

2011

Mathematics in the Age of Technology: There is a Place for Technology in the Mathematics Classroom

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Original Publication Citation

Crompton, H. (2011). Mathematics in the age of technology: There is a place for technology in the mathematics classroom. *Journal of the Research Center for Educational Technology*, 7(1), 54-66.

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JOURNAL OF THE
RESEARCH CENTER FOR EDUCATIONAL TECHNOLOGY

KENT STATE
UNIVERSITY

www.rcetj.org

ISSN 1948-075X

Volume 7, Number 1
Spring 2011

Edited by:

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Special Section
Learning Without Frontiers 2011: Mobile Research Strand





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Mathematics in the Age of Technology: There Is a Place for Technology in the Mathematics Classroom

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Abstract

In today's world of ubiquitous computing there are a number of technologies available to K-12 educators for teaching and learning mathematics. However, Koehler and Mishra (2008) have described how teaching and learning with such technologies presents a "wicked problem," as it can involve a number of variables, independent of each other and contextually bound, that need to be brought together. This article highlights the advantages technology offers for mathematics education and looks at some of the reasons behind the poor uptake, such as teacher beliefs and lack of training. A number of solutions are offered to address these issues, including the TPACK framework, and a case is made for using technology in the mathematics classroom.

Keywords

Technology; Technology Integration; TPACK; Mathematics; Math; Assessment; Pedagogy

Background

Today's society is one of great change, with rapidly evolving technologies for use in many aspects of our lives. This is highly evident as we watch the number of people of all ages not just using their cell phones to talk, but also to text, tweet, and surf the web. We can see it in the grocery store as we move through the self-checkout lines, or as we use a card to enter a building or start a car without a key. Many young people have adopted the use of technologies in their everyday lives (Milrad & Spikol, 2007). Although some of these technologies have migrated into K-12 classrooms, this global technology movement is still not reflected in the use of technologies in all schools across the country (Norris, Soloway, & Sullivan, 2002; Tatar, Roschelle, Vahey, & Penuel, 2003). This includes mathematics classrooms, the focus of this article.

Technology can offer new approaches for teaching mathematics in the classroom (Kennedy, Ellis, Oien, & Benoit, 2007; Kinney & Robertson, 2003; Niess, 2005; Vasquez, 2003). It offers other forms of instructional models such as distance learning (Cady & Rearden, 2009), hybrid instruction (Jator, 2010), and interactive television (Donlevy, 2006). These new methods of teaching are supported by "computers, software, interactive television, and the internet" (Kinney & Robertson, 2003, p. 1). Using the many technologies available, teachers can create bespoke lessons for the many different needs of students in the classroom.

People often think of calculators as the main technology used in mathematics classrooms, but the current literature reveals a number of different hardware, software, and web-based tools that are available for teaching mathematics to all ages. These include computational technologies and technologies which may not be specifically designed for the mathematics classroom but may be used as a teaching aid. For the

purpose of highlighting some of the hardware and software available, technologies have been grouped using a number of Roblyer and Doering's (2010) strategies for integrating technology into mathematics: implementing data-driven curriculum; using virtual manipulatives and allowing representation of mathematical principles; and motivating, skill building, and practice. Some technologies may fit in multiple categories, but they have been placed in categories where they have the best fit. One final category has been added to incorporate technologies used specifically for assessment purposes.

Implementing Data-Driven Curriculum

One popular web-based application, which often appears in the mathematics technology literature, is [Wolfram Alpha](#). Wolfram Alpha is not merely a search engine, but a Computational Knowledge Engine (Hindin, 2010; Wolfe, 2010). "Wolfram Alpha was developed by Wolfram Research not only to search for answers but also to involve embedded calculations in order to bring up the most relevant data, as well as data-related charts and visuals of a searched query" (Wolfe, 2010, p.186). Answers to a searched query can produce a wide range of data information. The data from Wolfram Alpha provide real statistics that support investigations that are both timely and relevant. Although Wolfram Alpha can be used by students of all ages, it is generally more suited toward high school and college students as it presents the data in a formal statistical fashion, which may confuse the novice learner.

Spreadsheets are a more accessible and familiar way to display and organize data. These data can be easily collected and manipulated to support the development of student data collection and analysis. Spreadsheets are generally more appropriate for middle school students upwards. Age-appropriate software has also been created for students in the elementary and middle school age range, including [Kidspiration](#), [Inspiration](#), and [InspireData](#), which can be used for data collection and manipulation.

Virtual Manipulatives and Allowing Representation of Mathematical Principles

Virtual manipulatives are defined by Moyer, Bolyard, and Spikell (2002) as "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). There are free resources on the Internet that provide virtual manipulatives, including the [National Library of Virtual Manipulatives](#) and the [National Library of Virtual Manipulatives at the Shodor Foundation](#), which are both based in the U.S., and the [Interactive Teaching Programs](#) from the U.K. All three sites have an extensive selection of virtual manipulatives that are appropriate for all mathematics classes for children ages 3-12.

Virtual manipulatives can improve students' active involvement in the teaching-learning process and encourage their reflections on the concepts and relations to be investigated (Bouch & Flanagan, 2010). The use of manipulatives not only increases students' conceptual understanding and problem solving skills (Dorward, & Heal, 1999; Ozmantar, 2005), but also promotes their positive attitudes towards mathematics since they provide 'concrete experiences' that focus attention and increase motivation (Durmus & Karakirik, 2006). Graphing calculators and Geometer's Sketchpad create representations of mathematical principles (Roblyer & Doering, 2010), providing environments where students are able to make discoveries and conjectures related to concepts in geometry (Garofalo, Stohl-Drier, Harper, & Timmerman, 2000).

Motivating, Skill Building, and Practice

In recent years, game-based learning, or edutainment (Costabile et al., 2008; Deegan & Rothwell, 2010), has become more and more popular. Games are highly engaging and motivational to students (Schneider, Bleimann, & Stengel, 2009), and they also encourage active learning (Garris, Ahlers, & Driskill, 2002). Many mathematics games are visually stimulating, and their fast pace is appealing to many students (Vorder, Bryant, Pieper, & Weber, 2006). Students enjoy challenges and obstacles as games allow new players to experience mastery and then increase the challenge as they continue to play, keeping the students' self efficacy intact (Klimmt & Hartmann, 2006). Many web sites, including

<http://www.softschools.com/math/games/> and <http://mathplayground.com/>, host free educational games that allow K-12 students to practice different mathematical concepts. Many of these games provide guided instructions within a structured learning environment.

Assessment

A highly important aspect of teaching and learning in mathematics is the assessment component. There have been a number of different automated options for use in the mathematics classroom such as The Online Judge (Cheang, Kurnia, Lim, & Oon, 2003) and Automatic Marker, a program that can be used through the [Sakai management system](#) (Suleman, 2008). Automated marking programs provide a fast turnaround on evaluation and feedback (Naudé, Greyling, & Vogts 2010), which is very useful for busy teachers.

Personal Response Systems (PRS) constitute another example of a supportive technology that can be used to conduct formative assessments (Roschelle, 2003). Clickers are one type of PRS, although they only serve this one purpose. Many other devices are multifunctional and are not dedicated solely to one PRS application (Deegan & Rothwell, 2010), such as the iPod Touch. This device has applications, such as [Poll Everywhere](#) (Lynch, 2008), [iVote](#) (Jones, Medina, Rao, Rathi, & Singh, 2004), and [iRespond](#), that will serve the same purpose as the clicker, but the iPod Touch can also utilize an ever-expanding variety of applications and web-based programs for mathematics classrooms for all ages.

Although technology may provide a quick and efficient method of assessment, it has not always proven to be effective and exact when it comes to scoring tests with individualized answers (Ben-Simon & Bennett, 2007; Dikli, 2006; Rudner, Garcia, & Welch, 2006). This may be an issue for teachers who want to gather results of non-numerical answers, or to obtain an idea of how students are solving problems or voicing mathematical thinking. This is changing with the creation of more recent technologies such as an automated scoring algorithm for differentiated answers on geometry problems (Masters, 2010). For grading constructed responses, there are also tools such as the [RUReady](#) parser technology, which has a superior grading capability to that of humans when detecting error patterns (Livne, Livne, & Wight, 2007, p. 302).

Earlier studies have shown how assessment technology can be supportive in some areas, but weaker in others. More recent literature shows a trend towards manufacturers listening to educators concerns about the deficiencies they have found, and resolving those problems where possible. For example, earlier math assessment programs lacked the ability to score tests with individualized answers (Ben-Simon & Bennett, 2007; Dikli, 2006; Rudner, Garcia, & Welch, 2006). Recently, new products have become available that can effectively eradicate this issue (Naudé, Greyling, & Vogts, 2010).

It should be noted, however, that having the technology does not guarantee that it will be used effectively. While a number of teachers have identified technologies that can be used to support both the teaching and learning of mathematical concepts, many others need to make a shift in their teaching practices to include technology. This is not always easy, as other factors may hinder this change, such as not knowing when is it appropriate to use technology, not understanding how technology can facilitate higher order thinking, having preconceived ideas about how mathematics should be taught, and being resistant to change in general. The following sections will unpack each of these issues in more detail.

When Technology Is Appropriate

In 1996, Waits and Demana posed a challenge to teachers. They described a time before technology, when the only option for teaching mathematics was a paper-and-pencil approach. They spoke about the influx of calculators and graphing calculators that provided teachers with other options for teaching, and stated “We must deal with the fact that computer symbolic algebra and computer interactive geometry are better—far better—tools than paper and pencil for doing many of the manipulations associated with mathematics” (p. 712). Waits and Demana concluded that teachers should spend less time with paper

and pencil methods and share the time spent on traditional methods with technologies such as Computer Algebra Systems.

Now, 15 years later and with many new technologies available, there has been little change in the way in which mathematics is taught. Some claim that this lack of change is due to teachers not having the knowledge to effectively integrate technology (Blubaugh, 2009; Ertmer, 2005). Also, Kirschner and Wopereis (2005) have reported that when technology is used in the classroom, it is not used to promote complex thinking but instead for repetitive, basic tasks. One thing that has changed since Waits and Demana's (1996) challenge is that there is now an even larger choice in technologies for teachers to contend with (Vanderlinde & van Braak, 2010).

Teachers need to understand that they do not need to choose exclusively between technology and a traditional paper-and-pencil approach. When technology is used, this does not eradicate the use of all other non-technology approaches. Technology should be combined with the traditional paper-and-pencil approach so teachers can use the most effective and appropriate method at a given time (Artigue, 2002; Drijvers, 2003; Kieran, 2007). Teachers often argue that the paper-and-pencil method is the best option for teaching mathematical concepts because they believe students must work through the full problem on paper every time in order to learn effectively (Piel & Gretes, 1992). This is contradicted by studies that show there are no connections between student reasoning and paper-and-pencil arithmetic, and that higher mathematics can be learned in a technology-rich environment (Kennedy & Chavkin, 1992; 1995). Other studies are more specific, stating that technology can better enhance the teaching and learning experience by allowing a deeper understanding of the concept and higher motivation, self-esteem, and engagement (Deaney, Ruthven, & Hennessy, 2003; Hennessy, Ruthven, & Brindley, 2005). Technology is creating a significant shift in the way mathematics can be taught, although not all teachers are choosing to make that shift.

How to Use Technology for Higher Order Thinking in Math

One place where technology is most appropriate is to facilitate higher-order thinking. Technologies can offer students an opportunity to actively participate and reorganize the way in which they understand mathematics (Stohl-Lee, Hollenbrands, & Holt-Wilson, 2010). Mathematical software, such as [TinkerPlots](#), provides an opportunity for students to construct various types of graphs. The program allows for an inquiry-driven nature of data analysis where charts may be instantaneously manipulated to focus on a number of different mathematical perspectives. As the math class discussion arrives at a "What if...?" question, this can be easily explored by manipulating the charts and graphs in the program. Students need not wait for the alternative chart or graph to be drawn by hand, but instead they can immediately see a different mathematical perspective, offering time to explore that particular concept in more depth. The representations created by the technology reveal different features and procedures which have the potential to affect students' thinking processes and learning (Heid, 2005). This example shows that by automating lower level tasks, technology allows for students to spend more time on activities that require and stimulate higher order thinking.

Similarly, there are times when it is appropriate to use technology to bypass certain parts of mathematical equations to gain the opportunity for higher order thinking. If students have mastered the initial skill on a multistep problem, technology could be used to compute the answer to the initial step, allowing students to advance to the more complex steps in the problem that they have not yet grasped. Sinclair, Renshawa, and Taylor (2004) stated in this respect that

A significant advantage of CAI (Computer Assisted Instruction) over more traditional formats, such as paper and pencil, is that the computer can automate routine tasks such as basic computations and graphing, removing these potential distractions and allowing the student to focus on higher order concepts (p.170).

In sum, while it is essential that students have time to practice the skills they have gained, once students have mastered how to compute the underlying problems on paper they are then able to bypass those problems and move on to more advanced concepts (Edwards, 2003; Mahoney, 2002).

Resistance to Change

There is much discussion about the need for technology in the teaching of mathematics, but many teachers—and even students—are resistant to this change. Teacher perceptions, teaching styles, technological competence, and many other factors influence this choice (Clifford, Friesen, & Lock, 2004; Jonassen, Peck, & Wilson, 1999; Keengwe, Pearson, & Smart, 2009). Teachers with little experience with technologies in general may feel apprehensive about using technology in the classroom (Gros, 2003; Rosas, 2003), and they may feel that this lack of technological literacy could undermine their authority with their students.

Even technology-savvy teachers may not be sure which skills are essential for students to master by hand, and when teachers can move to technology for learning support (Stacey, Kendal, & Pierce, 2002). This could be defined not as a lack of adequate supportive technologies, but as a lack of teacher training in the use of technology in the teaching of mathematics. One of the main necessities for technology training is the time it takes to explore technologies and become accustomed to their use (Russell, Bebell, O'Dwyer, & O'Connor, 2003), so school districts and school administrators must set aside time for this purpose.

Bennison and Goos (2010) conducted a large-scale study surveying mathematics teachers in 456 schools, and their findings indicated that many teachers may be reluctant to change if they do not feel they have the time to learn something new. Researchers also found that while the students are motivated by the use of technology in mathematics (Ng & Gunstone, 2002; Nugent, Soh, & Samal, 2006), they hold similar concerns to teachers in relation to the time it takes to learn new technologies (D'Souza & Wood, 2007; Ng & Gunstone, 2002; Pierce & Stacey, 2004). Data from a number of studies also indicate that the majority of the students, including those as young as 10, prefer to use the paper-and-pencil method over technologies (D'Souza & Wood, 2007; Lightstone & Smith, 2009; Price & Irons, 1995), because they believe it is more effective. This resistance to the use of technology in mathematics may be due to time constraints, although teachers and students could also be reluctant to change due to teaching and learning methods they have grown accustomed to.

Apprenticeship of Observation

The constructivist view is that students' prior knowledge and beliefs effect learning (Davis, 1983; Posner, Stike, Hewson, & Gertzog, 1982). This being the case, many elementary, middle, and high school pre-service teachers often walk through the door of their first undergraduate class with what Lortie (1975) termed the "apprenticeship of observation". They have watched their teachers for many years and feel they already know how to teach. "They develop ideas about the teacher's role, form beliefs about what works in teaching math, and acquire a repertoire of strategies and scripts for teaching specific content" (Loewenberg-Ball, 1988, p. 40). Sadly, this is still reflected in current research (Hart, 2002; Lubinski & Otto, 2004; Wilkins & Brand, 2004) with teachers modeling their own teaching using the methods they themselves were taught by.

With this in mind, many teachers have not been taught using technology, and this could greatly impede their desire to use technology to teach mathematics. This may also have damaging effects in other ways, as researchers suggest that teachers project their attitudes toward the use of technologies onto the students in their classes. If a teacher does not believe technology is effective for learning, students often adopt that idea as their own (Doerr & Zangor, 2000). It is essential that teachers remain aware that their negative perceptions can easily be transferred to their students. Also, effective pre-service and in-service teacher training programs can ameliorate negative attitudes towards technologies for mathematics (Bahr, Shaha, Farnsworth, Lewis, & Benson, 2004; Polly, Mims, Shepherd, & Inan, 2010).

Pedagogical change for teachers

Given what we know about technology use for teaching and learning, it is imperative that teacher training programs include training in the effective and appropriate use of technology, including time for teachers to review and evaluate the many technologies available. This will prepare teachers to better understand the affordances of various technologies, as well as when they should or should not be used in classroom instruction. Koehler and Mishra (2008) describe how teaching and learning with new technologies presents a “wicked problem” as it can involve a number of variables which are independent of each other and contextually bound. But there are initiatives which work toward training pre-service teachers in technology. For example, Preparing Tomorrow’s Teachers to Use Technology (PT3) has been successful in providing pre-service teachers with “opportunities to develop, implement, and evaluate their own instructional activities that utilize technology effectively and appropriately in authentic situations, to give them the myriad of tool necessary to integrate technology into teaching and learning activities” (Brush et al., 2003, p.59).

Initiatives akin to PT3 are especially important for pre-service teachers who have based their ‘apprenticeship of observation’ on the linear, synchronous and controlled style of teaching mathematics. Initially, pre-service teachers could connect with technologies that emulate this style, such as PowerPoint slides which follow the familiar step-by step teaching trend. Patrick-Kinney and Robertson (2003) describe how technology in the mathematics classroom can change the dynamics entirely. By effective use of available technologies, the linear and tightly controlled approach can be swapped for an approach in which students have a choice of “where, when, and how they study mathematics” (Patrick-Kinney and Robertson, 2003, p. 327).

Various organizations have recognized the need for teacher standards to explicate the skills needed to teach effectively with technology. For example, the International Society for Technology in Education (ISTE, 2008) created the National Educational Technology Standards and Performance Indicators for Teachers (NETS-T) for teachers to self-assess and evaluate what specific training they need. It is important to find training which is not just in the use of technology, but the correct pedagogical practice to use with that technology. A recent study showed that teachers who participate in professional development are more confident using technology and could better understand the benefits of using technology in the mathematics classroom (Bennison & Goos, 2010). These data show that teachers indicated a clear preference for professional development that teaches them how to integrate technologies that facilitate student learning of specific mathematical concepts.

There are many challenges to developing an effective and much-needed professional development model for pre-service and in-service teachers. Many dynamic factors and differing conditions need to be considered while designing a framework, such as the inherent instability of technologies (Borko, Whitcomb, & Liston, 2009). Technical Pedagogical Content Knowledge (TPACK) is one solution that incorporates a multifaceted framework of knowledge needed by teachers for the effective integration of technology into the curriculum. Originating from Shulman’s (1986) pedagogical content knowledge (PCK), Mishra and Koehler (2006) built on the PCK framework to include three main components: technology, content, and pedagogy. TPACK refers to the intersection between the three components, where the teacher is effectively negotiating all three areas simultaneously to influence how technology is used and to enhance students’ learning. This integrative body of knowledge is TPACK.

While much technology training is offered, it is often disconnected from the subject matter (Niess, 2005; Syh-Jong & Kuan-Chung, 2010). The TPACK framework can be used in both pre-service and in-service teacher training to help develop an overarching conception of the subject content with respect to technology in mathematics. As technology, teachers, students, and classroom contexts change, TPACK provides a dynamic framework for building a complete understanding that ensures the effective implementation of technology (Niess et al., 2009). More importantly, as teachers TPACK develops and they gain a better understanding of how and why technology is used in the math classroom, teachers will also foster positive attitudes toward the integration of technology to ensure it is utilized once the training is completed (Özgün-Koca, Meagher, & Edwards, 2010).

Implications and Conclusion

This paper shows that there is a place for technology in the mathematics classroom. Many different hardware, software, and web-based tools can offer new approaches for teaching and learning mathematics. However, having the technologies available does not mean that teachers and students understand how to use them effectively, or even choose to use them at all. Although technologies can open up many possibilities, they also offer many challenges. This article highlights elements that need to be taken into account to ensure technology is successfully integrated, such as effective pre-service and in-teacher training. Finally, TPACK has been identified as a framework to integrate technology, content, and pedagogy. As teachers learn to successfully navigate the dynamic interplay between these three components, this will enable them to use technology as an effective tool in developing students' mathematical knowledge.

Acknowledgements

The author would like to thank Ruth Clement for her support in writing this article.

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