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ANALYSIS AND OPTIMIZATION OF DYNAMIC SPECTRUM SHARING FOR COGNITIVE RADIO NETWORKS

by

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ABSTRACT

ANALYSIS AND OPTIMIZATION OF DYNAMIC SPECTRUM SHARING FOR COGNITIVE RADIO NETWORKS

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Old Dominion University, 2010
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The goal of this dissertation is to present the analysis and optimization of dynamic spectrum sharing for cognitive radio networks (CRNs). Spectrum scarcity is a well known problem at present. In order to deal with this problem, dynamic spectrum sharing (DSS) was proposed. DSS is a technique where cognitive radio networks dynamically and opportunistically share the channels with primary users. The major contribution of this dissertation is in analyzing the problem of dynamic spectrum sharing under different scenarios and developing optimal solutions for these scenarios. In the first scenario, a contention based dynamic spectrum sharing model is considered and its throughput analysis is presented. One of the applications of this throughput analysis is in finding the optimal number of secondary users in such a scenario. The problem is studied for fixed and random allocation of channels to primary users while secondary users try to opportunistically use these channels. Primary users contend for the channels, and secondary users try to use the channels only when primary users are not using it. These secondary users themselves contend for the opportunistic usage. The numerical formulas developed for finding the optimal number of secondary users have been carefully analyzed with the solutions obtained using the throughput model directly and finding the optimal number of secondary users. These two match very closely with each other and hence provide simple numerical formulas to calculate the optimal number. The second scenario studied is based upon the idea of pre-knowledge of primary user activity. For instance, the active broadcasting periods of TV channels can be obtained from past measurements as the TV channels activities are approximately fixed. In this scenario, time spectrum block (TSB) allocation for DSS is studied. Optimal TSB allocation is considered to minimize the total interference of the system and hence maximize the overall throughput of the system of community networks. The results obtained using the proposed ABCD algorithm follow very closely with the optimal results. Thus the simple algorithm developed can be used for time spectrum block allocation in practical scenarios.

This work is dedicated:

To the infinite consciousness that creates all that is, the silent observer within us that animates our existence, and the beautiful nature that provides us everything in abundance.

To my mother, Mrs. Saroj Wadhwa, who made me understand the importance of being persistent and calm at all times.

To my father, Dr. Bhim Sain Wadhwa, who instilled in me the pursuit of science and technology, and the importance of honesty and consistent effort.

To my sister, Deepti Wadhwa Makkar, my source of great love and affection.

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CHAPTER I

INTRODUCTION

The work proposed in this dissertation, as the title suggests, deals with the analysis and optimization of dynamic spectrum sharing in cognitive radio networks (CRNs). In order to develop a thorough understanding of the work presented in this dissertation, it is important to understand the concepts related to the work ahead. The two major concepts taken into consideration in this work are, (i) *Cognitive Radio Networks*, and (ii) *Dynamic Spectrum Sharing*. This chapter introduces these basic concepts and related ideas.

The radio spectrum is one of the most expensive natural resources that is highly regulated. Billions of dollars are spent by companies these days to buy a small chunk of spectrum band. This indicates as to how much valuable this resource has become. This is because most of the spectrum is already licensed. Through studies done by the FCC and other organizations, it has come to the forefront that a large part of radio spectrum goes unused. These spaces, also called *white spaces*, go unused for a significant amount of time and at various locations. In order to access these white spaces, dynamic spectrum sharing (DSS) was proposed. DSS has become a very important research area, and the reason is, spectrum scarcity caused by licensing. Spectrum bands are the most important assets for technology providers. Without available bands, there is no possibility that any device can communicate. DSS is an effective way to use otherwise unused spectrum bands spatially and temporally available for opportunistic usage. It is thus important to study the challenges faced in the implementation of DSS.

This dissertation follows the style of *IEEE Transactions*.

The dynamic spectrum sharing has been made possible by recent advances in *cognitive radio* technology [1, 2]. Cognitive radios can dynamically sense the primary user activity in a wide range of spectrum and tune to an unused band in real time. It must be noted that the terms *dynamic spectrum sharing* and *cognitive radio* are independent of each other. Cognitive radio is a broad paradigm that promises to change the world of machines by bringing cognition in machines. Dynamic spectrum sharing, on the other hand, is an application of cognitive radios.

1.1 COGNITIVE RADIOS AND COGNITIVE RADIO NETWORKS

The concepts of software-defined radio and cognitive radio were introduced by Mitola in 1991 and 1998 respectively. Software-defined radio, sometimes shortened to software radio, is generally a multiband radio that supports multiple air interfaces and protocols and is reconfigurable through software run on DSP or general-purpose microprocessors [3]. Cognitive radio, built on a software radio platform, is a context-aware intelligent radio potentially capable of autonomous reconfiguration by learning from and adapting to the communication environment [4].

Before getting into more technical discussion of the terms and technologies presented further in this dissertation, it will be interesting to look at a more fairytale picture of the vision for future cognitive radio networks. Cognitive radios, as the name suggests, represent a whole new world in the field of machines due to one simple reason, the introduction of cognition in machines. In near future, it is going to be possible for machines to actually think. The parallels of these thinking machines can be seen in the science fiction movie,

“The Matrix”. The time is not far when such thinking machines will be developed that can perform various tasks on their own similar to humans. It is thus obvious that the one link that is missing in machines, and which is not allowing them to perform tasks on their own, is cognition. Once cognition enters the world of machines, it will open a whole new world of opportunities and technologies. The dream world of “The Matrix” can be seen projected into the real world in the near future. Research and technology are on the verge of developing powerful tools that will be performing at the behest of humans. It is again upon us as to how wisely we handle this world of thinking machines.

I.1.1 Cognitive Radios

Very much similar to the way humans interact with their surrounding environment, cognitive radios are expected to perform thinking based on the input from the environment in which such radios work. Thus cognitive radios are aware of the environment in which they work, and based on that they learn and, hence, perform cognition. As [5] states, “If a radio were smart, it could learn services available in locally accessible wireless computer networks, and could interact with those networks in their preferred protocols, so you would have no confusion in finding the right wireless network for a video download or a printout. Additionally, it could use frequencies and choose waveforms that minimize and avoid interference with existing radio communication systems. It might be like having a friend in everything that’s important to your daily life, or like you were a movie director with hundreds of specialists running around to help you with each task, or like you were an executive with hundred assistants to find documents, summarize them into reports, and then synopsise the reports into an integrated picture.”

The following architecture components of a cognitive radio can be considered, though various such architectures are possible [6].

- Cognition functions: These provide monitoring and structuring knowledge of behavior patterns. The patterns under consideration are of the Self, the User and the Environment. This is the major component that provides information regarding experiential learning.
- Adaptation functions: These deal with adapting or responding to the variations in the environment.
- Awareness functions: Sensor domain information is fed to the radio and thus provides an awareness of the environment. This functioning is important in order to adapt to the environment.
- Perception functions: These functions continuously identify and track knowns, unknowns, and backgrounds in a given sensor domain.
- Sensory functions: In these domains fall anything that can be sensed, such as audio, video, time, temperature, ambient light level, sun angle, smell, barometric pressure, power, and anything imaginable.

Below is presented the cognition cycle based on which the cognitive radios act. As shown in Fig. 1, below are presented the phases of the cognition cycle through which the cognitive radio passes [6, 7]. The cognition cycle shown in Fig. 1 is simplified from [7].

- Observe Phase: In this phase, the cognitive radio is involved in perceiving the environment in many dimensions at the same time. It binds these stimuli together or in

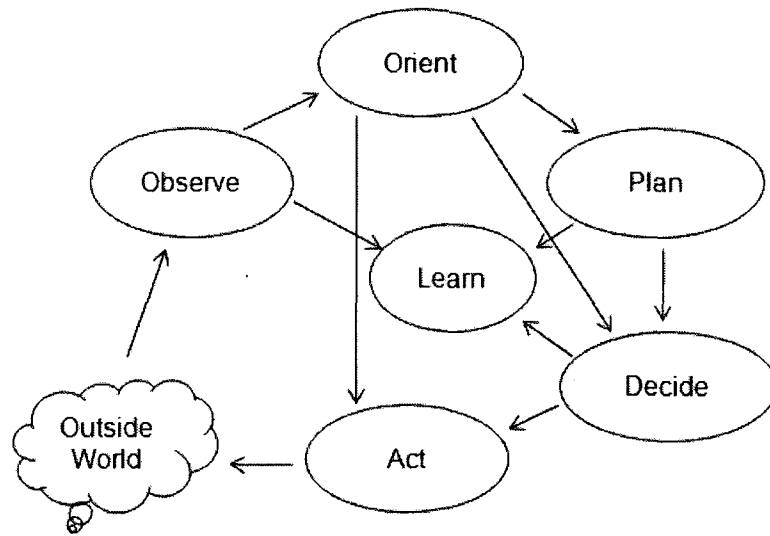


Fig. 1: Simplified Cognition Cycle in Cognitive Radios.

subsets to prior experiences. Based on these observations, cognitive radios generate time-sensitive stimuli and also generate plans for action.

- **Orient Phase:** In this phase, the previously known set of stimuli of a scene are matched with the current observation. Thus the matching is achieved based on stimulus recognition or by “binding”. Very much similar to human cognition, the ability to memorize something comes from observing similar facts over a longer period of time. Similar things happen in the orient phase, in which the external stimuli are fed to the radio, and it tries to form a memory over a long term by observing similar stimuli. Thus the orient phase is similar to what we have its parallel in short term memory in humans. Stimulus recognition is the exact match between prior experiences and observations and the current stimuli. Binding, on the other hand, is a nearly exact match between prior and current experiences.

- **Plan Phase:** In this phase, the stimuli are dealt with in a more planned and well thoughtout manner than responding reactively. The stimuli are deliberately dealt with by generating a plan.
- **Decide Phase:** In this phase, a decision is made among various candidate plans. The radio might either alert the user or defer interrupting the user until later.
- **Act Phase:** This phase brings out the plans and decisions made before this phase and, in doing so, initiates the selected processes. In this phase, either the actions are externally oriented or internally oriented. The actions that are spoken messages or sent as text messages using appropriate language are externally oriented actions. Internally oriented actions are about controlling the machine-controllable resources such as radio channels, etc.
- **Learn Phase:** Similar to human learning, this phase includes the functioning of most of the phases discussed above. Through observations, learning about new states, decisions, and through planning based on these stimuli comes learning.

Other phases reported in texts are self monitoring timing, retrospection, and reaching out. These deal with saving resources by restricting computation time, radio power, and other computational resources saving through sleep and prayer modes, and learning about the possible opportunities that were not resolved in sleep mode during prayer epoch, respectively. Having discussed the main components of CRs and how they learn through various phases, and what functions are they capable of performing, cognitive radio networks and their functions are discussed below.

I.1.2 Cognitive Radio Networks

Before beginning the discussion on cognitive radio networks, it is useful to clearly state the distinction between the terms cognitive radios and cognitive radio networks. Cognitive radio, as considered in the last section, is a node in a network that bases its decisions on cognition cycle, and thus it becomes aware of the environment in which it acts. Based on this awareness, it can make decisions based on the knowledge of environment and then act in a manner to accomplish user objectives. These experiences further develop the learning for future use [2, 5, 6]. A cognitive radio network is a network of nodes with the above mentioned cognitive capabilities [8, 9]. Thus, through awareness of each other, these nodes can combine their resources and expertise to perform as a team [10, 6].

This dissertation explores the possibility of dynamic spectrum sharing using cognitive radio networks [11, 12, 13], that are comprised of primary and secondary users to access the channel. In this dissertation, the work presented in Chapter IV deals with infrastructure-based cognitive radio networks. In the survey paper [14], Akyildiz et al. present the components of primary and CR networks as shown in Fig. 2. It gives the following basic components of primary networks:

- **Primary User:** A primary user has a license to operate in a certain spectrum band.

This access can only be controlled by the primary base station and should not be affected by the operations of any other CR users. Primary users do not need any modifications or additional functions for coexistence with CR base stations and CR users.

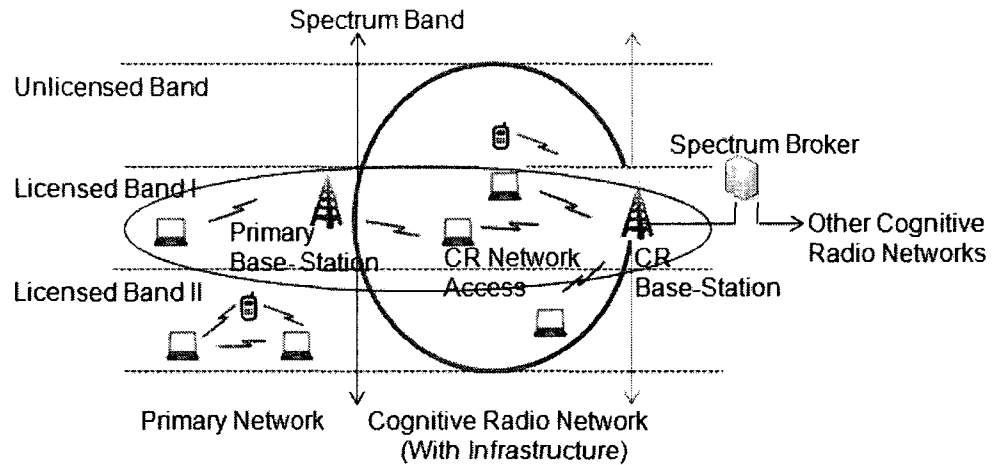


Fig. 2: Infrastructure-Based CR Network Architecture.

- **Primary base station:** A primary base station is a fixed infrastructure network component that has a spectrum license, such as a base station transceiver system (BTS) in a cellular system. In principle, the primary base station does not have any CR capability for sharing spectrum with CR users.

Similarly [14] gives the following basic elements of the CR network:

- **CR user:** A CR user has no spectrum license. Hence, additional functionalities are required to share the licensed spectrum band. In infrastructure-based networks, CR users may be able to only sense a certain portion of the spectrum band through local observations. They do not make a decision on spectrum availability and just report their sensing results to the CR base station.
- **CR base station:** A CR base station is a fixed infrastructure component with CR capabilities. It provides a single-hop connection without spectrum access licenses

to CR users within its transmission range and exerts control over them. Through this connection, a CR user can access other networks. It also helps in synchronizing the sensing operations performed by the different CR users. The observations and analysis performed by the latter are fed to the central CR base station so that the decision on the spectrum availability can be made.

- **Spectrum broker:** A spectrum broker (or scheduling server) is a central network entity that plays a role in sharing the spectrum resources among different CR networks. It is not directly engaged in spectrum sensing. It just manages the spectrum allocation among different networks according to the sensing information collected by each network.

I.2 DYNAMIC SPECTRUM SHARING

The radio frequency has been traditionally allocated by the spectrum management authority to licensed users for exclusive use. This results in two problems. First, the remaining spectrum available for future wireless services is being exhausted, called the *spectrum scarcity* problem. Second, the exclusive usage policy results in low overall utilization of the allocated spectrum [15][pp.9-16].

Authors in [16] present a survey of dynamic spectrum access. Dynamic spectrum access can be broadly categorized under three models [16], the dynamic exclusive use model, the open sharing model and the hierarchical access model, see Table 1.

As [16] presents a survey of the dynamic spectrum access taxonomy, below a brief introduction to the models is presented as follows.

Table 1: A Taxonomy of Dynamic Spectrum Access.

Dynamic Spectrum Access	
I. Dynamic Exclusive Use Model	1. Spectrum Property Rights 2. Dynamic Spectrum Allocation
II. Open Sharing Model (Spectrum Commons Model)	
III. Hierarchical Access Model	1. Spectrum Underlay (Ultra Wide Band) 2. Spectrum Overlay (Opportunistic Spectrum Access)

- **Dynamic Exclusive Use Model:** With this model, the spectrum bands are licensed for exclusive use to the services. As Table 1 shows, there are two approaches proposed under this model. The spectrum property rights approach allows licensees to sell and trade spectrum and to freely choose technology. The dynamic spectrum allocation approach is about allocating the spectrum to services for exclusive use in a given region at a given time [17]. Due to the bursty nature of wireless traffic, these approaches cannot eliminate white space.
- **Open Sharing Model:** In this model, peer users employ open sharing as the basis for managing spectral region. This model has also been referred to as spectrum commons [18, 19].
- **Hierarchical Access Model:** This model is based on a hierarchical structure where primary users are the license holders and secondary users can access the spectrum based on some specified etiquettes. This model thus provides an open access to licensed spectrum for secondary users. This open access to licensed spectrum is further based on two approaches: spectrum underlay and spectrum overlay. Spectrum

underlay is based on the idea that secondary users can communicate all the time along with primary users. This approach imposes severe constraints on the transmission power of secondary users, and thus must be below a specific threshold or noise floor. This approach makes sure that the primary users don't suffer from any interference from secondary users while they communicate. Spectrum overlay, also termed as opportunistic spectrum access in literature, is a technique that deploys the spatial and temporal usage of spectrum white space. Using this technique, secondary users must find such white spaces in time and space, also named as time-spectrum blocks [20] and then opportunistically access those bands in nonintrusive manner.

After a brief introduction of various dynamic spectrum access models, the major thrust of this dissertation will be on the spectrum overlay model, which is also called opportunistic spectrum access. After observing that a lot of spectrum space goes unused most of the time, both the spectrum management authority and the research community recently started to promote sharing the licensed spectrum bands between the primary (licensed) users and the secondary (unlicensed) users, termed *dynamic spectrum sharing* in this dissertation. Under dynamic spectrum sharing, also called *dynamic* or *opportunistic spectrum access* in the literature, primary users have the privilege to use their licensed spectrum band, and secondary users are allowed to access the band only when primary users are not using it. The background and the work related to dynamic spectrum sharing and more specifically to this dissertation will be discussed in Chapter II.

I.3 DISSERTATION STATEMENT

The main contributions of this work are presented as follows. The first contribution is the development of a throughput model for a contention-based dynamic spectrum sharing model. A mathematical model is developed in which is considered a system that is comprised of two types of users, primary users or licensed users of the spectrum, and secondary users or the unlicensed users of the spectrum. These users themselves contend for the channels with their own types, that is, primary vs. primary, secondary vs. secondary and also secondary users can only opportunistically use the channel. The throughput of the system is thus found by considering the throughput of primary and secondary users when secondary users try to opportunistically use the channels and primary users themselves contend with each other.

After developing a general throughput model, an application of this model is presented. Using the throughput model, the optimal number of secondary users that can dynamically share spectrum with primary users, is found. Thus, it provides the answer to the question, “How much dynamic spectrum sharing is optimal?” The answer to this question is provided in terms of finding the optimal number of secondary users that can dynamically access and share the channel with primary users. Two different numerical formulas have been presented as they have been developed using two different approaches.

The problem of channel allocation has been studied in great detail in past, but the problem of time spectrum block allocation is new to the research community. A simple way to attack the problem of TSB assignment is presented. The problem of TSB assignment is analyzed for community cognitive radio networks. Community cognitive networks are

like cellular networks, or WRAN networks, and aim to provide a wide coverage in terms of area and services. This dissertation presents an important practical step towards developing such TSB allocation algorithms that provide simple solutions for such networks.

A simple algorithm, which is named, **ABCD** (Allocation of time spectrum **B**locks in Cognitive radio networks for **D**ynamic spectrum sharing) algorithm is proposed, and using this algorithm, a near optimal solution to the problem of TSB assignment is presented. Comparing the algorithm results with optimal results, it is deduced that the algorithm provides near optimal results in all the cases. The complexity of the algorithm is also low and hence can be easily implemented in real time practical scenarios.

I.4 DISSERTATION OUTLINE

The remainder of this dissertation is organized as follows. Chapter II discusses the background of dynamic spectrum sharing and cognitive radio networks and discusses the protocols and algorithms related to cognitive radio networks. This Chapter also presents related work in the mentioned fields of research as well as their unique contributions and limitations. Chapter III presents the first contribution of this dissertation, the throughput analysis for a contention based dynamic spectrum sharing model. This Chapter also presents an application of this model in finding the optimal number of secondary users that can safely share the spectrum with primary users. Two different approaches for finding the solution for the optimal number of secondary users are presented. In Chapter IV, resource allocation based throughput optimization is considered. This Chapter deals with allocating time spectrum blocks to access points in community cognitive radio networks. A simple TSB

allocation algorithm, *ABCD* algorithm, is proposed. Chapter V concludes by discussing the contributions of this dissertation and presenting some areas of future work.

CHAPTER II

BACKGROUND AND RELATED WORK

This chapter discusses the background of dynamic spectrum sharing, cognitive radios and cognitive radio networks (CRNs), algorithms and protocols, and resource allocation in cognitive radio networks. The research work closely related to this dissertation, the objectives of this dissertation based on what has been accomplished, and the goal of this dissertation are discussed.

II.1 COGNITIVE RADIO NETWORKS

Cognitive radio (CR) is a new paradigm shift in the world of communications and cognition in machines. Cognitive Radios (CRs) are attracting the research community with every passing day. This is due in to the fact that this technology offers many more research challenges than any other technologies at work. Even if some research challenges on which CRs are being developed are akin to other technologies, that still forms a very little part of this vast field of research. Opportunistic Spectrum Access (OSA) is one of the most important applications of CRs.

Many researchers have devoted themselves religiously to exploring various tasks that cognitive radios can perform and their capabilities and limitations [21, 22, 23, 24, 25]. As J.Mitola et al. state in [21], “The outside world provides stimuli. Cognitive radio parses these stimuli to extract the available contextual cues necessary for the performance of its assigned tasks.” In [23], the authors suggest some input from primary users to aid cognitive

radios perform well. They suggest that if primary users transmit a pilot signal, the noise detection uncertainty can be overcome. Since even small noise uncertainty causes serious limits in detection, if the primary users send pilot signals it would aid towards dynamic spectrum sharing by cognitive radios. This kind of implementation is practically challenging. In their efforts for a peaceful coexistence between primary and secondary users, researchers are developing models, suggesting hardware implementations to accomplish that, and also simulating various scenarios. In [24] the authors study the largest rate at which the cognitive radio can reliably communicate under the constraint that no interference is created for the primary user, and the primary encoder-decoder pair is oblivious to the presence of the cognitive radio. The authors [25] have designed a biologically inspired cognitive engine with dynamic spectrum access (DSA) as one of its intended applications. Through software simulation they showed that using cognitive techniques, a 20 dB SINR improvement is achieved in an interference environment over that provided by current IEEE 802.11a service PHY standard. In [26], the authors present an interesting survey of vertical spectrum sharing in a cognitive radio network. They outline the recent information theoretic advances pertaining to the limits of such networks.

The two major diversifications of cognitive radio networks (CRNs) (and based on that, of the research communities) are the *infrastructure based* (or centralized) and *non-infrastructure based* (or decentralized or ad hoc) CRNs. In Chapter IV of the dissertation, infrastructure based (or centralized) CRNs are considered. A brief introduction of infrastructure based cognitive radio networks was presented in Chapter I. In the following sections the significant works in the fields of CRNs and DSS are discussed that provide an insight into the work that has already been done, and how it relates to the work in this dissertation.

II.2 DYNAMIC SPECTRUM SHARING

Dynamic spectrum sharing, as discussed in Chapter I, is the sharing of spectrum by secondary users that is otherwise licensed to primary users, under some predefined etiquettes. Dynamic spectrum access strategies can be broadly categorized under three models, the dynamic exclusive use model, the open sharing model, and the hierarchical access model [16]. In this dissertation, the overlay approach of hierarchical access model, also called the opportunistic spectrum access is considered. Dynamic spectrum sharing is a technique in which the spectrum, which is otherwise licensed to primary users, can be shared with another set of users called secondary users. The etiquettes that secondary users must follow are based on the criterion that primary users hold the monopoly over all the bands licensed to them. Thus when primary users want access to those bands, secondary users must leave those bands at that time.

Dynamic spectrum sharing has become an important issue for a simple reason that the spectrum space is limited, and lots of licensed bands are not in use most of the time. Thus it is wise to opportunistically use such bands. The problem of DSS has been studied from different perspectives and for different bands, e.g. licensed and unlicensed bands. While opportunistic access is being considered as an important technology for accessing licensed bands [27], it is nonetheless being studied for unlicensed bands [28]. At present, the primary users are not strictly defined, and hence only with the advent of this technology on a large scale would it be possible to clearly define primary users. Researchers have tried to study this problem for both licensed and unlicensed band technologies. Various multiple access technologies where primary users themselves contend for channels are

being targeted for theoretical analysis and providing optimal solutions [29]. Chapter III presents a throughput analysis and optimal solution for such kind of technologies where primary users themselves contend with each other.

There have been many research efforts on dynamic spectrum sharing [30, 31, 32, 33, 34, 35, 36]. Strategies have been developed to make decisions based only on the knowledge of the channel bandwidths or data rates as to what channels need to be probed [30]. Opportunistic spectrum access (OSA) implementation for mobile adhoc networks (MANETs) has been studied in [31] in which the author determines the transport capacity of OSA, thus providing insight into network design and topology control. In [32], OSA in cognitive radio networks has been studied to provide closed form analysis of secondary user performance, and to present a tight capacity upper bound. The spectrum sensing may take a periodic approach where the time is slotted and secondary users sense the spectrum at each time slot [33]. The throughput model discussed in Chapter III also assumes slotted time. In the following sections, the various studies are presented for the coexistence of primary and secondary users, 802.22 WRAN and the resource allocation specific to CRNs and DSS, and how they relate to this dissertation.

Before discussing resource allocation, a brief introduction to 802.22 WRAN is presented. Chapter IV considers community networks that use dynamic spectrum sharing for communication. One of the possible forms of such networks is the latest development called 802.22. 802.22 is the IEEE standard for Wireless Regional Area Networks (WRAN). IEEE 802.22 Working Group (WG) formed in November 2004 is the first world wide effort in the direction of defining wireless air interface standard based on Cognitive Radios in the license exempt bands (presently TV spectrum) [37]. TV bands in the US

span from channels 2 to 69 in the VHF and UHF bands of the radio spectrum. These channels are 6MHz wide and are spread over these VHF and UHF bands spanning 54-72 MHz, 76-88 MHz, 174-216 MHz, and 470-806 MHz. In the downstream, each channel of 6MHz would translate to 18 Mbps of data rate, with each base station covering 33 - 100 Km. WRAN is highly researched topic these days as researchers are trying to establish first working model using cognitive radio networks on a large scale.

Various research papers have been published in past that study various topics related to WRAN, for example, sensing, data transmission, coexistence, throughput optimization etc. In [38], the authors study the problem of how to efficiently schedule both channel sensing and data transmission for multiple adjacent WRAN cells. To tackle the problem, four different schemes are presented based on dynamic frequency hopping. The authors in [39] present the design and verification of IEEE 802.22 WRAN physical layer while [40] presents an overview of the IEEE 802.22 WRAN system. They further present an experimental study that was undertaken to characterize IEEE 802.22 WRAN interference limits into Advanced Television Systems Committee (ATSC) based DTV receivers. After discussing the experimental study, they present the implications of WRAN interference limits in terms of the maximum allowable radiated power and out-of-band emission limits that are imposed on WRAN end-user devices. Through this discussion, it is clear that various studies are underway to develop stable community cognitive radio networks that will open doors to bringing the technology to even rural areas. In Chapter IV, the motivation is to provide simple solution through the algorithm proposed for TSB allocation to APs. The idea is simple and implementation of the same in practical scenarios is cost effective.

II.3 RESOURCE ALLOCATION IN COGNITIVE RADIO NETWORKS

Resource allocation is one of the most challenging problems and many ways and methods can be devised in order to tackle with this problem. It depends upon the problem statement and the constraints thereof that determine the type of solution to this problem. Many attempts have been made in the direction of channel, rate, and power allocation in cognitive radio networks. This section thus presents various resource allocation techniques and approaches that various researchers have taken. It must be noted that in Chapter IV, the goal is TSB allocation. It thus becomes important to have an initial understanding of resource allocation related approaches. Even though this dissertation does not concentrate upon power allocation, it is nonetheless important to look at power allocation as one of the effective resource allocation techniques. For ease of assignment, a constant maximum power level for transmission is assumed in this dissertation. In fact in many papers, these resources, power, rate, and channel have been coupled together or considered all together. It must be noted however that considering all the resources together is a highly challenging task and many assumptions are made by various researchers even if they consider them together.

II.3.1 Power, Rate and Channel Allocation

The authors in [41] do channel assignment and power control in base stations (BSs) and customer-premise equipments (CPEs) in infrastructure-based cognitive radio networks with the goal of keeping the interference caused to primary users minimum. They present a game-theoretic model to analyze the non-cooperative behavior of the secondary users in

IEEE 802.22 networks. They conclude that the non-cooperative behavior of the players might result in a small number of supported CPEs and this can be solved by cooperative techniques, such as the Nash bargaining solution, which can significantly increase the efficiency of the opportunistic spectrum allocation. The authors in [42, 43, 44, 45] study the power allocation problem. In [42], the authors investigate the distributed multichannel power allocation problem as a non-cooperative game with coupled constraints to address both the selfish nature of secondary users and the interference temperature constraints imposed by the primary users. The authors in [43] study the problem of joint power control and beamforming in infrastructure based cognitive radios with the objective of minimizing the total transmitted power of the cognitive network such that the interferences at primary users remain below a threshold level; moreover the secondary users are guaranteed, with their signal to interference plus noise ratio (SINR) requirements. A method of power control in cognitive radio systems is based on using spectrum sensing side information as proposed in [44]. It must be noted that channels vary due to fading, and thus in a fading environment the secondary users may take advantage of this fact by opportunistically transmitting with high power when its signal, as received by the licensed primary receiver that is deeply faded [45]. All these approaches consider allocating power to secondary users in a very restrictive sense. These schemes are based on the idea that if secondary users operate with primary users using underlay approach, then their interference must remain below a specific threshold. Even for overlay approach, it is made sure that the power level of secondary users must be under control not to cause harmful interference to primary users in any case.

In order to maximize the total secondary user throughput under interference and noise impairments and maximize the sum rate, the authors in [46] develop a power allocation algorithm for infrastructure based cognitive radio networks. Joint admission control and rate/power allocation schemes have been suggested by [47], where the interference limits at primary receiving points are adapted depending on traffic load of the primary network. Finding channel gains among secondary links and between the secondary nodes and primary receiving points may not be easy to estimate. The authors derive outage probability for SIR constraints and violation probability for interference constraints considering fading dynamics of the wireless channel. It is important as to what assumptions are made about the channel gain information, is it that the secondary users are able to obtain instantaneous channel gains, or in average sense only. The authors in [48] propose a learning based approach which does not require cooperation or coordination and uses feedback information from collisions to assign channels to multiple secondary users in an unslotted primary network. In [49], the authors discuss the idea of a two-phase channel/power allocation scheme in order to avoid any excessive interference to primary users. The authors propose a two-phase resource allocation (TPRA) scheme that improves the system throughput by allocating channels and power to base stations (BSs) with the aim of maximizing their total coverage while keeping the total interference caused to each primary user (PU) below a predefined threshold. Based on the approach taken by [49], the authors in [50] propose a dynamic spectrum sharing model.

In [51], the main objective of the paper is to maximize the multiple cognitive user's weighted rate sum by jointly adjusting their rate, frequency, and power resource under the

constraints of multiple primary users' interference temperatures. The author in [52] suggests two models for interference temperature, the ideal model and the generalized model. Moreover, a question is posed as to whether the interference temperature reflects the unlicensed transceiver or the licensed receiver. These two interpretations are thus based on the criteria as to what is important or more appropriately, available to the researcher in terms of interference temperature related parameters. The authors in [51] develop an analytical framework for both the models discussed in [52]. A lot of this work considers interference temperature based underlay model. The reason is obvious, that is, that the concept of interference temperature and underlay model initially provided many interesting challenges, though lately it was realized that the concept was not workable. It would be almost impossible to track the secondary user offenders who cross interference thresholds. Thus the FCC discarded the idea of using the underlay model independently, though it may be used in combination with the overlay model. Needless to say, the concepts and mathematical models developed by most of these papers are relevant to other approaches and may be used for analyzing other models.

II.3.2 Channel and Time Spectrum Block Allocation

Channel allocation is a well studied problem in many other networking technologies such as cellular technology and WLAN channel assignment, but there are very few studies related to cognitive radio networks. The studies related to cellular networks and cognitive radio networks study this problem using integer programming and graph coloring techniques. In cellular networks, studies have been performed based on fixed channel allocation (FCA) [53, 54] and dynamic channel allocation (DCA) [55, 56, 57]. In [53], Sarkar

et al., present channel allocation algorithms with FCA where they considered the bounds on the offered traffic capacity given by McEliece et al. [58]. In order to satisfy the reuse constraints, Sarkar et al. [54], presented channel assignment based on co-channel and adjacent channel interference. In order to compute the blocking probability, Cimini et al. [56], studied the cellular systems with DCA with an ad hoc Erlang-B approximation for each cell. In the efforts to compute blocking probability, the interference conditions on different channels were not taken into consideration. In order to consider the impact of interference, Anand et al. [57], present a two-dimensional Markov chain model. In this paper, the authors studied linear and two dimensional circular cellular systems.

Co-channel and adjacent channel interference are two major factors that affect channel assignment. In cellular networks, the networks were studied taking into consideration these interference conditions. Cognitive radio networks are similar in their approach, but it is far more challenging with cognitive radio networks as the availability of the spectrum and the reuse constraints vary dynamically. The dynamic variation is caused by arrival and departure of primary users. Thus, when secondary users are planning to jump on a channel and opportunistically use it, a primary user may show up and secondary users have to move to another channel. It is a much more complex channel assignment problem comparatively. Thus, resource allocation is a challenging task in cognitive radio networks. Channel assignment using graph-theoretic approaches and integer programming approaches has been studied by researchers recently in last few years.

Time spectrum block (TSB) allocation is another way of looking at the problem of channel allocation. The concept of TSB was introduced by Yuan et al. [20] to model

spectrum reservation and to use the TSBs for theoretical formalization of spectrum allocation problem. In this dissertation, the concept of TSBs is used in Chapter IV, where the opportunities for secondary users are considered in the form of TSBs on different frequency bands. These TSBs are contiguous or discontinuous bands available for usage by secondary users during specific intervals of time. Considering TV spectrum bands for opportunistic usage, the availability of the white spaces in these bands is temporal and depends highly on the geographical location of the radio [59]. In [59], the authors develop a model called KNOWS (Kognitive Networking Over White Spaces). In this model they presented a hardware-software platform that includes a software-aware Medium Access Control (MAC) protocol and algorithms to deal with spectrum fragmentation. These protocol and algorithms, including the hardware implementation, deal with some of the challenging issues related to finding the unused portions of spectrum. It is thus important to dynamically assign these bands to the users. Chapter IV presents an algorithm that dynamically assigns time spectrum blocks to the users for each time interval. It provides a simple but very effective way to allocate time spectrum blocks to users.

There have been several studies on capacity limits, network bounds, and optimal solutions in dynamic spectrum sharing, e.g., see [60, 61]. The authors in [60] present the capacity limits for distributed cognitive radios based on two switch models to capture localized spectral activity and found that the correlation of the local spectral environment at transmitter and receiver predominantly determines the capacity. In [61], the authors consider the dynamic sharing of a set of channels between primary users and secondary users, with one primary user in each channel, and aim to find the optimal number of secondary users that can share spectrum with primary users. In this Chapter, the limitation of the

network model in [61] that each channel has a fixed primary user is relaxed. Specifically, in the scheme discussed in this Chapter, an arbitrary number of primary users are allowed to fairly contend for each channel, and each primary user can either be fixedly allocated to a channel, or dynamically selects a channel. Secondary users contend for a channel without primary user activity, which means that either there is no primary user in this channel or all primary users in this channel do not currently transmit data. An analytical model is derived to compute throughputs for both primary users and secondary users. Then, using the throughput analysis model, this chapter answers the question: what is the optimal number of secondary users that can share the spectrum with primary users when primary users themselves dynamically contend for each channel?

II.3.3 Resource Allocation Algorithms and Protocols

Many researchers have focused on developing protocols and algorithms to optimize the performance of dynamic spectrum sharing. In [34] the authors propose a primary-prioritized Markov approach for dynamic spectrum access. The interaction between primary users and secondary users is modeled as continuous-time Markov chains optimizing the access probabilities and thus efficiently and fairly sharing the spectrum resources. The authors in [35] develop a distributed algorithm that iteratively increases data rates for user communication sessions. The work in [62, 36] develops MAC protocols for secondary users in dynamic spectrum access. The protocol designed in [36] exploits the MAC protocol design for wireless networks and introduces C-MAC that operates over multiple channels and uses a slotted approach beacon period where nodes exchange information and

negotiate channel usage. The authors in [62] propose cognitive MAC protocols that optimize the performance of secondary users while limiting the interference perceived by primary users. The reactive cognitive radio algorithms have been used in unlicensed bands for coexistence between 802.11b and 802.16a networks [63]. These reactive coordination methods are used to reduce the mutual interference and improve link throughput.

Cognitive Radios are much more complex than other conventional radios. The reason is simple, cognitive radios possess the ability to adapt and think accordingly. This is one important factor that separates them from conventional radios. There are many important components that a MAC (Medium Access Control) protocol must possess. Envisioning future cognitive radios, MAC protocols must have the capability of decision making. The CRs, as the name suggests, must have intelligent decision algorithms. In [64], the authors investigate the characteristic features, advantages, and the limiting factors of existing CR-MAC protocols for both infrastructure-based and ad hoc networks. In order to efficiently utilize the spectrum, the authors in [65] propose a class of carrier sense multiple access (CSMA) based MAC protocols for the CRN while the primary system (PS) is also operating with widely-applied carrier sensing protocols. This approach of protocol designing is based on the idea that primary and secondary systems must be able to simultaneously communicate in the same channel. In [66], the authors design a cognitive radio that can coexist with multiple parallel WLAN channels abiding by interference constraint. The cognitive Medium Access (CMA) protocol that enhances WLAN coexistence based on sensing and prediction is derived from the above model. This CMA is based on a stochastic model for WLAN's packet transmissions. The authors in [67] investigate how to support user communication sessions by jointly performing power control, scheduling, and flow control for

an SDR-based multi-hop wireless network. Thus, power control has been considered as a part of the optimization space. The authors further develop a distributed optimization algorithm for multi-hop cognitive radio networks [68], using the protocol model for interference modeling as developed in [67]. The authors consider how to design distributed algorithm for future CR networks. The next section presents a brief background and motivation behind this dissertation.

II.4 DISSERTATION BACKGROUND AND MOTIVATION

This dissertation derived its motivation from the latest development in the field of wireless communications, that is, cognitive radio networks. This technology has the potential to completely change the way we look at technology at present. Within the sphere of this vast field of study, this dissertation concentrates upon one of the most relevant topics, and the present day challenges in front of the research community, the dynamic spectrum access. Due to spectrum scarcity, this problem is an important challenge for the research community, and hence this dissertation attacks some very relevant and important issues in this field. In dynamic spectrum sharing, there are various fields of study. The thrust of this dissertation is on the analysis and optimization of dynamic spectrum sharing. This is also due to the simple reason that optimization studies are at the heart of improving the efficiency and overall performance of a system. There are various optimization studies and related works as presented in the previous section, but this dissertation fills a very important void. This dissertation presents theoretical studies for throughput model, optimal solutions for spectrum sharing, and algorithms for dynamic spectrum sharing. One of the concepts used

in this dissertation is the time spectrum block allocation. This concept is newly introduced to the research community, and this dissertation presents simple allocation algorithm to attack the problem of time spectrum block allocation. Thus, this dissertation provides some very effective and important results that can be used for theoretical studies and practical implementation.

CHAPTER III

CONTENTION-BASED DYNAMIC SPECTRUM SHARING MODEL

This chapter studies the dynamic spectrum sharing problem under a contention-based scheme. The limitation of the network model in [61] that each channel has a fixed primary user is relaxed. Specifically, in the proposed scheme, an arbitrary number of primary users are allowed to fairly contend for each channel, and each primary user can either be fixedly allocated to a channel, or dynamically selects a channel. Secondary users contend for a channel without primary user activity, which means that either there is no primary user in this channel or all primary users in this channel are not currently transmitting data. An analytical model is derived to compute throughputs for both primary users and secondary users. Then, using the throughput analysis model, it answers the question: what is the optimal number of secondary users that can share spectrum with primary users when primary users themselves dynamically contend for each channel? This is closely related to how wisely the precious spectrum resource can be utilized, as a number below this optimal number will leave more wasted spectrum and a number above this will increase the chance of collision.

III.1 NETWORK MODEL AND DYNAMIC SPECTRUM SHARING

A network with both primary users and secondary users coexisting is considered. Each user may be treated as a communication session between a pair of nodes or among a set of nodes. Also, it is assumed that users are independent from each other, which is a general

assumption in most existing studies of dynamic spectrum access (e.g., see [30, 31, 32, 33, 34, 35]). The time is slotted¹. In each time slot, secondary users are able to dynamically detect the channels that are not used by primary users, termed *accessible channels* for secondary user. Furthermore, in each time slot, every primary/secondary user generates data to transmit with a probability p ². To make the model tractable, the traffic generation of primary/secondary users is assumed *independently and identically distributed* (i.i.d).

Denote the number of primary users and secondary users as N and \tilde{N} , respectively. The licensed spectrum of primary users are partitioned into M channels. The N primary users are allocated into the M channels, and the primary users in the same channel compete for this channel. The \tilde{N} secondary users conduct channel sensing before actually using the channel. In this Chapter, ideal channel sensing is assumed and hence there are no errors occurring due to channel sensing. Also, it is assumed that there is no delay involved in leaving or accessing the channel as soon as the channel is detected as busy or idle. The secondary users use Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) as used in 802.11 based Wireless LANs, in order to compete for idle channels. If secondary users sense the channel as busy, then the transmission is deferred for a random time. As in 802.11, secondary users may also use Request-To-Send (RTS) and Clear-To-Send (CTS), asking other secondary users to keep quiet for the duration of the main packet, further reducing the possibility of collision. The sensing time is assumed to be significantly smaller than the slot time [61]. With this assumption, the time spent in sensing can be neglected

¹Although the slotted time is not applicable to all primary users, this work can still give insights to the dynamic spectrum sharing under the scenario with non-slotted primary users, particularly for the relationship between the primary user traffic load/throughput and the optimal number of secondary users.

²This model can easily be extended to the scenario that the primary users and secondary users have different traffic transmission probabilities.

and it can be easily assumed that nearly all the slot time contributes towards throughput. Thus any overhead caused by the sensing time is neglected.

Two scenarios of allocating primary users into each channel are considered: fixed allocation and random allocation. In fixed allocation, the number of primary users allocated to a channel is fixed all the time, but the number of users in different channels may be different. In random allocation, each primary user dynamically and randomly selects a channel in each time slot. Let X_i ($1 \leq i \leq M, X_i \geq 0$) denote the number of primary users allocated to channel i . Then in the fixed allocation scheme, X_i is a constant over all time slots, and in the random allocation scheme, X_i is a random variable and changes in each time slot. A special case of fixed allocation is also considered, named “even allocation”, where X_i is equal to either $\lceil \frac{N}{M} \rceil$ or $\lfloor \frac{N}{M} \rfloor$ all the time. Clearly, the constraint $\sum_{i=1}^M X_i = N$ is necessary for all scenarios. Each of the \tilde{N} secondary users detects the accessible channels in each time slot, and randomly selects one of the accessible channels. The secondary users who select the same accessible channel compete to access this channel.

III.2 THROUGHPUT ANALYSIS

III.2.1 Fixed Allocation

In this scenario, the number of primary users allocated into each channel is a constant. By assumption, each of the X_i primary users in channel i has probability p to transmit data, and all primary users that have data to transmit compete for the channel. Thus the throughput of the primary users in channel i , denoted as $T(i)$, is

$$T(i) = X_i \cdot p \cdot (1 - p)^{X_i - 1}. \quad (1)$$

Note that Eq. (1) gives the throughput when the time slot is one time unit and the packet length is one unit. Thus, the total throughput of primary users, denoted as T , is

$$T = \sum_{i=1}^M T(i). \quad (2)$$

The throughput of secondary users in a time slot depends on the number of accessible channels, and the number of secondary users that have data to transmit, which are called *active secondary users* in this Chapter. Next, the probability distributions of accessible channels and active secondary users is derived. The random variable V denotes the number of accessible channels for secondary users. For each channel, the accessibility of this channel for secondary users is a Bernoulli trial, where the success means that all primary users in this channel have no data to transmit in this time slot, and thus the success probability for channel i is $(1 - p)^{X_i}$. For M channels, this is a repeated Bernoulli trial with heterogeneous success probability, since the number of primary users allocated into different channels may be different. A random variable Y_i is defined as follows,

$$Y_i = \begin{cases} 1, & \text{if channel } i \text{ is accessible for secondary users.} \\ 0, & \text{otherwise.} \end{cases}$$

The probability mass function (pmf) for Y_i is as follows,

$$f_{Y_i}(1) = \Pr(Y_i = 1) = (1 - p)^{X_i},$$

$$f_{Y_i}(0) = \Pr(Y_i = 0) = 1 - (1 - p)^{X_i},$$

By definition, $V = \sum_{i=1}^M Y_i$. Since the data transmission of primary users is independent of each other, Y_i ($1 \leq i \leq M$) is independent of each other. Thus the pmf of V is

$$f_V = f_{Y_1} * \cdots * f_{Y_M}, \quad (3)$$

where $*$ indicates convolution.

Let random variable Z denotes the number of active secondary users in a time slot. Since each secondary user has probability p to transmit data, Z follows the binomial distribution, and its pmf is

$$f_Z(k) = \binom{\tilde{N}}{k} p^k (1-p)^{\tilde{N}-k}. \quad (4)$$

Next, the secondary user throughput is derived, given h accessible channels and k active secondary users, denoted as $\tilde{T}(h, k)$. In each time slot, each active secondary user randomly selects an accessible channel for data transmission, i.e., with probability $\frac{1}{h}$ for each channel, given h accessible channels. The data transmission by an active secondary user will be successful if the user selects an accessible channel that is not selected by any other active secondary user. That is, the probability of successful data transmission for an active secondary user in a specific channel is

$$\frac{1}{h} \cdot \left(1 - \frac{1}{h}\right)^{k-1}, \quad (5)$$

given k active secondary users. Since there are h accessible channels, based on Eq. (5), the probability of successful data transmission for an active secondary user is

$$h \cdot \left[\frac{1}{h} \cdot \left(1 - \frac{1}{h}\right)^{k-1} \right] = \left(1 - \frac{1}{h}\right)^{k-1}. \quad (6)$$

Since there are k active secondary users, then $\tilde{T}(h, k)$, the secondary user throughput, is obtained as follows,

$$\tilde{T}(h, k) = k(1 - \frac{1}{h})^{k-1}. \quad (7)$$

Let $\tilde{T}(h)$ denote the secondary user throughput given h accessible channels. Based on Eq. (4) and (7),

$$\begin{aligned} \tilde{T}(h) &= \sum_{k=1}^{\tilde{N}} f_Z(k) \cdot \tilde{T}(h, k) \\ &= \sum_{k=1}^{\tilde{N}} \left[\binom{\tilde{N}}{k} p^k (1-p)^{\tilde{N}-k} \right] \cdot \left[k(1 - \frac{1}{h})^{k-1} \right] \\ &= p \cdot \tilde{N} \cdot (1 - \frac{p}{h})^{\tilde{N}-1}. \end{aligned} \quad (8)$$

Combining Eq. (3) and (8), the total throughput of secondary users in a time slot is obtained as follows,

$$\tilde{T} = \sum_{h=1}^M f_V(h) \cdot \tilde{T}(h). \quad (9)$$

With Eq. (2) and (9), the total throughput of both primary users and secondary users, denoted as \hat{T} , is

$$\hat{T} = T + \tilde{T} \quad (10)$$

As a special case of fixed allocation, the throughput analysis for even allocation is now presented. Let R denote the remainder of $\frac{N}{M}$. For this scenario, among the M channels, there are R channels that each has $\lceil \frac{N}{M} \rceil$ primary users, and other channels each has $\lfloor \frac{N}{M} \rfloor$ primary users. Denote $U = \lceil \frac{N}{M} \rceil$ and $L = \lfloor \frac{N}{M} \rfloor$. By Eq. (1) and (2), the total throughput of primary users is

$$T = RU p(1-p)^{U-1} + (M-R)L p(1-p)^{L-1}.$$

The throughput of secondary users can be derived similarly for the scenario of fixed allocation, but the probability distribution of accessible channels, $f_V(h)$, can be simplified. If a channel has U primary users, then the probability that it is accessible by secondary users, denoted as Q , is

$$Q = (1 - p)^U. \quad (11)$$

Correspondingly, if this channel has L primary users, then the probability that it is accessible by secondary users, denoted as \tilde{Q} , is

$$\tilde{Q} = (1 - p)^L. \quad (12)$$

The $f_V(h)$ is the sum probability that there are j accessible channels among the R channels that each has U primary users, and there are $h - j$ accessible channels among the remaining $M - R$ channels that each has L primary users, with $\max(0, h - M + R) \leq j \leq \min(R, h)$. That is,

$$f_V(h) = \sum_{j=\max(0, h-M+R)}^{\min(R, h)} \binom{R}{j} Q^j (1 - Q)^{R-j} \times \binom{M-R}{h-j} \tilde{Q}^{h-j} (1 - \tilde{Q})^{M-R-(h-j)}. \quad (13)$$

Substituting Eq. (13) into Eq. (9), we obtain \tilde{T} and T as in Eq. (10).

III.2.2 Random Allocation

In this scenario, X_i is a random variable. In each time slot, each primary user is randomly allocated into any of the M channels with the same probability, i.e., $\frac{1}{M}$. The pmf of $\mathbf{X} = [X_1, \dots, X_M]$ follows the multinomial probability, with

$$f_{\mathbf{X}}(n_1, \dots, n_M; N, \frac{1}{M}) = \Pr(X_1 = n_1, \dots, X_M = n_M) \\ = \frac{N!}{n_1! \dots n_M!} \left(\frac{1}{M} \right)^N.$$

Define the throughput of primary users T in Eq. (1) and throughput of secondary users \tilde{T} in Eq. (9) as functions of input parameters, with X_i replaced by n_i , i.e.,

$$T(n_1, \dots, n_M; M, N, p), \text{ and} \quad (14)$$

$$\tilde{T}(n_1, \dots, n_M; M, N, \tilde{N}, p).$$

Define the set

$$\mathcal{S} = \{x_1, \dots, x_M \mid \sum_{i=1}^M x_i = N\}.$$

The primary user throughput, denoted as $T'(M, N, p)$, can be derived similarly as $\tilde{T}(h)$ in Eq. (8) by substituting h by M , and \tilde{N} by N , thus

$$T'(M, N, p) = N \cdot p \cdot \left(1 - \frac{p}{M}\right)^{N-1}. \quad (15)$$

The secondary user throughput, denoted as $\tilde{T}'(M, N, \tilde{N}, p)$, is given as

$$\tilde{T}'(M, N, \tilde{N}, p) = \sum_{\{n_1, \dots, n_M\} \in \mathcal{S}} f_{\mathbf{X}}(n_1, \dots, n_M; N, \frac{1}{M}) \times \\ \tilde{T}(n_1, \dots, n_M; M, N, \tilde{N}, p). \quad (16)$$

The total throughput is

$$\hat{T} = T'(M, N, p) + \tilde{T}'(M, N, \tilde{N}, p).$$

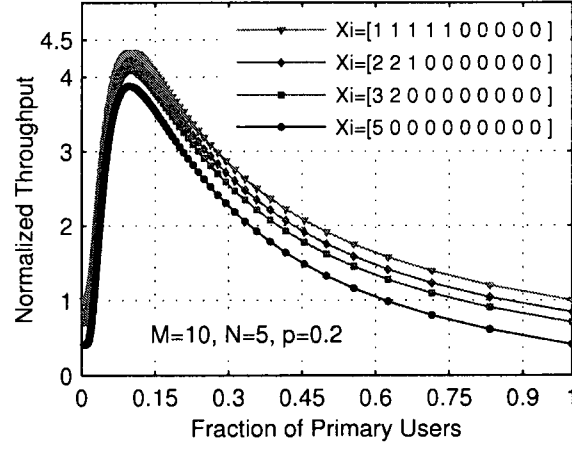
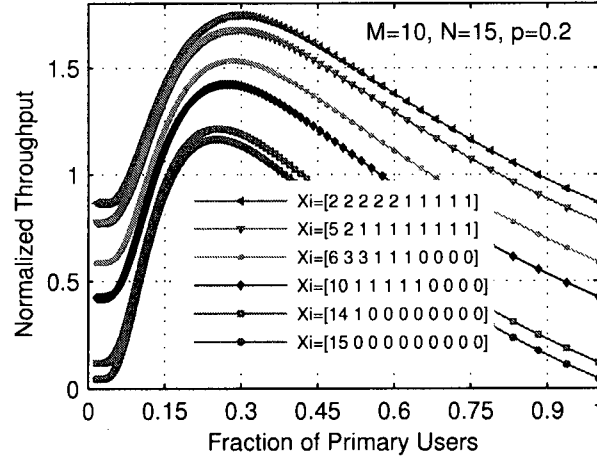
III.3 OPTIMAL SOLUTION AND NUMERICAL RESULTS

In this section, using the models developed in the previous section the performance of dynamic spectrum sharing is examined. The *normalized throughput* for a given number of primary and secondary users is examined. The normalized throughput is the total throughput of primary and secondary users divided by the traffic load of primary user (pN). For a given number of primary users, the goal is to find what is the optimal number of secondary users that can maximize the total throughput.

III.3.1 Fixed Allocation

For fixed M (number of channels) and N (number of primary users), let $\mathbf{X} = [X_1 \ X_2 \ \cdots \ X_M]$ with $X_i \geq 0$ and $\sum_{i=1}^M X_i = N$ denote a specific configuration of allocating primary users into the M channels. Note that there are potentially many configurations for the same M and N .

In Fig. 3, the normalized throughput is examined under several representative configurations for $M = 10$, $N = 5$, $p = 0.2$ (data transmission probability) and $M = 10$, $N = 15$, $p = 0.2$, respectively, and varying \tilde{N} (number of secondary users). To better illustrate the optimal throughput, the X-axis is represented as *the fraction of primary users*, defined as $\frac{N}{N+\tilde{N}}$. The case is considered when $N = 15$ and the observations are presented as follows. Note that when the fraction of primary users is equal to 1, the normalized throughput becomes the throughput of primary users only, since in this case the number of secondary users \tilde{N} must be 0. Another interesting observation is that the more even the allocation, i.e., the number of primary users in each channel is more balanced, the higher is the throughput. For example, the configuration $[2 \ 2 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1]$ has a higher throughput than the

(a) $M = 10, N = 5$ (b) $M = 10, N = 15$ Fig. 3: Normalized Throughput with Fixed Allocation, $M = 10$.

configuration $[5\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$, and the extremely uneven allocation such as $[14\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$ and $[15\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$ has the lowest throughput in Fig. 3.

To get a better understanding, it is better to particularly examine the throughput with the even allocation $[2\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 1]$, which has the highest throughput among fixed allocation as discussed above. Fig. 4 illustrates the normalized throughput under even allocation, with $M = 10$, and varying p and N , respectively.

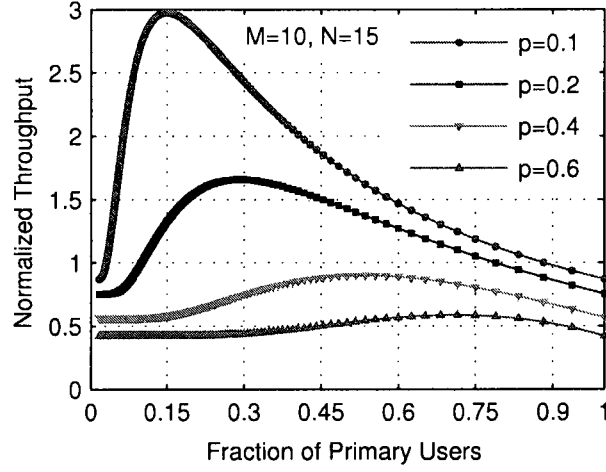
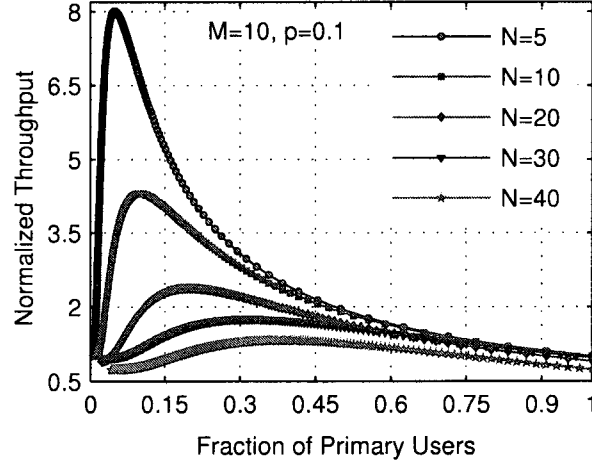
(a) Varying p (b) Varying N Fig. 4: Normalized Throughput with Even Allocation, $M = 10$.

Fig.4(a) shows the throughput with $N = 15$ and $p = 0.1, 0.2, 0.4$, and 0.6 , respectively. Fig. 4(b) shows the throughput with $p = 0.1$ and $N = 5, 10, 20, 30$, and 40 , respectively. From the data in Fig. 4, an interesting observation is that the fraction of primary users that obtains the maximum throughput is relevant to the average per-channel traffic load $\frac{pN}{M}$. The reason is as follows. When the per-channel traffic load decreases, a channel is less congested and is accessible to secondary users in more time. Thus the channel can

accommodate more secondary users, i.e., \tilde{N} can be increased, which makes the fraction of primary users $\frac{N}{N+\tilde{N}}$ decrease. Similarly, when the per-channel traffic load increases, the channel becomes more congested, and thus the number of secondary users that can be accommodated to obtain maximum total throughput should be reduced, which then makes $\frac{N}{N+\tilde{N}}$ increase.

With the above model to compute throughput, the optimal number of secondary users can be obtained that maximizes the total throughput for a given number of primary users and traffic generation probability p . Let \tilde{N}^* denote the optimal number of secondary users. This can be obtained as follows:

$$\tilde{N}^* = \arg \max_{\tilde{N}}(\hat{T}). \quad (17)$$

The \tilde{N}^* can be numerically computed. That is, for the given number of primary users and traffic generation probability, a set of throughputs \hat{T} for varying \tilde{N} is computed, and then the maximum throughput \hat{T}^* is identified. The \tilde{N}^* is the \tilde{N} corresponding to this throughput. Nevertheless the above method may be time consuming, thus a simpler and explicit formula to estimate the optimal number of secondary users that maximizes the total throughput is derived. The throughput of primary users does not depend upon the number of secondary users and thus the optimal throughput can be found by optimizing the secondary user throughput. To obtain the optimal number of secondary users, differentiate \tilde{T} in Eq. (9) with regard to \tilde{N} , and then let $\frac{\partial \tilde{T}}{\partial \tilde{N}} = 0$ to solve for \tilde{N} . Unfortunately there is no explicit expression for $f_V(h)$ so it is not possible to get an explicit expression for $\frac{\partial \tilde{T}}{\partial \tilde{N}}$. Thus, an approximation approach is used to obtain the optimal \tilde{N} in the following. From Eq. (9), the secondary user throughput is, $\sum_{h=1}^M f_V(h) \cdot \tilde{T}(h)$. First take the derivative of

$\tilde{T}(h)$ in Eq. (8) with respect to \tilde{N} , and then let it be equal to 0 to obtain the optimal number of secondary users for a given h , denoted as $\tilde{N}(h)$. In other words, solve

$$\frac{\partial \tilde{T}(h)}{\partial \tilde{N}} = (1 + \tilde{N} \cdot \log(1 - \frac{p}{h})) \cdot (1 - \frac{p}{h})^{\tilde{N}-1} = 0,$$

to obtain

$$\tilde{N}(h) = \frac{-1}{\log(1 - \frac{p}{h})}, \quad (18)$$

where $\log(x)$ is the natural logarithm of x . The optimal number of secondary users can then be estimated as the weighted average of $\tilde{N}(h)$ for $1 \leq h \leq M$ as follows.

$$\tilde{N}^* = \sum_{h=1}^M f_V(h) \cdot \tilde{N}(h) \quad (19)$$

Since the number of secondary users is an integer value, simply round up \tilde{N}^* to be an integer.

Fig. 5 shows the optimal number of secondary users \tilde{N}^* versus traffic generation probability p under fixed distribution, when $M = 10$, and $N = 5, 10$ and 40 , respectively. It can be observed that all plots of optimal solution given by Eq. (19) match very closely with the optimal values obtained through numerical simulation results obtained using the throughput model. The optimal number is obtained by Eq. (17) using the numerical computation method in the discussion related to Eq. (17). For instance, in Figs. 4 and 6, in the bell shaped curve for secondary user throughput, wherever the bell attains the maxima, the number of secondary users corresponding to the x-axis value is the optimal number of secondary users. The comparison is done between the optimal number of secondary

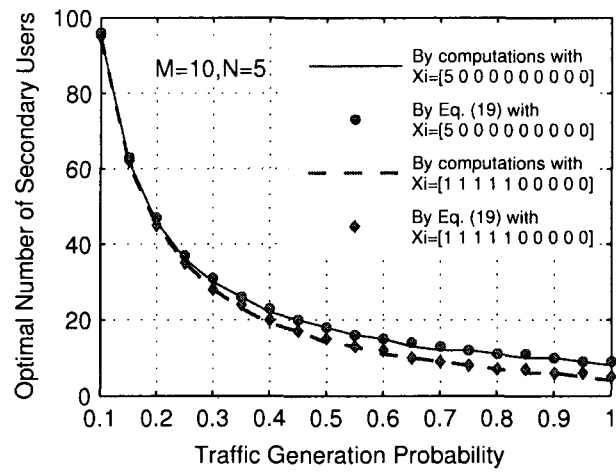
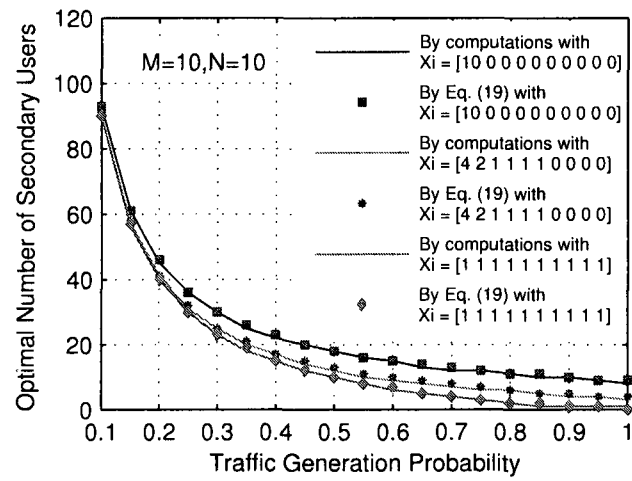
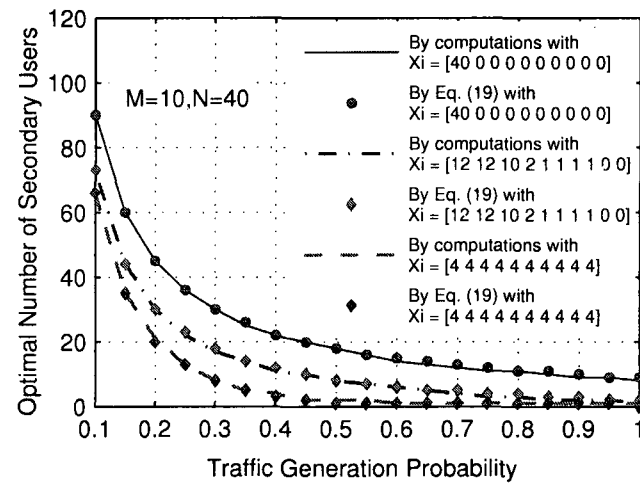
(a) $M = 10, N = 5$ (b) $M = 10, N = 10$ (c) $M = 10, N = 15$

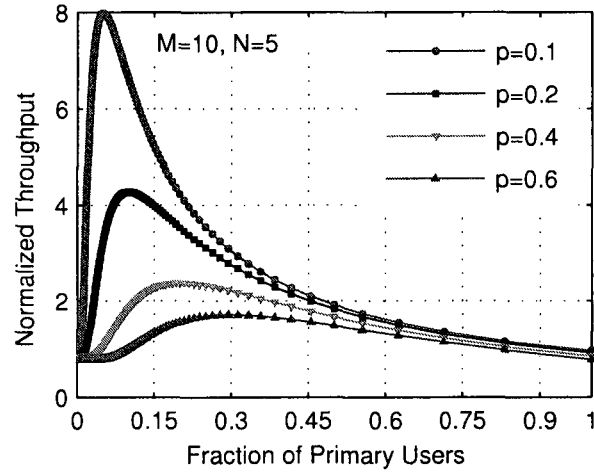
Fig. 5: Optimal Number of Secondary Users Obtained from Fixed Allocation and that from Eq. (19) with Varying Traffic Generation Probabilities, p , for Different Configurations, when $M = 10$ and $N = 5, 10$, and 40 , Respectively.

users obtained by numerical computation with the number of secondary users estimated by Eq (19).³ Extensive numerical computations were carried out under various scenarios, and the optimal number of secondary users estimated by Eq. (19) has been shown closely matching the numerical results obtained using the throughput model. Looking closer at the case when $M = 10, N = 40$. When $p = 0.1, \tilde{N}^* = 90$ for the configuration [40 0 0 0 0 0 0 0 0 0], which means almost 90 secondary users can be accommodated. A number less than 90 means wastage of channel resources, and a number above 90 means more chances of collisions. Also, it can be seen that, as the traffic generation probability increases, there is a drastic decrease in the number of secondary users. It can be noticed that, for p above 0.4, the number of secondary users lie somewhere between 10 and 20. Thus, for the applications where traffic is low, many secondary users can benefit from the opportunistic use and thus a lot of spectral space can be saved from getting wasted. Also, it can be observed that, when moving from one configuration to another, e.g., from [40 0 0 0 0 0 0 0 0 0] to [4 4 4 4 4 4 4 4 4 4] the number of secondary users that can be accommodated reduces. This is due to the fact that the number of available channels reduce. These expected number of available channels can further be given by $\hat{h}_f = \sum_{i=1}^M (1 - p)^{X_i}$. Using the formula for the expected number of available channels for each configuration, it can be easily understood as to why and how the number of secondary users that can opportunistically access a channel change. Thus, the more distributed are the primary users over all the channels, the lesser are the chances for the number of secondary users to actually be able to access channels opportunistically.

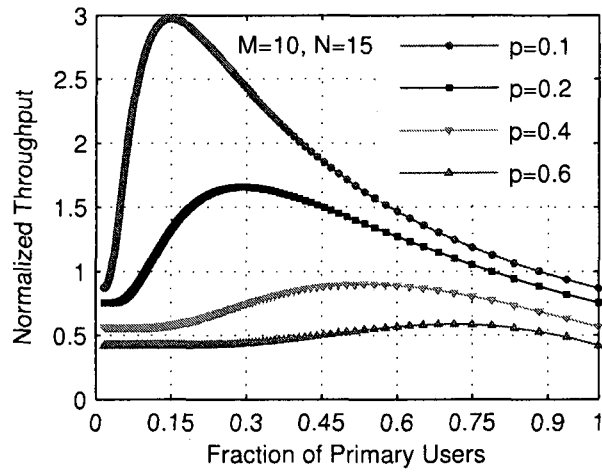
³ \tilde{T} calculated by Eq. (9) is manually examined for varying \tilde{N} , and then find the maximum \tilde{T} and the corresponding \tilde{N} , which is the optimal number of secondary users.

III.3.2 Random Allocation

In Fig. 6 plots are shown for the normalized throughput under random allocation, with $M = 10$, $N = 5$ or 15 , and varying p . From Fig. 6, a similar pattern can be observed as in the scenario of even allocation discussed above. Again for random allocation, it is observed that the fraction of primary users that obtains the maximum throughput is relevant to the average per-channel traffic load $\frac{pN}{M}$.



(a) $M = 10, N = 5$



(b) $M = 10, N = 15$

Fig. 6: Normalized Throughput with Random Allocation, $M = 10$.

Next follows a discussion of how to derive the optimal number of secondary users that maximizes the total throughput. A similar approach, as with the case of fixed allocation, is taken. First of all, the \tilde{N}^* in Eq. (19) is actually the optimal number of secondary users under a fixed distribution of primary users $\mathbf{X} = [n_1, \dots, n_M]$. It is denoted as $\tilde{N}^*(n_1, \dots, n_M; M, N, p)$, similarly as for $T(n_1, \dots, n_M; M, N, p)$ in Eq. (14). Then the optimal number of secondary users under random allocation is computed as follows.

$$\tilde{N}^*(M, N, p) = \sum_{\{n_1, \dots, n_M\} \in \mathcal{S}} f_{\mathbf{X}}(n_1, \dots, n_M; N, \frac{1}{M}) \times \tilde{N}^*(n_1, \dots, n_M; M, N, p). \quad (20)$$

Next the optimal number of secondary users is plotted versus the traffic generation probability p in Fig. 7, with $M = 10$ and $N = 5, 10, 15$ and 30 , respectively. The optimal solution estimated by Eq. (20) matches very closely with the one obtained through numerical computations using the throughput model. Since random allocation depends upon the random channel selection behavior of primary users, it is not possible to determine as to what the configuration is going to be in a specific time slot, the optimal number of secondary users that can opportunistically access the channels can be found using the formula as given by Eq. (20), given a specific set of M and N . As the traffic generation probability increases, the optimal secondary users reduce.

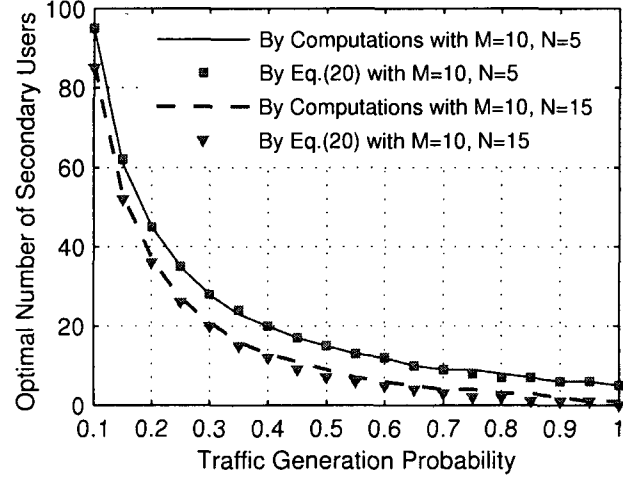
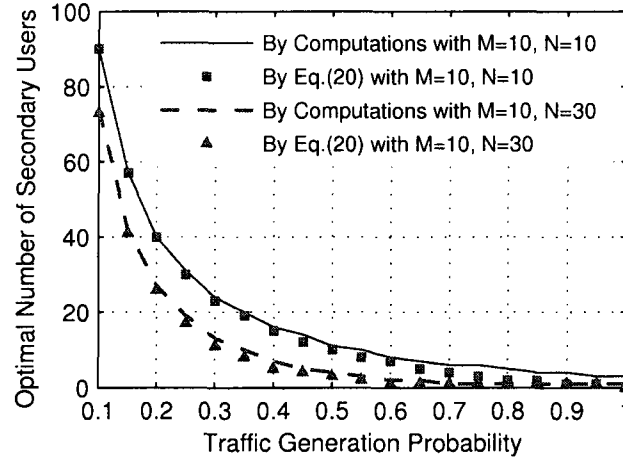
(a) $M = 10, N = 5$ (b) $M = 10, N = 15$

Fig. 7: Optimal Number of Secondary Users Obtained from Random Allocation and that from Eq. (20) with Varying Traffic Generation Probabilities, p , when $M = 10$ and $N = 5, 10, 15$ and 30 , Respectively.

III.4 OPTIMAL SOLUTION: AN ALTERNATIVE APPROACH

In this section an alternative approach to finding the optimal solution is presented. That is, another set of mathematical formulas have been provided to compute the maximum number of secondary users. Again, similar to section III.3, let \tilde{N}^* denote the optimal number of secondary users that maximizes the total throughput. The \tilde{N}^* can be numerically computed. That is, given N, M, p , compute a set of throughputs for varying \tilde{N} , and then identify the maximum throughput. The \tilde{N}^* is the \tilde{N} corresponding to the maximum throughput.

III.4.1 Fixed Allocation

In order to find \tilde{N}^* , use the same argument and equations as were used in section III.3. For a quick reference the equation are rewritten as follows.

$$(1 + \tilde{N} \cdot \log(1 - \frac{p}{h})) \cdot (1 - \frac{p}{h})^{\tilde{N}-1} = 0$$

$$\tilde{N} = \frac{-1}{\log(1 - \frac{p}{h})}. \quad (21)$$

In order to find the value of h that optimizes \tilde{N} , take a look at the distribution of h . Since h is random, Monte Carlo Simulations are done by performing the experiment over 10,000 random samples and then finding the distribution of h as shown in Fig. 8(a), (b) and (c) for $M = 10, N = 5$ and $p = 0.2, 0.5$ and 0.9 respectively and in Fig. 9a for $p = 0.4$ and Fig. 9b for $p = 0.9$ when $M = 10$ and $N = 30$. As can be seen from these figures, the distribution of h is heavily concentrated around the mean of h . After performing such

simulations with many other values of M , N and p , it is found that the distribution of h is concentrated around the mean of h , i.e., the probability of occurrence of available channels around the mean value of h is highest. The mean of h can be taken to obtain \tilde{N}^* .

To find the expected number of available channels, determine the probability of availability of each channel and then add all these probabilities as follows.

$$\overline{h_f} = \sum_{i=1}^M (1-p)^{X_i}. \quad (22)$$

Thus, write Eq. (19) as follows.

$$\tilde{N}^* = \frac{-1}{\log\left(1 - \frac{p}{\overline{h_f}}\right)}$$

Since the number of secondary users is an integer value, round it off and write \tilde{N}^* as follows.

$$\tilde{N}^* = \max\left(\left\lceil \frac{-1}{\log\left(1 - \frac{p}{\overline{h_f}}\right)} \right\rceil, 0\right) \quad (23)$$

III.4.2 Random Allocation

A similar approach is taken like in the case of fixed allocation as in Eq. (23). For a specific random distribution $\mathbf{X} = [n_1, \dots, n_M]$, denote the optimal number of secondary users as $\tilde{N}^*(n_1, \dots, n_M; M, N, p)$. Then the optimal number of secondary users under random allocation is computed as follows.

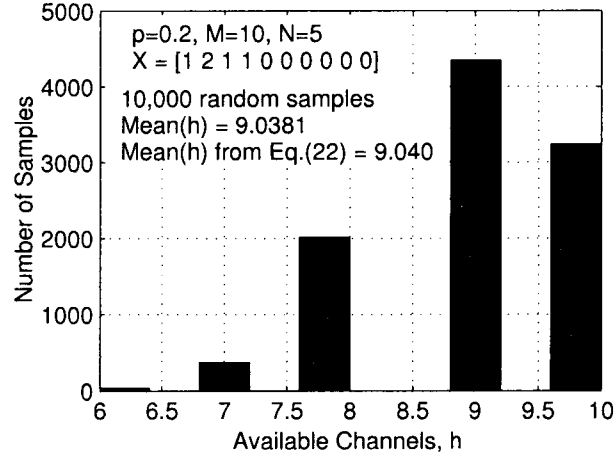
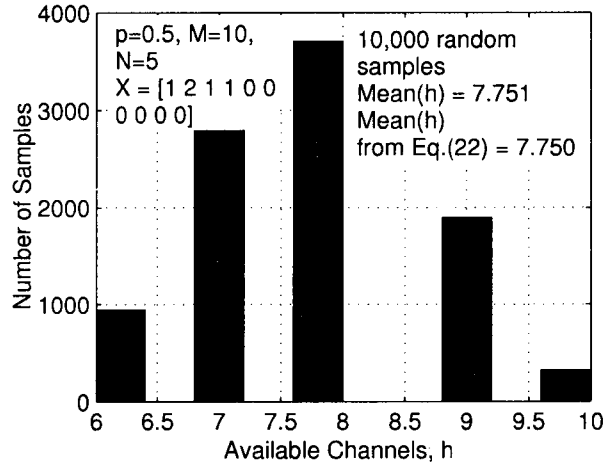
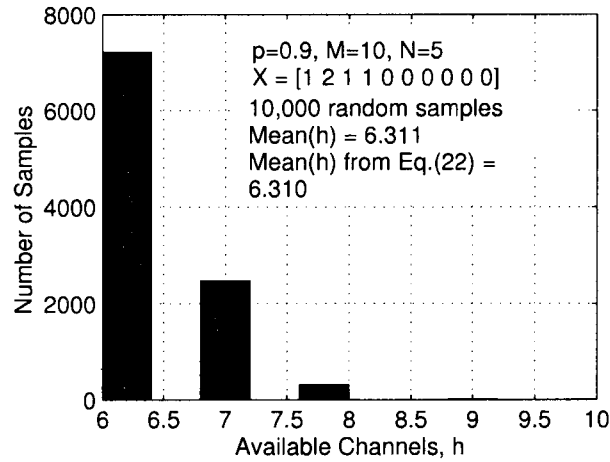
(a) $M = 10, N = 5, p = 0.2$ (b) $M = 10, N = 5, p = 0.5$ (c) $M = 10, N = 5, p = 0.9$

Fig. 8: Monte Carlo Simulations Performed to Find the Expected Number of Available Channels, when $M = 10, N = 5, p = 0.2$ (a), 0.5 (b) and 0.9 (c).

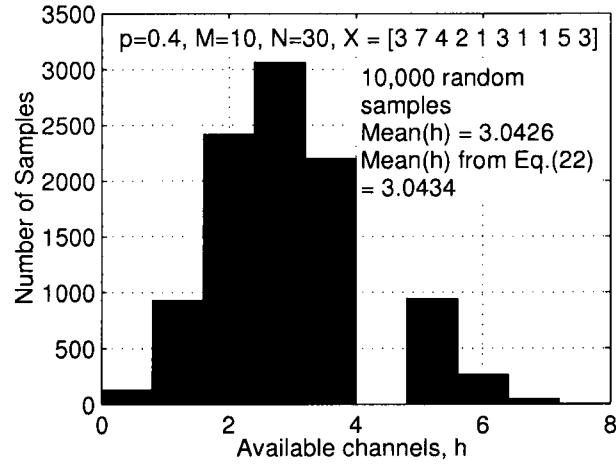
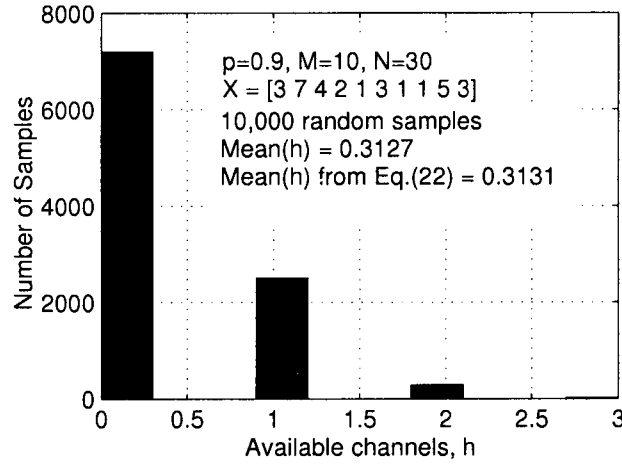
(a) $M = 10, N = 30, p = 0.4$ (b) $M = 10, N = 30, p = 0.9$

Fig. 9: Monte Carlo Simulations Performed to Find the Expected Number of Available Channels, when $M = 10$, $N = 30$, $p = 0.4$ (a) and 0.9 (b).

$$\tilde{N}^* = \sum_{\{n_1, \dots, n_M\} \in \mathcal{S}} f_{\mathbf{X}}(n_1, \dots, n_M; N, \frac{1}{M}) \times \tilde{N}^*(n_1, \dots, n_M; M, N, p). \quad (24)$$

Since the number of available channels is a random number based on the random distribution, the optimal solution can be obtained by finding the expected number of available

channels, denoted as $\overline{h_r}$, as follows:

$$\overline{h_r} = \sum_{j=1}^C \left(\sum_{i=1}^M (1-p)^{x_{ij}} \right) \cdot f_X(j), \quad (25)$$

where

$$f_X(j) = \frac{N!}{n_{1j}! \cdots n_{Mj}!} \left(\frac{1}{M} \right)^N$$

Here $f_X(j)$ represents the probability mass function of j th configuration out of total C configurations possible for a given M and N , where the value of C can be calculated using the method of finding the combinations with repetitions as follows.

$$C = \binom{N+M-1}{N}$$

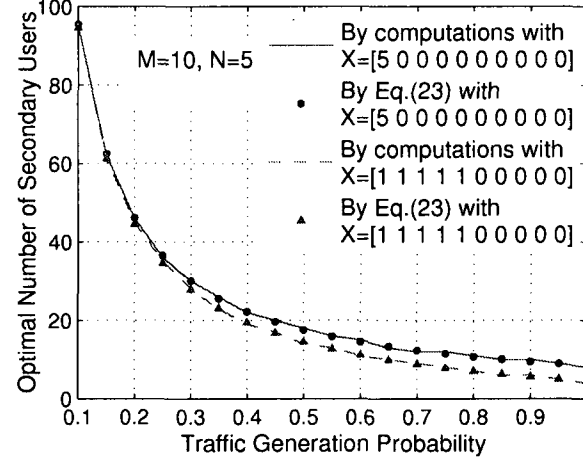
Since the problem of finding optimal solution for random allocation is similar to the one for fixed allocation, the optimal solution for random allocation can be written as follows.

$$\tilde{N}^* = \max \left(\left\lceil \frac{-1}{\log(1-\frac{p}{h_r})} \right\rceil, 0 \right) \quad (26)$$

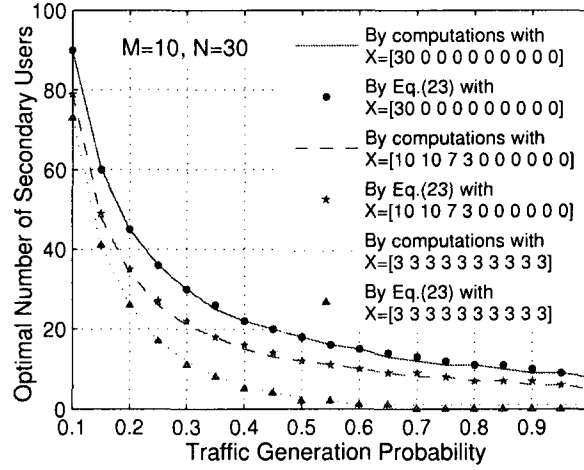
III.5 NUMERICAL RESULTS

Under the fixed allocation scheme, Fig. 10 illustrates the optimal number of secondary users versus the traffic generation probability with $M = 10$, $N = 5$ (Fig. 10a) and 30 (Fig. 10b). The optimal number of secondary users given by Eq. (23) follows very closely with the one obtained through numerical computation. Many plots (not shown due to space limits) were generated with varying values of M, N and p . It was found that the results

given by Eq. (23) and the results obtained through numerical approach match very well in all plots. Thus it is concluded that Eq. (23) gives a good estimation of the optimal number of secondary users that maximizes the total throughput.



(a) $M = 10, N = 5$

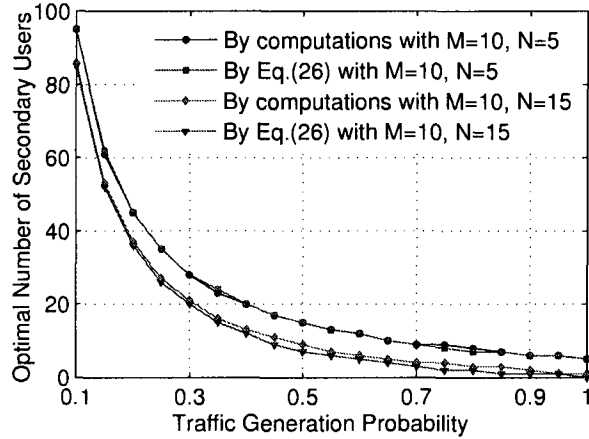


(b) $M = 10, N = 30$

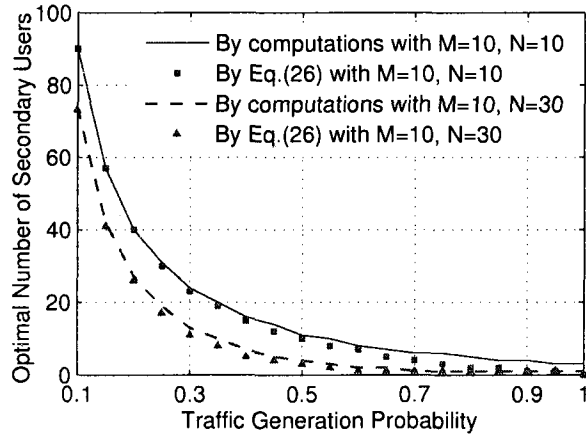
Fig. 10: Optimal Number of Secondary Users Obtained from Fixed Allocation and that from Eq. (23) with Varying Traffic Generation Probabilities for Different Configurations, when $M=10$ and $N=5$ (a), and $N=30$ (b).

In Fig. 11, the optimal number of secondary users versus the traffic generation probability is plotted with $M = 10, N = 5$ and 15 (Fig. 11a) and $M = 10, N = 10$ and 30 (Fig. 11b), under the random allocation scheme. The plots obtained for the optimal solution

follow very closely with the plots obtained through numerical computation. Based on the results of various other experiments with varying M, N and p , it is concluded that Eq. (26) is a good estimation for the optimal number of secondary users that maximizes the total throughput.



(a) $M = 10, N = 5$ and $M = 10, N = 15$



(b) $M = 10, N = 10$ and $M = 10, N = 30$

Fig. 11: Optimal Number of Secondary Users Obtained from Random Allocation and that from Eq. (15) with Varying Traffic Generation Probabilities for Different Configurations, when $M=10$ and $N=5, 15$ (a), and $N=10, 30$ (b).

III.6 COMPLEXITY ANALYSIS

The formulations obtained in using alternative approach for finding the optimal solution have less complexity as compared to the one derived in the first approach [29]. For fixed allocation, the optimal solution in [29] has a complexity of $O(M^2)$. The solution provided by this approach, as seen from Eqs. (22) and (23), has a complexity of $O(M)$. For random allocation, the optimal solution using first approach given in [29] has a complexity of $O(M^2C)$. On the other hand, using this approach, there is only one nested loop inside the main loop as given by Eqs. (25) and (26). Thus the complexity of the solution provided with this approach is $O(MC)$.

III.7 CONCLUSIONS

In this chapter the aim is to develop a throughput model, and using the optimal number of secondary users that may opportunistically access channels of primary users, in which primary users themselves contend for channels. To this end, two scenarios were considered for allocating primary users into channels. Analytical models were developed to calculate the throughput of both primary users and secondary users. Through these models, the optimal number of secondary users is derived that obtains the maximum total throughput for a given number of primary users, channels, and transmission probability. The maximum total throughput also implies that the throughput of secondary users is also maximized. The results obtained using the explicit formulas closely match with the numerical results obtained using the throughput models. As a future work, the plan is to design a protocol for achieving the optimal performance, based on the model developed in this Chapter, and

using the optimal number of secondary users obtained for each traffic generation probability. Also, eliminating a few of the assumptions made in the methodology related to the throughput analysis of contention-based dynamic spectrum sharing model would make it a more challenging and practical problem. For example, the model can be extended to the more practical cases where traffic generation probability for primary users and secondary users are different and also variable.

CHAPTER IV

TIME SPECTRUM BLOCK ASSIGNMENT

In this Chapter, time spectrum block assignment in community cognitive radio networks (CRNs) is considered. CRNs being opportunistic users present many technical challenges that were never presented by any other technologies. In WLANs and cellular networks, channel allocation is considered as one of the resource allocation problems. For resource allocation in CRNs, allocation of time spectrum blocks (TSBs), which are the chunks or blocks of frequency bands available for secondary users during specific intervals of time, is considered. In the following Chapter, a simple algorithm is presented to allocate TSBs to optimize the system performance based on interference minimization and throughput optimization.

IV.1 NETWORK MODEL AND PROBLEM FORMULATION

In dynamic spectrum access (DSA) networks, the activity of licensed or *primary users* (PUs) on a channel can be predicted in some scenarios. A channel indicates a band of spectrum, e.g., a TV channel of 6 MHz. For instance, the active broadcasting periods of TV channels can be obtained from past measurements as the TV channels activities are approximately fixed. The usage of a channel by PUs is characterized by alternating busy and idle periods. The SUs can utilize the idle periods for communication. An idle period on a channel is called a *time spectrum block* (TSB). Fig. 12 illustrates TSBs on 3 channels. The X-axis is the time dimension, and the Y-axis is the spectrum dimension. The shaded

area on a channel indicates that PUs are actively using the channel. A secondary user utilizes TSBs for communication, i.e., it needs to not only find a channel, but also find an idle period to carry out data communication.

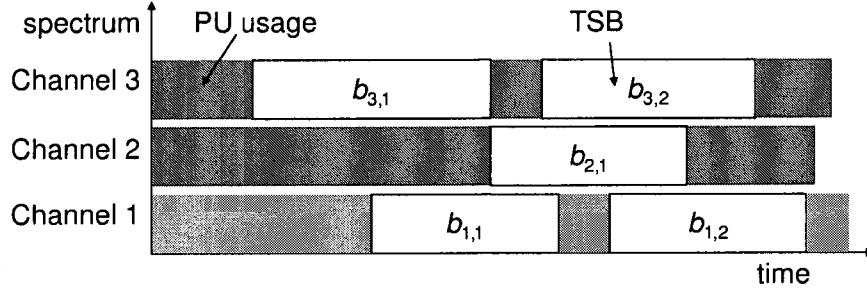


Fig. 12: Time Spectrum Blocks (TSBs).

The goal is to use optimization techniques to formulate TSB assignment and develop algorithms. The objective is to minimize co-channel interference between APs (and its nodes), and thus maximize throughput. Consider a scenario of DSA network illustrated in Fig. 13, where there are 3 APs. The solid circle illustrates the transmission range of an AP, i.e., the AP can send/receive packets to/from SU nodes within the circle, while the dotted circle illustrates the interference circle of an AP, i.e., the packet transmission at an AP interferes with all nodes in this circle even though the nodes may be in different networks. Each AP and the nodes within its transmission range form into a *wireless local network* (WLAN).

Fig. 14 illustrates a possible assignment of APs to TSBs in Fig. 12. It can be seen that the SU communication is carried out on TSBs, and the underlying communication frequency changes over the time. Note that the APs are distinguishable in TSB assignment. For instance, at time t_1 , if we assign AP 3 to TSB $b_{1,1}$, the purpose of interference

reduction cannot be achieved because the co-channel interference is not eliminated with this assignment. Due to the proximity between APs 1 and 2, AP 1 interferes node B (in AP 2's network), and so does AP 2 to node A.

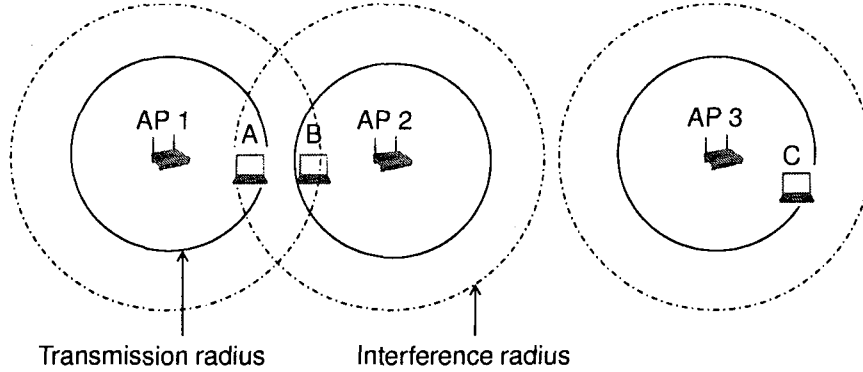


Fig. 13: A Scenario of DSA Networks. The Solid Circle is the AP Transmission Range, while the Dotted Circle is the AP Interference Range. Each AP with the Nodes within its Transmission Range forms into a WLAN.

Thus the throughput of WLANs formed by APs 1 and 2 is only half of the channel bandwidth. On the other hand, in Fig. 14, although APs 1 and 3 are in the same channel between time t_1 and t_2 , they do not interfere the WLAN of each other.

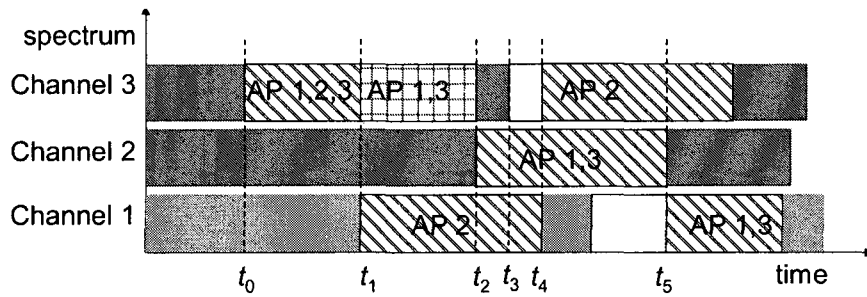


Fig. 14: Assignment of APs to TSBs. At time t_0 , TSB $b_{3,1}$ Appears and all APs are Assigned to $b_{3,1}$. At t_1 , another TSB, $b_{1,1}$, is Available on Channel 1, and AP 2 Picks this New TSB. This Reduces the Co-channel Interference between AP 2 and APs 1, 3. At t_2 , TSB $b_{3,1}$ expires and AP 1, 3 jump to TSB $b_{2,1}$. Similarly, at t_4 , TSB $b_{1,1}$ Expires and AP 2 Jumps onto $b_{3,2}$, and so on, and so forth.

In Fig. 12, some TSBs partially overlap with each other on time dimension, e.g., TSBs $b_{3,1}$ and $b_{1,1}$ overlap with each other between t_1 and t_2 . For the ease of modeling, convert such partially overlapping TSBs to TSBs which are either completely overlapping, i.e., with the same start/end times, or completely separate, the start time of one is larger than the end time of the other. This can be done by partitioning a TSB that partially overlaps with other TSBs into several smaller TSBs, at the start/end times of other TSBs. Fig. 15 illustrates the TSBs partitioned from the ones in Fig. 12. For instance, TSBs $b_{3,1}$ and $b_{3,2}$ in Fig. 12 are partitioned into 5 TSBs in Fig. 15, renamed to $b_{3,1}, \dots, b_{3,5}$.

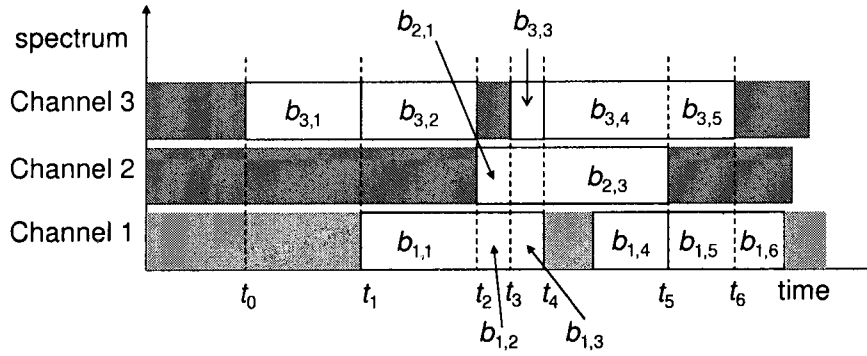


Fig. 15: Partition TSBs of Fig. 12 into TSBs that are Either Completely Separate or Completely Overlapping on Time Dimension.

Next the concept of *interference index* is introduced. Let $F(i, j)$ denote the interference index between APs i and j . Calculate $F(i, j)$ as the ratio of the intersection area of the two interference circles of APs i and j to the entire area of the two interference circles. Let d denote the distance of APs i and j . Let r_i and r_j denote the interference radius of APs i and j , respectively. Then the intersection area of the interference circles of APs i and j is given as

$$A = r_i^2 \arccos\left(\frac{d^2 + r_i^2 - r_j^2}{2 \cdot d \cdot r_i}\right) + r_j^2 \arccos\left(\frac{d^2 + r_j^2 - r_i^2}{2 \cdot d \cdot r_j}\right) - 0.5 \sqrt{(-d + r_i + r_j)(d + r_i - r_j)(d - r_i + r_j)(d + r_i + r_j)}. \quad (27)$$

Here, the unit of $\arccos(\bullet)$ is radian.

The total area of the two interference circles is trivially calculated as $\pi r_i^2 + \pi r_j^2 - A$.

Thus,

$$F(i, j) = \frac{A}{\pi r_i^2 + \pi r_j^2 - A}.$$

Let N denote the number of APs, and M denote the number of TSBs. Let $D_{i,m} = 1$ if AP i is assigned to TSB m , and $D_{i,m} = 0$ otherwise. Let $\{t_1, \dots, t_K\}$ denote the set of start times of all TSBs. Let J_k denote the set of TSBs that start at time t_k . The problem of TSB assignment can be formulated into an optimization problem as follows:

$$\min \sum_{i,j,m} D_{i,m} \cdot D_{j,m} \cdot F(i, j) \quad (28)$$

subject to:

$$\forall 1 \leq i \leq N \text{ and } 1 \leq k \leq K, \sum_{m \in J_k} D_{i,m} = 1 \quad (29)$$

Constraint (29) ensures that as long as there is any TSB available, each AP will be assigned to one and only one TSB available at that time. Objective (28) minimizes the total interference between all APs.

IV.1.1 Example

The above problem formulation can be better understood using the following example. In this example the resultant matrix obtained is shown when the problem was solved for 15 TSBs and 5 APs. Also, the set of TSBs, that is, J_k is given as $J_1 = \{1, 7\}, J_2 = \{10\}, J_3 = \{2, 7\}, J_4 = \{3, 8\}, J_5 = \{9, 12\}, J_6 = \{4, 13\}, J_7 = \{5, 14\}, J_8 = \{6, 15\}$, as shown in Fig. (16). Thus, it can be seen that, considering the above interference minimization criteria as given by Eq. (28) the condition as given by Eq. (29) is satisfied in the final result.

		TSB														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AP	1	0	0	0	0	0	1	1	0	1	1	1	1	1	1	0
	2	1	1	1	1	0	0	0	1	1	0	0	0	0	0	1
	3	0	0	0	0	0	1	1	0	1	1	1	1	1	1	0
	4	1	1	1	1	0	0	0	1	1	0	0	0	0	0	1
	5	0	0	0	0	0	1	1	0	1	1	1	1	1	1	0

$J_1 = \{1, 7\}, J_2 = \{10\}, J_3 = \{2, 11\}, J_4 = \{3, 8\}, J_5 = \{9, 12\},$
 $J_6 = \{4, 13\}, J_7 = \{5, 14\}, J_8 = \{6, 15\}$

Fig. 16: Matrix of TSB Assignment to Different APs for a Specific Interval of Time.

For example, consider $J_1 = \{1, 7\}$ as shown by circled entries. Observe that the sum of any two entries in 1 and 7 is always 1. Similarly consider $J_6 = \{4, 13\}$ as shown inside rectangular entries. Again, observe that the sum of any two entries in 4 and 13 is always 1. Similarly notice that this condition is always satisfied for all the cases. Thus, it ensures that an AP can be associated with one and only one TSB during J_k . Thus, the interference of the whole system is minimized based on the above formulation. The algorithm must also give a configuration that minimizes the interference and follows closely with

the interference minimization done using optimization methods. After understanding the problem formulation and what the expected final result is, the discussion of the algorithm is as follows.

IV.2 ABCD - ALGORITHM

This section discusses the proposed **ABCD** (Allocation of time spectrum **B**locks in **C**ognitive radio networks for **D**ynamic spectrum sharing) algorithm. Using the ABCD algorithm, TSBs are allocated to APs based on interference minimization. The algorithm is given in Fig. 17. There are three basic steps in the algorithm. Step 1 calculates the interference index, the cumulative interference, and sorts the cumulative interference values in descending order, as given by lines 2, 3 and 4 of the algorithm given by Fig. 17. In step 2, populate the TSBs associated with J_k , for example, if the k_{th} time interval has three TSBs available then these TSBs are populated one by one using step 2 of the algorithm. Firstly, it considers the APs that have highest cumulative interference with other APs. What is meant by having highest cumulative interference with other APs, is that this step ensures that first consider the APs that are heavily surrounded by other APs and nodes. Every time a new AP is to be allocated a TSB, its cumulative interference with the other APs present in each TSB is considered, and then the AP is allocated the TSB with minimum possible interference. This way all the TSBs available during J_k are allocated to the active APs. In step 3, the step 2 is repeated over all the J_k s. Thus, for each time interval, the ABCD algorithm allocates TSBs to active APs. This is done until all the TSBs are allocated over the time interval considered for allocation.

A centralized server assigns these TSBs to APs. The algorithm that generates optimal or near optimal TSB allocation is running on this centralized server that assigns TSBs to APs. The centralized server creates the interference index matrix $F(i, j)$ based on the information received from APs regarding transmission ranges and interference ranges. Thus $F(i, j)$ is created. ABCD algorithm that runs on centralized server is presented as follows.

- 1 Step 1. Calculate input parameters
- 2 Find $F(i, j)$, the interference of i_{th} AP with every j_{th} AP in the network.
- 3 Find $C(i)$, the cumulative interference of each AP with all other APs.
- 4 Find $L(i)$, the list of $C(i)$ sorted in descending order, thus the APs heavily surrounded by other APs are considered first for TSB allocation.
- 5 Step 2. Populate the TSBs during J_k
- 6 For first TSB in J_k , assign the TSB to the AP with highest cumulative interference $C(i)$.
- 7 The next AP in $L(i)$ is considered. This AP's sum interference with other APs already present is found on all the TSBs during J_k . The TSB that gives least interference is given to the AP under consideration.
- 8 The above procedure is repeated until all the APs are assigned the TSBs during J_k .
- 9 Step 3. Populate the TSBs over all J_k s.
- 10 Step 2 is repeated over all J_k .
- 11 If any of the TSBs are continuing from previous J_{k-1}^{th} interval, the condition is to keep most or all of the APs on the TSBs continuing from previous interval. Thus, while repeating Step 2, another condition of continuity is also checked.

Fig. 17: ABCD Algorithm for TSB Allocation.

IV.3 RESULTS AND ANALYSIS

This section presents the results that were obtained from TSB allocation using the optimal solution values and those using the algorithm values. In Figs.18 and 19, the distribution of APs and client nodes in XY-plane is shown. In Fig.18, 4 APs and 100 client nodes are considered, while in Fig. 19, 16 APs and 100 client nodes. Since community networks are considered, the APs are distributed on square grids and are equally spaced apart. It must be noted that the distribution can be considered similar to cellular networks in the form of hexagonal cells. The ABCD algorithm applies equally to other distributions.

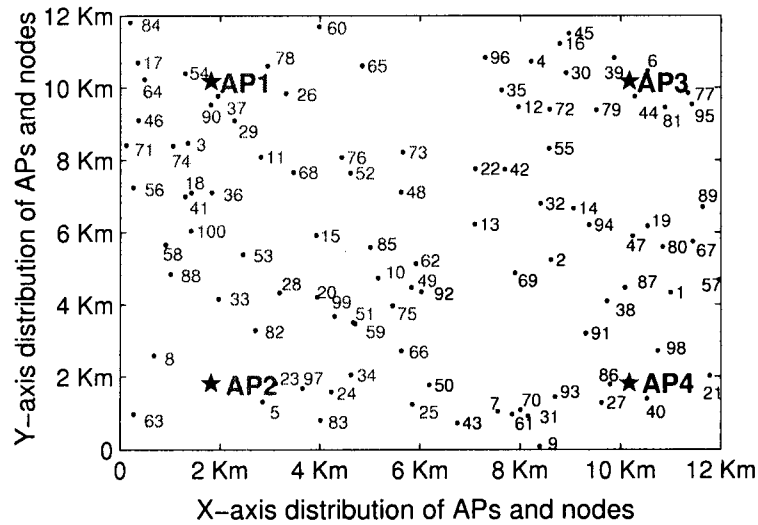


Fig. 18: Distribution of 4 APs and 100 Client Nodes in X-Y Plane. Client Nodes are Randomly Distributed.

It can be seen that the APs that lie in the center of Fig. 19 are more heavily interfered with as compared to those that lie at the edges. Thus, as discussed in the algorithm, those APs that are more heavily interfered are considered first for allocation. Also, if a single node is removed from Fig. 19, it becomes less symmetric, and hence different results can be expected as compared to those obtained for symmetric cases. Various parameter values

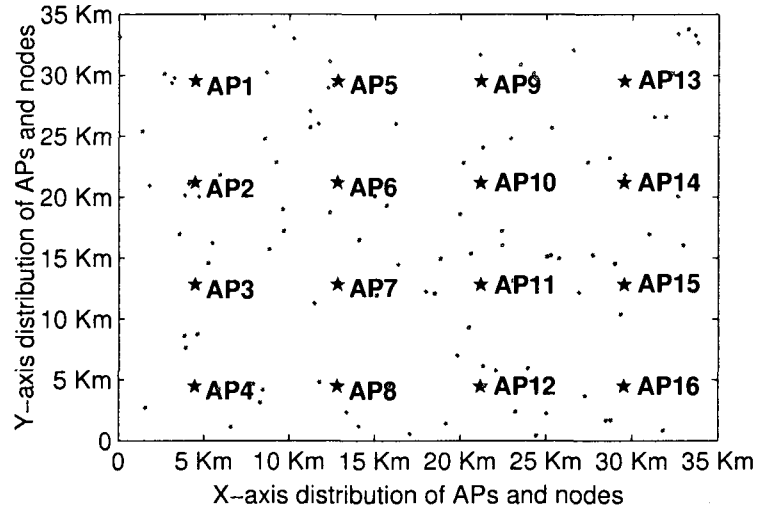


Fig. 19: Distribution of 16 APs and 100 Client Nodes in X-Y Plane. Client Nodes are Randomly Distributed.

related to the distribution of these APs and clients can be seen from Table 2 and Table 3.

Table 2: Community Cognitive Radio Network Parameters for WLAN.

Parameter	Value
Transmission Power	20dBm
Noise Power	-115 dBm
Receiver Sensitivity	-80 dBm
Frequency	400 MHz
Pathloss Factor	2
Transmission range	$5.9683e+003$ m = 6 km (approx.)
Distance between APs	8.356 km = 8.4 km (approx.)
Channel Bandwidth	6MHz

In Tables 2 and 3, two different simulation parameters are considered. Table 2 parameters are similar to parameter values considered for 802.11 b/g WLAN. Considering the transmission power and path loss effects, we get the range and other factors at 400 MHz TV band. The variation in transmission range and other factors that are affected by frequency can be taken into consideration similarly as we did for 400 MHz for any other frequency

band. A huge gain in the transmission range can be noticed if 802.11 WLANs work in TV bands instead of unlicensed 2.4 GHz ISM band. Table 3 gives parameter values considered for the latest development in wireless communication in TV bands using cognitive radios, that is, 802.22 WRAN. Here, in most of the results that follow, the results are obtained using the parameters considered in Table 2. Similar results can be obtained for 802.22 WRAN. While running the simulations it was found that ABCD algorithm works equally fine with WRAN related parameters. A comparative study of both of these is presented in further analysis and results and it can be seen that the algorithm is more or less equally applicable in the cases where AP distribution in space is varied.

Table 3: Community Cognitive Radio Network Parameters for WRAN.

Parameter	Value
Transmission Power	26 dBm
Noise Power	-115 dBm
Receiver Sensitivity	-90 dBm
Frequency	400 MHz
Pathloss Factor	2
Transmission range	$3.7747\text{e}+004 \text{ m} = 37.7 \text{ km (approx.)}$
Distance between APs	$5.2846\text{e}+004 = 52.8 \text{ km (approx.)}$
Channel Bandwidth	6MHz

In Figs. 20 and 21, bar graphs are shown representing the comparison between the optimal solution values, algorithm values and random allocation values obtained when different scenarios were run. These scenarios are presented in Table 4. A total of 6 scenarios are considered and the results are studied for two continuous time intervals, that is, J_1 and J_2 are considered with N and varying M , and values of TSBs in each time interval. The reason for studying the results for two continuous time intervals is to study the effect

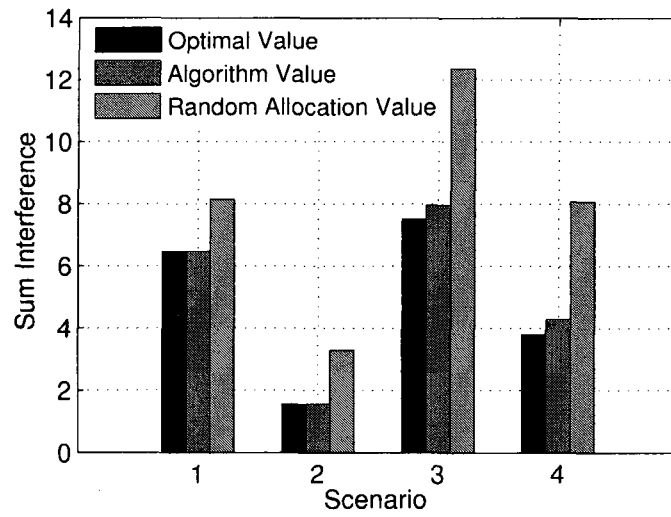


Fig. 20: Comparison of Sum Interference Values Obtained using Optimal Solution, with those Obtained using ABCD Algorithm, and also using Random Allocation, for Scenarios 1 to 4 as given in Table 4.

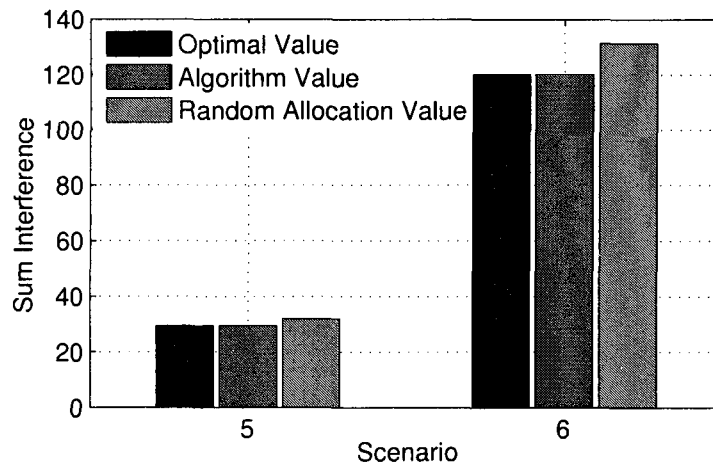


Fig. 21: Comparison of Sum Interference Values Obtained using Optimal Solution, with those Obtained using ABCD Algorithm, and also using Random Allocation, for Scenarios 5 and 6 as given in Table 4.

Table 4: Scenarios for Comparing Optimal vs. ABCD Algorithm vs. Random Allocation.

Parameter	Value
Scenario 1	
Total number of TSBs	3
TSB distribution	$J_1 = \{1, 2\}, J_2 = \{3\}$
Number of APs	4
Scenario 2	
Total number of TSBs	6
TSB distribution	$J_1 = \{1, 2, 4, 5\}, J_2 = \{3, 6\}$
Number of APs	4
Scenario 3	
Total number of TSBs	7
TSB distribution	$J_1 = \{1, 2, 4, 5\}, J_2 = \{3, 6, 7\}$
Number of APs	8
Scenario 4	
Total number of TSBs	10
TSB distribution	$J_1 = \{1, 2, 4, 5, 8, 10\}, J_2 = \{3, 6, 7, 9\}$
Number of APs	8
Scenario 5	
Total number of TSBs	3
TSB distribution	$J_1 = \{1, 2\}, J_2 = \{3\}$
Number of APs	8
Scenario 6	
Total number of TSBs	3
TSB distribution	$J_1 = \{1, 2\}, J_2 = \{3\}$
Number of APs	16

of time spectrum blocks continuing from J_1 to J_2 . One of the contributions of ABCD algorithm is to keep the APs on the TSBs continuing from the previous interval as much as possible, depending upon the availability of TSBs and the optimal low interference values obtained using the algorithm. This continuity will ensure less switching of APs from one band of spectrum to another. This will reduce the burden on the radio. Also, it will reduce any switching overhead that is caused due to switching, every time a new interval begins. This further ensures that instead of a random jump every time, the decision of staying in

the same band or hopping to another band in the next interval is based on the condition that if the TSB is continuing from previous interval or not.

Fig. 20 shows the first four scenarios as given in Table 4 and Fig. 21 shows the fifth and sixth. In order to separate huge variations in values caused by much higher values in scenarios 5 and 6, as compared to those in scenarios 1-4, and to make the plots aid towards better understanding, they are plotted separately. From Figs. 20 and 21, it can be observed that the algorithm gives optimal and near optimal solution. It is clear from the little variations in the sum interference values for scenarios 3 and 4 that the algorithm gives a different allocation compared to the optimal solution. Since the sum interference values are close, the allocation is good and near optimal.

Fig. 22 shows the plot between sum interference and number of APs when 2 TSBs are available during J_k under consideration. The Figure shows the plots for the two kinds of networks, that is WLAN and WRAN, considered in Tables 2 and 3. It can be seen from the Figure that the plots obtained from the algorithm overlaps with the one obtained using optimal. By increasing the number of APs for a given M, the affect of increasing the number of APs is observed. It must be noticed that the sum-interference shown in the figures is the interference obtained by adding the interference indices for TSB allocation obtained using optimal solution and the one obtained using the algorithm. After summing, the plots of the results for varying N are generated. It can be observed that the sum interference values for WLAN and WRAN are different and the results are as expected.

The sum interference values for WRAN are comparatively lower than WLAN values. This is due to much higher transmission range in case of WRANs and hence the sum interference reduces as the interference indices decrease due to decrease in overlapping

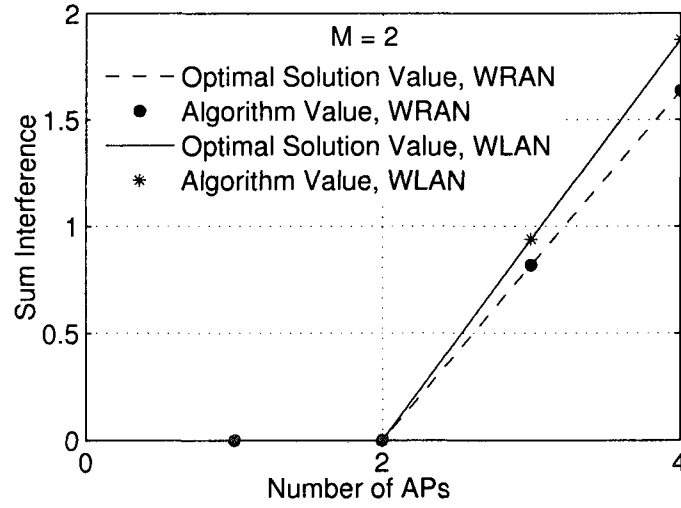


Fig. 22: Comparison of Sum Interference Values Obtained using Optimal Solution and Algorithm for Number of APs Varying from 1 to 4, when 2 TSBs are Available during J_k .

area compared to the overall area. Note that the distance between APs considered for both WLAN and WRAN is the same fraction of their transmission ranges considered in all the cases. Similarly the experiments are performed with other values of M and varying N . In Fig. 23, $M = 5$ and N varies from 1 to 16. It can be observed that the optimal and algorithm results overlap almost completely. It must be noticed further that in most of the cases, if not all, the WLAN and WRAN TSB allocations are different as expected (not given here due to space limitations), since the interference index values change, and also the transmission ranges and interference ranges change. In cases where there are not many APs and TSBs, as in Figure 22, the same allocation can be expected as there are not many possibilities. Thus in such cases, the optimal allocations and those obtained from algorithm are the same for both WLAN and WRAN for similar distribution of APs in space.

In Fig. 24, for $M = 10$ and N varies from 1 to 20, there is very small deviation from 11 to 16 APs and for 20 APs, which is not very significant for WRAN. It suggests that the

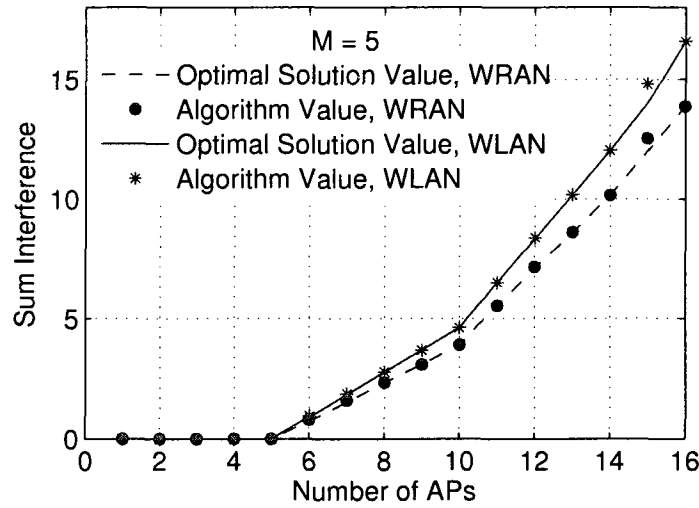


Fig. 23: Comparison of Sum Interference Values Obtained using Optimal Solution and Algorithm for Number of APs Varying from 1 to 16, when 5 TSBs are Available during J_k .

optimal is giving different configuration of TSB allocation as compared to the algorithm within 11 APs to 16 APs and for 20 APs. The algorithm is still giving TSB allocation very close to the one that gives least interference given by optimal solution. It can be observed from the comparison of WRAN and WLAN that, even if the distribution of APs in space is changed, the algorithm gives the allocations that are very close in interference values as those obtained using optimal solution allocation. It thus gives an important insight into how the algorithm provides near optimal solutions for varying distributions of APs.

In order to determine the overall system capacity, Shannon's Capacity Theorem was used given as follows.

$$C_s = \sum_{i=1}^{N_T} W \cdot \log\left(1 + \frac{P_i}{\eta + \sum_{j \neq i} I_{j,i}}\right) \quad (30)$$

The system capacity is given by C_s , where W is the width of the spectrum, P_i is the power received at i_{th} node from the AP to which it is associated. η is the noise floor and $I_{j,i}$ is the interference caused by j_{th} node on node i . Thus if j_{th} node belongs to the same AP as node i , $I_{j,i} = 0$.

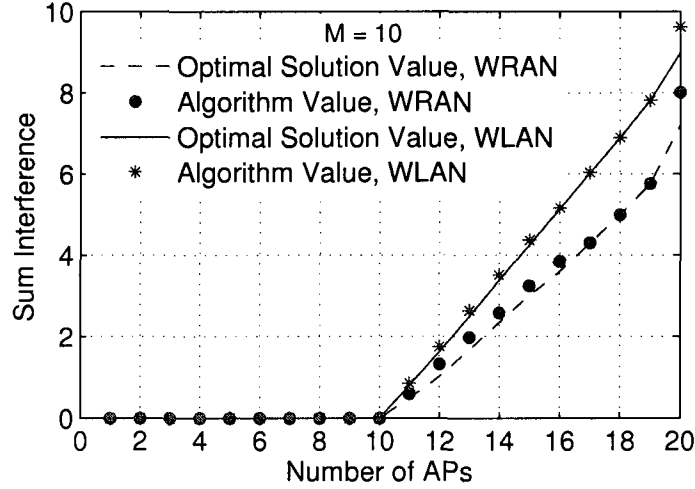


Fig. 24: Comparison of Sum Interference Values Obtained using Optimal Solution and Algorithm for Number of APs Varying from 1 to 20, when 10 TSBs are Available during J_k .

Next, the effect of TSB allocation configuration on the system capacity is studied. Based on the analysis of the plots obtained between normalized system capacity and TSB allocation configuration, it can be seen how it is important to use optimal results instead of using any random allocation. A specific TSB allocation configuration means the possible arrangement of APs in different TSBs. There are different arrangements or configurations possible depending upon M and N . Out of all possible configurations, few configurations were randomly selected to compare with the optimal configuration obtained using optimization tool. Also, the configuration with the lowest possible system capacity is considered, that is, the configuration with all 1s is considered in all the cases for comparison

between best and worst possible configurations. For example, consider the easiest case of $M = 2$ and $N = 2$. The possible configurations are only 2, that is the best configuration, which is $[1 \ 0; 0 \ 1]$ and the worst configuration, which is $[1 \ 1; 0 \ 0]$. In Fig. 25, for $M = 2$ and $N = 4$, it can be observed that the difference between the best and worst is around 40 percent. Other configurations, if randomly selected may result in a very low system throughput. It can be observed from Figs. 25, 26, and 27 that many other configurations give a significant degradation in the overall system throughput compared to that achievable through finding the optimal solution. It is thus important to consider optimal solution due to gain in overall system performance and the simple ABCD algorithm provides near optimal solution in all the cases.

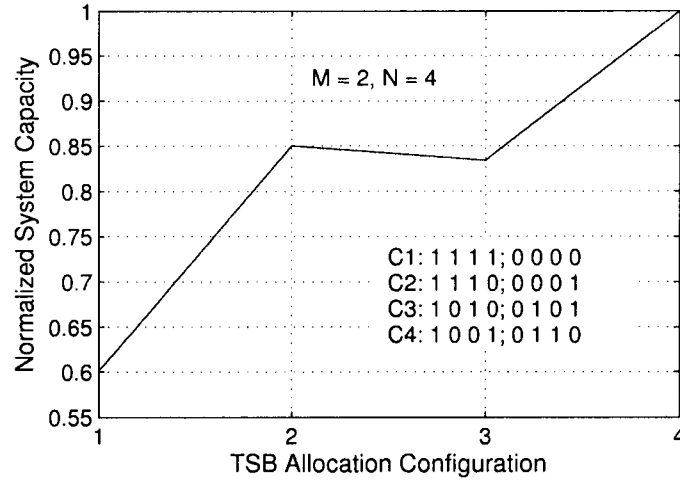


Fig. 25: TSB Allocation Configuration for $M = 2$, $N = 4$.

In Fig. 26, it was possible to accommodate the possible configurations, but due to lack of space, it is not possible to provide the configuration for Fig. 27. It would be better to discuss this case without providing the configuration values, since there are 16 APs and 3 TSBs, and it would take lot of space and would be difficult to interpret also. The important

point to understand in this case also is that there is more than a 50 percent difference between the best and the worst. It means that if the capacity per AP is say 100 Mbps, then the worst case provides only 50 Mbps. Thus a great deal of throughput can be gained by considering optimal or near optimal solution. With increasing number of M and N , the number of possible configurations greatly increases. Many configurations are possible that are closer to the worst case configuration of all 1s. Also, there are many configurations that lie very close to the optimal. The ABCD algorithm provides an optimal or near optimal solution.

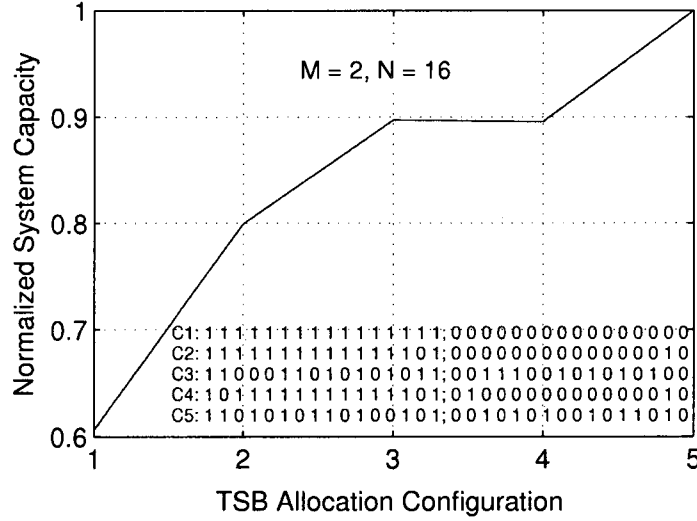


Fig. 26: TSB Allocation Configuration for $M = 2, N = 16$.

IV.4 ALGORITHM COMPLEXITY ANALYSIS

The algorithm has $O(N \times M)$ complexity over each J_k , that is, there are two nested loops to allocate M TSBs to N APs. Thus, if it runs over all the J_k s the complexity is $O(N \times M \times K)$, where K is the number of time intervals over which the TSB allocation is considered. For

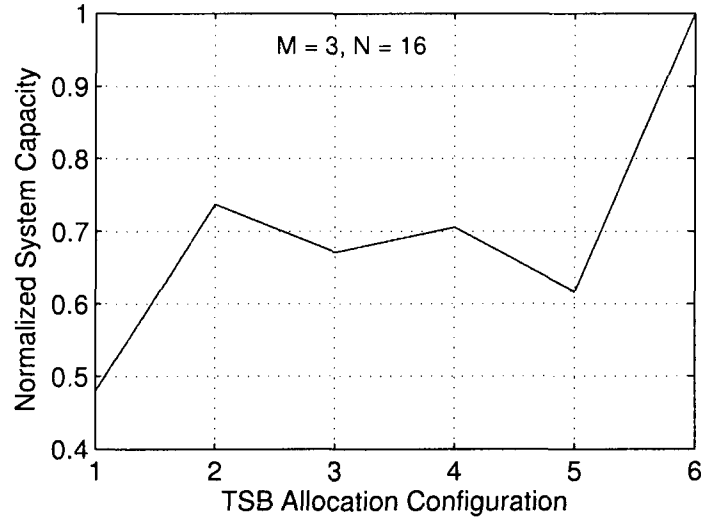


Fig. 27: TSB Allocation Configuration for $M = 3$, $N = 16$.

low density networks with small N and small K , the system complexity is small and is of the order similar to linear complexity. If the density of the network is high, which means that there is a large number of N , then the system complexity is predominantly determined by N . In the worst case, the overall algorithm complexity is not expected to be greater than $O(N^2)$. Since, M , N and K are independent of each other, it is not possible to determine a straightforward relationship. In short, the complexity of the system would also be $O(N \times M \times K)$ and by observing the values of M , N and K it can be said that the system complexity is very small for practical purposes. This simple algorithm can thus easily be implemented to practical systems without much complexity.

IV.5 CONCLUSIONS

This Chapter presented the throughput optimization based time spectrum block allocation in community cognitive radio networks. An **ABCD** (Allocation of time spectrum **B**locks

in Cognitive radio networks for Dynamic spectrum sharing) algorithm for throughput optimization based TSB allocation in community cognitive radio networks was proposed. Using this algorithm, a near optimal solution to the problem of TSB assignment was presented. Comparing the algorithm results with optimal results, it was found that the algorithm provides near optimal results in all the cases. Thus the algorithm can be easily implemented in practical scenarios. For future work, the system throughput optimization as considered can be analyzed based on extensive simulations using network simulator (for example Qualnet, NS2 or OPNET). Based on these simulations, more insight into the implementation of the algorithm can be gained for practical scenarios. In the system throughput optimization based TSB allocation, it was assumed that the TSB bands are fixed length spectrum bands and hence variable spectrum bandwidths are not considered. The problem formulation can be further extended by considering variable bandwidths. By dynamically varying the bandwidths shared between various APs, a more exhaustive understanding of the way varying bandwidths affect system throughput can be had.

CHAPTER V

CONCLUSIONS AND FUTURE DIRECTIONS

This dissertation has provided some very important and useful results in the area of optimization of dynamic spectrum sharing in Cognitive Radio Networks. This Chapter concludes with a summary of contributions and expounds on several possible directions for future research.

V.1 CONCLUSIONS

The main contributions of this work are as follows.

- The throughput analysis was presented for a contention-based dynamic spectrum sharing model. A system was considered that is comprised of two types of users, primary users or licensed users of the spectrum, and secondary users or the unlicensed users of the spectrum, and these users themselves contend for the channels with their own types (primary vs. primary, secondary vs. secondary). The throughput of the system was found by considering the throughput of primary and secondary users when secondary users try to opportunistically use the channels and primary users themselves contend with each other.
- Application of the throughput model was presented for contention-based DSS in finding optimal number of secondary users. After developing a general throughput model, an application of this model was presented. The throughput model was used in finding the optimal number of secondary users that can dynamically share

spectrum with primary users. Thus, this dissertation further answers the question, “How much dynamic spectrum sharing is optimal?” The answer to this question is provided in terms of finding the optimal number of secondary users that can dynamically access and share the channel with primary users. Two solutions were provided to attack this problem. Two different numerical formulas were presented as they using two different approaches.

- Throughput optimization based time spectrum block allocation in community cognitive radio networks was presented. An **ABCD** (**A**llocation of time spectrum **B**locks in **C**ognitive radio networks for **D**ynamic spectrum sharing) algorithm was proposed for throughput optimization based TSB allocation in community cognitive radio networks. Comparing the algorithm results with optimal results, it was found that the algorithm provides near optimal results in all the cases.

The following conclusions are drawn from the research work presented in this dissertation.

- For the contention based DSS model, using the formula for the expected number of available channels for each configuration, it can be easily understand as to why and how the number of secondary users that can opportunistically access a channel change. Thus, the more distributed are the primary users over all the channels, the lesser are the chances for the number of secondary users to actually be able to access channels opportunistically.

- Using the throughput model developed in Chapter III, it is easy to figure out the components that determine the total throughput of the system. Considering the traffic generation probability to be the same for both primary and secondary users, the throughput of the system at any time depends upon the traffic generation probability, the number of primary users in each channel, the number of active secondary users and the number of available channels. An interesting observation is that the more even the allocation of primary users, i.e., the number of primary users in each channel is more balanced, the higher the throughput is.
- For the contention based DSS model, when the per-channel traffic load decreases, a channel is less congested and is accessible to secondary users in more time. Thus the channel can accommodate more secondary users. Similarly, when the per-channel traffic load increases, the channel becomes more congested, and thus the number of secondary users that can be accommodated to obtain maximum total throughput should be reduced.
- Two different formulations for finding the optimal solution are presented. It was found that the second approach has comparatively much lower complexity for both fixed and random allocations.
- The ABCD Algorithm follows closely with the optimal solution obtained using optimization tool. It is a simple algorithm to provide effective solution for TSB allocation in community cognitive radio networks.

- The algorithm complexity is also low and hence it can be easily implemented in real time practical cases.

V.2 FUTURE DIRECTIONS

There are several ways to extend this research, which are briefly discussed below.

In Chapter III, the research can be further extended in the following directions.

- The throughput model can be further analyzed based on simulations using network simulator (for example, Qualnet, NS2 or OPNET). By using the results obtained from the optimal solution through the exhaustive search, and through the mathematical formulas, the simulation results can be compared to establish a more complete analysis.
- A protocol can be designed for achieving the optimal performance based on the throughput model we developed, and using the optimal number of secondary users obtained for each traffic generation probability.
- Also, few of the assumptions that were made in the methodology related to the throughput analysis of contention-based dynamic spectrum sharing model can be eliminated. For example, the model can be extended to the more practical cases where traffic generation probability for primary users and secondary users are different and also variable.

In Chapter IV, the research can be further extended in the following directions.

- The system throughput optimization as considered can be analyzed based on extensive simulations using network simulator (for example Qualnet, NS2 or OPNET). Based on these simulations, more insight into the implementation of the algorithm can be gained for practical scenarios.
- In the system throughput optimization based TSB allocation, it was assumed that the TSB bands are fixed length spectrum bands and hence variable spectrum bandwidths were not considered. The problem formulation can be further extended by considering variable bandwidths. By dynamically varying the bandwidths shared between various APs, a more exhaustive understanding of the way varying bandwidths effect system throughput can be had.

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