MIMO Radar Waveform Design and Sparse Reconstruction for Extended Target Detection in Clutter

Christopher Alan Rogers
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MIMO RADAR WAVEFORM DESIGN AND SPARSE RECONSTRUCTION FOR EXTENDED TARGET DETECTION IN CLUTTER

by

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B.Sc. May 2008, United States Coast Guard Academy
M.E. May 2014, Old Dominion University

A Dissertation Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

ELECTRICAL & COMPUTER ENGINEERING

OLD DOMINION UNIVERSITY
May 2019

Approved by:

Dimitrie C. Popescu (Director)
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Linda Vahala (Member)
Rafael Landaeta (Member)
ABSTRACT

MIMO RADAR WAVEFORM DESIGN AND SPARSE RECONSTRUCTION FOR EXTENDED TARGET DETECTION IN CLUTTER

Christopher Alan Rogers
Old Dominion University, 2019
Director: Dr. Dimitrie C. Popescu

This dissertation explores the detection and false alarm rate performance of a novel transmit-waveform and receiver filter design algorithm as part of a larger Compressed Sensing (CS) based Multiple Input Multiple Output (MIMO) bistatic radar system amidst clutter. Transmit-waveforms and receiver filters were jointly designed using an algorithm that minimizes the mutual coherence of the combined transmit-waveform, target frequency response, and receiver filter matrix product as a design criterion. This work considered the Probability of Detection ($P_D$) and Probability of False Alarm ($P_{FA}$) curves relative to a detection threshold, $\tau_{th}$, Receiver Operating Characteristic (ROC), reconstruction error and mutual coherence measures for performance characterization of the design algorithm to detect both known and fluctuating targets and amidst realistic clutter and noise. Furthermore, this work paired the joint waveform-receiver filter design algorithm with multiple sparse reconstruction algorithms, including: Regularized Orthogonal Matching Pursuit (ROMP), Compressive Sampling Matching Pursuit (CoSaMP) and Complex Approximate Message Passing (CAMP) algorithms. It was found that the transmit-waveform and receiver filter design algorithm significantly outperforms statically designed, benchmark waveforms for the detection of both known and fluctuating extended targets across all tested sparse reconstruction algorithms. In particular, CoSaMP was specified to minimize the maximum allowable $P_{FA}$ of the CS radar system as compared to the baseline ROMP sparse reconstruction algorithm of previous work. However, while the designed waveforms do provide performance gains and CoSaMP affords a reduced peak false alarm rate as compared to the previous work, fluctuating target impulse responses and clutter severely hampered CS radar performance when either of these sparse reconstruction techniques were implemented. To improve detection rate and, by extension, ROC performance of the CS radar system under non-ideal conditions, this work implemented the CAMP sparse reconstruction algorithm in the CS radar system. It was found that detection rates vastly improve with the implementation of CAMP, especially in the case of fluctuating target impulse responses amidst clutter or at low receive signal to noise ratios ($\beta_n$). Furthermore, where previous work considered a $\tau_{th} = 0$,
the implementation of a variable $\tau_{th}$ in this work offered novel trade off between $P_D$ and $P_{FA}$ in radar design to the CS radar system. In the simulated radar scene it was found that $\tau_{th}$ could be moderately increased retaining the same or similar $P_D$ while drastically improving $P_{FA}$. This suggests that the selection and specification of the sparse reconstruction algorithm and corresponding $\tau_{th}$ for this radar system is not trivial. Rather, a tradeoff was noted between $P_D$ and $P_{FA}$ based on the choice and parameters of the sparse reconstruction technique and detection threshold, highlighting an engineering trade-space in CS radar system design. Thus, in CS radar system design, the radar designer must carefully choose and specify the sparse reconstruction technique and appropriate detection threshold in addition to transmit-waveforms, receiver filters and building the dictionary of target impulse responses for detection in the radar scene.
ACKNOWLEDGEMENTS

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Furthermore, I would like to extend thanks and my utmost appreciation for the remainder of my advisory committee: Dr. Chunsheng Xin, Dr. Linda Vahala, and Dr. Rafael Landaeta who provided me with invaluable advice and coaching throughout this journey.

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For my father, Reginald I. Rogers (1948-2016), from whom I continue to learn many of life’s most treasured lessons.
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Chapter 1

INTRODUCTION

Radar systems are used for the identification and tracking of air, land, sea, and space targets. While originally exclusively used in the military to detect ships and aircraft, radar has since been adapted for commercial and public safety consumers. The applications are numerous, including but not limited to use aboard commercial ships and recreational vessels to ensure safe navigation, use in public safety to detect speeding vehicles or ice detection and imaging [1,2].

Modern radio detection and ranging, or radar, systems originate from the physical demonstration of James Clerk Maxwell’s electromagnetic theory by Henrich Hertz in 1888. It was not until the 1930’s that the U.S. Naval Research Laboratory (NRL) patented the very first continuous wave (CW) radar system using disparately located transmitters and receivers in what came to be known as a bistatic radar. These early radar implementations generally were limited to wavelengths greater than 60cm. In 1936, scientists at the NRL achieved monostatic radars where transmitters and receivers are collocated, using 60-MHz pulsed waveforms to detect aircraft, which proved vital to provide early warning to citizens of oncoming air raids. It was not until after World War II that radar systems began to operate in the microwave frequencies vice the commonly used 100-200 MHz range. The use of pulsed radar substantially improved spatial resolution and made determining range tractable.

Research and development in radar flourished following the war spawning several advancements and system improvements. These include: coherency, the advent of synthetic aperture radar (SAR) imaging and use of pulse compression in the 1950’s, integration of sophisticated antenna architectures such as phased arrays or multiple input multiple output (MIMO) designs, and ultrawideband radar implementations [1–4]. And research continues today in finding novel methods or approaches to detect, estimate, or track low radar cross section (RCS) and hard to detect targets in challenging radar environments with such degrading factors as noise and clutter (transmit signal dependent noise).
This chapter presents a review of radar basics and other fundamental concepts required to understand the goals of this dissertation. The chapter concludes with a presentation of relevant work and a summary of specific research contributions.

1.1 RADAR SYSTEM FUNDAMENTALS AND CONCEPTS

Radar involves the transmission of electromagnetic signals into space that illuminate targets in the scene only to have these signals reradiated back attenuated and phase shifted to the receiver for processing. A conceptual depiction of monostatic and bistatic radar systems is given by Fig. 1.

Signal processing at the receiver is used to extract such information as range, bearing, or velocity of a target, notwithstanding imaging applications [3]. The ability of a radar to detect a target or estimate its parameters are two common radar functions. Target detection refers to the radar’s ability to determine the presence of a target within a radar scene. Often in detection problems, some knowledge of target characteristics is required, as in the case of matched filter based detection schemes. Detection problems are typically posed as probabilistic, binary hypothesis tests [5]. Recent research in radar detection includes
the use of ultrawide band radar to assist with urban search and rescue operations. In [6], the ultrawideband radar is designed to detect periodic motion (breathing) of trapped victims in low signal to noise and clutter environments. Other detection problems of interest include the novel means to detect objects with low radar cross sections, such as drones. In [7], a multistatic single-antenna digital television passive radar is used successfully for drone detection. Estimation refers to the ability of the radar to extract information or characteristics of a target known to be present in the radar scene. Parameters such as luminosity, polarimetry, electromagnetic signature, or radar cross section are examples of what practical radar systems might estimate [8, 9]. Recent research in radar estimation includes estimating direction of arrival of surface clutter to improve the ability to characterize glaciological features. Other radar functions include radar tracking (detecting and tracking kinematic parameters), classification (determine type of target), and imaging (extract enough information of a radar scene to form an image) [8,10,11].

In addition to radar configuration (monostatic, bistatic, or multistatic) and function, radars are also characterized by the composition of their transmitted waveform, be it: continuous, pulsed, or digital; and by their main beam scanning type, such as: fixed beam, multibeam, oscillating or rotating (mechanical), electronic as well as hybrid (both mechanical and electrical). Furthermore, radar signal processing approaches designed to filter interference and improve system performance also vary. Examples include: constant false alarm rate (CFAR) processing, space time adaptive processing (STAP) and adaptive thresholding [8].

1.1.1 TARGETS

In classical radar theory, a point target is assumed to be a far-field reflector that returns a phase shifted and attenuated version of the transmitted signal. Juxtaposed to individual source point targets, extended radar targets are an amalgamation of multiple point targets and thus represent a larger, more complex physical manifestation. The consequence of detecting targets of multiple reflection centers is that extended targets reflect separate echoes at each of its reflection centers, creating resonance effects in the electric field. This creates a multipath-like effect in the received signal, complicating radar signal processing [9,12].
1.1.2 RADAR FREQUENCIES

Several factors must be considered when designing the operating frequency of a radar system to include: propagation characteristics, spatial resolution, ability to penetrate materials, electromagnetic or radiofrequency interference, and target phenomenology. This dissertation considers the use of an X-band, bistatic radar for detecting airborne targets. Radar frequency bands and their associated applications and areas of current research are presented in Table 1 [2].

Table 1. Radar Frequencies and Applications

<table>
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<tr>
<th>Frequency Band</th>
<th>Range</th>
<th>Applications</th>
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<tbody>
<tr>
<td>High Frequency</td>
<td>3-30 MHz</td>
<td>Over-the-Horizon Radar, Long range radar [2], ocean surface currents [13]</td>
</tr>
<tr>
<td>Very High Frequency</td>
<td>30-300 MHz</td>
<td>Foilage penetrating radar [2], Low radar cross section (RCS) targets (Stealth) [14]</td>
</tr>
<tr>
<td>Ultra High Frequency</td>
<td>300 MHz-1 GHz</td>
<td>Ballistic defense, airborne surveillance [2], low Radar Cross Section (RCS) targets (Stealth) [14]</td>
</tr>
<tr>
<td>L-Band</td>
<td>1-2 GHz</td>
<td>Air traffic control [2], sub-surface topography, ice sheet composition, soil freeze-thaw [4,15]</td>
</tr>
<tr>
<td>S-Band</td>
<td>2-4 GHz</td>
<td>Naval surface, weather [2], submarine surveillance [16] ground penetrating [4]</td>
</tr>
<tr>
<td>C-Band</td>
<td>4-8 GHz</td>
<td>Weather radar [2], iceberg detection [17], ground penetrating [4]</td>
</tr>
<tr>
<td>X-Band</td>
<td>8-12 GHz</td>
<td>Fire-control radar, ballistic missile tracking, air interceptor [2], drone detection [18], sea state monitoring [19]</td>
</tr>
<tr>
<td>Ku-Band</td>
<td>12-18 GHz</td>
<td>Air-to-Ground SAR [2], ice penetrating radar [20]</td>
</tr>
<tr>
<td>K-Band</td>
<td>18-27 GHz</td>
<td>Limited use (Absorption) [2], applications in monitoring ocean wave characteristics [21], vehicle Ultra Wideband (UWB) [4]</td>
</tr>
<tr>
<td>Ka-Band</td>
<td>27-40 GHz</td>
<td>Missile Seekers [2], cardiopulmonary remote sensing [22], vehicle UWB [4]</td>
</tr>
</tbody>
</table>
1.2 COMPRESSED SENSING RADAR

The resolution of detection and classification radar systems is reliant on both the transmission and reception of wide bandwidth signals such as a linear chirp or a pseudonoise (PN) sequence over a small observation time. These requirements drive the need for costly and complex wideband receivers that require speedy analog to digital (A/D) converters and depend upon pulse compression and matched filtering for detection [23, 24]. In fact, in conventional pulsed radar systems A/D converters are required to sample at a rate in the hundreds of megahertz, resulting in a considerable amount of data to be processed [3]. While theoretically optimal in the sense that the SNR is maximized, matched filter centric detection carries with it the underlying assumption that the receiver knows with exactness the characteristics of the reflected wave it is designed to detect. Furthermore, for extended targets, targets amidst clutter, and targets that are moving at an unknown velocity, multiple matched filters are used, each with a unique frequency bin. In this application, it is well understood matched filter banks are susceptible to sidelobe interference [25].

Capitalizing on the inherently sparse nature of radar scenes, Compressed Sensing (CS) techniques have been proposed to improve upon the constraints of conventional radar architectures, namely eliminating the need for matched filter detection while reducing the requisite A/D conversion bandwidth [23, 24]. Formalized by Candès, Tao, and Romberg [26, 27] and Donoho [28], CS refers to the process by which sparse signals are sampled and then reconstructed with substantially fewer measurements than would otherwise be required by the Nyquist sampling theorem [29]. The findings in [26–28] are preceded by earlier results [30, 31] with ℓ₁ minimization pertaining to the study of seismograms [32]. CS was first applied to radar by Baraniuk [24] for 1-dimensional radar imaging of point targets. Baraniuk successfully demonstrated the concept that a radar scene can be reconstructed with significantly fewer measurements than would otherwise be required using conventional sampling paradigms and were able to sample at rate much closer to the radar reflectivity’s information rate. The sparsity, or compressibility of a signal, is a requirement for CS and is often met in radar applications where the number of resolution cells in an illuminated area is much greater than the number of targets in a radar scene. CS enables a reduction in both
the sampling rate and required data rate while keeping the resolution fixed [24]. Refer to Appendix A for additional information regarding CS.

CS is, at its core, a non-linear reconstruction program, most often represented as an $\ell_1$ minimization problem. Many CS recovery algorithms have a variable regularization parameter that dictates the quality of the reconstruction result that should be tuned for optimal performance akin to that of Constant False Alarm Rate (CFAR) processors [23].

1.3 RELEVANT WORK

The design of radar waveforms to improve detection, false alarm rate, or other performance criteria is not itself novel. Bell [9] builds upon the pioneering works of Shannon [33] and Woodward [34] and develops an information theoretic based waveform design methodology to improve both the detection probability of a single extended target in additive noise, given a deterministic target impulse response and also target estimation performance, by maximizing the mutual information between the received radar waveform and random target ensemble. For target detection, the problem of waveform design was centered on maximizing the signal-to-noise radio at the receiver filter for optimal performance under the Neyman-Pearson or Bayes’ decision rule. This was done by placing as much energy as possible into exciting the mode of the target with the largest eigenvalue. Central to proceeding academic work, including this dissertation, was that Bell [9] was also the first to explicitly define the target impulse response for both deterministic and random targets.

The pre-cursor to cognition, or learning, in radar are fore-active radar systems [35, 36] that make use of radar scene observations to modify either the transmitted signal or receiver processing [37]. Cognitive radar systems are those that are intelligent and can adapt operating parameters autonomously to meet performance criteria under a highly dynamic radar channel. Cognitive radar stems from cognitive radio, where the idea of cognition as an extension to software defined radios was first introduced by Mitola [38]. In [39–41], Haykin introduces fully cognitive radar systems, where [41] was the first work to define a cognitive, fully adaptive and learning-based radar and did so by introducing a simple feedback loop. Since then, research in cognitive radar has flourished.

Adaptivity was implemented in radar to probabilistically detect and track multiple targets
via adaptive beamforming as part of a cognitive radar network comprised of two radars operating cooperatively [42]. However, research was limited to the consideration of point targets, only. An adaptive waveform design algorithm to detect a single extended target was considered in [43]. Next, Leshem [12] gave a MIMO radar waveform design algorithm for the estimation of multiple extended targets based on maximizing mutual information measures between targets and transmitted waveforms. This work incorporates a similar beamforming approach to the MIMO phased array radar and extended target models as in [12]. Additionally, Kay [44] presented a waveform design methodology to detect extended targets for a multistatic radar. This work includes a similar clutter model as that which is used in [44] and also incorporates the non-deterministic aspects of the target responses for extended targets, similar to what is considered in [44] and [9].

Furthermore, this work assumes some knowledge of characteristics of targets that may be present in the radar scene to perform compressed sensing and sparse reconstruction. If this knowledge base were imprecise or inaccurate regarding what is actually present in the radar scene, sparse reconstruction would be challenged due to a mismatch in basis. A similar problem was studied that pertained to sparse reconstruction where there is error introduced within the sensing matrix itself [45,46]. It was found that under such conditions, reasonable approximations of the sparse unknown signal are possible, provided that the mismatch is small.

Finally, Daniel [37,47,48] introduces an adaptive algorithm for the detection of multiple extended targets via the compressed sensing approach. This was the first work to introduce an adaptive joint waveform-receiver filter design algorithm, and it is based on the MIMO CS approach first proposed by Zhang [49] where minimizing the mutual coherence of the sensing matrix was used as the design criterion alternative to the designing the sensing matrix to explicitly satisfy the Restricted Isometry Property (RIP) to detect multiple point targets. For compressed sensing and sparse recovery, a necessary condition for the sensing matrix is that it must satisfy RIP, though is often computationally intractable for larger sensing matrices. Similarly, Chen [50] poses a transmit-waveform design approach for a compressed sensing based cognitive radar system to simultaneously estimate multiple extended targets.
As with [49], Chen chooses as a waveform design criterion to minimize the off-diagonal elements of a defined Gram matrix in order to minimize the mutual coherence of the sensing matrix. However, in [50], the authors did not consider MIMO arrays, clutter, multiple reconstruction methods, target ambiguity (e.g. Swerling targets) and was focused primarily on the radar estimation problem. In [51], a novel method was posed to design deterministic sensing matrices to minimize mutual coherence in electromagnetic compressive imaging applications where verifying RIP is otherwise cumbersome in these applications. Results indicated an improvement in sparse recovery for sensing matrices with minimum coherences. Furthermore, Zhang [52] constructed binary sensing matrices from low density parity check codes that maintained mutual coherence much lower than that of random Gaussian matrices and were found to display favorable recovery probabilities. Finally, in [53], it was shown that the probability of correct detection directly correlates to low mutual coherence for the compressed sensing based single-pixel imager. This work is inspired by these and similar findings where minimizing the mutual coherence of the sensing matrix correlates to improved sparse reconstruction or system performance.

The implementation of MIMO techniques enables an innate sparsity in the spatial domain of the radar scene, a requirement for CS, even amidst multiple extended targets and noise.

1.4 LIMITATIONS OF PREVIOUS WORK

Previous work [37,47,48] presented the novel, but academic approach to radar detection and estimation for a MIMO bistatic radar. Performance of this model was based on probability of detection ($P_D$) and probability of false alarm ($P_{FA}$) curves and error measurements following sparse reconstruction. These results were generated based solely on a fixed detection threshold, $\tau_{th} = 0$, rather than a variable threshold that can be adapted based on the physical characteristics radar scene or performance goals of the CS radar system. A $\tau_{th} = 0$ combined with the fixed sparsity level of the estimated signal caused flat $P_{FA}$ curves where even the smallest perturbations in the sparse estimate led to a false alarm. Rather than strictly $P_D$ and $P_{FA}$ curves, performance of radar systems are commonly characterized by Receiver Operating Characteristic (ROC) curves which are not present in previous work.

Next, [37,47,48] made two crucial simplifications, namely transmit-waveform dependent
clutter defined precisely by a Signal to Clutter Ratio ($\beta_c$) was not included in the system model; and second, the case of fluctuating targets was limited to fluctuations in magnitude, only and was assumed that the locations of these magnitudes were deterministic. In [37], clutter was represented as added noise into the target response term and did not correlate with specific instances of clutter in the radar scene. No evaluation was presented to characterize performance in realistic clutter, amidst realistic fluctuating targets, or in the algorithm’s ability to detect a completely unknown, fluctuating target that appears in the radar scene (e.g., the radar is designed to detect target $A$ and target $B$ is present in the scene).

Furthermore, the model presented in [37,47,48] did not formally consider additional sparse reconstruction techniques other than Regularized Orthogonal Matching Pursuit (ROMP), leaving open the question of whether $P_D$ and $P_{FA}$ performance gains can be realized with implementation of different techniques.

1.5 PROBLEM STATEMENT

This research answers the following question: Are CS radar techniques viable in a radar environment comprised of multiple extended targets amidst strong clutter and noise and where knowledge of the characteristics of targets present in the scene is imprecise? The goal of this dissertation is to present the joint waveform-receiver filter design algorithm as one piece of the larger CS radar system and then demonstrate performance gaps of the system’s current state in terms of commonly accepted radar performance characterization techniques when transmit-waveform dependent clutter and more realistic fluctuating targets are considered. Solutions to the radar detection problem to bridge these gaps in CS radar system design are proposed that enable improved system performance where these results were not previously tenable. Finally, the limitations of this CS radar system will be examined in considering the case of completely unknown target impulse responses present in the radar scene.

1.6 DISSERTATION CONTRIBUTIONS

This dissertation improves upon and rigorously evaluates a novel approach for MIMO
radar detection of multiple extended targets. To the best of this author’s knowledge, no other work comprehensively evaluates the MIMO compressed sensing radar system approach to extended target detection amidst clutter, including consideration of multiple fluctuating target types, multiple sparse reconstruction algorithms, and a variable detection threshold to minimize $P_{FA}$ while maintaining an acceptable $P_D$. The contributions of this work are summarized below.

1. A real-world implementation of transmit-waveform dependent clutter is defined and then implemented directly into the bistatic radar system model, making it more representative of actual conditions. This dissertation considers the problem of whether weak targets can be distinguished from relatively strong clutter located in the immediate vicinity of targets. (Chapters II and V)

2. The CS waveform design algorithm is treated as one part in the larger, proposed CS radar system. A non-trivial detection threshold is proposed to improve performance amidst clutter, noise, and suboptimal reconstruction due to the presence of more complicated fluctuating targets. (Chapter IV)

3. Next, the novel MIMO CS radar system is further expanded to include the consideration, implementation, subsequent evaluation of additional sparse reconstruction techniques such as Compressive Sampling Matching Pursuit (CoSaMP) and Complex Approximate Message Passing (CAMP) to improve $P_{FA}$ and $P_D$ performance, respectively, compared to previous work using ROMP. (Chapters IV and V)

4. ROC curves and mutual coherence measures of performance are presented to characterize system performance in addition to $P_D$, $P_{FA}$, and reconstruction error measures previously considered. (Chapter IV)

5. Next, fluctuating targets based on the Swerling I target model with fluctuating reflection center magnitudes and both fluctuating reflection center magnitudes/locations are defined and injected into the radar scene to test the ability of the CS radar system to adequately detect targets that are not explicitly part of its target dictionary. The
scenario of completely unknown target impulse responses is also evaluated. *(Chapters IV and V)*

6. Finally, results are translated to CS radar system design considerations, highlighting an engineering tradespace for CS radar system design when selecting appropriate detection thresholds and sparse reconstruction techniques or parameters given the confidence in the radar scene or performance goals of the CS radar system. *(Chapter VI)*

### 1.7 ORGANIZATION OF PAPER

The remainder of this dissertation is organized as follows:

- **Chapter II** will introduce the system model, including definitions for extended targets, waveforms, receiver filters, target impulse responses, reflection coefficients, noise and the novel transmit-waveform dependent clutter model.

- **Chapter III** describes the CS MIMO radar approach and formally presents the joint waveform-receiver filter design algorithm considered in [37,47,48].

- **Chapter IV** discusses CS Target Detection amidst clutter and noise and presents new performance measures used to assess the performance of the joint waveform-receiver filter design algorithm. Discussion of a variable decision threshold versus a statically defined threshold is also given. ROC and mutual coherence measures are introduced as additional means to characterize the performance of the transmit-waveform and receiver filter design approach in the posed MIMO CS radar system.

- **Chapter V** describes the simulation setup and presents results (ROC, $P_{FA}$, $P_{D}$, mutual coherence, and reconstruction error) for the joint waveform-receiver filter design algorithm. Results for extended target detection in the presence of realistic, transmit signal dependent clutter sources and additive noise are presented for both known, fluctuating, and completely unknown target impulse responses and using ROMP, CoSaMP, and CAMP for sparse reconstruction.
• **Chapter VI** includes a discussion on CS radar system design, highlighting the degrees of freedom available to the CS radar system designer and highlights the engineering trade-space that exists with design choices in sparse reconstruction algorithm/parameters and detection threshold used in conjunction with the joint waveform-receiver filter design algorithm.

• **Chapter VII** is reserved for concluding remarks and areas of future work.

• **Appendix A** presents a background and introduction of key concepts necessary for understanding this paper. Discussion of CS, sparse reconstruction techniques to include ROMP, CoSaMP and CAMP, as well as MIMO Radar are included.

• **Appendix B** gives the mathematical derivation of the statistics of the clutter model implemented in this dissertation.

• **Appendix C** gives a summary of the constant-\( \gamma \) clutter model and Monostatic Bistatic Equivalence Theorem used in previous work [54,55].

• **Appendix D** contains additional figures that were referenced but not included in the body of this work. These figures are included for completeness.
Chapter 2

MIMO RADAR SYSTEM MODEL

Radar detection problems are often posed as compressed sensing or sparse reconstruction problems given that there are generally few targets of interest in a much larger scene (implying sparsity). In [37, 47, 48], the novel joint waveform-receiver filter design algorithm was presented to improved compressed sensing-based target detection over static methods. This model is now extended to include consideration of transmit-waveform dependent clutter.

Consider a MIMO radar capable of simultaneously transmitting and receiving multiple beams as in [12]. First introduced in [47, 48], this system model uses transmit and receive beamforming is used to sparsify the bistatic radar scene, according to Fig. 2. The MIMO transmitter is comprised of an $N_T$ element phased array producing $T$ orthogonal transmit-waveform beams to detect multiple extended targets. The receiver array is comprised of an $N_R$ receive element phased array with $R$ receive beams. The radar scene is populated with $L$ targets and $Z_c$ clutter sources, both assumed static. The stationarity assumption is made only to simplify the mathematical representation and can otherwise be readily removed to include appropriate Doppler shift parameters, but such work is beyond the scope of this paper. Targets and clutter scatterers are also assumed extended enough in range such that they are fully encompassed by a single beam pair cell.

Extended targets in a bistatic MIMO model have multiple impulse response representations based on the orientation of a particular target with the transmitter and receiver arrays in addition to the physical characteristics of the target itself. Thus, when a target is said to be known or comprised of a impulse response, both its orientation and its is physical characteristics are known a priori. This assumption is removed when the case of fluctuating and unknown extended targets is considered and is similarly removed for clutter scatterers in this model.
**Figure 2.** Bistatic radar system model with targets and clutter sources. As shown, the radar scene is partitioned into a $T \times R$ sparse grid of targets and clutter sources using via transmit and receive MIMO arrays and beamforming techniques.

### 2.1 TIME DOMAIN REPRESENTATION

Let $h_\ell(t)$ represent the respective impulse response of individual extended targets, $\ell$, each comprised of $R_c$ unique reflection centers as in:

$$h_\ell(t) = \sum_{r_c=1}^{R_c} \iota_{r_c}^{(t)} \delta(t - \tau_{r_c}^{(t)}),$$

where $\iota_{r_c}$ and $\tau_{r_c}$ are the magnitude of the response for each reflection center, $r_c$, and associated time delay, respectively. Note that the time delay for each $r_c$ corresponds to its location in range space relative to the radar as per [37, 47, 48]. Consistent with previous work [37, 47, 48, 54, 55], this dissertation assumes that $h_\ell(t)$ is normalized to unit energy.

Transmit-waveforms, $s_d(t)$, are also normalized to unit energy with energy levels defined by $p_d$ for $d = 1 \ldots T$. Each waveform is multiplied by a beamforming vector $u_d \in \mathbb{C}^{N_T \times 1}$.
to focus the transmit-waveforms into the radar scene. Thus, the MIMO radar transmitted signal represents a linear combination of the set of all beamformed transmit-waveforms and associated energy levels similar to [12]:

$$s(t) = \sum_{d=1}^{T} u_d s_d(t) \sqrt{p_d} \quad (2)$$

Free-space pathloss is accounted for with $\alpha_T^{(\ell)}$ and $\alpha_R^{(\ell)}$ for transmit and receive paths pertaining to each individual radar target $\ell$, respectively. Given that each target is at azimuth angle $\tau_L$ relative to the transmit array, the signal reflected from the $\ell^{th}$ target can be defined as:

$$y_\ell(t) = h_\ell(t) \star [a_T^H(\tau_\ell)s(t)\alpha_T^{(\ell)}], \quad \ell = 1, \ldots, L, \quad (3)$$

where $a_T(\tau_L) \in \mathbb{C}^{N_T \times 1}$ is the array manifold vector in the direction of the target, $\star$ is the convolution operator, and $(\cdot)^H$ represents the Hermitian or complex conjugate transpose operation. When the reflected signal given by (3) reaches the MIMO receiver, the received signal is expressed as:

$$z_\ell(t) = \alpha_R^{(\ell)} a_R(\rho_\ell) y_\ell(t) = [\alpha_R^{(\ell)} a_R(\rho_\ell)a_T^H(\tau_\ell)\alpha_T^{(\ell)}]\sum_{d=1}^{T} u_d [h_\ell(t) \star s_d(t)] \sqrt{p_d}. \quad (4)$$

where $a_R(\rho_\ell) \in \mathbb{C}^{N_R \times 1}$ is the receive antenna array manifold vector from azimuth direction $\rho_\ell$. Considering noise and reflected signals from $L$ targets, the combined signal is given as the following:

$$z(t) = \sum_{\ell=1}^{L} z_\ell(t) + w(t). \quad (5)$$

$w(t)$ is a vector noise process and in this dissertation, assumed white and Gaussian. The power spectral density of the noise process is $Q_\epsilon(f) = \sigma_n^2$ for all receive elements $\epsilon = 1, \ldots, N_R$ and frequencies. At the receive MIMO array, (5) is processed through $r = 1/dots R$ receive beams with associated beamforming vectors, $v_r \in \mathbb{C}^{N_R \times 1}$. This gives the following expression
for the received signal across all receive directions:

\[
z_r(t) = v_r^H z(t) = \sum_{\ell=1}^{L} \sum_{d=1}^{T} \lambda^{(\ell)}_{rd} [h_{\ell}(t) * s_d(t)] \sqrt{p_d} + v_r^H w(t) \tag{6}
\]

In (6), the scalar quantities \(\lambda^{(\ell)}_{rd} = a_R^{(\ell)} v_r^H a_R(\rho_\ell) a_T^H(\tau_\ell) u_d \alpha^{(\ell)}_T\) are hereafter referred to as reflection coefficients. Reflection coefficients include all array manifold vectors, beamforming vectors, and pathloss coefficients for both the transmit and receive signal. Reflection coefficients are complex and not deterministic due to the uncertainty in the position of the target (angles \(\rho_\ell\) and \(\tau_\ell\) of each corresponding target \(\ell\) are unknown). In this model, \(\lambda_{rd}\) represents the sparse, unknown signal to be reconstructed via compressed sensing. \(\lambda_{rd}\) will be large when the respective transmit and receive beamforming vectors align with the relative direction of a target. Motion can be readily accounted for in the radar scene by including a Doppler parameter in (6) but is beyond the scope of this study. Note that these reflection coefficients are uncorrelated with each other when the spatial diversity condition defined further in [56] is satisfied [37,47,48].

The system model is expanded to include clutter. Clutter sources are defined in a similar manner to extended radar targets and injected directly into the target scene and with a corresponding \(\beta_c\). Defining clutter in this manner gives flexibility in testing the resilience of the proposed algorithm to adequately detecting targets surrounded by clutter sources. Including clutter sources \(\zeta = 1, \ldots, Z_c\), the noisy and cluttered received signal becomes:

\[
z_r(t) = \sum_{\ell=1}^{L} \sum_{d=1}^{T} \lambda^{(\ell)}_{rd} [h_{\ell}(t) * s_d(t)] \sqrt{p_d} + \sum_{\zeta=1}^{Z_c} \sum_{d=1}^{T} \gamma^{(\zeta)}_{rd} [x_{\zeta}(t) * s_d(t)] \sqrt{p_d} + v_r^H w(t) \tag{7}
\]

where \(\gamma^{(\zeta)}_{rd}\) is the random and complex clutter reflection coefficients defined similarly to \(\lambda^{(\ell)}_{rd}\) for radar targets with \(\gamma^{(\zeta)}_{rd} = \tilde{\alpha}^{(\zeta)}_R v_r^H a_R(\tilde{\rho}_\zeta) a_T^H(\tilde{\tau}_\zeta) u_d \tilde{\alpha}^{(\zeta)}_T\). For \(\gamma^{(\zeta)}_{rd} \), \(\tilde{\alpha}^{(\zeta)}_T\) and \(\tilde{\alpha}^{(\zeta)}_R\) define the pathloss coefficients corresponding to each clutter source \(\zeta\) for the transmit and receive paths, respectively. As before, \(\tilde{\tau}_\zeta\) and \(\tilde{\rho}_\zeta\) denote the azimuths where each clutter source
occurs relative to the transmit and receive arrays, respectively. Thus, $\gamma_{rd}^{(c)}$ is a combination of transmit and receive beamforming vectors, antenna array manifold vectors, and pathloss coefficients for free-space propagation. In this dissertation, clutter frequency responses are modeled as a stationary random process consistent with [44], where the clutter return is defined as the output of a linear time invariant (LTI) filter with a randomly defined impulse response and the transmit signal as the input.

What differentiates clutter from extended radar targets in (7) is that both clutter source impulse responses and reflection coefficients are unknown to the bistatic radar receiver. Note that as with extended targets, Doppler spreading due to clutter motion is not considered and all clutter returns are assumed stationary and independent from sensor to sensor as in [44]. It is noted that this assumption merely simplifies the mathematical representation and that it may easily be extended to include Doppler shift parameters to account for various motion types for each target as noted in [9, 12].

2.2 FREQUENCY DOMAIN REPRESENTATION

Taking the Fourier transform, (7) becomes:

$$Z_r(f) = \sum_{\ell=1}^{L} \sum_{d=1}^{T} \lambda_{rd}^{(c)} H_{\ell}(f) S_d(f) \sqrt{p_d} + \sum_{\varsigma=1}^{Z_c} \sum_{d=1}^{T} \gamma_{rd}^{(c)} X_{\varsigma}(f) S_d(f) \sqrt{p_d} + v_r^H W(f),$$

In (8), $W(f)$ is the frequency domain vector of random functions with statistics determined by the power spectral density of the bandpass and finite energy noise process $w(t)$. These characteristics imply that the sample functions of $w(t)$ are able to be Fourier transformed [9].

A vector representation of (8) is obtained by discretizing over $K$ distinct frequency bands. The Fourier transform of transmitted pulses and target impulse responses are assumed constant within the bandwidth with center frequencies $B_k = [f_k; f_k + \Delta f]$ chosen to be sufficiently narrow, consistent with [9]. This further implies that target and clutter frequency responses, transmit signals, and noise signal likewise remain constant [37, 47, 48]. The transmit and
receive signals are now represented as the following vectors of dimension $K$:

$$s_d = \begin{bmatrix} S_d(f_1) \\ \vdots \\ S_d(f_K) \end{bmatrix} \quad \text{and} \quad z_r = \begin{bmatrix} Z_r(f_1) \\ \vdots \\ Z_r(f_K) \end{bmatrix}$$

(9)

The respective target and clutter frequency responses for each target and clutter source become:

$$H_\ell = \text{diag}\{H_\ell(f_1) \ldots H_\ell(f_K)\},$$

$$\Xi_\zeta = \text{diag}\{X_\zeta(f_1) \ldots X_\zeta(f_K)\}$$

(10)

and are of dimension $K \times K$. Considering $K$ frequencies and $N_R$ receive antennas, the noise matrix is represented as the following $K \times N_R$ matrix:

$$W = \begin{bmatrix} W_1(f_1) & \ldots & W_{N_R}(f_1) \\ \vdots & \ddots & \vdots \\ W_1(f_K) & \ldots & W_{N_R}(f_K) \end{bmatrix}.$$ 

(11)

Now, the received signal with both composite clutter and noise is given by the expression:

$$z_r = \sum_{\ell=1}^L \sum_{d=1}^T \lambda_{\ell d}^{(t)} H_\ell s_d \sqrt{p_d} + \sum_{\zeta=1}^{Z_c} \sum_{d=1}^T \gamma_{\zeta d}^{(t)} \Xi_\zeta s_d \sqrt{p_d} + n_r,$$

$$r = 1, \ldots, R,$$

(12)

Here, the noise vector $n_r = Wv_r^*$ where denotes the complex conjugate transpose operation. $n_r$ is an amalgamation of the noise matrix and beamforming vector in the receive direction $r$. Given that the beamforming vectors are normalized to unit norm, elements of $w(t)$ can be considered uncorrelated. By extension, elements of $W(f)$ are uncorrelated over distinct frequencies which implies: $R_{n_r} = E\{n_r n_r^H\} = \sigma_n^2 I_K, \ r = 1, \ldots, R$ [9].

A compact and discretized form is obtained by first concatenating matrices. Waveform
energies are given by the $T \times T$ diagonal matrix, defined formally as:

$$P = \text{diag}\{p_1 \ldots p_T\}. \quad (13)$$

Next, the $T \times R$ matrices of reflection coefficients for all $\ell = 1, \ldots, L$ targets at position $(r, d)$ are given as:

$$\Lambda_\ell = \begin{bmatrix}
\lambda^{(\ell)}_1 & \cdots & \lambda^{(\ell)}_R
\end{bmatrix}, \quad \text{with } \lambda^{(\ell)}_r = \begin{bmatrix}
\lambda^{(\ell)}_{r1} \\
\vdots \\
\lambda^{(\ell)}_{rT}
\end{bmatrix}.$$

The $T \times R$ matrix of clutter source reflection coefficients for all $\zeta = 1, \ldots, Z_c$ clutter sources at position $(d, r)$ are defined as:

$$\Gamma_\zeta = \begin{bmatrix}
\gamma^{(\zeta)}_1 & \cdots & \gamma^{(\zeta)}_R
\end{bmatrix}, \quad \text{with } \gamma^{(\zeta)}_r = \begin{bmatrix}
\gamma^{(\zeta)}_{r1} \\
\vdots \\
\gamma^{(\zeta)}_{rT}
\end{bmatrix}.$$

The $K \times T$ transmit-waveform matrix is given as:

$$S = \begin{bmatrix}
s_1 & \cdots & s_T
\end{bmatrix}, \quad (14)$$

and the $K \times R$ noise matrix as:

$$N = \begin{bmatrix}
n_1 & \cdots & n_R
\end{bmatrix}. \quad (15)$$

Finally, the $K \times R$ received signal matrix is defined as:

$$Z = \begin{bmatrix}
z_1 & \cdots & z_R
\end{bmatrix}. \quad (16)$$
This yields the following simplified expression for the received signal:

$$Z = \sum_{\ell=1}^{L} H_\ell S P^{1/2} \Lambda_\ell + \sum_{\zeta=1}^{Z_c} \Xi_\zeta S P^{1/2} \Gamma_\zeta + N. \tag{17}$$

Finally, the discretized model given by (17) is made compact by defining the $K \times KL$ block matrix of target frequency responses as:

$$H = [H_1 | \cdots | H_L], \tag{18}$$

the $K \times KZ_c$ block matrix of clutter source frequency responses as

$$\Xi = [\Xi_1 | \cdots | \Xi_{Z_c}], \tag{19}$$

the $TL \times R$ block matrix of target reflection coefficients as

$$\Lambda = \begin{bmatrix} \Lambda_1 \\ \vdots \\ \Lambda_L \end{bmatrix}, \tag{20}$$

and the $TZ_c \times R$ block matrix of clutter reflection coefficients as:

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_{Z_c} \end{bmatrix}. \tag{21}$$

The Kronecker operator [57] $\otimes$ is applied to define the $KL \times TL$ matrix as

$$\bar{S} = I_L \otimes S, \tag{22}$$

and similarly the $KZ_c \times TZ_c$ matrix as:

$$\bar{S}_c = I_{Z_c} \otimes S. \tag{23}$$
For simplicity and no loss in generality, we assume that all transmit-waveforms are of equivalent energy. That is, \( P = pI_T \), where \( p \) is the energy of the transmitted waveform. Now, (17) reaches its most compact and simplified form, given by:

\[
Z = \sqrt{p}H\bar{S}A + \sqrt{p}\Xi\bar{S}_c\Gamma + N,
\]

(24)

The presence of a target, \( \alpha \), is indicated by its correlating reflection coefficient vector, \( \lambda_{rd}^{(\alpha)} \in \Lambda \). In a benign environment, \( \lambda_{rd}^{(\alpha)} = 0 \in \Lambda \) in the absence of each target \( \alpha \) and \( \lambda_{rd}^{(\alpha)} > 0 \) if a target is present. Absent clutter, the radar scene is sparse when \( L \ll TR \). Heuristically, sparsity in the reflection coefficient matrix is achieved when the number of targets and clutter sources in the scene is small as compared to the quantity of transmit-receive array beamforming pairs.

### 2.3 CLUTTER

Terrestrial radars, such as those considered in this paper, succumb to unwanted echo signals from terrain features, such as buildings, mountains or hills or from significant weather formations and chaff. Clutter sources are modeled similarly to extended targets with clutter frequency response and corresponding clutter reflection coefficient in keeping with the notion that clutter returns are akin to those from unwanted targets, as in [58]. This choice was made to provide a more realistic clutter representation in the MIMO bistatic air-search radar scene versus a generalized statistical term. In this way, clutter sources are injected around targets to stress the model and further challenge \( P_D, P_{FA} \), and ROC results.

This dissertation assumes that clutter impulse responses, \( x_{\zeta}(t) \), \( \zeta = 1 \ldots Z_c \), are WSS Gaussian random processes with zero mean \( (E\{x_{\zeta}(t)\} = \mu = 0 \ \forall \ t) \) and associated power spectral density, \( Q_{x_{\zeta}}(f) \), consistent with the model considered in [44]. As mentioned, clutter returns at distinct frequencies and different receive antennas are assumed independent from sensor to sensor, implying a zero-valued cross-correlation for clutter frequency responses. That is,

\[
E\{X_m(f_i)X_n(f_j)\} = 0 \ \forall \ m \neq n \text{ and } f_i \neq f_j.
\]

(25)
The cross-correlation and autocorrelation functions of the clutter frequency response are derived in Appendix B. It is noted that clutter responses based on other commonly used distributions such as the Compound Gaussian, Weibull, or K-distributions may be adapted to this model but is left for future work.

The WSS Gaussian clutter sources considered in this paper can be considered as composite vice isolated clutter sources. Composite sources are those that consist of multiple reflectors such as chaff, ground or sea clutter, and meteorological echoes. An isolated clutter source is merely a point target in space [59]. This paper’s consideration of composite clutter complicates the system model further and allows results to attained for non-ideal environments. It is noted that that while other clutter spectral densities can easily be implemented in to this system model, [60] demonstrates that spectral densities of weather echo autocorrelations have a Gaussian shape using the central limit theorem [61].

In addition, a $\beta_c$ is formally defined (Refer to Chapter V). It is noted that in practice, the return from an individual clutter source is small compared to that of an actual airborne target, but the total energy from the clutter sources collectively may be much larger than that of the target [58]. Thus, this dissertation explores whether relatively weak targets can be distinguished from strong clutter returns in the CS radar system context.

Further, this clutter definition fits the problem definition and system model of a bistatic air search radar where composite clutter sources are spread throughout the scene (wildlife, significant weather formations, etc.) as depicted in Fig. 2. Defining composite clutter as in (24) gives the flexibility in the implementation and positioning of clutter sources or defining radar scenes.

It is noted that previous work [54, 55] considered clutter returns based on the constant-$\gamma$ clutter model. While constant-$\gamma$ clutter is based on measurements from actual clutter environments and allowed consideration of actual radar parameters (such as operating frequency, pulse width, etc.), its implementation did not consider placement around targets or provide a strict definition for $\beta_c$. A review of constant-$\gamma$ clutter is included in Appendix C for completeness.
Chapter 3

CS MIMO RADAR AND JOINT WAVEFORM-RECEIVER FILTER DESIGN

3.1 CS MIMO RADAR

Due to the inherent sparsity common to many radar applications, detection can often be posed as a sparse reconstruction problem where some targets are present in a vast radar scene. As it relates to this system model, the radar scene is spatially partitioned via MIMO beamforming at the transmitter and receiver arrays as in Fig. 2, where the sparsity of the scene allows for the application of CS to perform target detections.

As described in Chapter II, the presence of a target in the MIMO radar scene is dictated by target’s corresponding, unknown and sparse $\lambda_{rd}$ matrix. With conventional radar, detection via matched filtering is challenged due to the stochastic nature of terms that comprise the reflection coefficients in the received signal. Rather than centering the detection problem on specific values of a target’s reflection coefficients, the posed CS radar system instead capitalizes on the support of the reflection coefficients to determine a target’s presence in a specific beam pair cell, which is by definition sparse.

The sparsity of $\lambda_{rd}$ allows the detection problem to be posed as sparse reconstruction problem. That is, for every target with a known frequency response, to reconstruct $\lambda_{rd}$ via sparse reconstruction and then use thresholding against the support of $\lambda_{rd}$ to determine a detection. Thus, the objective in optimizing detections in this CS context is to design the sensing matrix, $\Phi$, to best perform sparse reconstruction given a deterministic $H$. This implies that waveforms and linear receiver filters can be designed to optimize target detections of specific targets within this model, similar to Bell’s [9] pioneering work for radar waveform optimization but in the relation to CS.
3.2 JOINT WAVEFORM-RECEIVER FILTER DESIGN

The design goal of the joint waveform-receiver filter design algorithm is to optimize \( S \) and associated receiver filter, \( C \), for sparse reconstruction of \( \lambda_{rd} \) given a deterministic \( H_{\ell} \) such that detections are possible amidst both noise and clutter, \( \Gamma \). For the compressed sensing based radar, this means choosing transmit-waveforms and receiver filters that best satisfy the RIP for optimal sparse reconstruction. However, designing large sensing matrices to specifically satisfy this constraint represents an NP-complete design problem [62]. Given, for example, that the sensing matrix \( \Phi \in \mathbb{R}^{M \times N} \), confirming that \( \Phi \) satisfies RIP requires a search over \( \binom{N}{k} \) subspaces, where \( k \) represents number of nonzero entries of the sparse vector \( x \) in \( y = \Phi x \) (Refer to Appendix A). For \( N \geq 15 \), verifying this property is computationally exhaustive [53].

Rather than explicitly designing matrices to satisfy this constraint, an alternative design criterion is considered, namely minimizing the mutual coherence of the sensing matrix as in [63, 64]. In this context, compressed sensing based target detection can be performed nearly optimally so long as there is minimal coherence between the linear receiver filter, \( C \) (sampling matrix) and the combined transmit-waveform and target frequency response term, \( H\bar{S} \), provided that \( \Lambda \) is sufficiently sparse [47, 48, 54, 55]. It was further demonstrated in [49, 51–53] that sensing matrices with minimal mutual coherences display favorable sparse recovery results.

3.2.1 NOISELESS, CLUTTERLESS SCENE RECONSTRUCTION

Consider a noise and clutter free environment such that (24) simplifies to:

\[
Z = \sqrt{p}H\bar{S}\Lambda.
\]  

The first step in the joint waveform-receiver filter design algorithm is to design the transmit-waveform matrix, \( S \). Using \( H\bar{S} \) as an overcomplete dictionary assures that \( \Lambda \) contains at most \( L \) nonzero values. For a small \( H \) (very few targets of interest to detect), full row rank can be achieved by augmenting \( H \) with additional randomly selected rows. This gives the
noiseless, clutterless receive signal matrix as:

$$\tilde{Z} = \sqrt{p} \tilde{H} \tilde{S} \tilde{\Lambda}.$$  

(27)

where, $$\tilde{H} = [H_1|\cdots|H_L|H_{\text{aug}}]$$ and $$\tilde{\Lambda} = \begin{bmatrix} \Lambda \\ \Lambda_{\text{aug}} \end{bmatrix}$$. In (27), $$H_{\text{aug}}$$ is of dimension $$K \times KN_{\text{aug}}$$, $$\tilde{S} = I_{L+N_{\text{aug}}} \otimes S$$, and $$\tilde{\Lambda}_{\text{aug}}$$ is $$TN_{\text{aug}} \times R$$. $$N_{\text{aug}}$$ represents the number of additive, random block frequency response matrices that augment $$H$$ to ensure full row rank of the sensing matrix [37,47,48].

The radar scene (represented by (27) absent noise or clutter) is then simultaneously sampled and compressed after applying the linear $$\Upsilon \times K$$ receiver filter, $$C$$, where $$\Upsilon$$ specifies the number of CS measurements. The dimension $$\Upsilon$$ directly correlates to the an undersampling ratio, $$\delta_u = \frac{\Upsilon}{N_{\lambda}}$$, and relative sparsity, $$\rho = \frac{\Upsilon}{L}$$ (assuming only the non-zero entries in $$\tilde{\Lambda}$$ correspond to the $$L$$ targets present in the scene), where $$N_{\lambda}$$ is defined as the length of $$\tilde{\Lambda}$$ [23]. A requirement for CS is that $$\Upsilon$$ be at least greater than or equal to the intrinsic information of the unknown signal, or $$\Upsilon \geq L$$. Thus $$\delta_u$$ and $$\rho$$ values range from $$0 < \delta_u, \rho \leq 1$$. Note that a $$\delta_u = 1$$ implies that the length of $$y$$ and $$x$$ are equal in (51) and there is no reduction in dimensionality for the CS algorithm where it is noted that sensing by dimensionality reduction is one of the key benefits of CS approaches, as accurate reconstruction of lengthy signals is possible from very few measurements of a sparse signal.

What is considered the sensing matrix in CS theory, includes, in this instance: the transmit-waveforms, deterministic target frequency responses, and linear receiver filters. Thus, the objective of the joint waveform-receiver filter design algorithm is to design $$C$$ and $$\tilde{S}$$ such that the coherence between $$C$$ and $$\tilde{H}\tilde{S}$$ is minimized to improve the sparse reconstruction of $$\tilde{\Lambda}$$ and by consequence, target detection. This is equivalent to minimizing the mutual coherence of the sensing matrix, $$\Phi = C\tilde{H}\tilde{S}$$. Note that in the bistatic radar application, the transmitter produces $$\tilde{S}$$ and receiver linearly samples the scene according to $$C$$ that is comprised of targets with frequency responses $$\tilde{H}$$. The novelty of this approach also resides in the fact that no single component in the CS radar produces the entirety of the sensing matrix. Rather, the sensing matrix is formed with contributions spanning the entire CS radar system (absent clutter responses), including targets.
Applying the linear receiver filter, $C$, to the received signal yields the following matrix of CS measurements:

$$\tilde{D} = C\tilde{Z} = C(\sqrt{p}\tilde{H}\tilde{S}\tilde{A}).$$

(28)

The design goal of the joint waveform-receiver filter design algorithm is apparent. That is, transmit-waveform, $\tilde{S}$ and receiver filters, $C$, must be chosen to optimize reconstruction of $\Lambda$ given the $\Upsilon \times R$ measurements contained in $\tilde{D}$. Framing this problem as an underdetermined linear reconstruction problem, $\Lambda$ can be recovered via one of many proven methods given certain constraints on the sensing matrix $\Phi$ are met. For example, in the case of $L_1$ minimization, the reconstruction problem is posed as:

$$\min \|\tilde{\lambda}_r\|_1 \text{ subject to } \|\tilde{d}_r - \Phi\tilde{\lambda}_r\|_2^2 \leq \epsilon, \ r = 1, \ldots, R,$$

(29)

where $\tilde{\lambda}_r$ and $\tilde{d}_r$ correspond to the $r$-th column of the reflection coefficient and measurement matrices, $\tilde{D}$, respectively. This dissertation considers ROMP, CoSaMP, and CAMP for reconstruction of $\tilde{\Lambda}$.

For CS reconstruction, this implies that $\tilde{S}$ and $C$ should be designed for minimal coherence between $C$ and the overcomplete dictionary, $\tilde{H}\tilde{S}$ (in lieu of designing explicitly to satisfy RIP).

The mutual coherence of $\Phi$ is given by:

$$\mu(\Phi) = \max_{i \neq j} \left| \frac{g_{i,j}}{\sqrt{g_{i,i}g_{j,j}}} \right|$$

(30)

where $g_{i,j} = \phi_i^H\phi_j$ corresponds to an element in the Gram matrix defined by:

$$G = \Phi^H\Phi = (C\tilde{H}\tilde{S})^H(C\tilde{H}\tilde{S})$$

(31)

and $\phi_{i,j}$ correspond to the $i$-th or $j$-th column of $\Phi$, respectively [49].

Thus, minimizing the off-diagonal elements of the Gram matrix, e.g. $i \neq j$ will minimize coherence [49]. In the design algorithm, this means that $C$ and $\tilde{S}$ should be designed to this
end. Stated explicitly,

\[
\arg\min_{C,S} \left\| \mathbf{G} - \tilde{\mathbf{G}} \right\|_F^2 = \arg\min_{C,S} \left\| \tilde{\mathbf{S}}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{C}}^H \tilde{\mathbf{H}} \tilde{\mathbf{S}} - \tilde{\mathbf{G}} \right\|_F^2 \tag{32}
\]

with \( \tilde{\mathbf{G}} \) defined as \( \text{diag}\{g_{0,0}, \ldots, g_{i,i}, \ldots, g_{(L+N_{aug})T,(L+N_{aug})T}\} \). In (32) \( \| \cdot \|_F \) is the Frobenius norm.

This joint waveform-receiver filter design procedure, defined formally in [37, 47, 48] is based upon the design methodology given in [49], where (32) is alternatively framed in terms of a simpler minimization problem using \( \mathbf{U} \), a semiunitary matrix to be designed. That is:

\[
\arg\min_{C,S} \left\| \mathbf{G} - \tilde{\mathbf{G}} \right\|_F^2 = \arg\min_{C,S} \left\| \mathbf{C} \tilde{\mathbf{H}} \tilde{\mathbf{S}} - \mathbf{U} \tilde{\mathbf{G}}^{1/2} \right\|_F^2
\]

\[
= \arg\min_{C,S,U} \left\| \mathbf{C} \tilde{\mathbf{H}} (\mathbf{I}_{(L+N_{aug}) \otimes \mathbf{S}}) \tilde{\mathbf{G}}^{-1/2} - \mathbf{U} \right\|_F^2, \text{ such that } \mathbf{U}^H \mathbf{U} = \mathbf{I} \tag{33}
\]

It is noted that (32) and (33) are similar but not necessarily equivalent minimizations. These criterion are similar in that if (33) is small then so too is (32) [65]. This method is consistent with [65, 66] which use a similar minimization technique.

The waveform and receiver filter design algorithms begin by initializing either \( \mathbf{C} \) or \( \mathbf{S} \) to predefined values. Next, the algorithm determines \( \mathbf{U} \) based on the fixed values and then iteratively solves for \( \mathbf{S} \) or \( \mathbf{C} \) while the current and previous estimate of \( \mathbf{S} \) or \( \mathbf{C} \) are within a set tolerance value. Properties of this design algorithm such as its computational complexity are left for future work to formally derive. However, the transmit-waveform and receiver filter design methodology has been demonstrated to converge in [66] and again noted in [37, 49].

**Transmit-Waveform Design**

The joint waveform-receiver filter design algorithm first optimizes \( \mathbf{S} \), with a \( \mathbf{C} \) and \( \mathbf{U} \) is initialized to Gaussian random matrices because they are known to have favorable coherence properties. (33) can be restated in terms of \( \Psi_s \) and where \( (\cdot)^\dagger \) indicates the Moore-Penrose
pseudoinverse operation:

$$
\| (I_{L+N_{aug}} \otimes S) - (\text{CH})^\dagger U \tilde{G}^{1/2} \|_F^2,
$$

(34)

After further simplification, \( S \) is computed as:

$$
S = \frac{2}{\sum_{\ell=1}^{L+N_{aug}} \kappa_{\ell}} \sum_{\ell=1}^{L+N_{aug}} \kappa_{\ell} \Psi_s^{(\ell)},
$$

(35)

where \( \kappa_{\ell} \) corresponds to weight values assigned to each target to establish a target priority hierarchy and \( \ell = 1, \ldots, \ell, \ldots, (L + N_{aug}) \) denotes individual targets.

The Waveform Design Procedure is given by **Algorithm 1** and derived formally in [37].

---

**Algorithm 1** – Waveform Design

1. **Input:** Initialized \( C \), Initialized \( S \), \( \epsilon_1 \);
2. **while** \( \Delta_1 = \max_d |s_{d}^{(m)} - s_{d}^{(m-1)}| < \epsilon_1 \) **do**
3. **Compute** \( U = U_1 V^H \), where \( C \tilde{H}(I_{(L+N_{aug})} \otimes S)\tilde{G}^{-1/2} = U_1 \Sigma V^H \)
4. **Compute** \( S \) using

$$
S = \frac{2}{\sum_{\ell=1}^{L+N_{aug}} \kappa_{\ell}} \sum_{\ell=1}^{L+N_{aug}} \kappa_{\ell} \Psi_s^{(\ell)},
$$

(35)

5. **Normalize columns:** \( s_d = s_d / \|s_d\|, d = 1, \ldots, T \)
6. **end while**
7. **Output:** Optimized \( S \)

---

**Receiver Filter Design**

The receiver filter is designed by initializing \( S \) and \( U \) in (33). In the joint waveform-receiver filter design, \( S \) is initialized as the optimized \( S \) determined in **Algorithm 1**. Thus, the to be designed linear receiver filter, \( C \), must follow:

$$
\left( \frac{C \tilde{H}(I_{(L+N_{aug})} \otimes S)\tilde{G}^{-1/2}}{\Psi_c} \right) = U,
$$

(36)
where $C$ is calculated as:

$$C = \left(\left(\Psi_c^H U^H\right)^H\right)^H = U\Psi_c^H (\Psi_c \Psi_c^H)^{-1}. \tag{37}$$

The design process for the receiver filter as derived in [37] is presented in Algorithm 2.

**Algorithm 2 – Receiver Filter Design**

1: **Input:** Initialized $C$, Initialized $S$, $\epsilon_2$
2: **while** $\Delta_2 = \max_d |c^{(m)}_k - c^{(m-1)}_k| < \epsilon_2$ **do**
3: Compute $U = U_1 V^H$, where $C\check{H}(I_{(L+N_{aug})} \otimes S)\check{G}^{-1/2} = U_1 \Sigma V^H$
4: Compute $C$ using

$$C = \left(\left(\Psi_c^H U^H\right)^H\right)^H = U\Psi_c^H (\Psi_c \Psi_c^H)^{-1}. \tag{38}$$

5: Normalize columns: $c_k = c_k/\|c_k\|, k = 1, \ldots, K$
6: **end while**
7: **Output:** Optimized $C$

**Joint Waveform-Receiver Filter Design**

Algorithm 3 combines Algorithm 1 and Algorithm 2 for form the joint waveform-receiver filter design process. This approach is shown to have favorable convergence properties [67]. Iterations of Algorithm 3 are guaranteed to reach a fixed point and the algorithm halts when $\|G - \tilde{G}\|_F \leq \varepsilon$ where $\varepsilon$ is a predefined halting criteria [37].

### 3.2.2 NOISY SCENE RECONSTRUCTION

When noise is considered in the system model, the compressed received signal is expressed as:

$$\tilde{D} = C(\tilde{Z} + N) = C(\sqrt{p}\tilde{H}\tilde{S}\tilde{\Lambda}) + C(N) = \Phi_1\tilde{\Lambda} + \Phi_2N. \tag{40}$$
Algorithm 3 – Joint Waveform and Receiver Filter Design

1: **Input:** Initialized $C$, Initialized $S$, $T$, $R$, $L$, $K$, $\hat{H}_\ell$ $\ell = 1, \ldots, L$ (normalized to unit energy), $\kappa_\ell$, $\epsilon_1$, $\epsilon_2$, $\epsilon_3$

2: **while** $\Delta_3 = \|G - \tilde{G}\|_F^2 > \epsilon_3$ **do**

3: Optimize $S$ (Steps 4-8)

4: **while** $\Delta_1 = \max_d |s_d^{(m)} - s_d^{(m-1)}| < \epsilon_1$ **do**

5: Compute $U = U_1 V^H$, where $C \hat{H} (I_{(L+N_{aug})} \otimes S) \tilde{G}^{-1/2} = U_1 \Sigma V^H$

6: Compute $S$ using

\[ S = \frac{2}{\sum_{\ell=1}^{L+N_{aug}} \kappa_\ell} \sum_{\ell=1}^{(L+N_{aug})} \kappa_\ell \Psi_s^{(\ell)} \]

7: Normalize columns: $s_d = s_d/\|s_d\|$, $d = 1, \ldots, T$

8: **end while**

9: Optimize $C$ (Steps 10-14)

10: **while** $\Delta_2 = \max_d |c_k^{(m)} - c_k^{(m-1)}| < \epsilon_2$ **do**

11: Compute $U = U_1 V^H$, where $C \hat{H} (I_{(L+N_{aug})} \otimes S) \tilde{G}^{-1/2} = U_1 \Sigma V^H$

12: Compute $C$ using

\[ C = \left( [\Psi_c^H \Psi_c^H] \right)^{-1} = U \Psi_c^H \left( \Psi_c \Psi_c^H \right)^{-1}. \] (39)

13: Normalize columns: $c_k = c_k/\|c_k\|$, $k = 1, \ldots, K$

14: **end while**

15: **end while**

16: **Output:** Optimized $S$, $C$
with the signal to noise ratio defined as

$$\beta_n = \frac{p \text{Trace} \left[ \tilde{H} \tilde{S} \tilde{S}^H \tilde{H}^H \right]}{\text{Trace} \left[ E[NN^H] \right]} = \frac{p \text{Trace} \left[ \tilde{H} \tilde{S} \tilde{S}^H \tilde{H}^H \right]}{\sigma_n^2 K}. \quad (41)$$

(41) is the signal to noise ratio definition used in the following simulations of the radar system.

It is noted that $\Phi_2 N$ is both white and Gaussian if if $C$ is designed with orthogonal rows [37] under the stated assumptions given for the noise model.

In this instance, the parallel reconstruction problem is expressed as:

$$\min \| \tilde{\lambda}_r \|_1 \text{ subject to } \| \tilde{d}_r - \Phi_1 \tilde{\lambda}_r \|_2^2 \leq \epsilon, \ r = 1, \ldots, R, \quad (42)$$

where $\tilde{d}_r$ also includes the noise term. In order to meet the design criterion of the waveform and receiver filter design algorithm in minimizing the mutual coherence to approximate satisfying RIP, both $\Phi_1$ and $\Phi_2$ should both be designed to minimize mutual coherence. In the noiseless case, mutual coherence in $\Phi_1$ was minimized by minimizing the off-diagonal entries of $G$. Where noise is present, $\Phi_2$ is considered explicitly in the design algorithm. To minimize the mutual coherence of the compressed received signal, the linear receiver filter (sampling matrix) is therefore redesigned.

**Joint Waveform-Receiver Filter Design in Noise**

Coherence is minimized in $\Phi_2$ with modification to $C$. Starting with the singular value decomposition (SVD) $C = U_C \Sigma_C V_C^H$. $\hat{C} = U_C V_C^H$ is selected to minimize $\| C - \hat{C} \|_F^2$ such that $\hat{C} \hat{C}^H = I$. Since information contained in both eigenvectors is preserved in $\hat{C}$, $\Phi_1$ retains favorable coherence properties for sparse reconstruction [37].

The SVD step is inserted after $C$ is determined resulting in **Algorithm 4**.
Algorithm 4 – Joint Waveform and Receiver Filter Design

1: **Input:** Initialized $C$, Initialized $S$, $T$, $R$, $L$, $K$, $\hat{H}_\ell$ \( \ell = 1, \ldots, L \) (normalized to unit energy), $\kappa_\ell$, $\epsilon_1, \epsilon_2, \epsilon_3$

2: **while** $\Delta_3 = \|G - \hat{G}\|^2_F > \epsilon_3$ **do**

3: Optimize $S$ (Steps 4-8)

4: **while** $\Delta_1 = \max_d |s^{(m)}_d - s^{(m-1)}_d| < \epsilon_1$ **do**

5: Compute $U = U_1 V^H$, where $C \hat{H}(I_{(L+N_{aug})} \otimes S)\hat{G}^{-1/2} = U_1 \Sigma V^H$

6: Compute $S$ using

$$S = \frac{2}{\sum_{\ell=1}^{(L+N_{aug})} \kappa_\ell} \sum_{\ell=1}^{(L+N_{aug})} \kappa_\ell \Phi_\ell$$

7: Normalize columns: $s_d = s_d/||s_d||$, $d = 1, \ldots, T$

8: **end while**

9: Optimize $C$ (Steps 10-14)

10: **while** $\Delta_2 = \max_d |c^{(m)}_k - c^{(m-1)}_k| < \epsilon_2$ **do**

11: Compute $U = U_1 V^H$, where $C \hat{H}(I_{(L+N_{aug})} \otimes S)\hat{G}^{-1/2} = U_1 \Sigma V^H$

12: Compute $C$ using

$$C = \left( \left[ \Psi_c^H \right] \left[ U^H \right] \right)^H = U \Psi_c^H \left( \Psi_c \Psi_c^H \right)^{-1}.$$ (43)

13: Normalize columns: $c_k = c_k/||c_k||$, $k = 1, \ldots, K$

14: **end while**

15: **end while**

16: Compute $\hat{C} = U_C V_C$, where $C = U_C \Sigma_C V_C^H$

17: Normalize columns: $\hat{c}_k = \hat{c}_k/||\hat{c}_k||$, $k = 1, \ldots, K$

18: **Output:** Optimized $S$, $C$
Transmit signal dependent clutter is now considered. Implementing the novel clutter definition presented in Chapter 2 of this dissertation, the matrix of CS measurements is restated as:

\[
\tilde{D} = \tilde{C} (\sqrt{p} \tilde{H} \tilde{S} \tilde{A}) + \tilde{C} (\sqrt{p} \Xi \tilde{S}_c \Gamma + N) = \Phi_1 \tilde{\Lambda} + \Phi_2 (\Xi \tilde{S}_c \Gamma + N),
\]  

(44)

where \( \Phi_1 = \sqrt{p} \tilde{C} \tilde{H} \tilde{S} \) and \( \Phi_2 = \tilde{C} \) are the sensing matrices applied to the matrix of reflection coefficients and combined clutter and noise matrix, respectively. It is noted that under the stated assumptions for clutter and noise in this dissertation, \( \Phi_2 (\Xi \tilde{S}_c \Gamma + N) \) is also distributed according to the Gaussian distribution and is white when \( \Phi_2 = \tilde{C} \) designed with orthogonal rows. The signal to clutter ratio \( (\beta_c) \) is hereby defined as follows:

\[
\beta_c = \frac{\text{Trace} \left[ \tilde{H} \tilde{S} \tilde{S}^H \tilde{H}^H \right]}{\text{Trace} \left[ E \{ \Xi \tilde{S}_c \tilde{S}_c^H \Xi^H \} \right]}.
\]  

(45)

Here, the \( R \) parallel reconstruction problems become:

\[
\min ||\tilde{\lambda}_r||_1 \text{ subject to } ||\tilde{d}_r - \Phi_1 \tilde{\lambda}_r||_2^2 \leq \epsilon, \ r = 1, \ldots, R,
\]  

(46)

but where \( \tilde{d}_r \) now incorporates both noise and clutter terms. Similar to the noise only case, sparse reconstruction requires low coherence of both the \( \Phi_1 \) and \( \Phi_2 (\Xi \tilde{S}_c \Gamma + N) \) terms and \( \Phi_2 \) is designed such that \( \Phi_2 = \tilde{C} \) has orthogonal rows as in the noise only case. In this model, clutter is treated as a real-world consequence to the performance of the CS radar affecting the sparsity of the radar scene. Thus, there is no requirement to explicitly know a priori the clutter frequency response. It is noted that sparse reconstruction of the cluttered radar scene remains tenable considering different clutter models such that based on the constant-\( \gamma \) model [54,55] where no modification or provision is made within the joint waveform-receiver
filter design algorithm to specifically accommodate the chosen clutter model. If, however, the clutter frequency response is known, \( \Xi \) may be incorporated into \( \tilde{H} \) as \( H_{\text{aug}} \) and then detected or estimated as an extended target in this model. This is left for further research.

Following design of the transmit-waveforms and receiver filter, the radar scene is reconstructed with proven sparse reconstruction techniques. More specifically, the target reflection coefficient matrix is reconstituted with knowledge of \( \hat{C}, \tilde{H}, \tilde{S}, \) and \( \sqrt{p} \) and with measurements \( \tilde{D} \) in (44) or (28) for cluttered and non-cluttered scenes, respectively.

Previous work [37] used Regularized Orthogonal Matching Pursuit (ROMP) as the default reconstruction technique in this step, as the joint waveform-receiver filter design algorithm was not rigorously evaluated in the system context. It was previously suggested that there was a nominal or negligible performance difference between ROMP and CoSaMP reconstruction methods for the CS radar. This work formally characterizes the performance of the proposed system algorithm with ROMP, CoSaMP, and CAMP sparse reconstruction techniques with numerical results and simulations.

4.1 TARGET DETECTION

Target detection as it relates to the sparse estimate of the reflection coefficient matrix can be restated as the following set of \( L \times T \times R \) element-wise binary hypothesis tests corresponding to every cell in \( \Lambda \):  

\[
\mathcal{H}_0^{rd} : \hat{\lambda}_{rd}^\ell = \delta_{rd}^\ell < \tau_{th} \\
\mathcal{H}_1^{rd} : \hat{\lambda}_{rd}^\ell = \lambda_{rd}^\ell + \delta_{rd}^\ell \geq \tau_{th}
\]

(47)

\( r = 1, ..., R; d = 1, ..., T; \ell = 1, ..., L. \)

In (47) \( \hat{\lambda}_{rd}^\ell \) represents the estimate of the reflection coefficient corresponding target \( \ell \) in position \((r,d)\). \( \delta_{rd}^\ell \) is the undesired, impulsive perturbation in the reconstructed reflected coefficient due to noise and/or clutter present in the scene at the same location \((r,d)\).

If a target is present at a given \((r,d)\), \( \hat{\lambda}_{rd}^\ell \) exceeds the fixed threshold, \( \tau_{th} \), and the \( \mathcal{H}_1^{rd} \) hypothesis is true. Conversely, the null hypothesis, \( \mathcal{H}_0^{rd} \), is true when \( \tau_{th} \) is not exceeded.
at location \((r,d)\). This test is performed for all \(L \times T \times R\) hypothesis testing problems. It is noted that this detection statistic demonstrated the most optimal results compared to other commonly used detection methods. While the consideration of multiple detection methods is beyond the scope of this work, the joint waveform-receiver filter design algorithm is agnostic to the specific detection statistic chosen in the CS radar system.

Compressive sensing is, by its nature, a detection algorithm. Consider the sparse reconstruction of a radar scene absent noise and clutter. For each non-trivial value in the reconstructed \(\Lambda\), a target is determined to be present at the corresponding physical location as a \(\delta_{rd} = 0\) leaves only \(\hat{\lambda}_{rd}\) to be compared to \(\tau_{th}\) in (47). In this example, setting \(\tau_{th} = 0\) provides the best detection rate performance providing the ability to observe the most minute reflection coefficients in \(\Lambda\).

However, in this CS radar system and as illustrated by Fig. 3, target detection depends on \(\tau_{th}\), the sparsity of \(\hat{\lambda}_{rd}\), and the ability of the sparse reconstruction algorithm to accurately reconstruct \(\Lambda\) in the presence of clutter and additive noise. Here, \(\delta_{rd} \geq 0\). In contrast, conventional approaches to target detection require that only \(\tau_{th}\) be adjusted to provide continuous changes in \(P_{FA}\) or \(P_D\).

Fig. 3 is an illustration of the reflection coefficient matrix for 1 extended target as received on a single element in the MIMO receive array. The dashed horizontal lines correspond to the decision thresholds, \(\tau_{th} = [0.1, 0.6, 0.8]\). As observed, when \(\tau_{th} \approx 0\), noise and clutter contribute to the \(P_{FA}\), however small in magnitude. Increasing \(\tau_{th}\), reduces the effect of noise on the \(P_{FA}\). Increased too high, and the detection rate of the radar system may be hampered.

This issue is exacerbated when clutter or fluctuating target impulse responses are considered. Not only does including clutter inherently increase the \(P_{FA}\), but it also challenges the sparsity of the radar scene. Fluctuating or unknown target impulse responses challenge sparse reconstruction as the receiver has an imprecise sensing matrix definition from what was actually received due to a non-deterministic \(\tilde{H}\). This too causes errors in sparse reconstruction due to mismatch in basis. However, if mismatch between two different bases is small, the unknown sparse signal may still be approximated by performing sparse reconstruction in the nearby basis [45, 46]. Further, the error in sparse reconstruction increases
relative to the degree of perturbation in the sensing matrix [45]. This is due to the fact that perturbations in the sensing matrix result in multiplicative noise that is correlated with the unknown sparse vector to be recovered. In the case of known targets, $\Phi = \hat{C}\tilde{H}\tilde{S}$. When fluctuating targets are considered, $\tilde{H} = \hat{H} + E$, where $E$ represents an unknown perturbation in the target frequency response. In this instance, the perturbed sensing matrix used for sparse recovery becomes $\hat{\Phi} = \hat{C}(\hat{H} + E)\tilde{S}$, yet $\mathbf{A}$ was both sparse and sampled in the $\Phi$ basis. For these reasons, a $\tau_{th} = 0$ is inadequate and a variable threshold is proposed to improve upon previous work.

Previous work [37, 47, 48] used the same methodology as Anitori [23] and fixed $\tau_{th} = 0$ and a constant false alarm rate was observed for both the noiseless and noisy scene cases. This result stems from the fact that even the smallest perturbation in the received signal rendered a false positive by observing $H_1^{\ell_{rd}}$ in (47) absent $\hat{\lambda}_{\ell_{rd}}$ and instead with $\delta_{\ell_{rd}} > \tau_{th} = 0$. Furthermore, previous work implemented ROMP and specified a $k = 2$ sparsity for the sparse estimate of the unknown signal. This informed the algorithm to provide 2-sparse estimates of $\hat{\lambda}_{\ell_{rd}}\forall R$ receive elements, producing a relatively flat false alarm rate across all SNRs. In CAMP, sparsity is not an input to the reconstruction algorithm, rather, the sparse estimate is obtained after the complex soft thresholding function is applied to the noisy estimate of the unknown signal [23]. This dissertation considers the same $k = 2$ sparse ROMP (hereafter, referred to as ROMP) in addition to $k = 1$ sparse CoSaMP (hereafter, referred to as CoSaMP) and CAMP for target detection to characterize CS radar system performance and afford a direct comparison to previous work. CS radar performance using a similarly specified CoSaMP but with alternate target and clutter definitions is given in [54,55].

Consideration of additional sparsity levels is beyond the scope of this work, as one of the goals of this work is to highlight radar performance differences in the sparse reconstruction algorithm selected as part of the larger CS radar system and demonstrate that the joint waveform-receiver filter design algorithm improves the ability of the radar to make detections regardless of the spare reconstruction algorithm it is paired with. Specifically, CoSaMP was specified to reduce the maximum allowable $P_{FA}$, while CAMP was implemented to improve $P_{D}$ as compared to previous results.
Figure 3. $k = 25$ sparse sample reconstructed reflection coefficients as reconstructed from the measurement vector corresponding to a single receive element in the MIMO array.

4.1.1 DETECTION RATE AND FALSE ALARM RATE, DEFINED

The detection rate, $P_D$, is defined as the ratio of the quantity of true $\mathcal{H}_1^r$ decisions per beam pair cell $(r,d)$ to the number of targets present in the radar scene. The false alarm rate, $P_{FA}$, is the ratio of the quantity of false $\mathcal{H}_1^r$ decisions per beam pair cell $(r,d)$ to the total number of cells in the actual reconstructed reflection coefficient matrix in which the target is not present. The probability of miss, $P_M$, is defined as $1 - P_D$. $P_M$ curves, while a trivial extension to detection rate curves, are beyond the scope of this work. It is noted that these definitions of $P_D$ and $P_{FA}$ is the same used in previously published work [37, 47, 48, 54, 55] and consistent with definitions for $P_D$ and $P_{FA}$ used in by Anitori [23].

4.2 MEASURES OF PERFORMANCE

CS radar system performance is characterized by $P_D$ and $P_{FA}$ curves according to $\tau_{th}$,
ROC curves and reconstruction error results per each transmit-waveform and receiver filter pair and per each sparse reconstruction algorithm considered. Mutual coherence measures of the sensing matrix are calculated for each considered transmit-waveform and receiver filter pair to numerically further quantify design performance in addition to reconstruction error results alone.

4.2.1 RECEIVER OPERATING CHARACTERISTIC (ROC)

ROC analysis edifies the tradeoff between the detection and false alarm rates in signal detection. Previous work [37] fixed $\tau_{th}$ to zero and varied the SNR in the received signal, (40) in simulation to assess performance of the joint waveform-receiver filter design algorithm. Because this produced flat $P_{FA}$ for aforementioned reasons, ROC curves were untenable with this approach. This work assumes fixed $\beta_n$'s based on physical manifestations within the radar equation and varies the $\tau_{th}$ to attain $P_{FA}$ and $P_D$ curves that together enable ROC analysis and performance characterization of the joint waveform-receiver filter design algorithm. This is an important improvement in assessing the performance of the design algorithm within the larger radar system.

4.2.2 RECONSTRUCTION ERROR

The reconstruction error, $\Delta$, is defined as the difference between the actual reflection coefficient matrix, $\tilde{\lambda}_r$, and the sparse estimate of the reflection coefficient matrix, $\hat{\lambda}_r$, averaged over all $R$ columns of the reflection coefficient matrix [37, 47, 48]. The reconstruction error is stated formally in (48):

$$\Delta = \frac{1}{R} \sum_{r=1}^{R} |\tilde{\lambda}_r - \hat{\lambda}_r|.$$  

(48)

It is noted that while this statistic provides insight on the accuracy of sparse reconstruction, it does not necessarily correlate to improved radar detection performance. Rather, the detection rate is determined by the hypothesis test given by (47). Reconstruction errors are included in this dissertation for completeness and consistency with previously published work.
4.2.3 MUTUAL COHERENCE

The ability of a sensing matrix to recover sparse signals is often characterized by the mutual coherence measure of the sensing matrix [68]. A small mutual coherence implies that the columns of the sensing matrix are approximately orthogonal, similar to the characteristic of sensing matrices that satisfy RIP [53]. In other CS applications, improved performance was demonstrated when sensing mutual coherence is minimal [49, 51–53]. While reconstruction error measures will be presented in this work for completeness, this alternative performance metric is now considered to characterize system performance.

The Welch bound defines the lower bound of the mutual coherence for a given sensing matrix. For the sensing matrix \( \Phi \in \mathbb{C}^{M \times N} \), the Welch bound is given by:

\[
\mu_w(\Phi) \geq \sqrt{\frac{N - M}{M(N - 1)}}.
\]  

This metric is used to compare observed sensing matrix mutual coherences to their theoretical minimums [69].

As discussed in Chapter 3 of this dissertation, the primary design goal of the joint waveform-receiver filter design algorithm is to minimize the mutual coherence of the sensing matrix through the design of the both the transmit-waveforms and receiver filters for fixed target impulse responses. That is, the optimal transmit-waveform and receiver filter matrices in the CS radar context would be those that minimize (30). It is expected that the best performing transmit-waveform and receiver filter pair would be that which minimizes the mutual coherence in the sensing matrix to best satisfy RIP for sparse reconstruction as further described in Appendix A.
Chapter 5

SIMULATIONS AND NUMERICAL RESULTS

5.1 SIMULATION SETUP

A bistatic MIMO radar illuminates a radar scene comprised of a mixture of extended targets and composite clutter. \( L = 5 \) extended targets are placed at the following transmit-receive pair positions \((4, 7), (10, 6), (14, 15), (15, 13), \) and \((19, 12)\) and at a range of \( R_t = R_r = 35 \text{ km} \). Refer to Fig. 4 for a graphical depiction of the reflection coefficients as viewed on a single \( T \times R \) grid to compactly describe the simulated radar scene. As shown, the reflection coefficient matrix for each target is normalized to have a maximum magnitude of 1 [37, 47, 48, 54, 55].

A quantity of 5 unique extended targets are formed each of 5 distinct reflection centers (as in [37, 47, 48, 54, 55]) randomly scattered over the surface of the target. This comprises each target’s impulse response. Free space propagation is assumed with \( c = 3 \times 10^8 \text{ m/s} \). The Discrete Fourier Transform (DFT) is then applied to obtain the frequency response matrix corresponding to each target in all target scenarios and for all target impulse responses. This extended target model is consistent with [12] where similar descriptions were used to describe actual airborne objects.

In accordance with the system model given by (24), clutter is applied in a manner similar to how extended targets are implemented in this dissertation, or similarly, how targets are defined by Kay [44]. The clutter environment is adapted to this model via a clutter response that is described by both impulse response and reflection center. In this case, clutter frequency responses are assumed WSS Gaussian random processes and assigned to the radar scene with the reflection coefficient matrix, \( \Gamma \). In all simulations, the clutter response is assumed flat with \( Q_{\xi \zeta} = \sigma_c^2 \) \( \forall f \) and \( \forall \zeta = 1, \ldots, Z_c \) as in [44]. After setting the transmit-waveform power, \( \beta_c \) is obtained by fluctuating the clutter power spectral density. The following simulations consider a \( \beta_c = -6 \text{dB} \). \( Z_c = 15 \) clutter sources located at
the following transmit-receive positions centered around targets of interest are considered: 
(2, 7), (3, 8), (11, 5), (10, 7), (9, 6), (14, 14), (14, 13), (15, 15), (20, 11), (21, 10), (8, 14), (9, 15),
(13, 14), (10, 16), and (18, 12), as depicted on a single $T \times R$ grid in Fig. 5. This further
challenges the sparsity of the radar scene particularly around targets of interest and like-
wise challenges sparse reconstruction of the radar scene. Acknowledging the target model
presented in [44], it is noted that while $\tilde{H}$ represents the deterministic part of the target
response, no such assumption is made for clutter. Consideration of added localized clutter
sources or different clutter environments is beyond the scope of this work.

Fig. 6 depicts a single $T \times R$ grid containing all target and clutter reflection coefficients
describing the radar scene for the simulated example. As seen, clutter sources are positioned
in the immediate vicinity of targets to further challenge the ability of the radar discern
targets from clutter sources and make positive detections.
Figure 5. Single $T \times R$ grid of $Z_c = 15$ clutter reflection coefficients for the simulated radar environment.

Figure 6. Single $T \times R$ grid of $L = 5$ targets $Z_c = 15$ clutter reflection coefficients that comprise the simulated radar scene.
Algorithm 4 allows the ability for the radar designer to assign priorities to individual targets. In this simulation, all targets were given equal priority. Furthermore, the number of CS measurements was $\Upsilon = \lfloor TL/2 \rfloor$ yielding a $\delta_u = 0.28$ and $\rho = 0.08$ (under stated assumptions), halting criterion were set to $\epsilon_1 = \epsilon_2 = 0.01$ and $\epsilon_3 = 0.2$ and the MIMO bistatic radar was implemented with $K = 201$ individual frequencies and $N_{aug} = 4$.

The bistatic radar is comprised of phased arrays with $N_T = N_R = 25$ with 1/2 wavelength spacing. Beamforming is used for simultaneous transmission and reception of $T = R = 25$ beams centered at a 45° angle upward similar to what is depicted in Fig. 2. Together the beams cover a radial span of 85°. This simulation assumes a radar operating frequency of $f_c = 8GHz$.

The transmit and receiver filter design algorithm, Algorithm 4, was used to design waveforms and receiver filters to detect:

- **Multiple Known Extended Targets** - First, the case of completely known impulse responses of extended targets present in a scene is considered. This represents a best case scenario for the radar designer where the target impulse response is known a priori, allowing near-ideal design of the transmit-waveforms and receiver filters specifically suited for individual targets. Fig. 7 depicts the target impulse responses for $L = 5$ targets considered for this scenario. As shown, each extended target is comprised of 5 unique reflection centers spread randomly across a 30m surface.
Multiple Fluctuating Extended Targets - Second, the case of fluctuating extended targets is considered. In this test case, the location of the reflection centers along the target surface is assumed known but the magnitudes of the reflection centers vary exponentially about a deterministic mean akin to the Swerling I target model [70]. This scenario represents the case where a known target is at a slightly different aspect to the bistatic receiver, causing fluctuations in returns from reflection centers. This scenario explores the consequence and sensitivity of the posed CS radar system and design algorithm to basis mismatch [46] where, in this case, the actual sensing matrix differs from that used by the CS bistatic radar for scene reconstruction due to the variability in the actual target frequency response versus that used to both design waveforms/receiver filters and perform sparse reconstruction. Fig. 8 depicts the target impulse responses considered for this scenario. As in the case of known extended targets, each of $L = 5$ targets is comprised of 5 unique reflection centers spread randomly across a 30$m$ surface. Actual target impulse responses identified in Fig. 8 correspond
Figure 8. Target impulse responses in the fluctuating extended target detection scenario. Mean target impulse responses are depicted in blue whereas actual target impulse responses are in orange. $x$ is the distance along the target in meters.

to one instance and are again regenerated for each Monte Carlo simulation to quantify radar performance.

- **Multiple Fluctuating Extended Targets with Ambiguous Reflection Center Range**

This case considers the scenario where both the magnitudes of the reflection centers and locations of the reflection centers vary. In this instance, the magnitudes of the reflection centers vary exponentially about a deterministic mean as before, but the actual location of these reflection centers vary uniformly across neighboring range increments. It is noted that while reflection center locations are herein described in terms of increments from a deterministic mean that in actuality, reflections occur along a continuous range for extended, range-spread targets [71]. This scenario extends the previous case where the radar was designed to detect a particular target but the actual target varies slightly in aspect to the CS bistatic radar system causing the location of
Figure 9. Target impulse responses in the fluctuating extended target detection scenario and where locations of reflection centers vary. Mean target impulse responses are depicted in blue whereas actual target impulse responses are in orange. \( x \) is the distance along the target in meters.

• Multiple Unknown Extended Targets - Finally, the case of completely unknown impulse responses for extended targets is considered. Previous work assumed knowledge of each extended target’s mean where the actual value of the impulse response fluctuated exponentially about that mean in accordance with the Swerling I target model [70]. Here, any knowledge of the extended target to be detected is removed. This represents
Figure 10. Target impulse responses in the unknown extended target detection scenario. Dictionary target impulse responses are denoted in blue whereas actual target impulse responses are denoted in orange. $x$ is the distance along the target in meters.

a worst case scenario where the radar was designed to detect one group of targets and the scene is populated by entirely different types of targets. Fig. 10 depicts the target impulse responses for $L = 5$ targets considered for this scenario. As before, each extended target is made of 5 reflection centers spread randomly across a 30$m$ surface.

The scenarios with multiple known and multiple fluctuating target impulse responses are tested in a noisy environment and a noisy, cluttered environment at $\beta_n = 9dB, 12dB,$ and $15dB$ and with ROMP, CoSaMP, and CAMP sparse reconstruction methods. Next, the scenario with multiple fluctuating target impulse responses with reflection centers of ambiguous range is tested in a noisy environment and a noisy, cluttered environment at $\beta = 18dB$ and $21dB$ with ROMP, and CAMP sparse reconstruction methods to account for the requisite higher $\beta_n$ for to detect targets under these conditions. Performance at lower $\beta_n$’s may be inferred from the results in this section. Finally, the scenario with multiple unknown
target impulse responses is tested in a noisy environment, at a $\beta_n = 30 dB$ using the ROMP sparse reconstruction method to ascertain the ability to precisely make detections of unknown targets under near-ideal radar channel conditions. $\sigma^2_n = 0.01$ describes the noise all cases. The proposed radar system, comprised of the joint waveform-receiver filter design algorithm and multiple sparse reconstruction techniques were simulated in 200 Monte Carlo simulations for each $\tau_{th} = [0 : 0.1 : 3.0]$. Performance of designed waveforms, $(\hat{C}, S)$, were compared to that of statically designed waveforms. These include: randomly generated receiver filter and transmit-waveforms, $(C_r, S_r)$; and a randomly generated receiver filter paired with a matrix whose columns are cubic phase Alltop sequences, $(C_r, S_a)$. Both benchmarks are understood to have favorable coherence properties [72,73].

This study is limited to the aforementioned target impulse response types, $\beta_c$, and $\beta_n$ values associated with each scenario to concisely highlight CS radar system performance under a myriad of conditions. Consideration of additional target types, radar scenes, $\beta_n$’s, $\beta_c$’s, clutter types, clutter source locations, reconstruction algorithms, and design algorithm parameters are beyond the scope of this work left for future research. Simulation parameters defined in Table 2.

| $\varepsilon_1 = \varepsilon_2 = 0.01$ | Frequency Bands: $K = 201$ |
| $\varepsilon_3 = 0.02$ | $\sigma^2_n = 0.01$ |
| $T = R = 25$ | Qty Targets: $L = 5$ |
| WSS Gaussian Clutter ($\mu = 0, \sigma^2_c$) | Qty Clutter Sources: $Z_c = 15 \& Z_c = 0$ |
| TIR: Known, Fluctuating, & Unknown | Sparse Reconstruction Algorithm: ROMP, CoSaMP, & CAMP |
| $\beta_n = 9dB, 12dB, 15dB, 18dB, 21dB, 30dB$ | $f_c = 8GHz$ |
| $\beta_c = -6dB$ | $N_{aug} = 4$ |
| Target Locations $(r, d)$: (4, 7), (10, 6), (14, 15), (15, 13), (19, 12) | |
| Clutter Locations $(r, d)$: (2, 7), (3, 8), (11, 5), (10, 7), (9, 6) (14, 14), (14, 13), (15, 15), (20, 11), (21, 10), (8, 14), (9, 15) (13, 14), (10, 16), (18, 12) | |

**Table 2.** Simulation parameters
Applicability

\( \beta_n \)’s of 9\,dB, 12\,dB, and 15\,dB result from transmit powers of 25\,kW, 50\,kW, and 100\,kW peak power, respectively, and parameters given in Table 3. While the radar equation in (50) does not explicitly apply to extended or fluctuating targets or account for clutter power considered here, it indicates that tested \( \beta_n \) correlate to realistic radar or channel characteristics.

\[
\beta_n = \frac{P_r}{N} = \frac{P_t \tau G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_s R_t^2 R_r^2 L}
\]  

(50)

Table 3. Radar simulation parameters corresponding to \( \beta_n = 9\,dB, 12\,dB, \) and 15\,dB

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_t ) = 25,W, 50,kW, 100,kW Peak</td>
<td></td>
</tr>
<tr>
<td>System Temperature: ( T = 290K )</td>
<td></td>
</tr>
<tr>
<td>Reflection Center RCS: ( \sigma = 1m^2 )</td>
<td></td>
</tr>
<tr>
<td>Distance from Tx/RX: ( R_t = R_r = 35km )</td>
<td></td>
</tr>
<tr>
<td>( f_c = 8GHz )</td>
<td></td>
</tr>
<tr>
<td>( G_t = G_r = 35dB )</td>
<td></td>
</tr>
<tr>
<td>Pulse Width: 1,\mu s</td>
<td></td>
</tr>
<tr>
<td>System Loss: ( L = 5.72dB )</td>
<td></td>
</tr>
</tbody>
</table>

5.2 NUMERICAL RESULTS

\( \beta_n \)’s in Table 2 and target impulse response types were simulated according to the predefined radar scenarios. Cases with noise and without clutter and with noise and clutter are considered. The following are the mutual coherence results, \( P_D \), \( P_{FA} \), and ROC results and reconstruction error results.

5.2.1 MUTUAL COHERENCE

The average mutual coherence values, \( \bar{\mu}(\Phi) \), are given in Table 4 and depicted in Fig. 11 for 10,000 Monte Carlo simulations of random, Alltop, and designed waveforms/received filters for deterministic target impulse responses as defined in Fig. 7. The Welch bound for simulation parameters defined in Table 2 is \( \mu_w(\Phi) = 0.1083 \), where \( M = 62 \) and \( N = 225 \) in (49). A minimal mutual coherence using the proposed waveform and receiver filter design approach as compared to benchmark methods is observed.
Figure 11. Histograms fitted with a normally distributed PDF depicting distribution of mutual coherences for 10,000 iterations of the proposed, Alltop, and random transmit-waveform/receiver filters for CS radar.

Table 4. Average mutual coherences for sensing matrices

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Mutual Coherence, $\bar{\mu}(\Phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{C}, S$</td>
<td>0.6856</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>0.7696</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>0.7551</td>
</tr>
</tbody>
</table>

As shown in Table 4, the designed waveforms and receiver filters minimize the mutual coherence of the sensing matrix, meeting the overarching design goal of the design algorithm.
5.2.2 MULTIPLE KNOWN EXTENDED TARGETS

Case I: With Noise and No Clutter

Acknowledging the contributions made in [37,47,48], considered first is the case with noise but without clutter for multiple known extended targets. The system model given by (24) is implemented. The results below serve as a comparison baseline for the results that follow when the radar channel becomes more complex with the addition of WSS Gaussian clutter sources, fluctuating target impulse responses (varying the reflection center magnitude and both magnitude and location), or deliberately removing the assumption of a known target impulse response. Formal presentation of results are limited to the $P_D$, $P_{FA}$, and ROC curves for a receive $\beta_n$ of 15dB as it is understood that a test $\beta_n$ of 12dB or 9dB will demonstrate a decline in $P_D$, $P_{FA}$, and ROC performance and allows for emphasis on new contributions for the CS radar system. Appendix D contains the graphical results for the $\beta_n = 12dB$ and $\beta_n = 9dB$ cases, for completeness.

The goal for the proposed CS radar system is to attain results as close to those presented in this section as possible (or better!) when clutter or unknown target impulse responses are considered. Depicting results for the near-ideal scenario allow the reader to understand the degradation in performance due to clutter or target impulse response implementations.

Fig. 12, Fig. 13, and Fig. 14 depict resultant $P_D$, $P_{FA}$, and ROC curves for the 15dB case using ROMP, CoSaMP, and CAMP sparse reconstruction algorithms. Performance gains of the designed transmit-waveforms and receiver filters are evident and further improve when the $\tau_{th}$ is varied. Setting $\tau_{th} = 0.6$, for example, yields 98.5% and 97.0% detection rates for designed waveforms for ROMP and CoSaMP, respectively, and a $\tau_{th} = 0.3$ yields a 99.4% detection rate for CAMP. Designed waveforms afford a 40.3% increase in $P_D$ as compared to Alltop waveforms with ROMP reconstruction and a 47.4% increase in $P_D$ with CoSaMP reconstruction at this $\tau_{th}$. Designed waveforms paired with CAMP sparse reconstruction affords an increase in the detection rate of 11% over benchmark waveforms. Randomly generated waveforms and receiver filters displayed the least favorable detection results in this instance, though it is noted their detection rate performance was comparable.

At $\tau_{th} = 0$, the sparsity, $k$, of the sparse estimate algorithm is evident as the $P_{FA}$
of CoSaMP is approximately half of that of ROMP. CAMP yields a much higher $P_{FA}$ at $\tau_{th} = 0$ implying many fewer non-zero values in the sparse estimate per receiver sensor. This result is consistent for the remainder of this work.

$P_{FA}$'s were further improved when the detection threshold was increased. For a $\tau_{th} = 0.6$, designed waveforms and received filters gave a false alarm rate of $1.87 \times 10^{-2}\%$ with ROMP reconstruction and $1.59 \times 10^{-2}\%$ using CoSaMP. These results mark only a slight decrease in $P_{FA}$ as compared to randomly generated waveforms. A $\tau_{th} = 0.3$ and CAMP sparse reconstruction yields a $P_{FA}$ of $1.06 \times 10^{-1}\%$. In this instance, Alltop waveforms performed least favorably. Furthermore, juxtaposed to $P_D$ results, CoSaMP depicts the most favorable $P_{FA}$ performance.

$P_D$ and $P_{FA}$ results together map to the ROCs presented in Fig. 14. ROC results clearly display the performance advantages with designed waveforms and receiver filters versus the benchmark Alltop waveforms or random waveforms and random filters. In addition, the improved detection rate performance is evident where CAMP sparse reconstruction is used. As noted, this comes at a cost with an coinciding increase in the $P_{FA}$.

ROC results similar to those depicted in Fig. 14 are favorable. Ideal ROCs depict a steep (vertical) ascent to the maximum detection probability, 1. Results such as these indicate that a maximum probability of detection is reached at the minimum $P_{FA}$. 
Figure 12. Probabilities of detection for known targets at a $\beta_n = 15dB$ amidst noise and without clutter.
Figure 13. Probabilities of false alarm for known targets at a $\beta_n = 15dB$ amidst noise and without clutter.
Figure 14. Receiver operating characteristic for known targets at a $\beta_n = 15dB$ amidst noise and without clutter.
$P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 5. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of ($\hat{C}, S$) to the ($C_r, S_a$) and ($C_r, S_r$) benchmarks.

**Table 5.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 15dB$ with noise and without clutter for known targets

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0.6</td>
<td>98.5%</td>
<td>$1.87 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CoSaMP</td>
<td>0.3</td>
<td>99.4%</td>
<td>$1.06 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0.96</td>
<td>22.0%</td>
<td>$1.91 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CoSaMP</td>
<td>0.98</td>
<td>18.5%</td>
<td>$1.63 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0.49</td>
<td>60.0%</td>
<td>$1.03 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.83</td>
<td>27.0%</td>
<td>$1.87 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.8</td>
<td>23.1%</td>
<td>$1.56 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.48</td>
<td>62.0%</td>
<td>$1.08 \times 10^{-1}%$</td>
</tr>
</tbody>
</table>

Performance of individual sparse reconstruction algorithms is given by Table 6. In Table 6, the same $P_D$ is selected for the designed waveform and receiver filter pair per each sparse reconstruction algorithm considered in this paper. It is shown that for the same sample $P_D$, ROMP minimizes $P_{FA}$ under tested conditions. It is noted that these results depict a comparison of sparse reconstruction algorithms that reconstruct at different sparsity levels and comparison is made at a single $P_D$. Future work may include the testing of multiple reconstruction algorithms of the same sparsity level and at multiple $P_D$’s to characterize a best sparse reconstruction algorithm given the CS radar system model, clutter model, and simulation parameters.

**Table 6.** Sparse reconstruction algorithm performance comparison for ($\hat{C}, S$) at $\beta_n = 15dB$ with noise and without clutter for known targets

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>95.0%</td>
<td>$1.10 \times 10^{-2}%$</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>95.0%</td>
<td>$1.35 \times 10^{-2}%$</td>
</tr>
<tr>
<td>CAMP</td>
<td>95.0%</td>
<td>$1.96 \times 10^{-2}%$</td>
</tr>
</tbody>
</table>

Table 7 below depicts the observed reconstruction errors for the designed waveforms and receiver filters versus statically generated, benchmark waveforms and filters for known targets. Reconstruction errors for ROMP, CoSaMP, and CAMP sparse reconstruction methods
are presented for this radar system. It is noted that while reconstruction errors correlate with $P_{FA}$ performance of sparse reconstruction algorithms relative to one another (e.g. CAMP versus CoSaMP), they do not correlate with $P_{FA}$ performance of designed versus benchmark waveforms within each reconstruction method tested.

**Table 7.** Observed reconstruction error for known extended targets in noise for ROMP, CoSaMP, and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta, \beta_n = 9dB$</th>
<th>$\Delta, \beta_n = 12dB$</th>
<th>$\Delta, \beta_n = 15dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>$C, S$</td>
<td>2.27</td>
<td>1.16</td>
<td>0.62</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_a$</td>
<td>5.71</td>
<td>2.97</td>
<td>1.54</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>6.82</td>
<td>3.54</td>
<td>1.83</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C, S$</td>
<td>1.56</td>
<td>0.80</td>
<td>0.44</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_a$</td>
<td>3.83</td>
<td>2.03</td>
<td>1.06</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_r$</td>
<td>4.65</td>
<td>2.46</td>
<td>1.29</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C, S$</td>
<td>7.99</td>
<td>4.10</td>
<td>2.16</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_a$</td>
<td>13.32</td>
<td>6.76</td>
<td>3.49</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>18.35</td>
<td>9.25</td>
<td>4.70</td>
</tr>
</tbody>
</table>

While not directly correlated to ROC performance, a decreased reconstruction error using the designed $\hat{C}$ and $S$ is observed as compared to both $(S_a, C_r)$, and $(S_r, C_r)$ for ROMP, CoSaMP, and CAMP and sparse reconstruction methods. These reconstruction errors coincide with observed maximum $P_{FA}$ values, where CoSaMP results displayed the most favorable performance (smallest $P_{FA}$) and CAMP least favorable.

Mutual coherence results appear to coincide with the ROC performance via CAMP, where it is observed that the designed waveforms show improved performance over randomly generated waveforms which themselves demonstrate improved performance over the Alltop waveforms. This difference is less pronounced in this test case but is more apparent in the proceeding scenarios. It is noted that where reconstruction is governed by an input sparsity (e.g. ROMP and CoSaMP), this result does not hold true. Still, as with the observed mutual coherence values, performance of the random and Alltop waveforms are markedly similar.

**Case II: With Noise and Clutter**

The novel system model inclusive of both noise and transmit signal dependent clutter given by (24) is now fully implemented with WSS Gaussian clutter sources positioned around
targets of interest as per Fig. 6. The simulation is carried out for 200 iterations at each $\tau_{th}$ interval. As before, simulation parameters used are given by Table 2, considering $\beta_n = 15dB$, $12dB$, and $9dB$ and a $\beta_c = -6dB$.

Fig. 15, Fig. 16, and Fig. 17 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 15dB$ case, respectively. As shown, there is a moderate decrease in radar performance when the scene becomes cluttered as sparsity is affected. For the same $\tau_{th}$ as used in the preceding section, a detection rate of 78.6% was observed using ROMP, marking a 19.4% decrease in the ability of the CS radar to make a detection. For CoSaMP, this difference was more pronounced with a 25.9% decrease in the detection rate when clutter is added. For CAMP at a $\tau_{th} = 0.3$, the detection rate decreases by 10%. However, adjusting the $\tau_{th}$ to $\tau_{th} = 0.2$, a 95.0% detection rate is observed. A difference of 47.6% is observed in the steady state ROC for the the designed algorithms using ROMP and 45.3% with CoSaMP compared to the next best performing waveforms, ($C_r, S_a$). A difference of 8% between the design waveforms and benchmark waveforms was observed using CAMP. The ROC curves also depict the favorable $P_{FA}$ results shown in Fig. 16.
Figure 15. Probabilities of detection for known targets at a $\beta_n = 15dB$ amidst noise and clutter.
Figure 16. Probabilities of false alarm for known targets at a $\beta_n = 15dB$ amidst noise and clutter.
Figure 17. Receiver operating characteristic for known targets at a $\beta_n = 15dB$ amidst noise and clutter.
$P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 8. As before, like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks. As the $P_{FA}$ increases, the ROC increases less sharply and tends to gradually reach its steady state value. Very low $P_{FA}$’s correspond to sharp increases in the ROC. In the case of ROMP and CoSaMP and a for the sample $\tau_{th} = 0.5$, designed waveforms displayed the lowest $P_{FA}$ as compared to both Alltop and randomly generated waveforms. The CoSaMP reconstruction algorithm yielded the minimum peak $P_{FA}$ across designed and benchmark waveforms. It is noted that for $\tau_{th} < 0.6$, $(C_r, S_a)$ and $(C_r, S_r)$ waveforms outperform designed waveforms in terms of the false alarm rate.

**Table 8.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 15dB$ with noise and clutter for known targets

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0.6</td>
<td>78.6%</td>
<td>$6.31 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CoSaMP</td>
<td>0.6</td>
<td>71.0%</td>
<td>$3.86 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0.3</td>
<td>90.8%</td>
<td>$2.41 \times 10^{-1%}$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0.2</td>
<td>95.0%</td>
<td>$7.92 \times 10^{-1%}$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0.88</td>
<td>29.2%</td>
<td>$6.20 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CoSaMP</td>
<td>0.92</td>
<td>22.6%</td>
<td>$3.90 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0.33</td>
<td>63.6%</td>
<td>$7.92 \times 10^{-1%}$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.73</td>
<td>29.9%</td>
<td>$6.50 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.68</td>
<td>25.3%</td>
<td>$3.84 \times 10^{-2%}$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.33</td>
<td>67.0%</td>
<td>$7.95 \times 10^{-1%}$</td>
</tr>
</tbody>
</table>

Fig. 18, Fig. 19 and Fig. 20 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 12dB$ case. As the $\beta_n$ decreases, so too does the performance of the radar system. As before, the designed waveforms are shown to outperform Alltop transmit-waveforms and random receiver filters and random transmit-waveforms and random receiver filters in terms of the detection rate. In the case of ROMP and CoSaMP, designed waveforms display sub-optimal $P_{FA}$ performance for $\tau_{th} < 0.7$. Choosing a $\tau_{th} = 0.6$ and implementing ROMP and CoSaMP sparse reconstruction techniques, gives a detection rate of 62.9% and 51.2% designed waveforms, respectively. This marks a 278% improvement for ROMP and 300% improvement for CoSaMP using $(\hat{C}, S)$ versus the next best performing waveforms, $(C_r, S_a)$. Using this $\tau_{th}$, a $P_{FA}$ of
1.67 \times 10^{-1}\% and 9.63 \times 10^{-2}\% are observed for ROMP and CoSaMP, respectively. As before, it is observed that for $\tau_{th} < 0.8$ sub-optimal $P_{FA}$’s are observed for designed waveforms as compared to benchmark waveforms. Now, selecting a $\tau_{th} = 0.2$ and implementing CAMP sparse reconstruction, a detection rate of 90.6\% is observed for designed waveforms, a 24.1\% improvement over the next best performing waveforms, $(C_r, S_a)$. With this $\tau_{th}$, a $P_{FA}$ of 1.34\% is observed, marking a 57.6\% decrease compared to benchmark waveforms. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 9. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks. As $\beta_n$ decreases so too does $P_D$ and $P_{FA}$ performance and is recognizable in the ROC curve as a more gradual roll off is observed and to a diminished steady state value for all test cases as compared to the 15$dB$ case. This holds true for the ROMP, CoSaMP, and CAMP sparse reconstruction methods.

**Table 9.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results $\beta_n = 12dB$ with noise and clutter for known targets

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0.6</td>
<td>62.9%</td>
<td>1.67 \times 10^{-1}%</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CoSaMP</td>
<td>0.6</td>
<td>51.2%</td>
<td>9.63 \times 10^{-2}%</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0.2</td>
<td>90.6%</td>
<td>1.34%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0.66</td>
<td>18.5%</td>
<td>1.67 \times 10^{-1}%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CoSaMP</td>
<td>0.66</td>
<td>12.0%</td>
<td>9.63 \times 10^{-2}%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0.34</td>
<td>53.5%</td>
<td>1.32%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>15.2%</td>
<td>1.51 \times 10^{-1}%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0</td>
<td>9.8%</td>
<td>6.81 \times 10^{-2}%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.35</td>
<td>54.2%</td>
<td>1.32%</td>
</tr>
</tbody>
</table>
Figure 18. Probabilities of detection for known targets at a $\beta_n = 12dB$ amidst noise and clutter.
Figure 19. Probabilities of false alarm for known targets at a $\beta_n = 12\, dB$ amidst noise and clutter.
Figure 20. Receiver operating characteristic for known targets at a $\beta_n = 12dB$ amidst noise and clutter.
Fig. 18, Fig. 19 and Fig. 20 show that at an $\beta_n$ of 12dB and amidst clutter, tenable results are obtained as demonstrated by the steady state ROC detection values of 90.6%. Compared to the next best case Alltop waveforms with a steady state ROC value of 50.4%, this marks a 57% performance gain with the design algorithm approach.

Fig. 21, Fig. 22 and Fig. 23 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 9dB$ case. With a $\tau_{th} = 0.6$, detection rates of 38.6% and 28.9% and false alarm rates of $2.63 \times 10^{-1}\%$ and $1.37 \times 10^{-1}\%$ are observed for ROMP and CoSaMP reconstruction algorithms, respectively. A $\tau_{th} = 0.2$ yields a detection rate of 82.1% and false alarm rate of 2.16% using CAMP. As demonstrated, at low $\beta_n$’s, detections are challenged with whereas CoSaMP and CAMP affords a means to readily make detections, even amidst clutter. At $\beta_n = 9dB$, CAMP is shown to have a 52.9% increase in detection rate over ROMP and 64.8% improvement over CoSaMP. However, as before, the improved detection rates come at the cost of an increase in $P_{FA}$, a 93% increase as compared to CoSaMP, for example. This underscores the importance of establishing radar performance goals prior to making design choices in this CS radar system. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 10. As noted previously, diminished $P_{FAs}$ are viewable in the ROC plot with less sharp and more substantial roll offs to the steady state ROC value. The designed waveforms are shown to have adequate $P_D$ performance even at the lowest $\beta_n$ tested and amidst clutter whereas benchmark waveforms display diminutive performance.

Table 10. $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 9dB$ with noise and clutter for known targets

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.36</td>
<td>43.0%</td>
<td>2.17%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.34</td>
<td>42.5%</td>
<td>2.16%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.02</td>
<td>10.2%</td>
<td>2.63 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>6.4%</td>
<td>1.41 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0</td>
<td>3.9%</td>
<td>6.40 $\times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.6</td>
<td>38.6%</td>
<td>2.63 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.6</td>
<td>28.9%</td>
<td>1.37 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.2</td>
<td>82.1%</td>
<td>2.16%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.2</td>
<td>6.5%</td>
<td>1.37 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.02</td>
<td>10.2%</td>
<td>2.63 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.02</td>
<td>6.5%</td>
<td>1.37 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>6.4%</td>
<td>1.41 $\times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0</td>
<td>3.9%</td>
<td>6.40 $\times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.34</td>
<td>42.5%</td>
<td>2.16%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.36</td>
<td>43.0%</td>
<td>2.17%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.36</td>
<td>43.0%</td>
<td>2.17%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.36</td>
<td>43.0%</td>
<td>2.17%</td>
</tr>
</tbody>
</table>
Figure 21. Probabilities of detection for known targets at a $\beta_n = 9dB$ amidst noise and clutter.
Figure 22. Probabilities of false alarm for known targets at $\beta_n = 9dB$ amidst noise and clutter.
Figure 23. Receiver operating characteristic for known targets at a $\beta_n = 9dB$ amidst noise and clutter.
Performance of individual sparse reconstruction algorithms is given by Table 11, where as before, the same $P_D$ is selected for the designed waveform and receiver filter pair per each sparse reconstruction algorithm considered in this paper. It is shown that for the same, sample $P_D$, ROMP minimizes $P_{FA}$ under tested conditions for the $\beta_n = 15dB$ while for lower $\beta_n$’s, CAMP minimizes $P_{FA}$.

**Table 11.** Combined sparse reconstruction algorithm performance comparisons for $(\hat{C}, S)$ at $\beta_n = 15dB$, $12dB$, and $9dB$ with noise and clutter for known targets

<table>
<thead>
<tr>
<th>$\beta_n$</th>
<th>Algorithm</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15dB</td>
<td>ROMP</td>
<td>70.0%</td>
<td>$2.40 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CoSaMP</td>
<td>70.0%</td>
<td>$3.35 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CAMP</td>
<td>70.0%</td>
<td>$2.50 \times 10^{-2}%$</td>
</tr>
<tr>
<td>12dB</td>
<td>ROMP</td>
<td>50.0%</td>
<td>$2.57 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CoSaMP</td>
<td>50.0%</td>
<td>$2.54 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CAMP</td>
<td>50.0%</td>
<td>$2.05 \times 10^{-2}%$</td>
</tr>
<tr>
<td>9dB</td>
<td>ROMP</td>
<td>30.0%</td>
<td>$5.73 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CoSaMP</td>
<td>30.0%</td>
<td>$7.57 \times 10^{-2}%$</td>
</tr>
<tr>
<td></td>
<td>CAMP</td>
<td>30.0%</td>
<td>$5.11 \times 10^{-2}%$</td>
</tr>
</tbody>
</table>

Table 12 gives the observed reconstruction errors for unknown targets and without composite clutter. Results for ROMP and CoSaMP sparse reconstruction methods are presented. As before, designed waveforms are shown to have the lowest error amongst all sparse reconstruction techniques and test cases. Furthermore, it is demonstrated that the sparse reconstruction error does not directly correlate to the ability of the radar to make detections, but rather correlates with observed maximum $P_{FA}$’s with CoSaMP displaying most favorable results and CAMP displaying least favorable results, which is consistent with the reconstruction errors depicted in Table 12. However, it is noted that reconstruction errors do not correlate with $P_{FA}$ performance of designed versus benchmark waveforms for each reconstruction method.

As before, mutual coherence results coincide with the ROC performance via CAMP where it is observed that the designed waveforms show improved performance over randomly generated waveforms which themselves demonstrate improved performance over the Alltop waveforms. Note that where reconstruction produces an estimate of a specified sparsity, this result does not hold true, though ROC performance of $(C_r, S_a)$ and $(C_r, S_r)$ are similar.
Table 12. Observed reconstruction error for known extended targets in noise and clutter for ROMP, CoSaMP, and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta, \beta_n = 9dB$</th>
<th>$\Delta, \beta_n = 12dB$</th>
<th>$\Delta, \beta_n = 15dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>C, S</td>
<td>2.57</td>
<td>1.47</td>
<td>0.92</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_a$</td>
<td>6.42</td>
<td>3.70</td>
<td>2.32</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>7.36</td>
<td>4.12</td>
<td>2.49</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>C, S</td>
<td>1.74</td>
<td>1.02</td>
<td>0.64</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_a$</td>
<td>4.29</td>
<td>2.52</td>
<td>1.60</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_r$</td>
<td>4.99</td>
<td>2.83</td>
<td>1.71</td>
</tr>
<tr>
<td>CAMP</td>
<td>C, S</td>
<td>8.92</td>
<td>5.03</td>
<td>3.10</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_a$</td>
<td>14.83</td>
<td>8.33</td>
<td>5.10</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>19.33</td>
<td>10.28</td>
<td>5.83</td>
</tr>
</tbody>
</table>

5.2.3 MULTIPLE FLUCTUATING EXTENDED TARGETS

Case I: With Noise and No Clutter

The assumption for completely known target impulse responses is now removed. Considered now is the case of fluctuating extended targets, absent clutter, but otherwise using the same simulation parameters as before (refer to Table 2). Note that the results presented in this section denote those of fluctuating extended targets where the location of the reflection center is known but where the magnitude of that reflection center varies exponentially about a deterministic mean. This represents the scenario where a target present in the scene may differ in aspect slightly, causing fluctuations in the return from each reflection center. Simulations were performed for $\beta_n = 15dB, 12dB, \text{ and } 9dB$ and against a detection threshold that ranged from $[0, 0.1, 3]$. However, $P_D, P_{FA}$ and ROC curves presented in this section are limited to the $\beta_n = 15dB$ case to allow more focus on results where clutter is present in the radar scene. Appendix D contains the results for the $\beta_n$’s of $\beta_n = 12dB$ and $\beta_n = 9dB$ cases, for completeness.

Fig. 24, Fig. 25, and Fig. 26 depict the $P_D, P_{FA}$, and ROC curves for a $\beta_n = 15dB$. For a $\tau_{th} = 0.5$, detection rates of 85.2% and 80.0% and false alarm rates of $6.89 \times 10^{-2}\%$ and $4.82 \times 10^{-2}$ are observed using ROMP and CoSaMP sparse reconstruction methods. In the case of ROMP, an improvement of 56.4% is attained using the joint waveform-receiver filter.
design algorithm versus the next best performing waveforms, \((C_r, S_a)\). As before, CoSaMP displays the best false alarm rate performance in that its maximum false alarm rate is the smallest compared to that of other sparse reconstruction methods. Further, benchmark waveforms depict improved false alarm rates for \(\tau_{th} < 0.6\) when compared against designed waveforms and receiver filters. It was also observed that randomly generated waveforms and receiver filters performed most favorably in terms of \(P_{FA}\) with both ROMP and CoSaMP sparse reconstruction approaches. Choosing a \(\tau_{th} = 0.2\), a 95.9\% detection rate and \(4.67 \times 10^{-1}\%\) false alarm rate are observed via CAMP. These results mark a 17.9\% improvement in \(P_D\) and 73.2\% decrease in \(P_{FA}\) over benchmark waveforms. At this \(\tau_{th}\), designed waveforms depicted the most favorable \(P_{FA}\) performance as compared to benchmark waveforms.

Improved CS radar performance is also viewable in the ROC curves. Using any tested sparse reconstruction technique, it is shown that the designed waveforms outperform statically designed waveform benchmarks with higher peak values and sharper ascent to this maximum. \(P_D\) and \(P_{FA}\) results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 13. Like \(P_{FA}'\)s were selected per sparse reconstruction algorithm for performance comparison of \((\hat{C}, S)\) to the \((C_r, S_a)\) and \((C_r, S_r)\) benchmarks.
Figure 24. Probabilities of detection for fluctuating targets at a $\beta_n = 15 dB$ amidst noise and without clutter.
Figure 25. Probabilities of false alarm for fluctuating targets at a $\beta_n = 15dB$ amidst noise and without clutter.
Figure 26. Receiver operating characteristic for fluctuating targets at a $\beta_n = 15dB$ amidst noise and without clutter.
Table 13. \( \tau_{th} \)-based \( P_D \) and \( P_{FA} \) results at \( \beta_n = 15dB \) with noise and without clutter for fluctuating (fluctuating reflection center magnitudes, only)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>( \tau_{th} )</th>
<th>( P_D )</th>
<th>( P_{FA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, S</td>
<td>ROMP</td>
<td>0.5</td>
<td>85.2%</td>
<td>6.89 \times 10^{-2}%</td>
</tr>
<tr>
<td>C, S</td>
<td>CoSaMP</td>
<td>0.5</td>
<td>80.0%</td>
<td>4.82 \times 10^{-2}%</td>
</tr>
<tr>
<td>C, S</td>
<td>CAMP</td>
<td>0.2</td>
<td>95.9%</td>
<td>4.67 \times 10^{-1}%</td>
</tr>
<tr>
<td>C_r, S_a</td>
<td>ROMP</td>
<td>0.74</td>
<td>33.3%</td>
<td>6.85 \times 10^{-2}%</td>
</tr>
<tr>
<td>C_r, S_a</td>
<td>CoSaMP</td>
<td>0.75</td>
<td>26.7%</td>
<td>4.87 \times 10^{-2}%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>CAMP</td>
<td>0.34</td>
<td>59.4%</td>
<td>4.64 \times 10^{-1}%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>ROMP</td>
<td>0.5</td>
<td>30.7%</td>
<td>6.90 \times 10^{-2}%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>CoSaMP</td>
<td>0.08</td>
<td>23.2%</td>
<td>4.82 \times 10^{-2}%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>CAMP</td>
<td>0.34</td>
<td>62.6%</td>
<td>4.82 \times 10^{-2}%</td>
</tr>
</tbody>
</table>

As demonstrated, there are differences in \( P_D, P_{FA} \) and ROC performance of the proposed radar system based upon the sparse reconstruction algorithm selected in the CS radar system. With CAMP, designed waveforms outperform benchmark waveforms in terms of \( P_D \) and \( P_{FA} \) across all tested \( \tau_{th} \). With ROMP or CoSaMP, improved detection rates are observed but false alarm rates are improved at heightened \( \tau_{th} \) versus statically defined transmit-waveforms and receiver filters.

In this instance the implementation of ROMP affords a 9.17% increase in the steady state ROC value versus CoSaMP. CAMP affords a 9.74% increase in the steady state ROC value over ROMP for designed waveforms. This, however, comes at the cost of \( P_{FA} \) performance evidenced by the more gradual ascension to the peak ROC value for the CAMP ROC curve. These results are congruent with those presented in [55] where performance improvements were demonstrated in using CoSaMP versus ROMP.

While improved results are noted for the CS system with designed waveforms and receiver filters, detection rates of 95.9%, 85.2%, or 80% are not ideal for CAMP, ROMP, and CoSaMP. It is important to note, however, that these results highlight a best and worst case scenario to the radar designer. For this CS radar system, the radar can be tuned precisely to specific target impulse responses (and at multiple aspects). ROC results are near-optimal if the designed waveform is based on target impulse responses that match what is actually present in the radar scene. As demonstrated, sparse reconstruction and radar detection is possible if this is not the case but radar performance degrades.
Table 14 compares performance of the sparse reconstruction algorithms for the designed waveform and receiver filter pair and same $P_D$. It is shown that for the same, sample $P_D$, ROMP minimizes $P_{FA}$ under tested conditions.

**Table 14.** Sparse reconstruction algorithm performance comparison for $(\hat{C}, S)$ at $\beta_n = 15dB$ with noise and without clutter for fluctuating targets

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>75.0%</td>
<td>$1.55 \times 10^{-2}$%</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>75.0%</td>
<td>$2.28 \times 10^{-2}$%</td>
</tr>
<tr>
<td>CAMP</td>
<td>75.0%</td>
<td>$1.87 \times 10^{-2}$%</td>
</tr>
</tbody>
</table>

Table 15 gives the observed reconstruction errors for fluctuating targets without clutter present. Results for ROMP, CoSaMP, and CAMP sparse reconstruction methods are presented. It is again observed that while reconstruction errors correlate with $P_{FA}$ performance of sparse reconstruction algorithms relative to one another (e.g. CAMP versus CoSaMP), they do not correlate with $P_{FA}$ performance of designed versus benchmark waveforms within each reconstruction method tested. As before, a decreased reconstruction error using the designed $\hat{C}$ and $S$ is observed as compared to both $(S_a, C_r)$, and $(S_r, C_r)$.

**Table 15.** Observed reconstruction error for fluctuating extended targets in noise for ROMP, CoSaMP, and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta$, $\beta_n = 9dB$</th>
<th>$\Delta$, $\beta_n = 12dB$</th>
<th>$\Delta$, $\beta_n = 15dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>$C, S$</td>
<td>2.34</td>
<td>1.23</td>
<td>0.67</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_a$</td>
<td>5.72</td>
<td>3.02</td>
<td>1.62</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>6.83</td>
<td>3.58</td>
<td>1.91</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C, S$</td>
<td>1.63</td>
<td>0.89</td>
<td>0.50</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_a$</td>
<td>3.85</td>
<td>2.08</td>
<td>1.15</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_r$</td>
<td>4.66</td>
<td>2.50</td>
<td>1.37</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C, S$</td>
<td>8.00</td>
<td>4.11</td>
<td>2.17</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_a$</td>
<td>13.35</td>
<td>6.80</td>
<td>3.52</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>18.39</td>
<td>9.29</td>
<td>4.74</td>
</tr>
</tbody>
</table>

**Case II: With Noise and Clutter**

The following are the results given a radar scene comprised of fluctuating extended targets present in WSS Gaussian clutter positioned according to Fig. 6, $\beta_c = -6dB$ and simulation
parameters given in Table 2. Fig. 27, Fig. 28 and Fig. 29 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 15dB$ case. As in previous results, designed waveforms significantly outperform benchmark waveforms with fluctuating targets and amidst clutter. For the ROMP and CoSaMP sparse reconstruction methods, selecting a $\tau_{th} = 0.5$ yields detection rates of 62.6% and 53.4% and false alarm rates of $1.21 \times 10^{-1}$% and $7.46 \times 10^{-2}$%, respectively. A $\tau_{th} = 0.1$ yields a detection rate of 92.5% and false alarm rate of 2.60% for CAMP. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 16. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks.

**Table 16.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 15dB$ with noise and clutter for fluctuating targets (fluctuating reflection center magnitudes, only)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0.5</td>
<td>62.6%</td>
<td>$1.21 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CoSaMP</td>
<td>0.5</td>
<td>53.4%</td>
<td>$7.46 \times 10^{-2}$%</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0.1</td>
<td>92.5%</td>
<td>2.60%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0.65</td>
<td>18.3%</td>
<td>$1.21 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CoSaMP</td>
<td>0.67</td>
<td>13.6%</td>
<td>$7.42 \times 10^{-2}$%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0.18</td>
<td>61.5%</td>
<td>2.60%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0.36</td>
<td>17.6%</td>
<td>$1.21 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0.01</td>
<td>12.0%</td>
<td>$7.40 \times 10^{-2}$%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.18</td>
<td>64.3%</td>
<td>2.46%</td>
</tr>
</tbody>
</table>

As demonstrated the presence of fluctuating targets and clutter challenges the CS radar. Using CAMP affords the system 32.3% increase in the detection rate versus previous results attained solely with ROMP [37, 47, 48]. Improved detection does come at a cost of a higher false alarm rate that must be acceptable in CS radar design. Still, the proposed design algorithm offers a 16.2% improvement in the steady state ROC values for CAMP, a 67.0% increase for ROMP, and a 70.2% increase for CoSaMP sparse reconstruction methods. Reduced $P_{FA}$’s are viewable within the ROC plot with a more pronounced roll off to the steady state ROC value (less steep).
Figure 27. Probabilities of detection for fluctuating targets at a $\beta_n = 15dB$ amidst noise and clutter.
Figure 28. Probabilities of false alarm for fluctuating targets at a $\beta_n = 15dB$ amidst noise and clutter.
Figure 29. Receiver operating characteristic for fluctuating targets at a $\beta_n = 15dB$ amidst noise and clutter.
Fig. 30, Fig. 31, and Fig. 32 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 12dB$ case. As shown, both the detection rate performance of both the ROMP and CoSaMP sparse reconstruction algorithms suffers with the diminishing $\beta_n$, the presence of clutter, and fluctuating target reflection center magnitudes. At $\tau_{th} = 0.5$, the designed waveforms using ROMP display a mere 45.1% detection rate. This is dismal compared to the still robust detection rate using CAMP of 92.2% at a $\tau_{th} = 0.1$. Still, with $\tau_{th} = 0.1$, CAMP is able to achieve a false alarm rate of 3.45%. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 17. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of ($\hat{C}, S$) to the ($C_{r}, S_a$) and ($C_{r}, S_r$) benchmarks.

**Table 17.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 12dB$ with noise and clutter for fluctuating targets (reflection center magnitudes, only)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0.5</td>
<td>45.1%</td>
<td>$2.24 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CoSaMP</td>
<td>0.5</td>
<td>36.0%</td>
<td>$1.18 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0.1</td>
<td>92.2%</td>
<td>3.45%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0.33</td>
<td>11.0%</td>
<td>$2.25 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CoSaMP</td>
<td>0.5</td>
<td>7.1%</td>
<td>$1.18 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0.17</td>
<td>52.9%</td>
<td>3.55%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>8.8%</td>
<td>$1.5 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CoSaMP</td>
<td>0</td>
<td>6.1%</td>
<td>$6.85 \times 10^{-2}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0.19</td>
<td>54.8%</td>
<td>3.4%</td>
</tr>
</tbody>
</table>
Figure 30. Probabilities of detection for fluctuating targets at a $\beta_n = 12dB$ amidst noise and clutter.
Figure 31. Probabilities of false alarm for fluctuating targets at a $\beta_n = 12dB$ amidst noise and clutter.
Figure 32. Receiver operating characteristic for fluctuating targets at a $\beta_n = 12dB$ amidst noise and clutter.
Fig. 33, Fig. 34 and Fig. 35 give the $P_D$, $P_{FA}$, and ROC curves for the $\beta_n = 9dB$ case. The detection and false alarm rate and ROC performance of both the ROMP and CoSaMP sparse reconstruction algorithms continue to diminish as $\beta_n$ decreases in the presence of clutter and fluctuating targets. However, designed waveforms still are able to make detections with a detection rate of 28.9% using ROMP. In this instance, benchmark waveforms detect at a rate consistent of a random guess given this problem size. Furthermore, at $\beta_n = 9dB$, performance of the CS radar using designed waveforms paired with CAMP reconstruction also begins to diminish. For a $\tau_{th} = 0.1$, the a $P_D$ of 73.1% at a $P_{FA}$ of 4.38% was observed, suggesting a built-in resilience to relatively non-sparse radar scenes. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 18. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks.

Table 18. $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 9dB$ with noise and clutter for fluctuating (fluctuating reflection center magnitudes, only)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, S</td>
<td>ROMP</td>
<td>0.5</td>
<td>28.9%</td>
<td>2.92 $\times 10^{-1}$%</td>
</tr>
<tr>
<td>C, S</td>
<td>CoSaMP</td>
<td>0.5</td>
<td>20.3%</td>
<td>1.47 $\times 10^{-1}$%</td>
</tr>
<tr>
<td>C, S</td>
<td>CAMP</td>
<td>0.1</td>
<td>73.1%</td>
<td>4.38%</td>
</tr>
<tr>
<td>C_r, S_a</td>
<td>ROMP</td>
<td>0</td>
<td>6.9%</td>
<td>2.63 $\times 10^{-1}$%</td>
</tr>
<tr>
<td>C_r, S_a</td>
<td>CoSaMP</td>
<td>0</td>
<td>4.1%</td>
<td>1.44 $\times 10^{-1}$%</td>
</tr>
<tr>
<td>C_r, S_a</td>
<td>CAMP</td>
<td>0.18</td>
<td>42.9%</td>
<td>4.28%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>ROMP</td>
<td>0</td>
<td>4.1%</td>
<td>1.42 $\times 10^{-1}$%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>CoSaMP</td>
<td>0</td>
<td>2.3%</td>
<td>6.23 $\times 10^{-2}$%</td>
</tr>
<tr>
<td>C_r, S_r</td>
<td>CAMP</td>
<td>0.19</td>
<td>46.3%</td>
<td>4.35%</td>
</tr>
</tbody>
</table>
Figure 33. Probabilities of detection for fluctuating targets at a $\beta_n = 9dB$ amidst noise and clutter.
Figure 34. Probabilities of false alarm for fluctuating targets at a $\beta_n = 9dB$ amidst noise and clutter.
Figure 35. Receiver operating characteristic for fluctuating targets at a $\beta_n = 9dB$ amidst noise and clutter.
Table 19 depicts the reconstruction performance of individual sparse reconstruction algorithms. As before, the same \( P_D \) is selected for the designed waveform and receiver filter pair per each sparse reconstruction algorithm considered in this paper. It is shown that for the same, sample \( P_D \), CAMP minimizes \( P_{FA} \) for the lowest tested \( \beta_n \), consistent with previous test cases.

Table 19. Combined sparse reconstruction algorithm performance comparisons for \( (\hat{\mathbf{C}}, \mathbf{S}) \) at \( \beta_n = 15dB, 12dB, \) and \( 9dB \) with noise and clutter for fluctuating targets

<table>
<thead>
<tr>
<th>( \beta_n )</th>
<th>Algorithm</th>
<th>( P_D )</th>
<th>( P_{FA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15dB</td>
<td>ROMP</td>
<td>50.0%</td>
<td>( 3.66 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CoSaMP</td>
<td>50.0%</td>
<td>( 4.16 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>50.0%</td>
<td>( 3.65 \times 10^{-2} )%</td>
</tr>
<tr>
<td>12dB</td>
<td>ROMP</td>
<td>30.0%</td>
<td>( 2.86 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CoSaMP</td>
<td>30.0%</td>
<td>( 3.68 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>30.0%</td>
<td>( 3.23 \times 10^{-2} )%</td>
</tr>
<tr>
<td>9dB</td>
<td>ROMP</td>
<td>15.0%</td>
<td>( 4.58 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CoSaMP</td>
<td>15.0%</td>
<td>( 5.76 \times 10^{-2} )%</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>15.0%</td>
<td>( 3.69 \times 10^{-2} )%</td>
</tr>
</tbody>
</table>

As shown in this section, CS radar performance is degrades in the case of fluctuating targets amidst clutter, particularly at low \( \beta_n \) values. Choice of the sparse reconstruction algorithm for the CS radar system enables improved detection performance where before detection rates were unreliable. Paired with the novel design algorithm, and detection rates and false alarm rates continue to improve.

Table 20 gives the observed reconstruction errors for fluctuating targets amidst clutter. Results for ROMP, CoSaMP, and CAMP sparse reconstruction methods are presented. It is again noted that while reconstruction errors correlate with \( P_{FA} \) performance of sparse reconstruction algorithms relative to one another, they do not correlate with \( P_{FA} \) performance of designed versus benchmark waveforms within each reconstruction method tested.

5.2.4 MULTIPLE FLUCTUATING EXTENDED TARGETS WITH AMBIGUOUS REFLECTION CENTER RANGE

The following are the results given a radar scene comprised of fluctuating extended targets that now have reflection centers at ambiguous ranges (actual reflection centers vary slightly in
Table 20. Observed reconstruction error for fluctuating extended targets in noise and clutter for ROMP, CoSaMP, and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta, \beta_n = 9dB$</th>
<th>$\Delta, \beta_n = 12dB$</th>
<th>$\Delta, \beta_n = 15dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>C, S</td>
<td>2.61</td>
<td>1.52</td>
<td>0.97</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_a$</td>
<td>6.43</td>
<td>3.73</td>
<td>2.37</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>7.39</td>
<td>4.15</td>
<td>2.54</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>C, S</td>
<td>1.79</td>
<td>1.08</td>
<td>0.70</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_a$</td>
<td>4.31</td>
<td>2.55</td>
<td>1.66</td>
</tr>
<tr>
<td>CoSaMP</td>
<td>$C_r, S_r$</td>
<td>5.00</td>
<td>2.85</td>
<td>1.78</td>
</tr>
<tr>
<td>CAMP</td>
<td>C, S</td>
<td>8.89</td>
<td>5.01</td>
<td>3.06</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_a$</td>
<td>14.83</td>
<td>8.33</td>
<td>5.12</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>19.33</td>
<td>10.28</td>
<td>5.83</td>
</tr>
</tbody>
</table>

physical location) in addition to having an impulse response that varies exponentially about a deterministic mean, as before. Simulation parameters are given in Table 2. This scenario considers the case without clutter, at $\beta_n = 18dB$ and $21dB$, and using ROMP and CAMP sparse reconstruction techniques only. It is noted that the added complexity of uniformly distributed reflection centers across neighboring range bins challenges the CS radar to make detections and thus higher $\beta_n$ values are evaluated. Furthermore, CoSaMP reconstruction is left out of this analysis, as it has been demonstrated to perform consistently or slightly less optimally as compared to ROMP in the preceding sections in terms of $P_D$. These results extend previous work [37, 47, 48] where fluctuating targets only consisted of random target impulse response magnitudes and where their corresponding physical locations along the target were still known. This cases resembles the real world scenario of the CS radar being tuned for a particular set of targets where the actual target is the same but is rotated slightly, varying both the reflection center magnitude and the distance of the reflection center is away from the radar receiver.

Case I: With Noise and No Clutter

Fig. 36, Fig. 37, and Fig. 38 depict the $P_D$, $P_{FA}$, and ROC curves for a $\beta_n = 21dB$. Considering a $\tau_{th} = 0$ to maximize $P_D$, detection rates of 56.8% and 84.3% are observed for ROMP and CAMP methods, respectively. For both ROMP and CAMP, designed waveforms
depict greatly improved detection rates over benchmark waveforms. For CAMP, a 10.2% increase in the $P_D$ is realized. For ROMP, a 60.2% increase is noted. For CAMP, a false alarm rate of 11.4% is observed at this detection threshold. This, of course, can be controlled via $\tau_{th}$ but at the cost of $P_D$. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 21. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks. ROC curves appear less steep due to higher false alarm rates and reach a smaller maximum value compared to previous scenarios. This is a direct result of error introduced in the sensing matrix due to the presence of ambiguous, fluctuating targets. Still, reliable detections remain possible with this CS radar system when paired with an appropriate spare reconstruction algorithm and provided that the incurred $P_{FA}$ cost is acceptable.

**Table 21.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 21dB$ with noise and without clutter for fluctuating targets (fluctuating reflection center magnitude and location)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_r, S$</td>
<td>ROMP</td>
<td>0</td>
<td>56.8%</td>
<td>$4.09 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0</td>
<td>84.3%</td>
<td>11.4%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>26.4%</td>
<td>$2.64 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0</td>
<td>75.7%</td>
<td>10.2%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>20.6%</td>
<td>$1.44 \times 10^{-1}$%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0</td>
<td>70.0%</td>
<td>11.4%</td>
</tr>
</tbody>
</table>
Figure 36. Probabilities of detection for fluctuating targets at a $\beta_n = 21 dB$ amidst noise where reflection centers have an ambiguous range.
Figure 37. Probabilities of false alarm for fluctuating targets at a $\beta_n = 21dB$ amidst noise where reflection centers have an ambiguous range.
Figure 38. Receiver operating characteristic for fluctuating targets at a $\beta_n = 21dB$ amidst noise where reflection centers have an ambiguous range.
Fig. 39, Fig. 40, and Fig. 41 depict the $P_D$, $P_{FA}$, and ROC curves for a $\beta_n = 18dB$. As before, selecting a $\tau_{th} = 0$ to maximize $P_D$, detection rates of 44.3% and 78.8% are observed for ROMP and CAMP methods. Designed waveforms paired with ROMP display a 60.3% increase in the detection rate, whereas those paired with CAMP display 16.39% detection performance increase over the next best benchmark waveform, $(C_r, S_a)$ for both cases. A false alarm rate of 11.4% is observed at this detection threshold for CAMP and $4.15 \times 10^{-1}\%$ for ROMP. As before these can be adjusted via $\tau_{th}$ but at the cost of $P_D$. The steeper the $P_D$ curve, the more costly the adjustment is in terms of $P_D$ to minimize $P_{FA}$. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 22. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks.

Table 22. $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 18dB$ with noise and without clutter for fluctuating (fluctuating reflection center magnitude and location)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0</td>
<td>44.3%</td>
<td>$4.15 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0</td>
<td>78.8%</td>
<td>11.4%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0</td>
<td>17.6%</td>
<td>$2.65 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0</td>
<td>67.7%</td>
<td>10.2%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>10.4%</td>
<td>$1.37 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0</td>
<td>63.3%</td>
<td>11.4%</td>
</tr>
</tbody>
</table>
Figure 39. Probabilities of detection for fluctuating targets at a $\beta_n = 18dB$ amidst noise where reflection centers have an ambiguous range.
Figure 40. Probabilities of false alarm for fluctuating targets at a $\beta_n = 18dB$ amidst noise where reflection centers have an ambiguous range.
Figure 41. Receiver operating characteristic for fluctuating targets at a $\beta_n = 18dB$ amidst noise where reflection centers have an ambiguous range.
In Table 23, reconstruction performance of individual sparse reconstruction algorithms is depicted for the same $P_D$ and for the designed waveform and receiver filters.

**Table 23.** Combined sparse reconstruction algorithm performance comparisons for $\hat{C}, S$ at $\beta_n = 21dB$ and $18dB$ with noise for fluctuating targets in ambiguous range

<table>
<thead>
<tr>
<th>$\beta_n$</th>
<th>Algorithm</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21dB</td>
<td>ROMP</td>
<td>56.0%</td>
<td>$3.95 \times 10^{-1}%$</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>56.0%</td>
<td>$3.77 \times 10^{-1}%$</td>
</tr>
<tr>
<td>18dB</td>
<td>ROMP</td>
<td>44.0%</td>
<td>$3.98 \times 10^{-1}%$</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>44.0%</td>
<td>$6.62 \times 10^{-1}%$</td>
</tr>
</tbody>
</table>

Table 24 gives the observed reconstruction errors for fluctuating (reflection center magnitude and location) targets without clutter present. Results for ROMP and CAMP sparse reconstruction methods are presented. It is again observed that while reconstruction errors correlate with $P_{FA}$ performance of sparse reconstruction algorithms relative to one another, they do not correlate with $P_{FA}$ performance of designed versus benchmark waveforms within each reconstruction method tested. Reconstruction errors continue to diminish as $\beta_n$ increases. In this case, these results paired with the $P_D$ and $P_{FA}$ curves suggest that the reconstructed reflection coefficient matrix has entries that are close to zero.

**Table 24.** Observed reconstruction error for fluctuating extended targets with reflection centers in ambiguous range in noise for ROMP and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta$, $\beta_n = 18dB$</th>
<th>$\Delta$, $\beta_n = 21dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>$C, S$</td>
<td>0.54</td>
<td>0.40</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_n$</td>
<td>1.04</td>
<td>0.69</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>1.19</td>
<td>0.77</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C, S$</td>
<td>1.28</td>
<td>0.80</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_n$</td>
<td>1.99</td>
<td>1.18</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>2.57</td>
<td>1.45</td>
</tr>
</tbody>
</table>

**Case II: With Noise and Clutter**

The following are the results given a radar scene comprised of fluctuating extended targets that have reflection centers at ambiguous ranges in addition to having an impulse response that varies exponentially about a deterministic mean and amidst WSS Gaussian clutter with a $\beta_c = -6dB$ and located around targets as per Fig. 6. As before, simulation parameters are
given in Table 2 for $\beta_n = 18 dB$ and $21 dB$ and using ROMP and CAMP sparse reconstruction methods. These results continue to extend previous work [37, 47, 48] where clutter is now considered along with more complex fluctuating targets with reflection centers that vary in magnitude and location than that what the radar waveforms and receiver filters were designed for and what the receiver expects for sparse reconstruction.

Fig. 42, Fig. 43, and Fig. 44 depict the $P_D$, $P_{FA}$, and ROC curves for a $\beta_n = 18 dB$. As before a $\tau_{th} = 0$ is considered to maximize $P_D$. Detection rates of 28.5% and 69.9% are observed for ROMP and CAMP methods, respectively. For both ROMP and CAMP, designed waveforms depict greatly improved detection rates over benchmark waveforms. For CAMP, an 20.2% increase in the $P_D$ is realized and a 63.1% increase is observed for ROMP. A $\tau_{th} = 0$ yields a false alarm rate of $4.33 \times 10^{-1}\%$ and 10.3% for ROMP and CAMP, respectively. $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 25. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of ($\hat{C}, S$) to the ($C_r, S_a$) and ($C_r, S_r$) benchmarks. Amidst clutter, reliable detections remain possible with this CS radar system when an appropriate sparse reconstruction algorithm is used and with a high enough $\beta_n$.

**Table 25.** $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 21 dB$ with noise and clutter for fluctuating targets (fluctuating reflection center magnitude and location)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, S</td>
<td>ROMP</td>
<td>0</td>
<td>28.5%</td>
<td>$4.3 \times 10^{-1}%$</td>
</tr>
<tr>
<td>C, S</td>
<td>CAMP</td>
<td>0</td>
<td>69.9%</td>
<td>10.3%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0</td>
<td>10.5%</td>
<td>$2.59 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0</td>
<td>55.8%</td>
<td>10.2%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>7.6%</td>
<td>$1.92 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0</td>
<td>49.7%</td>
<td>10.8%</td>
</tr>
</tbody>
</table>
Figure 42. Probabilities of detection for fluctuating targets at a $\beta_n = 21dB$ amidst noise and clutter where reflection centers have an ambiguous range.
Figure 43. Probabilities of false alarm for fluctuating targets at a $\beta_n = 21dB$ amidst noise and clutter where reflection centers have an ambiguous range.
Figure 44. Receiver operating characteristic for fluctuating targets at a $\beta_n = 21dB$ amidst noise and clutter where reflection centers have an ambiguous range.
Fig. 45, Fig. 46, and Fig. 47 depict the $P_D$, $P_{FA}$, and ROC curves for a $\beta_n = 18dB$. Detection rates of 24.6% and 66.0% are obtained at a $\tau_{th} = 0$ for ROMP and CAMP, respectively. Designed waveforms show improved detection rates over benchmark waveforms. Designed waveforms using CAMP for sparse reconstruction afford a 63.1% increase in the detection rate over that using ROMP (and at the cost of $P_{FA}$). $P_D$ and $P_{FA}$ results for all tested sparse reconstruction algorithms and sample thresholds are given in Table 26. Like $P_{FA}$’s were selected per sparse reconstruction algorithm for performance comparison of $(\hat{C}, S)$ to the $(C_r, S_a)$ and $(C_r, S_r)$ benchmarks. Detections remain possible with this CS radar system in the presence of clutter with a high enough $\beta_n$. A $\beta_n = 18dB$ appears to approach a feasibility or practicality limit for the CS radar in this clutter environment and under these target conditions. Diminishing the $\beta_n$ too far beyond and reliable detections would not be possible.

Table 26. $\tau_{th}$-based $P_D$ and $P_{FA}$ results at $\beta_n = 18dB$ with noise and clutter for fluctuating targets (fluctuating reflection center magnitude and location)

<table>
<thead>
<tr>
<th>Filter, Waveform</th>
<th>Algorithm</th>
<th>$\tau_{th}$</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C, S$</td>
<td>ROMP</td>
<td>0</td>
<td>24.6%</td>
<td>$4.3 \times 10^{-1}%$</td>
</tr>
<tr>
<td>$C, S$</td>
<td>CAMP</td>
<td>0</td>
<td>66.0%</td>
<td>10.2%</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>ROMP</td>
<td>0</td>
<td>8.4%</td>
<td>$2.61 \times 10^{-10}%$</td>
</tr>
<tr>
<td>$C_r, S_a$</td>
<td>CAMP</td>
<td>0</td>
<td>52.6%</td>
<td>10.2%</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>ROMP</td>
<td>0</td>
<td>5.0%</td>
<td>$1.77 \times 10^{-10}%$</td>
</tr>
<tr>
<td>$C_r, S_r$</td>
<td>CAMP</td>
<td>0</td>
<td>47.3%</td>
<td>10.9%</td>
</tr>
</tbody>
</table>
Figure 45. Probabilities of detection for fluctuating targets at a $\beta_n = 18dB$ amidst noise and clutter where reflection centers have an ambiguous range.
Figure 46. Probabilities of false alarm for fluctuating targets at a $\beta_n = 18dB$ amidst noise and clutter where reflection centers have an ambiguous range.
Figure 47. Receiver operating characteristic for fluctuating targets at a $\beta_n = 18dB$ amidst noise and clutter where reflection centers have an ambiguous range.
In Table 27, reconstruction performance of individual sparse reconstruction algorithms is depicted for the same $P_D$ and for the designed waveform and receiver filters. It is shown that CAMP minimizes $P_{FA}$ for the sample $P_D$ under tested conditions.

**Table 27.** Combined sparse reconstruction algorithm performance comparisons for $(\hat{C}, S)$ at $\beta_n = 21dB$ and $18dB$ with noise and clutter for fluctuating targets in ambiguous range

<table>
<thead>
<tr>
<th>$\beta_n$</th>
<th>Algorithm</th>
<th>$P_D$</th>
<th>$P_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21dB</td>
<td>ROMP</td>
<td>28.0%</td>
<td>$4.01 \times 10^{-1}%$</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>28.0%</td>
<td>$2.11 \times 10^{-1}%$</td>
</tr>
<tr>
<td>18dB</td>
<td>ROMP</td>
<td>24.0%</td>
<td>$4.13 \times 10^{-1}%$</td>
</tr>
<tr>
<td>-</td>
<td>CAMP</td>
<td>44.0%</td>
<td>$2.24 \times 10^{-1}%$</td>
</tr>
</tbody>
</table>

Table 28 gives the observed reconstruction errors for fluctuating targets (reflection center magnitude and location) with clutter present. Results for ROMP and CAMP sparse reconstruction methods are presented. In this instance, it is again observed that while reconstruction errors correlate with $P_{FA}$ performance of sparse reconstruction algorithms relative to one another, they do not correlate with $P_{FA}$ performance of waveforms within each reconstruction method tested. Reconstruction errors continue to diminish as $\beta_n$ increases. As before, these results paired with the $P_D$ and $P_{FA}$ curves suggest that the reconstructed reflection coefficient matrix has entries that are close to zero, albeit those entries are slightly increased as compared to that where clutter is not present.

**Table 28.** Observed reconstruction error for fluctuating extended targets with reflection centers in ambiguous range in noise and clutter for ROMP and CAMP sparse reconstruction methods

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Filter, Waveform</th>
<th>$\Delta, \beta_n = 18dB$</th>
<th>$\Delta, \beta_n = 21dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROMP</td>
<td>$C, S$</td>
<td>0.81</td>
<td>0.68</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_a$</td>
<td>1.43</td>
<td>0.77</td>
</tr>
<tr>
<td>ROMP</td>
<td>$C_r, S_r$</td>
<td>1.81</td>
<td>1.42</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C, S$</td>
<td>2.13</td>
<td>1.66</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_a$</td>
<td>3.60</td>
<td>2.83</td>
</tr>
<tr>
<td>CAMP</td>
<td>$C_r, S_r$</td>
<td>3.79</td>
<td>2.75</td>
</tr>
</tbody>
</table>
5.2.5 MULTIPLE UNKNOWN EXTENDED TARGETS

The following are the results given a radar scene comprised of unknown extended targets. This case resembles the real world scenario of the CS radar being tuned for one target where a completely different target is present in the uncluttered scene. Simulation parameters are given in Table 3. This scenario considers the case without clutter, at $\beta_n = 30\, dB$ and using the ROMP sparse reconstruction technique. This choice was made to more precisely highlight what can be considered both a limitation and an advantage of the proposed CS radar system. As with ambiguous range, fluctuating targets, the added complexity of unknown targets challenges the CS radar system to make detections. These results extend previous work [37, 47, 48] where only fluctuating targets were considered, and extends those in [55] where transmit-waveforms and receiver filters were designed for one particular target, but a different target was present in the scene. In [55], the target impulse response was assumed known to the receiver for sparse reconstruction. However, this scenario removes that assumption and considers the case of a target that is unknown in waveform design and at the receiver.

Fig. 48 depicts the ROC curves for a $\beta_n = 30\, dB$, where otherwise near-optimal performance is expected in the case of known or fluctuating targets for ROMP [47, 48]. A flat ROC curve is observed. This implies that the CS radar cannot adequately reconstruct the scene without an accurate description of the actual sensing matrix, $\Phi$. While this is not ideal in the context of performing radar detections on any target present in the radar scene, regardless of its description in $\tilde{H}$, it highlights the uniqueness of the sparse solution (56) to (51). This implies that when detections are made, there is a reasonable certainty, an associated probability, that a particular detection corresponds to a particular target (and aspect) within $\tilde{H}$. It is noted that with ROMP, detections are possible but the results below suggest these detections would be consistent with random chance.
Figure 48. Receiver operating characteristic for unknown targets at a $\beta_n = 30dB$ amidst noise and without clutter.
Chapter 6

CS RADAR SYSTEM DESIGN

Based on the system model and detection results in high noise or clutter environments or those pertaining to fluctuating targets, the following steps are presented to design the posed CS radar system:

1. **Determine Priorities.** The first step in designing the CS radar system as proposed in this dissertation is to determine the performance goals for the radar itself. Priorities for the radar need to be set. In the given CS radar application, is \( P_D \) significantly more important than \( P_{FA} \)? Or, is the opposite true? Are \( P_D \) and \( P_{FA} \) of equal importance? These performance goals will guide specific design choices within the CS radar system as they relate to the engineering tradespace in prioritizing \( P_D \) or \( P_{FA} \) with tunable parameters.

2. **Define Targets.** The second step in designing the CS radar system is to determine the targets or types of targets the radar will be optimized to detect. Once determined, target frequency responses should be explicitly defined to form part of the overcomplete dictionary, \( \tilde{H} \). As it as been shown that fluctuating nature of targets affect detection and false alarm rate performance, \( \tilde{H} \) should include multiple aspects of the same target to improve the radar’s ability to detect at any aspect to the transmitter or receiver.

3. **Determine Radar Channel Characteristics.** Next, the sparsity of the radar channel should be evaluated. This evaluation should consider, at minimum, the presence or absence of clutter (weather, etc.), presence of undefined targets in the radar scene, and an estimated channel noise. An estimate of the inherent sparsity of the radar scene is important as it will inform selection of the sparse reconstruction method. As shown in Chapter 5, there are significant performance differences between sparse reconstruction methods under different channel and clutter conditions. While CAMP, for example, offers improved detection performance across all tested \( \beta_n \)'s, it comes at a cost: namely
an increased \( P_{FA} \). This cost might be acceptable given the design priorities of the radar determined in Step 1 or the potentially cumbersome nature the radar channel.

4. **Set Tunable Parameters.** Table 29 depicts the tunable parameters available as degrees of freedom in the CS radar design. These should be set to meet pre-determined performance priorities/ criteria of the CS radar.

<table>
<thead>
<tr>
<th>Table 29. Tunable design parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design algorithm stopping criterion: ( \varepsilon_1, \varepsilon_2 &amp; \varepsilon_3 )</td>
</tr>
<tr>
<td>Number of CS measurements: ( \Upsilon )</td>
</tr>
<tr>
<td>Number of transmit and receive elements: ( T, R )</td>
</tr>
<tr>
<td>Reconstruction Algorithm: (CAMP, ROMP, CoSaMP, etc.)</td>
</tr>
<tr>
<td>Sparsity of estimate: ( k ) (for ROMP, CoSaMP, etc.)</td>
</tr>
<tr>
<td>Detection threshold: ( \tau_{th} )</td>
</tr>
<tr>
<td>Operating frequency: ( f_c )</td>
</tr>
<tr>
<td>Transmit Power: ( P_t )</td>
</tr>
<tr>
<td>Antenna Gains: ( G_t, G_r )</td>
</tr>
<tr>
<td>Target frequency responses: ( \tilde{H} )</td>
</tr>
</tbody>
</table>

User selectable and tunable parameters to the posed novel MIMO CS radar system are further discussed below:

- \( \varepsilon_1, \varepsilon_2 \& \varepsilon_3 \) are tolerances used as stopping criterion in the design of \( \tilde{S} \) and \( \tilde{C} \). As these tolerances become small, so too do the off diagonal entries of the Gram matrix, \( G = \Phi^H \Phi = (\tilde{C} \tilde{H} \tilde{S})^H(\tilde{C} \tilde{H} \tilde{S}) \), minimizing the coherence of \( \Phi \). This comes at the cost of increased run time for the joint waveform- receiver filter design algorithm given in (Algorithm 4).

- \( \Upsilon \) is the number of measurements taken of the unknown \( \lambda_{r,d} \) (or, equivalently the length of \( y \) in (51)). This dissertation implemented a \( \Upsilon = \lfloor TL/2 \rfloor \). It is noted that the value of \( \Upsilon \) must, at minimum, exceed the intrinsic information of the unknown signal (at least \( \geq \) the quantity of targets) to enable adequate sparse reconstruction. The value of \( \Upsilon \) as it relates to the length of the reconstructed reflection coefficient matrix is one of the hallmarks and benefits to CS approaches. That is,
the radar scene can be accurately reconstructed with many fewer measurements that otherwise would be required (as implied by the Nyquist rate). $\Upsilon$ dictates both the undersampling ratio, $\delta_u$, and the relative sparsity, $\rho$, in the CS regime. The smaller the $\delta_u$ the fewer measurements are taken. A $\delta_u = 1$ implies that the number of measurements equals the length of the unknown signal. Diminutive undersampling ratios or large relative sparsities diminish performance, particularly as compared to that of matched filtering [23]. While the number of targets in the overcomplete dictionary may not be adjustable in design, $N_{aug}$ should be reduced to the minimum required to ensure full row rank of the sensing matrix as this too increases $\delta_u$. The effect of smaller or larger values of $\Upsilon$, $\delta_u$ or $\rho$ were not studied in this dissertation and are left for future work.

- The sparse reconstruction method, as demonstrated, directly impacts ROC results. Additionally, ROMP and CoSaMP have an additional tunable parameter, the sparsity of the estimate, $k$. This dissertation considered ROMP and CoSaMP algorithms with a defined $k$, meaning that each column of $\hat{\Lambda}$ has exactly $k$ non-trivial values. Consideration of these or sparse reconstruction methods with additional values for $k$ or a multitude of alternate sparse reconstruction algorithms tuned to produce non-sparse estimates is left for future work. It is noted, however, that increasing the value of $k$ not only increases the maximum allowable $P_{FA}$ but also gives the CS radar additional opportunities to make a correct detection. CAMP does not require $k$ to be specified and rather uses the complex soft thresholding function to generate the sparse estimate. Still, the CS radar system enables a maximum $P_{FA}$ to be set with the specification of the sparse reconstruction technique should this parameter be of importance. Furthermore, it is noted that setting $\tau_{th} = 0$ and controlling radar detection performance with $k$ would stand to increase the $P_{FA}$ forcing $k - \mathcal{H}_1^l$ decisions per receive sensor, even when no target is present.

- Next, $\tau_{th}$ directly correlates to $P_D$ and $P_{FA}$ performance. Too high a $\tau_{th}$ and the radar loses its ability to make detections but minimizes false alarm rate. Too low a $\tau_{th}$ and the $P_{FA}$ is maximized but detections are made more readily. These
results are codified in Chapter 5. To determine the best $\tau_{th}$, the received $\beta_n$ should be estimated for the most likely or most important targets defined in $\tilde{H}$ and simulations should inform best $\tau_{th}$ given each scenario (using $P_D$, $P_{FA}$, and ROC figures similar to those presented in Chapter 5). Alternatively, $\tau_{th}$ may be adapted based on the radar scene, forming a CFAR CS radar system. This is left for future work.

- Radar parameters such as $P_t$, $G_t$, $G_r$, $f_c$ should also be taken into account in design. $P_t$, $G_t$, and $G_r$ dictate the attainable receive $\beta_n$ given channel losses, system losses, and the target RCS, as per (50). It is noted that while increasing $P_t$ directly correlates to an increase in $\beta_n$, in practice, it also may increase $\beta_c$ and challenge sparse reconstruction by impeding sparsity too greatly. The operating frequency $f_c$ is another tunable parameter. This parameter dictates the type of radar (e.g. X-band, S-band, etc.) and specific use cases for the radar. The type and use of the radar may also dictate the type or strength of clutter source that the CS radar will encounter, informing the designer of the inherent sparsity of the scene or lack thereof.

- Lastly, the radar designer specifies the target dictionary $\tilde{H}$. It is noted, that while this work considered targets with 5 distinct reflection centers, previous work [54, 55] considered targets with 10 distinct reflection centers. Furthermore, this work did not consider design of $\tilde{H}$ to minimize $\mu(\Phi)$. It is left open whether the order of block target frequency responses is significant in $\tilde{H}$.

5. Design Waveforms and Filters. After the CS radar design parameters are determined, $\hat{C}$ and $\hat{S}$ may be designed according to Algorithm 4. Radar performance should be constantly re-evaluated and the parameters given in Table 29 should be consistently updated to ensure that CS radar performance criteria is maximized, given the radar scene. It is noted that cognition in the proposed CS radar system is left for future work.
Chapter 7

CONCLUSION

In this dissertation, a CS radar system was presented and simulated. Advancing the foundational work in [37, 47, 48], this dissertation expands the system model to include a commonly used transmit-waveform dependent clutter model adapted to the vector channel model presented in this paper. Clutter was then placed in the immediate vicinity of actual targets and bound by a $\beta_c$. In addition, target models were expanded to include the cases of fluctuating targets comprised of reflection centers of an ambiguous range and completely unknown targets. The joint waveform-receiver filter design algorithm was implemented in a novel CS radar system where a decision threshold was varied to minimize adverse performance impacts of clutter, additive noise, and target ambiguity. For sparse reconstruction, CoSaMP and CAMP were evaluated against ROMP in each extended target scenario. It was found that, as implemented, CoSaMP affords reduced peak $P_{FA}$’s whereas CAMP significantly improved $P_D$ performance as compared to ROMP. For CAMP in particular, $P_{FA}$ performance was greatly improved by increasing $\tau_{th}$ while sustaining improved $P_D$ performance.

Performance of designed waveforms and receiver filters in this novel radar system were compared to that of cubic-phase Alltop transmit-waveforms and random receiver filters and random transmit-waveforms and random receiver filters. Waveform and receiver filter design performance was calculated numerically in the mutual coherence calculation and averaged over 10,000 iterations. It was found that designed transmit-waveforms and receiver filters displayed improved mutual coherence values compared to benchmark, statically generated waveforms and that the radar’s ability to perform detections using each waveform-filter pair correlated the respective average mutual coherence values.

Over 6,000 iterations were run per $\beta_n$ per target scenario per the sparse reconstruction method, accounting for 270,000 total Monte Carlo simulations to support the findings of this work. Designed waveforms and receiver filters were shown to outperform statically generated waveforms and receiver filters in terms of $P_D$, $P_{FA}$, and the newly implemented ROC
analysis which is common to performance characterization of conventional radar systems in all tested sparse reconstruction methods. This held true for both known and fluctuating targets (reflection center magnitude and reflection center magnitude and range). Additional numerical results for reconstruction error were presented and correlated to observed $P_{FA}$ trends, the presence of clutter, and improved $\beta_n$'s.

Finally, this work concluded with discussion on CS radar system design for the proposed system. Tunable parameters were presented and their associated design benefit or consequence, highlighting an engineering tradespace in defining system parameters for the CS radar. Attributes such as the $\tau_{th}$ and sparse reconstruction method should be selected in concert with the $P_D$ or $P_{FA}$ performance requirements of the radar.

7.1 FUTURE RESEARCH

Numerous areas of future research were acknowledged throughout this dissertation. Of note, more work needs to be completed to fully comprehend the relationship between $\Upsilon$, $\tau_{th}$, and the sparsity level of matching pursuit algorithms as they relate to $P_D$, $P_{FA}$, and ROC performance in radar detection for this system model. For instance, Baraniuk [24] states that significantly fewer measurements are required for radar detection vice other modalities such as radar imaging. It would be interesting to rigorously investigate the trade off between $\delta_u$ and detection performance to observe how few measurements are actually required for sufficient performance. Left for future work is quantifying and observing radar performance degradation due to an increased $\mathbf{E}$, the error in the sensing matrix due to basis mismatch that may occur due to target fluctuations. Future work may also include more formal analysis of the sensitivity of the posed CS radar system to greater fluctuations in the location of reflection centers in target impulse responses to characterize radar performance for targets with reflection centers that vary between those considered Fig. 9 and those considered in Fig. 10. Additionally, this novel CS radar detection approach should be extended to include an additional dimension in the radar scene taking into account target or clutter motion, as it would be interesting to fit this CS radar approach to additional applications. While this dissertation and previous work [37, 47, 48, 54, 55] focused on the pairing of the joint waveform-receiver filter design algorithm with a bistatic radar, its extension to a multistatic
radar may improve the radar’s ability to distinguish clutter from targets by associating a confidence level or probability to targets or clutter detected at multiple, geographically dispersed receiver sensors. This way detections corroborated at multiple receivers would have a higher confidence measure than sporadic detections made at receivers due to noise or clutter. Finally, future research may include steps to make this CS radar system cognitive and able to adjust its parameters based on meeting certain performance criterion in a dynamic radar scene similar that in [23]. A near term example of such a mechanism may be the implementation of CFAR processing, as the advent of a variable $\tau_{th}$ provides the ability for the detection threshold to be varied according to the radar scene.
Appendix A

COMPRESSION SENSING

This appendix reviews compressed sensing and sparse reconstruction methods, which are vital for understanding the behavior, performance, and constraints of the proposed compressed sensing MIMO radar system.

A.1 MOTIVATION

Typical sampling methods require a sampling rate of at least twice the highest frequency present in the signal or image to avoid aliasing. This rate, commonly known as the Nyquist-Shannon sampling theorem is a fundamental requirement for perfect reconstruction for the set of all bandlimited functions. Its prevalence cannot be understated. From telephonic or compact disc audio to medical imaging, the Nyquist-Shannon sampling theorem is the guiding precept of so much of today’s technology [29].

However, perfect reconstruction of a signal is still possible with significantly fewer measurements if the signal retains certain properties, namely sparsity. Compressed sensing involves the recovery of a sparse signals from a limited number of observations and represents a method to both sense and compress data simultaneously by exploiting innate sparsity in certain datum [26,27,29,74].

While innately sparse signals are encountered in the physical world, oftentimes real signals are compressible. A compressible signal is made \( k \)-sparse if the magnitudes of individual elements of a signal decay rapidly and can otherwise be set to be zero after the \( k \) largest elements if those remaining elements are very close to zero [75]. This is precisely what is done in transform domain compression techniques such as JPEG where a sparsifying transform is first applied to image content (e.g. wavelet or direct cosine transform) and only the most significant coefficients are retained [76]. This practice underscores the idea that signals may not be inherently sparse but can be made sparse if represented in the appropriate basis, \( \Psi \).
### A.2 COMPRESSED SENSING

Given the non-sparse vector $x$ that has a sparse expansion in an orthonormal basis, it can be rewritten as $x = \Psi x_k$, where $\Psi$ is an $N \times N$ matrix and $x_k$ are the coefficients corresponding to columns of $\Psi$. In image compression, small coefficients in the vector $x_k$ are set to zero and only the $k$ most significant are retained, leaving an $k$-sparse vector. For the set of all compressible signals, the error given by, $\|x - x_k\|_{\ell_2}$, is small [29].

A signal is said to have a sparse representation if it can be expressed in more compact or succinct form (perhaps represented in appropriately chosen basis). In other words, the term *sparsity* characterizes the notion that certain subsets of signals are able to be represented in a form that is much less than what is required by the sampling theorem given its bandwidth (for continuous time signals) or length (discrete time signals) [29].

Given that:

$$y = \Phi x \in \mathbb{R}^M,$$  \hspace{1cm} (51)

where $\Phi \in \mathbb{R}^{M \times N}$ is as amalgamation of unit norm column vector elements, $[a_1, \ldots, a_N]$ and $x$ represents a vector of unknown scalar coefficients corresponding to each vector element in $\Phi$. In (51), $\Phi$ is commonly referred to as an $N$-element dictionary [3]. $y$ is a vector of measurements that are observed on the system $\Phi x$.

Define:

$$\text{supp}(x) := \{i \in [n] : x_i \neq 0\},$$  \hspace{1cm} (52)

where $n$ denotes the set $\{1, \ldots, n\}$. A vector, $x$ is said to be $k$–sparse if $k < n$ where $|\text{supp}(x)| \leq k$ [74].

If $M > N$ in the system described by (51), then the system contains more equations than coefficients and is therefore the system of equations is deemed to be *overdetermined* and can be trivially solved via least squares [3]. Stated explicitly,

$$\hat{x} = \arg \min_x \|y - \Phi x\|_2^2.$$  \hspace{1cm} (53)
For sparse representations where \( M < N \) or \( M \ll N \), (51) becomes an underdetermined problem. Here, there are fewer equations than there are unknowns, and \( \Phi \) therefore has a non-trivial null-space. This means there are infinitely many candidate solutions to the ill-problem posed in (51). However, the goal of sparse recovery in compressed sensing is to recover the sparse signal from an undersampled set of measurements where the sparsest signal that satisfies (51) is unique given sufficient sparsity in the unknown signal and incoherence in the sensing matrix. Exact reconstruction is possible provided that the signal of interest is sufficiently sparse and that the sensing matrix meets the restricted isometry property (RIP) [29, 77].

One approach to solving this underdetermined problem is to apply the least squares method as before, giving:

\[
\hat{x} = \arg \min_x \|y - \Phi x\|_2 = (\Phi^t \Phi)^{-1} \Phi y.
\] (54)

This gives the minimum energy solution. Unfortunately, while it does minimize energy, \( \ell_2 \) minimization does not make any guarantee of sparseness of a particular solution. Rather than finding a sparse solution (few large non-zero coefficients), performing \( \ell_2 \) minimization results in many small coefficients [3, 29]. Thus, an alternative approach to solving (51) might be to search the nullspace of \( \Phi \) for the solution which is sparsest. Stated formally, this would imply the following approach involving the \( \ell_0 \) (quasi-) norm:

\[
\hat{x} = \arg \min_x \|x\|_0 \text{ s.t. } y = \Phi x
\] (55)

While giving the most sparse solution to (51), this optimization problem represents a combinatorial search. It is therefore not solvable in polynomial time (\( NP \)-hard) [78] and has been further demonstrated to be difficult to estimate [79]. Ridge regression [80] offers another option for reliably determining a unique solution, \( \hat{x} \) to (51) but this estimate is generally non-sparse [74].

However, if the solution to (51) is sufficiently sparse and the sensing matrix satisfies RIP (necessary conditions), the sparse solution resulting from (55) is equal to that using \( \ell_1 \) minimization. Furthermore, the \( \ell_1 \) norm is convex. Otherwise referred to as Basis Pursuit
in [81], \(\ell_1\) minimization is in fact computationally tractable and solvable in polynomial time [3,82]. One particularly successful algorithm for solving (51) is Basis Pursuit Denoising (BPDN) (known also as the least absolute shrinkage and selection operator or LASSO [83]) given in its constrained form in (56) which is based on the \(\ell_1\) norm [23,47,81]:

\[
\hat{x} = \arg\min_x \|x\|_1 \text{ s.t. } \|\Phi x - y\| \leq \varepsilon
\]  

(56)

Alternatively BPDN can be expressed in its unconstrained, \(\ell_1\) penalized least squares form (Lagrangian formulation), given by (57) where (57), \(\lambda \geq 0\) is a tuning parameter.

\[
\hat{x}(\lambda) = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \|x\|_1
\]  

(57)

The solution given in (57) is sparse for the underdetermined case. This implies that compressed sensing offers a way to recover a length \(N\) sparse \(x\) from a length \(n\) vector of measurements, \(y\) where \(n < N\) in (51), enabling both sensing via dimensionality reduction and accurate reconstruction.

In addition to \(\ell_1\) minimization approaches such as Linear Programming (LP) algorithms, which themselves are solvable in polynomial time, greedy methods have also been introduced to solve the ill-posed inverse problem in (51) [84]. These methods pursue an adequate global solution via a series of locally optimal decisions [32]. These iterative algorithms afford a low storage cost and a significantly reduced computational complexity as compared to LP approaches, which become inordinately complex and expensive to solve for large problem sizes and thus also many practical applications. Message passing algorithms, such as Approximate Message Passing (AMP), provide similar performance guarantees as LP methods while also providing an equal sparsity-undersampling tradeoff [84,85]. A sampling of those algorithms implemented and evaluated in the pose MIMO CS radar system will be discussed in greater detail below.

Whereas, the requirement of sparsity has already been introduced, the other necessary condition for the application of compressed sensing yet to be discussed is incoherence. Incoherence is inferred by the RIP and is a property that resides in the sensing matrix, \(\Phi\).
A.3 RESTRICTED ISOMETRY PROPERTY

Candès and Tao specify a necessary condition for the sensing matrix, \( \Phi \), for successful sparse recovery, called the Uniform Uncertainty Principle (UUP) or synonymously, the RIP [86, 87]. The RIP is analogous to the UUP found in harmonic analysis and is a guarantee for sparse recovery [88]. It requires that every set of the columns of \( \Phi \) with cardinality less than \( \text{supp}(x) \) be generally orthogonal. This implies the need for incoherence between \( \Phi \) and \( \Psi \), the associated basis [27].

Stated formally, the isometry constant of sensing matrix \( \Phi \) is the minimum \( \delta_k \) s.t. (58) holds for the set of all \( k \)-sparse vectors \( x \) [29].

\[
(1 - \delta_k)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_k)\|x\|_2^2
\]

In it generally held that sensing matrix, \( \Phi \), satisfies the RIP of order \( m \) if \( \delta_k \ll 1 \). The RIP is similar to the Johnson-Lindenstrauss lemma [89] in its ability to preserve distances between points, meaning that \( x \) cannot be in the null space of \( \Phi \) [27]. (58) requires that every set of \( k \) columns in \( \Phi \) are nearly orthonormal (full orthonormality requires equal number of columns as rows in \( \Phi \)). This would imply that disparate \( k \) sparse vectors can be distinguished from their respective measurements \( y \) when the sensing matrix preserves their Euclidean lengths.

However, designing sensing matrices to explicitly satisfy RIP represents an NP-complete design problem for large sensing matrices. In many CS applications, random matrices with Independent Identically Distributed (IID) entries are used because they follow the UUP with high probability and are therefore incoherent with most basis matrices [64,90]. It was also found that partial Fourier, Bernoulli, or Reed-Muller code based matrices can be used in the measurement matrix for sparse recovery (via LASSO) [23,82,88,91].

RIP implies the following: The greater the degree of incoherence between the measurement matrix and associated basis, the better the recovery algorithm is able to accurately recover the sparse signal in question. This is the motivation behind the MIMO compressed sensing based joint waveform-receiver filter design algorithm in [47,48,54,55] and this dissertation.
A.4 SPARSE RECONSTRUCTION METHODS

Sparse reconstruction algorithms can be generalized into three broad categories, namely: convex relaxations, combinatorial algorithms, and greedy pursuits. Convex relaxation is the category of sparse recovery that encompasses the set of all minimization methods used to solve a convex program to approximate the sparse signal, including that presented by Candès and Tao [27]. They require very few measurements but suffer from being computationally exhaustive. Combinatorial algorithms implement group testing to reconstruct the sparse signal. Examples include Heavy Hitters on Steroids (HHS) pursuit and Fourier sampling. These algorithms, while very fast, require specialized samples that may not be practical nor feasible in the real world. Finally, greedy pursuits offer another alternative to convex relaxation. With close ties to approximation theory, greedy algorithms that approximate the sparse signal use a step-wise approach that makes locally optimal approximations of individual elements of the unknown signal at each iteration. They are computationally tractable and able to perform recovery in polynomial time because the complexity within each iteration amounts to solving a a least squares problem. Orthogonal Matching Pursuit (OMP) [92] is an example of a greedy algorithm, though its success is limited when partial Fourier matrices are used as measurement matrices. With greedy approaches, both computational complexity and sampling requirements fall in the middle of the convex relaxation and combinatorial sparse recovery types, respectively [75, 88, 93].

A.4.1 REGULARIZED ORTHOGONAL MATCHING PURSUIT

ROMP, a variant of OMP, as the name suggests, combines elements of greedy sparse recovery techniques with convex relaxation programs and with similar performance guarantees, given a suitable measurement matrix (satisfies RIP), measurement vector, sparsity of the approximation, and halting criterion. In the noiseless and clutterless case, ROMP is able to achieve exact recovery provided that RIP is satisfied with the parameters $(k, \delta_k) = (2k, 0.03/\sqrt{\log k})$ vice $(2k, \sqrt{2} - 1)$, which is what is otherwise required for exact recovery using convex relaxation. In [94], ROMP was used in a synthetic aperture radar to distinguish radar target signatures amongst clutter [88]. With ROMP, performance
degrades gracefully where noise and clutter are present in the radar scene and with the same bound for stability as convex approximations. Stability refers to the ability of the sparse reconstruction algorithm to reconstruct the unknown signal amidst noisy measurements. This property is highly desirable in CS applications such as that tested in this dissertation, where strong clutter is present in the radar scene [88]. The ROMP algorithm is summarized below as Algorithm 5 and derived formally in [88].

Algorithm 5 – Algorithm 5: ROMP Algorithm (Summarized)

1: Input: $y$, $\Phi$, $k$
2: Initialize: Initialize with measurement vector, index set $I$, and residual, $r = y$.
3: Identify: Select the set $J$ largest magnitude nonzero coordinates of the observation vector, $\hat{x} = \Phi^*r$ or all nonzero coordinates, whichever is smaller.
4: Regularize: Select the subset $J_0$ with maximal energy among all subsets of $J$ with similar coordinates
5: Update: Add $J_0$ to the index set and update $r$ according to:
6: $\hat{x} = \arg \max_{z \in \mathbb{R}^I} \|y - \Phi z\|_2$; $r = y - \Phi \hat{x}$.
7: For: $k$ iterations or until $\|I\| \leq 2k$.
8: Output: $\hat{x}$

A.4.2 COMPRESSIVE SENSING MATCHING PURSUIT

CoSaMP is best categorized as a greedy pursuit algorithm, though it incorporates some elements characteristic of both combinatorial algorithms and convex relaxation methods. As with ROMP, absent additive noise and clutter, CoSaMP is able to precisely recover $k$-sparse signals. However, CoSaMP achieves these similar results without the need for a logarithmic factor to be imposed in the RIP and therefore improves error bounds and reduces the stringency in RIP for exact recovery [88, 93] with performance guarantees analogous to that of Subspace Pursuit in [95]. CoSaMP was used in [96] to classify surface ships or submarines based on the sparse reconstruction of propeller tonals and a block version of CoSaMP was used in [97] to track multiple targets in an Orthogonal Frequency Division Multiplexing (OFDM) radar. Inputs to the CoSaMP algorithm are the same as what is required for ROMP. That is, a suitable measurement matrix (satisfies RIP), measurement vector, and sparsity of the approximation to be produced are all required [75]. The CoSaMP algorithm is described heuristically in Algorithm 6 but derived formally in [75].
Algorithm 6 – Algorithm 6: CoSaMP Algorithm (Summarized)

1: **Input:** $y$, $\Phi$, $k$

2: **Identification:** The algorithm is initialized with a trivial initial approximation of the sparse signal. This step forms a signal proxy based on the approximation and then calculates the residual and identifies its largest components.

3: **Support merger:** This step unifies the supports of the new and current approximations.

4: **Estimation:** Estimate signal with least squares to obtain values of for coefficients in merged support.

5: **Pruning:** The algorithm keeps only the $k$ most significant coefficients in the new approximation.

6: **Sample update:** Samples are updated to account for the new approximation of the sparse signal.

7: **Until:** Halting criterion is true. Halting criterion are defined further in [75].

8: **Output:** $k$-sparse estimate $\hat{x}$

Both ROMP and CoSaMP require a sparsity parameter as an input to the algorithm. It is understood that testing various sparsity levels can be tried so as to minimize the error $\|\Phi\hat{x} - y\|$ would not significantly increase runtime [88].

### A.4.3 COMPLEX APPROXIMATE MESSAGE PASSING

The CAMP recovery algorithm is a fast converging complex extension to the AMP algorithm, solving the complex valued LASSO (c-LASSO) problem posed in (57) where the $\|x\|_1$ term is comprised of both real and imaginary components. Although similar to AMP, the CAMP algorithm has unique features inherent to complex signals [98].

CAMP first adapted to CS radar design by Anitori [23] and is motivated by the fact that in many practical applications signals are complex in nature.

The Median CAMP algorithm is given in **Algorithm 7** [23,98,99]. Ideal CAMP [23] requires knowledge of the sparse unknown vector, $x$, in $y = \Phi x \in \mathbb{C}^m$ to calculate the standard deviation of the error signal, $\sigma_t = \hat{x} - x$. However, Median CAMP estimates $\sigma_t$ via $\tilde{\sigma}_t = \sqrt{\frac{1}{\ln2}\text{median}(|\hat{x}^t|)}$. The error of the median estimate is bounded by:

$$\frac{\hat{\mu} - \mu^*}{\sigma_*} \leq \frac{|\ln(1-\epsilon)|}{2\sqrt{\ln2}}.$$  

(59)
where \( \sigma_* \triangleq \lim_{t \to \infty} \sigma_t \), \( \sigma_t = \text{std}(w^t) \), and \( w^t = \tilde{x}^t - x \). In (59), \( \mu^* \) is the median of \( w^t \) and \( \hat{\mu} \) is the estimate \([23]\). As stated in (59), the upper bound remains independent of the locations of the non-sparse elements of \( x \). The inputs to CAMP are the measurement vector \( y \), sensing matrix \( \Phi \), and \( \text{tol} \). \( \text{tol} \) defines the acceptable mean square error (MSE) between estimates of the signal \( x \), \( \hat{x}^t \), at different iterations, \( t \), of the algorithm. \( \hat{x}^t \) is made sparse via the soft thresholding of the non-sparse estimate of \( x \), \( \tilde{x}^t \). The soft thresholding function that is often implemented in CAMP is the complex soft thresholding function \( \eta(u; \lambda) \) and is applied element-wise to the vector \( u \), where

\[
\eta(u; \lambda) \triangleq (|u| - \lambda)e^{j\angle u}1(|u| > \lambda),
\]

In (60), \( 1 \) is an indicator function and \( \angle \) denotes the phase angle.

\textbf{Algorithm 7 – Algorithm 7: Median CAMP Algorithm}

1: \textbf{Input:} \( y, \Phi, \text{tol} \) 
2: \textbf{Initialization:} \( \hat{x}^0 = 0, z^0 = y, t = 0 \) 
3: \textbf{Repeat:} 
4: \hspace{1em} \( t = t + 1 \) 
5: \hspace{1em} \( \tilde{x}^t = \Phi^H z^{t-1} + \hat{x}^{t-1} \) 
6: \hspace{1em} \( \hat{\sigma}_t = \sqrt{\frac{1}{ln2} \text{median}(|\tilde{x}^t|)} \) 
7: \hspace{1em} \( z^t = y - \Phi \hat{x}^{t-1} + z^{t-1} \frac{1}{2\hat{\sigma}_t} \left( \langle \frac{\partial u^R}{\partial x} (\tilde{x}^t; \hat{\sigma}_t) \rangle + \langle \frac{\partial u^I}{\partial x} (\tilde{x}^t; \hat{\sigma}_t) \rangle \right) \) 
8: \hspace{1em} \( \hat{x}^t = \eta(\tilde{x}^t; \hat{\sigma}_t) \) 
9: \hspace{1em} \textbf{Until:} \( \|\tilde{x}^t - \hat{x}^{t-1}\|_2 < \text{tol} \) 
10: \textbf{Output:} \( \tilde{x}, \hat{x} \)

Note that the implementation of CAMP sparse reconstruction into this CS radar system model also extends previous CS radar results in \([23]\), where CAMP was implemented in a model that considered point targets only, rather than extended targets as in this dissertation.
Appendix B

CLUTTER AUTOCORRELATION AND CROSS CORRELATION

B.1 PROOF THAT $R_\zeta$ IS DIAGONAL

Following in a similar method as used in [37] to describe the autocorrelation and cross correlation of noise, consider the vector of random variables comprised to the diagonal elements of $\Xi_\zeta$.

$$\Xi_\zeta = \begin{bmatrix} X_\zeta(f_1) \\ \vdots \\ X_\zeta(f_2) \\ \vdots \\ X_\zeta(f_K) \end{bmatrix}$$  \hspace{1cm} (61)

where the $K$ random variables in (61) correspond to $K$ samples of $X_\zeta(f) = \mathcal{F}\{x_\zeta(t)\}$ and $x_\zeta$ denotes the impulse response for $\zeta = 1$ clutter source of $\zeta = 1 \ldots Z_c$ total clutter sources present in the radar scene. It is assumed that clutter returns are independent from sensor to sensor and uncorrelated at distinct frequencies.

Given the cross-correlation,

$$\Xi_\zeta(i,j) = E\{X_{\zeta i}, X_{\zeta j}^*\} = E\{X_\zeta(f_i)X_\zeta(f_j)^*\}$$  \hspace{1cm} (62)

and given that clutter impulse responses, $x_\zeta(t)$ $\zeta = 1 \ldots Z_c$, are WSS Gaussian random processes with zero mean implies that

$$E\{X_\zeta(f_i)X_\zeta(f_j)^*\} = \delta_{ij}$$  \hspace{1cm} (63)
where $\delta_{ij}$ correspond to Kronecker $\delta$ operator defined with respect to $i, j$ so that:

$$
\Xi_{\zeta}(i, j) = E\{X_\zeta(f_i)X_\zeta(f_j)^*\}\delta_{ij}
$$

$$
= \begin{cases} 
E\{|X_\zeta(f_i)|^2\}, & i = j \\
0, & i \neq j, 
\end{cases}
$$

(64)

implies a diagonal correlation matrix for each clutter source, $\zeta = 1, \ldots, Z_c$ and 0 valued cross correlation of frequency responses at distinct antennas $(m, n)$ and frequencies $(f_i, \ldots, f_K)$, respectively. That is:

$$
R_{\zeta} = \text{diag}\{E[|X_\zeta(f_i)|^2], \ldots, E[|X_\zeta(f_K)|^2]\}
$$

(65)

and

$$
E\{X_m(f_i)X_n(f_j)\} = 0 \quad \forall \quad m \neq n \text{ and } f_i \neq f_j.
$$

(66)

Next, define the autocorrelation function of clutter source, $\zeta$ as $R_{\zeta} = E[x_\zeta(t)x_\zeta^*(t + \tau)]$. The PSD is then obtained after applying the Fourier Transform to $R_{\zeta}$, that is: $Q_{x_\zeta}(f) = \mathcal{F}\{R_{\zeta}(\tau)\}$.

As shown, $R_{\zeta}(\tau)$ depends only on time difference, $\tau \forall t, \tau$ affirming the conditions of wide sense stationarity. Now, assuming that $Q_{x_\zeta}(f_i)$ is flat (power is uniformly distributed across
all frequencies) and has the same autocorrelation function for all clutter sources, $\zeta = 1, \ldots, Z_c$ implies that:

$$Q_{x\zeta}(f_i) = \sigma_c^2 \quad \forall \quad \zeta = 1, \ldots, Z_c, f_i = f_1, \ldots, f_K, \quad (72)$$

where $R_{\zeta}(\tau) = \sigma_c^2 \delta(\tau) \quad \forall \quad \zeta = 1, \ldots, Z_c$. This affirms that clutter samples at any two different time instances are uncorrelated. Thus,

$$E\{|X_{\zeta}(f_i)|^2\} = Q_{x\zeta}(f_i) = \sigma_c^2 \quad \forall \quad f_i = f_1, \ldots, f_K. \quad (73)$$

Thus, the correlation matrix for each clutter source is a scaled identify matrix, $R_\zeta = \sigma_c^2 I_K, \zeta = 1, \ldots, Z_c$. 
Appendix C

CONSTANT-$\gamma$ CLUTTER

Echo strength from extended clutter is often described by the cross section per unit of intercepted area given by the clutter reflectivity, $\sigma^0$. $\sigma^0$ is typically independent of the breadth of the illuminated composite clutter source. $\gamma$ is the normalized reflectivity parameter that is used to characterize the echo signal return. It is derived from empirical data for various terrain types. $\sigma^0$ and $\gamma$ are related by:

$$\sigma^0 = \gamma \sin \psi,$$  

(74)

where $\psi$ is the grazing angle. This clutter model, referred to as the Constant-$\gamma$ model, is named as such because the $\gamma$ parameter remains constant for across the entire clutter source. For low grazing angles, constant-$\gamma$ clutter model is less accurate. Correction factors are introduced in the case of large bistatic angles.

$\gamma$ values are calculated for 3ft sea state, 5ft sea state, woods, metropolitan and rugged mountain for a default operating frequency of 10GHz in [100]. This paper considers an X-band radar with an operating frequency of 8GHz. Therefore, in order to correctly apply the clutter model, the $\gamma$ must be scaled according to:

$$\gamma = \gamma_0 + 5 \log \frac{f}{f_0},$$  

(75)

where $f_0 = 10GHz$, $f$ is the designated operating frequency of the radar system, and $\gamma_0$ is the empirical value of $\gamma$ calculated at $f_0$ [100, 101]. Table 30 provides $\gamma$ values for various terrain types at both $f_0 = 10GHz$ and $f = 8GHz$.

Constant-$\gamma$ clutter assumes that the terrain is homogeneous, free space propagation, stationarity in the clutter during the entire coherence time, the radar system maintains a constant speed and height, and that the radar is monostatic.

In [54, 55] the final constraint of this clutter model was circumvented via application of the Monostatic Bistatic Equivalence Theorem (MBET). It is noted that while the accuracy
Table 30. $\gamma$ values for various terrain types

<table>
<thead>
<tr>
<th>Terrain</th>
<th>$\gamma, f_0$</th>
<th>$\gamma, f = 8GHz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3ft Sea State</td>
<td>-40dB</td>
<td>-40.4846dB</td>
</tr>
<tr>
<td>5ft Sea State</td>
<td>-30dB</td>
<td>-30.4846dB</td>
</tr>
<tr>
<td>Woods</td>
<td>-15dB</td>
<td>-15.4846dB</td>
</tr>
<tr>
<td>Metropolitan</td>
<td>0dB</td>
<td>-.4846dB</td>
</tr>
<tr>
<td>Rugged Mountain</td>
<td>0dB</td>
<td>-.4846dB</td>
</tr>
</tbody>
</table>

of the MBET decreases for large bistatic angles, the composite clutter remains dependent on the transmit-waveform and power as is the case in the physical world.

The further consideration of constant-$\gamma$ clutter and MBET or other clutter models is beyond the scope of this dissertation and is left for future work.

C.1 MONOSTATIC-BISTATIC EQUIVALENCE THEOREM

The MBET approximates the bistatic radar cross section from the monostatic radar cross section and the bistatic angle, $\theta$. $\theta$ is the angle that lies between the clutter target centroid, the transmitter and the receiver. According to the MBET, the bistatic radar cross section can be estimated from the monostatic radar cross section as measured on the bisector of the bistatic angle and at a frequency scaled by $\cos(\theta/2)$. The key insight is that as the bistatic angle approaches zero, the radar becomes increasingly monostatic and the monostatic radar cross section becomes a more accurate characterization of the bistatic clutter [54,55,102].

C.2 PRELIMINARY RESULTS

In [54,55] the constant-$\gamma$ based model was applied in the bistatic radar system was applied via the MBET into the clutter response, $\sqrt{pH_\xi S_\xi} \Gamma$. It is noted that this too challenged the sparsity of the radar scene and thus, sparse reconstruction, as is the case with multiple localized scatterers (e.g. undesired targets). A randomly generated matrix defined $H_\xi$, consistent with the the system model and the target impulse response term. The constant-$\gamma$ model specified $\Gamma$. Together, these terms described the response of the clutter source (similar
to the model for targets in [44]). The radar scene was reconstructed using CoSaMP [54] and CAMP [55] sparse reconstruction algorithms and the performance of the joint waveform-receiver filter design in CS radar was compared to that of statically designed waveforms and receiver filters.
Appendix D

ADDITIONAL FIGURES

D.1 KNOWN TARGETS, NO CLUTTER

Figure 49. Probabilities of false alarm for known targets at a $\beta_n = 12dB$ amidst noise and without clutter.

Figure 50. Probabilities of detection for known targets at a $\beta_n = 12dB$ amidst noise and without clutter.
Figure 51. Receiver operating characteristic for known targets at a $\beta_n = 12dB$ amidst noise and without clutter.

Figure 52. Probabilities of false alarm for known targets at a $\beta_n = 9dB$ amidst noise and without clutter.
Figure 53. Probabilities of detection for known targets at a $\beta_n = 9dB$ amidst noise and without clutter.

Figure 54. Receiver operating characteristic for known targets at a $\beta_n = 9dB$ amidst noise and without clutter.
D.2 FLUCTUATING TARGETS, NO CLUTTER

![Figure 55.](image)

(a) ROMP, CoSaMP  
(b) CAMP

**Figure 55.** Probabilities of false alarm for fluctuating targets at a $\beta_n = 12dB$ amidst noise and without clutter.

![Figure 56.](image)

(a) ROMP, CoSaMP  
(b) CAMP

**Figure 56.** Probabilities of detection for fluctuating targets at a $\beta_n = 12dB$ amidst noise and without clutter.
Figure 57. Receiver operating characteristic for fluctuating targets at a $\beta_n = 12dB$ amidst noise and without clutter.

Figure 58. Probabilities of false alarm for fluctuating targets at a $\beta_n = 9dB$ amidst noise and without clutter.
Figure 59. Probabilities of detection for fluctuating targets at a $\beta_n = 9dB$ amidst noise and without clutter.

Figure 60. Receiver operating characteristic for fluctuating targets at a $\beta_n = 9dB$ amidst noise and without clutter.
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