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Comparative analysis of Bragg fibers

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Abstract: In this paper, we compare three analysis methods for Bragg fibers, viz. the transfer matrix method, the asymptotic method and the Galerkin method. We also show that with minor modifications, the transfer matrix method is able to calculate exactly the leakage loss of Bragg fibers due to a finite number of H/L layers. This approach is more straightforward than the commonly used Chew's method. It is shown that the asymptotic approximation condition should be satisfied in order to get accurate results. The TE and TM modes, and the band gap structures are analyzed using Galerkin method.

OCIS Codes: (060.3310) Fiber optics, (260.2110) Electromagnetic theory

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1. Introduction

The confined modes in a circular Bragg fiber with air core and periodic coaxial claddings were first analyzed by Yeh *et al.* [1]. This kind of fiber is of particular interest in guiding light through a hollow waveguide in UV and far-infrared regions [2]. Recently it has attracted much attention in many applications because of its extraordinary properties and significant experimental and theoretical studies. Unlike the conventional index-guiding fiber, light in Bragg fiber is confined in a low index core due to a photonic band gap (PBG) by Bragg reflection. The fundamental mode is a non-degenerate TE_{01} mode without azimuthal dependence. The TE_{01} mode has the lowest loss since the Fresnel reflection for TE components is larger than TM components, and the mode confinement is always the best in the case of TE_{01}. In addition, it effectively eliminates the polarization mode dispersion [3, 4]. By employing a hollow core, the absorption loss due to fiber materials is reduced significantly. However, the leakage loss introduced by imperfect PBG confinement or by a finite number of layers is not negligible. The leakage loss can be controlled by proper design of the structure, and it is largely different for different modes, which can effectively differentiate various modes [3, 4]. Hence asymptotic single mode propagation can be achieved in a Bragg fiber even with a very large air core [5].

The invention of perfect dielectric mirror [6-9] enables great improvements in the properties of Bragg fibers, especially in reducing the leakage loss by several orders of magnitude. By applying high index/low index (H/L) materials with a high index contrast (both refractive indices are much higher than the refractive index of air), one avoids the Brewster angle where no reflection occurs for TM components, and a complete band gap for all polarizations is formed. In the band gap, light with any polarization at any incident angle will be reflected perfectly. Such Bragg fibers, also called Omniguide fibers, have been fabricated successfully [6] and used for CO_2 laser guiding at 10.6μm [10], with a loss <1.0dB/m, which is much lower than the intrinsic material losses. Theoretical study shows that in hollow Omniguide fiber, the leakage loss can be reduced further than the lowest loss (0.2dB/km) in conventional glass fibers [5], leading to potential applications in long distance communications.

Several numerical approaches have been used to analyze the modal properties of Bragg fibers. Yeh *et al.* analyzed Bragg fibers successfully using transfer matrix method [1], where

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the Bragg modes were considered as quasi-modes with minimum radiation loss. In [11], the photonic band gap concept was used in the transfer matrix method. It obtained the band gap by searching for the fast increasing solutions. The increasing numerical errors make the field calculation very sensitive to the propagation constants. In [12-15], periodic alternate layers were approximated by planar Bragg stacks using asymptotic approximation of Bessel functions; therefore, Bloch theorem can be used to obtain an analytical eigen-equation. In Ref. [9], plane wave expansion for photonic crystal calculation was used along with the supercell concept. Since Bragg fiber is not a strictly photonic crystal, plane wave method is not quite suitable or efficient. In [16], we proposed a full vectorial Galerkin method to treat circular symmetric fibers with arbitrary index profile and the dispersion relations of TE modes in Bragg fibers were obtained.

In this paper, we briefly review the calculation methods for Bragg fibers including transfer matrix method, asymptotic method and Galerkin method. Even though Chew's method [17] has been prevalent in treating the leakage loss in Bragg fibers [4, 5, 18-22], we will show that the transfer matrix method can be modified for this purpose in a more straightforward way. Our work on Galerkin method is extended to include the calculations of the modes and the band gap structure.

2. Transfer matrix method

In each uniform cylindrical layer, the four field components (E_z, H_ϕ, H_z, E_ϕ) can be expressed as a linear combination of any two types of Bessel functions [1]. In matrix form, they can be expressed as:

$$\begin{bmatrix} E_z \\ H_\phi \\ H_z \\ E_\phi \end{bmatrix} = \begin{bmatrix} J_m(k_i r) & Y_m(k_i r) & 0 & 0 \\ \frac{i\omega\varepsilon}{k_i} J'_m(k_i r) & \frac{i\omega\varepsilon}{k_i} Y'_m(k_i r) & -\frac{m\beta}{k_i^2 r} J_m(k_i r) & -\frac{m\beta}{k_i^2 r} Y_m(k_i r) \\ 0 & 0 & J_m(k_i r) & Y_m(k_i r) \\ -\frac{m\beta}{k_i^2 r} J_m(k_i r) & -\frac{m\beta}{k_i^2 r} Y_m(k_i r) & -\frac{i\omega\mu}{k_i} J'_m(k_i r) & -\frac{i\omega\mu}{k_i} Y'_m(k_i r) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \quad (1)$$

where k_i and n_i are the transverse propagation constant and the refractive index of the *i*th layer, respectively. $k_i = k_0\sqrt{n_i^2 - \beta^2/k_0^2}$, k_0 is the wave vector in free space, β is the propagation constant and β/k_0 is the effective index of the mode; *A*, *B*, *C* and *D* are the coefficients, and *m* is the azimuthal modal number. When *m*=0, the modes are decoupled into two polarizations, TE and TM modes.

2.1 Bragg fibers with infinite number of H/L layers

If we assume the Bragg fiber has an infinite number of layers, the propagation constant β and the coefficients *A*, *B*, *C* and *D* are all real numbers, leading to perfectly confined modes. In each layer, the field is a standing wave and the power flux of the incoming and outgoing waves are equal [1].

The four field components are continuous across the interfaces. Using this boundary condition, a transfer matrix [**T**] for a β-ω pair (or λ-n_{eff}) is obtained to relate the coefficients in the innermost layer (*i*=1) and the outermost layer (*i*=*N*) [1, 11]:

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \begin{bmatrix} A_N \\ B_N \\ C_N \\ D_N \end{bmatrix} \quad (2)$$

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Giving arbitrary values to the coefficients for the 1st layer or the last layers, the coefficients for the last layer or first layer can be easily evaluated. It is also known that $B_1=D_1=0$ since $Y_m(r)$ goes to infinite at the origin.

Since the effective indices (n_{eff}) of Bragg modes are in the range of [0,1], for a given wavelength, all possible modes with n_{eff} in this range are evaluated. If the mode is inside the band gap, the mode increases rapidly due to the increasing numerical error.

As an example, we use the same parameters in [11]; the fiber has an air core with a radius of 5.0μm, refractive indices of the alternate layers are $n_H=2.0$ and $n_L=1.0$, along with widths $d_H=1.0$μm and $d_L=1.0$μm, respectively. Fifteen periods ($N=30$) are used in the calculation. To confirm if a state is inside the gap or not, we use the value of $\sqrt{A_1^2+B_1^2}\big/\sqrt{A_N^2+B_N^2}$ to ascertain whether the field increases fast. A rough value to compare with is $(n_H/n_L)^{N/2}$ for TE (~10^5) and a smaller number (~10^2) for TM, based on their different Fresnel reflection. The band gap and the Bragg modes for TE and TM polarizations are shown in Fig. 1. The curves with 'x' in Fig.1 are allowed modes in the cylindrical fiber that satisfy the condition of $B_1=0$ and $D_1=0$. The white area is the band gap. A TE_{01} and TM_{01} modes are supported in the fundamental gap, and one TM and three TE modes are supported in the second gap.

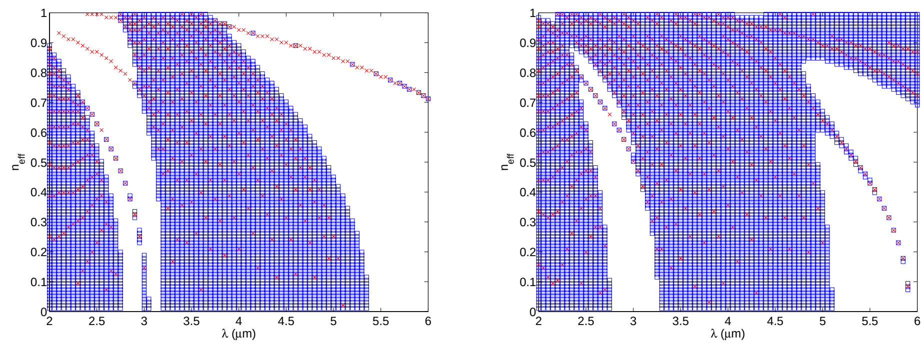

Fig. 1. Bragg modes and band gap structures calculated by transfer matrix method. Left: TE, Right: TM.

The calculation of mode field of the Bragg modes in the gap is sensitive to the propagation constants. In order to get a decaying mode field, the propagation constant should have a high accuracy, which can be obtained by finding the root to satisfy $B_1=0$ and $D_1=0$. When $k_0=1.2$, the effective indices are 0.7859081 for TE_{01} and 0.5270 for TM_{01}. The calculated fields for $k_0=1.2$ are shown in Fig. 2, which are normalized to its maximum. Both the effective indices and the mode fields agree well with results in [11][1].

[1] Note: In both Fig. 9 and 10 of [11], there are several minor mistakes: the solid lines are actually for $r^{1/2}H_z$, and dashed lines are for $r^{1/2}E_z$, and the β values given in [11] are actually the effective indices n_{eff}.

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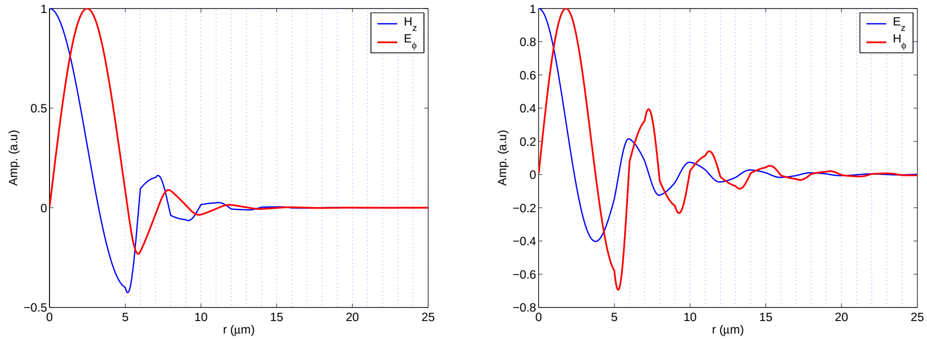

Fig. 2. TE_{01} and TM_{01} at k=1.2 in the Bragg fiber, calculated by transfer matrix method. Left: TE, Right: TM.

2.2 Bragg fibers with finite number of H/L layers

Bragg fibers, in practice, have a finite number of H/L layers, and the outermost layer is a large uniform cladding. In this case, the guiding modes are leaky since the confinement is not perfect, and leakage loss induced by the imperfect structure is a very important parameter for practical applications.

A complex propagation constant is introduced to take into account of the leakage loss. In most published works, Chew's method [17] was used [4, 5, 19, 20], where only E_z and H_z were used to track the field in each layer. A 2x2 reflection matrix and a 2x2 transmission matrix were used to relate adjacent layers. This approach is not so straightforward. Here we show that with minor modification of the transfer matrix method, the complex propagation constants can be calculated accurately.

All layers except the outermost cladding are treated the same way as in Eq. (1). The coefficients and propagation constants are not real numbers any more, indicating a nonzero net power flux across the interfaces. In the outermost cladding, there will be no incoming waves since there is no reflection from outside, and we rewrite the four field components equivalently in Hankel functions:

$$\begin{bmatrix} E_z \\ H_\phi \\ H_z \\ E_\phi \end{bmatrix} = \begin{bmatrix} H_m^I(k_i r) & H_m^{II}(k_i r) & 0 & 0 \\ \frac{i\omega\varepsilon}{k_i} H_m'^I(k_i r) & \frac{i\omega\varepsilon}{k_i} H_m'^{II}(k_i r) & -\frac{m\beta}{k_i^2 r} H_m^I(k_i r) & -\frac{m\beta}{k_i^2 r} H_m^{II}(k_i r) \\ 0 & 0 & H_m^I(k_i r) & H_m^{II}(k_i r) \\ -\frac{m\beta}{k_i^2 r} H_m^I(k_i r) & -\frac{m\beta}{k_i^2 r} H_m^I(k_i r) & -\frac{i\omega\mu}{k_i} H_m'^I(k_i r) & -\frac{i\omega\mu}{k_i} H_m'^{II}(k_i r) \end{bmatrix} \begin{bmatrix} A_N \\ B_N \\ C_N \\ D_N \end{bmatrix} \quad (3)$$

where $H^I = J + iY$ and $H^{II} = J - iY$. H^I and H^{II} represent the outgoing and incoming wave, respectively. A similar transfer matrix [**T'**] is obtained to relate the coefficients in the first and last layers:

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} & T'_{13} & T'_{14} \\ T'_{21} & T'_{22} & T'_{23} & T'_{24} \\ T'_{31} & T'_{32} & T'_{33} & T'_{34} \\ T'_{41} & T'_{42} & T'_{43} & T'_{44} \end{bmatrix} \begin{bmatrix} A_N \\ B_N \\ C_N \\ D_N \end{bmatrix} \quad (4)$$

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$$[T'] = [T] \times \begin{bmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{bmatrix} \tag{5}$$

where [T] is the transfer matrix obtained in the same way as in the case of infinite fiber.

Using $B_1=D_1=0$ and $B_N=D_N=0$, we obtain:

$$\begin{bmatrix} T'_{21} & T'_{23} \\ T'_{41} & T'_{43} \end{bmatrix} \begin{bmatrix} A_N \\ C_N \end{bmatrix} = 0 \tag{6}$$

The nontrivial solution condition is:

$$\det \begin{bmatrix} T'_{21} & T'_{23} \\ T'_{41} & T'_{43} \end{bmatrix} = 0 \, . \tag{7}$$

The complex propagation constant is the root of above equation. Since the imaginary part is much less than the real part, we expect that the real part of the propagation constant to be close to that of the infinite fiber. Using it as the initial value in a multi-variable root finder, for example, the '*fsolve*' function in MATLAB's optimization toolbox, the complex propagation constant can be calculated easily.

Using the example of Fiber A in [20], which has an air core (n=1.0) of radius 1.3278μm, followed by 16 pairs of alternating H/L layers (n_H=1.49, d_H=0.2133μm and n_L=1.17, d_L=0.346 μm), and an infinite medium of the higher index (n=1.49) as the outer cladding, we have obtained identical results as by Chew's method. At λ=1.0μm, the values of n_{eff} are $0.8910672175+1.422605\times10^{-8}i$ and $0.7920859031+1.819323\times10^{-3}i$ for TE_{01} and TE_{02} modes, respectively[2]. The loss in dB/m can be calculated from the imaginary part of the effective index using the relation [22]:

$$Loss = \frac{40\pi}{\lambda \ln 10} \operatorname{Im}(n_{eff}) \tag{8}$$

And the loss is 7.76387×10^{-1}dB/m for TE_{01} and 9.92897×10^{3}dB/m for TE_{02}.

3. Asymptotic method

According to Erdogan *et al*. in [3], TE/TM and other low order azimuthal hybrid modes are very close to modes in 1D planar stacks. Asymptotic method approximates the Bessel functions for the cladding layers with their asymptotic expressions [3, 14, 15, 23]; therefore , the periodic cylindrical claddings are approximated as planar Bragg stacks. In this approach, only the core and the first several layers are evaluated by analytical form, which is able to reduce computation time significantly. The advantage of this method is that it provides an analytical eigen-equation for the propagation constant in the case of air core Bragg fibers. The calculated results at k_0=1.2 (λ=5.2360μm) for the same Bragg fiber as in section 2.1 are: n_{eff}=0.79935 (TE_{01}), 0.57850 (TM_{01}).

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[2] Two extra digits are given here to differentiate from the results in Ref. [20]. The loss in dB/m for TE_{02} in Table 1a of Ref. [20] has a typo.

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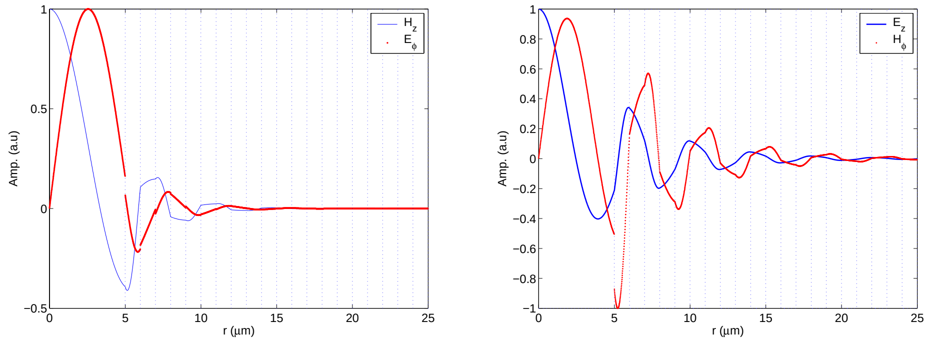

Fig. 3. Mode field of TE_{01}, TM_{01} at k=1.2 by asymptotic method. Left: TE, Right: TM.

The calculated mode fields are shown in Fig. 3. E_ϕ or H_ϕ from H_z or E_z has some discontinuities at the first several interfaces that might lead to incorrect results. In this example, n_L of the low-index material is close to the index of air core, and $k_i r$ in the first low-index layer is not large enough (~5 for TE mode) and asymptotic approximation of Bessel functions introduces an error of ~4%. When the asymptotic approximation condition is satisfied well, for example the Omniguide fiber with n_L much higher than the index of air, this method is generally able to provide reliable mode properties.

In Ref. [24], Xu *et al*. extended the asymptotic method to include the leakage loss. For the same Fiber A as in [20], the asymptotic method gives: n_{eff}=0.9091923789-6.86126×10^{-9}i and 0.7869166381-3.56600×10^{-3}i for TE_{01} and TE_{02} respectively. Though the real parts are close to the accurate values obtained by transfer matrix method, the imaginary parts have much large errors with even a negative sign. The same reason as mentioned above introduced this error.

4. Galerkin method

Galerkin method makes use of a set of orthogonal associated Laguerre-Gauss functions to approximate the guided modes, and it has been used to calculate LP modes in conventional circular fibers with small index differences [25-28]. Vectorial Galerkin methods using double series sine functions [29] or Hermite-Gauss functions [30,31] were employed to solve the transverse fields in fibers with large index differences in a two-dimensional Cartesian coordinate, however, these methods did not make use of the fiber's circular symmetry. In [16], we proposed a vectorial Galerkin method to analyze TE/TM modal properties in circular fibers with arbitrary index profiles. By adding an imaginary uniform cladding with index close to zero, the Bragg fiber was approximated as a conventional fiber with microstructures. The calculated TE modes in Bragg fibers were in good agreement with other methods. Here, we extend our work to include the modes and the band gap structure.

Assuming $E_\phi(r,\phi) = f(r)$ for TE modes, and $H_\phi(r,\phi) = g(r)$ for TM modes, the wave equations in cylindrical coordinate are [32]:

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + \left(k_0^2 n^2 - \beta^2 - \frac{1}{r^2}\right) f = 0 \tag{9}$$

$$\frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} + \left(k_0^2 n^2 - \beta^2 - \frac{1}{r^2}\right) g - \frac{d \ln n^2}{dr}\left(\frac{dg}{dr} + \frac{1}{r} g\right) = 0 \tag{10}$$

Since the Bragg modes decay fast in the first several layers, we terminate the periodic cladding after a finite number of layers with an infinite uniform cladding. The refractive index n_{cl} of the imaginary cladding is close to 0, so that all the modes with effective indices larger

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than n_{cl} will be confined by total internal reflection. Thus, we expect that it can obtain the gap structure in addition to the Bragg modes. Figure 4 shows the same Bragg fiber as in section 2.1 terminated by an imaginary cladding after 10 periods. The core radius is *a* and the core includes all microstructures. The index profile of the fiber is *n*(r). The refractive indices of the core and cladding are n_{co} and n_{cl}. n_{co} could be an arbitrary value different from n_{cl}.

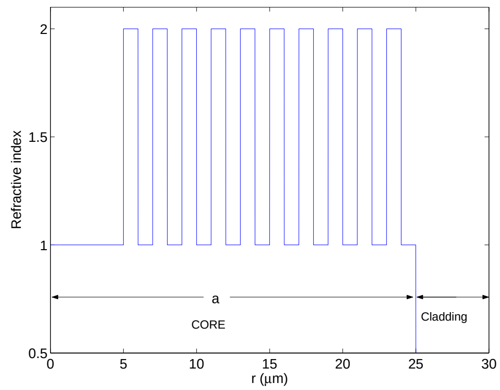

Fig. 4. The index profile of a Bragg fiber in Galerkin method.

We define several normalized parameters:

$$x = \sigma r^2 / a^2 , \quad h(r) = \frac{n^2(r) - n_{cl}^2}{n_{co}^2 - n_{cl}^2} , \tag{11}$$

$$V^2 = k_0^2 a^2 \left(n_{co}^2 - n_{cl}^2\right), \quad b = \frac{(\beta/k_0)^2 - n_{cl}^2}{n_{co}^2 - n_{cl}^2} \tag{12}$$

where σ is an arbitrary positive number, and *h*(r) is the normalized profile. *V* number and normalized propagation constant *b* are defined as in the case of conventional optical fiber. V^2 could be negative if n_{co} is chosen to be less than n_{cl}. h(r) is zero in the cladding region. The choice of σ may affect the computation and convergence. A larger σ leads to a longer computation time but a better convergence, therefore there is a tradeoff. σ is chosen to be *a* in our cases and a satisfactory speed and convergence are achieved.

Expanding the mode fields using orthonormalized m^{th} order (m=1 for TE/TM modes) associated Laguerre-Gauss functions [33],

$$f(x) = \sum_{i=0}^{N-1} a_i \varphi_i(x), \; g(x) = \sum_{i=0}^{N-1} b_i \varphi_i(x) \tag{13}$$

$$\varphi_i(x) = \sqrt{\frac{i!}{(i+m)!}} e^{-x/2} x^{m/2} L_i^{(m)}(x) \tag{14}$$

$$L_i^{(m)}(x) = \sum_{k=0}^{i} \frac{(i+m)!}{(i-k)!(k+m)!k!} (-x)^k \tag{15}$$

where φ(x) is the Laguerre-Gauss function, $L_i^{(m)}(x)$ is the associated Laguerre polynomial, and *N* is the number of Laguerre-Gauss functions used.

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The Galerkin method [16] transforms Eq (9) or (10) into a system of eigenvalue equations:

$$[\mathbf{M}][\mathbf{A}]=b[\mathbf{A}] \tag{16}$$

where [**A**] is the coefficient eigenvector and [**M**] is a square matrix with a dimension of NxN. For a given wavelength, the normalized propagation constants of all allowed modes are calculated. The profile dependent matrices are calculated only once for different wavelengths.

One important advantage of this method is its versatility in treating circular fibers with arbitrary index profiles, and it is not limited in treating step index profile. The calculated band gap structure and Bragg modes are shown in Fig. 5 using 300 functions. Both the gap and Bragg modes have a good agreement with the results from transfer matrix method (See Fig 1). When the mode is close to the cutoff, which is shown in the figure as the region of low effective index, the mode is close to the radiation mode of the introduced waveguide and Galerkin method gives complex propagation constants.

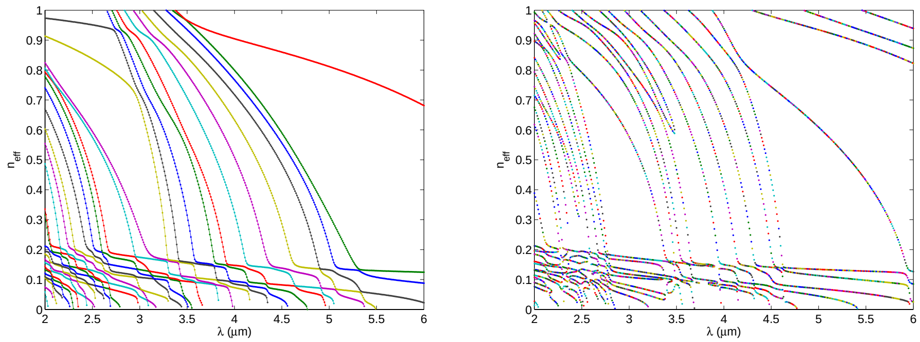

Fig. 5. Band gap and Bragg modes obtained by Galerkin method. Left: TE, Right: TM.

The mode fields for TE and TM modes at k_0=1.2 are also calculated as shown in Fig. 6. The TE mode matches very well with the result obtained using transfer matrix method. H_ϕ of TM mode also has a good match in the core and the first cladding, but it decays faster than it should. This is because H_ϕ is not smooth and the high frequency components are not fully included when using a finite number of functions.

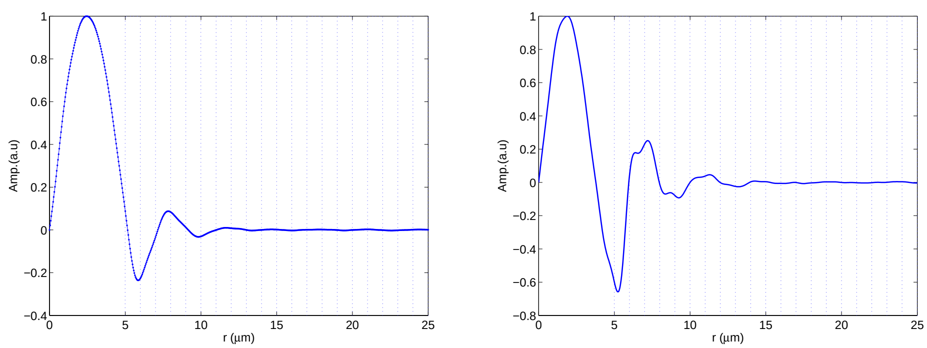

Fig. 6. Mode fields of Bragg mode by Galerkin method. Left: TE, Right: TM.

We illustrate the calculated effective indices of the TE/TM modes for the fiber in section 2.1 by these three methods in Table 1. The errors are evaluated with respect to the results by transfer matrix method.

Table 1. Comparison of calculated effective indices by three methods at k_0=1.2

	TE_{01}	Error	TM_{01}	Error
Transfer matrix method	0.7859080	-	0.5270	-
Asymptotic method	0.79935	1.7×10-2	0.5785	9.8×10-2
Galerkin method	0.7858	-1.4×10-4	0.5335	1.2×10-2

5. Conclusions

In conclusion, we have reviewed and compared the calculation methods for Bragg fibers. Transfer matrix method is most accurate, but it has no explicit form to obtain the propagation constants. In order to obtain the field distribution of Bragg modes, highly accurate propagation constants may be needed. Asymptotic method provides an explicit form for the propagation constants, and is generally stable if the asymptotic condition is satisfied. The transfer matrix method can be modified to calculate the leakage loss due to a finite number of H/L layers, which is more straightforward than the commonly used Chew's method. Galerkin method is generally stable and gives good results when the mode is away from the cutoff region. The main advantage of this method is that it is able to analyze circular fibers with arbitrary index profiles without modifications.

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