Multidisciplinary Sensitivity Analysis and Design Optimization of Flexible Wings Using the Euler Equations on Unstructured Grids

Arunkumar Satyanarayana
Old Dominion University

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MULTIDISCIPLINARY SENSITIVITY ANALYSIS
AND DESIGN OPTIMIZATION OF FLEXIBLE WINGS
USING THE EULER EQUATIONS ON UNSTRUCTURED GRIDS

by

Arunkumar Satyanarayana

Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirement for the Degree of

DOCTOR OF PHILOSOPHY
MECHANICAL ENGINEERING
OLD DOMINION UNIVERSITY
December 1999

Approved by:

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______________________________
Stephen Cupschalk (Member)

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ABSTRACT

MULTIDISCIPLINARY SENSITIVITY ANALYSIS AND DESIGN OPTIMIZATION OF FLEXIBLE WINGS USING THE EULER EQUATIONS ON UNSTRUCTURED GRIDS

Arunkumar Satyanarayana
Old Dominion University, 1999
Director: Dr. Gene J.-W. Hou

Aeroelasticity is a classical discipline. However, even with recent advancement in computational technology, it still remains a challenging discipline. This is particularly true when aeroelasticity problems are solved in a loosely coupled manner. The advantage of a loosely coupled scheme is that the legacy codes of computational fluid dynamics (CFD) and computational structural mechanics (CSM) can be preserved and used as independent modules in solving the desired aeroelasticity problems. In such a scheme, maintaining proper data transfer (load transfer and deformation tracking between CFD and CSM codes) is crucial in ensuring successful coupled solutions. This can be a challenging task because the interface (the wetted area or the outer mold line) may be discretized differently as required by different levels of computational domains, which leads to mismatch and even gaps. Most published works overcome these drawbacks by transferring the aerodynamic loads and elastic deformation through projection and curve-fitting. The projection is used to find the “host” node, element or Gaussian point from a CSM mesh to the associated CFD mesh, or vice versa. It is then followed by local or global curve-fitting so that the nodal values on the projected surface can be extracted from interpolation. Most of these works can not guarantee a “consistent and conservative” load transfer. Further, they have not adequately demonstrated their availability to support coupled sensitivity analysis.
A new remeshing scheme that can guarantee consistent and conservative load transfer and smooth deformation tracking between CFD and CSM is proposed here not only for coupled analysis, but also for coupled sensitivity analysis. The method will introduce an artificial interface structure that is confined with the aerodynamic surface mesh and is supported at the structural surface nodes. This structure is used to redistribute the aerodynamic load as well as the structural deformation. With the help of this artificial structure, the same design parameter that guides the remeshing processes of the structural mesh can be used to guide the remesh processes of the artificial interface structural mesh as well as the aerodynamic interior mesh. This particular feature of the proposed remeshing scheme allows the aerodynamic sensitivity coefficients of a rigid wing to be predetermined and later used for coupled sensitivity analysis that includes only the structural sensitivity code in an iterative routine.

A flexible wing with a 3-dimensional Euler flow and a linear finite element model is considered in the present work to demonstrate the proposed scheme for coupled analysis and sensitivity analysis. Preliminary results obtained from the optimization process are presented to substantiate the efficiency of proposed schemes.
This thesis is dedicated to my parents.
ACKNOWLEDGMENTS

First and foremost, I would like to express my gratitude and heartfelt appreciation to my advisor and mentor Dr. Gene J.-W. Hou. His technical expertise and guidance has helped me throughout my research. He was there to help answer my questions, and has always channeled me in the right direction. I feel honored to have worked with him.

I would like to thank my committee members, Drs. Sebastian Bawab, Duc Nguyen and Stephen Cupschalk, for the useful and encouraging discussions during my graduate studies. I would also like to thank Dr. Chaturvedi and Dr. Tiwari for their support, without which I would not have completed my research. Their guidance and suggestions have always been a great help to me. I also would like to thank Dr. Newman III, J. C., for providing me with the technical aid during the initial stages of my research work.

I would like to thank my father (my role model) Mr. Satyanarayan, and my mother Satyalakshmi for having provided me with strong support and encouragement. I could not have come this far without them.

I would like to thank my wife Nalina for her moral support and for tolerating all my idiosyncrasies during my various ups and downs in the research, and also for allowing me to work late hours.

I would also like to thank my brother-in-law, Dr. Manjunath K. Ramarao, and my sister, Suma, for having initially interested me in furthering my education, and also for their constant support. Thanks also to my brother Madhu who has been there for me.

I would like to take this opportunity and thank my friends Dr. Satish Boregowda, Srikanth Pidugu, Dinesh Kaushik and Ajit Mandgi, for their positive influence and moral support throughout my graduate studies.
Last but not least, I would like to thank the Almighty for providing me with the opportunity to attain this level. Thanks again to everyone who has helped me throughout my journey.

Arunkumar Satyanarayana
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<tr>
<td>AAO</td>
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<td>CFD</td>
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<td>CST</td>
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<td>DOT</td>
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<td>IDF</td>
<td>Individual Discipline Feasible</td>
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<td>MDA</td>
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<td>MDF</td>
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<td>MPC</td>
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<td>NURBS</td>
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\( U \) Triangular Matrix
\( U_a \) Aerodynamic Mesh Displacements
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\( V_f \) Approximated Displacement Vector for Aerodynamic Surface Mesh Movement
\( V_s \) Approximated Displacement Vector for Structural Mesh Movement
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\( V_{fi}, V_f \) Basic Vectors of Aerodynamic Surface Wing
\( V_{si}, V_s \) Basic Vectors of Structural Wing
\( X_a \) Aerodynamic Mesh
\( X_{a0} \) Initial Aerodynamic Mesh
\( X'_{a} \) Derivative of Aerodynamic Mesh
\( Y \) Intermediate Solution Vector in Cholesky Factorization
\( a_{\infty} \) Speed of Sound
\( b \) Design Variable
\( b_{\text{min}} \) Lower Limit of Design Variable
\( b_{\text{max}} \) Upper Limit of Design Variable
\( e \) Element
\( e_0 \) Total Energy per Unit Mass
\( f \) Body Force
\( i \) Nodes
\( k \) Number of Faces
\( k_{se} \) Structural Element Stiffness Matrix
\( \hat{n} \) Unit Normal
\( \hat{n}_x, \hat{n}_y, \hat{n}_z \) Unit Normal Along x, y, z Directions Respectively
\( p \) Pressure
\( p' \) Derivative of Pressure
\( q \) Element Displacement Vector
\( r_s \) Global Structural Force Vector
\( r'_s \) Derivative of Structural Force Vector
\( s \) Surface
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<td>$\delta$</td>
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CHAPTER I

LITERATURE SURVEY

1.1 OVERVIEW

On one hand, the competition of commercial and military markets and the human desire for exploration of the unknown have forced scientists and engineers to analyze and simulate larger and more sophisticated systems and phenomena. These sophisticated systems and phenomena are in general, the results of interaction between various governing equations, each of which was the focus of a prior disciplinary study. For example, the design of wing structure was usually completed based upon the aerodynamic load calculated by assuming a rigid wing. Thus, the aerodynamic load was calculated by the aerodynamic code alone, without interference from the elastic deformation. Recently, however, it has become desirable to directly consider the elastic interaction in calculation of aerodynamic load that results in the coupling of elasticity and aerodynamics.

On the other hand, the advances in computer technology have now enabled scientists and engineers to investigate the insight of these sophisticated systems and phenomena by solving these coupled governing equations in numerical terms. To this end, however, scientists and engineers face two choices. They can either generate the computational model and code from scratch based upon the best tools and algorithms available, or they can coordinate and modify the "legacy" disciplinary codes to achieve the same purpose. For example, Ghattas and Li [1] established and solved the coupled fluid-structural equations with the same numerical discretization, while Walsh et al [2] combined different disciplinary codes for aeroelastic analysis. The former method can usually...
generate a code that can achieve better accuracy and efficiency, though it takes more resources and expertise to develop such a code. The motivation of the latter method, however, is to reduce the code development time by taking advantage of the "legacy" codes that have been validated and used for many complicated disciplinary problems. The challenge of the latter method is, therefore, to coordinate the "legacy" codes with minimal code modification for multidisciplinary applications. This research undertaking represents an effort to respond to such challenge. Particularly, this research will use a flexible wing as an example to study interface methodology that can lead to an efficient method to coordinate legacy codes for multidisciplinary applications.

A flexible wing is a typical example of static aero-elastic problems that include disciplines of grid generation, structural mechanics and aerodynamics. Thus, the nature of a flexible wing has attracted the attention of many researchers in the field of multidisciplinary analysis and design.

The organization of this document is as follows. Sections 1.2 to 1.4 will summarize the literature survey. In Chapter 2, the coupled analysis methodology, along with a new method for load transformation are discussed. Chapter 3 is dedicated to the coupled sensitivity analysis. Here, the introduction of basic vectors to approximate elastic deformation, as well as shape changes have led to decoupling of aero-structural sensitivity analysis. Chapter 4 outlines a multidisciplinary design optimization strategy and demonstrates the effectiveness of load transferring and deformation tracking methods, used in flexible wing shape optimization, with the aid of numerical results. Finally, Chapter 5 ends this thesis with conclusions and indicates the direction for future research.
1.2 COUPLED DESIGN OPTIMIZATION

Recent works of Arian, Shubin and Haftka et al [3, 4, 5, 6] used aeroelastic problems as examples to discuss various solution strategies for multidisciplinary analysis (MDA) and design optimization (MDO). Particularly, Shubin [5] investigated computational characteristics of three alternatives: multidisciplinary feasible (MDF); individual discipline feasible (IDF); and all-at-once (AAO) approaches. The MDF approach will perform optimization at the converged multidisciplinary solution, while the IDF approach will perform optimization with converged disciplinary solution. On the other hand, the AAO approach, which is the same as the simultaneous analysis and design optimization (SAND) method, treats the disciplinary state equations as equality constraints in formulation. Shubin suggested that the AAO approach is preferable for large scale problems. Haftka and Sobieszczanski-Sobieski recognized the importance of Jacobian matrix of the system equations, which are called the global sensitivity equation (GSE) matrix, in determining the coupling procedure in MDO. Finally, it was Arian [4, 7] who analyzed the Jacobian matrix of the system equations with respect to the design variables, and the Hessian matrix of the system equations with respect to the state variables, to derive mathematical conditions that can determine whether MDA, MDF and MDO can be solved in a "loosely coupled" manner.

Recent works done by Reuther et al [8] and Walsh et al [2] are typical examples of utilizing the MDF approach for aeroelastic optimization, whereas the work done by Raveh and Karpel [9] is a mix of the IDF and the AAO approaches. They halted the progress of aerodynamic analysis for structural optimization and trim correction in their study. All of the above mentioned papers used high fidelity computational fluid dynamic
(CFD) and finite element structural codes. Particularly, references [8] and [9] illustrated that their aeroelastic results are in agreement with testing data, while reference [2] placed emphasis on integrated software development. Other works of MDO in early aerospace applications can be found in reference [10].

1.3 COUPLED ANALYSIS

This research employs the MDF approach to investigate static aeroelastic problems. Particularly, the research will use the generalized Gauss-Seidel (GGS) approach [11] for coupled analysis, in which the disciplinary analysis will be performed in a sequential manner to achieve a multidisciplinary solution. The challenge encountered in such a loosely coupled procedure is to maintain proper data transfer between the disciplines. The aerodynamic analysis tends to faithfully represent the aircraft geometry including details such as flaps, pylons and nacelles, whereas the structural analysis concerns mainly the strength members. Therefore, it is quite often that, depending upon the degree of fidelity used in the disciplinary analysis, their associated surface meshes contain mismatches and even gaps. These incongruences can cause numerical difficulties in dealing with aerodynamic load transfer and elastic deformation update. All of the papers surveyed here have proposed remedial procedures to address such difficulty in aeroelastic analysis.

The interface methodologies suggested in the surveyed papers may be collectively categorized into two approaches: curve-fitting and mapping, and artificial interface structure.

The first step in the methods of curve-fitting and mapping is to find the projected or mapped point on the structural surface. The fluid or structural points involved can be at
the vertices, the center or the Gauss point. The projection may be accomplished by determining the shortest distances between the points [12] or based upon the normal projection [13]. The displacement of the structural point can be transferred to the fluid surface point through a rigid element connecting the involved points [12-15]. Once the displacements on the selected fluid surface points are calculated, they may either be curve-fitted to obtain the displacement field of the entire wetted surface [9,12,16,17] or interpolated locally to obtain the displacements at the desired points in the neighborhood [14, 18].

Once the relationship between the structural surface displacements and the fluid surface displacements is identified, the loads applied to the structural surface nodes may be obtained as a function of the aerodynamic loads through the consistency of the virtual work. That is, the work done by the structural load is the same as that of the aerodynamic load. However, this procedure does not guarantee the conservative aspect of load transferring method. For example, the resultant aerodynamic loads may not necessarily be equal to the resultant structural loads. Cebral and Lohner [14] thus developed a conservative load projection method to transfer aerodynamic load. Although their new method preserves the magnitude of the loads, it fails to guarantee the consistency of the loads.

Samareh [19-22] considered the design representation in his curve-fitting procedure. In his works, a non-uniform rational B-spline (NURBS) representation is first constructed to model the wing or aircraft. The structural displacement is not projected onto the aerodynamic surface mesh; instead, it is projected onto the NURBS model. A new NURBS is then constructed to represent the deformed geometry with which the new
aerodynamic surface mesh may be established. The load transfer can then be accomplished in a similar manner. Nevertheless, this method may not be consistent or conservative.

From the design point of view, Samareh's method offers a distinct advantage. Since the geometry of the NURBS surface is regulated by the control points, the coordinates of those points become a natural choice in shape design variables. Furthermore, since the NURBS representation is a linear function of the coordinates of the control points, the shape derivatives of the NURBS surface are readily available. Thus, the shape derivatives of the load transfer and deformation tracking necessary in coupled aeroelastic sensitivity analysis can be obtained without difficulty [23].

The second group of methods [24-27] introduces an artificial structure to cover the interface between the structural model and the aerodynamic model. This artificial structure is discretized into a mesh pattern which is the same as that of the aerodynamic surface mesh. To update the elastic deformation, the artificial structure is forced to be deformed according to the structural deformation. This is achieved by introducing Lagrange multipliers in the minimization process. The corresponding structural loads can be calculated from the aerodynamic loads based upon the condition that the virtual work is conserved in load transfer.

1.4 COUPLED SENSITIVITY ANALYSIS

The coupled sensitivity equation may be derived directly from the coupled state equation or based upon the input and output responses of each discipline involved in the coupled analysis. The sensitivity equations described by the latter approach are called the global sensitivity equations (GSE). Only limited literature on aeroelastic problems has
elaborated on the coupled sensitivity analysis. Arslan and Carlson [28] have derived the GSE for coupled sensitivity analysis, which was solved using Newton's method. The terms in the sensitivity equations associated with structural disciplines were generated by finite difference. Similar to the previous work, Giunta and Sobieszczanski-Sobieski [29] also have derived the GSE, in which the completed right-hand side terms are calculated using finite-difference scheme. Particularly, Giunta [30] introduced modal coordinates to approximate the elastic displacement vector in order to reduce the size of the GSE. Issac and Kapania [31] solved the flutter equation using the code FAST [32] in which the modal variables are calculated by MSC/NASTRAN [33]. The sensitivities of the flutter velocity were calculated by the ADIFOR-enhanced code of FAST. The seed matrices submitted to the ADIFOR-enhanced code are the derivatives of modal variables with respect to geometrical design variables, which are obtained by finite differencing. Ghattas and Li [1] have constructed the coupled sensitivity equations by differentiating the coupled state equations and solving them using domain decomposition methods. By selecting a proper preconditioning method, the solution procedure proposed in this paper becomes identical to the Generalized Gauss-Seidel (GGS) method, which is the method used in the current work. Newman et al [34] has used the complex variable approach to obtain coupled sensitivity derivatives. The regular aeroelastic code has first been modified to accommodate complex variables. The imaginary part of the output function obtained from the coupled analysis, in which the imaginary part of the new design is considered as the design perturbation, and is equal to the product of the derivative and the perturbed design.
CHAPTER II
AEROELASTIC WING ANALYSIS

2.1 INTRODUCTION

Solving an aeroelastic problem (the subject of this research) consists of performing aerodynamic and structural analysis simultaneously. Many researchers working in the field of multidisciplinary design optimization have recognized the necessity of coupled analysis to obtain the aerodynamic coefficients accurately, thus arriving at an optimal design. Based upon the procedures adopted for coupling, a coupling can be classified as loosely coupled or strongly coupled. A strongly coupled approach solves aerodynamic and structural equations simultaneously, while a loosely coupled approach solves each problem in a sequential, iterative mode. Both approaches face the challenge of data transfer between the coupled aerodynamic and structural disciplines. The two major issues of interface data transfer are, (i) transferring forces from aerodynamic grid points to structural node points and, (ii) transferring structural wing displacements to aerodynamic mesh.

2.2 LINEAR STRUCTURAL ANALYSIS

Analysis of a linear structure consists of determining the structural responses due to the external forces acting on the structure. The external load may be concentrated forces, pressure forces, surface traction, temperature, etc. For a simple structure, the system responses can be obtained by directly solving the basic equilibrium equations of the elasticity while satisfying all of the imposed boundary conditions. This approach is called the exact solution method. This type of solution method can be applied only to simple geometries and load conditions. For structures which have complex geometric shapes and
complicated load conditions, it is nearly impossible to find the exact solution. Therefore, approximate solution methods are developed to solve such complex structural problems using potential energy or variational methods [35,36].

The potential energy of an elastic body can be represented as:

$$\pi = U - W,$$  \hspace{1cm} (2.1)

where $U$ is the internal energy of the elastic structure and $W$ is the work done by the external load acting on the structure. For linear elastic structure, the internal energy stored in the body and the work done by the external forces can be defined as:

$$U = \frac{1}{2} \int_{\Omega} \sigma^T \varepsilon \, dv$$

and

$$W = \int_{\Omega} u_s^T f \, dv + \int_{s} u_s^T T \, ds + \sum_{i} u_{si}^T r_{si}.$$

where $\sigma$ is the stress induced in the structure due to external load and $\varepsilon$ is the strain induced in the structure. In the above expression, $u_s$ is the displacement of the structure and $f$, $T$, $r_s$ and $v$ are the body forces, surface traction, point forces and volume of the structure respectively.

The principle of potential energy states that the amount of work completed by the internal stress and the external forces must be minimized in order for the structure to be in equilibrium. In other words, the potential energy of the system should be minimized in order for the system to be in equilibrium.

Finite element analysis involves finding an approximate solution for the displacement field of the structure subjected to the external load. Hence, the displacement-based
finite element method consists of discretizing the whole structure into finite elements. Potential energy in each element must be maintained at a minimum state to ensure equilibrium in the deformed structure. In displacement-based finite element analysis, nodal displacements are the primary unknowns. Therefore, a set of simultaneous algebraic equations for $u_s$ may be obtained by expressing the potential energy function in terms of nodal displacements $u_s$, utilizing appropriate interpolation, and minimizing the potential energy functional with respect to $u_s$.

The potential energy expression for the discretized structure can be written as:

$$\pi = \sum_e \frac{1}{2} \int_{v_e} \sigma^T \varepsilon dv - \sum_e \int_{v_e} q^T f dv - \sum_e \int_{v_e} q^T T ds - \sum_{v_e} u_{si} r_{si}.$$ 

For linear material, stress and strain can be expressed as $\sigma = DBq$ and $\varepsilon = Bq$, where $D$ and $B$ are known as material matrix and element strain displacement matrix, respectively. The symbols $e$ and $v_e$ indicate element and the elemental volume, respectively. Substituting for stress and strain in the potential energy expression and considering only the point load acting on the structure, the potential energy equation takes the form:

$$\pi = \sum_e \frac{1}{2} q^T k_{se} q - \sum_i u_{si} r_{si},$$ 

where $q$ is the elemental displacement vector, $k_{se}$ is known as the element stiffness matrix and is denoted as $k_{se} = \int_{v_e} B^T DBdv$, $u_{si}$ is the global displacement vector, and $r_{si}$ is the global load acting at point $i$. The potential energy of the complete structure can be written as:

$$\pi = \frac{1}{2} u_s^T K_s u_s - u_s^T r_s,$$
where $K_s$ is global stiffness matrix, $u_s$ is the global displacement vector, and $r_s$ is the global load vector. Minimizing potential energy of the complete structure leads to the expression:

$$K_s u_s - r_s = 0.$$  \hfill (2.2)

Equation (2.2) is known as the equilibrium equation of linear structures in finite element form.

### 2.2.1 Solution of Equilibrium Equations

A solution to the finite element equation Eq.(2.2) can be obtained in three steps. The first step is to factorize the stiffness matrix $K_s$ into lower and upper triangular matrices, which is known as Cholesky factorization. Since linear structural stiffness matrix is symmetric in nature, the elements in the lower and upper triangular matrix will be the same. Therefore, Eq.(2.2) can be written as:

$$U^T U u_s = r_s .$$

The second step involves solving the following equation for $Y$, which is called forward substitution:

$$U^T Y = r_s .$$

The third step consists of solving the following equation for the real displacements, $u_s$. This step is known as backward substitution:

$$U u_s = Y .$$

### 2.2.2 Linear Structural Model

In the current work, a generic finite element analysis code [37] is used to compute the system response of the structural wing. The structural wing (Fig.(2.2)) is discretized...
using 162 Constant Strain Triangular (CST) elements, which include the surface, horizontal and vertical girders of the structure. The structure is also reinforced with 28 3-D truss members along the edges of the wing, as well as along the center line of the top and bottom surfaces of the wing. Figure 2.3 represents the network of reinforcements and truss members inside the structural wing.

A truss element is a one-dimensional bar or rod that is assumed to deform under axial stretching only. These elements are pin joined at their nodes, and therefore only translational displacements and the initial position vector at each node are used in computation. This element has a single material coordinate and has two node points. Each node has 3 degrees of freedom implying that it can translate along x, y and z directions.

A CST element is a two-dimensional triangular element with 3 nodes and 3 straight sides. Each node has 3 degrees of freedom, while the element as a whole has 9 degrees of freedom. This element has constant thickness and maintains constant strain in the domain.

2.3 NONLINEAR AERODYANAMIC ANALYSIS

Aerodynamic analysis consists of solving fluid dynamic equations in the given flow domain to compute flow parameters such as velocities, pressure, etc. These parameters are computed at each and every point in the flow domain. In the current work, a three-dimensional flow domain is considered, which encompasses an aeroelastic wing fixed at its root (see Fig.(2.4)). The flow field is discretized into tetrahedral cells, and Euler flow is considered in the fluid dynamic analysis.

The governing equation for a time-dependent Euler flow can be written in the conservative form as:
\[
\frac{\partial}{\partial t} \iiint_{\Omega} Q \, dv + \int_{\partial \Omega} F(Q) \cdot \hat{n} \, ds = 0. 
\] (2.3)

The above equation describes a relationship in which the time rate of change of the state vector \( Q \) within the domain \( \Omega \) is balanced by the net flux \( F \) across the boundary surface \( \partial \Omega \). In Eq.(2.3), \( Q \) and \( F \) are known as the state vector and the flux vector respectively, and are represented as:

\[
Q = \begin{bmatrix} 
\rho \\
\rho u \\
\rho v \\
\rho w \\
e_o
\end{bmatrix}, \quad F(Q) \cdot \hat{n} = (V \cdot \hat{n}) + p \begin{bmatrix} 
0 \\
\hat{n}_x \\
\hat{n}_y \\
\hat{n}_z \\
0
\end{bmatrix}.
\]

The equations are non-dimensionalized with a reference density \( \rho_{\infty} \) and the speed of sound \( a_{\infty} \). Here \( \hat{n}_x, \hat{n}_y \) and \( \hat{n}_z \) are the Cartesian components of the exterior surface unit normal \( \hat{n} \) on the boundary \( \partial \Omega \). The Cartesian velocity components are \( u, v \) and \( w \) in the \( x, y \) and \( z \) directions, respectively. The term \( e_o \) is the total energy per unit volume. Using the ideal gas assumption, the pressure can be expressed as:

\[
p = (\gamma - 1) \left[ e_o - \frac{1}{2} \rho (u^2 + v^2 + w^2) \right],
\]

where \( \gamma \) is the ratio of specific heat. Equation (2.3) describes the relationship between the time rate of change of the state vector \( Q \) within the domain \( \Omega \), and the net flux balance \( F \) across the boundary surface \( \partial \Omega \). The governing equation also expresses the conservation of mass, momentum and energy for an inviscid gas. Equation (2.3) is solved by time marching algorithm and by adopting linearized backward Euler time differencing scheme.
which yields a system of linear equations for the solution at each step such as:

\[ [A]^n \{ \Delta Q \}^n = \{ R \}^n, \]  

(2.4)

where

\[ [A]^n = \frac{A}{\Delta t} I + \frac{\partial R^n}{\partial Q}, \]

and \( A \) is the area of the cell, and \( R \) is the steady state residual vector given by:

\[ R = -\oint_{d\Omega} F \cdot \hat{n} d\Omega = \sum_{j = k(i)} F_{i,j} A_{i,j} \]

where \( A_{i,j} \) is the area of the face \( j \) for cell \( i \) through which the flux passes, and \( F_{i,j} \) is the flux across the face \( j \) for cell \( i \). The symbol \( k(i) \) represents the number of faces of cell \( i \). The residual is calculated using the trapezoidal integration method by summing the fluxes over each of the faces that comprise the control volume. In the current work, the inviscid flux vector across the cell’s face and the Jacobian \( \frac{\partial R^n}{\partial Q} \) are computed using Roe’s flux-difference splitting and Van Leer flux-vector splitting techniques, respectively. In Roe’s flux-difference splitting, the flux across each cell face \( k \) is computed using Roe’s numerical formula [38]

\[ F_k = \frac{1}{2} [F(Q_L) + F(Q_R) - (|\Delta \tilde{F}_1| + |\Delta \tilde{F}_2| + |\Delta \tilde{F}_3|)]_k \]

where subscripts \( L \) and \( R \) are the state variables belonging to the left and right cells of the interface \( k \), and \( \Delta \tilde{F}_1, \Delta \tilde{F}_2, \Delta \tilde{F}_3 \) are given as:
\[ |\Delta \tilde{F}_{1} | = | \tilde{u} | \left( \frac{\Delta \rho - \Delta \rho}{\tilde{a}^2} \right) \left[ \begin{array}{c} 1 \\ \tilde{u} \\ \tilde{v} \\ \tilde{w} \\ \left( \tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2 \right) / 2 \end{array} \right] + \tilde{\rho} \left[ \begin{array}{c} 0 \\ \Delta u - \hat{\alpha}_x \Delta U \\ \Delta v - \hat{\alpha}_y \Delta U \\ \Delta w - \hat{\alpha}_z \Delta U \\ \tilde{u} \Delta u + \tilde{v} \Delta v + \tilde{w} \Delta w - \tilde{U} \Delta U \end{array} \right] \]

\[ |\Delta \tilde{F}_{2,3} | = | \tilde{U} \pm \tilde{a} | \left( \frac{\Delta \rho \pm \tilde{\rho} \tilde{a} \Delta U}{2 \tilde{a}^2} \right) \left[ \begin{array}{c} 1 \\ \tilde{u} \pm \hat{\alpha}_x \tilde{a} \\ \tilde{v} \pm \hat{\alpha}_y \tilde{a} \\ \tilde{w} \pm \hat{\alpha}_z \tilde{a} \\ \tilde{h}_0 \pm \tilde{U} \tilde{a} \end{array} \right] \]

where

\[ \tilde{\rho} = \sqrt{\rho_L \rho_R} \]
\[ \tilde{u} = (u_L + u_R \sqrt{\rho_R / \rho_L}) / (1 + \sqrt{\rho_R / \rho_L}) \]
\[ \tilde{v} = (v_L + v_R \sqrt{\rho_R / \rho_L}) / (1 + \sqrt{\rho_R / \rho_L}) \]
\[ \tilde{w} = (w_L + w_R \sqrt{\rho_R / \rho_L}) / (1 + \sqrt{\rho_R / \rho_L}) \]
\[ \tilde{h}_0 = (h_{OL} + h_{OR} \sqrt{\rho_R / \rho_L}) / (1 + \sqrt{\rho_R / \rho_L}) \]
\[ \tilde{a}^2 = (\gamma - 1) (\tilde{h}_0 - (\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2) / 2) \]
\[ \Delta \rho = \rho_R - \rho_L \]
\[ \Delta u = u_R - u_L \]
\[ \Delta v = v_R - v_L \]
\[ \Delta w = w_R - w_L \]
\[ \Delta p = p_R - p_L \]
\[ \ddot{U} = \ddot{u} \hat{n}_x + \ddot{v} \hat{n}_y + \ddot{w} \hat{n}_z \]

and

\[ \Delta U = \hat{n}_x \Delta u + \hat{n}_y \Delta v + \hat{n}_z \Delta w. \]

The Jacobian matrix, \( \frac{\partial R}{\partial Q} \), is expressed in terms of the split Van Leer flux Jacobians.

Expressions for the Van Leer flux Jacobians can be found in references [39 and 40], and are therefore not included in this thesis. The order of accuracy of the aerodynamic analysis, however, is determined through evaluation of the residual vector. In this study, a first-order Jacobian has been found sufficient enough to converge with the implicit scheme.

The solution of Eq.(2.4) may be obtained by utilizing several methods. The first method is by direct inversion of matrix \( A \). This method is expensive in terms of computer memory and time. Hence, an iterative method known as the Gauss-Seidel procedure is employed in this work. This procedure works by first grouping the terms of matrix \( A \) into three matrices representing diagonal, subdiagonal and superdiagonal terms such as:


The current work has adopted a Gauss-Seidel procedure, which is known for its good convergence rate. The associated iterative procedure can be presented as:

\[ [D]\{\Delta Q\}^{n+1} = [\{R\}^n - [M]^n\{\Delta Q\}^{n+1} - [N]^n\{\Delta Q\}^n], \]

where the latest values of \( \{\Delta Q\} \) from the subdiagonal terms are immediately used on the right-hand side of the iteration equation. This algorithm is implemented by sweeping sequentially through each mesh cell and simply using the latest values of \( \{\Delta Q\} \) for all off-diagonal terms which have been taken to the right-hand side. Detailed information

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regarding the solution strategy can be found in references [39 and 40].

2.4 AERODYNAMIC LOAD APPROXIMATION

In a coupled analysis, two major steps are aerodynamic and structural analysis. In the current work, aerodynamic analysis is accomplished by solving nonlinear Euler flow equations on a three-dimensional unstructured computational mesh, known as aerodynamic mesh, $X_a$. This mesh encompasses the fluid domain surrounding the wing. The aerodynamic pressure acting on the wing surface is then converted to forces acting at the structural node points, so that linear structural analysis may be performed. The aerodynamic surface of the wing usually contains more grid points than the structural surface and is referred as aerodynamic surface mesh, $X_f$. Unlike aerodynamic analysis, structural analysis does not require a finer mesh on the structural wing surface to compute accurate wing deformation; moreover, a fine mesh requires more computational effort during structural analysis. Generally, a coarser mesh is used to represent the wing surface for the structural analysis. This surface is known as structural wing surface and the mesh on this surface as structural surface mesh, $X_s$. The equivalent forces acting on the structural nodes must be approximated from the force acting on aerodynamic grid points. This is due to the fact that two different mesh sizes have been adopted for construction of the wing surfaces. That is, fine mesh has been used for aerodynamic analysis, while coarse mesh has been used for structural analysis.

There are two methods of transferring the pressures from the aerodynamic grid to the structural nodes. In the first method, pressures on the aerodynamic wing surface are interpolated onto the structural nodes, and are later integrated to obtain the forces at the structural nodes. Tzong et al [17] opinioned that this conversion procedure may be
improper because of the inconsistency between the aerodynamic and structural meshes. In the second method, the forces at the aerodynamic surface grid points are calculated by using the aerodynamic grid information, and are then transferred to the structural nodes. This method is more accurate and is easier to implement.

Even though the above mentioned methods have been used quite effectively to serve their purpose, researchers are still attempting to obtain a better, simpler and more accurate method for load transfer. In the current work, a different approach called the "Reaction Force Method" has been used successfully in the load approximation. Moreover, this approach was appropriate in this work because of the fact that the structural nodes form a subset of the aerodynamic surface grid points. Forces at structural nodes are obtained simply by performing finite element analysis of an artificial shell structure that covers the surface of the wing with the same mesh as aerodynamic surface mesh. The artificial shell structure is fixed at the nodes, which are indicated by dots in Figures 2.5 and 2.6. These dots correspond to the structural nodes in all degrees of freedom. Once the displacements are obtained, the forces acting at the structural nodes are obtained as the reaction forces acting at the fixed points and are noted with a negative sign. Hence, the equivalent forces acting at the structural nodes are determined by solving a system of equations representing the artificial shell structure as:

\[
\begin{bmatrix}
K_{fI} & K_{fB} \\
K_{fBI} & K_{fBB}
\end{bmatrix}
\begin{bmatrix}
\tilde{U}_f \\
\tilde{U}_s
\end{bmatrix} = \begin{bmatrix}
F \\
r_s
\end{bmatrix},
\]

(2.5)

where \(\tilde{U}_f\) is the displacements of the interior nodes in the artificial shell structure, and \(\tilde{U}_s\) is the displacement of the boundary nodes in the artificial shell structure that
corresponds to the structural nodes. The notation \( F \) represents the forces acting at aerodynamic surface grid points, while \( r_s \) is the reaction forces at the fixed boundary nodes. As the boundary nodes in the artificial shell structure are fixed in all directions, the displacements \( \bar{U}_s \) at the structural nodes are zero. Therefore, the first equation becomes:

\[
K_{fi} \bar{U}_f = F.
\]  

(2.6)

which is a linear structural equation. Upon being solved, this equation yields the displacements of the interior node points of the aerodynamic surface mesh. Once \( \bar{U}_f \), the symbol for the interior displacements, is found, the second equation may be solved for the reaction forces \( r_s \). This equation is the second of the Eq.(2.5) set, and is read as:

\[
K_{fb} \bar{U}_f = r_s.
\]  

(2.7)

By multiplying the reaction forces by a negative sign, equivalent forces acting on the structural nodes are discovered.

2.5 DEFORMATION TRACKING

The second important facet of a coupled analysis is the updating of aerodynamic surface mesh from the structural wing displacements. It is noted that many of the researchers have used one of the interpolating schemes such as (1) the infinite-plate spline, (2) finite-plate spline, (3) multiquadric-biharmonic, (4) thin-plate spline, (5) inverse isoparametric mapping and (6) non-uniform B-splines for displacement mapping. Byun and Guruswamy [27], have used a parallel multi-block, moving grid method for displacement approximation, which is based on the transfinite interpolation scheme. In this method, the transfinite interpolation scheme is used to perturb the interior grid points following the surface boundary deformation.
Deformation tracking consists of two steps. The first step is to transform structural wing displacements to aerodynamic surface displacements. The second step is to transfer the aerodynamic surface displacements to the interior grid points of the aerodynamic mesh. In step 1, the corresponding aerodynamic surface deformation, \( U_f \), is obtained by solving the following set of equations where the structural displacement, \( u_s \) (which is already determined), is used as the prescribed boundary movement at the nodes of the artificial shell structure which correspond to structural nodes.

\[
\begin{bmatrix}
K_{flI} & K_{flB} \\
K_{fBI} & K_{fBB}
\end{bmatrix}
\begin{bmatrix}
U_f \\
u_s
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (2.8)

The first equation in the above set yields:

\[ K_{flI}U_f = -K_{flB}u_s. \] (2.9)

It is possible to apply this type of boundary movement to the artificial shell structure because the structural nodes form a subset of the aerodynamic surface mesh points.

The second step in deformation tracking is transferring aerodynamic surface displacements to the aerodynamic interior grid points. This can be accomplished by following the same strategy adopted for the aerodynamic surface mesh. In this strategy, the artificial spring structure covers the aerodynamic domain, in which the unstructured aerodynamic grids are connected on a network of springs. The network is constructed by representing each edge of each triangle by a linear spring. The stiffness of the spring is assumed to be inversely proportional to the length of its edge and can be written as:

\[ K_{aij} = 1.0 / \left[ (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \right]^{p/2}, \]

where \( p \) is a parameter used to control the stiffness of the spring. Thus, a matrix equation
similar to Eq.(2.8) can be established as:

\[
\begin{bmatrix}
K_{all} & K_{alB} \\
K_{alI} & K_{aBB}
\end{bmatrix}
\begin{bmatrix}
U_a \\
U_f
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

(2.10)

where \(K_{aij}\) is the spring stiffness. The movement of the interior grid points is a result of the equation:

\[
K_{all}U_a = -K_{alB}U_f,
\]

(2.11)

where \(U_f\) is the solution of Eq.(2.9). The resulting set of linear systems, Eq.(2.11) may be solved for the displacements of each node using several Jacobian iterations to save the computer memory requirement. The positions of the interior nodes are then updated using the determined displacements of the aerodynamic interior mesh points, \(U_a\). This method does not require a large amount of memory, but does require an initial guess for the displacements. In the current research work, the initial guess at the start of the grid adaptation process for the current design is the final displacements of the adapted mesh from the previous design. This unstructured grid adaptation method does produce an acceptable mesh in four to six Jacobian iterations.

2.5.1 Conservative System

It is important to demonstrate that the transfer of forces from aerodynamic surface mesh to structural mesh is consistent and conservative. Conservativeness of the process may be proven by illustrating that the summation of forces and moments on aerodynamic surface is equal to that on the structural surface. The consistency of the transfer process is demonstrated by showing that the work done by the structural wing is equal to the work done by the aerodynamic wing structure. Work done by the structural wing can be defined
as $-u_s^T r_s$. The displacement of aerodynamic surface mesh points is determined by the finite element analysis of the artificial shell structure (see Fig.(2.5)) as discussed in Section 2.5. Equation (2.8) basically consists of two equations, where the first equation yields a linear relationship between $U_f$ and $u_s$ as:

$$K_{fl} U_f = - K_{fb} u_s,$$

(2.12)

which yields

$$U_f = K^{\perp}_{fl} K_{fb} u_s.$$  

The next step is to calculate the reaction force. Equation (2.6) can be expressed as:

$$\tilde{U}_f = K^{\perp}_{fl} F.$$  

Substituting for $\tilde{U}_f$ in Eq.(2.7), $r_s$ can be rewritten as:

$$r_s = K_{fb} \tilde{U}_f = K_{fb} K^{\perp}_{fl} F.$$  

then

$$-u_s^T r_s = -u_s^T (K_{fb} K^{\perp}_{fl} F)$$

$$= - (K^{\perp}_{fl} K_{fb} u_s)^T F$$

$$= U_f^T F.$$  

(2.13)

Equation (2.13) implies that work done by the structural loading is equal to the work done by the aerodynamic load. Hence, the consistency is maintained in the force approximation approach used in this work.

2.5.2 Basic Vectors for Deformation Tracking

The deformation tracking procedure explained in Section 2.5 is required to be performed every time the structural wing analysis takes place, in each and every coupled analysis cycle. An alternative to the deformation tracking is to introduce a set of indepen-
dent basic vectors that can span the structural displacements. The methodology behind the calculation of basic vectors for deformation tracking is explained in the next paragraph.

It is assumed that the structural wing deformation can be approximated by a number of basic patterns whose amplitudes are treated as weighted coefficients. Each of the linear combination of the basic vectors and their corresponding weighting coefficients should approximate the structural deformation with a high degree of accuracy. There are many ways to select basic vectors. In this study, the basic patterns of the structural wing are obtained by performing finite element analysis with a unit force applied at each of the selected structural nodes (see Fig.(2.7)). These selected nodes include: six assigned to the nodes along the central line of the top surfaces; the same amount for the bottom surface (A to F); three at G, H and I along the leading edge; J, K and L along the trailing edge; two at M and N along the leading edge; O and P along the trailing edge; four design variables Q and R at one of the wing sections; and S and T at the tip of the wing. A unit force along the Z-direction at nodes A to F and M to P, a unit force along the Y-direction at node Q to T, and a unit force along X-direction at G to L were applied sequentially during the finite element analysis.

The deformations of the structural wing for all the load cases mentioned above are known as the basic displacement vectors, $V_{si}$, or simply, the basic vectors of the structural wing. That is,

$$K_s V_{si} = I_i, \quad (2.14)$$

where $I_i$ has only one non-zero component corresponding to the selected degree of freedom. Once this vector is obtained, the set of basic displacement vectors of the aerodynamic surface mesh, $V_{fi}$, are obtained by using the basic vectors of the structural wing.
wing as the imposed boundary movement in the finite element analysis of the artificial shell structure. This procedure, which essentially consist of solving Eq.(2.9), has already been explained in Section 2.5. Next, the set of basic vectors of the aerodynamic mesh, \( V_{ai} \), are obtained by using the basic vectors of the aerodynamic surface mesh as the input to the mesh-moving strategy as explained in Section 2.5. This procedure basically consists of solving Eq.(2.11) with the basic vectors of the aerodynamic surface mesh, \( V_{fi} \), as an input.

It should be noted that the finite element analysis technique adopted in updating deformation of aerodynamic surface mesh and the mesh-moving strategy for updating aerodynamic mesh deformation are perfectly linear, and are used only to obtain the corresponding set of basic vectors. Hence, the movements of various meshes are maintained by considering the products of the associated basic vectors with the same weighting coefficients as:

\[
V_a = \sum \alpha_i V_{ai}, \quad V_f = \sum \alpha_i V_{fi}, \quad V_s = \sum \alpha_i V_{si}, \quad (2.15)
\]

where \( V_s \), \( V_f \) and \( V_a \) represent the changes in structural, aerodynamic surface and aerodynamic interior meshes, respectively. Consequently, the movement of the structural mesh, \( V_s \), induces the movement of the aerodynamic surface mesh, \( V_f \), which in turn, induces the movement of the aerodynamic interior mesh, \( V_a \).

An alternative method for deformation tracking discussed at the beginning of Section 2.5 can be devised by selecting the weighting coefficients of \( \alpha_i \) so that \( V_s \) in Eq.(2.15) can approximate the structural elastic deformation. Once this is complete, the corresponding changes in aerodynamic meshes, \( V_f \) and \( V_a \), are readily available without further computation. The approximate displacement can be written in the equation form as:
where $V$ is an $n_s \times n_a$ matrix, where $n_s$ is the number of structural degrees of freedom and $n_a$ is the number of basic vectors. The objective of this exercise is to find a set of $\alpha$ that can minimize the difference between the elastic deformation, $u_s$ and $V_s$. The error can be estimated as:

$$E = u_s - V_s = u_s - V\alpha.$$  \hspace{1cm} (2.17)

The weighted square of the error can be represented as:

$$\psi = E^T K_s E$$

$$= (u_s - V\alpha)^T K_s (u_s - V\alpha)$$

$$= (\alpha^T V^T - u_s^T) K_s (u_s - V\alpha)$$

$$= ((\alpha^T V^T - u_s^T) K_s u_s - (\alpha^T V^T - u_s^T) K_s V\alpha)$$

$$= (2\alpha^T V^T K_s u_s - u_s^T K_s u_s - \alpha^T V^T K_s V\alpha),$$

where $K_s$, the positive definite structural stiffness matrix, is used here as the weighting matrix. A minimization problem is established to find $\alpha$ as:

$$\min_\alpha \psi.$$

The necessary condition of the above problem yields a set of linear equations:

$$\frac{\partial \psi}{\partial \alpha} = 0,$$

which implies
\[ 2(V^T K_s u_s)^T - 2(V^T K_s V)^T \alpha = 0, \]

or

\[ (V^T K_s V)\alpha = V^T K_s u_s. \]  

(2.18)

Here, it should be noted that \( K_s \) is the stiffness matrix that is modified with boundary conditions, and therefore is considered positive definite. As a result, a zero of \( \psi \) implies that the exact and approximated deflections are equal. The right-hand side of Eq.(2.18) gives

\[ V^T K_s u_s = V^T r_s, \]

where \( r_s \) is the loading that causes \( u_s \). The left-hand side of Eq.(2.18) can be written as:

\[ V^T K_s V = [\delta_{ij}]_{i,j=1}^{ton_s}, \]

where \( V_i \) and \( V_j \) are the basic vectors corresponding to unit forces \( l_i \) and \( l_j \). Furthermore,

\[ \delta_{ij} = V_i^T K_s V_j = V_i^T l_j = V_i^j, \]

where \( l_j \) is unit force applied at the node corresponding to design variable \( j \). Only one component in \( l_j \) is one, while the rest are zero. Therefore, \( V_i^T l_j \) results in a value of \( V_i^j \), which is the \( j^{th} \) component of \( V_i \). The \( j^{th} \) component corresponds to the non-zero component in \( l_j \). Note that \( V_i^j = V_i^j \) as a result of reciprocal theorem. Therefore Eq.(2.18) becomes:

\[ (V^T K_s V)\alpha = V^T K_s u_s \]

\[ [\delta] \alpha = V^T r_s \]

(2.19)

where

\[ \delta_{ij} = V_i^j. \]
Equation (2.19) can be solved to find $\alpha$ with which the deformations of the structural mesh, aerodynamic surface mesh and the aerodynamic mesh can be approximated by solving Eq.(2.15). Once $V_a$ is calculated, the aerodynamic mesh is updated as $V_a + X_a^o$.

### 2.6 COUPLED ANALYSIS

#### 2.6.1 Formulation

Aeroelastic wing analysis consists of solving aerodynamic and structural equations simultaneously until equilibrium condition is reached, for which the aerodynamic and structural disciplines must be coupled at the boundary interfaces. This coupling can be referred to as aerodynamic-structural interaction. Coupling of these disciplines requires the transfer of the aerodynamic load from the aerodynamic surface mesh to the structural mesh, and updating the movement of aerodynamic mesh caused by the structural wing deformations in an accurate and efficient manner.

The governing equations for a coupled analysis can be represented as:

$$K_s(X_s)u_s = -r_s(Q, X_a)$$

$$R(Q, X_a) = 0.$$  \hspace{1cm} (2.20) \hspace{1cm} (2.21)

Equations (2.20) and (2.21) represent the equilibrium equations for a linear structural analysis and nonlinear aerodynamic state equations, respectively. Sections 2.2 and 2.3 explain in detail the procedure for solving the structural and aerodynamic state equations. A general outline of the steps involved in a coupled analysis is presented here, along with a flowchart (see Fig.(2.1)).

1. Read the initial aerodynamic mesh $X_a^o$.
2. Solve Eq.(2.4) to obtain forces at aerodynamic surface grid points.
3. Approximate forces at aerodynamic surface nodes to structural nodes, \( r_s \), by solving Eqs.(2.6) and (2.7).

3. Substitute \( r_s \) into Eq.(2.2) and subsequently solve for \( u_s \).

4. Interpolate the deformation of the structural wing, \( u_s \), to obtain the deformation on the aerodynamic surface mesh, \( U_f \), and subsequently, the aerodynamic mesh, \( U_a \), by solving Eqs.(2.9) and (2.11) respectively. Update the aerodynamic mesh as:

\[
X_a^n = X_a^o + U_a.
\]

5. Go back to step 2 and repeat steps 3 and 4 until convergence is met.

Step 4 may be replaced by solving Eq.(2.18) for \( \alpha \) and then using Eq.(2.15) to obtain \( V_f \) and \( V_a \). In this case, one may calculate basic vectors \( V_{si} \), \( V_{fi} \) and \( V_{ai} \) in Eq.(2.15) using Eqs.(2.14), (2.9) and (2.11) before beginning the coupled analysis.

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Figure 2.1: Flow Chart of Coupled Analysis Cycle
2.6.2 Results and Discussions

Aeroelastic analysis of a flexible wing with strong coupling between aerodynamic and structural disciplines is performed successfully and is illustrated through Figures 2.8 to 2.20. A more accurate method known as “Reaction Force Method” was used for transformation of the load from the aerodynamic surface mesh to the structural mesh. In addition, this method was proved to be consistent and conservative. This nature of the load transformation method is required to prove that the transformed loads are accurate. The aerodynamic mesh updating used in the current work is computationally efficient. The computational time taken in deformation tracking is minimized as the new aerodynamic surface mesh and the aerodynamic mesh are updated by solving Eqs.(2.15) and (2.11).

The fluid flow analysis over the aerodynamic wing was performed at a free stream Mach number of 0.85. The wing is placed at an angle of attack of 2.5°. The flow analysis is performed for a cruising condition at an altitude of 35,000 ft. The fluid properties such as density of air and velocity of sound are taken as 7.382 E-04 slug/ft.³ and 973.14 ft./sec., respectively. The broad dimensions of the wing are 65.0 ft. long along the span wise direction and 32.5 ft. wide at the root.

For the structural analysis, the wing is considered to be made of Aluminum 6061-T6, which has a modulus of elasticity, Poission’s ratio and density of 107 lbs/in.², 0.33, and 0.097 lbs/in.³, respectively. The structural wing is discretized using CST and Truss elements. The cross-sectional area of 0.005 in.² is considered for the truss members and the CST elements are considered to have a thickness of 0.03 in.

Two types of wings were considered for the aerodynamic analysis. In the first case, a rigid wing was considered, and in the second case, a flexible wing was considered.
Assumption of a rigid wing indicates that the wing does not deform from the initial configuration during the fluid dynamic analysis. In case of a flexible wing, the wing is allowed to deform during the fluid flow analysis, and therefore requires the coupling between the fluid dynamics and structural disciplines. Since a strong coupling is considered, the interaction between the aerodynamic and the structural disciplines were maintained at every iteration of the coupled analysis cycle.

The results of aerodynamic analysis on both the rigid wing and flexible wing mainly consist of computational performance and flow variable prediction characteristics. Figure 2.8 illustrates the CFD residual history plot versus the number of iterations for the rigid wing and aeroelastic wing. This plot indicates that the number of iterations taken by coupled analysis is the same as the rigid wing analysis, which basically illustrates that although the flexible wing is allowed to deform during the analysis, it does not require additional iterations to reach the equilibrium state. It should be noted that even though the number of iterations taken by the rigid and flexible wing analyses are the same, the time taken for the previously mentioned analyses are not the same. In fact, the flexible wing analysis takes 50% more time than the rigid wing analysis. This is due to the fact that the flexible wing analysis consists of structural wing analysis, as well as data exchange between the aerodynamic wing and the structural wing. Figure 2.9 represents the lift coefficient history. It is clear that the coefficient of lift predicted by rigid wing analysis is much smaller than that of the flexible wing. As the flow condition adopted for the aerodynamic analysis is a transonic flow, high wing displacements are expected to occur, which in turn provides a relatively high lift coefficient. Hence, this figure confirms that the rigid wing analysis will lead to an incorrect calculation of lift coefficient, and drag coefficient for
transonic flow conditions.

Convergence history of the root-mean-square value of weighting coefficients ($\alpha_i$) in terms of iteration history plot is illustrated in Fig.(2.10). This figure implicitly explains the behavior of the structural wing during the iterative process of reaching the equilibrium position. It is clear that the structural wing has reached the equilibrium position, thereby indicating that the change in weighting coefficient is negligibly small.

Figures 2.11 and 2.12 show cross-section-wise deflections of the rigid wing and the aeroelastic wing. Figure 2.13 represents the deflection of the aerodynamic wing surface from the initial position. The flow condition considered for the flexible wing analysis induced a maximum wing deflection of about 12% of the maximum dimension of the wing (65 ft. along the span-wise direction). Figures 2.14 to 2.18 represent the pressure distributions obtained from the rigid wing analysis and the aeroelastic wing analysis. These results are plotted at sections measured from the root, at a distance of 95.23 in., 188.23 in., 277.19 in., 360.58 in. and 506.10 in., respectively, along the span-wise direction of the wing (y-coordinate). These figures clearly illustrate the difference in the pressure values predicted by the rigid wing analysis and the flexible wing analysis. Figures 2.19 and 2.20 show pressure and Mach contours, respectively. Once again, it is undeniably visible from these plots that the rigid wing analysis may mislead in the required computation of pressures when calculating coefficients of lift and drag. As coefficients of drag and lift are considered as objective and constraint functions in the optimization process, the choice of the rigid wing in the aerodynamic analysis would have provided a different optimum design, which need not be an accurate one.
Figure 2.2: Sectional View of the Structural Wing Model

Figure 2.3: Network of Reinforcements and Truss Members in the Structural Wing Model

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Figure 2.4: Aerodynamic Mesh Configuration

Figure 2.5: Artificial Shell Structure

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Figure 2.6: Cross-Sectional View of the Artificial Shell Structure

Figure 2.7: Structural Wing Model Identifying Design Variables

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Figure 2.8: Residual History Plot

Figure 2.9: Coefficient of Lift History Plot
Figure 2.10: Convergence History of Weighting Coefficients ($\alpha_i$)

Figure 2.11: Deflection of Rigid Wing Cross-Sections

Figure 2.12: Deflection of Aeroelastic Wing Cross-Sections
Figure 2.13: Aerodynamic Wing Deflection from Initial Position

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Figure 2.14: Pressure Distribution at Span Section, $y = 95.23$ in
Figure 2.15: Pressure Distribution at Span Section, $y = 188.23$ in
Figure 2.16: Pressure Distribution at Span Section, $y = 277.19$ in
Figure 2.17: Pressure Distribution at Span Section, $y = 360.58$ in
Figure 2.18: Pressure Distribution at Span Section, $y = 506.10$ in
Figure 2.19: Comparison of Pressure Contours

Figure 2.20: Comparison of Mach Contours

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CHAPTER III
SHAPE SENSITIVITY ANALYSIS

3.1 INTRODUCTION

In any design optimization process, minimization of an objective function can be achieved by two methods. These methods are known as function-based and gradient-based methodologies. In a function-based optimization process, the objective function and constraints are computed several times in order to arrive at the optimum design. In case of a gradient-based optimization process, the objective function is minimized faster than in the previous method by making use of the gradient information.

Therefore, the optimization methods which are widely used in solving multidisciplinary optimization (MDO) problems are usually gradient-based methods in which a specific objective function is minimized. In this process, the gradients of the objective function, with respect to design variables, are used to update the design variables in such a way that the value of the objective function is reduced systematically to a local minimum. An important step in this process is the determination of these gradients, which are also referred to as sensitivity derivatives. The computation of sensitivities for a coupled aero-structural problem has components of both aerodynamic and structural quantities. To obtain accurate sensitivities of the design problem, the coupling terms cannot be neglected. Moreover, the strong interdependency between aerodynamic and structural disciplines make the computation of derivatives a difficult task. There are several techniques employed for obtaining sensitivities, which can be categorized as the adjoint, direct, and finite-difference approach. In the adjoint approach, the sensitivities of aero-structural functions are obtained by solving a set of coupled adjoint equations. The straightforward
alternative to the adjoint approach is to use finite difference approximation. This requires separate solutions of the coupled system for each perturbation of design variables, which is computationally expensive for a coupled aeroelastic wing. In the current work, a direct approach is used in computing analytical coupled derivatives in which the mesh sensitivity is done with the concept of the basic vectors discussed in Section 2.5.2.

3.2 SHAPE SENSITIVITY ANALYSIS OF FLUID FLOW EQUATIONS

Finite difference, complex variable, automatic differentiation, adjoint variable and direct differentiation are commonly used methods for sensitivity analysis. The finite difference method is time consuming, and its accuracy depends strongly upon the step size used in the calculation. The complex variable method can provide accurate sensitivities which are not sensitive to the step size. However, prior to applying the method, the computational code must be modified so that it can handle complex variables [41]. The automatic differentiation (AD) method relies on the computer compilers to interpret the code line by line, and subsequently generate an AD-enhanced code that can calculate the derivatives [42]. The automatic differentiation method has proven to be an accurate method for aerodynamic sensitivity analysis [43,44]. However, it requires computer memory and time. The adjoint variable method has been applied to the continuous form of the aerodynamic equations to find the derivatives of aerodynamic functionals in references [45-49]. Finally, the direct differentiation method has been applied to the discretized aerodynamic equations to find the derivatives of the flow variables in references [50-53]. Since the direct differentiation method will be used in this study, the derived aerodynamic sensitivity equations will be similar to those derived in references [50-53]. It should be noted that the sensitivity analysis code generated here is based upon the Euler code originated by New-
man [40]. A general review of the sensitivity analysis methods may be found in reference [54].

The governing equation of a 3-D Euler flow in the discrete residual vector form for a steady state solution can be written as \( R(Q,X_a) = 0 \), which upon differentiation with respect to the design variable, generates a sensitivity equation which can be represented as:

\[
R' = 0
\]

or

\[
\frac{\partial R}{\partial Q} Q' + \frac{\partial R}{\partial X_a} X'_a = 0.
\] (3.1)

In the above equation, \( Q' \) and \( X'_a \) are known as the derivatives of the state variable vector and the aerodynamic mesh, with respect to the design variable. The terms \( \frac{\partial R}{\partial Q} \) and \( \frac{\partial R}{\partial X_a} \) are the Jacobian matrices evaluated at a converged flow solution. The Jacobians can be constructed by two methods. The first method is to derive analytical expressions for the Jacobians of typical fluid cells that are then assembled together to become the global Jacobian matrices. The second option is to apply an automatic differentiation (AD) tool to the residual term, \( R \), to obtain the Jacobians. In this work, an AD tool known as ADIFOR [42] is used successfully to obtain the Jacobians. ADIFOR is used here selectively to minimize the memory requirement while maximizing the computational efficiency of the ADIFOR-enhanced code. The first step involves the collection of the FORTRAN subroutines that compute the residual \( R_e \) of a typical cell. These subroutines are then input to ADIFOR to obtain the derivatives of the residual \( R_e \) with respect to the user-defined independent vari-
able. In this case, the independent variables are $Q$ and $X_a$. The output of ADIFOR gives a collection of new subroutines that can compute $R_e$ as well as $\left( \frac{\partial R_e}{\partial Q_e} \right) S_e$ and $\left( \frac{\partial R_e}{\partial X_a} \right) S_e$.

where $S_e$ is a user-specified seed matrix. In this step, $S_e$ is fixed as an identity matrix with which the ADIFOR-produced code computes $\frac{\partial R_e}{\partial Q_e}$ and $\frac{\partial R_e}{\partial X_a}$ as two, two-dimensional arrays. The second step is to generate a new subroutine to compute the products, $\frac{\partial R_e}{\partial Q_e} S_Q$ and $\frac{\partial R_e}{\partial X_a} S_X$, where $S_Q$ and $S_X$ are parts of the user specified seed matrices, $Q'$ and $X'_a$, respectively. The third step is to assemble the element arrays produced in the previous step into the global arrays to represent Eq.(3.1).

The solution method adopted for solving the nonlinear flow in Eq.(2.3) is an incremental iterative scheme, which can be expressed as:

$$\frac{\partial R^n}{\partial Q} \Delta Q^n = -R^n \quad (3.2)$$
$$Q^{n+1} = Q^n + \Delta Q^n; \quad n = 1, 2, 3, \ldots \quad (3.3)$$

The term $\frac{\partial R^n}{\partial Q} \Delta Q^n$ in Eq.(3.2) represents the matrix operator which is an approximation to the exact Jacobian matrix operator that is associated with Newton iteration. The iterative scheme involved in solving Eqs.(3.2) and (3.3) has already been mentioned in Chapter 2. The same solution methodology has been used in solving aerodynamic sensitivity, Eq.(3.1), which can be written in an incremental iterative form as:
\[
\frac{\partial R^n}{\partial Q} (\Delta Q')^m = -(R')^m \tag{3.4}
\]

\[
(Q')^m + 1 = (Q')^m + (\Delta Q')^m; \quad m = 1, 2, \ldots \tag{3.5}
\]

where \( R' \) is represented by Eq.(3.1). In Eq.(3.2), the left-hand side coefficient matrix \( \frac{\partial R^n}{\partial Q} \) represents any computationally convenient approximation of the exact Jacobian matrix. In particular, the identical approximate left-hand side operator and algorithm that are used to solve the nonlinear flow equation can be used here to solve the linear sensitivity, Eq.(3.4). Comparing Eqs.(3.2) and (3.3) with that of Eqs.(3.4) and (3.5), reveals that the sensitivity equation is solved by interchanging the right-hand side of Eq.(3.2) with that of Eq.(3.4), and using the steady state value obtained during flow analysis for the left-hand side operator.

Hence the steps involved in solving Eq.(3.4) can be summarized as follows:

1. Obtain the converged flow solution \( Q \) from the aerodynamic analysis.
2. Obtain the right-hand side vector of Eq.(3.4) by multiplying the Jacobians, given in Eq.(3.1), by the seed matrix \( S_Q \) and \( S_X \), which are part of \( Q' \) and \( X_a' \).
3. Solve Eq.(3.4) with Gauss–Seidel iterative scheme until convergence is met.

3.3 SENSITIVITY ANALYSIS OF STRUCTURAL EQUATION

The equilibrium equation of a linear static structure in the finite element form can be written as \( K u_s = r_s \). Upon differentiating this equation with respect to the design variable, one can obtain the sensitivity equation as:

\[
K_s' u'_s = r'_s - K_s' u_s, \tag{3.6}
\]

where \( K_s' \), \( u'_s \) and \( r'_s \) are known as derivatives of the structural stiffness matrix, the struc-
tural displacements and the structural force, respectively, with respect to the design variable. These can be represented as:

\[
K'_s = \left( \frac{\partial K_s}{\partial X_s} \right) X'_s
\]

\[
u'_s = \left( \frac{\partial u_s}{\partial X_s} \right) X'_s
\]

\[
r'_s = \left( \frac{\partial r_s}{\partial X_s} \right) X'_s
\]

The solution procedure adopted in solving Eq.(3.6) to obtain \(u'_s\) is to compute the right-hand side of Eq.(3.6), and then to perform backward substitution, as matrix \(K_s\) has already been factorized during the calculation of the displacement field, \(u_s\). Hence, solving sensitivity equations remains a difficult computational task, as many new derivative terms are needed to be constructed as parts of the right-hand sides of the sensitivity equations. The new terms include \(K'_s\) and \(r'_s\). The term, \(r'_s\) involves aerodynamic load, and is discussed in Section 3.4. Hereafter, the discussion focuses on computing \(K'_s\) only. The derivative of the stiffness matrix with respect to the design variable, \(K'_s\), can be computed by applying ADIFOR to the subroutines that compute the elemental stiffness matrix \(K_e\) [55]. The output of ADIFOR gives a new subroutine that computes \(K_e\) and \(\left( \frac{\partial K_e}{\partial X_s} \right) S_e\), where \(S_e\) is a user-specified seed matrix. As mentioned earlier in the previous section, \(S_e\) is taken to be an identity matrix in this case also, with which the ADIFOR enhanced code computes
\( \left( \frac{\partial K_s}{\partial X_s} \right) \) as a three-dimensional array. The derivative matrix is then multiplied with the seed matrix, \( S_e \), which is part of the derivatives of nodal coordinates with respect to the independent design variable, \( X'_s \). The derivative term is again multiplied with the displacement vector, \( u_s \), which will have been determined by solving the linear structural equation \( K_s u_s = r_s \). Once the right-hand side of Eq.(3.6) is computed, then the equation is solved by backward substitution, as \( K_s \) would have been factorized during the calculation of displacement field \( u_s \). Hence, the sensitivity analysis of a linear structure can be summarized as follows:

1. Solve equation \( K_s u_s = r_s \) to obtain \( u_s \).
2. Compute the first term on the right-hand side of Eq.(3.6) assuming \( r'_s \) is given.
3. Solve Eq.(3.6) by backward substitution, as \( K_s \) is already factorized during the calculation of displacement field, \( u_s \).

### 3.4 AERODYNAMIC FORCE DERIVATIVE

The aerodynamic force, \( F \), acting at the center of an aerodynamic surface cell, is calculated by the expression:

\[
F = cpA, \tag{3.7}
\]

where \( A \) is the surface area of the cell, \( p \), the pressure which is part of the solution vector, \( Q \), and \( cp \) is a constant that makes force a dimensional quantity. Differentiation of Eq.(3.7) with respect to a shape design variable, \( b_i \), provides an expression for obtaining the derivative of aerodynamic force acting at the center of the aerodynamic surface cell as:

\[
F' = cp'A + cpA'. \tag{3.8}
\]
In the above expression, \( p' \) is obtained by solving Eq.(3.1) for \( Q' \), while \( A' \) is the derivative of the aerodynamic surface cell area with respect to design variables. The force derivatives acting at a vertex of each cell are extrapolated by considering one-third of the force derivatives acting at the cell center.

Once the force derivatives at the aerodynamic surface grid points are obtained, the force derivatives at the structural nodes (\( r' \)) can be obtained by solving the sensitivity equation of the artificial shell structure, which is used in force approximation and is explained in Section 2.4. The following equations are obtained after differentiating Eqs.(2.6) and (2.7) with respect to a design variable:

\[
K_{fl1} \ddot{U} = -K'_{fl1} \dot{U} + F'
\]  
(3.9)

and

\[
\dot{r}_s = K'_{fl1} \dot{U} + K_{fl1} \ddot{U} + F'
\]  
(3.10)

The boundary conditions acting on this artificial shell structure for sensitivity analysis remain the same as that of coupled analysis. The derivative of the stiffness matrix of the artificial shell structure is obtained by submitting the subroutine that calculates the stiffness matrix to an automatic differentiator known as ADIFOR. This, in turn, provides a FORTRAN 77 code that can compute the stiffness matrix, as well as the derivatives of the stiffness matrix with respect to the independent variables. The implementation process of this automatic differentiator in obtaining derivatives is explained in Section 3.3. The displacement vector of the artificial shell structure can be obtained by first solving Eq.(2.6), which is repeated here as:

\[
K_{fl1} \ddot{U} = F.
\]  
(3.11)
In the above equation, $K_{fl}$ is the stiffness matrix of the artificial shell structure, while $\ddot{U}_f$ is the displacement of the artificial shell structure due to the aerodynamic force, $F$, acting on it. The aerodynamic force used in solving Eq.(3.11) is already obtained during the coupled analysis. Force derivative $F'$ in Eq.(3.9) is obtained from Eq.(3.8). Therefore, Eq.(3.9) can be solved using the same procedure explained in Section 3.3. After obtaining $\ddot{U}_f'$, $r'_s$ is obtained simply by computing the right-hand side of Eq.(3.10), in which all the variables are known.

3.4.1 Conservative System

Similar to the load itself, the derivatives of the load converted from the aerodynamic surface mesh to the structural mesh should be consistent and conservative in nature. The conservativeness of the scheme can be proved by showing that the summation of force derivatives on the aerodynamic surface mesh is equal to the summation of the force derivatives on the structural mesh. The consistency of this scheme can be demonstrated by showing that the derivative of work done by the structural wing is equal to the derivative of the work done by the aerodynamic wing structure.

The derivative of work done by the structural wing can be defined as:

$$-(u'_s)^T r_s - (u_s)^T r'_s.$$ 

In order to prove that this is equal to the work done by an aerodynamic wing structure, one can first begin by calculating the derivative of aerodynamic surface deformation by differentiating Eq.(2.9) with respect to the design variables, which can be represented as:

$$K'_{fl} U_f + K_{fl} U'_f = - K'_{flB} u_s - K_{flB} u'_s. \quad (3.12)$$

The derivative of structural force, $r'_s$, and the aerodynamic force, $F'$, can be obtained by

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differentiating Eqs.(2.7) and (2.6), respectively, as:

\[ r'_s = K'_{f_Bl} \tilde{U}_f + K_{f_Bl} \tilde{U}'_f \quad (3.13) \]

\[ F' = K'_{f_Bl} \tilde{U}_f + K_{f_Bl} \tilde{U}'_f. \quad (3.14) \]

Then the derivative of work done by the structural wing can be presented after substituting for \( r_s \) and \( r'_s \) from Eqs.(2.7) and (3.13) as:

\[-(u'_s)^T r_s - (u_s)^T r'_s = -(u'_s)^T (K_{f_Bl} \tilde{U}_f) - (u_s)^T (K'_{f_Bl} \tilde{U}_f + K_{f_Bl} \tilde{U}'_f).\]

Rearranging the terms in the above equation, one obtains

\[-(u'_s)^T r_s - (u_s)^T r'_s = -(u'_s)^T K_{f_Bl} - (u_s)^T K'_{f_Bl} \tilde{U}_f - (u_s)^T K_{f_Bl} \tilde{U}'_f.\]

In the above equation, the first term on the right-hand side can be substituted by the left-hand side of Eq.(3.12), as \( K_{f_Bl} \) is equal to \( K_{f_Bl} \) for the artificial shell structure, and the second term by the left-hand side of Eq.(2.9). After rearranging the terms, the derivative of work done by the structural wing becomes

\[-(u'_s)^T r_s - (u_s)^T r'_s = (\tilde{U}'_f K_{f_Bl} + \tilde{U}_f K'_{f_Bl})^T U_f + (\tilde{U}_f K_{f_Bl})^T U'_f.\]

Substituting Eqs.(3.14) and (2.6) for first and second terms respectively, the right-hand side of the above equation yields

\[-(u'_s)^T r_s - (u_s)^T r'_s = (F')^T U_f + (F)^T U'_f,\]

which implies that the derivative of work done by the structural wing is equal to the work done by the aerodynamic wing. Hence, the consistency is maintained in the derivative of the force approximation scheme used in this work.
3.5 COUPLED SHAPE SENSITIVITY ANALYSIS

Coupled shape sensitivity analysis consists of solving the aerodynamic sensitivity equations and linear structural equations simultaneously. Sections 3.2 and 3.3 explained the procedure to solve aerodynamic sensitivity equations and structural sensitivity equations without considering the interactions between the disciplines. In other words, the two disciplines in these sections were treated independently when obtaining their corresponding derivatives. Realistically, these two disciplines are interdependent in nature and hence, the derivatives should be computed considering these two disciplines as one system. This leads to solving the sensitivity equations of aerodynamic discipline and structural discipline simultaneously, with additional terms arising due to the interdependent nature of these disciplines. Therefore, the coupled sensitivity equations are obtained by differentiating the coupled aeroelastic Eqs.(2.20) and (2.21) with respect to the design variables, $b$.

The coupled sensitivity equations after algebraic manipulation can be represented as:

\[ K_s \frac{\partial u_s}{\partial b_i} = - \left( \frac{\partial K_s}{\partial b_i} \right) u_s + \frac{\partial r_s}{\partial b_i} \]  
(3.15)

\[ \frac{\partial R}{\partial Q} \frac{\partial Q}{\partial b_i} + \frac{\partial R}{\partial X_a} \frac{\partial X_a}{\partial b_i} = 0 \]  
(3.16)

In Eqs.(3.15) and (3.16), the terms as $\frac{\partial u_s}{\partial b_i}$ and $\frac{\partial Q}{\partial b_i}$ are known as the coupled derivatives.

These derivatives form a coupling link between Eqs.(3.15) and (3.16), which implies that in order to solve Eq.(3.15), solution of Eq.(3.16) is required, and vice versa.

Moreover, Eqs.(3.15) and (3.16) can be further simplified as:

\[ K_s u'_s = - K'_s u_s + r'_s \]  
(3.17)
\[ \frac{\partial R}{\partial \beta} Q' + \frac{\partial R}{\partial X_a} (X'_a) = 0. \] (3.18)

The above sensitivity equations are coupled and linear in terms of \( u'_s \) and \( Q' \). The seed matrix, \( X'_a \), is the derivative of aerodynamic mesh with respect to a design variable. Because of deformation tracking, the aerodynamic mesh is updated each time when the elastic deformation occurs. That is:

\[ X_a = X'_a + U_a, \]

where \( U_a \) is the mesh movement of the aerodynamic interior points, which in turn is caused by elastic deformation, \( u_s \). The seed matrix is then equal to:

\[ X'_a = (X'_a)' + U'_a. \]

The coupling between Eqs. (3.17) and (3.18) is thus established through the term \( r_s' \) which is a function of \( Q' \) and the term \( X'_a \) which involves the derivative of elastic structural deformation.

### 3.5.1 Basic Vectors for Shape Design Change

The coupled sensitivity equations may be decoupled by introducing the concept of basic vectors. In Section 2.5.2, the basic vectors have been introduced to represent the elastic structural deformation. Thus,

\[ u_s = V_s \alpha, \]

which results in the movement of the aerodynamic mesh as \( U_f = V_f \alpha \) and \( U_a = V_a \alpha \), where \( V_s, V_f \) and \( V_a \) are the matrices of the basic vectors. The same concept can be extended here to represent the shape change also. That is, the change in structural mesh...
due to the shape change of the base line design can be expressed as:

$$\Delta X_s^o = \Phi_s b,$$

where $\Phi_s$ is a matrix of linearly independent vectors called basic vectors, and $b$ provides the scale factor for each of the basic vectors. In design consideration, $b$ is the vector of the design variables. The new mesh after the design change is then given as:

$$X_s = X_s^o + \Delta X_s^o = X_s^o + \Phi_s b,$$

and the derivative of $X_s$ with respect to a design variable is found to be:

$$X'_s = \Phi_{si}.$$

The basic vectors, $\Phi_{si}$, can be selected in various ways. The eigenmodes, the prescribed patterns, as well as those displacements obtained due to unit load are examples of valid candidates. The change in aerodynamic meshes due to change in the basic line design can then be obtained as:

$$\Delta X_f^o = \Phi_f b$$

$$\Delta X_a^o = \Phi_a b,$$

which result in the new aerodynamic meshes due to the design change as:

$$X_f = X_f^o + \Delta X_f^o$$

$$X_a = X_a^o + \Delta X_a^o.$$

The columns of $\Phi_f$ and $\Phi_a$ in the above equations are obtained as the solutions of respective artificial structural equations

$$K_{flf} \Phi_{fi} = - K_{flb} \Phi_{si} \quad (3.19)$$

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\[ K_{all}\Phi_{ai} = -K_{allB}\Phi_{fi}. \] (3.20)

Equation (3.19) is the same as Eq.(2.9) and Eq.(3.20) is the same as Eq.(2.11). The seed matrix of the aerodynamic mesh due to change in design can then be given as:

\[ X'_{fi} = \Phi_{fi} \]

\[ X'_{ai} = \Phi_{ai}. \]

In an aero-structural problem, the new meshes in various disciplines resulting from design change and deformation tracking can now be obtained as:

\[ X_s = X_s^0 + \Phi_s b + u_s \]
\[ = X_s^0 + \Phi_s b + V_s \alpha \]

\[ X_f = X_f^0 + \Phi_f b + U_f \]
\[ = X_f^0 + \Phi_f b + V_f \alpha \]

\[ X_a = X_a^0 + \Phi_a b + U_a \]
\[ = X_a^0 + \Phi_a b + V_a \alpha. \] (3.21)

where the structural deformation is approximated by the basic vectors, \( V_s \), and the vector \( \alpha \) is the solution of

\[ (V_s^T K_s V_s)\alpha = V_s^T K_s u_s. \] (3.22)

### 3.5.2 Decoupling the Coupled Sensitivity Equations

The seed matrix, \( X'_a \), which accounts for both design change and deformation update in Eq.(3.18) can be obtained, based upon Eq.(3.21), as:
\[ X'_a = \Phi_{ai} + V_a \alpha', \quad (3.23) \]

where \( \alpha' \) is the solution of

\[ (V_s^T K_s V_s) \alpha' = V_s^T K_s u'_s. \quad (3.24) \]

Equation (3.24) is obtained by differentiating Eq.(3.22) in which \( V_s \) is a constant matrix and \( K_s \) is a weighting matrix which again remains a constant. Therefore, \( V_s \) and \( K_s \) are independent of design changes. In fact, Eq.(3.24) can be solved by minimizing the difference between the approximated structural displacement derivative, \( \tilde{u}'_s = V_s \alpha' \), and the exact one \( u'_s \) as:

\[ \psi' = (\tilde{u}'_s - u'_s)^T K_s (\tilde{u}'_s - u'_s). \]

Substituting Eq.(3.23) in Eq.(3.18), one obtains

\[ \frac{\partial R}{\partial Q} Q' + \frac{\partial R}{\partial X_a} (\Phi_{ai} + V_a \alpha') = 0. \quad (3.25) \]

Since Eq.(3.25) represents a linear equation with respect to the basic vectors, it can be solved in two steps. The first step is to solve

\[ \frac{\partial R}{\partial Q} Q'_{ai} + \frac{\partial R}{\partial X_a} (\Phi_{ai}) = 0 \quad (3.26) \]

and the second step is to solve

\[ \frac{\partial R}{\partial Q} Q'_{aj} + \frac{\partial R}{\partial X_a} (V_{aj}) = 0; j = 1 \text{ to } n_a. \quad (3.27) \]

Note that Eqs.(3.26) and (3.27) are the aerodynamic sensitivity equations for a rigid wing with \( \Phi_{ai} \) and \( V_{aj} \) as seed matrices. Therefore, a disciplinary aerodynamic sensitivity analysis code may be used here repeatedly or in a parallel computational environment to
find $Q'_{ai}$ and $Q'_{aj}$. The second step is to calculate the coupled sensitivity $Q'$ which is given as:

$$Q' = Q'_{ai} + [Q'_{aj}] \alpha'.$$

(3.28)

Solving the coupled sensitivity equations based upon Eqs.(3.26) to (3.28), instead of Eqs.(3.17) and (3.18), enjoys several advantages. A disciplinary aerodynamic sensitivity analysis code can be used here to prepare the data needed for coupled sensitivity analysis of Eq.(3.28). Once $Q'$ is calculated, the structural load derivatives, $r'_s$, can be obtained following the procedure given in Section 3.4, with which Eq.(3.17) can be solved using the disciplinary structural sensitivity analysis code by itself. At the end, it becomes evident that the major advantage of introducing the basic vectors is to enable the disciplinary sensitivity analysis codes to be directly used to support coupled aero-structural sensitivity analysis with very limited code modifications. In other words, as long as the coupled sensitivity analysis can be accomplished by the disciplinary sensitivity codes, there is no need to build a new code for coupled sensitivity analysis.

### 3.5.3 Solution Procedure for Coupled Sensitivity Analysis

Following the coupled sensitivity equations and their solution procedures discussed earlier, one can summarize the solution methodology for coupled sensitivity analysis as follows:

1. Perform the coupled aeroelastic analysis to obtain converged solution, $Q^*$ and $X^*_a$, where $X^*_a = X^0_a + U_a$.

2. Select basic vectors, $\Phi_a$. In this study, $V_a$ is set to be $\Phi_a$.

3. Solve Eqs.(3.26) and (3.27) for each of the basic vectors.
4. Calculate the aerodynamic force derivative, $F'$, acting on the aerodynamic wing surface by using Eq.(3.8).

5. Solve for $r'$ from Eq.(3.10). Note that the seed matrix for $K'^{Bl}$ is always $V'^i_f$.

6. Calculate $\alpha'$ by Eq.(3.24), which is known as the derivative of weighting coefficients.

7. Update $Q'$ by solving Eq.(3.28).

8. Continue Steps 4 and 5, in which the seed matrix is $[V'^i_f]$. This is because $p$, $A$, and $K'^{Bl}$ of the aerodynamic surface wing are functions of the deformed mesh.

9. The convergence of the coupled sensitivity analysis is obtained when the value of $|\alpha'|$ stabilizes; otherwise, steps 4 and 5 must be repeated.

3.6 RESULTS AND DISCUSSIONS

The objective of coupled sensitivity analysis is to obtain the derivative of aerodynamic coefficients, $C'_{L}$ and $C'_{D}$, which are usually necessary in the design optimization formulation. The most important part of the optimization problem lies in the evaluation of accurate sensitivity derivatives. The accuracy of the solution procedure adopted in this work to solve the coupled sensitivity equations is demonstrated by perturbing the magnitude of the basic vector of the design variable "D" on the surface mesh (see Fig.(2.7)) by 2%, 5% and 10%. The derivatives obtained by the coupled sensitivity analysis are tabulated in Table 3.1 along with the derivatives obtained by the finite difference method. The table illustrates that the errors in derivative values calculated by the finite difference method for 2% and 5% are less than two percent, whereas the error in derivative values obtained by the finite difference method for 10% is around 10% change. Figures 3.1 and 3.2 show function
values of $C_L$ and $C_D$, respectively, obtained for different perturbation of design variables and their corresponding derivatives obtained from the analytical procedure adopted in the current work. The line drawn by joining all the derivative data represents a perfect tangential curve to the one obtained by joining the functional data points. This is indeed another proof that the proposed coupled sensitivity analysis procedure is accurate in predicting accurate derivatives of lift and drag coefficients. Table 3.1 indicates that the derivatives predicted by the finite difference method deteriorate as the change in design increases. Table 3.2 lists the derivatives of $C_L$ and $C_D$, with respect to all the design variables. Figure 3.3 shows the convergence history of the root mean square value of the derivative of design variable vector, $\alpha'$. It can be inferred from this plot that a few coupled sensitivity analysis iterations are required to obtain couple derivatives. Unlike coupled analysis, coupled sensitivity analysis can be considered faster if the time taken for creating the database of $Q'$ vectors (solution of Eqs.(3.26) and (3.27)) is neglected. Figure 3.4 shows the distributions of pressure derivatives on the wing surface obtained through coupled sensitivity analysis and rigid sensitivity analysis. It is evident from this figure that the pressure derivative value obtained by the rigid sensitivity analysis is very small compared to the one obtained by coupled sensitivity analysis. This characteristic can be attributed to the drawback of the rigid wing model.
Figure 3.1: Coefficient of Lift vs Change in Aerodynamic Mesh

Figure 3.2: Coefficient of Drag vs Change in Aerodynamic Mesh
Figure 3.3: Convergence History of $\alpha'$

Figure 3.4: Distribution of Pressure Derivatives
Table 3.1: Comparison of Derivatives Obtained by Finite Difference and Analytical Methods

<table>
<thead>
<tr>
<th>Change in design variable D</th>
<th>$\frac{dC_L}{da}$</th>
<th></th>
<th>$\frac{dC_D}{da}$</th>
<th></th>
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</thead>
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<tr>
<td></td>
<td>Central Finite Difference</td>
<td>Analytical</td>
<td>Central Finite Difference</td>
<td>Analytical</td>
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<tr>
<td>2%</td>
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<td>-0.39539</td>
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<tr>
<td>10%</td>
<td>-0.36600</td>
<td>-0.39539</td>
<td>-0.06456</td>
<td>-0.07342</td>
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</table>
Table 3.2: Derivatives of Coefficient of Lift and Drag for All the Design Variables

<table>
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<th>Design Variables</th>
<th>$dC_L/da$</th>
<th>$dC_D/da$</th>
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</thead>
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<td>A</td>
<td>-3.9566E-02</td>
<td>-7.3456E-03</td>
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<tr>
<td>B</td>
<td>-1.0848E-02</td>
<td>-1.9929E-03</td>
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<td>D</td>
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<tr>
<td>T</td>
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CHAPTER IV

MULTIDISCIPLINARY SHAPE DESIGN OPTIMIZATION

Optimization indicates a process of finding the best possible solution for a given problem by satisfying all the restrictions (constraints) imposed on the system. Therefore, it is understood that the objective and constraints of multidisciplinary optimization involve two or more disciplines. In the current work, optimization of an aeroelastic wing is performed such that the structural and aerodynamic disciplines are treated in a coupled fashion. In Chapters 2 and 3, coupled analysis and sensitivity analysis techniques have been discussed in detail. Therefore, in this chapter, the design optimization portion of the current work will be discussed.

4.1 FORMULATION OF A SHAPE DESIGN OPTIMIZATION PROBLEM

An optimization problem basically consists of minimizing or maximizing the given objective function, as the case may be, while satisfying all the constraints. The minimization or maximization algorithms for both one-and multi-dimensional problems can be classified into three distinct categories: zeroth-, first- and second-order methods. In the zeroth-order method, only the values of the objective and the constraints are used during the minimization process. The first-order method employs values of the objective and constraint functions, as well as their first-order derivatives, with respect to the design variables. The second-order method uses the values of the objective and constraints as well as their first- and second-order derivatives. The drawback of zeroth-order method is that the objective and constraint functions are evaluated at many design points. For problems such as the aero-elastic wing, these evaluations consume a great deal of computational time. Hence, this method is considered to be slow and costly in computational terms. The first-
and second-order methods are the best choice for optimizing the aero-elastic wing, provided the gradients or the Hessian matrix can be computed efficiently. Compared to the first-order method, the second-order method again requires more computational effort in obtaining second-order derivatives of the objective and constraint functions. Therefore, in the current work, the first-order method has been selected for functional minimization.

The particular design optimization problem for a flexible wing studied here can be casted in the form:

$$\min C_D,$$

subjected to $$C_L \geq C_L^0$$, with the side constraints on design variables $$b_{\min} \leq b \leq b_{\max}$$. Here, the coefficient of drag, $$C_D$$, is considered the objective function, while the coefficient of lift, $$C_L$$, as the constraint function. The symbol $$C_L^0$$ is the lift coefficient of the initial configuration of the aerodynamic wing. The vector "$$b$$" represents the vector of design variables which are involved in the minimization process, and $$b_{\min}$$ and $$b_{\max}$$ are the lower and upper limits of the design variables respectively, between which the design variables can be changed during the optimization process. In the current problem, the magnitude of basic vectors associated with design points (A-T) as shown in Figure 2.7, are allowed to be changed approximately $$\pm 10\%$$ of their initial values. The aerodynamic wing model considered for optimization is 65 ft. long in the spanwise direction and 32.5 ft. wide at the root. The fluid flow domain, including the aerodynamic wing surface, consists of 3,867 node points, of which 1,073 lie on the aerodynamic wing surface. The flow field is discretized using tetrahedral cells. The wing model used in the structural analysis consists of 61 node points, and has the same dimension as that of the aerodynamic wing. The struc-
tural wing is reinforced with truss and CST finite elements. The material of the structural wing was considered to be made up of Aluminum 6061-T6, which has a modulus of elasticity, Poission's ratio and density of 107 lbs/in.$^2$, 0.33 and 0.097 lbs/in.$^3$, respectively.

4.2 OPTIMIZATION PROCESS

An optimization process consists of three important steps. These steps are: (i) calculating objective and constraint functions (in this case, they are $C_D$ and $C_L$); (ii) computing first-order derivatives of $C_L$ and $C_D$ with respect to the design variables; and (iii) applying a minimization algorithm to minimize the objective function while satisfying the constraints. The first two steps are important in obtaining better results during the third step, and have been explained in Chapters 2 and 3. The flow chart of the design optimization process is presented in Fig.(4.1).

In the current work, the software Design Optimization Tool (DOT) [56] has been used as the minimization algorithm. This tool allows users to build the optimization problem and call the optimizer to perform minimization or maximization of the objective functions, as the case may be. In this regard, this tool has been interfaced with the coupled analysis module, and the minimization process is carried on using the gradient-based method. The DOT provides three methods for constrained optimization. Among these methods, the first, known as “Modified Method of Feasible Direction”, is used in the current work. This method is a gradient-based one, and primarily consists of two steps. The first step is to determine a search direction, which defines how the design variables will be changed. The idea is to change the design variables simultaneously, in a fashion that will improve the design. The second step is to determine how far to move in this direction, which can be called one-dimensional search. This process of finding a search direction is repeated until
optimum convergence is reached. The gradient information required by the optimizer is obtained from coupled sensitivity analysis, which is not interfaced with the optimizer at the current stage of work.

4.3 RESULTS AND DISCUSSIONS

The results obtained from the first design optimization cycle are presented in both graphical and tabular forms. This portion of the current work was intended to demonstrate the feasibility of the interfacing algorithms, as well as the use of the coupled sensitivity analysis methodology developed in this work, to arrive at the optimum design. Therefore, the design optimization of the wing is limited for one iteration run, in which the initial design and the coupled derivatives remain unchanged throughout the design optimization iteration.

Table 4.1 lists the optimization results for one iteration. During this iteration, the optimizer evaluated the objective and constraint functions 77 times, and used the gradient information 5 times, to find a new design point. The first optimization cycle was completed because the change in the objective function was within the tolerance limit set in the optimization algorithm. Since the main objective of this optimization process is to reduce drag and increase the lift of the wing, the ratio of lift to drag is a good indicator in providing a measure of the optimized level. Table 4.1 indicates that the lift to drag ratio of the initial and the final configurations are 4.2082 and 4.2734, respectively. This is a good improvement in the value for just one optimization cycle. In this particular design, $C_L$ and $C_D$ are 0.1754 and 4.1055E-02, respectively, in which $C_L$ has not violated the constraint bound, $C_L^0 = 0.1759$. Table 4.2 indicates the optimum values of the design variables. From this table, it is apparent that although none of the design variables have reached the limit,
the design variable associated with design point "J" is very close to reaching the upper limit with a value of 0.08. The lower and upper limit on this variable was set to -0.1 and 0.1, respectively.

Figure 4.2 indicates the initial and final configurations of the aerodynamic wing. This illustrates that the size of the final configuration of the wing is decreased along the spanwise direction, and that there is a major shift in the chord-wise direction from the center to the tip of the wing. Figure 4.3 indicates pressure distribution on the initial configuration (left-hand side) and the final configuration (right-hand side) of the wing. Although the difference in the pressure distribution is not glaring in this plot, there is still a significant difference around the tip. Figures 4.4 to 4.8 present the pressure distribution on the initial configuration and final configuration at stations along the spanwise direction (y-coordinate), 95.23 in., 188.23 in., 277.19 in., 360.58 in. and 506.1 in., respectively. In Fig. (4.4), the station represented is close to the root of the wing, therefore there is little change in the shape of the cross-sections between the final and initial configurations. However, there is a clear change in the pressure distribution along the leading tip and the top sections. The change in pressure distribution becomes more pronounced in Figs. (4.5) to (4.8), whereas the change in the cross-section is not significant in any of these figures. More pronounced changes can be expected if one updates the design variables, repeats the coupled sensitivity analysis and continues more optimization runs.
Figure 4.1: Flow Chart of One Design Optimization Cycle
Figure 4.2: Initial and Final Configurations of Aeroelastic Wing
Figure 4.3: Distribution of Pressure Contours
Figure 4.4: Pressure Distribution at Span Section, $y = 95.23$ in.
Figure 4.5: Pressure Distribution at Span Section, \( y = 188.23 \) in.
Figure 4.6: Pressure Distribution at Span Section, $y = 277.19$ in.
Figure 4.7: Pressure Distribution at Span Section, \( y = 360.58 \) in.
Fig 4.8: Pressure Distribution at Span Section, y = 506.10 in.
Table 4.1. Optimization History

<table>
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<tr>
<th>Optimization Cycle</th>
<th>Initial $C_L/C_D$</th>
<th>Final $C_L/C_D$</th>
<th>Function Evaluations</th>
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<td>5</td>
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Table 4.2. Optimized Value of the Design Variables

<table>
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<th>Design Points</th>
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<th>Optimized Value</th>
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</thead>
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<td>3</td>
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</tr>
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CHAPTER V

CONCLUSIONS

5.1 SUMMARY

The main objective of this work was to develop an efficient and accurate methodology for coupled aeroelastic analysis and sensitivity analysis, while causing minimal changes to the existing disciplinary codes, with the idea of obtaining an effective methodology to perform multidisciplinary optimization in a loosely coupled environment. It is a well-established fact in the aeroelastic research community that there are two difficult tasks in the coupling process which have drawn the attention of several researchers. These tasks can be categorized as aerodynamic load transfer and deformation tracking. The most common methodology used for load transfer and deformation tracking are projection and curve fitting, which do not guarantee a "consistent and conservative" load transferring process. Therefore, in the current work, aerodynamic load transfer has been accomplished by a different approach called "Reaction Force Method". In this method, an artificial shell structure was introduced to transfer the aerodynamic load to structural load. The Reaction Force Method has been proven to be conservative and consistent, thereby establishing the accuracy of the methodology used in the current work. The same artificial shell structure was also used in the current work to transfer the structural displacements to the aerodynamic surface displacements. To complete the deformation tracking, the change in the aerodynamic interior mesh, caused by the change in the aerodynamic surface mesh, is obtained by the finite element analysis of an additional artificial spring structure. Although the deformation tracking method discussed above does not consume significant computational effort, further simplification can be achieved by introducing the basic vectors to
approximate the elastic structural deformation and the change in the aerodynamic surface mesh. This method is particularly advantageous in coupled shape sensitivity analysis. In coupled shape sensitivity analysis, the basic vectors used in deformation approximation have been extended to approximate the shape change, caused by the change in design variables. The basic vectors referred to here can be generated in several forms such as eigenvectors, prescribed displacement patterns and the displacement vectors obtained from finite element analysis of the structural wing, with unit force acting at the design points. This concept of using basic vectors to approximate deformation and shape changes leads to a new method of performing coupled shape sensitivity analysis in a decoupled manner. As demonstrated by the example problem, the new method does provide a great deal of flexibility in coordinating disciplinary codes for coupled analysis and sensitivity analysis.

Lastly, the optimization was performed on the aeroelastic wing to demonstrate the effectiveness of the procedures and schemes adopted in the current work. In this regard, a DOT algorithm was used to minimize the drag coefficient. The optimization was carried on for just one iteration, because the main objective in this work was to illustrate that the proposed schemes and procedures adopted in coupled analysis and coupled sensitivity analysis work effectively. Hence, from the results of optimization, appreciable improvement in the lift-to-drag ratio was noticed. A modest change in the shape of the wing was also noticed in the chord-wise and span-wise directions.

5.2 FUTURE WORK

There are several issues which have not been addressed extensively in this work, due to the fact that they require additional research. These issues are listed as follows.
1. The multidisciplinary shape design optimization of the flexible wing for one design cycle was performed in the current work with some level of human interaction between the coupled analysis and coupled sensitivity analysis. Therefore, these two modules must be interfaced with the optimizer in order to complete the optimization cycle in an automatic manner.

2. Another important issue to be addressed is the selection of basic vectors. Several forms of the basic vectors such as eigenvectors, prescribed displacement patterns and the one used in the current work can be used for approximating displacements and shape changes. A basic guideline must be established to select the proper number and the best forms of basic vectors for approximation. These issues require additional research.

3. The new concept of introducing the basic vectors for approximating wing deformations and the shape change has opened a way for parallelization. In parallel processing, the rigid wing aerodynamic sensitivity analysis code can be used to calculate $Q'$ for each of the basic vectors on separate processors, simultaneously. Since computing the derivatives is a time consuming process, such parallelization will certainly improve the efficiency of coupled shape design optimization.

4. Another aspect which also requires additional research is the load approximation method. In the current work, Reaction Force Method has been limited to an example problem in which the nodes on the structural wing formed a subset of the aerodynamic surface grid points. However, it is expected that the Reaction Force Method is still applicable to those cases in which the structural nodes do not form a subset of the aerodynamic surface grid points. In those cases, multiple point constraints (MPC) can
be employed to relate the constraints on the structural nodes to the surrounding aero-
dynamic surface grid points, providing the structural nodes are in contact with the
aerodynamic surface.

There are other types of applications where the structural nodes are not in contact
with the aerodynamic surface. In other words, there are mismatches and gaps between
the aerodynamic and structural wing surfaces. In these cases, the gaps can be filled
with a three-dimensional elastic media. The elastic media can be discretized into finite
element form and the compatible loads and displacements can still be transferred with
the help of this elastic media, as performed by the artificial shell structure. The imple-
mentation of this technique will be addressed in future work as well.
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VITA

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