2012

A Wealth of Numbers: An Anthology of 500 Years of Popular Mathematics Writing, by Benjamin Wardhaugh. Princeton University Press: Princeton, 2012 (Book Review)

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Original Publication Citation

Adam, J. A. (2012). A Wealth of Numbers: An Anthology of 500 Years of Popular Mathematics Writing, by Benjamin Wardhaugh. Princeton University Press: Princeton, 2012 (Book Review). *American Journal of Physics, 80*(8), 745-746. doi:10.1119/1.4729608

In Chapter 11, the authors foresee econophysics "making it possible to perform a scientifically rigorous reappraisal of the consequences of economic growth, and even whether growth is desirable under all circumstances." On what is this hope based? There is no substantial treatment of economic growth in the entire preceding ten chapters. Models of sharing or competing for wealth and services are purely zero-sum.

Mainstream economics may indeed be seriously flawed, but econophysics (as presented here) does not offer a serious alternative. All the statistical explorations, unrealistic assumptions, and gratuitous appeals to physics, taken together, do not even add up to a coherent discipline.

[1]See also Martin Schaden's review in this journal of a book on "quantum finance" that lacks quantum mechanics: Am. J. Phys. **78**(6), 664–665 (2010).

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A Wealth of Numbers: An Anthology of 500 years of Popular Mathematics Writing. Benjamin Wardhaugh. (ed.) 388 pp. Princeton U.P., 2012. Price: $45.00 (cloth) ISBN 978-0-69114775-8. (John A. Adam, Reviewer.)

To describe the landscape encompassed by this book I can do no better than to quote the dust jacket: "*A Wealth of Numbers* includes recreational, classroom, and work mathematics; mathematical histories and biographies; accounts of higher mathematics; explanations of mathematical instruments; discussions of how math should be taught and learned; reflections on the place of math in the world; and math in fiction and humor." More such details can be found on the Princeton University Press website. I shall use this as a point of departure to describe the highlights of my own trajectory through the book.

Not being a huge fan of puzzles, I skimmed Chapter One ("Sports and Pastimes, Done by Number: Mathematical Tricks, Mathematical Games") to find some fascinating tidbits in the next ("Much Necessary for All States of Men: From Arithmetic to Algebra.") Who can fail to be intrigued by J. E. Thompson's 1931 article on "Cubic Equations for the Practical Man"!

The book contains a plethora of items from sources devoted to women, though at times they appear somewhat patronizing. In chapter 3, for example, Wardhaugh selects some questions from *The Ladies' Diary*—which ran from 1701 to 1841. The first of these (and seemingly the simplest, with three alternative solutions submitted by readers) is: *What two numbers are those whose product, difference of their squares, and the ratio or quotient of their cubes, are all equal to each other*? (The Golden Ratio is the smaller of the two.) Other questions concern areas of fields, the distance between summits of hills, and length of tree shadows, to name but a few. My particular favorite is: *"I have seen a sheep leap from a bridge very high, into water, and swim out. Now, if a globe, whose weight is 112 pounds, and one foot in diameter, fall from an eminence ten yards high, how deep must the water be, just to destroy all the globe's velocity, supposing the density of air, water, and the globe to be in the ratios 1.2:1000:10,000 respectively."* I wonder how appreciated such a question might be on a "Physics 101" final examination.

I liked the inclusion of *Introduction to Spherical Geometry* (Horatio Nelson Robinson, 1854) and *Napier's Rules* (Alan Clive Gardner, 1956) in the fourth Chapter ("Drawyng, Measuring and Proporcion": Geometry and Trigonometry). One of my favorite Chapters was the next (Maps, Monsters, and Riddles: The Worlds of Mathematical Popularization). It includes the entry *Newton for the Ladies* (Francesco Algarotti, 1739) and *Einstein's Real Achievement* (Oliver Lodge, 1921)—a brief discussion of special relativity. There is also an excellent piece, *Saddles and Soap Bubbles* by Iakov Isaevich Khurgin (1974).

Passing over Chapter Six ("To Ease and Expedite the Work: Mathematical Instruments and How to Use Them") we next encounter "How Fine a Mind: Mathematicians Past". An anonymous entry from *The Children's Picture-Book of Good and Great Men* (1860) features a "charming, though not exactly accurate" portrayal of Isaac Newton. A piece containing items about the great mathematicians Emmy Noether (*Her Absolute, Incomparable Uniqueness*, B. L. van der Waerden, 1935), Leonhard Euler (*Analysis Incarnate*, Carl Boyer, 1968) and Ramanujan (*Hardy and Littlewood Rummage*, Robert Kanigel, 1991) close out this very interesting Chapter. Chapter Eight is devoted to extremely practical matters, as its name suggests ("By Plain and Practical Rules: Mathematics at Work"). This includes topics on sailing, high-pressure engines, the strength of materials, plumbing and hydraulics, and automobile mechanics.

The loquacious-sounding Chapter Nine ("The Speedier Expedition of Their Learning: Thoughts on Teaching and Learning Mathematics") contains some real gems. I enjoyed *The Idea of Velocity* (1760) by Euler in his *Letters to a German Princess*, and *A Mother Explains Comets* (Catherine Vale Whitwell, 1823). In this latter entry, a dialogue between a mother and her daughter, I was very impressed by the daughter's questions, and would certainly encourage her to pursue a degree in the astronomical sciences (or perhaps to teach both science and English to my students!) [Thus: "*I have heard various conjectures respecting the cause of the tails of the comets; will you have the goodness to mention to me the most plausible opinion?*"]. For me, the highlight of this Chapter was *Higher Mathematics for Women* (Mrs. Henry Sidgwick, 1912). She was Principal of the all-female Newnham College, Cambridge, and a promoter of university education for women. In citing this article, Wardhaugh writes "Her warning against teaching and learning mathematics only 'because it is useful for something else' retains its force in the twenty-first century." Excellent stuff.

Chapter Ten ("So Fundamentally Useful a Science: Reflections on Mathematics and Its Place in the World") was the one to which I first turned, but containing as it does items from

1481 through to 1978, I found, not surprisingly, the most recent entries the easiest to read and absorb. The jewel in the crown for me, and I suspect for many readers of AJP, was Feynman's 1965 essay *The Character of Physical Law*, but a (fairly distant) second was *Our Invisible Culture* (Allen L. Hammond, 1978) about the elusive human face of mathematics. I think that more than three decades later, developments in mathematics and its applications to biology in particular may render some aspects of this entry dated, but then that is to be expected in a volume spanning 500 years of mathematical writing!

The final Chapter ("The Mathematicians Who Never Were: Fiction and Humor") contains several amusing items, but I gravitated immediately to an extract from Edwin Abbott's celebrated 1884 classic, *Flatland* (here subtitled *A Sight of Thine Interior*). The joys of "dimensional interface phenomena" are many and varied! In fact, several times in his Scientific American essays Martin Gardner (1914–2010) discussed the dimensional analogy, so I am astonished that there is no mention of him in this book (at least, I could find none). He was a well-known and highly regarded popular mathematics and science writer, specializing in recreational mathematics. His obituary in the NY Times described him as "puzzler and polymath," and the tribute to him in the Scientific American noted that "For 25 years, he wrote *Scientific American*'s Mathematical Games column, educating and entertaining minds and launching the careers of generations of mathematicians." In my opinion this certainly qualifies him for inclusion in this eclectic survey of popular mathematics writing.

In summary, this book contains much hard-to-find material that will be of particular interest to those who teach the history of mathematics, but also to those who wish to broaden their mathematical horizons in time (and space!), despite the significant omission of some of Gardner's prolific writings. There is much here worth delving into, and that is probably the best way to approach the book, rather than reading it through directly.

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A First Course in Loop Quantum Gravity. Rodolfo Gambini and Jorge Pullin. 191 pp. Oxford U.P., New York, 2011. Price: $62.95 (hardcover) ISBN 978-0-19-959075-9. (Seth Major, Reviewer.)

Once we have it, quantum gravity will explain the microstructure of space-time. Is space continuous? What about time? Does space-time geometry make sense near the initial singularity? What is the description of physics deep inside a black hole? What is the dimension of space-time on the smallest length scales? These are the sort of questions, the theory of quantum gravity is expected to answer. The essence of our search for the theory is an exploration of the quantum foundations of space-time.

The theory of quantum gravity is a fascinating subject full of wonderful conceptual and technical challenges. It is no wonder that Rodolfo Gambini and Jorge Pullin wish to introduce it to undergraduates with a "light and nimble" (p. iv) presentation in **A First Course in Loop Quantum Gravity.**

General relativity makes a strong statement: geometry is physical, rather than a mere array of background coordinates over which field and particle dynamics unfolds. At times called general coordinate invariance, or diffeomorphism invariance, the background independence of general relativity makes sense for the theory of the space-time geometry with which and on which stuff happens. It is also radically different from other well-tested physical theories that typically assume a great deal of background structure.

Loop quantum gravity is an attempt, as yet incomplete, to quantize Einstein's theory of general relativity in four space-time dimensions while retaining its background independence. In the Hamiltonian approach taken, the kinematics (the quantum geometry of space) is quantized before the dynamics (the evolution of spatial geometry). To date, loop quantum gravity has succeeded at the first step. Building on methods from field theory, an essentially unique quantum theory of spatial geometry emerges. Loop quantum gravity predicts discrete spectra of familiar geometric quantities such as length, area, volume, and angle. Acting like quantum mechanical angular momentum operators, these operators give the physical geometry. The scale of the granularity in geometry is set by the fantastically remote Planck length, or about 10^{-35} m.

As one would expect from a quantum theory of geometry, the state space is suitably weird. For instance, there exist states for which surfaces have area, but every closed surface encloses vanishing volume. The state space is also suitably familiar. The space contains states that closely approximate our apparent Euclidean geometry of daily life.

Loop quantum gravity has also achieved success in applications to cosmology, black holes, and even the nascent field of quantum gravity phenomenology. However, before the dynamics is complete, the theory must be regarded as tentative.

In presenting this fascinating but incomplete theory, there are two immediate issues to settle—the reader's background knowledge and the expected depth of understanding. Wisely choosing not to include general relativity in the required background, the authors state that they assume electrodynamics, Lagrangian and Hamiltonian mechanics, special relativity, and quantum mechanics. In fact, the requirements go a bit beyond this with such topics as index notation and complex analysis. But the gaps between core subjects in the undergraduate physics major and the book could be bridged in a class setting. As to the expected depth of understanding, the authors announce their modest goal of hoping that