was adjusted to the number of data points (when the record was truncated by filtering) to keep the number of degrees of freedom at 18 or better.

Coherence of Wind and Current Events

Having been assured that the data was valid each ten minute record was then treated as a single data point representing an average value of the current for that hour. You will notice in Figures 10 through 17 (pp. 27 through 34) that there is a five hour period with no usable data as explained earlier. In view of behavior of the data, and in order to get a better look at the lower frequencies the five hour period was replaced by a linear function between the adjacent values. Examination of Figures 6, 8 and 10 (pp. 23, 25 and 27) shows

Fig. 26. A sample of the fluctuating component of the onshore current that has been smoothed with a two hertz low pass filter.

Autocorrelation function of the onshore bottom Fig. 27. current that has been filtered by removal of the mean and a 1-minute sliding average.

Spectrum of the onshore bottom current Fig. 28. that has been filtered by removal of the mean and by a 1-minute sliding average.

the obvious coherence of the overall, low-passed data; barometric pressure, orthogonal components of wind and orthogonal components of current (the orthogonal components of the wind are also low passed; being integrated hourly). Figures 29 through 36 (pp. 53 through 60) are representative of the final sequence of analyses. Shown is spectra of the low-passed components of the surface wind and surface current without removal of any specific components. It was found that the quadratic trend functions (Figures 18 and 19 [pp. 36 and 37]) were so similar that a better correlation would be obtained without their removal. This decision seems justified by Figures 37 through 40 (pp. 61 through 64) which show the coherence of all of the possible combinations of the normalized orthogonal components of wind and surface current. It is very obvious that the coherence between the wind perpendicular to the coast and the alongshore current is striking. Even though the highest coherence is in a frequency band of low evergy, the overall coherence is significant. The phase relationship between these components of high coherence is shown in Figure 41 (p. 65). The distinct difference in phase between events of periods above and below ten hours indicate that there is a different mechanism involved between these events. Short period events may be direct wind-drift while longer term events may be the result of geostrophic balance; the former being more instantaneous than the latter. It may also involve the seabreeze effect (Sonu et al., 1973).

The confidence limits for the fast fourier analysis (Hinich and Clay, 1968) are computed as follows:

$$
\sigma
$$
 (f_k)² = $\frac{1}{2r} \cdot \frac{3}{8} [1 - \gamma_{xy} (f_k)]^2$

 σ (f_k)² = variance at a particular frequency k

 $r =$ number of sets of values treated with FFT (e.g. 4 sets of $32 = 128$)

 $\frac{3}{8}$ = factor introduced by employment of hanning smoothing $\gamma_{xy}(f_k)$ = relative coherence between x and y at a frequency k.

e.g. if $\gamma_{xy} = 0.8$ at a particular frequency then

or

$$
\sigma (f_k)^2 = \frac{1}{8} \cdot \frac{3}{8} [1 - (.8)^2]^2
$$

$$
= 0.006075
$$

$$
(f_k) = 0.078
$$

for 95% confidence (2 \geq 0.16); . value lies between 0.816 and 0.784.

Phase relationship between the spectral Fig. 31. energies of the orthogonal components of the wind.

of the low-passed surface current record.

of the low-passed surface current.

components of spectral energies of the surface current.

components of the low-passed record of the wind.

Fig. 36. Phase relationships between the spectral energies in the orthogonal components of surface current.

Coherence between the spectral energies
in the alongshore wind and surface Fig. 37. current.

Coherence between the spectral energies Fig. 38. of the onshore wind and the onshore current.

current.

Fig. 40. Coherence between the spectral energies
of the onshore wind and the alongshore.

Fig. 41. Phase relationships between the spectral energies in the onshore wind and the alongshore current.

Chapter 6

CONCLUSIONS

In the intermediate coastal zone between the breaker zone and the remainder of the coastal boundary layer the greatest coherence between wind and current events is that between the wind component perpendicular to the coast and the current component parallel to the coast for events whose period is about one week or less.

In the relationship between the component of the wind perpendicular to the shore and the component of the current parallel to the shore, there is a distinct difference in mechanism between those events that occur more and those that occur less often than every ten hours.

There exists a component primarily in the alongshore current having a period of about ten minutes.

The electromagnetic current meter is well suited for studies in a wave-dominated environment.

The pier at the Coastal Engineering Research Center field facility at Duck, North Carolina, is an outstanding research resource.

The data acquisition system organized for this study performed virtually flawlessly throughout the study.

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APPENDICES

APPENDIX A

CALIBRATION AND SCALING FACTORS OF THE CURRENT METER AND DESCRIPTION OF THE EQUIPMENT SUPPORT TOWER

Prior to the delivery of the Marsh McBirney model 511 current meter it was calibrated at the company. This is accomplished by measurements made in a small flume which are matched to a standardized meter in the same flume at the same speed. It is not possible to do an absolute calibration in a flume unless the cross-section exceeds a square meter. This is true because the meter probe creates a magnetic field. If this field is distorted by a nearby discontinuity it may give a slightly modified response (Hecker, 1970).

Upon receipt, the meter was checked out in the Department of Oceanography flume. Calibration against an accurate pygmy current meter (as well as lagranginan measurements of flume velocity) confirmed that the meter was operating properly. It also confirmed the above statement about absolute calibration (see Figure 42). Also at this time noise measurements were made with sensitive strip chart recorder and oscilliscope. The noise amplitude was equivalent to 0.5 cm/sec at 30 cm/sec. At this time the calibration of the meter was measured and recorded. No attempt was made to adjust the output to the exact specification. When in the "calibrate" position the signal output should be exactly 0.500 v. For the best accuracy this should be measured by a sensitive digital voltmeter (which became a

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Fig. 42. Check of operation of meter in the flume at Old Dominion University.

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permanent fixture in the portable lab). If one simply accepts that the output meter needle is in the black portion of the "calibrate" mark on the face of the meter, the actual deviation from 0.500 volts can be enough to make the difference between a net flow on or offshore (as indeed was true in this study).

During the flume tests the output of the meter was fed into the PDP-8. A system was developed to read back the last value in the data queue when the "halt" was pressed by reading the contents of that address as shown by the binary light counter on the computer. (One also quickly developed the ability to read the holes in the tape and compare them to the output of the meter as read by digital voltmeter). It was at this point that the inability of the DEC 10 system to read the paper tape at that time came to light.

During the flume tests it was again noted that if the pygmy and the E/m meter are operated together, the meter will register the impulses created as the pygmy's rotor turns. A test was therefore made of the accelerometers (induction type) that were to be attached to the support structure. These devices created no interference of any kind.

Because the meter was on relatively short-term loan the planned two-day rehearsal period at Duck had to proceed on schedule. To conduct the entire program without such a test, followed by data analysis and possible test plan revision, is to invite serious problems and justified charges of "mindless monitoring." One final step was taken, however before mounting the rehearsal. As previously mentioned, the current meter has a built in calibration that is a positive check of the electronics but tells nothing about the sensor.

Furthermore, in a high energy environment, or difficult equipment installations, the sensor and cable can develop problems that may not be evident until one tries to analyze the data (Ludwick, 1977). In order to provide an on-site check of the condition and constancy of the sensor a device was made to permit a controlled operational check of the sensor (see Figure 43). The sensor is attached to the bottom of a pendulum which was swung through the same arc and depth of water Since this measurement is a relative check of the operaeach time. tion of the whole system, the first check was done at the University laboratory. To insure similarity, the water in the test tank was adjusted to a salinity of 30% o. One will notice when employing an electromagnetic current meter that the higher the salinity the higher the signal to noise ratio. The record made by the strip chart recorder was preserved to check against each subsequent check.

Upon installation at the site the same calibration and noise checks were made. The inexperienced investigator is apt to panic at this stage until he becomes familiar with grounding problems ("ground loops," etc.). The experience at the end of the pier at Duck was almost unique in this respect because it turned out to be so quiet electrically. Not only was the noise level very low, but isolation transformers that were taken along were not used. It was also discovered that operation of the meter in the external power mode (charging battery) was just as noise-free as on battery.

During the progress for the data acquisition the meter calibration check was made frequently and found to be essentially constant on both channels $(x = 5.007; \sigma = 0.003$ and $y = 4.988; \sigma = .003)$. The amplification factor was also checked frequently and found to

Fig. 43. On-site equipment system check

vary slightly. It was in fact readjusted when the power supply was changed. These values were frequently recorded and entered as corrections at the appropriate time.

At the conclusion of the rehearsal sampling it was planned to run a complete analysis of data collected. The problems of reading the paper tape remained unresolved. As a desperation measure, almost one full minute of the paper tape was read optically, converted to velocity and plotted. This record was compared to the analog record made at the same time. The results, Figure 44 confirms that the system was operating properly and the full field program was mounted on schedule and as planned. Again the field "calibrations" were done before and after installation. Each of the five field calibrations of the sensor were identical which proved that the meter was neither drifting nor changing calibrated output.

Immediately upon return to Norfolk a complete open water calibration of the meter was conducted in a reversing tidal flow without nearby boundaries. The results of this test are shown in Figure 45. The x-direction channel was found to be within specifications (± 1.97 cm/sec) but the y-direction was not. Time did not allow further tests because the borrowed meter had to be returned to the manufacturer. In retrospect it is felt that the skiff-mounted constant speed, calm water method (Hecker, 1970) is superior and logistically simpler.

Upon return of the meter to the company it was rechecked and found to be in specification.

In summary, concerning the performance of the meter, it was found to give outstanding performance within specification in

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Fig. 44. Comparison between the analog record upper, with the output of the current meter and on to the paper tape.

Fig. 45. Open water calibration of current meter
against a mechanical meter of known performance.

every way. It represented water movement between zero and four Hertz adequately and gave an output of \pm 1.000 (\pm 0.007) volts for 3.048 (\pm 0.021) m/s (\pm 10.00 (\pm 0.07) ft/sec). It had essentially zero drift and very low noise.

Amplification of Sigual and Conversion of Output to cm/sec

The current meter furnished by Marsh McBirney was designed to produce \pm 1.0 volt to represent \pm 10.00 1/sec (\pm 3.048 m/sec). The PDP-8 analogue-to-digital converter was designed to accept \pm 2.50 v for full-scale input. In order to obtain maximum sensitivity therefore it was necessary to amplify the signal. In order to determine the correct amplification factor, the maximum expected instantaneous velocity was estimated. Assuming that such a velocity would be generated by a large gravity wave: assume a 10.0 ft wave and calculate the maximum horizontal component of velocity

$$
U_m = \frac{2E_b}{h_b}
$$
; $E = \frac{gH^2}{8}$; $\frac{h}{L}$ 0.05

$$
\therefore \quad U_m = \frac{1}{2} \quad \frac{g}{h_b} \quad H
$$

where $H = 10'$; $h = 25'$

$$
U_m = \frac{1}{2} \frac{32.2 \text{ ft/sec}^2}{25 \text{ ft}} \cdot 10 \text{ ft}
$$

Such a velocity would create a meter output of 0.567 volts. Therefore an amplification of the output of the meter by approximately 3 would seem appropriate.

After a discussion with Professor Thornton of the Department of Electrical Engineering Technology it was decided to construct an instrumentation amplifier based on a Burr-Brown 3662 low offset voltage instrumentation amplifier. In addition to providing amplification the device has another desirable feature; quenching any noise pick-up in the cable between the signal processor and the PDP-8. The amplifier was assembled by the School of Science instrument shop. Four amplifications were provided 0.5, 1.0, 2.0, 3.0, 4.0 for each of 4 channels (to provide two spare channels). A balance control for each channel was provided to balance positive and negative amplification. The device was made to be battery powered, but was converted in the field to electronic regulated power supply.

The PDP-8 has the capability of dividing any signal range into 256 steps; -128 to +128. In this case each step was equal to 0.01953 volts which is round off to 0.0195 to conform to the \pm 0.007 volt accuracy of the current meter. The data is now stored on magnetic tape as signed integers from -128 to +128 units. Conversion of this number to cm/sec is as follows:

$$
\frac{\text{cm}}{\text{sec}} = \left[\frac{\text{no. units}}{\text{amplification factor}} \times 0.0195 \frac{\text{volts}}{\text{unit}} \right)
$$

+ $\frac{Zero\ Set}{correction}$ $\frac{304.8\ cm/sec}{1.0\ volts}$

Zero correction for x-direction was -0.007 volts and for y-direction was +0.012 volts. It should be noted that the standard deviation of the so called zero drift was 00.6%. This deviation is in fact not a drift so much as variation in reading the digital voltmeter during calibration. You will notice that the zero set correction in the y-direction corrects the raw data by 3.66 cm/sec in the onshore direction. Of course the instrument can be reset internally to give a lower correction but this was considered unnecessary because it was possible to determine the exact value to an acceptable degree. The bottom line is that we believe the output data to be a true representation of water motion to \pm 2 cm/sec.

The Equipment Support Tower (Figures 46 thru 50)

In order to obtain the desired current profiles with one meter it was necessary to construct a structure that would span the 16.46m (54 ft) from the pier deck to the bottom that would permit movement of the meter from one level to another. The structure required reasonable strength and rigidity but must have no vibrational mode (including harmonic) over 4 Hz. The structure had to be portable and capable of being easily installed. The structure was build of recycled steel pipe in three triangular sections of 4.57m (15 ft) and one of 3.66m (12 ft) each section weighing about 36 Kg (80 pounds). The sections were bolted together during installation with stainless steel bolts. At the bottom was a steel plate 0.6m (2 ft) square (limited by limber holes in pier deck). The instrument was clamped to a dolly which ran up and down the structure. The dolly had five spring loaded, contoured, ball bearing skate board wheels clamping

Fig. 46. Erecting structure

Fig. 47. Bottom section, bed plate and carriage

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Fig. 48. Sensor clamped to carriage

 \sim

Top of mechanism showing electric Fig. 49. winch (recycled timing gear and Cadillac window operating motor)

 ~ 140 km s $^{-1}$.

Fig. 50. Installing accelerometers on structure

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 $\frac{1}{4}$

it completely around the tower. On the seaward side a strong pyramid stand off mounting structure to which the sensor was clamped to keep it 1.0m (3.3 ft) away from the tower to keep from distorting its magnetic field and to minimize the vortex shedding turbulent effect of the structure on the meter. The dolly was moved up and down the structure by a stainless steel wire with counterblance inside the triangular cross-section of the structure and driven by a reversable electric winch at the top which was controlled from inside the portable lab. A positive stop on the cable positioned the sensor exactly 1.00m (3.28 ft.) above the bed at the bottom.

The theoretical forces on the structure under attack by a three meter, ten second-period wave and the resulting deflection were calculated. A static load test verified that it defiected less than half an inch.

After construction it was assembled and given a series of shaker tests to determine its fundamental resonent frequencies. **It** was found to have a resonant frequency of approximately 0.9 Hz in the x and y lateral planes and 3.25 Hz in the torsional mode.

In a later phase of the field work, not reported herein, accelerometers were attached to the structure in the field. They revealed no significant structure movement and no tendency to vibrate as a resonant structure. This mechanism was an unqualified success. It operated essentially faultlessly for the 170 round trips it made during the ten day period it was installed.

APPENDIX B

ELECTED COMPUTER PROGRAMS

This appendix contains the principle computer programs written by the author for the purpose of this dissertation. Each program begins with a statement of the purpose of the program.

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TUDDE OF

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STOP
END

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 \mathcal{L}_{eff}
THIS PROGRAM IS AN INTERACTIVE PROGRAM TO CORRECT
THE AVERAGED X & Y COMPONENTS OF THE CURRENT
FOR ZERO INDEX AND FOR MAGNIFICATION FACTORS. IT IS
DESIGNED TO ALLOW SEPARATE CORRECTIONS FOR INDEX AND
FOR MAGNIFICATION. THE $\mathbf c$ \overline{c} $\ddot{\mathbf{c}}$ $rac{c}{c}$

and a single

L.

 $\ddot{}$

REAL MX, MY

j

 $C4$

INTEGER A, B, C, D

READ(21,1)A, B, C, D WRITE(5,1)A, B, C, D
FORMAT(212, 213) READ(21,*)AVGX,AVGY

WRITE(5,*)AVCX,AVCY

READ(5,*,END=99)MX \mathbf{c}

AVGX=(AVGX*0.02/2.92-0.007)*304.8

ACCEPT *, HY $\mathbf c$

AVGY=(AVGY*0.02/2.96+0.012)*304.8

ACCEPT *, H $D=D+5$ WRITE(22,4)B,C,D,AVGX,AVGY,H

FORMAT(3(12,1X),2F7.3,1X,F4.2) 4

GO TO 10 CLOSE(UNIT=21,FILE="TRKAVG.DAT")
CLOSE(UNIT=22,FILE="AVGCUR.QDI") 99 **STOP**

FND

```
\mathbf{C}A PROGRAM BY GEORGE TO EXTRACT AND WRITE OUT THE SEMI-DI-
\frac{c}{c}URNAL TIDE COMPONENT FROM A RECORD OF WATER CURRENT OR
          SURFACE HEIGHT ACCORDING TO DOODSON'S SEMI-DIURNAL BAND
          PASS FILTER.
          DINENSION FX(500), FY(500), GX(500), GY(500), HX(500), HY(500),
         PX(500), PY(500), QX(500), QY(500), RX(599), RY(599), SX(500),
    \cdot 2 SY(500)
          WRITE(5,11)
11
          FORMAT("OFILE SPECS TO OPEN FOR RECORD DATA IN (UNIT=21),
          EXAMPLE - < BOTTON-DAT >")
          OPEN(UNIT=21,ACCESS="SEQIN", HODE="ASCII", DIALOG)
         WRITE(5,22)<br>FORMAT("OFILE SPECS FOR TIDAL COMPONENT FILE(UNIT=22),<br>EXAMPLE - <BOTIDE=DAT >")
22OPEN(UNIT=22, ACCESS="SEQUUT", MODE="ASCII", DIALOG)
          WRITE(5,33)
33
          FORMAT(1X, TYPE IN N (NUMBER OF DATA)")
          ACCEPT*, N
         00 5 1=1, N<br>READ(21, 1) A, B, C, FX(I), FY(I), D<br>EOBMAT(3(I), 1X), 2(F7, 3), 1X, E4
\mathbf{C}c1FORMAT(3(I2,1X),2(P7.3),1X,F4.2)
         READ(21,1)PX(1), PY(1)\mathbf{1}FORMAT(2F7.3)
c<br>5
         WRITE(5,2)FX(I),FY(I)
         CONTINUE
         NN=12K = 0DO 10 I=1, NN
         K = K + 1GX(I)=FX(I+12)+FX(I)GY(I)=PY(I+12)+FY(I)10
         CONTINUE
         NN = K - 6\bulletK=0DO 20 I=1, NN
         K = K + 1HX(I)=GX(I+6)-GX(I)HY(I)=GY(I+6)-GY(I)20
         CONTINUE
         NN = K - 4K=0
         DO 30 I=1, NN
         K = K + 1PX(I)=-RX(I)+RX(I+4)PY(I)=-HY(I)+HY(I+4)
30
         CONTINUE
         NN = K - 4
```
90

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DO 40 I=1, NN

$K = K + 1$ $QX(1)=PX(1)+PX(1+4)+PX(1+2)$ QY(I)=PY(I)+PY(I+4)+PX(I+2)

 $\ddot{}$

CONTINUE 40

> $NN = K - 2$ $K=0$

DO 50 I=1, NN

 $K = K + 1$

 $RX(I)=QX(I)+QX(I+2)$ $RY(I)=QY(I)+QY(I+2)$

CONTINUE 50

DO 60 I=1,K

 $SX(I)=RX(I)/48.0$
 $SY(I)=RY(I)/48.0$

CONTINUE 60

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DO 70 I=1,K
WRITE(22,2)SX(I),SY(I)
FORMAT(2F7.3) $\frac{2}{70}$ CONTINUE WRITE(5,*)K

CLOSE(UNIT=21)
CLOSE(UNIT=22)

STOP

END

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 $\ddot{\cdot}$

DO 40 I=1, NN $K = K + 1$ QX(I)=PX(I)+PX(I+4)
QY(I)=PY(I)+PY(I+4) $40.$ CONTINUE $NN = K - 2$ $K=0$ DO 50 I=1, NN $K = K + 1$ $RX(I)=QX(I)+QX(I+2)$ $RY(I)=QY(I)+QY(I+2)$ 50 CONTINUE DO 60 $I=1,K$ SX(I)=RX(I)/32.0
SY(I)=RY(I)/32.0 60 CONTINUE DO 70 I=1,K
WRITE(22,2)SX(I),SY(I)
FORMAT(2F7.3)
CONTINUE $\frac{2}{70}$ WRITE(5,*)K CLOSE(UNIT=21)
CLOSE(UNIT=22) **STOP BND**

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AX=DA/DD BX=DB/DD $CX = DC/DD$ WRITE(5,*)AX,BX,CX $\mathbf c$ COEFFICIENTS FOR Y FIT: $HY = 0 - 00$ $PY = 0 - 00$ $QY = 0 - 00$ DO 80 I=1, N $HY=HY+Y(1)$ $PY=PY+Y(1)*I$ QY=QY+Y(I)*I*I 80 **CONTINUE** WRITE(5,*) HY,PY,QY
USING CRAMERS RULE TO FIND AY,BY&CY;THE COEFF FOR
FIT IN Y-DIRECTION \mathbf{C} C \mathbf{C} DA=HY*E*G+PY*F*E+D*F*QY-E*E*QY-F*F*HY-D*PY*G DB=N*PY*G+D*QY*E+HY*F*E-E*PY*E-F*QY*N-HY*D*G DC=N*E*QY+D*PY*E+D*F*HY-HY*E*E-PY*F*N-D*D*QY WRITE(S,*)DD,DA,DB,DC c $\ddot{}$ AY=DA/DD **BY=DB/DD** $CY=DC/DD$ WRITE(5,*)AY,BY,CY SUBROUTINE SUBTRACT $\mathbf c$ $T(1)=0.00$ DO 90 I=1, N $T(I)=1.00*FLUAT(I-1)$

-
- \mathbf{C} WRITE(5,*)DD,DA,DB,DC
- DC=N*E*QX+D*PX*E+D*F*HX-HX*E*E-PX*F*N-D*D*QX
- DB=N*PX*G+D*QX*E+HX*F*E-E*PX*E-F*QX*N-HX*D*G
- DA=HX*E*G+PX*F*E+D*F*QX-E*E*QX-F*F*HX-D*PX*G
- DD=N*E*G+2*D*F*E-E*E*E-D*D*G-N*F*F
- $\begin{array}{c} c \\ c \\ c \end{array}$ WRITE(5,*) HX, PX, QX USING CRANERS RULE TO FIRD AX, BX&CX;THE COEFF FOR FIT IN X-DIRECTION
- $QX = QX + X(I) * I * I$ 60 CONTINUE

UXDIFF(I)=X(I)-(CX*T(I)*T(I)+RX*T(I)+AX)
UYDIFF(I)=Y(I)-(CY*T(I)*T(I)+BY*T(I)+AY)

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WRITE(JOUT,2)UXDIFF(I),UYDIFF(I)
FORMAT(2F7.3)

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CONTINUE 90

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CLOSE(UNIT=JIN)
CLOSE(UNIT=JOUT)

STOP

END

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CLOSE(UNIT=21)
STOP
END

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THIS IS A PROGRAM BY GEORGE TO NORMALIZE A RECORD $\mathbf c$ BY DIVIDING BY SOME CONSTANT SUCH AS THE MAX.
VALUE OR BY THE TOTAL ENERGY ETC. THE INPUT MUST BE
AB ORDERED PAIR....USUALLY ORTHOGONAL COMPONENTS $\frac{c}{c}$ DIMENSION X(150), Y(150), PX(150), QY(150) WRITE(5,11)
FORMAT("OFILE SPECS TO OPEN FOR RECORD DATA IN (UNIT=21), 11 EXAMPLE - < 11DAT.DAT >")
OPEN(UNIT=21,ACCESS="SEQIN",MODE="ASCII",DIALOG) \bullet WRITE(5,12) FORMAT("OFILE SPECS TO OPEN FOR RECORD DATA OUT (UNIT=22),
EXAMPLE – < 11NRMZ.DAT >")
OPEN(UNIT=22,ACCESS="SEQOUT",MODE="ASCII",DIALOG) 12 TYPE 22 22 FORMAT(1H, "TYPE IN SQRT OF TOT.X-ENERGY, ETC. TOT.Y-ENERGY") ACCEPT*, EX, EY DO 88 I=1,128 READ(21,999)X(I),Y(I) 999 FORMAT(2F7.3) $PX(I)=X(I)/EX$ $QY(I)=Y(I)/EY$ ÷. WRITE(22,999)PX(I),QY(I) 88 **CONTINUE** CLOSE(UNIT=21)

CLOSE(UNIT=22) **STOP END**

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APPENDIX C

SPECTRAL ANALYSIS

Autocorrelation

The objective of the analysis is to identify periodic components from a finite, time series of a random process sampled at a specific time inverval, Δt . The process being observed must be "stationary" meaning the result will be statistically the same regardless of the exact time of the beginning of the sample. The sample must have a limited range and a zero mean.

First, find the mean value of the record and subtract it from all values to obtain the fluctuating component of the record. Second, find the autocorrelation function to determine whether or not the record includes components which have some periodicity. The autocorrelation is created as follows: multiply the amplitude of the first value sampled by the amplitude of the value τ second later, then repeat the process for the second value times the value the same number of seconds later. Continue this process until the end of the record is reached. Find the average of the products found in this manner. This will be one ordered pair on the auto correlation function (τ, R) . The next larger increment of τ is now used and the products taken and averaged again. Mathematically

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$$
R_{x}(\tau) = \frac{1}{T} \sum_{t=0}^{T} x(t) x(t + \tau);
$$

 $T = total length of record$

where $x = f(t)$ is the amplitude

 τ = the lag number

represents one point on the autocorrelation function. If the conditions mentioned above (stationarity and zero mean) have been met, and the sample does indeed contain periodic components, the normalized plot of $R_x(\tau)$ will look like a damped cosine function whose amplitude will uniformly decay to a value of 0.02.

Third the autocorrelation function will be fourier transformed from time space into frequency space by

$$
G_{\mathbf{x}}(f) = 2\Delta t \left[R_0 + 2 \sum_{\tau=1}^{m=1} R_{\mathbf{x}}(\tau) \cos \frac{\pi m f}{\Delta f} \right]
$$

$$
+ R_m \cos \frac{\pi \tau f}{\Delta f} \,]
$$

 $G_{\mathbf{x}}(f)$ is the "power spectral density" function (PSD). $R_{\mathbf{0}}$ is the estimate of the autocorrelation function at zero $\texttt{lag.}$ $\texttt{R}_{\texttt{m}}$ is the estimate of the autocorrelation function at the last lag. m is the total number of lags. Af is the "resolution frequency" or "bandwidth" of the PSD function, where

$$
\Delta f = \frac{1}{2\Delta t} ,
$$

to give the "raw spectral estimates." That is, for each of m frequencies of bandwidth, the amplitude represents the energy per frequency unit (usually Hertz). This is called "raw" because the energy associated with each Af does not appear only at

$$
\tau \Delta f \pm \frac{\Delta f}{2}
$$

but will have "side lobes" which in effect "leak" into adjacent frequency bands (Af units away). To minimize this effect "Hanning smoothing" is applied; one type of "lag window." The fourier transform of the lag window is a spectral window that supresses the leakage, and effectively reduces the variances of the spectral estimates by a factor of approximately 3/8ths. For each frequency examined (the center frequency of the bandwidth mentioned) the corrected value of the spectral estimate (smoothed spectral estimate) is determined by smoothed estimate

$$
G_X^{\dagger}(f) = \frac{1}{4} G_X(f - \Delta f) + \frac{1}{2} G_X(f) + \frac{1}{4} G_X(f + \Delta f)
$$

Reliability of Smoothed Spectral Estimates

Having assumed that our sample is a gaussion distribution, it can be described in terms of the "Chi-squared" function. Therefore how closely the amplitude of the frequency components compare to the amplitudes of the chi-squared function will be a measure of the reliability of the PSD function found. The ability to "solve" any system of equations (such as the component frequency function) depends

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on the number of independent relationships that exist. The number of linearly independent squared terms of equal size into which the quadratic forms in guassian variables can be divided is called the degrees of freedom. In general the number of degrees of freedom is a measure of stability equal to twice the square of the average value divided by the variance. In the application of the autocorrelation analysis, the number of degrees of freedom is twice the number of dara points divided by the number of lags used.

To evaluate the error bounds for the value of any particular frequency band with F degrees of freedom one finds that the error in the value is less than $-\epsilon_1\%$ and $+\epsilon_2\%$ with a selected probability p (use table 26.7, page 978 of the Handbook of Mathematical Functions, AMS 55, National Bureau of Standards, 1964, for values of less than 30), where $\varepsilon_{\textbf{i}}$ is the chi-squared value corresponding to the table values of $0.5 \pm p/2$. It is more convenient to make up a set of curves of F vs ε_i for selected degrees of freedom, but a representative example will illustrate how the reliability was obtained for the PDS. With 4800 data points (N) and 600 lags (m) the degrees of freedom is

$$
F = \frac{2N}{m} = \frac{(2)(4800)}{600} = 16;
$$

(It should be pointed out that this is an unusually large number of lags to use. It was used in this case to produce an autocorrelation function which decayed satisfactorily.) For 80% confidence,

$$
\mathscr{Z} \varepsilon_{i} = 1 - \frac{\chi^{2}}{F} ;
$$

$$
0.5 + \frac{.8}{2} = 0.90
$$

0.5 + $\frac{p}{2}$:

$$
0.5 - \frac{.8}{2} = 0.10
$$

÷.

is found for $p = 0.10$ and 0.90 by interpolation of table 26.7.

$$
x^2.10 = 23.57
$$

$$
x^2.90 = 9.31
$$

and

$$
\varepsilon_1 = (1 - \frac{23.57}{16}) = -0.47
$$

$$
\varepsilon_2 = 1 - \frac{9.31}{16} = 0.42
$$

Therefore, for a smooth spectral estimate of $2 \text{ cm}^2/\text{sec}^2/\text{Hz}$, with 80% confidence, the value would lie between 1.06 and 2.84.

Power Spectral Density By Fast Fourier Transform
(Hinich and Clay, 1968)

If $x_i = f(t_i)$ represents the data as a time series, we compute N values for the function

$$
X_k = \sum_{i=0}^{N-1} X_i e^{-\frac{j2\pi ik}{N}}; k = 0, 1, 2 \ldots, N-1
$$

 e^{-j} is complex notation

 $N =$ number of data points such that N is an integral power of 2; adding zeros before and after the data set to achieve this condition. $N \leq 1024$

The PSD function may then be obtained from

$$
S_k = \frac{2\Delta t}{N} |X_k|^2
$$
; k = 0, 1, 2 ... N-1

where Δt is the time interval between data points; the frequency interval $(0, 1/(2\Delta t))$ is broken down into N/2 parts and the bandwidth $f = 1(N \Delta t)$. In order to obtain a good balance between frequency resolution (small Δf) and variance (more trials averaged), the frequency record is broken up into r non-overlapping pieces in which m corresponding frequency ordinates are averaged and $m = N/r$. Then the m numbers are computed for each $p = 1, \ldots, r$:

$$
S_k = \frac{1}{r} \sum_{p=1}^{r} |A_k^{(p)}|^2
$$
; $k = 0, \ldots, \frac{m}{2} - 1$

Hanning smoothing is again used to minimize leakage. If V_k represents the smoothed values of S_t ,

$$
V_k = \frac{1}{4} S_{k-1} + \frac{1}{2} S_k + \frac{1}{4} S_{k+1}
$$

for each $K = 1, 2, 3...$, $\frac{m}{2} + 1$.

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Reliability of the PSD by Fast Fourier Transform (FFT) The variance of V_k is given by

Var(V_k) =
$$
\left[\left(\frac{1}{4} \right)^2 + \left(\frac{1}{2} \right)^2 + \left(\frac{1}{4} \right)^2 \right]
$$
 $\frac{m}{N} S_k^2 = \frac{3}{8} \frac{1}{r} S_k^2$

Coherence

We may study the two-dimensional structure of some phenomenon if we have two concurrent time series originating from the same or related systems. We proceed as before except that instead of a convolution with itself, as $|x_k|^2$ we perform a convolution between the concurrent time series from $(X_tY_{t+\tau})$ with the product of the mean values removed. This process will produce a cross-spectra between the two series selected. Comparison of the value of this function at a particular frequency with the product of values from the spectra of each component at the same frequency gives us two things: a coefficient of how well they compare (coherence) and the relative phase of one with respect to the other. If the magnitude of $S_{xy}(f)$ and $(S_x(f)S_y(f))^{\frac{1}{2}}$ are equal, the coherence is 1.0; if not the coherence will be less than 1.0. The coherence coefficient is inversely related to the variance of the observed phase difference between the corresponding frequency components of the two time series. If the coherence is 1.0, then the components are in phase, with a specified phase difference, but if the coherence is near zero the phase difference will have such a large variance that the phase difference has no meaning. The cross spectrum is

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$$
S_{x4}(f) = \Delta t \sum_{\gamma = -\infty}^{\infty} \rho_{\gamma} x_4 e^{-2\pi i \gamma \Delta f}
$$

 \bullet

a complex relation where the Yth cross-covariance is

$$
\rho_{\gamma}^{xy} = E(X_t Y_{t+\tau}^*) - \mu_x \mu_y^*
$$

 μ_x = mean value of X_t

 μ_y = mean value of Y_t

 \ast indicates complex conjugate when they are complex.

The coefficient of coherence of the two records compared is given by

$$
\gamma_{xy}(f) = \frac{|S_{xy}(f)|}{(S_x(f)S_y(f))^{\frac{1}{2}}},
$$

 $0 \leq \gamma_{xy}(f) \leq 1$

where S_x and S_y are values from the power spectra of $X(t)$ and Y(t). The average phase difference is

$$
\phi(f) = \tan^{-1} \frac{\text{Im}(S_{xy}(f))}{\text{Re}(S_{xy}(f))};
$$

Im and Re signify Imaginary and Real parts of S_{xy} .

Reliability of the Coefficient of Coherence

The variance of the coefficient of coherence is given by

$$
\sigma^2(\gamma_{xy}) \doteq \frac{1}{2r} (1 - \gamma_{xy}^2 (f_k))^2
$$

and the phase by

$$
\sigma^2(\phi(f)) \triangleq \frac{1}{2r} \left(\frac{1}{\gamma^2(f_k)} - 1 \right)
$$

For example, if we took a sample of 128 data points (after truncating or adding zeros if necessary so that the number is an integral power of 2) and divide them into 4 sets of 32, we have $r = 4$ and $m = 32$. From our coherence relation we pick off value, say 0.8:

or
$$
\gamma_{\text{xy}}
$$
 (f_k) = 0.8

therefore
$$
\sigma^2 = (\frac{3}{8}) \frac{1}{8} (1-(0.8)^2)^2;
$$

the 3/8 factor is from Hanning smoothing

$$
= 0.0061
$$

$$
\sigma = 0.78
$$

therefore, for 95% confidence (2 σ = 0.16), the value lies between 0.816 and 0.784.

AUTOBIOGRAPHICAL STATEMENT

George Milton Hecker

Born: 25 May 1923; Baltimore, Maryland Education: B.S. June 1944, U.S. Naval Academy, Annapolis, Maryland M.S. August 1968, Rensselaer Polytechnic Institute, Troy, New York M.S. August 1970, Old Dominion University Honors: U.S. Army Corps of Engineers Fellowship (1969-1970) Science Teaching Awards, 1966, 1968 Sigma Xi, 1975 Appointments and Positions: U.S. Navy - Enlisted USNR in May 1940; retired as Commander in October 1964. Secondary Science teacher (physics, chemistry and mathematics) -November 1964-June 1969, September 1970-June 1977, September 1978-June 1980. Norfolk and Virginia Beach, Virginia. Old Dominion University - Part-time faculty, 1968-1975; teaching assistant, 1977-1978. Publication: Use of the overhead projector in teaching wave properties.

Physics teacher. September 1968.

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