Summer 6-2020

Two Essays on the Microstructure of the Housing Market: Agents' Diffused Effort and Sellers' Behavior Bias

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TWO ESSAYS ON THE MICROSTRUCTURE OF THE HOUSING MARKET: AGENTS' DIFFUSED EFFORT AND SELLERS' BEHAVIOR BIAS

by

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A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

BUSINESS ADMINISTRATION

OLD DOMINION UNIVERSITY
June 2020

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Abstract

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For the first essay, we generalize the classic Williams [1998 RFS] brokerage model by introducing the diffused effort. That is, the agent can cross-utilize effort spending on one listing to another one. Besides, the agent can manage heterogeneous housing assets. One counterintuitive finding in Williams' paper is the absence of the agency problem. As a special case in our model, we recover the agency problem. We examine the positive externality due to the diffused effort and show that it depends on the agent's inventory size. Hence there exists a trade-off between agents' effort spending on existing listings and on finding a new listing.

For the second essay, we model a home seller's pricing decision under a generally defined prospect value function. We show a simple disposition effect is caused by reference dependence, but it only exists when the agent is risk neutral. Diminishing sensitivity will lead to a two-way disposition effect by generating a local reverse disposition effect, a range in which the seller's asking price decreases with increasing potential loss. Loss aversion tends to magnify the disposition effect and hence mitigates the reverse disposition effect. One direct implication is that acclaimed tests on loss aversion such as Genesove and Mayer [2001] and Pope and Schweitzer [2011] are likely invalid. We present evidence consistent with the model by using multiple listing service data from Virginia. Our findings suggest that studies which predominantly focus on a one-way disposition effect can be overly simplistic and misleading as it depends on the strong assumption of risk neutrality.

Acknowledgement

Firstly, I would like to express my sincere gratitude to my advisor Prof. Mohammad Najand for the continuous support of my Ph.D. study for his patience, generosity, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D. study.

Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Walter D'Lima and Prof. Haiwen Zhou, for letting my defense be an enjoyable moment. Not only your insightful comments and encouragement but also your hard questions and brilliant suggestions incentive me to widen my research from various perspectives.

I would also like to express my special thanks of gratitude to one of my coauthor, Prof. Hua Sun, at Iowa State University, who provided insight and expertise that greatly assisted my research. Without his constant help, this dissertation would not have been possible. Also, I would like to thank Prof. Michael Seiler, in College of William & Mary, who generously shares his Hampton Roads database to me.

Last but not least, I must express my very profound gratitude to my parents for supporting me spiritually throughout writing this thesis and my life in general. This accomplishment would not have been possible without them. Thank you.

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Chapter 1

Introduction of the Housing Market

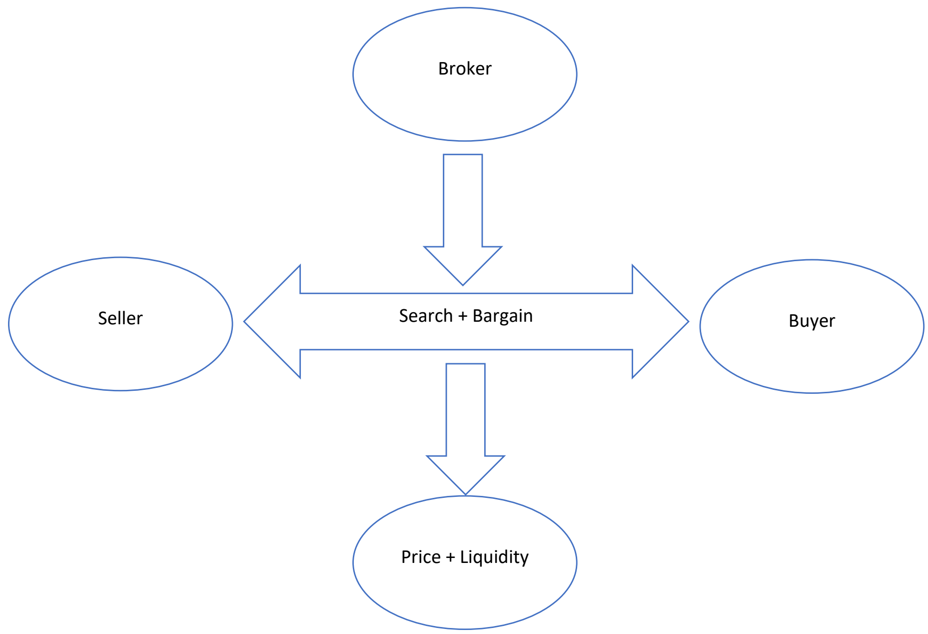
Real estate assets account for a significant part of the capital market. According to Geltner et al. [2014], in the United States, they amount to about 42% of the total market value of all assets and generate over 25% of US gross domestic product (GDP). Also, the housing market generates nearly 70% of local government revenue from property tax and creates nearly 9 million jobs within the US. Moreover, the housing market is the largest element of a household's portfolio; more than 22% of total households asset is housing. With housing's large capital and high riskiness, it has become an essential driver of the business cycle. During recent years, housing has played an especially large and unhappy role in the global recession.

Unlike the stock market, three aspects of housing are most important in our study. First, houses are heterogeneous. It is a naturally localized monopoly because of no substitute. Also, this is true because of many characteristics of housing (i.e., square feet, lot size, number of bedrooms, bathrooms, and age) and the neighborhood amenities and local public goods that jointly impact housing prices. Second, housing transactions could be taken under uncertainty. The search-bid system under the housing market is taken within a narrow local market, and the mismatch is quite often. Thus, both buyer and seller have to spend much effort on each other. Even when they meet, the buyer and seller are likely unable to make a successful agreement of price. As most participants in the housing market are individuals, their decision-making process is full of bias and uncertainty. Third, the housing market has many significant market frictions. The search cost under the housing market is economically costly. Besides, transaction costs, including brokerage commission fees, moving costs, title insurance, and taxes, are also costly. Thus, unlike the perfectly competitive market, the housing market needs time to clear the price and inventory.

In this study, we will focus on the microstructure of housing markets. Figure 1.1 present a general framework of all the related studies involved in this study. Unlike other perfect competitive markets, broker (or agent)[1] plays an important role in the final transaction

[1] In this study, we are using the term "broker" and "agent" interchangeably. It is widely used for many housing market studies. In fact, under the formal legal definition, the licensing process for brokers are more

Figure 1.1: Microstructure of the Housing Market



Source: Han, Lu, and William C. Strange. "The Microstructure of Housing Markets: Search, Bargaining, and Brokerage." Handbook of Regional and Urban Economics. Vol. 5. Elsevier, 2015. 813-886.

price and liquidity, often measured by time-on-market (TOM). Most studies focus on how these outcomes are affected by a range of market and strategic forces, including market conditions, house characteristics, and strategic decisions by sellers and buyers. Besides, the broker's performance, which may be affected by ownership types, compensation plans, and incentive structures.

The analysis of housing markets must begin with the fact that housing is a unique good. This means that the analysis of other markets cannot simply be brought into the housing market without modification. For the first essay, we focus on an economic model of principal-agency problem, and an agent's externality called diffused effort. We show that the results from Williams [1998 RFS] are wrong. We find both agent problems and diffused effort have a significant impact on housing market prices and liquidity. For the second essay, we focus on the sellers' behavior bias. We find that sellers are pricing their assets under three

strict. The broker is required for both additional experiences as an agent and additional education.

major assumptions of prospect theory: reference dependence, marginal diminishing, and loss aversion. We build a model and show that the asking price is highly related to these factors. Our empirical and calibration results also confirm this. Also, we show that, unlike the studies in the stock market (Barberis and Xiong [2009, 2012]), reference dependence alone can generate a disposition effect.

Chapter 2

Essay 1: Diffused Effort and Real Estate Brokerage

2.1 Introduction

In the real estate market, brokers play a key role in facilitating transactions. A 2016 report from the National Association of Realtors (NAR) documents that brokers sell 88 % of single-family dwellings [1]. Rutherford et al. [2005] report that total brokerage commission fees add up to more than $65.5 billion a year. Real estate brokers are specialists in collecting and sharing market information, providing financial expertise, and marketing properties.

There is a vast body of theoretical literature that looks at the commission structure and the principal-agent relationship in real estate brokerage[2]. Comprehensive reviews on brokerage literature can be found in Yavas [1994] and Zietz & Sirmans [2011]. Most of the papers in the literature consider three commission systems: fixed commission percentage, flat fee, and net listing. The general conclusion is that none of the systems perfectly aligns the broker's interest with those of sellers and that the industry is inefficient (Han & Hong 2011; Barwick & Pathak 2015). However, Williams [1998] has pointed out that little research has considered interactions of externalities driven by multiple key characteristics in brokerage: multiple brokers, each with multiple tasks; the costly search for both buyers and new tasks (sellers); and the competitive equilibrium among brokers. As a result, he proposes a search model that explicitly incorporates these aspects. Contradicting the findings in most papers, Williams [1998] claims that the agent's optimal search effort and reservation price are independent of the commission rate. Hence, there is no agency problem under this compensation structure. Despite the absence of agency problem, Williams shows that an agent who commits (dilutive) effort to searching for new clients generates a negative externality to the existing sellers; hence, the perfectly competitive equilibrium without an agency problem

[1] https://www.nar.realtor/reports/highlights-from-the-profile-of-home-buyers-and-sellers

[2] An incomplete list includes Yinger [1981]; Zorn & Larsen [1986]; Miceli [1989,1991]; Anglin & Arnott [1991]; Arnold [1992]; Geltner, Kluger & Miller [1992]; Yavas [1995a, 1995b]; Williams [1998]; Miceli, Pancak & Sirmans [2007]; Fisher & Yavas [2010]; Han & Hong [2016], etc.

is not the first-best.

In a different setting, Fisher & Yavas [2010] study a “winner-takes-all" externality in brokerage. The idea is that in the multiple listing service, as brokers pool their listings, multiple brokers can commit competing for the effort to the same listing. However, only those brokers who ensure a successful transaction get compensated eventually. The authors show that competition among brokers can induce each broker to commit more marketing efforts. Nevertheless, in the Fisher & Yavas model, unlike in the Williams [1998] model, a less than 100% commission rate still induces the broker to spend insufficient effort on selling a house. They continue and show that under a unique set of parameters, these two competing forces can offset and bring back the first-best outcome.

In this paper, we extend Williams’ brokerage model by introducing a new type of *diffused effort* externality. The original Williams's model, as well as most of the brokerage models, assumes that an agent’s effort committed to different houses is mutually exclusive. For example, for each listing property, a broker spends efforts to find a potential buyer. However, if an agreement is not reached, the buyer then exits, and the effort spending dissipates. In our paper, with a positive probability, we allow the broker to have the same buyer subsequently visit other listing properties that are available in the network of the same broker with no additional cost. Unlike the “winner-takes-all" externality, where multiple agents commit competing effort towards the same task, the diffused effort externality discussed in this paper emphasizes the potential for an agent to (partially) cross-utilize effort on one listed house to other listings because of the close substitutability.

This diffused effort externality is different from what is modeled in the common agency literature, such as in Bernheim and Whinston [1986] and Laussel and Le Breton [2001], etc. The common agency models often focus on how the principals cooperatively attempt to influence the decision of a common agent. In the housing market, such cooperation among multiple sellers is rarely seen. Instead, our model focuses on the brokers’ optimal choice under multiple tasks. As shown in this paper, the fact that the effort committed on one task (listing) can potentially benefit others generates a positive but size-dependent externality that improves the expected payoff to both principals and agents. Furthermore, we demonstrate a trade-off for the agent between utilizing diffused effort among the existing listings and the potential to increase network size (i.e., by finding another seller), and we examine its property in great detail in the paper.

In addition to diffused effort, we further generalize the Williams model by incorporating asset heterogeneity. That is, we allow brokers to manage two different types of assets at the same time. The scope of asset heterogeneity considered in our study is flexible. For example, we allow an agent to simultaneously sell a pool of listings from customers (with the commission rate less than 1) and for herself (with a commission rate of 1) while holding other characteristics the same. This allows us to examine the agency problem in a way that is close to reality. Alternatively, we can make buyer arrival process type-dependent in order to model for potential heterogeneous demand; or to assume heterogeneous bidding distributions as proxies for different classes of housing assets.

Regarding the commission rate, one counterintuitive finding in Williams [1998] is the absence of agency problem, as the author shows that the broker's effort level and the expected selling proceeds are independent of the commission rate. As our model embeds Williams's model as a special case, we investigate his model and show that part of the proof and consequently, the major conclusions in Williams [1998] turn out to be problematic. In particular, within Williams's original setting on a single type asset, our model reveals positive relations between an agent's effort/reservation price and commission rate. The positive relation continues to hold when we allow two types of assets that have different commission rates to be simultaneously managed by a broker.

Our findings are consistent with much of the documented empirical evidence offered by previous research. For example, several studies find that agents are able to sell their own houses (hence a higher implied commission rate) at a premium compared to otherwise similar houses from their clients (Rutherford et al. [2005, 2007]; Levitt & Syverson [2008]; Bian et al. [2017]). With respect to the expected time to sell, our model predicts that it is negatively related to the commission rate. Notably, Jia & Pathak [2010] finds that a higher commission rate is associated with a higher likelihood of a sale, and it also reduces days on the market. Bian et al. [2017] also find that client properties competing with agent-owned properties remain on the market 30% to 46% longer. These findings are in tune with the prediction from our paper: due to the lower commission, a broker tends to deliver a longer time on the market and a lower transaction price for a client's property than for her property.

On-demand strength, we show that, compared to the other type of asset, for the asset that is easier to solicit potential buyers, a broker tends to commit lower search effort on an existing listing. Nevertheless, a broker commits more effort on finding a new listing in order to increase the network size, in order to strengthen the diffused effort externality. At the same time, a broker tends to set a higher reservation value and experience a bigger selling likelihood.

Finally, we examine the impact of the valuation heterogeneity driven by either variance or mean of the underlying bidding distribution. Our model shows that when diffused effort externality is weak, an agent may focus more on low variance assets and deliver a better payoff for the sellers of this type. When diffused effort externality is strong enough, high variance assets will become more attractive to the agent. That is because as agent's reservation value will go above the mean of the bidding distribution, and the likelihood of sale is higher for high variance type due to the fatter upper tail. Regarding the level of average willingness to pay, we show that when valuation distribution of one type stochastically dominates the other, a broker tends to ask for a higher price, commits more efforts that increase the likelihood of sale, and also commit more effort on finding a new listing on the high-value type asset.

We organize the paper as follows. Section 2.2 first briefly restates the Williams model and then extends the Williams model to allow for diffused effort and heterogeneous assets. Section 2.3 presents our major findings. Section 2.4 discusses our model limitations and suggest potential directions for future extension. We conclude the paper in section 2.5. To help readers in comparing Williams's [1998] work and this paper, whenever possible, we adopt the same symbolic conventions that were used in the original Williams [1998] paper whenever relevant.

2.2 Model Set Up

2.2.1 Williams Model

Consider a large market with a countably infinite number of brokers competing for business. These brokers are homogeneous in the sense that they have the same productivity in searching for both potential buyers and new sellers, which will be discussed later. However, they may not have the same number of clients at a given time. Once a broker successfully solicits a new seller, she will sign an exclusive contract with the seller[3]. This contract has

[3] The model implicitly assumes that it is always optimal for a seller to hire a broker with full delegation. It can be satisfied by suitably assume the outside option of a seller and a broker's production function of search. See Williams [1998] for a detailed discussion on it.

infinite maturity, and the contractual compensation is a common, constant percentage[4] of the asset's sale price, payable upon sale. All properties are ex-ante identical in the model.

To sell a property, a broker must commit effort to searching for a potential buyer, who arrives in an independent Poisson process. For broker i with n clients, we define the time spent for client j's property as x_{ijn}. During the time window of Δt, a buyer arrives with the probability of $F(x_{ijn}, \overline{x})\Delta t$, where $\overline{x}$ refers to the average search effort across all brokers. Given this setup, no buyer arrives with the probability of $1 - F(x_{ijn}, \overline{x})\Delta t$, and more than one buyer arrives with a probability of smaller than the order of Δt. The function F has the following properties. First, it is twice continuously differentiable everywhere. Second, F is increasing and strictly concave in x_{ijn}, and decreasing in $\overline{x}$. The former assumption implies decreasing marginal productivity on the broker's searching ability. The latter assumption reflects the negative externality of searching buyers due to competition among brokers. To prevent corner solutions, we also assume $F(0, \cdot) = 0$ and $F'(0, \cdot) \equiv \frac{\partial F(x,\overline{x})}{\partial x}|_{x=0} = \infty$. In a large market with many brokers and assets, the interaction between effort spent on current assets and the market average is minimal[5]. Third, F is homogeneous. It implies a constant elasticity of the broker's own effort: $x_{ijn}F'(x_{ijn}, \overline{x})/F(x_{ijn}, \overline{x}) = \eta > 0$. To guarantee a positive reservation price in equilibrium, we set its value at $0 < \eta < 1$.

The new seller's arrival rate also follows an independent Poisson process. Let $\overline{n}$ be the concurrent number of average clients per broker and $\overline{y}$ be the average search effort expended on new clients across all brokers. During the time window of Δt, a new seller arrives with the probability of $\alpha\overline{n}(y_{in}/\overline{y})\Delta t$, where y_{in} refers to the time spent by broker i with n current clients. In this case, the chance of no new seller arriving has the probability of $1 - \alpha\overline{n}(y_{in}/\overline{y})\Delta t$, and the chance of more than one new sellers arriving has a probability of smaller than the order Δt. As pointed out by Williams [1998], the proportionality of the average number of assets is consistent with the arrival rate of buyers. In the steady state, the average arrival rates of both new sellers and buyers are proportional to the aggregate number of assets in the market. This implies the independence of the arrival rate of buyers and the proportionality of the average arrival rate of new sellers. $y_{in}/\overline{y}$ measures the relative allocation of time to new clients. Because we assume the broker's time spent on searching for new sellers is purely dissipative, the number of new sellers in the market will not depend on the total searching time spent by all brokers. Besides, competition among brokers requires

[4]Based on the Federal Trade Commission's (FTC) 2007 report, the real estate commission rate in the US is around 5% to 6% of the sale price. More recently, Jia & Pathak [2010] finds that there is little variation in commission rates over time despite the increased penetration of the internet and new technologies.

[5]See Williams [1998] appendix (page 247) for a detailed discussion on why it must be the case.

that the marginal productivity of the search must be constant across time. As a result, the arrival rate of new clients is homogeneous on degree zero on y_{in} and $\overline{y}$. Therefore it is reasonable to assume the arrival rate is proportional to $y_{in}/\overline{y}$.

For broker i with n current clients, the total fraction of time spent on searching is $w_{in} = \sum_{j=0}^{n} x_{ijn} + y_{in}$. The time not spent on searching is spent on leisure. During a time window of Δt, the working time incurs a cost of $\theta H(w_{in})\Delta t$. θ is a positive scaling factor and must satisfy some technical requirement to generate the competitive equilibrium results[6]. As is typical in agency models, we assume function H to be twice differentiable, increasing, and strictly convex in x_{in}. Furthermore, we assume $H^{'}(0) = 0$ and $H^{'}(1) = \infty$ to prevent any corner solutions. A usual assumption is that x_{in} is not verifiable, and hence the contract cannot be enforced upon the effort level.

Once a potential buyer arrives for property j, he inspects this property only and determines the quality of match to his own preference. Depending on the matching quality, the buyer offers a price of p_{ijn}. We assume p_{ijn} is a random draw from some distribution G. If there is no match at all, $p_{ijn} = 0$. If there is a perfect match, $p_{ijn} = 1$. Therefore, G has a finite support [0,1]. Furthermore, we assume the buyer's final offer is independent of other possible offers that may come from other buyers, other units, and a different time. The distribution of G could be very general. The only requirement is that the hazard function, $g/(1-G)$ is nondecreasing everywhere. Here we do not consider the uncertain nature of the housing market. Hence G does not shift over time. The broker chooses a reservation price r_{ijn} for the jth property. If $p_{ijn} \geq r_{ijn}$, the transaction is closed, and the seller pays p_{ijn}. Thus, the probability of a successful agreement is $1 - G(r)$. As argued by Williams [1998], this simple matching mechanism can be motivated from a model of sequential search and bilateral bargaining game, which is seen in Williams [1995].

Based on the model we have thus far introduced, for broker i with n current clients, her

[6]See Williams [1998] for detail.

objective function is to solve the following problem:

$$
\begin{aligned}
V(n) \quad = \quad & \underbrace{max}_{r_{ijn}, x_{ijn}, y_{in}} \quad e^{-\iota \Delta t} \Bigg\{ \sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*) \int_{r_{ijn}}^{1} b^* p dG(p) \Delta t \\
+ \quad & \sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})] V(n-1) \Delta t \\
+ \quad & \alpha \overline{n}^* (y_{in}/\overline{y}^*) V(n+1) \Delta t - \theta H(\sum_{j=1}^{n} x_{ijn} + y_{in}) \Delta t \\
+ \quad & \Big[1 - \sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})] \Delta t - \alpha \overline{n}^* (y_{in}/\overline{y}^*) \Delta t \Big] V(n) \Bigg\} \\
+ \quad & o(\Delta t), \qquad (2.1)
\end{aligned}
$$

Subject to $V(0) = \gamma$, $0 \leq r_{ijn} \leq 1, x_{ijn}, y_{in} \geq 0$, and $\sum_{j=1}^{n} x_{ijn} + y_{in} \leq 1$ for $j = 1, ..., n$ and $i, n = 0, 1....$

In Equation (2.1), ι is the discount rate, and b^* is the common commission rate on the market. Here $\overline{n}^*$, $\overline{x}^*$ and $\overline{y}^*$ are average values of the market equilibrium counterparts, and we assume that in a large and steady market, each broker correctly regards b^*, $\overline{n}^*$, $\overline{x}^*$ and $\overline{y}^*$ as fixed values that do not depend on her own choices. Over the time interval of Δt, the broker maximizes the discounted expected payoff from the following decision problem. First, it is possible that one buyer will arrive and bid a higher price than r_{ijn}. In that case, the expected commission benefit forms the first summation term. The second term in the brackets reflects the possibility that the broker may successfully find a new seller. This will increase the current number of clients by 1. The broker will thus face a similar decision problem with state variable $n+1$. The third term measures the total search cost spent by broker i. In the fourth term, $\sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})]\Delta t$ refers to the probability of successfully selling one property. This will decrease the current number of clients by 1. Finally, the probability of remaining in the current state is $1 - \alpha \overline{n}^* (y_{in}/\overline{y}^*)\Delta t - \sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})]$. In this case, the broker will repeat the current decision problem $V(n)$, as shown by the last term in the brackets. We should note that the terms in the brackets correctly consider all possible events that could happen during the period Δt. The other events can occur only with the probability of a smaller order than Δt. For example, the probability of both selling one current property and finding a new seller is $\sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})] * \alpha \overline{n}^* (y_{in}/\overline{y}^*) \Delta t^2$. Those terms are collected as $o(\Delta t)$, which is neglectable in the maximization.

Equation (2.1) has three desirable features. First, it reflects the multiple tasks of agency nature in real estate brokerage. Multiple sellers must compete for the broker's exclusive search effort. Second, it explicitly models the fact that a broker has the freedom to search for new principals (sellers). Third, as Williams [1998] shows, Equation (2.1) implies that under the steady state, the likelihood of sale must equal to the speed of finding new clients :

$$\alpha = F(x^*, \overline{x}^*)[1 - G(r^*)] \tag{2.2}$$

The proof is straightforward. In the steady state, the expected rate of changing asset's size from n to $n+1$ is the same as from n to $n-1$, i.e., $E(\Delta n = 0)$. Thus, $E\left\{\sum_{j=1}^{n} F(x_{ijn}, \overline{x}^*)[1 - G(r_{ijn})]\right\} = E\left\{\alpha \overline{n}^*(y_{in}/\overline{y}^*)\right\}$. On the market average, as all the assets are identical, Equation (2.2) must hold.

There are two technical conditions for V. First, we assume a fully competitive market for brokerage. As a result, the supply for brokerage has infinite elasticity. Suppose the entry cost for a broker is γ. In equilibrium, the entry profit for a new broker must be equal to the cost of entrance. As a result, $V(0) = \gamma$. In addition, to keep the stationary nature of the value function, the common transversality condition implies that $\lim_{n\to\infty} V(n+1) - V(n) \leq \delta$ for some positive and finite δ.

We refer to Equation (2.1) as the benchmark case. The next section extends this model to allow for diffused effort and heterogeneous assets. We then characterize the equilibrium and discuss whether agency problems exist.

2.2.2 Extended Model with Diffused Effort and Heterogeneous Assets

The benchmark model assumes that an agent's effort committed to competing tasks is mutually exclusive with each other. This can be seen in Equation (2.1), in which the spent effort x_{ijn} benefits only the task j (i.e., to solicit a potential buyer for house seller j). As a result, if $p_{ijn} < r_{ijn}$, no transaction occurs, and the potential buyer for house j exits the model.

This can be an extreme assumption. In reality, if the buyer is not satisfied with the house that is shown first[7], very likely, the broker will bring her to another house in the broker's network without extra search cost, and this sequential touring process will end either when a transaction is realized for some house down the road or when she has visited all of the houses in the broker's inventory in which the buyer might have an interest.

Motivated by this phenomenon, we introduce ρ, a new parameter that ranges from 0 to 1, as the probability that the solicited buyer will continue to look at another house with the broker if the current visit fails to yield a transaction. The direct implication from this extension is that a potential buyer could have multiple bids within one round of search[8]. As a result, the total arrival rate for a buyer looking for asset j, $F(x_{ijn}, \overline{x})$, now changes to the $F(x_{ijn}, \overline{x})$ plus the sum of all the other arrival rate diffused from the other assets. For example, one buyer comes for the asset j with the agent's effort x_{ijn}, the arrival rate is $F(x_{ijn}, \overline{x})$. And then, if this buyer is not satisfied with the price and the transaction is closed (the probability is $G(r_{ijn})$), the agent could introduce the buyer with the second asset j', with the probability of ρ. Moreover, if this buyer is still not satisfied with the price and the transaction is closed (the probability is $G(r_{ij'n})$), the agent could introduce the buyer with the third asset, with total probability of $F(x_{ijn}, \bar{x}^*) \times [1 + \rho G(r_{ijn}) + \rho^2 G(r_{ijn}) G(r_{ij'n})]$. So on and so forth, the original effort on the asset j could be diffused to all the rest of $n-1$ asset with a diffusion coefficient. With the same story, efforts from all the other assets $j' \neq j$ could finally transfer to the asset j with other diffusion coefficients. As all the assets are identical, $r_{ijn} = r_{in}$, this coefficient is actually $\rho G(r_{in})$. Thus, the effective arrival rate for the asset j is as following:

$$\begin{aligned} & F(x_{ijn}, \bar{x}^*) \times \left[1 + \rho G(r_{in}) + ... + (\rho G(r_{in}))^{n-1}\right] \\ =& F(x_{ijn}, \bar{x}^*) \times \frac{1 - [\rho G(r_{in})]^n}{1 - \rho G(r_{in})} \end{aligned} \tag{2.3}$$

In the similar spirit, agent's final selling probability for house j changes from $[1 - G(r)]V(n-1)\Delta t$ to $[1 - G(r)]\frac{1-[\rho G(r_{in})]^n}{1-\rho G(r_{in})} V(n-1)\Delta t$.

[7] The unsatisfaction is equivalent to a low willingness to pay, which implies $p_{ijn} < r_{ijn}$.

[8] As we assume all houses are identical, so a buyer's offering price is a random draw of the bidding distribution G.

Incorporating diffused effort, Equation (2.1) is now generalized to :

$$\begin{aligned}
V(n) \quad &= \quad \underbrace{max}_{r_{ijn},x_{ijn},y_{in}} \quad e^{-\iota\Delta t}\Bigg\{ \sum_{j=1}^{n} F(x,\overline{x}^{*})\frac{1-(\rho G)^{n}}{1-(\rho G)}\int_{r_{ijn}}^{1} b^{*}pdG(p)\Delta t \\
&+ \quad \alpha\overline{n}^{*}(y/\overline{y}^{*})V(n+1)\Delta t - \theta H(w_{in})\Delta t \\
&+ \quad \sum_{j=1}^{n} F(x,\overline{x}^{*})(1-G)\frac{1-(\rho G)^{n}}{1-(\rho G)}V(n-1)\Delta t \\
&+ \quad \left[1-\alpha\overline{n}^{*}(y/\overline{y}^{*})\Delta t - \sum_{j=1}^{n} F(x,\overline{x}^{*})(1-G)\frac{1-(\rho G)^{n}}{1-(\rho G)}\Delta t\right] V(n)\Bigg\} \\
&+ \quad o(\Delta t)
\end{aligned} \tag{2.4}$$

Compared with Equation (2.1), Equation (2.4) has two new features. First, it contains the original Williams model as a special case when we turn off the effort diffusion by setting $\rho = 0$. Second, the network size (total assets number n) plays a more direct effect. That is because searching for a new seller, while costly, is also beneficial as it improves the arrival rates on existing listings. Thus, there is a trade-off between the efforts on the current assets and the effort on finding a new listing.

Further, we can extend the diffused effort model by introducing two types of assets. Consider a broker i who has n_1 type 1 assets and n_2 type 2 assets. The maximization problem now becomes:

$$
\begin{aligned}
\widehat{V}(n_1,n_2) \;=\;& \underbrace{max}_{r_{1ijn},x_{1ijn},y_{1in},r_{2ijn},x_{2ijn},y_{2in}} e^{-\iota\Delta t}\Bigg\{\sum_{j=1}^{n_1} F_1(x_{1ijn},\overline{x_1}^*)\frac{1-(\rho G_1)^{n_1}}{1-(\rho G_1)}\int_{r_{1ijn}}^{1} b_1{}^* p dG_1(p)\Delta t \\
+\;& \sum_{j=1}^{n_1} F_1(x_{1ijn},\overline{x_1}^*)[1-G_1(r_{1ijn})]\frac{1-(\rho G_1)^{n_1}}{1-(\rho G_1)}\widehat{V}(n_1-1,n_2)\Delta t \\
+\;& \sum_{j=1}^{n_2} F_2(x_{2ijn},\overline{x_2}^*)\frac{1-(\rho G_2)^{n_2}}{1-(\rho G_2)}\int_{r_{2ijn}}^{1} b_2{}^* p dG_2(p)\Delta t \\
+\;& \sum_{j=1}^{n_2} F_2(x_{2ijn},\overline{x_2}^*)[1-G_2(r_{2ijn})]\widehat{V}(n_1,n_2-1)\Delta t \\
+\;& \alpha_1\overline{n_1}^*(y_{1in}/\overline{y_1}^*)\widehat{V}(n_1+1,n_2)\Delta t+\alpha_2\overline{n_2}^*(y_{2in}/\overline{y_2}^*)\widehat{V}(n_1,n_2+1)\Delta t \\
-\;& \theta H(\sum_{j=1}^{n_1} x_{1ijn}+\sum_{j=1}^{n_2} x_{2ijn}+y_{1in}+y_{2in})\Delta t \\
+\;& \Big[1-\sum_{j=1}^{n_1} F_1(x_{1ijn},\overline{x_1}^*)(1-G_1)\frac{1-(\rho G_1)^{n_1}}{1-(\rho G_1)}\Delta t-\alpha_1\overline{n_1}^*(y_{1in}/\overline{y_1}^*)\Delta t \\
&\quad -\sum_{j=1}^{n_2} F_2(x_{2ijn},\overline{x_2}^*)(1-G_2)\frac{1-(\rho G_2)^{n_2}}{1-(\rho G_2)}\Delta t-\alpha_2\overline{n_2}^*(y_{2in}/\overline{y_2}^*)\Delta t\Big]\hat{V}(n_1,n_2)\Bigg\} \\
+\;& o(\Delta t) \qquad (2.5)
\end{aligned}
$$

subject to $0 \leq r_{1ijn}, r_{2ijn} \leq 1, x_{1ijn} \geq 0, x_{2ijn} \geq 0$, and $\sum_{j=1}^{n_1} x_{1ijn} + \sum_{j=1}^{n_2} x_{2ijn} + y_{1in} + y_{2in} \leq 1$. Here the asset type can be differentiated by one or more of the following characteristics: commission rate b^*, buyer search efficiency η, and valuation distribution $G(r)$. This set up offers a flexible structure to better match empirical patterns. For example, the literature testing agency problem often looks at the case when a broker sells his/her own house while selling for clients'. To model this phenomena, we can simply set $b_1^* < 1$, $b_2^* = 1$, $n_2 = 1$ and $y_2 = 0$, but maintain a common buyer arrival process and bidding distribution. The existence of agency problem will be evidenced if we can show that $r^*_{b=1} > r^*_{b=0.06}$ and $x^*_{b=1} > x^*_{b=0.06}$ both hold.

2.3 Results

2.3.1 Williams Model: a Revisit

As the purpose of this paper is to examine the potential agency issue associated with real estate brokerage, we re-solve Williams [1998] model (i.e., Equation 2.1) for benchmark comparison.

In the steady state and under some boundary condition of θ, a unique set of constant, symmetric solution exists for Equation (2.1), by applying the modified Blackwell sufficiency condition in Stockey & Lucas [1989], Theorem 4.12 [9]. Meanwhile, the value function for the broker is in the form:

$$V(n) = \gamma + nb^*r^* \tag{2.6}$$

For completeness, and to facilitate comparison, we first re-produce all findings that we agree with Williams [1998] in Proposition 0. For parts that we observe a discrepancy, they are summarized in Proposition 1, and we offer detailed discussions on why we believe the original findings proposed in Williams paper is incorrect.

Proposition 0. *In the steady state with delegation, with $0 < \eta < 1$, $\theta = \theta^*$, $r^*_{ijn} = r^*$, $x^*_{ijn} = x^* = \overline{x^*}$ and $y^*_{in} = w^* - nx^*$, the solution of Equation (2.1) satisfies the following conditions:*

$$S1: \quad \frac{r^*[1 - G(r^*)]}{\int_{r^*}^1 [1 - G(p)]dp} - \frac{\alpha}{\iota}(1 - \eta) = 0 \tag{2.7}$$

$$S2: \quad \frac{1 - \eta}{\eta\iota} x^*\theta^* H'(w^*) - b^*r^* = 0 \tag{2.8}$$

$$S3: \quad \alpha - F(x^*, \overline{x}^*)[1 - G(r^*)] = 0 \tag{2.9}$$

$$S4: \quad w^*\theta^* H'(w^*) - \theta^* H(w^*) - \gamma\iota = 0 \tag{2.10}$$

$$S5: \quad \frac{\eta\iota}{\eta\iota + (1 - \eta)\alpha} w^* - \overline{n}^* x^* = 0 \tag{2.11}$$

[9] See page 272 in Williams [1998] for more discussion on it.

For $i = 1, ...; j = 1, ..., n;$ and $n = 0, 1..., n^* = [\frac{w^*}{x^*}] = [\frac{\overline{n}^*}{\pi}]$. Here, n^* is the largest integer not bigger than the real value $\frac{w^*}{x^*}$. Hence, $y^*_n > 0$ when $n \leq n^* - 1$; $y^*_{n^*} \geq 0$ if $n = n^* = [\frac{w^*}{x^*}]$; and $y^*_{n^*+1} = 0$.

The proof is in Appendix A.

One counter-intuitive conclusion from Williams [1998] is the absence of the agency problem, i.e., $r^*_{b=1} = r^*_{b=0.06}$ and $x^*_{b=1} = x^*_{b=0.06}$. What leads Williams [1998] to "prove" the absence of an agency problem? The spirit of his argument on no agency problem is that, from Equation (2.7), we agree that you can solve the r^*. Furthermore, take this r^* into Equation (2.9), you can also solve x^*, and these two equations have no b^* inside. Thus, r^* and x^* are not related to the commission rate[10], which indicates no agency problem. Economically, as discussed in Han & Strange [2015], no agency under Williams [1998] implies that the agent's marginal productivity of marketing effort, which is equal to the marginal productivity of searching for new listings, must be proportional to the commission rate. Only in this case, the optimal effort per listing can be independent of b^*.

It turns out that, mathematically, Williams [1998] misuses the regular partial derivative argument when variables are non-independent. Although there is no b^* appears in Equation (2.7) and (2.9), these two equations still have another common parameter α. Williams's conclusion would be correct only if we could meaningfully take $\frac{\partial r^*}{\partial b^*}$ and $\frac{\partial x^*}{\partial b^*}$, by holding other factors (i.e., α, ι, η and γ) constant.[11] Unfortunately, it is impossible. To see it note that the claim of no agency problem would imply that α, the likelihood of sale, must be independent of the buyer's arrival process (η), agent's discount rate (ι), entry cost (γ) and commission rate (b^*). That is unrealistic and would directly contradict the steady state conditions outlined by Equations (2.7) to (2.11) [12].

Chiang & Wainwright (2005, Chapter 8) discuss the correct way to conduct comparative-static analysis under this circumstance; hence our treatment follows their procedure. Given five conditions in the steady state, in addition to x^*, r^* and y^* (hence w^*), we need to find two more free parameters to make the system compatible. Due to the homogeneity of $F(x)$ and discount rate, η and ι should be constant. Thus, we let α (related to agent's marketing efficiency and the likelihood of sale via Equation (2.9)) and γ (related to the entry barrier

[10] See the first paragraph on page 258 in Williams [1998].

[11] For a non-technical discussion, see MIT course lecture: Partial Differentiation with Non-independent Variables, Example 1, available at https://bit.ly/2PBwqkY.

[12] For a rigorous proof on such a contradiction, see the first paragraph of Appendix B.

of brokerage industry) vary when conducting comparative statics analysis on b^* (the equilibrium compensation level). As a result, throughout the paper, we adopt this choice and choose α and γ as free parameters. We now take $\frac{\partial r^*}{\partial b^*}$ and $\frac{\partial x^*}{\partial b^*}$ by holding other factors like discount rate (ι) and arrival elasticity (η) constant.

Given the system of equations (2.7) to (2.11), and by the property of comparative-static analysis of general function model (Chiang & Wainwright (2005, Chapter 8)), because they all have continuous partial derivatives with respect to all variables, the following Jacobian determinant $|J|$ is nonzero.

$$|J| = \left| \frac{\partial(S1, S2, S3, S4, S5))}{\partial(r^*, x^*, \alpha, w^*, \gamma)} \right| \neq 0 \tag{2.12}$$

Under the condition of the Jacobian determinant $|J|$ is nonzero, by Cramer's rule, we can obtain the first-order derivative. With all technical details in the Appendix B, the following proposition characterizes the equilibrium effort and the reservation price in the steady state.

Proposition 1. *In the steady state with delegation, we have the following:*

$$\frac{\partial r^*}{\partial b^*} > 0 \tag{2.13}$$

$$\frac{\partial x^*}{\partial b^*} > 0 \tag{2.14}$$

$$\frac{\partial w^*}{\partial b^*} > 0 \tag{2.15}$$

$$\text{For some integer } N \geq \overline{n}, \quad \frac{\partial y^*}{\partial b^*} \begin{cases} > 0 & n \leq N \\ \leq 0 & n > N \end{cases} \tag{2.16}$$

$$\frac{\partial \alpha}{\partial b^*} = \frac{\partial \{F(x^*, x^*)[1 - G(r^*)]\}}{\partial b^*} > 0 \tag{2.17}$$

The proof is in the Appendix B.

Proposition 1 bring back the agency problem. In particular, the partial derivative of r^* with respect to b^* is positive. This prediction is consistent with Rutherford et al. [2005, 2007] and Levitt & Syverson [2005], who find that brokers tend to sell their own houses at a premium range between 3 and 7% compared with when they sell on behalf of their clients.

Equation (2.14) implies that a broker will also spend more effort on marketing her own property than her clients. Because $F(x)$ is concave, it must be the case that $F(x^*)_{b=1} > F(x^*)_{b=0.06}$ and $F'(x^*)_{b=1} < F'(x^*)_{b=0.06}$. On the other hand, Equation (2.16) shows that the incentive/disincentive of soliciting additional seller depends on the current network size. When the existing network size is not too large (i.e., when $n \leq N$), and when the commission rate increases, the agent will commit more effort to searching for a new seller. In contrast, when an agent's existing network is big enough, increasing compensation to the agent tends to disincentivize her from finding a new seller.

Another prediction is about the time on the market (TOM). Based on Equation (2.17), the likelihood of sale, $F(x^*, \overline{x}^*)[1-G(r^*)]$, is the probability of buyers' arrival rate multiplied by the probability of successful matching. On the one hand, with a higher commission rate, a broker will commit more effort in searching for potential buyers, which increases the buyers' arrival rate. On the other hand, a broker will also ask for a higher price, which reduces the matching probability. Overall, it turns out that the first force always dominates and hence leads to a higher likelihood of selling. Because the TOM is the inverse of the likelihood of sale, the model predicts that the TOM and hence expected duration would be shorter when the commission rate increases.

Several recent empirical studies support our theoretical predictions. For example, Turnbull & Dombrow [2007] conduct a direct test on $\frac{\partial r^*}{\partial b^*} > 0$. Using single-family transaction data in Baton Rouge, Louisiana, they find that the realized transaction price is 2.3% higher when a single broker is both the listing and selling agent.[13] Jia & Pathak [2010] find that likelihood of sale is positively related to the agent's commission and that TOM is negatively related. Bian et al. [2017] document that when clients' assets compete with agent-owned assets, properties remain on the market, 30% to 46% longer and are sold for 1.8% less. These findings also provide indirect evidence on the marketing effort that is consistent with the prediction that $\frac{\partial x^*}{\partial b^*} > 0$. As discussed before, given a bidding distribution G, a higher price per se implies a lower matching likelihood. Therefore, the only way to get a shorter TOM comes from a more intensive search for potential buyers.

[13] And hence captures the whole 6% commission rather than splitting it with another agent.

2.3.2 Model Results with Diffused Effort

We now consider the general case with diffused effort, which gives a potential buyer a certain probability to have sequential bids within a Δt period before either a successful match or the exhaustion of all inventory. This extension complicates the tractability of the model, because of the $[\rho G(r)]^n$ term in Equation (2.4). Here we impose a similar assumption as is used by Williams [1998], i.e.,$\Delta V_n = \Delta V_{n-1}$. This assumption in turn implies $[\rho G(r)]^n \sim 0$. Given this assumption, Equation (2.4) can be simplified to:

$$\begin{aligned}
V(n) &= \underbrace{max}_{r_{ijn},x_{ijn},y_{in}} \quad e^{-\iota\Delta t}\{\sum_{j=1}^{n} F(x,\overline{x}^*)\frac{1}{1-(\rho G)}\int_{r_{ijn}}^{1} b^* p dG(p)\Delta t \\
&+ \alpha\overline{n}^*(y/\overline{y}^*)V(n+1)\Delta t - \theta H(w_{in})\Delta t \\
&+ \sum_{j=1}^{n} F(x,\overline{x}^*)(1-G)\frac{1}{1-(\rho G)}V(n-1)\Delta t \\
&+ \left[1-\alpha\overline{n}^*(y/\overline{y}^*)\Delta t - \sum_{j=1}^{n} F(x,\overline{x}^*)(1-G)\frac{1}{1-(\rho G)}\Delta t\right]V(n)\} \\
&+ o(\Delta t)
\end{aligned} \tag{2.18}$$

In the steady state, the likelihood of sale, although it is still equals to α, changes to the following form:

$$\alpha = \frac{F(x^*,\overline{x}^*)[1-G(r^*)]}{1-\rho G(r^*)} \tag{2.19}$$

Recall that in the case without diffused effort, $\alpha = F(x^*,\overline{x}^*)[1-G(r^*)]$. The denominator in Equation (2.19) reflects the positive impact from the diffused effort externality on selling likelihood.

Solving the maximization problem on r^*, x^*, y^* in the case of diffused effort yields similar relations between the key variables and b^*. In addition, new results on relations between these key variables and diffused effort via ρ emerge. We summarize the results in the following proposition and the subsequent simulations.

Proposition 2. *In the steady state with delegation and for $0 \leq \rho \leq 1$, we have the following:*

$$\frac{\partial r^*}{\partial b^*} > 0 \tag{2.20}$$

$$\frac{\partial x^*}{\partial b^*} > 0 \tag{2.21}$$

$$\frac{\partial w^*}{\partial b^*} > 0 \tag{2.22}$$

$$\textit{For some integer } N(\rho) \geq \overline{n}, \quad \frac{\partial y^*}{\partial b^*} \begin{cases} > 0 & n \leq N(\rho) \\ \leq 0 & n > N(\rho) \end{cases} \tag{2.23}$$

$$\frac{\partial \alpha}{\partial b^*} = \frac{\partial \{\frac{F(x^*, \overline{x}^*)[1-G(r^*)]}{1-\rho G(r^*)}\}}{\partial b^*} > 0 \tag{2.24}$$

$$\frac{\partial r^*}{\partial \rho} > 0 \tag{2.25}$$

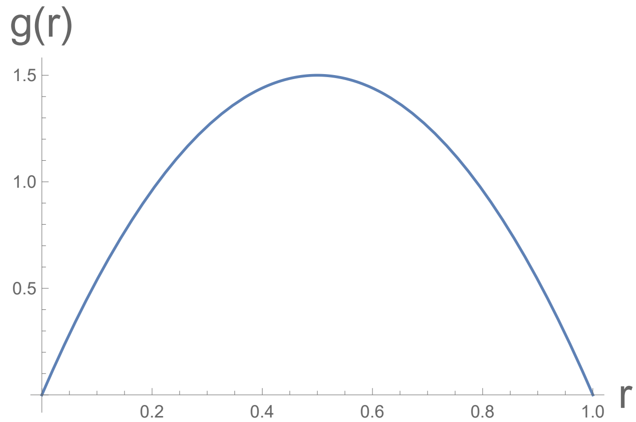
The proof is in the Appendix C.

Equation (2.20) to (2.24) re-affirm that all the previous findings on the existence of the agency problem without diffused effort (i.e., $\rho = 0$) pass to an arbitrary ρ that is between 0 and 1. Equation (2.25) further shows that the fact that an agent's effort on one task can potentially benefit others provides a network externality that encourages agents to increase the reservation price.

Solving Equation (2.18) enables us to obtain the relation between x^* and ρ and y^* and ρ as well. Unfortunately, the messier mathematical form makes its directional implications unclear. As a result, from now on, we make our model more parameterized. In particular, we assume that the bidding distribution $g(r) = G'(r) = 6r - 6r^2$ follows the symmetric Beta(2,2) function, as plotted in Figure 2.1.

We first examine whether the existence of diffused effort improves the expected selling proceeds, which in turn generates a Pareto-improvement on both the seller and agent welfare. With details of proof abstracted in Appendix C, Equation (2.26) shows that the expected gross proceeds from each asset, which equals to the likelihood of sale multiplied by the

Figure 2.1: The Density Function of Beta (2,2)



reservation price, increases as the diffused effort externality becomes bigger. [14]

$$\frac{\partial \alpha r^*}{\partial \rho} = \frac{\partial \{ \frac{F(x^*, \overline{x}^*)[1-G(r^*)]}{1-\rho G(r^*)} r^* \}}{\partial \rho} > 0 \tag{2.26}$$

Thus far, we have shown that diffused effort generates a positive externality that incentivizes agent to mark up the reservation price (Equation 2.25) and improves the welfare for both the seller and agent (Equation 2.26). We now explore the relations between the diffused effort (ρ) and the effort on creating a new task $y^* = y^*(\rho)$, the effort on existing task $x^* = x^*(\rho)$, and the likelihood of sale $\alpha = \alpha(\rho)$. Hereafter we assume $F(x, \overline{x}) = x^\eta$ and $H(w) = 1 - \sqrt{1 - w^2}$. Furthermore, we fix the parameter values by setting $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$ and $n = \overline{n} = 20$.

The parameter that warrants special attention is η, the elasticity of the buyer's arrival function $F(x) = x^\eta$. It measures the ease of soliciting a potential buyer in the market. Thus, given the same effort level, a smaller value of η would lead to a higher arrival rate. Hence η can serve as a proxy for demand strength. We plot the numerical results in Figure 2.2.

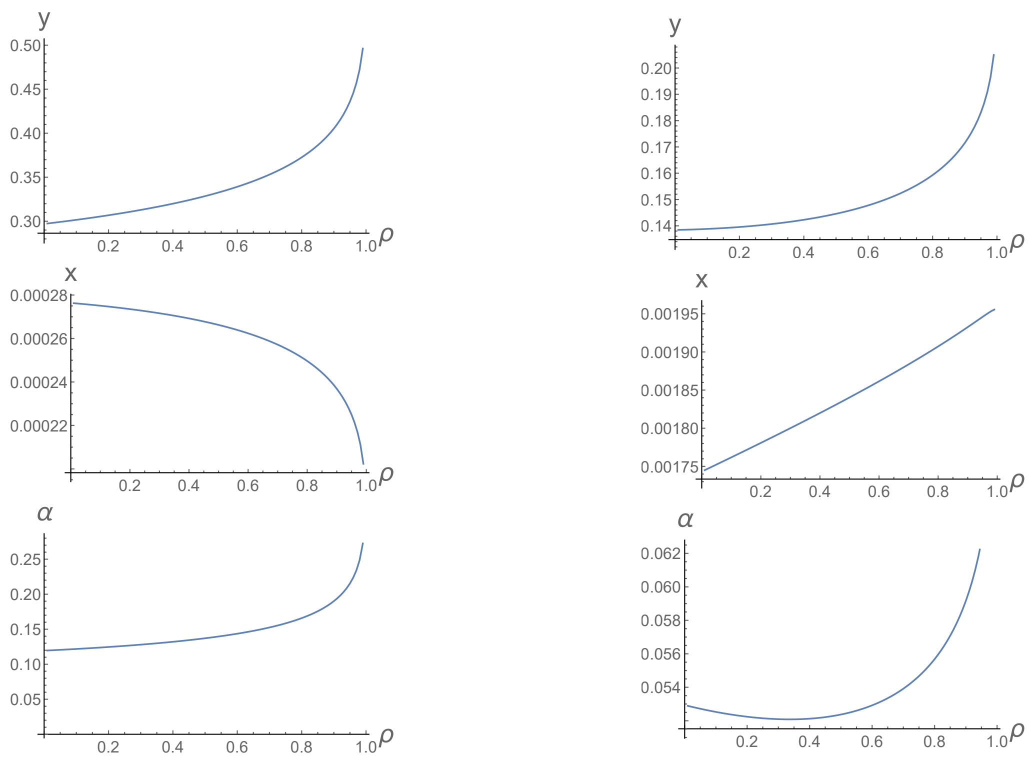
There are several major takeaways from Figure 2.2. First, a stronger diffused effort externality stimulates agents to commit more effort on searching for new sellers (y^*). The intuition is that, when effort pooling becomes easier, the extent of this positive externality not only depends on ρ but also depends on n, the network size. Hence it is in an agent's self-interest to enlarge the size of the network to maximize the potential gains from cross-

[14]Although we cannot prove the validity of Equation (2.26) for the whole family of Beta functions for G, our case by case algebraic proofs show that it holds for a wide class of Beta functions, in addition to Beta(2,2). These additional results are available upon request.

Figure 2.2: Other Comparative Statics on Diffused Effort (ρ)

Panel A: Stronger Arrival $\eta = 0.1$

Panel B: Weaker Arrival $\eta = 0.4$



Notes: For Panel A and Panel B, we set $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$ and $n = \overline{n} = 20$. η comes from the elasticity of the buyer's arrival function $F(x) = x^{\eta}$.

utilizing efforts from other tasks.

Second, as ρ becomes larger, an agent's marketing effort on each existing task (x^*) tends to decrease in a stronger arrival market, but an increase in a weaker arrival market. Hence, in a strong market, diffused effort externality seems to lead agents to shift more effort to find a new seller at the expense of existing customers. However, as shown in Equation (2.26), existing customers can indirectly benefit from the larger network size, and their expected payoffs still increase in ρ. Regarding total effort w^*, although not displayed, it can be shown that in both cases, a stronger diffused effort externality motivates the agents to commit more w^*, which equals $nx^* + y^*$.

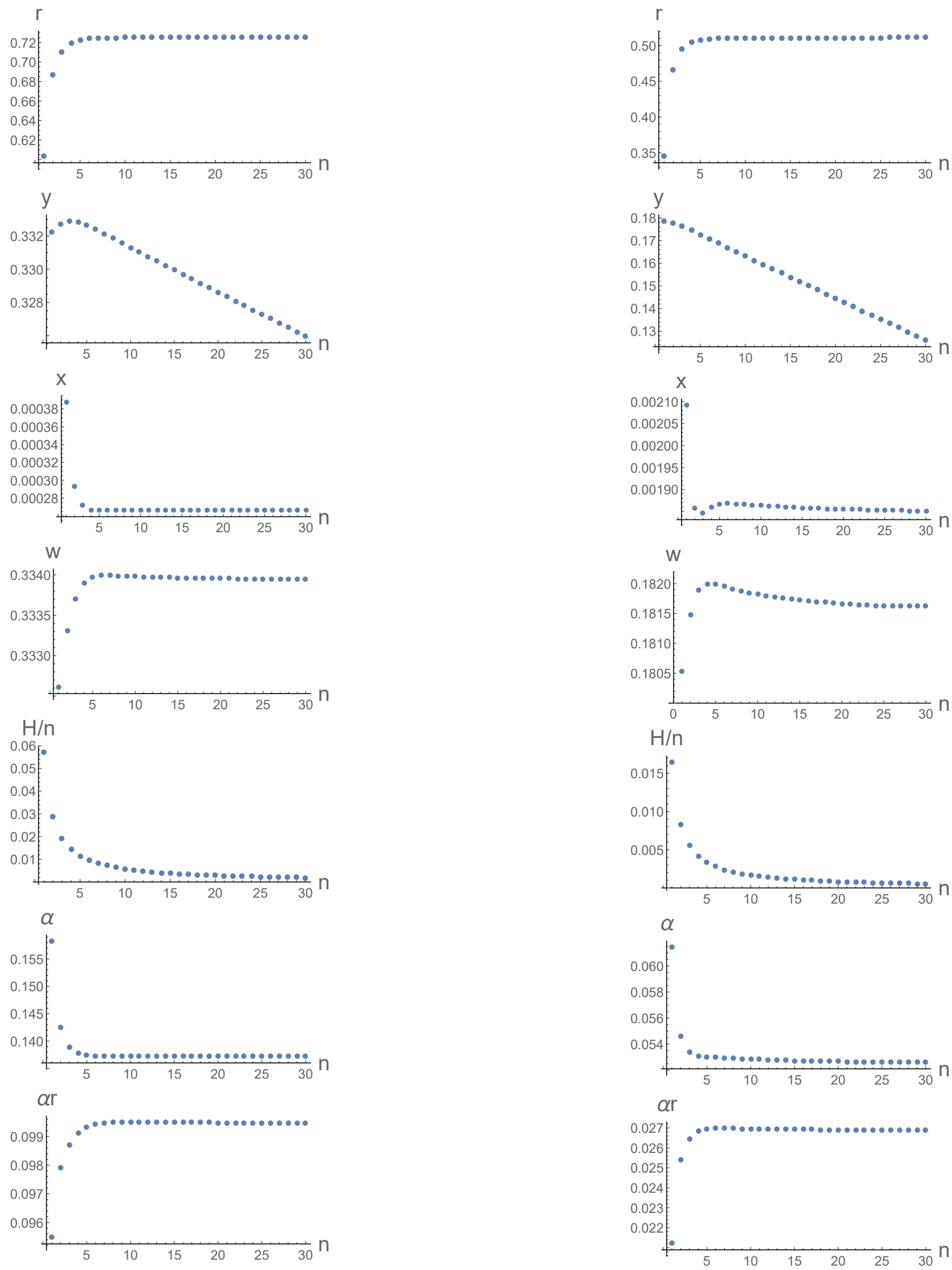
Third, while diffused effort greatly improves the matching efficiency hence increases the arrival rate, it does not always lead to a shorter TOM. As seen from Panel B, when the arrival process is weak, the likelihood of a sale first declines (despite increasing marketing effort) with ρ, then increases with ρ when it continues to increase. The reason a higher arrival rate is not sufficient to shorten TOM is that TOM is jointly determined by the arrival (hence x^*) and reservation price (r^*). Furthermore, our numerical results show that, in a weak market, the impact from $\frac{\partial r^*}{\partial \rho} > 0$ can dominate and hence increase TOM.

We now turn our attention to the impact of network size (n) on agent behavior. We assume a moderate diffused effort potential by setting $\rho = 0.5$ and maintain the same values as we did for the other parameters. The results are plotted in Figure 2.3.

First, note that for both cases, the optimal reservation price r^* initially increases with network size, and it quickly converges to a stable level with a moderate network size. The relation between the optimal y^* and n is interesting. Based on Panel A, in a strong arrival market, the time allocated by the agent on searching for new sellers tends to increase when the network size is relatively small, then decreases as the size of inventory further grows. The rationale for the initial increase in y^* is that, given the smallest network size of $n = 1$, the immediate benefit on y^* is low as there is no diffusion externality. In the meantime, with a strong market arrival process, the immediate benefit of finding a buyer on the existing listing is high. Hence, the agent is more willing to trade away some effort on y^* for x^*. When n=2, the potential benefit of further increasing network size is bigger, hence agent is willing to spend more time to create a new task and hence expand the network size for the future. With a weak arrival process, the incentive to trade y^* on x^* becomes less when n is small, as it is harder to find a buyer anyway, and it is beneficial to have a larger network size for

Figure 2.3: The Effect of Network Size n

Panel A: Stronger Arrival $\eta = 0.1$ | Panel B: Weaker Arrival $\eta = 0.4$



Notes: For Panel A and Panel B, we set $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$, $\overline{n} = 20$, and $\rho = 0.5$. η comes from the elasticity of the buyer's arrival function $F(x) = x^\eta$.

the future by committing more y^*. It is worth noting that this differential effect of network size on y^* can only happen when n is small. That is because, with large n, as w and x^* become stable along n, and as $y^* = w^* - nx^*$, the $\frac{\partial y^*}{\partial n}$ tends to converge to a negative constant.

Regarding the effort spent per task (x^*), we observe a huge drop when n increases from 1 to 2, reflecting a dilution effect. With strong arrival, x^* continues and quickly converges to its steady state level as n further grows; and it first increases and then decreases gradually when the arrival process is weak. For the total effort of $w^* = nx^* + y^*$, we find that they all increase dramatically when n is small. Meanwhile, for a strong arrival case, it seems to converge faster. However, the average cost function $H(w^*)/n$ is decreasing dramatically when n is small, which leads to the great cost advantages on the economies of scale. When n is large, the average costs are all converge to the constant levels, which means that this benefit is marginal diminishing.

In terms of overall marketing outcome, the common pattern is that as n grows, the likelihood of sale tends to decline and converges to the steady-state level; additionally, the reservation price tends to increase and converge to the steady-state level. Finally, there is a significant increase in the expected revenue (αr^*) per unit when n is small and growing, which also converges to its steady-state value as n further grows. Recall that much of our discussion in this section is based on the assumption that $[\rho G(r)]^n \sim 0$. The fast convergence rate (around n=10) observed in Figure 2.3 suggests that it reasonably characterizes the majority of the cases.

The relevant studies in the literature on the agent's network size are Brastow et al. [2012] and Bian et al. [2015]. Bian et al. [2015] show that taking on additional inventory results in the dilution of an agent's selling effort. Furthermore, they document empirical evidence that a larger inventory tends to lead to longer TOM, which is equivalent to a lower likelihood of sale. These two findings are consistent with our model. Nevertheless, these authors also find empirically that larger agent inventory tends to reduce the selling price, whereas our model predicts either a positive (when n is small) or insignificant (moderate to large n) association. We defer the discussion on the likely cause of this discrepancy in a later section on model limitation and extension.

2.3.3 Model Results with Heterogeneous Assets

Thus far, we focus on the single type asset case. Now let us allow a broker to hold two different types of listings at the same time, as specified by Equation (2.5), and examine her effort allocation and pricing strategy.

As previously discussed in section 2.2.2, the empirical literature testing agency problem typically looks at the case when a broker sells for clients' houses (type 1) while selling for her own (type 2). According to Equation (2.5), it is equivalent to the case when $b_1^* < 1$, $b_2^* = 1$, $n_2 = 1$ and $y_2 = 0$. To maintain an apple to apple comparison with the original Williams model, we set ρ as 0 for now. Hence, Equation (2.5) with two asset types collapses to the Williams model with n+1 listings when we set $b_1^* = b_2^*$. The following proposition re-affirms the existence of the agency problem.

Proposition 3. *In the heterogeneous assets model as specified by Equation (2.5), assuming type 1 and type 2 assets are otherwise identical except that* $b_1^* < 1$, $b_2^* = 1$. *Further assume* $n_2 = 1$, $y_2 = 0$ *and* $\rho = 0$. *Then,*

$$\widehat{r}^* > r^* \tag{2.27}$$

$$\widehat{x}^* > x^* \tag{2.28}$$

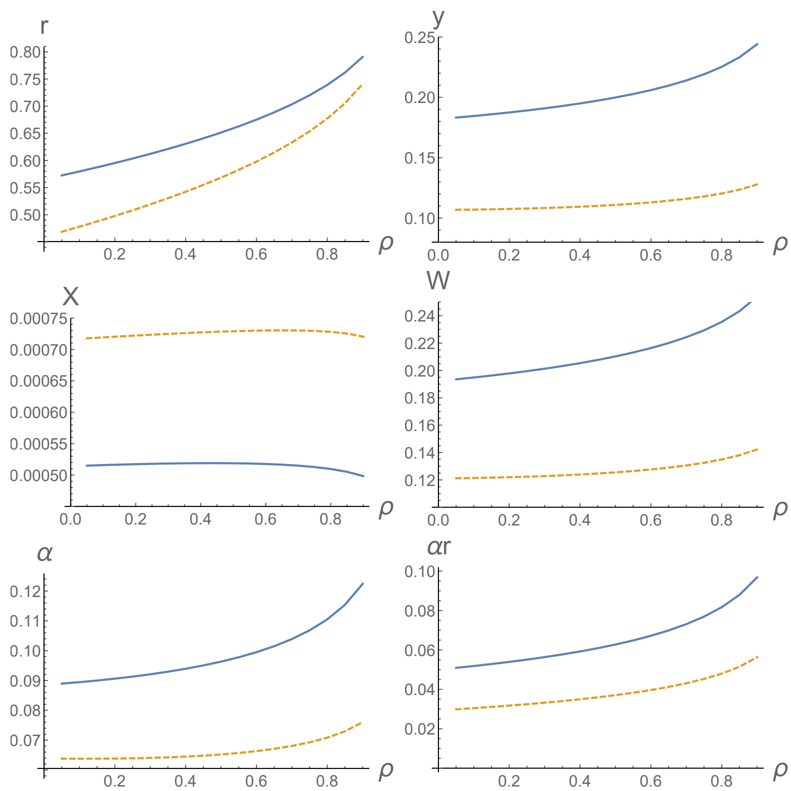
where r^*, x^* *are optimal solutions for* $b_1^* < 1$ *assets, and* $\widehat{r}^*$, $\widehat{x}^*$ *are optimal solutions for* $b_2^* = 1$ *asset.*

The proof is in the Appendix D.

Proposition 3 re-affirms the existence of the agency problem in a more realistic setting than Proposition 1. That is because, in Proposition 1, we implicitly assume that all assets have the same commission rate, and then look at the agency problem from a comparative static perspective on how effort may change when market-wide commission rate increases. Here, we show that given a level of market commission rate (say, $b_1^* = 0.06$), as it must be smaller than 1, a broker will always commit more search effort and ask for a higher price when she sells her own property.

We now consider asset heterogeneity on other dimensions, and with diffused effort. In the following numerical simulation, we maintain the same parameter values for two assets on $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$, and $n_1 = \overline{n}_1 = n_2 = \overline{n}_2 = 20$. First, we look that the heterogeneity on-demand strengthen, as measured by the buyer arrival rates. We examine how the key parameters respond differently to the presence of diffused effort externality. As discussed before, a smaller value of η reflects a stronger buyer arrival process. We plot the results in Figure 2.4:

Figure 2.4: Asset Heterogeneity on η



Notes: We set $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$, $n_1 = n_2 = \overline{n} = 20$, and $\rho = 0.5$. Solid lines refer to assets with $\eta = 0.2$, and dashed lines refer to assets with $\eta = 0.3$.

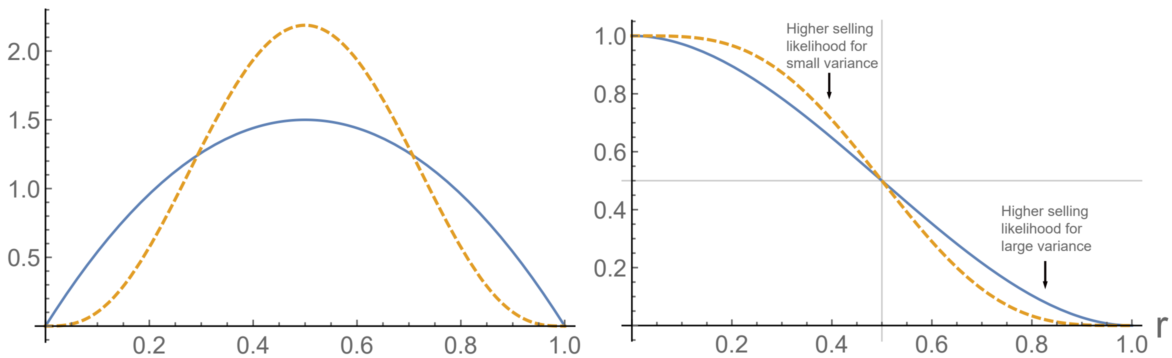
For a given asset type, the patterns of the key parameters along diffused effort externality is similar to Figure 2.2. Hence, our major focus now is on the impact of the heterogeneous arrival processes. Here we assume the level of ρ is common for both types of assets.[15] When a broker simultaneously manages two types of listings, she asks for a high price and spends more total effort on the assets that have stronger demand, which should be intuitive. Interestingly, the model reveals that at the individual level, the effort committed to an existing listing is less for a high demand type. Therefore, relatively speaking, the broker has a much stronger incentive to increase the network size of the high type. While the marketing effort

[15]We can easily relax and allow each type to have its own level of ρ, but we will have to plot a 3-d surface to observe the pattern. The current choice on a common ρ allows us to isolate the impact from η alone.

on an existing listing is lower for the high type, the selling likelihood and the expected payoff are still higher, because of the stronger arrival process and diffused effort externality.

Another type of heterogeneity is the valuation distribution of the assets. We first consider bidding distribution heterogeneity caused by variance differences only. As an example, we assume type 1 assets have Beta (2,2) bidding distribution, and type 2 assets have Beta (4,4) distribution. In Figure 2.5, we plot the density function (i.e., $g(r)$) and selling likelihood function (i.e., $1 - G(r)$) for both distributions.

Figure 2.5: Bidding Distribution Heterogeneity on the Variance

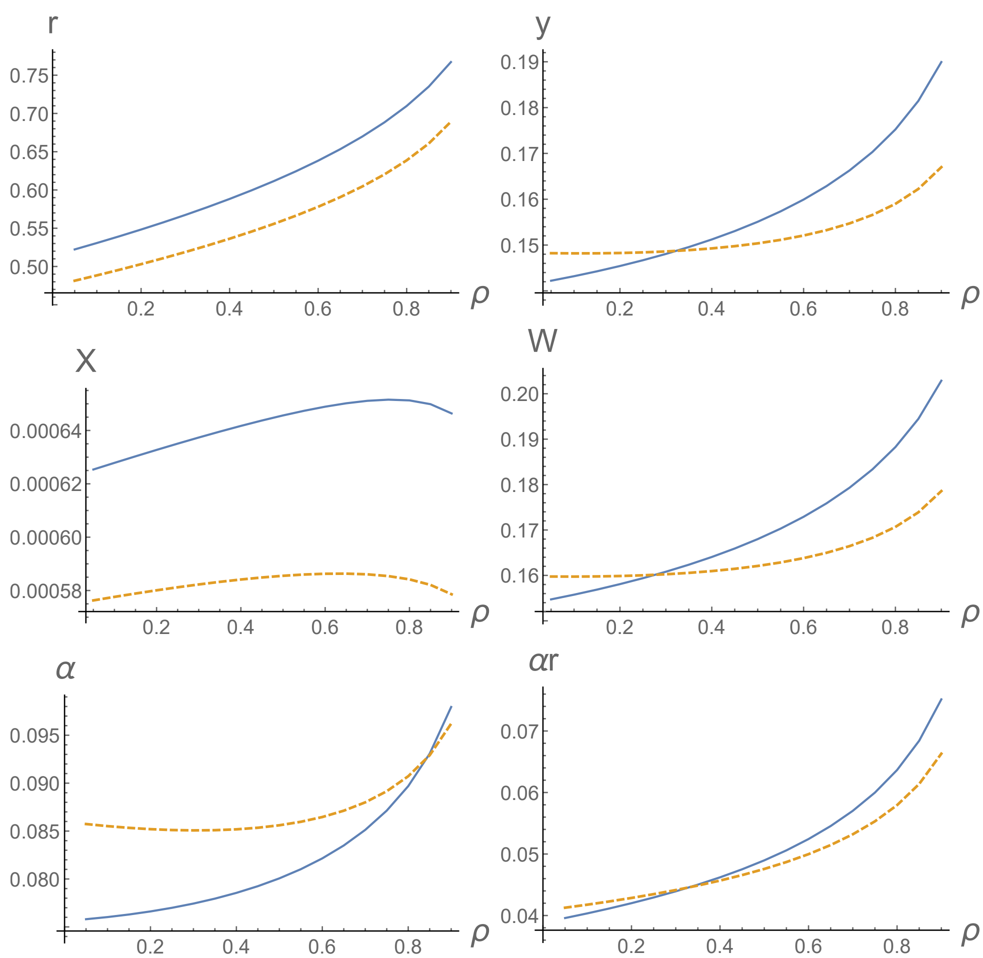


Notes: 1. Panel A: Density functions: g(r). Panel B: Selling likelihood function: $1 - G(r)$.
2. Solid curve: $Beta(2, 2)$, with variance of 0.05. Dashed curve: $Beta(4, 4)$, with variance of 0.028.

As seen in Figure 2.5, both valuation distributions have the same mean of 0.5, but the variance for type 1 asset is larger (0.05) than type 2 (0.028). Using the same parameter settings, we plot the simulation results in Figure 2.6.

According to Figure 2.6, for low variance asset, an agent always asks for a lower price. Interestingly, for low variance type and when diffused effort externality is relatively weak (i.e., a small ρ), although the agent also always commits less marketing effort on existing listings, she may spend more total effort and deliver a higher expected payoff. Hence the higher total effort is driven by the incentive of finding a new seller (y^*). With stronger diffused effort externality, the agent starts to allocate more effort to high variance assets. The graph on reservation price in Figure 2.6 sheds some light on the reason for the shifting focus by the agent. When diffused effort externality is weak, the optimal reservation price for low variance asset turns out to be smaller than the mean of the valuation distribution. Given a reservation price that is smaller than the mean, as seen from Panel B of Figure 2.5, lower variance distribution actually has a high selling likelihood. Hence it incentivizes the agent to spend more total effort on this type of asset. However, as diffused effort externality gets stronger, the optimal reservation price for low variance assets eventually will bypass

Figure 2.6: Valuation Heterogeneity on the Variance



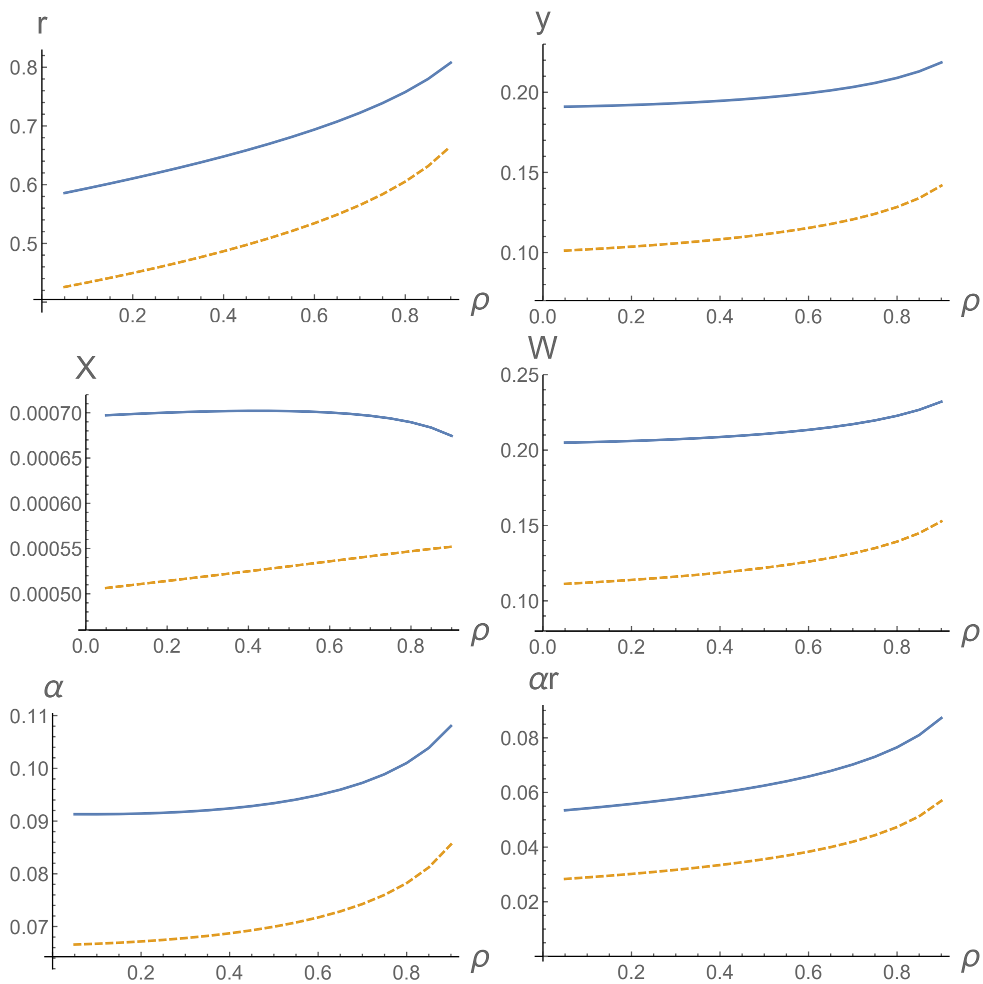
Notes: We set $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$, $n_1 = n_2 = \overline{n} = 20$, $\rho = 0.5$ and $\eta = 0.25$. Solid lines: $Beta(2, 2)$, with variance of 0.05. Dashed lines: $Beta(4, 4)$, with variance of 0.028. Both of bidding distributions have the same mean 0.05.

the mean, beyond which it loses its comparative advantage on selling likelihood. Hence, eventually, agents will switch the preference and focus more on the high variance assets.

Bidding valuations can also differ on the mean. To isolate the effect from mean difference only, we require that the valuation distribution with high mean stochastic dominates the one with low mean, and both distributions have the same variance. As a numerical example, we set the bidding distribution as $beta(2, 3)$ for low mean (0.4) type, and as $beta(3, 2)$ for high mean (0.6) type. It is easy to verify that these two distributions satisfy the retirements. All baseline parameters remain the same as before. We plot the results in Figure 2.7:

The impact of the heterogeneity on the means of valuation distributions is straightforward. As high mean distribution stochastic dominates the low mean distribution, holding

Figure 2.7: Valuation Heterogeneity on the Mean



Notes: We set $b^* = 0.06$, $\iota = 0.02$, $\theta = 1$, $n_1 = n_2 = \overline{n} = 20$, $\rho = 0.5$ and $\eta = 0.25$. Solid lines: $Beta(3, 2)$, with mean is 0.6. Dashed lines: $Beta(2, 3)$, with mean is 0.4. Both of bidding distributions have the same variance 0.04.

other factors the same, it is always more beneficial for the agent to focus more on high mean assets. Hence agent's optimal response is to ask for a higher price and to commit more effort on both existing listings and new sellers on high mean assets.

2.4 Limitations and Future Extensions

While we have presented a model that supports a range of empirical findings documented in the literature, there are some serious limitations remained.

First, one prediction from our model is that the network size of an agent is positively associated with reservation price. This turns out to be opposite to what is found in some empirical studies. A likely reason is that our model ignores the potential interactions among agents. In a completely different setting, Deng, Seiler, and Sun [2019] build a model demonstrating that in a bigger brokerage firm, two agents are more likely to engage in an internal transaction, which lowers the transaction price. Hence agent heterogeneity and interaction could play important roles in shaping their trading strategies.

Second, as pointed out by Piazzesi, Schneider, and Stroebel [2020], housing search models and empirical studies often ignore the buyer side, partially due to data constraint. Our model is no exception as we put little structure on the buyer in the search process, except for an exogenous arrival process. It is unrealistic to assume that a buyer is equally likely to search all the houses in a market. Also, it is unlikely that an agent will find a buyer who is willing to look at all the assets from the entire agent's network. More likely, buyers will perform their pre-screening based on the information from the advertisement, and they would only search within a sub-market which meets their preference. Nowadays, the most important source of information can be found relatively easily on the Internet. Typically, buyers do the initial search on-line and learn about the house and neighborhood from websites like Zillow.com, etc. As a result, buyers who come to the agents for physical searches often focus mainly on features that are unobservable from the web sites. It would be very interesting to incorporate this kind of buyer heterogeneity and clientele effect into the search model.

Third, when the seller's broker receives the commission fee, under the MLS guidelines, her then (equally or not equally) splits the commission fee with the cooperating broker as the buyer's side. This is because historically, the cooperating broker was viewed as a subagent of the seller and represented the seller's interest. Nowadays, in some cases, the broker working with the buyer is no longer a subagent of the seller, but rather a separate broker of the buyer. However, Miceli et al. (2000) suggested that such an arrangement still has not addressed the agency problems associated with buyer brokers being compensated based on the sales price. Moreover, this agency's incentive issues become severe when buyers and sellers are represented by the same brokerage firm or even the same person. This is so called in-house transaction or dual agency transaction. Han and Hong [2016] claims that about 20% of residential real estate transactions in North America are in-house transactions, for which the same brokerage represents buyers and sellers. Many studies have tested the effect of dual agency: For example, Gardiner et al. (2007) find that dual agency reduced the sales price and the time-on-market. Besides, they find that both reduction effects were weaker after law enforcement in Hawaii in 1984 that required full disclosure of dual agency. Our model has the potential to include the dual agency scenario into consideration. Suppose the sellers' broker has two different assets, type 1 is the normal houses, but type 2 is the dual agency houses. Thus, compared with type 1 houses, type 2 houses should have a higher arrival rate function (as the broker have incentives to sell the type 2 houses first), a larger ρ (as the in-house transaction makes sequential visit process more easily) and, a lower commission fee (as the broker does not need to split it).

2.5 Conclusion

Real estate brokerage forms a very special principal-agent relationship. First, a broker executes multiple tasks on behalf of a group of sellers as principals. Second, the broker's limited effort is split among the existing tasks and on finding new sellers to achieve the desired network size. Third, the compensation structure is typically a constant commission rate upon the realized sale price. Williams [1998] proposes a brokerage model that incorporates all of the above features. Building upon his model, we propose a multiple tasks agency model with both diffused effort and asset heterogeneity

Because our general model setup embeds the Williams model as a special case, we first revisit the benchmark Williams model. One of Williams' major conclusions is that there is no agency problem (i.e., an agent's marketing effort and reservation price are independent of the commission rate), and it is counter-intuitive. It turns out that part of the proof in Williams [1998] is incorrect. We first prove that in equilibrium, agency problem does exist in the original Williams model. For example, a broker will set a lower reservation value and spend less effort on marketing when the commission rate declines. We further show that the agency problem remains with the presence of diffused effort and asset heterogeneity.

Next, we examine the properties of diffused effort. In particular, we show that the existence of diffused effort generates a positive but size-dependent externality, which motivates the agent to ask for a higher price and yields a higher likelihood of a sale. Furthermore, a stronger diffused effort potential also stimulates agents to spend more resources on expanding network size. Regarding the impact of network size on agent's behavior, we find that in general, with the existence of diffused effort, a larger network size tends to dilute an agent's marketing effort on the existing listing and also reduce her incentive to further expand the size of the network. Nevertheless, a larger network improves marketing outcomes by yielding a quicker sale at a higher price and hence improves the welfare of both brokers and sellers. The marginal impact of network size is stronger when the initial size of the network is small.

Finally, our model shows that asset heterogeneity can play a significant role in shaping a broker's trading strategies, and we explore the impacts driven by different sources of heterogeneities such as commission rate, demand strength, and valuation distribution.

Chapter 3

Essay 2: Prospect Theory and a Two-way Disposition Effect: Theory and Evidence from the Housing Market

3.1 Introduction

The seminal works on prospect theory by Kahneman and Tversky [1979] and Tversky and Kahneman [1991, 1992] propose three major components to help explain the decision-making process of individuals under uncertainty. The first component is reference dependence, in which people derive utility over gains and losses relative to a reference value such as a prior acquisition price or an initial endowment. In the second component, diminishing sensitivity, the marginal value of both gains and losses declines with size. In this setting, people will be risk averse in the domain of gains but risk seeking in the domain of losses. In the third component, the loss aversion effect, people treat losses and gains asymmetrically in their value functions. In particular, a loss looms larger than an equal-sized gain. Barberis [2013] offers a comprehensive review of the broad applications of prospect theory in economics.

Although conceptually intuitive, much evidence that supports prospect theory comes from experimental studies (e.g., Kahneman, Knetsch, and Thaler [1991], Tversky and Kahneman [1991], Knetsch, Tang and Thaler [2001], Haigh and List [2005] and Imas [2016]). Not surprisingly, finding non-experimental evidence is an active research topic. Pope and Schweitzer [2011] find that golfers make their birdie putts 2% points less often than they make comparable par putts. They argue that the finding is consistent with loss aversion in the sense that players invest more focus when putting for par to avoid encoding a loss than putting for a birdie to achieve an “equal size" gain. Ellen et al. [2016] investigate bunching behavior from the marathon data and find strong evidence that marathon runners have reference points of round number finishing times.

As a classic study based on transactional data, Genesove and Mayer [2001] examine the home seller’s behavior and find a disposition effect. That is, compared to potential gainers, a seller subject to a larger potential loss, as measured by realization utility, sets a higher asking price, exhibits lower sales hazard, and obtains a higher transaction price if the house is sold. They further find that the marginal mark-up on price declines with the size of a seller's potential loss exposure. Genesove and Mayer interpret the disposition effect as a test of the loss aversion effect and the latter finding as a test of diminishing sensitivity. Their proposed connection between prospect theory and seller behavior has become popular in the literature. Follow-up studies using either the home listing or transaction prices to test loss aversion include Bokhari and Geltner [2011] and Anenberg [2011]. Both find similar results as in Genesove and Mayer [2001] and view them as evidence of loss aversion. Chan [2001] and Engelhardt [2003] test the loss aversion effect by examining factors that influence household mobility. Both find that potential losses have a negative relation with a household's mobility, which is consistent with the behavior that a seller subject to a larger potential loss will set a higher price when selling a house. Beggs and Graddy [2009] test the loss aversion effect using painting auction data and fail to find significance in a regression of auction price on seller's loss exposure, suggesting there is no loss aversion.

Field evidence supporting prospect theory is also documented in the finance literature. Many studies document a similar disposition effect: relative to the purchase price, investors have a higher propensity to sell stocks that have risen in value rather than those that have fallen (e.g., Shefrin and Statman [1985], Odean [1998], Grinblatt and Keloharju [2001], Feng and Seasholes [2005], Linnainmaa [2010], and Chang, Solomon and Westerfield [2016]). However, there is an active debate concerning which component of prospect theory drives the observed disposition effect in the stock market (Barberis [2013]). Barberis and Xiong [2012] show that reference dependence under realization utility can generate the disposition effect if the discount rate is sufficiently positive. Bodnaruk and Simonov [2016] find that mutual fund managers with a higher degree of loss aversion tend to exhibit a stronger disposition effect. As their paper does not discuss much on reference dependence and diminishing sensitivity, it seems to echo Genesove and Mayer [2001] that it is loss aversion that drives the disposition effect. Some researchers, however, argue that it is diminishing sensitivity that generates the disposition effect (e.g., Shefrin and Statman [1985], Li and Yang [2013]). Contrary to Bodnaruk and Simonov [2016], the simulation results from Li and Yang [2013] suggest that, in general, loss aversion tends to mitigate the disposition effect. Ingersoll and Jin [2013] also argue that an S-shaped utility function does not create the disposition effect; it actually reduces it.

The review above clearly shows the split views in the literature on the relation between the components of prospect theory and empirical trade patterns. This paper aims to examine how prospect theory affects agents' trading behavior. We illustrate this by building a simple search-match model in an asset (housing) market under heterogeneous valuations, which incorporates a generally defined prospect utility as a special case. We choose the housing market over the stock market as a modeling device for several reasons. First, the subject under prospect theory is easier to contextualize in the housing market than in the stock market, which makes model implications more clearly. As stocks are constantly traded in and out of portfolios, it is less clear what the unit of analysis should be (i.e., the portfolio as a whole or each stock individually?). The basis for the reference value is also unclear because the same company's stock can be purchased at different time and prices (Meng and Weng [2017]). Another challenge concerns the interactions of stocks within a portfolio. If an agent draws prospect utility from each stock individually[1], how will the response from one stock interact with that from another stock in a portfolio context? The popular measure used in the stock market on loss/gain positions, as proposed in Odean [1998], makes a simplifying assumption and essentially counts the proportion of losing stocks in a portfolio as a measure of the loss/gain position faced by an agent. This measure, while easy to construct and conceptually intuitive, is dichotomous (i.e., an overall incidence of gain or loss), and it overlooks the extent of gain/loss[2]. In contrast, the subject of analysis is easier to define in the housing market due to the capital-intensive and indivisible nature of housing assets. In reality, we seldom observe an individual home seller attempting to buy and sell multiple houses simultaneously. The second reason is that a model based on the housing market allows us to compare our results with Genesove and Mayer [2001] in order to obtain direct insights into the potential conceptual mismatch discussed above. Finally, owner-occupied housing units totaled approximately $27 trillion in 2017Q1, making residential real estate one of the most important asset classes in the United States[3].

This study contributes to the literature by isolating each component of prospect theory and its unique empirical implication. As discussed below, our model offers a wide range of empirical predictions and helps reconcile some seemingly contradicting findings documented in the literature that cause confusion and debate. In parallel with Barberis and Xiong [2012], we are among the first to independently show in a model that reference dependence alone

[1]See Barberis and Xiong [2009] for one such example.

[2]According to Odean [1998], for a stock in the investor's portfolio on the day that is sold, a "realized gain" is counted if the stock price exceeds the average price at which the shares were purchased, and a "realized loss" is counted otherwise. Similarly, for a stock in the investor's portfolio on the day that is not sold, a "paper gain" is counted if the stock price exceeds the average price at which the shares were purchased, and a "paper loss" is counted otherwise. Hence, we can define $\frac{RealizedGains}{RealizedGains+PaperGains}$ = Proportion of Gains Realized (PGR) and $\frac{RealizedLosses}{RealizedLosses+PaperLosses}$ = Proportion of Losses Realized (PLR). Then, for investors in a trading period, if they experience two realized gains and two paper gains, PGR=1/2. If they experience one realized loss and two paper losses, PLR=1/3. In this case, as PGR > PLR, people then interpret the finding as supporting the disposition effect.

[3]See Table B.100 entitled "Balance Sheet of Households and Nonprofit Organizations" in the Federal Reserve's Flow of Funds Report, which can be found at http://www.federalreserve.gov/releases/z1/current/.

generates the disposition effect.[4] However, our result is more general than Barberis and Xiong [2012]. Unlike the simulation-based study in Barberis and Xiong [2012], we adopt a generally characterized prospect value function equivalent to Wakker and Tversky [1993] and prove analytically that reference dependence alone generates the disposition effect. Moreover, their model requires a discount rate to be high enough to generate the disposition effect, while ours has no such restriction. Furthermore, Barberis and Xiong [2012] run their simulation under a realization utility framework, which requires the reference value to be the initial purchasing price. There is no such requirement in our model as the reference value is just a generic parameter according to the prospect theory.

The finding that reference dependence alone is enough to generate disposition effect offers profound empirical insights. For example, it implies that, contrary to the argument made in Genesove and Mayer [2001], their empirical findings do not have a direct relation with loss aversion. In fact, we show that what they find can be perfectly compatible with a value function that is loss neutral and has a marginally increasing sensitivity in both the gain and loss domains. Hence there seems to be a conceptual mismatch in Genesove and Mayer [2001] between the two stylized findings and their theoretical counterparts. A clarification on this point is much needed because, as reviewed above, many subsequent studies perceive the effect of loss aversion in the same way as in Genesove and Mayer [2001].

Regarding the component of diminishing sensitivity, we show that it distorts the disposition effect to two-way by inducing a local reverse disposition effect. In particular, when a seller is subject to a range of moderately sized losses, her asking price will be decreasing when the potential loss is increasing. Therefore, our model implies a non-monotonic (up-down-up) pricing curve along with agents' potential gain/loss positions, with the non-monotonicity arising only in the loss range. It is worth noting that Barberis and Xiong [2012] focus exclusively on a linear value function and predict a one-way disposition effect only. Interestingly, in a different context, Barberis and Xiong [2009] examine an investor's portfolio choice problem under a prospect value function with marginal diminishing sensitivity. When they match using Odean [1998] data, they indeed find a reverse disposition effect when the expected return is high. However, the authors view this finding as somewhat unexpected and propose using an alternative choice of reference point to recover the disposition effect. Our model offers a potential explanation as to why there seems to be a contradiction between the findings in these two papers.

While relatively rare, there have been findings documented in the literature of a reverse

[4] To our knowledge, the earliest draft that leads to Barberis and Xiong [2012] was presented in October 2007 at NYU Stern Five-Star Conference on Research in Finance. An earlier draft of our paper was presented in January 2018 at the ASSA-AREUEA conference by one of the co-authors. A slightly newer 2008 draft can be found in chapter 2 from https://open.library.ubc.ca/cIRcle/collections/ubctheses/24/items/1.0066841.

disposition effect. However, the proposed causes, if any, vary and are often against using prospect theory to explain the disposition effect. For example, many studies simply cite Barberis and Xiong [2009]'s finding on reverse disposition effect as an example of the weak relation between prospect theory and the disposition effect (see An [2015], Birru [2015], Heimer [2016] and Bian et al. [2017]). In the seminar study of Odean [1998], although the author shows a robust presence of a disposition effect, he also reports that stock trading in December behaves oppositely and exhibits a reverse disposition effect. Odean attributes this fact to tax-loss selling. Ben-David and Hirshleifer [2012] find a reverse disposition effect for buying additional shares without any realization conditions. They suggest that prospect theory is not the key to the (reverse) disposition effect. Cici [2012] and Lu, Sugata, and Teo [2016] find that fund managers show a tendency towards a reverse disposition effect and argue that it arises because these professional managers serve as potential mitigators of distortions caused by the behavioral biases of retail investors who are more likely subject to a prospect value function.

Our study helps reconcile the literature by showing that, as gain/loss positions change, the disposition effect and the reverse disposition effect can co-exist. We show it is the diminishing sensitivity, hence the underlying risk seeking behavior of the agent, that leads to such a two-way disposition effect. This finding also suggests studies that predominantly focus on a one-way disposition effect can be overly simplistic and misleading.

On loss aversion, we show that it tends to magnify the disposition effect. This offers a theoretical foundation supporting the findings of Bodnaruk and Simonov [2016]. This result also echoes Ray, Shum, and Camerer [2015], which examines asymmetric demand elasticity around a reference price. Although merely a discussion, the authors argue that it is the reference dependence effect that causes asymmetric demand elasticity, while loss aversion magnifies this asymmetry. As loss aversion intensifies the disposition effect, we show that it also mitigates the reverse disposition effect and suppresses the range of the reverse effect among potential losers.

Jointly, our model further predicts that the price dispersion in a cold market is higher than in a hot one. This is because in a cold market, there will be more home sellers subject to potential losses, and as discussed above, loss aversion will come into play and magnify the disposition effect, leading to more heterogeneous asking prices among sellers of similar homes. Finally, as more sellers ask for extreme prices in a distressed market, the selling hazard rate declines, which further reduces the transaction volume. Hence, our model helps to explain the positive price-volume relation observed in housing markets. Using multiple listing service data from Virginia, we find evidence mostly consistent with the predictions made by prospect theory.

The remainder of the paper is structured as follows. Section 3.2 provides an exposition of our model of a house seller's decision problem, which incorporates a generally characterized prospect utility as a special case, and discusses its implications. The empirical predictions are tested in section 3.3. Section 3.4 continues to conduct calibration exercises that shed light on the parameter values of the prospect value function. We conclude the paper in section 3.5. All the other materials are given in Appendix E, F, G , H and, I. which contains the proofs of Proposition 4 and 5, results of price-dispersion effect, calibration of the vary α, and some supplemental tables.

3.2 Model Setup and Results

We propose a simple search/match model of a home seller's pricing strategy under a prospect value function.

3.2.1 General Setup

Consider a large housing market with a countably infinite number of potential sellers[5]. Each seller has an ex-ante identical house for sale. To sell it, she must commit an effort to search for a potential buyer, who arrives via an independent Poisson process. For seller i, we define the proportional time spent on searching as t_i, which ranges from 0 to 1. During time window Δt, a buyer arrives with probability $B(t_i)\Delta t$. As a result, no buyer arrives with probability $1 - B(t_i)\Delta t$, and more than one buyer arrives with a probability that is smaller than the order of Δt. When a potential buyer arrives due to seller i's search effort, the buyer inspects i's house and decides whether the quality matches his preference. Depending on the matching quality, the buyer decides the highest possible price of p_i that he is willing to pay. We assume that p_i is a random draw from distribution G. If there is no match, $p_i = 0$. If there is a perfect match, $p_i = 1$, which is only a normalization. Therefore, G has a finite support [0,1]. Furthermore, we assume that p_i is independent across buyers, houses, and time. The distribution of G could be very general in this regard. One typical assumption is that $(1 - G)/g$, the inverse of the hazard function, is both non-increasing and marginally non-decreasing. The seller chooses an asking price of r_i for her house. If $p_i \geq r_i$, the transaction is closed and the seller pays r_i.

During time window Δt, the search effort incurs a cost of $H(t_i)\Delta t$. We assume that the arrival function B has the following properties. First, it is twice continuously differentiable everywhere. Second, B is increasing and strictly concave in t_i. This assumption implies a

[5]Our model of the search/match process shares some features with that of Williams [1998].

marginally decreasing productivity of a seller's searching ability. To prevent corner solutions, we also assume that $B(0) = 0, B'(0) = \infty$. Consistent with the literature, we further assume the search cost function H to be twice differentiable, increasing, and strictly convex in t_i. Next, we assume that $H'(0) = 0$ and $H'(1) = \infty$ to prevent any corner solutions. To match the normalization on G, we assume that $H(t)$ also has a domain of $[0, 1]$.

Collectively, seller i's objective is to solve the following maximization problem:

$$U^{v_i,c_i} = \underbrace{max}_{t_i,r_i} \quad e^{-\beta\Delta t}\{B(t_i)[1 - G(r_i)]\Delta t W(r_i, v_i) - H(t_i)\Delta t \\ +[1 - B(t_i)[1 - G(r_i)]\Delta t](U^{v_i,c_i} - c_i\Delta t)\} + o(\Delta t) \tag{3.1}$$

subject to $0 \leq t_i \leq 1$ and $o \leq r_i \leq 1$, for $i = 1, 2......$

In Equation (3.1), β is a positive discount rate, and c_i reflects the per-period net cost of staying in the current house, possibly due to the disutility of not being able to sell the house in the current period. A positive value of c_i means that the seller has an incentive to move sooner due to some attractive outside option. However, we acknowledge the possibility that for some sellers, c_i could be negative, which implies that remaining in the current house is more attractive than moving. This idea generates the positive effect of spatial lock-in. In this case, fishing by asking a higher than expected market price is a natural response for sellers, since they must ask potential buyers to compensate for giving up the benefit from superior matching. To rule out this trivial case, henceforth, we only consider the case in which c_i is non-negative.

Conditional on a successful sale, seller i draws utility as measured by value function $W(r_i, v_i)$. To minimize structural restriction, we adopt a general characterization of $W(r_i, v_i)$ according to Wakker and Tversky [1993][6].

Def 1: W is a value function that has the following properties:

(1.1) *Reference dependence*: $W(r_i, v_i) = W(x)$ with $x = r_i - v_i$ and $W(0) = 0$. $W(x)$ is continuous everywhere, except possibly at zero, $W'(x)$ exists and is positive.

(1.2) *Marginal diminishing sensitivity*: $W''(x) \leq 0$ for all $x > 0$ and $W''(x) \geq 0$ for all $x < 0$.

(1.3) *Loss aversion*: $W(x)$ is steeper for losses than for gains. That is, for all $x_1 > x_2 \geq 0$, $W(x_1) - W(x_2) \leq W(-x_2) - W(-x_1)$.[7]

While prospect theory proposes marginal diminishing sensitivity, for completeness, we

[6]Also see Tversky and Kahneman [1992] and Loewenstein and Prelec [1992] for similar characterizations.

[7]Kahneman and Tversky [1979] suggest loss aversion is defined as for all $x > 0$, $W(x) \leq -W(-x)$, which is more restrictive than as defined here.

define in a similar way the marginal increasing sensitivity as:

(1.2') *Marginal increasing sensitivity*: $W''(x) \geq 0$ for all $x > 0$ and $W''(x) \leq 0$ for all $x < 0$.

We now discuss the implication outlined in Equation (3.1). Over the time interval Δt, the seller maximizes the discounted expected payoff from the following decision problem. First, it is possible that one buyer will arrive who is willing to pay r_i. The first term in brackets in Equation (3.1) measures this effect. Similarly, the second term measures the total search cost expended by seller i. In the third term, $1 - B(t_i)[1 - G(r_i)]\Delta t$, refers to the probability of remaining in the current state. In this case, the seller incurs a waiting cost, $c_i\Delta t$, and will repeat the current decision problem U^{v_i,c_i}. It should be noted that the terms in brackets correctly consider all possible and non-trivial events that could occur during the period Δt. The other events can only occur with a probability of a smaller order than Δt. Those terms are collected as $o(\Delta t)$.

By Taylor expansion on $e^{-\beta \Delta t}$, Equation (3.1) can be rewritten as:

$$\begin{aligned} 0 = \underbrace{max}_{t_i, r_i} \quad & \{B(t_i)[1 - G(r_i)]\Delta t[W(r_i, v_i) + c_i\Delta t - U^{*v_i,c_i}] \\ & -H(t_i)\Delta t - c_i\Delta t\} - \beta U^{*v_i,c_i}\Delta t \end{aligned} \tag{3.2}$$

Dividing the above Equation by Δt, taking the limit as $\Delta t \to 0$ and re-organizing the terms yields:

$$U^{v_i,c_i} = \frac{B(t_i)[1 - G(r_i)]W(r_i, v_i) - H(t_i) - c_i}{B(t_i)[1 - G(r_i)] + \beta} \tag{3.3}$$

Taking the first-order condition with respect to t_i and r_i yields:

$$B'(t_i^*)[1 - G(r_i^*)][W(r_i^*, v_i) - U^{*v_i,c_i}] - H'(t_i^*) = 0 \tag{3.4}$$

$$[1 - G(r_i^*)]W_{r_i}(r_i^*, v_i) - g(r_i^*)[W(r_i^*, v_i) - U^{*v_i,c_i}] = 0 \tag{3.5}$$

where the subscript directs us to take the partial derivative with respect to the corresponding variable. By replacing U^{*v_i,c_i} with (3.3), equations (3.4) and (3.5) fully characterize the equilibrium solutions, since we have two equations to solve for two unknowns: t_i^* and r_i^*. A proof of the existence and a unique solution for this type of searching problem can be found in Williams [1998].

3.2.2 Results

3.2.2.1 Comparative Statics

Although the solving process is straightforward, equations (3.4) and (3.5) are too general to provide any clear implications of the relation between a seller's asking price and the reference value. First, let's only assume reference dependence in the value function. Hence $W(r,v) = W(x)$, where $x = r - v$ and $W_r = W_x$. To obtain more concrete results, substituting equations (3.3) into Equation (3.4) and (3.5) yields:

$$W(x) + \frac{H(t^*) + c}{\beta} - F(r^*)W'(x) = 0 \tag{3.6}$$

$$B'(t^*)\left[\beta W(x) + H(t^*) + c\right] - H'(t^*)\left[B(t^*) + \frac{\beta}{1-G}\right] = 0 \tag{3.7}$$

Here, we define $F(r^*) = \frac{(1-G)}{\beta g}[B(1-G)+\beta]$ to simplify the expression. As $B(t)$ and $G(r)$ are positive, $[B(1-G)+\beta]$ is decreasing in r. Coupled with the fact that $(1-G)/g$ is non-increasing in r, we can show that $F'(r^*) < 0$. Further note that the asking price has a functional form of $r^* = r^*(v_i, c_i)$ and effort has a functional form of $t^* = t^*(v_i, c_i)$. Recall that v_i and c_i are independent, and $W'(x) \neq 0$ because of reference dependence. Use the formula for derivative of implicit function on Equation (3.6) and (3.7) separately. While we leave the technical details for the Appendix E, after some re-arrangeing, we show that:

$$\frac{\partial t^*}{\partial v} = \frac{-\beta B' W'(x)}{-B''\left[\beta W(x) + H + c\right] + H''(B + \frac{\beta}{1-G})} < 0 \tag{3.8}$$

$$\frac{\partial r^*}{\partial v} = \frac{1 - F(r^*)\frac{W''(x)}{W'(x)}}{1 - F(r^*)\frac{W''(x)}{W'(x)} - F'(r^*)} \tag{3.9}$$

From Equation (3.8), as $B' > 0$, $W' > 0$, $B'' < 0$, and $H'' > 0$, $\frac{\partial t^*}{\partial v} < 0$ always holds. It is clear from Equation (3.9) that $\frac{\partial r^*}{\partial v}$ is non-zero, in general. As $W'(x) > 0$, the sign of this comparative static depends on the sign of $W''(x)$, or the risk attitude of the agent.

First, let us consider the impact of reference dependence only by turning off the marginal diminishing (or increasing) sensitivity. This implies a linear value function with $W''(x) = 0$. Then, we could see that $\frac{\partial r^*}{\partial v} = \frac{1}{1-F'(r^*)} > 0$.

Second, with marginally diminishing sensitivity as an incremental component of W, we further have $W''(x) \leq 0$ for all $x > 0$ and $W''(x) \geq 0$ for all $x < 0$. Thus, in the gain area where $x > 0$, $\frac{\partial r^*}{\partial v} > 0$. The sign can be indefinite in the loss area, as it depends on the magnitude of $F'(r^*)$ and $F(r^*)\frac{W''(x)}{W'(x)}$.

We now summarize the comparative statics on v in the following proposition.

Proposition 4. *For a value function W that satisfies reference dependence (Def. 1.1), and diminishing sensitivity (Def. 1.2) or increasing sensitivity (Def. 1.2') when relevant, we have $\frac{\partial t^*}{\partial v} < 0$. Meanwhile, $\frac{\partial r^*}{\partial v}$ has the following property:*

$\frac{\partial r^*}{\partial v}$	*Gain*	*Loss*
Risk Neutral	> 0	> 0
Marginal Diminishing Sensitivity	> 0	*Non-monotonic: +/-/+*
Marginal Increasing Sensitivity	*Non-monotonic: +/-/+*	> 0

The proof of Proposition 4 is provided in the Appendix E.

There are several major takeaways from this proposition. First, as Proposition 4 holds without using the property of Def 1.3, loss aversion plays no role in determining the sign of $\frac{\partial r^*}{\partial v}$. Indeed, with reference dependence but without marginal diminishing (or increasing) sensitivity, $\frac{\partial r^*}{\partial v} = \frac{1}{1-F'(r^*)} > 0$ always holds. Hence a positive association between a seller's asking price and the reference value, which is essentially the key finding in Genesove and Mayer [2001][8], which has no direct connection with loss aversion. Nevertheless, the disposition effect is a valid test of reference dependence. If this were not the case, then from Equation (3.3), we would know that U^{*v_i,c_i} is no longer a function of v_i. Therefore, it must be the case that $\frac{\partial r^*}{\partial v} = 0$ for all v_i. The link between the disposition effect and reference dependence is intuitive. If the reference value plays no role in affecting seller utility, it should not have any predictive power for the optimal asking price a seller chooses. Recall that Barberis and Xiong [2012] show that with a sufficiently positive discount rate, linear reference-dependent realization utility can generate the disposition effect. Here we have proven it in a more general setting as there is no restriction on the magnitude of the positive discount rate, and we have not made any additional assumption on whether the reference value needs to be the initial purchasing price, which is how the realization utility is defined.

Although the risk neutral case provides direct insight into the potential disconnection between the empirical test of loss aversion in Genesove and Mayer [2001] and its theoretical counterpart, its strong assumption on a linear value function may not be realistic. We now consider the general case with marginal diminishing (or increasing) sensitivity.

Without risk neutrality, the optimal asking price depends on the agent's loss/gain exposure. In particular, with marginally diminishing sensitivity, when an agent is subject to

[8] To be more precise, Genesove and Mayer [2001] find a positive relation between the asking price and a seller's potential loss exposure, $v_i - \overline{P}$, where $\overline{P}$ means the expected market price. However, as $\overline{P}$ is ex-post fixed in our theory, this finding is equivalent to a positive relation between r_i^* and v_i.

a higher potential loss (i.e., as v_i increases), the optimal asking price first increases, then declines and eventually increases again. This phenomenon suggests that the agent's risk attitude can generate a local reverse disposition effect, a range in which the optimal asking price is decreasing when the reference value is higher. Moreover, it holds independent of loss aversion. The fact that the non-monotonicity of $\frac{\partial r^*}{\partial v}$ is only observed in the loss range with marginally diminishing sensitivity and only in the gain range with marginally increasing sensitivity suggests that it is the risk seeking behavior that generates the local reverse disposition effect. According to the prospect theory, only potential losers are risk seeking. Hence, the finding that the reverse disposition effect only occurs among potential losers can serve as evidence to rule out marginally increasing sensitivity.

To gain additional insights on the two-way disposition effect, and to obtain the comparative statics over loss aversion, we need to add more concrete parametric structure to the value function. From now on, we incorporate a functional form $W(r_i, v_i)$ as suggested by Tversky and Kahneman [1992]:

$$W(r_i, v_i) = \begin{cases} (r_i - v_i)^\alpha & \text{if } r_i - v_i \geq 0, \\ -\lambda(v_i - r_i)^\alpha & \text{if } r_i - v_i < 0 \end{cases} \tag{3.10}$$

where $\lambda, \alpha > 0$. This parametric characterization is flexible. On the one hand, when $v_i = 0$ for all i and $\alpha = \lambda = 1$, Equation (3.1) reduces to the traditional search model in which risk-neutral sellers attempt to maximize the expected selling proceeds. On the other hand, $W(r_i, v_i)$ also incorporates Kahneman and Tversky's prospect utility as a special case when $v_i \neq 0$ (reference dependence), $0 < \alpha < 1$ (marginal diminishing sensitivity) and $\lambda > 1$ (loss aversion). As loss aversion refers to a behavior where a loss looms larger than an equal-sized gain, it is clear that $\lambda > 1$ measures this asymmetric response. It is easy to verify that (3.10) is compatible with the generally characterized prospect value function from Def. 1. Kahneman and Tversky [1979] and Tversky and Kahneman [1992] define λ as the coefficient of loss aversion. Their studies propose that λ should be approximately 2.25 and α near 0.88.

Plugging in (3.10) and (3.6) into Equation (3.9), under the $\alpha < 1$ and the loss ($v_i - r^* > 0$) scenario, it yields:

$$\frac{\partial r^*}{\partial v_i} = \frac{1 - \frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha}}{1 - \frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha} - \alpha F'(r^*)} \tag{3.11}$$

Let us define $\rho(r^*) \equiv \frac{1}{1-\alpha F'(r^*)}$. It can be shown that for different values of loss $(v_i - r^*)$, the numerator and denominator of Equation (3.11) have no definite signs. That leads to a non-monotonic relation between the asking price and reference value. In particular, as shown in the Appendix E:

$$\frac{\partial r^*}{\partial v_i} = \begin{cases} > 0 & \text{if} \quad 0 < \lambda(v_i - r^*)^\alpha < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta} \\ < 0 & \text{if} \quad \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta} < \lambda(v_i - r^*)^\alpha < \frac{(H+c)(1-\alpha)}{\beta} \\ > 0 & \text{if} \quad \lambda(v_i - r^*)^\alpha > \frac{(H+c)(1-\alpha)}{\beta} \end{cases} \tag{3.12}$$

Equation (3.12) shows that the reverse disposition arises when the magnitude of agent's value function $W(r^*, v_i) = \lambda(v_i - r^*)^\alpha$ is within the loss range of $\left[\rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}, \frac{(H+c)(1-\alpha)}{\beta}\right]$. Recall that $\rho(r^*) \equiv \frac{1}{1-\alpha F'(r^*)} < 1$ is a factor related to the marginal arrival rate. Moreover, arrival rate is decreasing in r^*, or $F'(r^*) < 0$ implies that $\rho < 1$. Further, $H + c$ is the sum of searching and waiting costs, β is the discount rate, and $(1 - \alpha)$ is related to the agent's risk preference. Thus, $\frac{(H+c)(1-\alpha)}{\beta}$ is a measure of the discounted total cost multiply by the level of risk seeking.

Intuitively, $\rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$ demonstrates the trade-off between the cost of not selling and potential benefit of marking-up the asking price r^*. On the one hand, for sellers subject to potential loss, asking a higher price could mitigate the loss if the house is successfully sold. On the other hand, a higher asking price will also reduce the probability of realizing a sale, increase the time on the market, and incur a cost to the seller. Equation (3.12) reveals that, when the loss is relatively small, the benefit of asking for a higher price (and hence improving the value function if realized) dominates, and the agent will first exhibit the disposition effect by asking for a higher price with a larger loss. However, when the loss further increases beyond $\rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$, the second effect could dominate the first due to the non-trivial waiting costs and marginally diminishing sensitivity to loss. Because of the already high asking price, the selling hazard rates for these sellers, should they choose to increase the asking price further, could be too low. As a result, the effective benefit of asking for more is relatively trivial when it is unlikely to result in a sale. In this range, the agent would rather ask for a lower price in exchange for avoiding the high cost of not selling. Finally, when the potential loss to house seller is large enough, successfully selling at a "low" price in the current period is itself painful to a seller. When a home seller perceives that the magnitude of the prospect (loss) valuation function is greater than risk-adjusted total cost, she attempts to avoid realizing this loss and again exhibits the disposition effect by marking-up the price. Further, observe that in the following Equation (3.12), λ appears as the multiplier in the prospect loss valuation function. Hence, among potential losers, greater loss aversion (hence, a larger λ) leads to a narrower range of the reverse disposition effect.

We present the comparative statics on the waiting cost c and loss aversion λ in the following proposition:

Proposition 5. *Given the prospect value function as defined in (3.10), we find* $\frac{\partial r^*}{\partial c} < 0$ *and* $\frac{\partial t^*}{\partial c} > 0$*. Meanwhile, within the loss range,* $\frac{\partial r^*}{\partial \lambda} > 0$*,* $\frac{\partial t^*}{\partial \lambda} < 0$*, and* $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$*.*

The proof of Proposition 5 can be found in Appendix F.

Regarding the waiting cost, the optimal asking price r^* is negatively associated with the waiting cost c. This is intuitive because when an agent faces a higher cost of waiting, she has the incentive to lower down the asking price and spend more effort, as doing so will increase the likelihood of being able to sell the house in the current period.

The fact that $\frac{\partial r^*}{\partial \lambda} > 0$ implies that, among potential losers, more loss aversion always leads to a higher asking price. Further, $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$ indicates that compared with the loss neutral case, loss aversion leads to a steeper slope of the pricing curve, hence to a stronger disposition effect. Conversely, because of the upward force generated by both $\frac{\partial r^*}{\partial \lambda} > 0$ and $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$, loss aversion tends to mitigate the extent of the reverse disposition effect.

While it is conceptually appealing to benchmark on the loss neutral case to discuss loss aversion, Proposition 5 sheds much light on the empirical challenge of testing the loss aversion effect directly. This is because loss aversion itself only carries an incremental effect on asking price. When λ is positive, $\frac{\partial r^*}{\partial \lambda} > 0$ and $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$ always hold, and the property of $\frac{\partial r^*}{\partial v}$ remains the same. Hence, there is nothing unique about whether $\lambda < 1$, $\lambda = 1$, or $\lambda > 1$ as far as the trading strategy is concerned. Empirically, we can only say that as λ increases, the agent becomes less loss tolerating (which is not equivalent to loss aversion), her asking price tends to be higher, and the disposition effect tends to be stronger. This is very different from a test on whether an agent is loss averse. Unless the counterfactual for when $\lambda = 1$ is observable, loss aversion will be empirically untestable given the relative nature of the impact by λ .

3.2.2.2 The Case of Costly Search: Simulation

In this section, we conduct a series of simulations to help visualize and better understand Propositions 1 and 2. We first parameterize our model by specifying the functional forms of search productivity $B(t)$, cost $H(t)$, the distribution of potential buyers' valuations $G(r)$, and the joint distribution of sellers' reference values and net waiting costs $F(v, c)$. Hereafter, we will assume:

$$B(t) = \sqrt{2t - t^2} \tag{3.13}$$
$$H(t) = 1 - \sqrt{1 - t^2} \tag{3.14}$$

Both $B(t)$ and $H(t)$ are quarter portions from unit circles. A convenient feature of this specification is that it satisfies all the assumptions we make concerning the productivity and cost functions. The terms are both monotonically increasing and twice continuously differentiable. Furthermore, we can easily verify that they also satisfy our curvature assumptions. Furthermore, we assume the v_i and c_i are independent variables. In other words, the reference value and net waiting cost for individual sellers are uncorrelated.

A more realistic assumption concerning potential buyers' bidding distribution should have a greater central tendency than the uniform distribution. Since $g(r)$ has a finite support of [0,1], we cannot assume the usual normal distribution for $g(r)$. Specifically, we use the $Beta(2,2)$ (quasi-triangular) distribution:

$$\begin{cases} G(r) = 3r^2 - 2r^3 \\ g(r) = 6r - 6r^2 \qquad for \quad 0 \leq r \leq 1 \end{cases} \tag{3.15}$$

Similar to a normal distribution, a $Beta(2,2)$ distribution is also symmetric and peaked at its center. The simulation results presented below are based on this inverse-U shape curve distribution. However, when conducting parallel studies on a uniform distribution, we obtain very similar results.

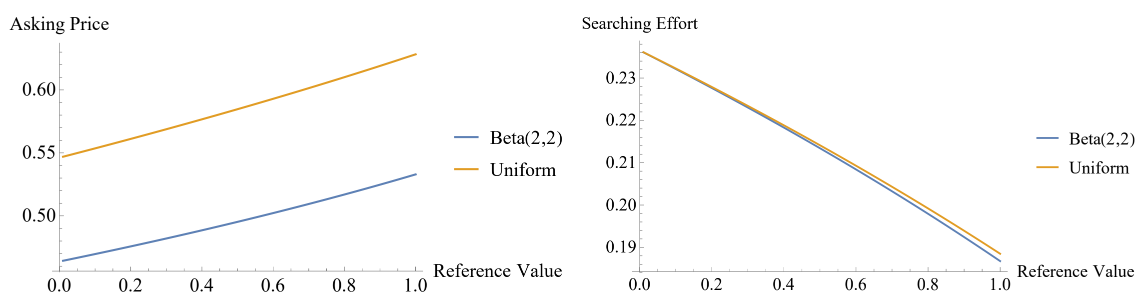
Equipped with specifications for $G(r), B(t)$, and $H(t)$, we can substitute them into equations (3.4) and (3.5). For a given set of v_i, c_i, β, α and λ, we can solve the system numerically to obtain t_i^* and r_i^*. In particular, we assume that β is 5 percent and c_i is 0.1[9]. Hereafter, we maintain these parameter values unless stated otherwise. To show that finding one from Genesove and Mayer [2001]'s abstract has no necessary relation with the loss aversion effect, we set $\lambda = 1$. We also assume that $\alpha = 1$ to rule out any potential effects from diminishing sensitivity in the value function. Hence, $W(r_i, v_i) = r_i - v_i$.

First, we examine the relations among a seller's asking price, search effort, and reference value. The results are presented in Figure 3.1. In both cases, a seller's asking price is increasing in the reference value, which is consistent with finding one in Genesove and Mayer [2001]'s abstract. Since we set $\lambda = 1$, we can confirm the prior conclusion that finding one has minimal power when testing the loss aversion effect. The impact obtained from reference dependence is intuitive. Because the reference value is regarded as an un-sunk cost by sellers, the higher the reference value is, the more compensation the seller will ask for because the higher the reference value, the more likely the seller is to realize a loss when selling. Another finding is that with more heterogeneous valuations by potential buyers (uniform case), a seller tends to increase her asking price, ceteris paribus. This is true for all possible reference values. The intuition is that because potential buyers are more heterogeneous in terms of

[9]We also considered other cases in which c_i is random and found very similar results. The advantage of using a constant c_i is that, conditional on it, we can examine the relation among a seller's asking price, her reference value, and the expected market price.

their preferences, the probability of meeting a buyer who is willing to pay higher matching premium increases. As a result, it is more attractive for a seller to ask for a higher price in the market. Finally, we find that the search effort decreases with the asking price. This implies that, on the one hand, a seller may want to fish in the market by asking for a higher price. On the other hand, she may also choose to spend less effort in searching for potential buyers. One obvious force that helps to generate this finding is the role of the reference value. The higher the reference value is, the higher the asking price because they are conditional on a given holding cost. Since the reference value is also a cost component in addition to searching, an increase in the reference value may depress the seller's search incentive.

Figure 3.1: Asking Price, Search Effort and Reference Value ($\lambda = 1$)



In Genesove and Mayer [2001], and many other studies, a seller's asking price is examined with respect to her potential loss exposure, which is the difference between a seller's reference value and the expected market price. To make a more direct comparison, we need to define the expected market price.

Equations (3.4) and (3.5) show that the equilibrium conditions are functions of v_i and c_i. Suppose that the C.D.F of its joint distribution is $F(v,c)$ and that conditional on $F(v,c)$, the expected market price can be defined as the average of all transaction prices, weighted by the probability of realizing such sales. As a result,

$$\overline{P} = \frac{\iint r^*(v,c)B(t_i^*(r_i^*))[1-G(r_i^*)]f_{v,c}(v,c)dvdc}{\iint B(t_i^*(r_i^*))[1-G(r_i^*)]f_{v,c}(v,c)dvdc} \tag{3.16}$$

In the simulation, we assume that the seller's reference values are distributed uniformly in [0,1]. Each time we randomly draw 1,000 sellers from this distribution and calculate the expected market price following Equation (3.16). We then define $Loss_i = v_i - \overline{P}$, consistent with extant empirical studies.

To isolate the unique predictions of prospect theory, we consider three types of value functions in our simulation. We first set $\alpha = 1.2$, implying an increased sensitivity in contrast to prospect theory. We then set $\alpha = 1$, implying a risk-neutral case. Finally, we set $\alpha = 0.88$[10], as proposed by Kahneman and Tversky in prospect theory. Within each α setting, we compare cases both with and without loss aversion. Based on equations (3.4) and (3.5), by introducing the distribution function $G(r)$ and assigning the parameters the same values, we can identify the relation between r and loss exposure ($Loss = v_i - \overline{P}$) at different values of λ. Figures 3.2 and 3.3 show the results.

Figure 3.2: Asking Price with Different Value Functions $\alpha = 1.2$ and $\alpha = 1$

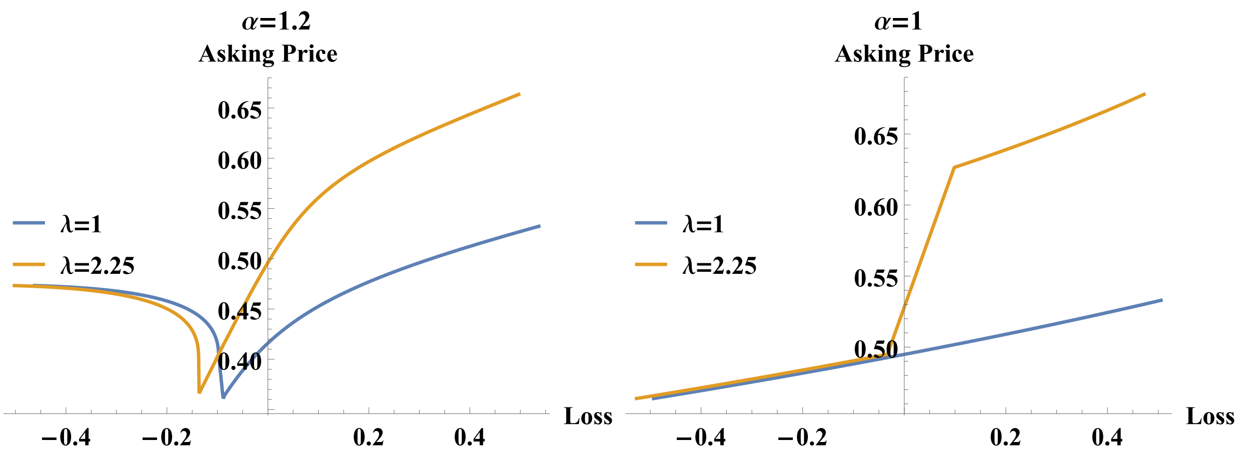
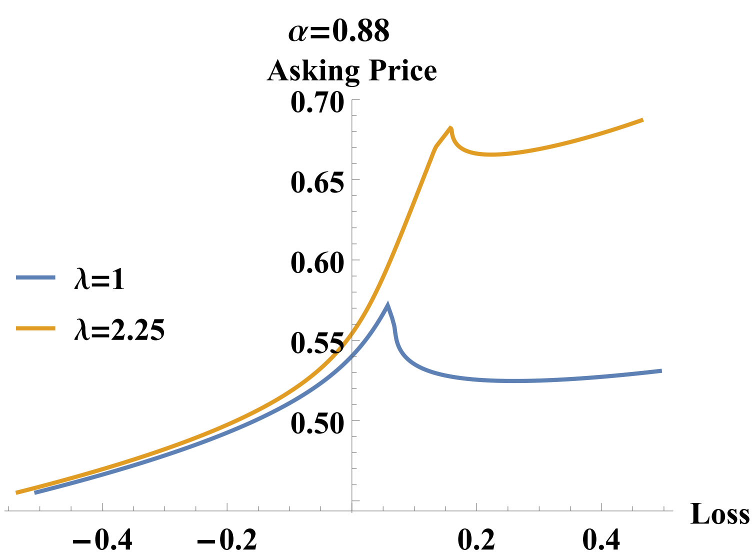


Figure 3.3: Asking Price with Different Value Functions ($\alpha = 0.88$)



Genesove and Mayer [2001] conclude that sellers subject to a greater potential loss ask for a higher price, and they find that the marginal mark-up declines as the size of the potential

[10]Results are qualitatively similar when choosing other values of α.

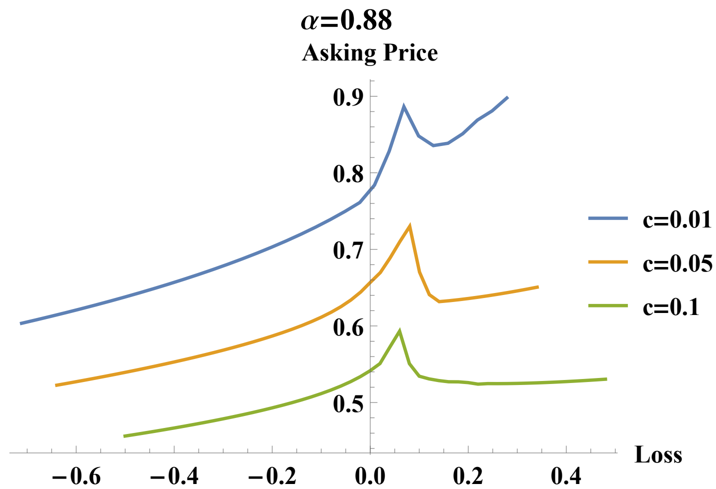
loss increases. They interpret the first finding as evidence of loss aversion (i.e., $\lambda > 1$) and the second finding as evidence of a marginal diminishing effect (i.e., $\alpha < 1$). It is now clear why Genesove and Mayer [2001] lack testing power for both claims. From the left graph of Figure 3.2, we can see that for losers (i.e., $v_i - r^* > 0$), $\frac{\partial r^*}{\partial v} > 0$ and $\frac{\partial^2 r^*}{\partial v^2} < 0$ both hold for home sellers with $\lambda = 1$ and $\alpha > 1$. Therefore, these findings serve neither as evidence that loss aversion exists nor as evidence that there is a marginal diminishing effect.

Thus, what are the real impacts generated by loss aversion and marginal diminishing sensitivity? According to Proposition 5, $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$ for all positive α. Hence, if the loss aversion effect holds (i.e., $\lambda > 1$ if $v - r^* > 0$), we anticipate that the pricing curve along v is steeper than when agents are loss neutral (i.e., $\lambda = 1$) under the loss area. Thus, it is the stronger disposition effect, not the disposition effect itself, that is linked to the loss aversion effect. Our simulation further shows that the slope run-up starts when the seller is subject to a small perceived gain, which is the difference between $\overline{P}$ and v. Further, as discussed after Proposition 5, as loss aversion incentivizes potential losers to ask for higher prices, it partially suppresses the reverse disposition effect. Figure 3.3 reveals that an agent with loss aversion exhibits a more moderate reverse disposition effect, as reflected by a less negative slope of the pricing curve. Moreover, the reverse disposition effect also appears in a narrower range in the presence of loss aversion.

To test diminishing sensitivity (i.e., $\alpha < 1$), we first notice from Proposition 4, Figure 3.2 and Figure 3.3 that the non-monotonicity of the pricing curve only holds when agents are not risk neutral (i.e., $\alpha \neq 1$). Furthermore, if $\alpha < 1$, $\frac{\partial r^*}{\partial v}$ becomes non-monotonic only for losers (i.e., $v_i - r^* > 0$). The prediction reverses when the agent exhibits marginally increasing sensitivity $\alpha > 1$, in which non-monotonicity will arise only for gainers. Therefore, diminishing sensitivity uniquely predicts a universal disposition effect for gainers and a mix of a disposition and local reverse disposition effect among potential losers, due to the non-monotonic pricing strategy.

The fact that the reverse disposition effect is guaranteed to exist when agents are not risk-neutral and that where it appears depends on whether α is above or below 1 is very interesting. We discussed some economic intuition for this after Proposition 4. Another factor that influences a seller's pricing behavior is the waiting cost. Clearly, a higher waiting cost makes the price mark-ups less attractive, since fishing will decrease the hazard rate of selling a home. Eventually, if the waiting cost is high enough, the stress of selling a home will dominate the incentive to fish and earn higher conditional proceeds. Therefore, the asking price should decrease in the waiting cost. To verify this intuition, we apply several waiting costs and plot the results in Figure 3.4. To isolate the waiting cost effect from the loss aversion (λ) effect, we present the findings with a $Beta(2, 2)$ bidding distribution by holding $\lambda = 1$.

Figure 3.4: Asking Prices When Waiting Costs Are Different



Consistent with our intuition and Proposition 4, we find that the asking price tends to decrease when the waiting cost increases. Moreover, the slope becomes flattered when the cost is higher, which indicates the weakening effect of a reference value, as sellers will want to ask a price that facilitates a sale as soon as possible.

3.2.2.3 The Impact of Valuation Heterogeneity on the Asking Price

In the previous section, our primary focus is a fixed bidding distribution like $Beta(2, 2)$. An interesting perspective is the spread of the bidding distribution. In particular, what is its impact on a seller's optimal asking price when we hold the mean of the bidding distribution constant but vary the bidding heterogeneity among buyers? To answer this question, we now introduce a series of Beta functions with equal mean but different variances. Specifically, we use Beta functions from $Beta(1, 1)$ (a uniform distribution) to $Beta(7, 7)$ as plotted in Figure 3.5.

We observe that as parameters of the *Beta* distribution increase, its tail becomes thinner, which means that buyers tend to have less heterogeneous house valuations and hence a smaller likelihood of receiving an offer that is significantly above the mean. We then plug in different Beta functions and solve for optimal asking prices. The parameter setting is the same as in the previous section. We next compute the average asking price, r, out of 1,000 randomly selected reference values, v_i, and plot the results in Figure 3.6.

Figure 3.6 shows that as the variance increases, so does the optimal asking price. The intuition is that when potential buyers have a bidding distribution with higher variance, the probability of receiving an upper-tail offer increases, which encourages sellers to be consistent

Figure 3.5: Beta Functions with the Same Mean but Different Variances

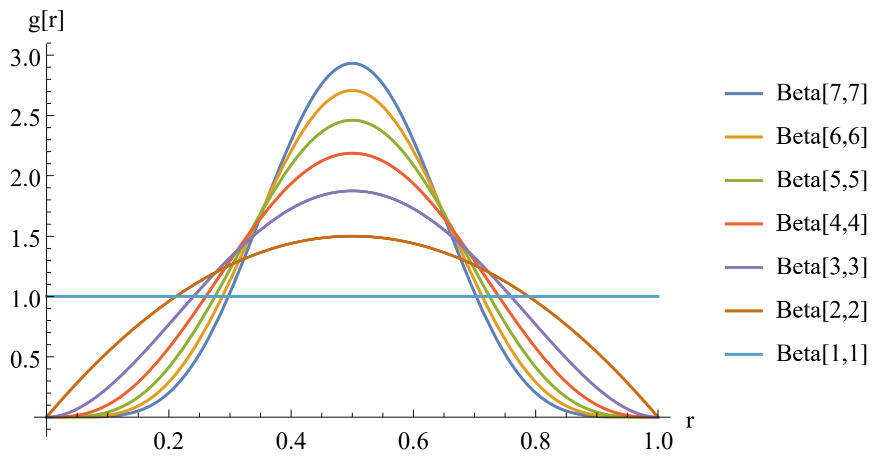
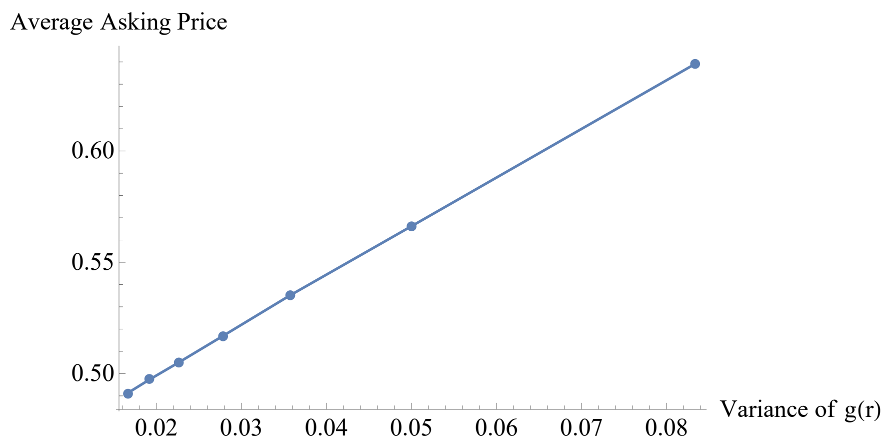


Figure 3.6: Variance of Bidding Distribution and the Asking Price



with their asking prices. To confirm that our finding is not driven by the loss aversion (λ) effect, we also consider a loss “neutral" case ($\lambda = 1$) and find a similar result.

One stylized finding in the housing market is a strong positive correlation between home prices and transaction volume. The heterogeneity of pricing behavior between sellers with low versus high reference values naturally leads to a positive price-volume relation. The intuition is simple. Conditional on a given distribution of reference values, when the market becomes hot, the willingness to pay off potential sellers will increase. As a result, in a hot market, the proportion of sellers who have low reference values relative to the market price increases. Because these sellers are not subject to potential equity loss, the incentive for them to mark-up the price is relatively low. As increasingly more sellers in the market choose to sell at a moderate price, as reflected by the lower and flatter asking price curve, it is clear that the probability of a successful sale will increase, which in turn generates a higher transaction volume. Furthermore, a flatter asking price curve for sellers with low reference values may shed light on the extent of price dispersion under different market conditions. As previously mentioned, when a market becomes hot, the proportion of sellers who have low reference values relative to market price increases. With a high market price, as more and more sellers cluster in the range of low reference values, differences among their asking prices becomes smaller. In aggregate, we should expect that ceteris paribus, in a hot market, the observed transaction prices will be less dispersed with respect to the expected market price than in a cold market. We refer the readers to Appendix G for more technical details on how our model generates these predictions.

3.3 Empirical Tests

We first list the four key findings from the model: 1) As shown in Figure 3.3, the three components of prospect theory jointly predict a non-monotonic (up-down-up) pricing curve along with a seller's potential loss/gain exposure, with a local reverse disposition effect appearing in the loss range; 2) the asking price decreases when the waiting cost increases; 3) the asking price tends to increase with greater bidding heterogeneity. In this section, we test these three hypotheses using a comprehensive housing transaction dataset from Hampton Roads, a region of southeastern Virginia composed of several counties and eight cities such as Virginia Beach, Norfolk, Portsmouth, Chesapeake, Hampton, Newport News, Suffolk, and Williamsburg.

3.3.1 Data

Our housing transaction data are based upon the complete record of single-family transactions in Hampton Roads over the period 1993(Q1)-2013(Q1), as provided by the Real Estate Information Network (REIN). Due to the strength of the data, which includes detailed records of housing characteristics in terms of structure and neighborhood information, we can obtain a more accurate estimate of the expected market price when using a hedonic model. We drop observations with missing hedonic characteristics, resulting in 226,389 listing and selling records during the sample period. Within the data, 31,969 observations reflect transactions involving homes with at least two selling records. Hence, we can use these repeat sales for subsequent tests. Besides of the hedonic variables, we generate a *Vintage* dummy for houses with more than 120 years. For each quarter, we generate the price index based on the 1993Q1. The results are shown in Figure 3.7. From the figure, we can see that Hampton Roads experience a great price recession after 2008 and the housing market is still under recovery after 2010.

Table 3.1: Summary Statistics for Key Variables

	Count	Mean	SD	Min	Max
Asking Price	226389	12.01019	.6963668	4.976734	16.75467
Selling Price	156790	11.95246	.6555323	9.220291	15.73243
Moving Hazard	225022	8.051885	4.996146	1.08e-31	47.56719
Lag Volatility	226389	.0944664	.0742729	0	.3353927
Price Index	226223	172.9299	59.69367	95.19187	264.5465
Last Residue	31933	-.0615411	.2883628	-2.689037	2.098295
Loss	31969	-.2916937	.4099128	-3.126657	2.071548
Age	225213	25.96711	23.84644	0	181
Vintage	226389	.0073193	.0852392	0	1
Baths_Full_	226389	1.907712	.7062155	0	12
Baths_Half_	226389	.5412321	.5373087	0	14
Bedrooms_	226389	3.371745	.8254281	1	16
Fireplaces_Number_	226389	.7319834	.6586105	0	14
Square_Feet_Approx_	226389	1899.852	938.0276	100	55000
Stories_Number_	226389	1.615913	.5337507	1	4
Waterdummy	226389	.084907	.2787438	0	1
ATTdummy	226389	.157998	.3647399	0	1
PoolDummy	226389	.0591239	.235857	0	1
SewerDummy	226389	.8880555	.3152988	0	1
Heater_EleDummy	226389	.5639055	.4959004	0	1
WaterCityDummy	226389	.9182248	.2740225	0	1

3.3.2 Methodology

Define V_{it} as unit i's expected log market value at time t:

$$V_{it} = X_i\beta + \delta_t \tag{3.17}$$

where X_i is a vector of hedonic characteristics, and δ_t is a time dummy for period t. However, in reality, we cannot observe this expected market value. Instead, what we observe is the selling price at time t, in log form P_{it}, which we express as:

$$P_{it} = X_i\beta + \delta_t + e_{it} = V_{it} + e_{it} \tag{3.18}$$

Where the additional component e_{it} is the amount that is over- or under-paid by the buyer. In the theory section, we assumed that housing units are ex-ante identical in terms of structural characteristics and quality. That is, all housing units should have the same expected market price. To control for quality differences in real data, we perform a two-stage process. In stage 1, we estimate a hedonic regression through which we can generate the expected market price for each unit. In stage 2, using the log of the asking price L_{it} as our dependent variable, we then regress it on the seller's loss/gain exposure, after controlling for the expected market prices of different housing units. One key prediction from our theory is that L_{it} depends on the potential loss/gain exposure, which reflects the seller's heterogeneous reference value. We measure it using a variable called $Loss_{ist}$, to be consistent with Genesove

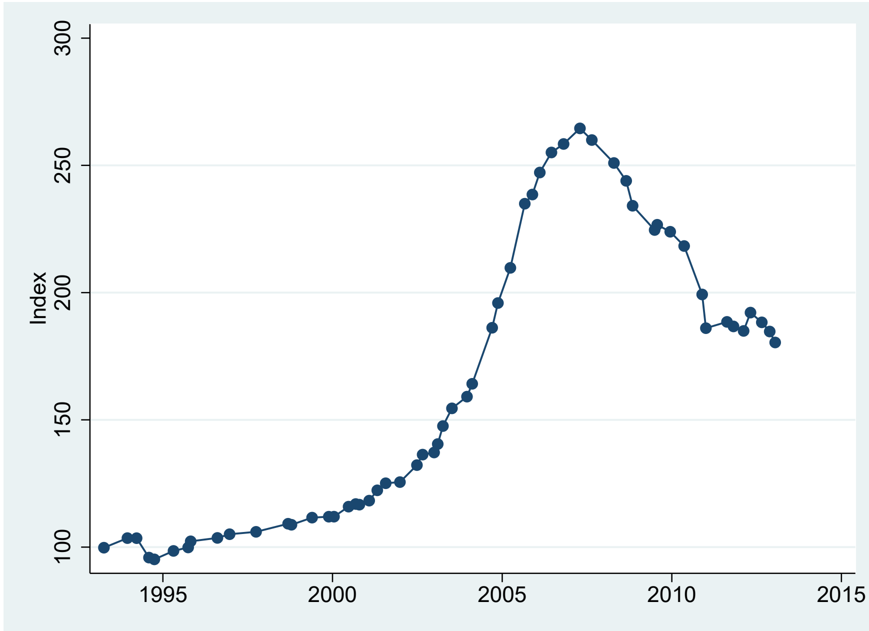


Figure 3.7: Price Index from 1993-2013

and Mayer [2001]. The relation is specified as:

$$L_{it} = \alpha V_{it} + f(Loss_{ist}) + \phi c_{it} + \gamma \sigma_t + \varepsilon_{it} \tag{3.19}$$

Where ϵ_{it} is the error term with the usual assumptions, c_{it} refers to the seller's waiting cost upon the sale, and σ_t measures the perceived period t bidding heterogeneity in the market. As is typically done in the literature, we use the original log purchase price P_{is} at time s as a reference value, and hence, $Loss_{ist}$ is defined as the difference between the log prior transaction price and the log of the currently expected value[11]:

$$Loss_{ist} = P_{is} - V_{it} = \delta_s - \delta_t + e_{is} \tag{3.20}$$

Substituting Equation (3.20) into Equation (3.19) yields our ideal econometric specification:

$$L_{it} = \alpha V_{it} + f(\delta_s - \delta_t + e_{is}) + \phi c_{it} + \gamma \sigma_t + \varepsilon_{it} \tag{3.21}$$

[11] In Genesove and Mayer [2001], the authors censor the potential gainer's loss exposure at zero and only examine the behavior of the potential losers. However, the theoretical justification for this censoring treatment is unclear. As our model covers a full range of potential losses ($loss > 0$) and gains ($loss < 0$), here, we choose to conduct our empirical test without censoring. Hence, a negative value of $Loss_{ist}$ indicates a potential gain to house seller i.

Our theory in section 1 predicts that the coefficient associated with the holding cost, ϕ, should be negative. One difficulty in testing the waiting cost effect is that one is unable to observe an individual seller's moving pressure. Nevertheless, holding other factors constant, a smaller waiting cost should imply less stress with regard to moving and longer duration in the current home. Fortunately, for each repeat-sale seller, we have information on the length of time between her initial purchase and her next move, so we can measure tenure in the current house before a successful sale is realized. Ideally, if we were able to observe the demographic information of home sellers, we could use it to estimate the moving pressure conditional on this demographic information. However, such information is unavailable in our data. Because we have detailed information on housing characteristics, we use it as a proxy for household characteristics and can derive the systematic relations between housing attributes and a household's expected duration time. Accordingly, we conduct a Cox proportional hazard model to estimate a seller's hazard rate of moving, conditional on the given housing characteristics that are used in our hedonic regression. We then use the predicted hazard rate as a proxy for the underlying holding cost of each seller. Since a higher hazard rate implies a greater likelihood of moving and a higher holding cost, we should expect a negative coefficient when we regress the realized transaction price on this rate. The results from the first-stage hedonic and Cox proportional hazard regressions are reported in the Appendix I.

With regard to the perceived time t bidding heterogeneity, σ_t, we construct a measure using a GARCH model. In particular, from our stage-one hedonic model, we first generate a quarterly return index for the overall Hampton Roads housing market with 1993Q1 as the base quarter. We then estimate a simple GARCH (1,1) model on the return series and use the implied volatility as the proxy for σ. The intuition is that when the implied market return volatility is higher, there is a greater chance of observing extreme values, an indication of greater bidding heterogeneity. To ensure that our measure of perceived bidding heterogeneity is forward-looking, we use the lag of volatility throughout our analysis. Our theory predicts a positive impact of bidding heterogeneity on the asking price; thus, we anticipate the coefficient associated with our implied volatility measure to be positive.

In the second-stage regression, as shown in Equation (3.21), in order to remove outliers, we drop 1.25 percent of the loss/gain observations at both tails. Furthermore, we omit the observations that have more than one sale within a year to rule out potential home flippers. This screening leaves 27,564 observations for use in stage 2 of the analysis.

3.3.3 Estimation Results

3.3.3.1 OLS Analysis

We first perform a simple two-stage analysis using Ordinary Least Squares (OLS). Our empirical second-stage regression Equation is $L_{it} = \alpha V_{it} + f(Loss_{ist}) + \phi c_{it} + \gamma \sigma_t + \psi Control_{it} + e_{it}$. The difference between the empirical second stage and the ideal specification in Equation (3.21) is that we add additional control variables (via the vector $Control_{it}$) to account for empirical irregularities caused by unobserved housing quality, general market price level and neighborhood effects. Table 3.2 presents our stage 2 result on the relation between the asking price and a seller's potential loss and other control variables. Here, we use a polynomial functional form $F(Loss_{it}) = \sum_{j=1}^{n} \gamma_j Loss_{ist}^j$, where n is 1, 2 and up to 7 in the regression.

Table 3.2: Asking Price and Loss Exposure
OLS Regressions, Dependent Variable: Log of Asking Price

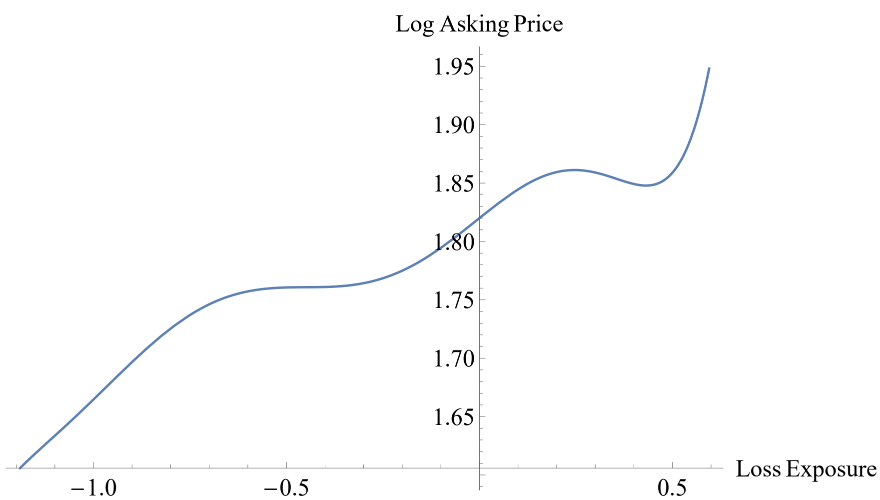
Explanatory Variables	Model 1	Model 2	Model 3	Model 4
Expected Log Selling Price	0.8058***	0.8057***	0.8071***	0.8269***
	(0.0074)	(0.0074)	(0.0073)	(0.0072)
Moving Hazard Rate	-0.0159***	-0.0158***	-0.0158***	-0.0142***
	(0.0007)	(0.0007)	(0.0007)	(0.0007)
Market Volatility (Lag)	0.1842***	0.1864***	0.1653***	
	(0.0199)	(0.0199)	(0.0206)	
Market Index When Listing	0.0017***	0.0017***	0.0016***	
	(0.00005)	(0.00005)	(0.00005)	
Last Residual	0.2582***	0.2580***	0.2558***	0.2615***
	(0.0080)	(0.0080)	(0.0079)	(0.0079)
Loss	0.1284***	0.1217***	0.2510***	0.2644***
	(0.0047)	(0.0082)	(0.0209)	(0.0213)
Loss^2		-0.0119	-0.0664	-0.0398
		(0.0102)	(0.0649)	(0.0665)
Loss^3			-1.3707***	-1.4470***
			(0.2674)	(0.2603)
Loss^4			-0.7657*	-0.9155**
			(0.3978)	(0.3958)
Loss^5			2.8054***	2.8326***
			(0.6597)	(0.6393)
Loss^6			3.5009***	3.6421***
			(1.1479)	(1.1186)
Loss^7			1.1374**	1.1921**
			(0.5077)	(0.4972)
Constant	1.9568***	1.9580***	1.9580***	1.8202***
	(0.1034)	(0.1028)	(0.1028)	(0.1035)
Neighborhood Fixed Effect	Yes	Yes	Yes	Yes
Quarterly Fixed Effect	No	No	No	Yes
Observations	27,564	27,564	27,564	27,564
R^2	0.8405	0.8405	0.8411	0.8451

1) *Significant at the 0.10 level. **Significant at the 0.05 level. ***Significant at the 0.01 level.
2) Robust standard errors are in parentheses.
3) Model 1 and 2 repeat the empirical tests from Genesove and Mayer (2001) and find the similar results.
4) Model 3 and 4 run the septic OLS regression. In Model 3, we include the market volatility and index. In Model 4, we include the quarterly fixed effect. As it is highly correlated with the market volatility and index, we exclude them from the model. We plot the septic loss function in Figure 3.8.

In Model 1, we include only the linear loss exposure term. As our loss exposure variable is not censored at zero, a negative value indicates a potential gain. The coefficient is 0.1284 and is significant at the 1 percent level, which implies that a 1 percent increase in potential loss (or 1 percent decrease in potential gain) will increase the asking price by 0.1284 percent. We then add the quadratic loss term in model 2. The quadratic $loss^2$ term is negative but not significant, and the joint test on $loss$ and $loss^2$ is significant at the 1 percent level. Finally, in Models 3 and 4, we estimate a septic polynomial model. The joint test on all polynomial terms is also significant at the 1 percent level. A legitimate concern when using the moving hazard rate as a proxy is that it may be correlated with some unobservables that are also correlated with transaction prices. To check for this potential endogeneity, based on model 4, we run the Durbin-Wu-Hausman test for our waiting cost proxy. The test yields $t = -0.57$, $p = 0.567$, suggesting that endogeneity is unlikely to be a problem.

To delve deeper into the joint curvature of selling price on reference values, in Figure 3.8, we plot the predicted selling price based on the septic polynomial fit in Model 4, conditional on different loss/gain exposures.

Figure 3.8: Joint Curvature of Asking Price from the Septic Polynomial Fit in Model 4.



Notes: The functional form here is $f(loss) = 0.2644\ loss - 0.0398\ loss^2 - 1.4470\ loss^3 - 0.9155\ loss^4 + 2.8326\ loss^5 + 3.6421\ loss^6 + 1.1922\ loss^7 + 1.8202$.

Both Proposition 4 and the simulation show that, under prospect theory, the optimal asking price tends to increase as the potential gain declines, and as the potential loss increases. However, when the potential loss increases further, a reverse disposition effect arises, and the optimal asking price decreases. Eventually, when the potential loss increases further and becomes large enough, the optimal asking price begins to increase again. The septic polynomial fit, as seen in Model 4, matches this pattern of a two-way disposition effect well, which supports our model prediction under marginally diminishing sensitivity. Further, it is clear from Models 1 and 2 in Table 3.2 and Figure 3.8 that empirical specifications that consider only linear and quadratic terms for loss/gain may overlook the underlying non-monotonicity. Had we only considered up to Model 2, our results would be consistent with Genesove and Mayer [2001] and support a (one-way) disposition effect with marginally declining mark-up.

As discussed in the simulation section 3.2.2.2, the loss aversion effect (i.e., when $\lambda > 1$) implies an increasing slope in the seller's pricing curve around the break-even point. It is clear from Figure 3.8 that the slope of the pricing curve is steeper in this range, as predicted. With regard to the holding cost, the coefficient on the moving hazard rate is negative and significant in all models. Consistent with theory, this implies that people with a higher probability of moving tend to sell their homes at lower prices. Furthermore, the positive and significant coefficient on the implied return volatility echoes our model prediction that when the market is subject to greater bidding heterogeneity, home sellers tend to mark-up their asking prices.

Our control variables are also consistent with the extant literature. For example, the residual from the last sale, which controls for potential unobservable house quality, shows as expected a significant positive effect on the current asking price. A positive last residual means that the seller was willing to pay a higher than expected market price when she purchased the unit. Hence, it is very likely that the house may have an unobservable quality premium, which may make the current asking price high. Finally, a positive and significant coefficient on Market Index When Listing indicates that when the anticipated market price is high, in general, sellers tend to ask for a correspondingly higher price.

Our simple OLS analysis in this section is subject to some econometric challenges. First, as our measures of potential loss/gain, holding cost (via the moving hazard rate), and bidding heterogeneity (via implied market return volatility) are all based on estimates from auxiliary regressions, we need to consider the potential estimation error in order to make valid statistical inferences. Second, using a high-order polynomial to fit a non-linear curve, such as in Model 4, may be inaccurate and overfitting, and any bias in the mean and standard error of loss may be amplified in the high-order term, leading to high oscillations in the regression. Furthermore, using a high-order polynomial model may only fit our specific sample rather than generalize to the overall population (Good and Hardin [2012]).

3.3.3.2 Two-stage Bootstrap Analysis

To address both the "generated regressor" problem and the concern with using a simple polynomial to fit a non-linear pricing curve, in this section, we conduct our analysis in a two-stage bootstrap setting. In stage 1, we run hedonic, Cox proportional hazard and GARCH (1,1) regressions in order to obtain all needed control variables. In stage 2 (i.e., for

$L_{it} = \alpha V_{it} + f(Loss_{ist}) + \phi c_{it} + \gamma \sigma_t + \psi Control_{it} + e_{it}$), we adopt spline and semi-parametric regression techniques, two methods widely used in non-linear model fitting. The standard errors for linear parameters are constructed through a two-stage housing unit stratified bootstrap procedure with 1,000 replications.

First, we estimate spline regressions (see Silverman [1985]), which allow us to estimate the range-specific behavior of a seller's pricing strategy. We report the results in Table 3.3. Here, we examine a seller's initial listing price in Models 5 and 6, and the realized transaction price in Models 7 and 8, where Models 6 and 8 include quarterly fixed effects. The findings are largely consistent with the predicted two-way disposition effect. Across all models, in the gain range (see terms up to Loss4), we observe positive and significant coefficients for most loss terms, which supports the disposition effect. The pattern continues with moderate loss, although the positive coefficients on Loss5 are significant only in Models 5 and 6 on asking price. Interestingly, from Models 5 and 6, when potential loss further increases, Loss8 ($0.355 < loss < 0.397$) exhibits a significant downward slope, with coefficients of -1.5613 and -1.5405, respectively. The implication is that within this range, as the potential loss increases by 1%, the log asking price decreases by 1.5613% and 1.5405%, holding other factors constant. A similar pattern holds in Models 7 and 8 for the realized transaction price, although the coefficients on Loss8 (-0.2694 and -0.1966) imply a weaker reverse disposition effect when measuring the realized price. Notably, the relevant range of the reverse disposition effect now becomes broader ($0.239 < loss < 0.507$).

There is a vast literature purporting the advantages of using the semi-parametric analysis to model non-linear specifications (Yatchew [2003]; Simar and Wilson [2007]; Verardi and Debarsy [2012]). Here, we use a partially linear semi-parametric model first proposed by Robinson [1988] in our stage 2 regression. In particular, we maintain the linear specification for all parameters except $f(Loss_{ist})$, which is left non-parametrized. Table 3.4 reports the coefficients for the linear component of the stage 2 regression with 1,000 replications.

The parametric results in Table 3.4 are consistent with the previous findings from the OLS and spline regressions, with the coefficients being significant and carrying the expected signs. The curvature of the semi-parametric fit again exhibits non-linearity and an up/down/up trending pricing curve along the seller's potential loss exposure, consistent with model predictions. To test whether our previous septic polynomial fits reasonable well, we plot the non-parametrically fitted pricing curve in Figure 3.9 and compute Härdle and Mammen's [1993] specification test to assess whether the nonparametric fit can be approximated by a 7-order polynomial. The underlying p-value is 0.80, suggesting no significant difference between the two curve fittings.

Table 3.3: Asking Price, Transaction Price and Loss Exposure
Spline Regressions

	Model 5 Lask	Model 6 Lask	Model 7 Lsell	Model 8 Lsell
Expected Selling Price	0.8215***	0.8211***	0.8357***	0.8442***
	(0.0102)	(0.0106)	(0.0115)	(0.0116)
Last Residual	0.2627***	0.2664***	0.2821***	0.2612***
	(0.0080)	(0.0079)	(0.0094)	(0.0091)
Moving Hazard Rate	-0.0146***	-0.0146***	-0.0157***	-0.0152***
	(0.0024)	(0.0023)	(0.0026)	(0.0024)
Market Volatility (Lag)	0.1133*		0.0540	
	(0.0640)		(0.0576)	
Market Index	0.0017***		0.0017***	
	(0.0001)		(0.0001)	
Loss1	0.3982***	0.3973***	0.3727***	0.4011***
	(0.0542)	(0.0548)	(0.0668)	(0.0694)
Loss2	0.1487***	0.1496***	0.1520***	0.1642***
	(0.0162)	(0.0163)	(0.0175)	(0.0177)
Loss3	0.0313	0.0219	0.0614***	0.0718***
	(0.0196)	(0.0196)	(0.0220)	(0.0215)
Loss4	0.2720***	0.2619***	0.2644***	0.2936***
	(0.0322)	(0.0330)	(0.0403)	(0.0409)
Loss5	0.1646***	0.1692***	-0.1247	-0.0501
	(0.0577)	(0.0587)	(0.1342)	(0.1338)
Loss6	-0.0204	-0.0089	-0.0634	0.0500
	(0.2834)	(0.2785)	(0.1835)	(0.1807)
Loss7	0.0872	0.0881	1.0993	1.0983
	(0.1662)	(0.1656)	(1.5492)	(1.5416)
Loss8	-1.5613***	-1.5405***	-0.2694**	-0.1966*
	(0.5569)	(0.5726)	(0.1098)	(0.1108)
Loss9	0.6761***	0.6559***	0.0541	0.0442
	(0.0980)	(0.1024)	(0.1338)	(0.1417)
Constant	2.1752***	2.2829***	1.9688***	1.9089***
	(0.2722)	(0.3542)	(0.3638)	(0.4613)
Neighborhood Fixed Effect	Yes	Yes	Yes	Yes
Quarterly Fixed Effect	No	Yes	No	Yes
Observations	27,564	27,564	19,737	19,737
Bootstrap Replications	1,000	1,000	1,000	1,000

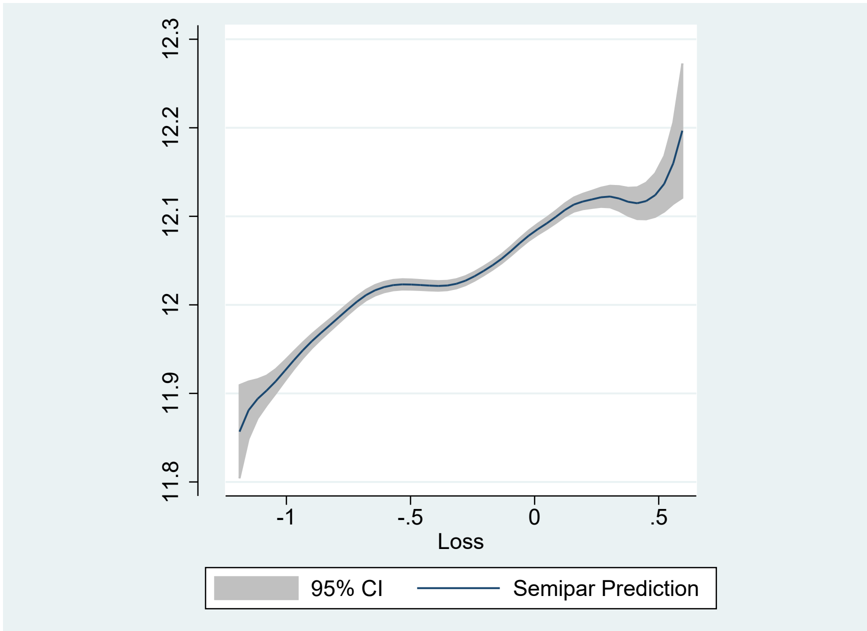
1) *Significant at the 0.10 level. **Significant at the 0.05 level. ***Significant at the 0.01 level.
2) Reported standard errors are constructed by a two-stage housing unit stratified bootstrap.
3) The 8 knots for Models 5 and 6 are: -0.9, -0.5, -0.2, 0, 0.17, 0.22, 0.355, 0.397. For example, Loss1 corresponds to the range of (.,-0,9), Loss2 is to the range (-0.9,-0.5), and so forth.
4) The 8 knots for Models 7 and 8 are -0.9, -0.5, -0.2, 0, 0.1, 0.224, 0.239, and 0.507.

Table 3.4: Asking Price, Transaction Price and Loss Exposure:
Semi-parametric Model

Variables	Log Asking Price	Log selling Price
Expected Selling Price	0.8279***	0.8443***
	(0.0106)	(0.0115)
Moving Hazard Rate	-0.0145***	-0.0153***
	(0.0024)	(0.0025)
Last Residual	0.2671***	0.2600***
	(0.0079)	(0.0092)
Neighborhood Fixed Effect	Yes	Yes
Quarterly Fixed Effect	Yes	Yes
Observations	27,564	19,737

1) * Significant at the 0.10 level. ** Significant at the 0.05 level. *** Significant at the 0.01 level.
2) The reported standard errors are constructed through a two-stage housing unit stratified bootstrap procedure with 1000 replications.

Figure 3.9: The Joint Curvature of Asking Price from the Semi-Parametric Fit



Our findings, based on the realized transaction price, are qualitatively similar to those associated with the asking price. Figure 3.10 plots the semi-parametrically fitted pricing curve on the transaction price.

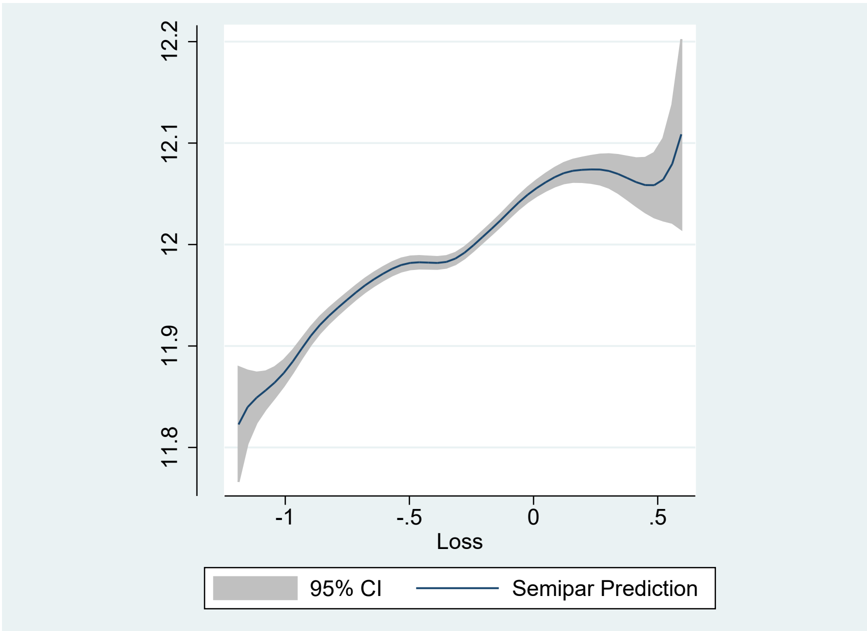
The last prediction from our theory relates to the price dispersion effect. If we interpret the difference between the realized transaction price and expect market price as noise, our theory predicts that the higher the expected market price, the smaller the noise should be since there would be fewer and fewer losers in the market. In Appendix G, we present evidence supporting this prediction.

3.4 Calibration

As discussed earlier, it is empirically challenging to test the loss aversion effect directly using observational data, as there is no way to measure the benchmark pricing curve from counterfactual loss neutral sellers. As an alternative, in this section, we calibrate our model on α, λ and other parameters to optimize the matching between the theoretical pricing curve and the empirically estimated curve using our Hampton Roads data.

While our theoretical model assumes that all houses are identical, this is not the case in reality. As a result, before we conduct the calibration exercise, we first standardize our theoretical and empirical measures by taking the log difference in order to construct percentage measures. We use $(\ln v - \ln \overline{P})$ to measure the potential loss/gain exposure

Figure 3.10: The Joint Curvature of the Realized Transaction Price from the Semi-Parametric Fit



and $(\ln r - \ln \overline{P})$, which we call fishing, to measure the spread of the asking price. In the calibration, we re-generate a theoretical pricing curve similar to Figure 3.3 using these measures. Moreover, we estimate an empirical septic polynomial model similar to Model 4 in Table 3.2. We then conduct our calibration by minimizing the Mean Squared Error (MSE) between our theoretical and empirical pricing curves [12]. To test the significance of α and λ, we follow the bootstrap calibration method suggested by Diebold, Ohanian, and Berkowitz [1998]. We maintain $\beta = 0.05$ and perform 1,000 bootstrap replications. In each replication i, our calibration works as follows:

1. Estimate Model 4 using the bootstrap sample generated in the current replication[13]. Then, use the coefficients from the 7-order polynomials to generate a pricing curve of fishing on loss/gain exposure.

2. Simulate 10,000 sellers with various reference values. Instead of assuming a uniform distribution when drawing reference values as we did in section 1.2.2, here we match the density of sellers' loss/gain positions with the distribution observed in the underlying bootstrap sample.

[12] See Bakshi et al. [1997] and Hilpisch [2015] for details of the MSE minimization procedures in this context.

[13] The only exception is that we now use fishing as the dependent variable and, therefore, no longer have Expected Log Selling Price as a regressor.

3. Select the optimal parameter vector ($\hat{\alpha_i}$, $\hat{\lambda_i}$ and $\hat{c_i}$) that minimizes the MSE[14] between the theoretical and empirical pricing curves, subject to the condition that the ranges of fishing and loss/gain from the theoretical curve match their empirical counterparts. As the empirical pricing curve is come up to an unknown intercept shift, in selecting the parameter vector, we posit the empirical curve by choosing an intercept such that, given the chosen parameter vector, it crosses the theoretical pricing curve at the break-even point (i.e., $Loss = 0$).

4. Start the next replication by repeating the process.

The calibrated values of α and λ are obtained by the same procedure applied to the full data sample. We then calculate the standard errors and test the hypotheses of $H_0 : \alpha \geq 1$ and $H_0 : \lambda \leq 1$, using the distribution of $\hat{\alpha_i}$ and $\hat{\lambda_i}$, $i = 1, ..., 1000$. The results are reported in Table 3.5.

Table 3.5: Bootstrap Calibration Result

Parameter	α	λ
	0.8880***	1.9667***
	(0.0462)	(0.2504)
Replications	1,000	1,000

1) Null hypotheses are based on $H_0 : \alpha \geq 1$ and $H_0 : \lambda \leq 1$.
2) ** Significant at the 0.05 level. *** Significant at the 0.01 level.
3) Standard errors are constructed from the bootstrap procedure.

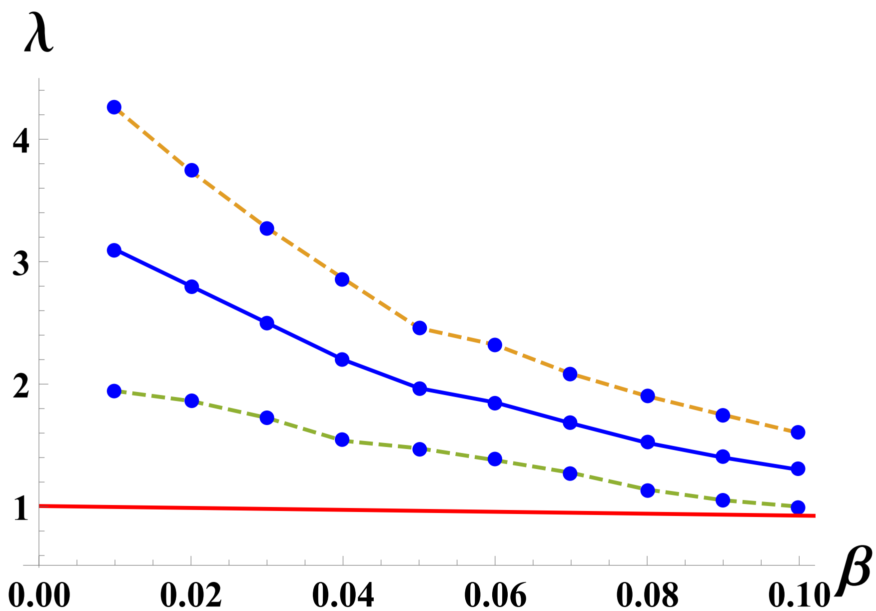
Our calibrated parameter values, $\alpha = 0.8880$ and $\lambda = 1.9667$, are both significantly different from 0. Meanwhile, α is significantly smaller than 1 at the 1% level, and λ is significantly larger than 1 at a 1% level. The coefficient on the marginal diminishing sensitivity is close to the value of 0.88 estimated by Tversky and Kahneman [1992], but it is larger than the value reported by some later experimental studies (Fennema and van Assen [1999], Abdellaoui et al. [2007]). The loss aversion coefficient of 1.9667 suggests that a one-unit loss looms approximately 1.9667 times larger than an equal-sized gain.

While we use $\beta = 0.05$ in the above calibration, our findings are qualitatively similar under a wide range of discount rates. To show this, in Figure 3.11, we repeat the above calibration exercise by maintaining all other parameters the same but letting β vary from 0.01 to 0.1. We also plot the 95% confidence interval along with the point estimate of λ. With this wide range of β, the calibrated value of λ is consistently and significantly larger than 1, supporting loss aversion effect. Further, a larger discount rate tends to reduce the magnitude of λ, which should not be surprising. When the discount rate is high, postponing

[14] We calculate the inverse probability-weighted MSE (based on 10,000 simulated sellers) in order to place equal weights on segments within the observable domain of the pricing curve.

a loss to the future becomes more appealing per se, even without loss aversion. Hence, an agent with a lower extent of loss aversion still has incentives to mark-up the price.

Figure 3.11: Bootstrap Calibration λ under Varying Discount Rates



We have thus far documented a strong connection between our prospect theory-driven model predictions and empirical findings. However, a closer comparison of the overall curvature between the theoretical pricing curve (i.e., Figure 3.3) and the empirical one (i.e., Figure 3.9) still reveals some notable differences, especially when “Loss” and “Gain” are large. For example, home sellers in large loss positions seem to mark-up their asking price at a rate faster than as predicted by a static prospect theory-based model. As the incremental (marginal) effect may be affected by α, one potential implication is that α might not be constant across all loss/gain positions. To assess whether a changing α can potentially help explain the empirical discrepancy, we perform another calibration by utilizing power-law transformation on the prospect value function. This allows α to differ depending on the different regions of the loss/gain positions. We cover the details of this extended calibration in Appendix H. To summarize, the result suggests that agents tend to be more risk averse when potential gain is larger, and they are slightly more risk seeking with increasing losses. The finding that an agent's risk preference seems to vary along with the extent of loss/gain echoes several experimental and empirical studies in the literature.

The closest lab study that supports our findings on varying α is by Bouchouicha and Vieider [2017]. By experimenting with different outcomes on financial rewards, the authors notice that the respondents are more risk averse when gain size is larger. Our empirical results are qualitatively consistent with Bouchouicha and Vieider [2017]. For example, in the gain range, we reject the null hypothesis that $\alpha_1 \geq \alpha_2$ in the one-way test at the 5% level. Meanwhile, Bouchouicha and Vieider [2017] do not find any significant changes in risk-seeking preference with the magnitude of losses. Within the loss range, we also fail to reject the null hypothesis that $\alpha_3 = \alpha_4$ at the 5% level. Another important study is Wang, Yan, and Yu [2017]. By investigating risk-return trade-off in the stock market, the authors find that the risk aversion effect seems to be more significant among investors experiencing larger capital gains while the risk seeking effect seems to be more significant among those who experienced large capital losses.

3.5 Conclusion

Our study aims to contribute to the ongoing debate in the literature on the connection between prospect theory and the observed trading behavior in asset markets. We build a search model to examine a home seller's pricing decision under a generally characterized prospect value function. This setup allows us to examine the precise correspondence between each component of prospect theory and its unique empirical implications. Our model shows that reference dependence alone generates a disposition effect when sellers are risk neutral. We show that the seller's risk seeking attitude will distort the one-way disposition effect by introducing a local reverse disposition effect, a range in which the seller's asking price tends to decrease with the reference value. According to decreasing sensitivity from the prospect theory, it implies that the reverse disposition effect must occur in the loss range. Further, loss aversion tends to magnify the disposition effect but mitigates the magnitude and the range of the reverse disposition effect. To the best of our knowledge, this study is the first that shows the possibility of a reference-dependent two-way disposition effect.

While our model predictions are consistent with some well documented empirical findings, we argue that there is a conceptual mismatch in many empirical studies on prospect theory. For example, a stream of literature following Genesove and Mayer [2001] associates the disposition effect with loss aversion and the marginally diminishing disposition effect with decreasing sensitivity. We show that these findings need not have any such direct relations. Moreover, the co-existence of disposition and reverse disposition effect suggests that empirical studies dominated by focusing on a one-way disposition effect, as often seen in financial and housing studies, can be overly simplistic and incomplete.

We test our model predictions by using a detailed set of multiple listing service data with both listing and transaction information on house sellers in Virginia. Our empirical results are broadly consistent with the model predictions. Using spline and semi-parametric regression techniques, we find significant evidence of a local reverse disposition effect, in addition to the well documented overall disposition effect. We further calibrate the pricing curve generated from our model with the empirical data and find significant evidence on the loss aversion effect.

Concerning the coefficient that governs an agent's risk preference, we use power-law transformation to estimate marginal sensitivity and find it seems to vary with the extent of an agent's loss/gain. However, we note that, unlike many insightful studies on changing risk preferences in other contexts, such as the habit model by Campbell and Cochrane [1999], we offer no proposal on what forces may generate such a reference-dependent α. Therefore, we welcome follow-up research on the underlying mechanism that leads to heterogeneous risk preferences in the domain of losses and gains under prospect theory.

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Appendix A

Proof of Proposition 0

Proof. The proof of Equation (2.6) to (2.11) can be found in the appendix of Williams [1998] paper, page 272-273. Here, we briefly repeat this.

By Taylor expansion on $e^{-\iota\Delta t}$, and using integration by parts on $\int_{r_{ijn}}^{1} pdG(p)$, Equation (2.1) can be re-written as:

$$\begin{aligned}
V(n) &= \underbrace{max}_{r_{ijn},x_{ijn},y_{in}} e^{-\iota\Delta t}\Big\{\sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)t\int_{r_{ijn}}^{1} b^*(1-G(p))dp\Delta t \\
&+ \alpha\overline{n}^*(y_{in}/\overline{y}^*)\Delta V_n^*\Delta t - \theta H(w_{in})\Delta t \\
&+ \sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)[1-G(r_{ijn})][b^*r_{ijn}-\Delta V_{n-1}^*]\Delta t\Big\} \\
&- \iota V(n)\Delta t + V(n) \qquad \text{(A.1)}
\end{aligned}$$

where $\Delta V_n^* = V(n+1) - V(n)$.

Now, by dividing Equation (A.1) by Δt and taking the limit as Δt approaches zero, we get

$$\begin{aligned}
\iota V(n) &= \underbrace{max}_{r_{ijn},x_{ijn},y_{in}} \{\sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)\int_{r_{ijn}}^{1} b^*(1-G(p))dp \\
&+ \alpha\overline{n}^*(y_{in}/\overline{y}^*)\Delta V_n^* - \theta H(w_{in}) \\
&+ \sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)[1-G(r_{ijn})][b^*r_{ijn}-\Delta V_{n-1}^*]\} \qquad \text{(A.2)}
\end{aligned}$$

We set $M(r) = \int_r^1 1 - G(p)dp$. Now take the first-order conditions for x_{ijn}, r_{ijn}, and y_{in}:

$$\Delta V_{n-1}^* = b^*r_{ijn}^* \qquad \text{(A.3)}$$

$$b^*F'(x_{ijn}^*,\overline{x}^*)M(r_{ijn}^*) = \theta H'(w_{in}^*) \qquad \text{(A.4)}$$

$$\theta H'(w_{in}^*) = \frac{\alpha \overline{n}^* \Delta V_n^*}{\overline{y}^*} \tag{A.5}$$

Williams [1998] appendix shows that for this class of problem, a set of constant, unique solution exists. As a result, we know $x_{ijn}^* = x^*$, $r_{ijn}^* = r^*$,and $y_{in}^* = y^*$. Equation (A.3) implies $b^* r^* = \Delta V_{n-1}^* = \Delta V^*$. Thus, it means $V(n) = \gamma + n b^* r^*$. Now, putting Equation (A.3), (A.4), and (A.6) back to Equation (A.2), we find:

$$\iota V(n) = \iota\gamma + \iota n b^* r^* = n b^* F(x^*, \overline{x}^*) M(r^*) + \alpha \overline{n}^* \frac{y^*}{\overline{y}^*} b^* r^* - \theta H'(w^*) \tag{A.6}$$

As $y^* = w^* - nx^*$ and Equation (A.6) holds for all the possible n, with the steady state assumption, we could prove Proposition 1 results, from S1 to S5.

Appendix B

Proof of Proposition 1

Proof. From Equation (2.7) to (2.11) (S1 to S5), we get a set of equations of r^*, x^*, w^*, α and γ in the following form:

$$\begin{aligned}
&S1(r^*, \alpha : \iota, \eta) = 0 \\
&S2(r^*, x^*, w^* : b^*, \iota, \eta, \theta) = 0 \\
&S3(r^*, x^*, \alpha : b^*) = 0 \\
&S4(w^*, \gamma : \iota, \theta) = 0 \\
&S5(x^*, \alpha, w^* : \iota, \eta, \bar{n}) = 0
\end{aligned} \tag{B.1}$$

We have two methods to prove that the solutions of r^*, x^*, and w^* are related to the b^*. First, these five equations are coming from the three first order conditions from Equation (2.1), steady state equilibrium Equation $\alpha = F(x^*)(1 - G)$ and $V(0) = \gamma$. In order to solve r^*, x^*, and w^* directly, we may replace $\alpha = F(x^*)(1 - G)$ into Equation (2.7) and (2.11). Thus, S1 to S5 are now simplified to:

$$SS1: \quad \frac{r^*}{M(r^*)} - \frac{F(x^*, \overline{x}^*)}{\iota}(1 - \eta) = 0 \tag{B.2}$$

$$SS2: \quad \frac{1 - \eta}{\eta\iota} x^* \theta^* H'(w^*) - b^* r^* = 0 \tag{B.3}$$

$$SS3: \quad \frac{\eta\iota}{\eta\iota + (1 - \eta)F(x^*, \overline{x}^*)[1 - G(r^*)]} w^* - \overline{n}^* x^* = 0 \tag{B.4}$$

From the above equations, we can see SS1, SS2 and SS3 are three equations with three unknown variables r^*, x^*, and w^*. We can directly solve this by allowing other parameters fixed. Also, we find that, as SS2 has a b^* inside, thus, w^* should be related to the b^*. Therefore, all the r^*, x^*, and w^* should be related to b^* because they all appear in SS3. After we get the solution of r^*, x^*, and w^*, take them back to S3 and S4, we can get the α and γ. So far, all the five variables r^*, x^*, w^*, α, and γ are all related to the b^*.

Second, we can go directly to find the relations between r^* and b^* from S1 to S5. As we are not able to directly get solutions like $r^* = r^*(b^*, \iota, \eta, \theta, \bar{n})$, we have to find another way to get the partial derivatives. First, the Jacobian Determinant $|J|$[1] is:

$$|J| = \begin{vmatrix} S1_{r^*} & 0 & S1_\alpha & 0 & 0 \\ S2_{r^*} & S2_{x^*} & 0 & S2_{w^*} & 0 \\ S3_{r^*} & S3_{x^*} & S3_\alpha & 0 & 0 \\ 0 & 0 & 0 & S4_{w^*} & S4_\gamma \\ 0 & S5_{x^*} & S5_\alpha & S5_{w^*} & 0 \end{vmatrix}$$

$$= -S4_\gamma S2_{w^*} \times \begin{vmatrix} S1_{r^*} & 0 & S1_\alpha \\ S3_{r^*} & S3_{x^*} & S3_\alpha \\ 0 & S5_{x^*} & S5_\alpha \end{vmatrix} - S4_\gamma S5_{w^*} \times \begin{vmatrix} S1_{r^*} & 0 & S1_\alpha \\ S2_{r^*} & S2_{x^*} & 0 \\ S3_{r^*} & S3_{x^*} & S3_\alpha \end{vmatrix} \tag{B.5}$$

Now, we start from the proof of $S1_{r^*} > 0$. Use the same method with Williams [1998] page 274. We set $M(r) = \int_r^1 1 - G(p)dp$ and $L(r^*) \equiv log\frac{[r^*(1-G)]}{M}$. The problem has transformed to prove the partial derivative of $L(r^*)$ against r^*. Therefore, $L'(r^*) = \frac{1}{r^*} + \frac{1-G}{M} - \frac{g}{1-G} = [1 + \frac{\alpha}{\iota}(1-\eta)]\frac{1}{r^*} - \frac{g}{1-G}$. Following the same step in Williams's [1998] paper page 274, with the nondecreasing hazard function $\frac{g}{1-G}$, $L'(r^*)$ is nonincreasing, so $L'(1)$ must be its minimum value. By using L'Hospital's rule, $\lim_{r\to 1} L'(r^*) = 1 + \lim_{r\to 1} \frac{1-G}{M} - \lim_{r\to 1} \frac{g}{1-G} = 1 + \lim_{r\to 1} \frac{g}{1-G} - \lim_{r\to 1} \frac{g}{1-G} = 1 > 0$. we prove the following:

$$S1_{r^*} = \frac{\partial \frac{r(1-G)}{M}}{\partial r} = \frac{(1 - G - r^*g)M + r[1-G]^2}{M^2} > 0 \tag{B.6}$$

This also proves that the solution of r^* is unique under the constrain of $0 < r^* < 1$. Thus, other solutions like x^*, w^* and y^* are also unique under the constrain $(0, 1)$. Other terms in Equation (B.5) are straightforward to derive. Taking them all into Equation (B.5), we could get:

$$\begin{aligned} |J| = &- S4_\gamma S2_{w^*}(S1_{r^*}S3_{x^*}S5_\alpha + S1_\alpha S3_{r^*}S5_{x^*} - S1_{r^*}S3_\alpha S5_{x^*}) \\ &- S4_\gamma S5_{w^*}(S1_{r^*}S2_{x^*}S3_\alpha + S1_\alpha S2_{r^*}S3_{x^*} - S1_\alpha S2_{x^*}S3_{r^*}) > 0 \end{aligned} \tag{B.7}$$

[1]Here, $S1_{r^*}$ means $\frac{\partial S1}{\partial r^*}$.

Now, based on Crammer's rule:

$$\frac{\partial r^*}{\partial b^*} = \frac{\begin{vmatrix} 0 & 0 & S1_{\alpha} & 0 & 0 \\ -S2_{b^*} & S2_{x^*} & 0 & S2_{w^*} & 0 \\ 0 & S3_{x^*} & S3_{\alpha} & 0 & 0 \\ 0 & 0 & 0 & S4_{w^*} & S4_{\gamma} \\ 0 & S5_{x^*} & S5_{\alpha} & S5_{w^*} & 0 \end{vmatrix}}{|J|} = \frac{|J_{rb}|}{|J|} > 0 \tag{B.8}$$

Using the same method, we could prove

$$\frac{\partial x^*}{\partial b^*} = \frac{\begin{vmatrix} S1_{r^*} & 0 & S1_{\alpha} & 0 & 0 \\ S2_{r^*} & -S2_{b^*} & 0 & S2_{w^*} & 0 \\ S3_{r^*} & 0 & S3_{\alpha} & 0 & 0 \\ 0 & 0 & 0 & S4_{w^*} & S4_{\gamma} \\ 0 & 0 & S5_{\alpha} & S5_{w^*} & 0 \end{vmatrix}}{|J|} = \frac{|J_{xb}|}{|J|} > 0 \tag{B.9}$$

$$\frac{\partial w^*}{\partial b^*} = \frac{\begin{vmatrix} S1_{r^*} & 0 & S1_{\alpha} & 0 & 0 \\ S2_{r^*} & S2_{x^*} & 0 & -S2_{b^*} & 0 \\ S3_{r^*} & S3_{x^*} & S3_{\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 & S4_{\gamma} \\ 0 & S5_{x^*} & S5_{\alpha} & 0 & 0 \end{vmatrix}}{|J|} = \frac{|J_{wb}|}{|J|} > 0 \tag{B.10}$$

Using the same method, we could prove $\frac{\partial \alpha}{\partial b^*} > 0$. For the relation between y^* and b^*, we first notice $y^* = w^* - nx^*$. Substituting w^* from Equation (2.11), we get $y^* = (\overline{n} - n)x^* + \frac{1-\eta}{\eta\iota}\overline{n}\alpha x^*$. Because we could prove $\frac{\partial(\alpha x^*)}{\partial b^*} > 0$, when $n \leq \overline{n}$,

$$\frac{\partial y^*}{\partial b^*} > 0. \tag{B.11}$$

Meanwhile, we notice that as n increases to larger than $\overline{n}$, the partial derivative $\frac{\partial y^*}{\partial b^*}$ will be smaller or even negative. This indicates there exists some integer $N \geq \overline{n}$ that satisfies $\frac{\partial y^*}{\partial b^*} > 0$ when $n \leq N$ and $\frac{\partial y^*}{\partial b^*} \leq 0$ when $n > N$.

Finally, we prove the relation between likelihood of sale $F(x^*, \overline{x}^*)[1 - G(r^*)]$ and commission rate b^*. We firstly note that under our steady state model, $x^* = \overline{x^*}$. Also, $\alpha = F(x^*, \overline{x}^*)[1 -$

$G(r^*)]$ because finding new assets has the same rate with selling current assets. Thus, we prove the relation between the likelihood of a sale and the commission rate by:

$$\frac{\partial\{F(x^*,x^*)[1-G(r^*)]\}}{\partial b^*} = \frac{\partial \alpha}{\partial b^*} = \frac{\begin{vmatrix} S1_{r^*} & 0 & 0 & 0 & 0 \\ S2_{r^*} & S2_{x^*} & -S2_{b^*} & S2_{w^*} & 0 \\ S3_{r^*} & S3_{x^*} & 0 & 0 & 0 \\ 0 & 0 & 0 & S4_{w^*} & S4_{\gamma} \\ 0 & S5_{x^*} & 0 & S5_{w^*} & 0 \end{vmatrix}}{|J|} = \frac{S4_{\gamma} S5_{w^*} S1_{r^*} S2_{b^*} S3_{r^*}}{|J|} > 0 \tag{B.12}$$

□

Appendix C

Proof of Proposition 2

The term $\frac{1-[\rho G(r_{ijn})]^n}{1-\rho G(r_{ijn})}$ looks messy because we need to take derivative against r. Here, we introduce a notation $N(r,\rho)$ by:

$$N(r,\rho) \times G'(r) = \partial \frac{1-[\rho G(r_{ijn})]^n}{1-\rho G(r_{ijn})}/\partial r \tag{C.1}$$

and $D(r^*,\rho)$:

$$D(r^*,\rho) = \frac{1-(\rho G)^n}{1-(\rho G)} \frac{\frac{1-(\rho G)^n}{1-(\rho G)}}{\frac{1-(\rho G)^n}{1-(\rho G)} - N(r^*,\rho)(1-G)} \tag{C.2}$$

Also, we use the same notation of $M(r) = \int_r^1 1 - G(p)dp$ as the Williams paper. The equations after picking maximum outcome against r^*, x^*, and y^* are:

$$\Delta V_{n-1}^* = br^* - \frac{bM(r^*)N(r^*,\rho)}{\frac{1-(\rho G)^n}{1-(\rho G)} - N(r^*,\rho)(1-G)} \tag{C.3}$$

$$\theta H'(w^*) = b^* F'(x^*, \overline{x}^*) M(r^*) D(r^*,\rho) \tag{C.4}$$

$$\theta H'(w^*) = \alpha \overline{n} \frac{\Delta V_n}{\overline{y}} \tag{C.5}$$

The final outcome $V(n)$ must have following expression with all possible n:

$$\iota V_n = b^* n F(x^*, \overline{x}^*) M(r^*) D(r^*,\rho) - \theta H(w^*) + \alpha \overline{n} \frac{y^*}{\overline{y}} \Delta V_n \tag{C.6}$$

In addition, Equation (2.9), which indicates a steady state of $E(\Delta n) = 0$ now changes to:

$$\alpha - F(x^*, \overline{x}^*)[1 - G(r^*)] \frac{1-[\rho G(r^*)]^n}{1-\rho G[r^*]} = 0 \tag{C.7}$$

We are unable to find the partial derivative relation between variables from the above equations, because we have five equations (C.3), (C.4), (C.5), (C.6) and (C.7) but six independent variables here. However, under the limiting condition of $[\rho G(r)]^n \sim 0$, the function $N(r,\rho)$ and $D(r,\rho)$ will be simplified to:

$$N(r^*,\rho) = \frac{\rho}{[1-\rho G(r^*)]^2} \tag{C.8}$$

$$D(r^*,\rho) = \frac{1}{1-\rho} \tag{C.9}$$

Equations from (C.3) to (C.6) will change to:

$$\Delta V^* = br^* - \frac{bM(r^*)\rho}{1-\rho} \tag{C.10}$$

$$\theta H'(w^*) = b^* F'(x^*, \overline{x}^*) M(r^*) \frac{1}{1-\rho} \tag{C.11}$$

$$\theta H'(w^*) = \alpha \overline{n} \frac{\Delta V^*}{\overline{y}} \tag{C.12}$$

The final outcome $V(n)$ must have following expression with all possible n:

$$\iota V_n = b^* n F(x^*, \overline{x}^*) M(r^*) \frac{1}{1-\rho} - \theta H(w^*) + \alpha \overline{n} \frac{y^*}{\overline{y}} \Delta V \tag{C.13}$$

Now, we can solve these equations with the limiting condition $[\rho G(r)]^n \sim 0$ and compare them with the original Williams's model from Equation (2.7) to (2.11) :

$$V(n) = \gamma + nb^* \left[r^* - \frac{M(r^*)\rho}{1-\rho} \right] \tag{C.14}$$

$$S6: \frac{[r^*(1-\rho) - M(r^*)\rho][1-G(r^*)]}{[1-\rho G(r^*)]M(r^*)} - \frac{\alpha}{\iota}(1-\eta) = 0 \tag{C.15}$$

$$S7: \frac{1-\eta}{\eta\iota} x^* \theta^* H'(w^*) - b^* \left[r^* - \frac{M(r^*)\rho}{1-\rho} \right] = 0 \tag{C.16}$$

$$S8 : \alpha - F(x^*, \overline{x}^*)\frac{1 - G(r^*)}{1 - \rho G(r^*)} = 0 \tag{C.17}$$

$$S9 : w^*\theta^* H'(w^*) - \theta^* H(w^*) - \gamma\iota = 0 \tag{C.18}$$

$$S10 : \frac{\eta\iota}{\eta\iota + (1-\eta)\alpha} w^* - \overline{n}^* x^* = 0 \tag{C.19}$$

The results show that the agency problem holds because we have five equations but three independent variables. Thus we cannot tell r^* has no relation to b^* because of absence b^* in Equation (C.15). Here we also take α and γ vary, and other factors like η, ι, θ and $\bar{n}^*$ hold constant. First, the derivation $S6_{r^*}$ is similar to our previous model. We notice that (C.15) could be transformed to:

$$\frac{\alpha}{\iota}(1-\eta) = \frac{1}{1-\rho G(r^*)}\left[\frac{r^*[1-G(r^*)]}{M(r^*)}(1-\rho) - \rho[1-G(r^*)]\right] \tag{C.20}$$

We have already proved that the term $\frac{r(1-G)}{M}$ increases against r from (B.6). Now, as the term $-\rho(1-G)$ and $\frac{1}{1-\rho G}$ are also increasing against r, the whole right-hand side is increasing against r, thus we could prove $S6_{r^*} > 0$. After this, take it into the Jacobian determinant of the Equation set $S6$ to $S10$, we will also find that the Jacobian determinant $|J'| > 0$. Finally, the relations between r^*, x^*, w^*, y^* and b^* are the same with our previous model:

$$\frac{\partial r^*}{\partial b^*} > 0 \tag{C.21}$$

$$\frac{\partial x^*}{\partial b^*} > 0 \tag{C.22}$$

$$\frac{\partial w^*}{\partial b^*} > 0 \tag{C.23}$$

$$\text{For some integer } N(\rho) \geq \overline{n}, \quad \frac{\partial y^*}{\partial b^*} \begin{cases} > 0 & n \leq N(\rho) \\ \leq 0 & n > N(\rho) \end{cases} \tag{C.24}$$

with the prerequisite condition $[\rho G(r)]^n \sim 0$.

For $\frac{\partial r^*}{\partial \rho}$, we know that

$$\frac{\partial r^*}{\partial \rho} = \frac{|J_{r^*\rho}|}{|J'|} = \begin{vmatrix} -S6_\rho & 0 & S6_\alpha & 0 & 0 \\ -S7_\rho & S7_{x^*} & 0 & S7_{w^*} & 0 \\ -S8_\rho & S8_{x^*} & S8_\alpha & 0 & 0 \\ 0 & 0 & 0 & S9_{w^*} & S9_\gamma \\ 0 & S10_{x^*} & S10_\alpha & S10_{w^*} & 0 \end{vmatrix} /|J'|$$

$$= -\frac{S9_\gamma S7_{w^*}}{|J'|} \times \begin{vmatrix} -S6_\rho & 0 & S6_\alpha \\ -S8_\rho & S8_{x^*} & S8_\alpha \\ 0 & S10_{x^*} & S10_\alpha \end{vmatrix} - \frac{S9_\gamma S10_{w^*}}{|J'|} \times \begin{vmatrix} -S6_\rho & 0 & S6_\alpha \\ -S7_\rho & S7_{x^*} & 0 \\ -S8_\rho & S8_{x^*} & S8_\alpha \end{vmatrix} \tag{C.25}$$

From Equation (C.25), because ρ must be in the domain $\rho \in [0, 1]$, and with the previous result of increasing $\frac{r(1-G)}{M}$ against r^*, by carefully input each partial derivative coming from (C.15) to (C.19), we could derive that both 3×3 determinants are positive and thus $\frac{\partial r^*}{\partial \rho} > 0$.

The proof of Equation (2.26) is following: we firstly notice that:

$$\frac{\partial \{\frac{F(x^*,\overline{x}^*)[1-G(r^*)]r^*}{1-\rho G(r^*)}\}}{\partial \rho} = \frac{\partial(\alpha r^*)}{\partial \rho} = \alpha \frac{\partial r^*}{\partial \rho} + r^* \frac{\partial \alpha}{\partial \rho} \tag{C.26}$$

$$= -\frac{S9_\gamma S7_{w^*}}{|J'|} \times \left(\alpha \begin{vmatrix} -S6_\rho & 0 & S6_\alpha \\ -S8_\rho & S8_{x^*} & S8_\alpha \\ 0 & S10_{x^*} & S10_\alpha \end{vmatrix} - r^* \begin{vmatrix} S6_{r^*} & 0 & -S6_\rho \\ S8_{r^*} & S8_{x^*} & -S8_\rho \\ 0 & S10_{x^*} & -S10_\rho \end{vmatrix} \right)$$

$$- \frac{S9_\gamma S10_{w^*}}{|J'|} \times \left(\alpha \begin{vmatrix} -S6_\rho & 0 & S6_\alpha \\ -S7_\rho & S7_{x^*} & 0 \\ -S8_\rho & S8_{x^*} & S8_\alpha \end{vmatrix} - r^* \times \begin{vmatrix} S6_{r^*} & 0 & -S6_\rho \\ S7_{r^*} & S7_{x^*} & -S7_\rho \\ S8_{r^*} & S8_{x^*} & -S8_\rho \end{vmatrix} \right)$$

Based on equations (C.15) to (C.19), we could solve the x^*, w^* and α by the function forms of r^* and ρ. Then, we input the function form of $G(r) = 3r^2 - 2r^3$, thus we can find that the minimum value of Equation (C.26) is positive. This proves (2.26).

Our previous results of r^*, x^*, w^*, α and γ are free with current assets number n, which means these variables (except y^*) are fixed along with different number of assets if n is smaller than its maximum integer $n^* = \left[\frac{w^*}{x^*}\right]$. These results are all based on our assumption that $(\rho G)^n \sim 0$. If the current asset number is small, this assumption will not hold. For the case of $(\rho G)^n \gg 0$, by substituting Equation (C.3) into (C.5) and (C.6), we have four equations (C.4), (C.5), (C.6) and (C.7) but five independent variables. We notice that unlike other variables, the entry cost γ should have no relation to the current number of assets n. By this assumption, we replace the current γ here with the solutions of the γ under the case $[\rho G(r^*)]^n \sim 0$. Thus, we have four equations and four variables and we can solve them together.

Appendix D

Proof of Proposition 3

For the agency problem test case, we set two different assets ($b^* < 1$ and $\widehat{b}^* = 1$). The Equation (2.5) is simplified as following:

$$
\begin{aligned}
\widehat{V}(n,1) &= \underbrace{max}_{r_{ijn},x_{ijn},y_{in},\widehat{r}_{in},\widehat{x}_{in}} \quad e^{-\iota\Delta t}\Big\{\sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)\int_{r_{ijn}}^{1} b^* p dG(p)\Delta t \\
&+ \sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)[1-G(r_{ijn})]\widehat{V}(n-1,1)\Delta t \\
&+ F(\widehat{x}_{in})\int_{\widehat{r}_{in}}^{1} p dG(p)\Delta t \\
&+ F(\widehat{x}_{in})[1-G(\widehat{r}_{in})]\hat{V}(n,0)\Delta t \\
&+ \alpha\overline{n}^*(y_{in}/\overline{y}^*)\widehat{V}(n+1,1)\Delta t - \theta H(\sum_{j=1}^{n} x_{ijn} + \widehat{x}_{in} + y_{in})\Delta t \\
&+ \Big[1 - \sum_{j=1}^{n} F(x_{ijn},\overline{x}^*)[1-G(r_{ijn})]\Delta t - F(\widehat{x}_{in})[1-G(\widehat{r}_{in})]\Delta t - \alpha\overline{n}^*(y_{in}/\overline{y^*})\Delta t\Big]\hat{V}(n,1)\Big\} \\
&+ o(\Delta t)
\end{aligned} \tag{D.1}
$$

Here, $\widehat{r^*}$ is for the asset with $b^* = 1$. we realize that the marginal benefit for general asset should be the same with the broker's own asset, and they should all equal to the marginal cost. Thus, from Equation (A.3), (A.4), and (A.6), we should have the similar results:

$$\widehat{V}_{n,1}^* - \widehat{V}_{n,0}^* = \widehat{r}^* \tag{D.2}$$

$$F'(\widehat{x}^*,\overline{x}^*)M(\widehat{r}^*) = b^* F'(x^*,\overline{x}^*)M(r^*) = \theta H'(\widehat{w}^*) \tag{D.3}$$

$$
\begin{aligned}
\iota\widehat{V}(n,1) &= nb^* F(x^*,\overline{x}^*)M(r^*) + F(\widehat{x}^*,\overline{x}^*)M(\widehat{r}^*) + \alpha\overline{n}^*\frac{y^*}{\overline{y}^*}b^* r^* - \theta H'(\widehat{w}^*) \\
&= \iota\gamma + \iota n b^* r^* + \iota\widehat{r}^*
\end{aligned} \tag{D.4}
$$

Here, $w^* = nx^* + \widehat{x}^* + y^*$. Similar with Equation (A.6), Equation (D.4) holds for all the possible number of assets. Thus, the formula of r^* and $\widehat{r}^*$ here should also satisfy the S1 and S3 in the Proposition 1. That is actually S11: $F(x,\overline{x}) = \frac{\iota r}{(1-\eta)M(r)}$. As $F(x,\overline{x}) \equiv x^\eta$, we get $x^\eta = \frac{\iota r}{(1-\eta)M(r)}$ and $\widehat{x}^\eta = \frac{\iota \widehat{r}}{(1-\eta)M(\widehat{r})}$. Put them into Equation (D.3), we could find:

$$b^* r^{1-\frac{1}{\eta}} M(r)^{\frac{1}{\eta}} = \widehat{r}^{1-\frac{1}{\eta}} M(\widehat{r})^{\frac{1}{\eta}} \tag{D.5}$$

As $\frac{dM(r)}{dr} = -(1-G(r)) < 0$, $0 < \eta < 1$. For any $0 < b^* < 1$, as $r^{1-\frac{1}{\eta}} M(r)^{\frac{1}{\eta}} > \widehat{r}^{1-\frac{1}{\eta}} M(\widehat{r})^{\frac{1}{\eta}}$, we could find that $\widehat{r}^* > r^*$. Back to $x^\eta = \frac{\iota r}{(1-\eta)M(r)}$, we prove that $\widehat{x}^* > x^*$.

Also, for any $0 < b_1^* < b_2^* < 1$, Equation (D.5) could transform to:

$$b_1^* r_1^{1-\frac{1}{\eta}} M(r_1)^{\frac{1}{\eta}} = b_2^* r_2^{1-\frac{1}{\eta}} M(r_2)^{\frac{1}{\eta}} \tag{D.6}$$

As $b_1^* < b_2^*$, we get $r_1^{1-\frac{1}{\eta}} M(r_1)^{\frac{1}{\eta}} > r_2^{1-\frac{1}{\eta}} M(r_2)^{\frac{1}{\eta}}$. Thus, we can also find $r_1^* < r_2^*$. It indicates that for any asset with lower commission rate, agent will ask for a lower price, ceteris paratus. Meanwhile, we could get $x_1^* < x_2^*$. $\square$

Appendix E

Proof of Proposition 4

From Equation (3.3), the utility function of our model is:

$$U = \frac{B(t_i)[1 - G(r_i)]W(r_i, v_i) - H(t_i) - c_i}{B(t_i)[1 - G(r_i)] + \beta} \tag{E.1}$$

We define the first order conditions against t and r in A1 and A2. Here, $W(x) = W(r^* - v_i)$:

$$A1 = W(x) + \frac{H(t^*) + c}{\beta} - F(r^*)W'(x) = 0 \tag{E.2}$$

$$A2 = B'(t^*)\left[\beta W(x) + H(t^*) + c\right] - H'(t^*)\left[B(t^*) + \frac{\beta}{1 - G}\right] = 0 \tag{E.3}$$

From (E.2) and (E.3), we can derive $A1_t = A2_r = 0$. As $B' > 0$, $W' > 0$, $B'' < 0$, and $H'' > 0$, the two remaining terms of the Jacobian Matrix are:

$$A1_r = W' - F'W' - FW'' \tag{E.4}$$

$$A2_t = B''(\beta W + H + c) - H''(B + \frac{\beta}{1 - G}) < 0 \tag{E.5}$$

Thus, by using the formula of implicit function derivative, we find:

$$\frac{\partial r^*}{\partial v} = \frac{1 - F(r^*)\frac{W''(x)}{W'(x)}}{1 - F(r^*)\frac{W''(x)}{W'(x)} - F'(r^*)} \tag{E.6}$$

$$\frac{\partial t^*}{\partial v} = \frac{\beta B'W'(x)}{B''\left[\beta W(x) + H + c\right] - H''(B + \frac{\beta}{1-G})} < 0 \tag{E.7}$$

From Equation (E.7), $\frac{\partial t^*}{\partial v} < 0$ always holds. However, the sign of $\frac{\partial r^*}{\partial v}$ is still unknown. Moreover, in order to maximize the utility, the second order conditions must be:

$$U_{rr} = -\frac{B\beta g}{[B(1 - G) + \beta]^2} A1_r < 0 \tag{E.8}$$

$$U_{tt} = \frac{1-G}{[B(1-G)+\beta]^2} A2_t < 0 \tag{E.9}$$

Equation (E.9) always holds and the second order interaction term $U_{rt} = 0$. Thus, to maximize the utility function (E.1), U_{rr} must be negative, which indicates $A1_r$ must be positive. First, if we assume $A1_r > 0$, the denominator of Equation (E.6) is positive. Thus, the sign of $\frac{\partial r^*}{\partial v}$ depends on the sign of $1 - F(r^*)\frac{W''(x)}{W'(x)}$. After incorporating the prospect theory value functions in Equation (3.10), in the case of $\alpha < 1$, we have the following: If $v_i - r^* < 0$, i.e., the gain area, we have:

$$\frac{\partial r^*}{\partial v_i} = \frac{1 + \frac{(H+c)(1-\alpha)}{\beta}(r^* - v_i)^{-\alpha}}{1 + \frac{(H+c)(1-\alpha)}{\beta}(r^* - v_i)^{-\alpha} - \alpha F'(r^*)} > 0 \tag{E.10}$$

In the loss area, or $v_i - r^* > 0$, we get:

$$\frac{\partial r^*}{\partial v_i} = \frac{1 - \frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha}}{1 - \frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha} - \alpha F'(r^*)} \tag{E.11}$$

As we assume $U_{rr} < 0$, or $A1_r > 0$, the denominator is positive and we can find that:

$$\frac{\partial r^*}{\partial v_i} = \begin{cases} > 0 & \text{if} \quad v_i - r^* < 0 \\ < 0 & \text{if} \quad \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta} < \lambda(v_i - r^*)^{\alpha} < \frac{(H+c)(1-\alpha)}{\beta} \\ > 0 & \text{if} \quad \lambda(v_i - r^*)^{\alpha} > \frac{(H+c)(1-\alpha)}{\beta} \end{cases} \tag{E.12}$$

As $F'(r^*) < 0$, we define $\rho(r^*) \equiv \frac{1}{1-\alpha F'(r^*)} < 1$. It is debatable whether a solution exists within the third area when waiting cost c is very large. In practice, the waiting cost could not be very large. The total discounted cost, $\frac{(H+c)(1-\alpha)}{\beta}$, should be smaller than the possible maximum value function $\lambda(v - r)^{\alpha}$. Thus, we could always find a solution that matchs the condition $\lambda(v_i - r^*)^{\alpha} > \frac{(H+c)(1-\alpha)}{\beta}$.

Second, in the previous discussion, we assumed $A1_r > 0$. If $A1_r \leq 0$, we find that it must be the missing area $0 < \lambda(v - r)^{\alpha} < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$ in the previous results (E.12). Now, we are going to find a local maximum in this area under the $A1_r \leq 0$ constraint. By using the Kuhn-Tucker conditions, we find that the local maximum is just at the boundary, which is $A1_r = 0$. After incorporating the specific prospect theory function, it is:

$$B1 = (v_i - r^*)[1 - F'(r^*)] - F(r^*)(1-\alpha) = 0 \tag{E.13}$$

Thus, B1 (E.13) Combined with Equation A2 (E.3) are our new first order conditions under the $0 < \lambda(v-r)^{\alpha} < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$ area. In this area, with $B1 = 0$, we find that $U_r < 0$ always holds. That is, the utility function is always decreasing in this area, and the corner conditions are $r^* = v_i$ or $r^* + \frac{F(1-\alpha)}{1-F'} = v_i$. The second condition gives a smaller r^*. Thus, the r^* solution from $B1 = 0$ gives a local maximum of U in this area.

Now, we can derive the $\frac{\partial r^*}{\partial v}$ and $\frac{\partial t^*}{\partial v}$ in this area. By using B1 (E.13) and A2 (E.3), we find that $B1_v > 0$, $B1_r < 0$, $B1_t = 0$, $F2_v < 0$, $F2_r < 0$, and $F2_t < 0$. Finally, using the formula of implicit function derivative, we can prove $\frac{\partial r^*}{\partial v} > 0$ and $\frac{\partial t^*}{\partial v} < 0$ within this area.

In sum, for the case of marginal diminishing sensitivity ($\alpha < 1$), $\frac{\partial t^*}{\partial v} < 0$ and the results of $\frac{\partial r^*}{\partial v}$ are as following:

$$\frac{\partial r^*}{\partial v_i} = \begin{cases} > 0 & \text{if} \quad v_i - r^* < 0 \\ > 0 & \text{if} \quad 0 < \lambda(v_i - r^*)^{\alpha} < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta} \\ < 0 & \text{if} \quad \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta} < \lambda(v_i - r^*)^{\alpha} < \frac{(H+c)(1-\alpha)}{\beta} \\ > 0 & \text{if} \quad \lambda(v_i - r^*)^{\alpha} > \frac{(H+c)(1-\alpha)}{\beta} \end{cases} \tag{E.14}$$

The derivation process for $\frac{\partial r^*}{\partial v}$ under $\alpha > 1$ is similar.

Appendix F

Proof of Proposition 5

(1) For the relation between r^* and c, when $\alpha < 1$ and $v_i - r^* < 0$, the denominator of $\frac{\partial r^*}{\partial c}$ is the same as $\frac{\partial r^*}{\partial v_i}$. Only the numerator is different:

$$\frac{\partial r^*}{\partial c} = \frac{-\frac{1}{\beta}(r^* - v_i)^{1-\alpha}}{1 + \frac{(H+c)(1-\alpha)}{\beta}(r^* - v_i)^{-\alpha} - \alpha F'(r^*)} < 0 \tag{F.1}$$

When $\alpha < 1$ and $v_i - r^* > 0$, and if $\lambda(v - r)^\alpha > \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$, the denominator is always positive. By using the first order conditions, A1 (E.2) and A2 (E.3), we obtain the same results:

$$\frac{\partial r^*}{\partial c} = \frac{-\frac{1}{\lambda\beta}(v_i - r^*)^{1-\alpha}}{1 - \frac{(c+H)(1-\alpha)}{\lambda\beta}(v_i - r^*)^{-\alpha} - \alpha F'(r^*)} < 0 \tag{F.2}$$

If $0 < \lambda(v - r)^\alpha < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$, as we have done before, we use different first order conditions, B1 (E.13) and A2 (A.2). We find that $\frac{\partial r^*}{\partial c} = 0$. However, within all the other areas, an decreasing waiting cost c would lead to an increasing asking price r^*. This implies that the area of $0 < \lambda(v - r)^\alpha < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$ should also change because r^* changes. As v_i must meet the increase in r^*, and c is on the righthand side, when c is decreasing, this area is increasing. Our simulation result also confirms this. From Figure 3.4, if we set the x axis reference point v_i but not $Loss = v_i - \overline{P}$, all three curves should have the same starting point ($v_i = 0$) and ending point ($v_i = 1$). We will find that compared with the $c = 0.1$ curve, $c = 0.05$ curve will move to the right and the $c = 0.01$ curve will move ever further. Thus, we can conclude that $\frac{\partial r^*}{\partial c}$ is still negative here. The derivation process for $\frac{\partial r^*}{\partial c}$ when $\alpha > 1$ is similar. Meanwhile, from Equation (A.2), we observe that as $A2_v < 0$ but $A2_c > 0$, the result of $\frac{\partial t^*}{\partial c}$ is always the opposite against $\frac{\partial t^*}{\partial v} < 0$. Therefore, $\frac{\partial t^*}{\partial c} > 0$.

(2) For the relation between r^* and λ, we first notice that it only matters when $loss > 0$ ($v_i - r^* > 0$). Moreover, compared with $\frac{\partial r^*}{\partial c}$ when $v_i - r^* > 0$, the expression of $\frac{\partial r^*}{\partial \lambda}$ has the same denominator but a positive numerator. Therefore, under $\alpha < 1$, if $\lambda(v - r)^\alpha >$

$\rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$, we find:

$$\frac{\partial r^*}{\partial \lambda} = \frac{\frac{c+H}{\beta\lambda^2}(v_i - r^*)^{1-\alpha}}{1 + \frac{(c+H)(\alpha-1)}{\lambda\beta}(v_i - r^*)^{-\alpha} - \alpha F'(r^*)} > 0 \tag{F.3}$$

$$\frac{\partial^2 r^*}{\partial v \partial \lambda} = \frac{-\alpha F'(r^*)\frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha}}{\lambda^2[1 - \frac{(H+c)(1-\alpha)}{\beta\lambda}(v_i - r^*)^{-\alpha} - \alpha F'(r^*)]^2} > 0 \tag{F.4}$$

Also, under $0 < \lambda(v-r)^\alpha < \rho(r^*)\frac{(H+c)(1-\alpha)}{\beta}$, the entire area is increasing when λ is increasing. Thus, $\frac{\partial r^*}{\partial \lambda} > 0$ and $\frac{\partial^2 r^*}{\partial v \partial \lambda} > 0$ hold. The derivation process when $\alpha > 1$ is similar. Meanwhile, from Equation (A.2), we observe that under the loss area, as $W = -\lambda(v-r)^\alpha$, we have $A2_v < 0$ and $A2_\lambda < 0$, the result of $\frac{\partial t^*}{\partial \lambda}$ is always the same as $\frac{\partial t^*}{\partial v} < 0$. Therefore, $\frac{\partial t^*}{\partial \lambda} < 0$.

Appendix G

Price Dispersion Effect and Price Volume Relation

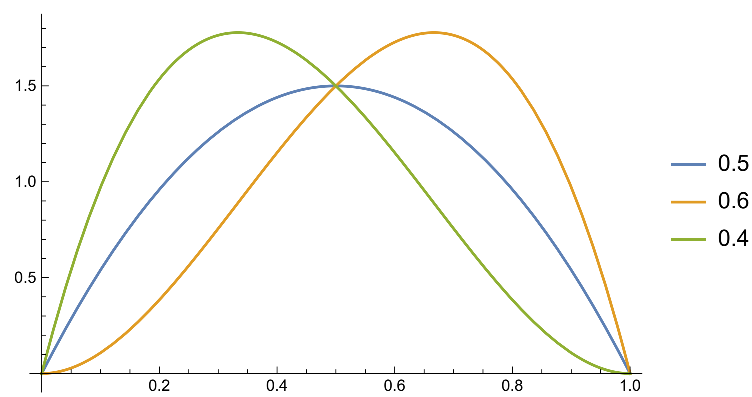
One puzzling finding in the housing market is a strong positive correlation between home prices and transaction volume. For example, using national data, Stein [1995] finds that a 10 percent decline in prices is associated with a reduction in transaction volume by over 1.6 million units in the United States. Another study by Ortalo-Magne and Rady [2006] shows a similar relation in the U.K. In this section. We demonstrate that, with prospect utility, the decision problem, as outlined in Equation (3.1), can help explain this phenomenon. As in Figure 3.3, with prospect utility, although home sellers tend to mark-up their asking price along with reference values, the positive slope of the asking price curve is much flatter for sellers who have low reference values than for sellers who have high reference values. This is due to the loss aversion effect. Compared to the expected market price, for sellers who have high reference values, it is more likely that they will encounter a loss, which yields a greater disutility due to the asymmetric response in the value function. As a result, they have a greater incentive to inflate the asking price to mitigate this disutility.

The heterogeneity of pricing behavior between sellers with low versus high reference values naturally leads to a positive price-volume relation. The intuition is simple. Conditional on a given distribution of reference values, when the market becomes hot, the willingness to pay off potential sellers will increase. As a result, in a hot market, the proportion of sellers who have low reference values relative to the market price increases. Because these sellers are not subject to potential equity loss, the incentive for them to mark-up the price is relatively low. As increasingly more sellers in the market choose to sell at a moderate price, as reflected by the lower and flatter asking price curve, it is clear that the probability of a successful sale will increase, which in turn generates a higher transaction volume.

Furthermore, a flatter asking price curve for sellers with low reference values may shed light on the extent of price dispersion under different market conditions. As previously mentioned, when a market becomes hot, the proportion of sellers who have low reference values relative to market price increases. With a high market price, as more and more sellers cluster in the range of low reference values, differences among their asking prices becomes smaller. In aggregate, we should expect that ceteris paribus, in a hot market, the observed transaction prices will be less dispersed with respect to the expected market price than in a cold market.

To confirm the implication of the price-volume relation and dispersion effect from our model, we simulate different market conditions and then compare the corresponding transaction volume and variance in asking prices within these markets. In particular, we define the market fundamental as the mean of the buyer's bidding distribution. Therefore, in the last section, the market fundamental for a $Beta(2,2)$ distribution with support of [0,1] is 0.5. In the following simulation, we expand the $Beta(2,2)$ distribution by gradually increasing the market fundamental from 0.3 to 0.9. Here, we use the set of Beta distributions to change the mean of the market fundamental. For example, Beta distribution $Beta(2,2)$ is the exact quasi-triangular distribution we used above. $Beta(2,3)$ has a mean of 0.4, and $Beta(3,2)$ has a mean of 0.6.

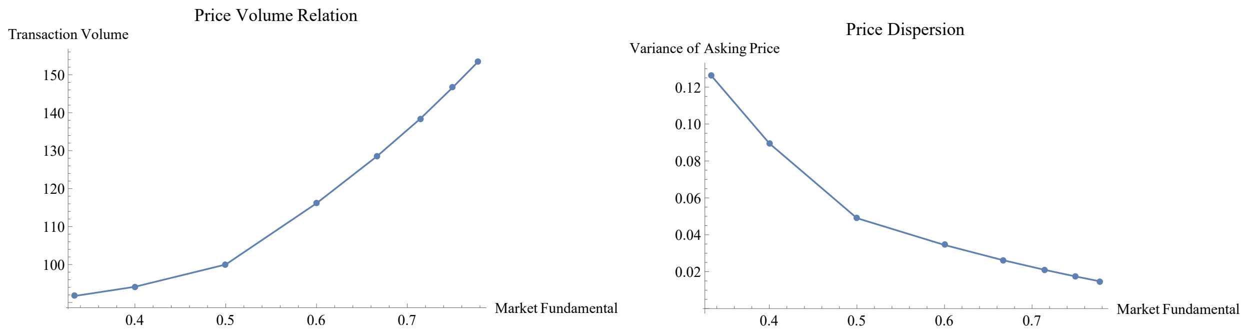
Figure G.1: Beta distributions with Different Means



The simulation process remains the same as above. That is, we assume the seller's reference values are distributed uniformly within [0,1]. Each time we randomly draw 1,000 sellers from this distribution. Again, we set $\beta = 0.05$, $c_i = 0.1$, $\alpha = 0.88$ and $\lambda = 2.25$. We normalize the transaction volume per unit period as 100 when the market fundamental equals 0.5. Doing so makes it easier to infer the percentage change in transaction volume with the expected market price. When calculating the variance of asking prices under each expected market price, we weight every seller's asking price by the corresponding selling probability. The simulation results are presented in Figure G.2.

As expected, the left panel of Figure G.2 reveals a positive price-volume relation. Conditional on a given pool of potential sellers in the marketplace, when the market fundamental increases from approximately 0.5 to 0.74, the transaction volume also increases by roughly 38 percent. The right panel of Figure G.2 also follows our expectation when the market fundamental increases from approximately 0.5 to 0.74, the variance of the asking price drops

Figure G.2: Price-volume Relation and Dispersion Effect



from 0.05 to 0.02.

For the empirical test, if we interpret the difference between the realized transaction price and expect market price as noise, our theory predicts that the higher the expected market price, the smaller the noise should be since there would be fewer and fewer losers in the market. To test this prediction, in each quarter, we first compute the variance of stage 1 hedonic residuals and then regress them on the level of the price index in that given quarter. Note that our theory has different implications for existing home sales versus sales from a developer. Individual sellers generally bought their houses at different times and are thus subject to different initial purchase prices, even after controlling for quality. That is, there is greater heterogeneity in repeat sellers' reference values. As a result, we should expect greater price dispersion for repeat sellers. However, for developers, this may not be true. First, a new house is typically sold by real estate developers, instead of individual households, and prospect theory is more relevant for an individual decision-making process. Second, even under the assumption that real estate developers follow the same decision-making process as individual sellers, we should still expect less price dispersion from these sellers because developers should face very similar costs in terms of construction materials, financing, labor, and so forth, which represents possibly similar reference values for developers. However, conditional on a common v_i, they will also have the same asking price. In this case, our model implies a much weaker price dispersion effect. Accordingly, to test for a price dispersion effect, we calculate two variances for the hedonic residuals in each quarter. One is for the repeat sellers, and the other is for new home transactions, under the assumption that these homes were sold by developers. We also include a dummy variable that is equal to 1 if transactions occurred after 2007. Finally, we drop the observations that have tenure durations of less than one year to rule out potential home flippers. Table G.1 reports the regression results for both groups. The standard errors are calculated via the Biased-Reduced Linearization (BRL) method (Bell and McCaffrey [2002]), which is robust to both heteroskedasticity and small sample size.

Table G.1: Price-dispersion Effect
Dependent Variable: Variance of Hedonic Residuals in Each Quarter

Variables	Individual Sellers	Developers
House Price Index	-0.0002**	-0.0003
	(0.0001)	(0.0003)
Post-2007	0.0629***	-0.0104
	(0.0171)	(0.0159)
Constant	0.0970***	0.1113***
	(0.0833)	(0.0503)
Observations	80	80
R^2	0.4085	0.1384

1) * Significant at the 0.10 level. ** Significant at the 0.05 level. *** Significant at the 0.01 level.
2) Biased-Reduced Linearization (BRL) corrected standard errors are reported.
3) Variance is generated for quarters have at least 10 transactions from the corresponding group.

Consistent with our theoretical prediction, the coefficient on the house price index from the full model for individuals is significantly negative at 5 percent, which implies that when the market price increases, market noise (as measured by the variance in the hedonic residuals) decreases. Moreover, as expected, we observe no significant dispersion effect for developers. The R^2 in the developer regression is also much lower than the case for individual sellers, indicating a much less systematic pattern of price dispersion. Taking them together, the overall finding suggests that for developers, the dispersion of the realized price is much less sensitive to the level of the expected market price, due to the lack of reference value heterogeneity among developers. Concerning the post-2007 dummy, the coefficient is positive and significant at the 1 percent level in the individual sellers' sample and insignificant for the developers' sample. Hence, the data imply that after the year 2007, average price dispersion increased significantly. This is not surprising given that many home sellers are subject to losses, which tends to increase the mark-up when they sell.

Appendix H

On Potential Heterogeneity in Risk Preferences

We have thus far documented a strong connection between our prospect theory-driven model predictions and empirical findings. However, a closer comparison of the overall curvature between the theoretical pricing curve (i.e., Figure 3.3) and the empirical one (i.e., Figure 3.9) still reveals some notable differences, especially when "Loss" and "Gain" are large. For example, home sellers in large loss positions seem to mark-up their asking price at a rate faster than as predicted by a static prospect theory-based model. As the incremental (marginal) effect may be affected by α, one potential implication is that α might not be constant across all loss/gain positions.

Note that we are not the first in the literature to suggest a possibly varying α. Von Gaudecker et al. [2011] estimate the distribution of risk preferences among their 1,422 Dutch experimental respondents. They find that risk preferences in their population tend to be heterogeneous, ranging from risk averse to risk seeking, and only a small portion could be explained by socioeconomic factors, such as age, gender, income, and education. In a non-experimental study, Yao and Li [2013] investigate data used by Odean [1998] that include trading records for 10,000 accounts at a large discount brokerage house. The authors argue that a smaller α in the gain region compared to the loss region can better explain the observed disposition effect. Quantitatively, Abdellaoui et al. [2008] estimate that α is approximately 0.86 in the gain area and 1.06 in the loss area.

To assess whether a changing α can potentially help explain the empirical discrepancy, we perform another calibration in which we allow α to differ depending on the different regions of the loss/gain positions in the prospect value function. To model it, we adopt a smoothly double power law transformation approach. Mathematically, the classic prospect value function (i.e., constant α) has a power law form (x^{α}). Often, when we expect some varying curvature, a double power law-with two different exponents, α_1 and α_2, performs better. For example, to describe the distribution of income, Reed [2003] suggests that, compared with a single power law, a double power law function generates a good fit of upper and lower tails over the whole range of the data. Barro and Jin [2011] also uses a double power law function to match the size distribution of macroeconomic disasters such as on consumer expenditure and GDP shocks. As our maximization problem involves a continuous first order derivative on value function $W(r, v)$, we use a smoothly double power law function separately in the gain and loss region to ensure the continuity of the first order derivative. With exponents of α_1 and α_2 under the gain area and α_3 and α_4 under the loss area, a smoothly double power law transformation takes the following form:

$$W(r,v) = \begin{cases} S\left[(\frac{r-v}{\tau})^{-\alpha_1} + (\frac{r-v}{\tau})^{-\alpha_2}\right]^{-1} & \text{if } r-v>0, \\ -\lambda S\left[(\frac{v-r}{\tau})^{-\alpha_3} + (\frac{v-r}{\tau})^{-\alpha_4}\right]^{-1} & \text{if } r-v<0 \end{cases} \tag{H.1}$$

where τ reflects the transition point on changing curvature, and S is a scale factor in the transformation. A nice feature of this smoothly double power law transformation is that it embeds the classic prospect value function as a special case. For example, with the scale factor $S = 2\tau^{(\alpha_1+\alpha_2+\alpha_3+\alpha_4)/4}$, the classic (single α) prospect value function in Equation (3.10) is equivalent to Equation (H.1) with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$.

This class of smoothly double power-law function is widely used in natural science research. For example, Beuermann et al. [1999] and Uemura et al. [2003] use this form to fit the gamma-ray burst light curve because the curve has two stages and the transition between the two stages is smooth. Aguilar et al. [2015] use a smoothly double power law and show that it describes their proton flux data from the International Space Station well.

Table H.1: Varying α Calibration

Area	Gain: (0.25, .)	Gain: (0, 0.25)	Loss: (0, 0.25)	Loss: (0.25, .)
α	0.715***	0.980	0.910*	0.875**
	(0.119)	(0.103)	(0.076)	(0.074)

1) For consistency, we assume that λ is 1.9667 and c is 0.08 to match our single α calibration in section 3.

2) Here we use a smoothly double power law function with a transition point at $\tau = 0.25$.

3) Null hypotheses are based on $H_0 : \alpha_i \geq 1$.

4) * Significant at the 0.1 level. ** Significant at the 0.05 level.

5) Standard errors are constructed from the bootstrap procedure.

In Table H.1, we run our calibration by segmenting the loss/gain domain into four regions and allow each region to have a different value for α. We set τ to be 0.25, and S is defined as above. The calibrated values of α exhibit an inverse V-shape pattern, with lower values of α on the larger gain/loss, although the variation is much bigger in the gain range. The interpretations of α_1, α_2, α_3, and α_4 are as following[1]: with a potential gain larger than 0.25, the marginal sensitivity coefficient is closer to α_1 of 0.715. When it is smaller than 0.25,

[1] As suggested by Stanek et al. [1999], the mathematical intuition could be explained as following: The function (H.1) describes a power law $(r - v)^{\alpha_1}$ curve for large gainers ($r - v \gg \tau$) and another power-law

the marginal sensitivity coefficient is closer to α_2 of 0.98. Similar interpretations can also be applied in the loss region. Our result differs from Abdellaoui et al. [2008] in that as the reference value decreases, we find losers become slightly less risk seeking, but not risk averse as suggested by their study. We also run 1,000 bootstrap replications to generate standard errors. The results show that α_1 and is significantly smaller than 1 at the 1% level,α_4, and is significantly smaller than 1 at the 5% level, and α_3 is significantly smaller than 1 at the 10% level.

As expected, when we allow α to change, we can achieve a better match between the theoretical and empirical pricing curves. The MSE is reduced by over 10% when we relax the constant α assumption. Thus, jointly there appears to be evidence that the marginal diminishing effect, a key component of prospect theory, could differ depending on an agent's loss/gain position. Moreover, our multi-α calibration implies that while people tend to be risk averse in the gain domain and risk seeking in the loss domain, it becomes more so when the magnitude of gain (or loss) becomes larger.

The closest lab study that supports our findings on varying α is by Bouchouicha and Vieider [2017]. By experimenting with different outcomes on financial rewards, the authors notice that the respondents are more risk averse when gain size is larger. Our empirical results are qualitatively consistent with Bouchouicha and Vieider [2017]. For example, in the gain range, we reject the null hypothesis that $\alpha_1 \geq \alpha_2$ in the one-way test at the 5% level. Meanwhile, Bouchouicha and Vieider [2017] do not find any significant changes in risk-seeking preference with the magnitude of losses. Within the loss range, we also fail to reject the null hypothesis that $\alpha_3 = \alpha_4$ at the 5% level. Another important study is Wang, Yan, and Yu [2017]. By investigating risk-return trade-off in the stock market, the authors find that the risk aversion effect seems to be more significant among investors experiencing larger capital gains while the risk seeking effect seems to be more significant among those who experienced large capital losses.

$(r-v)^{\alpha_2}$ curve for small gainers $(0 < r - v \ll \tau)$. Also, there is a power-law $(v-r)^{\alpha_3}$ curve for small losers $(0 < v - r \ll \tau)$ and another power-law $(r-v)^{\alpha_4}$ curve for large losers $(v - r \gg \tau)$.

Appendix I

Supplemental Tables

Table I1: Hedonic Regression:
Dependent Variable: Log of Transaction Price

age	-0.0059*** (0.0002)
age^2	0.0000198*** (0.00000288)
age^3	-1.94e-08*** (6.44e-09)
$age >= 120$	0.3498*** (0.05035)
Baths_Full_	0.1665*** (0.006219)
Baths_Half_	0.0807*** (0.0037)
Bedrooms_	0.0532*** (0.0044)
Fireplaces_Number_	0.0890*** (0.0031)
Square_Feet_Approx_	0.00014*** (0.000014)
Stories_Number_	-0.0079*** (0.0027)
Cool_CENT	0.1261*** (0.0025)
Cool_WIN	0.0807*** (0.0048)
Cool_OTHER	0.1917*** (0.0509)
Waterdummy	0.1233*** (0.0042)
ATTdummy	-0.3015*** (0.0039)
PoolDummy	0.0773*** (0.0032)
SewerDummy	-0.0633*** (0.0055)
Heater_EleDummy	-0.0187*** (0.0019)
WaterCityDummy	-0.0243*** (0.0058)
Parkingscale= 1	0.0648*** (0.0030)
Parkingscale= 2	0.1434*** (0.0027)

Parkingscale= 3	0.1652***
	(0.0131)
Viewscale= 1	0.0336***
	(0.0019)
Viewscale= 2	0.1373***
	(0.0070)
Viewscale= 3	0.1497***
	(0.0044)
Floorscale= 2	0.0078
	(0.0160)
Floorscale= 3	0.1355***
	(0.0098)
Floorscale= 4	0.2554***
	(0.0101)
Floorscale= 5	0.2915***
	(0.0135)
Floorscale= 6	0.1692***
	(0.0113)
Floorscale= 7	0.2792***
	(0.0143)
Constant	10.2605***
	(0.0508)
Neighborhood Fixed Effect	Yes
Quarterly Fixed Effect	Yes
Observations	155,933

1) * Significant at the 0.10 level. ** Significant at the 0.05 level. *** Significant at the 0.01 level.
2) Robust Standard errors are in parentheses
3) Cool: Cent = Central air conditioning; WIN = Window unit
4) Floor: 1=worst; 7=best

Table I2: Cox Proportional Hazard Regression

Independent Variables	Coefficients
age_list	0.0019***
	(0.0005)
age>= 120	-0.6817*
	(0.3560)
Baths_Full_	0.0566***
	(0.0181)
Baths_Half_	0.0562***
	(0.0169)
Bedrooms_	0.0141
	(0.0142)
Fireplaces_Number_	0.0704***
	(0.0108)
Square_Feet_Approx_	-0.00058***
	(0.00002)
Stories_Number_	0.1123***
	(0.0192)
Cool_CENT	-0.0526***
	(0.0202)
Cool_WIN	-0.1154***
	(0.0369)
Cool_OTHER	-42.1492
	(.)
Waterdummy	-0.0882**
	(0.0377)

ATTdummy	0.1928***
	(0.0236)
PoolDummy	-0.0452
	(0.0344)
SewerDummy	0.0812
	(0.0554)
Heater_EleDummy	0.0286*
	(0.0164)
WaterCityDummy	0.2440***
	(0.0595)
Parkingscale== 1	-0.0708***
	(0.0241)
Parkingscale== 2	-0.0803***
	(0.0232)
Parkingscale== 3	-0.7980*
	(0.4124)
Viewscale== 1	0.0424 *
	(0.0225)
Viewscale== 2	0.3037***
	(0.0854)
Viewscale== 3	-0.0427
	(0.0413)
Floorscale== 2	0.4016***
	(0.1077)
Floorscale== 3	0.1659***
	(0.0589)
Floorscale== 4	0.0698
	(0.0599)
Floorscale== 5	0.1397
	(0.1187)
Floorscale== 6	0.2083**
	(0.0832)
Floorscale== 7	0.1998 *
	(0.1182)
Neighborhood Fixed Effect	Yes
Observations	147,055

1) * Significant at the 0.10 level. ** Significant at the 0.05 level. *** Significant at the 0.01 level.
2) Robust Standard errors are in parentheses
3) Cool: Cent = Central air conditioning; WIN = Window unit.
4) Floor: 1=worst; 7=best

Vita

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Education:

Ph.D. in Finance	June 2020 (Expected)	Old Dominion University
M.B.A	2015	Old Dominion University
B.S. in Physics	2011	University of Science & Technology of China

Selected Papers:

1. Zhaohui Li, Qiang Li, Hua Sun, Sun Li, Diffused Effort and Real Estate Brokerage
Revise and resubmit at the 3rd round at *Real Estate Economics*.

2. Zhaohui Li and Hua Sun, Is Tiger Woods Loss Averse? Comment. Working paper.

3. Zhaohui Li, Michael J. Seiler and Hua Sun, Prospect Theory and a Two-way Disposition Effect: Theory and Evidence from the Housing Market. Working paper.

This paper has been presented in:
ARUAEA National Conference in Washington DC, June 2017
The Asian Real Estate Society (AsRES) in Taichung, Taiwan, July 2017
AREUEA-ASSA Conference, Philadelphia, PA, Jan 2018

Teaching Experience:

2016 Fall: BUSN110, Intro to Contemporary Business (Online Course)
2017 Fall: FIN323, Intro to Financial Management
2018 Fall: ECON200S, Basic Economics
2018 Fall: ECON201S, Principles of Macroeconomics
2019 Spring: FIN319, Principles of Real Estate
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Professional Services:

The anonymous referee of the Journal of Real Estate Research for three papers
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Programming: STATA, Python, Mathematica, SAS, EViews, C language
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Office Productivity: LaTeX, MS Office (Word, Excel, PowerPoint)