A QUALITY SYSTEMS ECONOMIC-RISK DESIGN THEORETICAL FRAMEWORK

by

Abdallah S. Thefeid
B.S. Industrial and Systems Engineering, King Fahd University
M.S. Engineering Management, King Abdulaziz University

A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

ENGINEERING MANAGEMENT AND SYSTEMS

ENGINEERING OLD DOMINION UNIVERSITY
August 2022

Approved by:

T. Steven Cotter (Director)

C. Ariel Pinto (Member)

Resit Unal (Member)

Stephen J. Fehr (Member)

ABSTRACT

A QUALITY SYSTEMS ECONOMIC-RISK DESIGN THEORETICAL FRAMEWORK

Abdallah S. Thefeid
Old Dominion University, 2022
Director: Dr. T. Steven Cotter

Quality systems, including control charts theory and sampling plans, have become essential tools to develop business processes. Since 1928, research has been conducted in developing the economic-risk designs for specific types of control charts or sampling plans. However, there has been no theoretical or applied research attempts to combine these related theories into a synthesized theoretical framework of quality systems economic-risk design. This research proposes to develop a theoretical framework of quality systems economic-risk design from qualitative research synthesis of the economic-risk design of sampling plan models and control charts models. This theoretical framework will be useful in guiding future research into economic-risk quality systems design theory and application.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my dissertation advisor, Dr. T. Steven Cotter, for his continuous support throughout my doctoral study and research. For his patience, help, and immense knowledge, I am extremely grateful. He provided me with the appropriate path and directions to start and complete my dissertation successfully. Under his supervision, I was able to overcome many difficulties and challenges to accomplish this tough task. I would like to thank him for his kind guidance, passion, and motivation, which have always inspired me to do my best. In addition, I would like to thank my dissertation committee members: Dr. C. Ariel Pinto, Dr. Resit Unal, and Dr. Stephen J. Fehr for their insightful comments, support, and encouragement.

I would like to thank my parent, Saleh and Mariam, and my brothers and sisters for their endless and unconditional love, support, and prayers. Without them, I would not be who I am today. I am grateful to my beloved wife, Hanadi Mahdi. She is my best friend, my soulmate, and the mother of my children. She always stood by me, with continuous love and support. She taught me lessons in patience, perseverance, and dedication. She is a great wife, great mom, and great friend. I cannot ask for more. To my daughters, Lana, Jwana, and Seba, I do love the way you color my life.

I would like to express my special thanks to my sponsor, the Government of Saudi Arabia, for its tremendous support throughout this journey, particularly to its representative here in the United States, Saudi Arabian Cultural Mission (SACM).

TABLE OF CONTENTS

Page

LIST OF TABLES vii

LIST OF FIGURES viii

Chapter

1. INTRODUCTION 1
    - 1.1 THEORITICAL FORMULATION 5
    - 1.2 PURPOSE OF THE STUDY 7
    - 1.3 PROBLEM STATEMENT 7

2. BACKGROUND OF THE STUDY 9
    - 2.1 REVIEW OF THE DESIGN LITERATURE 9
    - 2.2 LIMITATIONS OF EXISTING THEORY AND APPLICATION 17

3. METHODOLOGY 19
    - 3.1 VARIANCE COMPONENTS ANALYSIS 19
    - 3.2 POWER ANALYSIS 21
    - 3.3 MODELS ADMISSION CRITERIA 24
    - 3.4 RESEARCH SYNTHESIS 25
    - 3.5 EVALUATION AND VALIDATION 32

4. ANALYSES AND RESULTS 33
    - 4.1 ANALYSIS OF CONTROL CHARTS ECONOMIC-RISK MODELS 33
    - 4.2 ANALYSIS OF SAMPLING PLANS ECONOMIC-RISK MODELS 40
    - 4.3 INTEGRATED ANALYSIS OF ECONOMIC-RISK VARIANCE COMPONENTS EFFECT ON OC-CURVE 45
    - 4.4 EVALUATION OF INTEGRATED ANALYSIS OF QUALITY SYSTEMS ECONOMIC-RISK MODELS 74

5. DISCUSSION 76
    - 5.1 OVERVIEW OF FINDINGS 76
    - 5.2 RESEARCH IMPLICATIONS 77
    - 5.3 RESEARCH LIMITATIONS 78

6. CONCLUSIONS AND RECOMMENDATIONS 81
    - 6.1 PRIMARY CONTRIBTIONS OF THIS STUDY 81
    - 6.2 WIDENING THE SCOPE 81
    - 6.3 RECOMMENDATIONS FOR FUTURE RESEARCH 82

Page

REFERENCES 84

APPENDICES
A. QUALITATIVE ECONOMIC-RISK VARIANCE COMPONENTS ANALYSIS FOR CONTROL CHARTS DESIGNS 94
B. QUALITATIVE ECONOMIC-RISK VARIANCE COMPONENTS ANALYSIS FOR SAMPLING PLANS DESIGNS.. 114
C. THE CALCULATION OF POWER FROM QUALITY CONTROL CHARTS OC CURVE 122

VITA ……………... 132

LIST OF TABLES

Table Page

1. Research Synthesis Stages. 31

2. Admitted Papers of Control Charts Model. 35

3. Conceptual Coding of Control Charts Variance Components Effects. 36

4. Conceptual Coding of Control Charts Economic Effects. 37

5. Admitted Papers of Sampling Plans Model. 41

6. Conceptual Coding of Sampling Plans Variance Components and Economic Effects. 44

7. Conceptual Coding Based on Frequency of Occurrence of Integrated Variance Components. 46

8. Conceptual Coding Based on Frequency of Occurrence of Integrated Economic Effects. 46

9. Different Control Charts Economic Models 52

10. Effect Analysis of Quality Parameters to Power 53

11. The Comparison Analysis of the Signs of First Derivatives of ARL with Respect to n among the Different Studied Cases. 57

12. The Comparison of Fitting Sampling Plans OC Curve. 69

13. Assessment of the Proposed Theoretical Frameworks. 75

LIST OF FIGURES

Figure Page

1. Operating Characteristic (OC) Curve. 3
2. Hierarchical Bottom-up Approach for Integration Process. 29
3. The Admission Process of Control Charts Articles. 35
4. The Admission Process of Sampling Plans Articles. 42
5. Extracted Relationships of Control Charts Economic-Risk Design. 48
6. Extracted Relationships of Sampling Plans Economic-Risk Design. 48
7. OC-Curves for $\bar{X}$ Chart for Different Sample Sizes and Constant Control Limits Width. 54
8. OC-Curves for p-Chart for Different Sample Sizes and Constant Control Limits Width. 55
9. OC-Curves for $\bar{X}$ Chart for Different Control Limits Widths and Constant Sample Size. 56
10. OC-Curves for p-Chart for Different Control Limits Widths and Constant Sample Size. 56
11. CLD of Integrated Control Charts Economic-Risk Design. 58
12. The Ideal OC Curve of Sampling Plans. 62
13. The Effect of Changing Sample Size and Acceptance Number on the OC Curve (Constant Ratio). 63
14. The Effect of Changing Acceptance Number on Sampling Plans OC Curve. 63
15. The Effect of Changing Sample Size on Sampling Plans OC Curve. 64
16. AQL-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size. 66
17. LTPD-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size. 67
18. AOQL-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size. 68
19. CLD of Integrated AQL-Sampling Plans Economic-Risk Design. 70

Figure Page

20. CLD of Integrated LTPD-Sampling Plans Economic-Risk Design. 71

21. CLD of Integrated AOQL-Sampling Plans Economic-Risk Design. 72

CHAPTER 1

INTRODUCTION

Over its brief 100-year history, quality management systems have contributed to the enhancement of productivity through the reduction of internal and external quality costs. The essential tools of quality management systems are control charts and sampling schemes.

Control charts, also known as Shewhart charts, are simply statistical tools used to determine whether processes are in a stable state of control or out of control. Given stable in-control operation, control chart data can be used to predict process capability and the future performance of the process. When supplemented with out-of-control action plans (OCAPs), control charts enhance organizational decision making about the quality of its processes and products. Variables control charts are used when process or product characteristics are measured on a continuous scale, and attribute charts are used when process or product characteristics are measured as pass-fail conformance or as nonconformities counts per unit. In practice, the original control charts are the $\bar{X}$ chart, R chart, and s chart for continuous variable characteristics, the p-chart and np chart for pass-fail attributes, and the c-chart and u-chart for nonconformities counts per unit. For continuous characteristics, the $\bar{X}$ chart is used to control process or product characteristic location (centering), and either a R chart or a s chart is used to control the variation (spread). During the last half of the 20th century, the cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) control charts were developed to monitor small process trends and steps not efficiently detected by $\bar{X}$ control charts. The g and h control charts were developed to control nonconformity counts with non-Poisson, over dispersed or rare events distributions. Profile monitoring charts control critical-to-quality characteristics that are functionally dependent on one or more explanatory variables. Instead of observing a single measurement on each product sample, a set of measurements with values over a prescribed range take on a required product profile.

The T-square and generalized variance control charts are extensions of the $\bar{X}$ and *s* control charts to jointly control the location and variation of multiple variables joint performance, and the multivariate exponentially weighted moving average (MEWMA) is the natural extension of the univariate EWMA control chart to monitor joint trends and steps in multivariate characteristics. Multiple-stream control charts were developed to monitor and control the joint variation of multiple stream production processes. Principal components control charts were developed to monitor collinear, multivariate process, and product characteristics. Univariate and multi-variate autoregressive control charts extend EWMA control charts to processes that exhibit natural stable cyclic location (mean) variation over time.

The ability of all control charts to detect changes in product proportion nonconforming or the mean operating point or increased variance of the process statistic are described by their operating (OC) characteristic curves. A general OC curve for control charts is illustrated in Figure 1. All control chart OC curves are expressed as the beta probability, β, of failing to reject that a change in product proportion nonconforming, shift in the process mean, or increase in the process variance has occurred. The power of the test, $1 - \beta$, expresses the ability of a given control chart to detect a change in product proportion nonconforming or a change in the process mean or increase in the process variance given the change occurred. When product proportion nonconforming is operating at the expected population proportion nonconforming or process quality is operating at the expected population mean and variance, the beta error equals the alpha error. That is, $1 - \beta_0 = \alpha_0$. The allowable alpha error and process nonconforming, change in process mean, or increase in process variance is set based on economic-risk tradeoff analysis of rejecting that a change in the product proportion nonconforming, change in the process mean, or increase in the process variance has occurred when, in fact, no change has occurred. Once the alpha error and its process change point is set, the beta error and its process change point are set based on economic-risk tradeoff analysis of failing to reject that a change in the product proportion nonconforming or process mean or increase in the process variance has occurred when, in fact, it has occurred. Hence, two points are required to describe the discrimination of a control chart OC curve: the P(α error | allowable no change point) and the P(β error | detection change point). Once, P(α error | allowable no change point) is set, the sample size necessary to achieve the P(β error | detection change point) is determined.

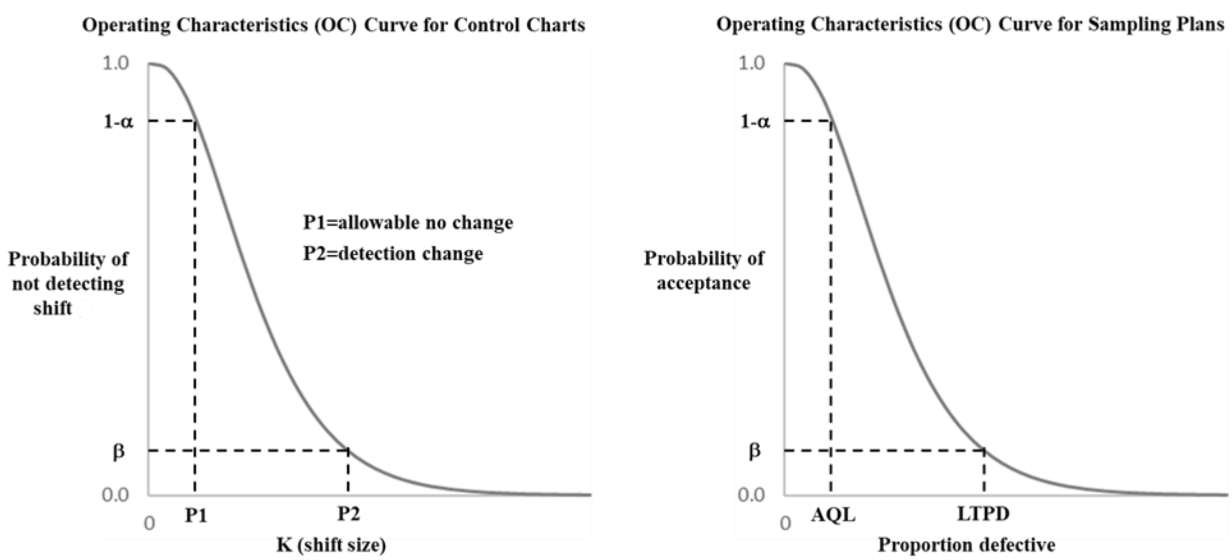

Figure 1. Operating Characteristic (OC) Curve.

On the other hand, acceptance sampling is used to disposition lots of product. The lot acceptance or rejection decision is dependent upon the sampling plan that specifies the estimated lot proportion nonconforming versus an *a priori* probability of acceptance (*Pa*) operating characteristic (OC) curve established by AQL-alpha and LTPD-beta economic-risk tradeoff probabilities (AQL = acceptable quality level, alpha = Type I error probability of rejecting an acceptable lot, LTPD = lot tolerance percent defective, and beta = Type II error probability of accepting a rejectable lot). Lot acceptance sampling systems OC curves' AQL-alpha point and LTPD-beta points are established by the economic-risk cost tradeoffs of rejecting truly acceptable lots of acceptably low proportion nonconforming product and failing to reject truly rejectable lots of unacceptably high proportion nonconforming product. A general OC curve for acceptance sampling plans is illustrated in Figure 1.

To minimize production and inspection costs, different types of sampling are available. Single, double, multiple, sequential, and continuous sampling plans may be designed to minimize average total inspection costs of unit-by-unit or lot production. Single sample acceptance plans, the most common and easiest to use, tests one sample from a lot. Double and multiple sampling plans are used when the results of the first sample are not conclusive regarding accepting or rejecting the lot. In sequential sampling plans, individual samples are taken from a lot in sequence. Alpha α and beta β error risks are controlled by the sequential probability ratio test (SPRT) developed by Wald (1947). Continuous sampling plans, invented by Dodge in 1943, provide a given Average Outgoing Quality (AOQ) when the production is continuous rather than in lots. If a sequence of *i* units inspected are found acceptable, inspection switches to inspection one out of a fraction *f* of units.

Typically, acceptance sampling is useful when surveillance testing is costly or destructive. Acceptance sampling, however, can be applied only for lot-by-lot disposition and not for the control of process or product quality. Conversely, control charts are used to maintain current control of a process or product's proportion nonconforming, location of its mean, or spread of its variance. The $1 - \beta$ discrimination power of both are determined by their respective OC curves.

1.1 THEORITICAL FORMULATION

In general, control chart economic-risk tradeoff models are formulated using a total cost per unit time function, which describes the relationships between the control chart design parameters, P(α error | allowable no change point) and the P(β error | detection change point), and the different types of costs. These costs are categorized as 1) the costs of sampling and testing, 2) the costs of investigation of an action signal and repair or correction of any assignable causes found, and 3) the costs of production of defective items (Montgomery, 1980). In general, the expected cost per unit time is modeled by

$$E(A) = \frac{E(C)}{E(T)} \tag{1}$$

when *E(T)* is the expected length of a sampling cycle and *E(C)* is the expected total cost incurred during a sampling cycle. Optimization techniques are applied to minimize *E(C)* per unit of *E(T)* for a specific control chart type ($\bar{X}$, *R*, *s*, *p*, etc.). Minor variations of equation (1) have been proposed by various authors. Some authors replace *E(T)* with *E(N)* the expected number of units produced during a cycle resulting in expected cost per unit.

Duncan (1956) set forth two theoretical models for the purpose of determining optimum control chart design. The first model allows the process to continue in operation during the search for the assignable cause, assuming the cost of repair (including possible shutting down of the process) and the cost of bringing the process back to a state of control after the discovery of the assignable cause will not be counted against the net income from the process. The second theoretical model allows the process to shut down immediately following the discovery of a point outside the control limits, and for the charging of the cost of repair to the net income from the process. Both models used the design criterion of maximizing the expected net income per sampling cycle.

A body of research focused in finding the optimum design of quality control charts followed Duncan’s (1956) model, which finds the optimum values of *n*, the sample size, *h*, the interval between samples, and *k*, the control limits for the $\bar{X}$ chart. The main purpose of calculating the model’s parameters based on these three variables is to determine the minimum economic-risk loss cost, which is equivalent to maximizing the expected net income per sampling cycle.

In sampling economic-risk modeling, numerous studies consider the problem of setting up an economical inspection plan whose purpose is the determination the optimum decision related to the detection of unacceptable levels of nonconforming products. The interest in the detection of unacceptable levels of nonconforming products is shared by two parties: the producer and the consumer (Dodge & Romig, 1928). Sampling inspection design seeks to minimize the average total inspection cost (ATI) given stated *a priori* set producer’s risk, α and AQL point and the consumer’ risk, β and LTPD point. Note that producer’s risk is defined as a Type I error, and consumer’s risk is defined as a Type II error.

For control chart design, the E[A] = E[C] / E[T] relationship incorporates the P(α error | allowable no change point) and the P(β error | detection change point) economic-risk 1 – β discrimination power of a control chart’s OC curve. For sampling plan design, minimizing ATI costs incorporates the AQL-alpha point and LTPD-beta point economic-risk 1 – β discrimination power of the sampling plan OC curve. The E[A] and ATI cost minimization perspectives can be used to develop a theoretical framework of economic-risk cost tradeoffs in the design of quality systems.

1.2 PURPOSE OF THE STUDY

The purpose of this research is to synthesize a general theory on the economic-risk design of quality systems from sampling and control chart economic design models as into a theoretical framework of quality systems economic-risk design useful in guiding future research into economic-risk quality systems design theory and application.

1.3 PROBLEM STATEMENT

Although 70 years theory specifying control chart and sampling economic design exists, there is no overarching theory of quality systems economic-risk design. Prior research into the economic design of quality control charts have yielded useful theoretical economic tradeoff models assuming constant alpha and beta errors. Likewise, almost 100 years of research into the economic design of sampling plans have yielded useful economic tradeoff models of sampling schemes and systems. Like the economic design of control charts, these economic tradeoff sampling plans, assume that an alpha error can be set, and a sufficiently large sample size can be attained to achieve the desired beta error. The designed alpha and beta errors were assumed to be held constant across resultant sampling schemes and systems. In practice, constant alpha and beta errors are rarely attainable due to external competitive environmental and intellectual property constraints, organizational policies and capabilities constraints, product and process technological constraints, and process logistical constraints. Hence, the problem quality systems design must be considered from the broader perspective of the economic-risk tradeoffs of nonconstant alpha and beta error risks in addition to the economic factors previously considered in the economic design of control charts and sampling plans. Further, research has not been conducted to synthesize the economic-risk design of control charts and sampling plans into a unified theoretical framework of the economic-risk design of quality systems. This yields the following research questions.

1. What economic-risk model structures are consistent among control chart economic-risk models and sampling plan economic-risk model?
2. What is the theoretically defensible synthesis of the consistent E[A] economic-risk design of control charts and the ATI cost minimization of sampling plans model structures into a theoretical framework of quality systems economic-risk design?

CHAPTER 2

BACKGROUND OF THE STUDY

2.1 REVIEW OF THE DESIGN LITERATURE

This chapter presents a review of various scientific publications in the field of quality systems economic-risk design using different methodologies and numerous applications. This review is divided into two sections. The first section presents a review of research into economic-risk design of quality control charts. The second section presents a review of research into economic-risk design of quality sampling plans.

2.1.1 Literature Review of Control Charts Economic-Risk Design

Montgomery (1980) published a literature review of the pioneering papers in the economic design of quality control charts. This literature review included papers by Girshick and Rubin (1952), Weiler (1952), Duncan (1956), Savage (1962), Bather (1963), Taylor (1965), Ross (1971), and White (1977). In Montgomery (1980), a body of significant studies were well reviewed covering different types of control charts. Moreover, he categorized the literature review in according to two main criteria, single assignable cause models and multiple assignable cause models, and the assumption of whether to continue or stop the process during the search for assignable causes.

Duncan (1956) pioneered the study of the economic design of $\overline{\mathrm{X}}$ charts. His simple and practical model determines the optimal design of an $\overline{\mathrm{X}}$ chart, proposing a procedure of how to determine the sample size n, the interval between samples h, and the control limits k that yield approximately maximum average net income per sampling cycle. Duncan's theory, which can be applied to any type of control chart such as an R chart, p-chart or c-chart, was the stimulus for various subsequent research in this area.

Bather (1963) used dynamic programming in his model that led to a functional equation which determined the optimal decision rule for overhauling the process. Taylor (1965) generated theory of control of replacement processes that provides a counter example showing the Girshick-Rubin solution to be in error. Ross (1971) urged for future research on Girshick-Rubin model under both an average and discounted cost criterion. White (1977) studied generalized formulation of modified versions of the quality control problems such as presented by Girshick and Rubin and Ross. He posited that this generalization of all previous studies of the production problem provided informative data to be considered in the decision-making process.

Ho and Case (1994) presented more detailed literature review of the papers developed in economic design of process control charts for the period from 1981 to 1991. Their summary not only focused on unified approaches to the economic design of process control charts but also presented computer programs used to solve different models for different situations. Both Montgomery (1980) and Ho and Case (1994) are excellent references in studying and reviewing the economic designs and applications of quality control charts model. Discussions of the advantages and disadvantages of using economic designs for control charts can be found in Woodall, Lorenzen, and Vance (1986).

Following the pioneering works, considerable attention was devoted to developing economic models of control charts. For instance, the $\bar{X}$ chart was considered in many studies intended to develop, extend, compare, review, and critique its economic design models. Recent studies (Alexander et al., 1995; Chen & Yeh, 2006, 2011; Prabhu et al., 1997; Surtihadi &

Raghavachar, 1994; Vommi & Seetala, 2007) proposed the optimum $\bar{X}$ control charts design assuming continuous process during the search for a single assignable cause.

On the other hand, Bai and Lee (1998), and Chen (2004) considered a single assignable cause model in their economic design of $\bar{X}$ control charts when the process is stopped during the search for the assignable cause. Economic design of $\bar{X}$ control charts when there are multiple assignable causes was presented by Chen and Yang (2002), assuming the continued processing while the search for the assignable cause and by Yang (1997), and Yu and Hou (2006), assuming the process was stopped while the search for the assignable cause.

Montgomery and Klatt (1972) presented a method to determine the optimal sample size, interval between samples, and critical region parameter for the Hoteling T^2 control chart. Their model is a multivariate analog of several well-known models for the univariate $\overline{X}$ chart. A production model for an np-chart was proposed by Chiu (1975) to control the number of defectives. The method led to an algorithm for the determination of the most economic control parameters. Additionally, Chiu's model can be applied to any type of control charts.

Saniga (1977) developed an expected cost model for a process whose mean is controlled by an $\overline{X}$ chart and whose variance is controlled by an R chart. This joint economically optimal design used a search procedure to determine the sample size, interval between samples and control limits for both charts that minimize the expected cost. The expected cost comprises the fixed and variable costs of sampling, the cost of investigating and correcting the process when the process parameters have shifted. Duncan (1978) Studied the economic design of p-charts with respect to the expected size of the process average shift.

Montgomery et al. (1995), and Serel and Moskowitz (2008) studied the economic design of the exponentially weighted moving average (EWMA) control charts. Both models were extended to economic-statistical design by adding constraints associated with in-control and out-of-control average run lengths. Including constraints on average run length of the model led to a decrease in the optimal sampling interval when a small shift in mean or variance occurred. Therefore, the economic statistical designs have good statistical performance.

The economic design of Cumulative Count of Conforming (CCC) control charts was presented by Xie et al. (1997). Yu and Wu (2004) presented an economic design for single variable sampling interval (VSI) moving average (MA) control charts. They concluded that the economic design of a VSI MA control chart performs better than the conventional fixed-sampling interval (FSI) scheme in terms of the loss cost. Besides, they noticed that there was no significant difference in the loss cost by applying two or more than three sampling-interval lengths in the VSI MA control charts.

Lorenzen and Vance (1986) considered a general process model that can apply to all control charts, regardless of the statistic used. In developing their model, they used in-control and out-of-control average run length instead of using the Type I and Type II error probability risks. Saniga (1989) presented an improved method to economically design control charts that have bounds on Type I and Type II error probabilities. It can be readily adapted to design any Shewhart-type control charts.

2.1.2 Literature Review of Sampling Plans Economic-Risk Design

In parallel to research in the economic-risk design of control charts, the pioneering work of Dodge and Romig (1928) initiated research into the economic-risk design of sampling plans, considering average outgoing quality limit (AOQL) and lot tolerance percent defective (LTPD). Comprehensive reviews were presented by Wetherill and Chiu (1975), and Wall and Elshennawy

(1989), covering various fundamental concepts, principles of acceptance sampling schemes, and economic aspect. Tippett (1958) presented a useful introduction of how to use acceptance sampling plan, discussing most economical sampling schemes such as operating characteristic (OC) curve. Savage (1962) presented a detailed discussion of an economic model for the surveillance of a production process and optimal policies for making inspections of and adjustments to the process. He developed models and mathematical procedures for handling the problem of determination of qualitative and quantitative properties of the optimal strategy for surveillance procedures and rules for deciding when to make repairs.

Using economic design, remarkable research studying different types of sampling plans based on attribute parameters were presented. Wetherill (1960), Hald (1960, 1965, 1967, 1968, and 1971), and Guenther (1971) used single sampling attribute plan in their models. Govindaraju and Bebbington (2015) considered the case of isolated lot inspection and examine the consumer risk, economic sample design, and errors in the inspection process. Bayesian design was used in modeling the consumer's case for zero number acceptance sampling. An economic, multi-attribute acceptance sampling model was developed by Schuler (1967), Hald and Keiding (1972), Schmidt and Bennett (1972). In their model, Schmidt and Bennett determined the expected total cost of quality control per lot under the assumption that rejected lots are scrapped. The most economical sequential sampling design was developed jointly by Wetherill (1959), Cox (1960), and Wortham (1971).

On the other hand, the economic variables sampling plans were studied in many research papers. Campling (1968) performed serial sampling acceptance by variable approach. Schleifer (1969) and Dayananda and Evans (1973) proposed economic schemes for multiple acceptance sampling plans with known variables. A double acceptance sampling plan with known parameters was developed by Aslam and Jun (2010). In their model, the minimum sample sizes of the first and second samples were determined to ensure that the true median life is longer than the given life at the specified consumer's confidence level. Moreover, the minimum such ratios were obtained to lower the producer's risk at the specified level.

A body of research that considers both attribute and variables data in modelling sampling plans has been published. Examples of this approach can be found in Johansen (1970), Wetherill (1961), Hald and Keiding (1969), Stange (1963), Pendrill (2008), Li et al. (2011), and Klufa (2010, 2014). Furthermore, the economic design of sampling plans that includes various types of sampling was developed. Stephens and Dodge (1976) compared chain sampling plans with single and double sampling plans. In their study, the effect of changing some model's parameters were discussed.

Using single sample, Fink and Margavio (1994) developed economic models to examine the profitability of different inspection policies and to determine the preferred 100 percent inspection plan or acceptance sampling plan. Besides, these models employed the quadratic loss function to represent the economic cost of quality from external failures. Pendrill (2008) introduced and exemplified the inclusion of cost models in sampling when using inspection by variable and attribute. He illustrated optimum strategies for the supplier in terms of minimizing production and testing costs, while at the same time maintaining satisfactory levels of customer satisfaction.

Manikandan et al. (2009) used quality function deployment (QFD) to design an economical sampling plan, based on customer demands and involving all members of the producer or supplier organization. The proposed QFD matrix, easily extended to include further customer requirements from a quality improvement program for effective implementation, provided a more coherent and consistent approach to quality improvement in production systems.

Nezhad and Yazdi (2014) designed an economic model to determine the optimal sampling plan which minimizes the producer's loss plus the consumer's loss, while satisfying both the producer's and the consumer's risk requirements. The proposed sampling plan was based on the Markov modeling. A continuous Taguchi loss function was used to obtain loss of deviations between the value of quality characteristics and its target. An optimization model was developed for obtaining control tolerances and the corresponding critical acceptance and rejection thresholds based on the geometric distribution, which minimizes the loss function for both producers and consumers.

Various statistical schemes contributed to economic design of sampling plans. Baklizi and El Masri (2004) studied the acceptance sampling plans using the Birnbaum-Saunders model. They found the minimum values of sample size which satisfies inequality along with the probability of acceptance and the minimum ratio of true median life to specified median life.

Aslam and Shahbaz (2007) constructed economic reliability plans using the generalized exponential distribution. They concluded that the reliability test plans obtained from the GED are economically best as compared to the plans obtained from the log-logistic distribution in terms of saving time and energy. Aslam and Kantam (2008) developed the reliability acceptance sampling plan based on truncated life test, assuming the lifetime of a product follows the Birnbaum-Saunders (BS) distribution. They found the termination ratio for various values of producer’s risk, acceptance number, and sample size. Their proposed plan was compared with the acceptance sampling plans given by Baklizi and EI Masri (2004). The proposed plan was useful in minimizing the producer’s risk as well as saving the time and cost of the experiment to reach the final decision about a lot of the product. Aslam (2008) also developed the reliability acceptance sampling plan, assuming the lifetime of a product follows the generalized Rayleigh distribution with known value of the shape parameter. His proposed plan was compared with the acceptance sampling plans given by Tsai and Wu (2006), and the termination ratio of the proposed test plans was found to be smaller.

Rao (2010) proposed a group acceptance sampling plans (GASP) based on truncated life tests for Marshall-Olkin extended Lomax distribution with known shape parameter. He found the minimum number of groups required decreased as test termination time multiplier increased, and the operating characteristics values increased more rapidly as the quality improved. GASP can be used when a multiple number of items at a time are adopted for a life test, and it might be beneficial in terms of test time and cost when a group of items are tested simultaneously. A group acceptance sampling plan was developed by Rao (2009) based on truncated lifetimes when the lifetime of an item follows a generalized exponential distribution. His model calculated the values of operating characteristic function for various quality levels as well as the minimum ratios of the true average life to the specified life at given producer's risk.

Mughal et al. (2010) developed economic reliability test plan (ERTP) for a truncated life test when the lifetime of an item follows Marshall-Olkin extended Lomax distribution. The minimum termination time was found, for given sample size, acceptance number and producer’s risk. The proposed plan was compared with Rao (2010), and it was found that the proposed plan was more economical in the sense of saving cost, time, and energy.

Fernández and Pérez-González (2012) presented a procedure to incorporate prior information on the proportion of nonconforming units when constructing optimal failure-censored sampling plans for log-location–scale lifetime distributions. The proposed method of determining optimal reliability sampling plans has the advantages of familiarity and applicability and allows the analyst the flexibility to delimitate the range of nonconforming units and to include prior neutrality between producer and consumer.

Al-Nasser and Al-Omari (2013) considered exponentiated Fréchet distribution as a model for a lifetime random variable when the life test is truncated at a pre-assigned time. They found a minimum sample size necessary to assure a certain average life when the life test is terminated at a pre-assigned time and when the observed number of failures does not exceed a given acceptance number.

Fernández (2017) considered sampling inspection based on Poisson defect counts. He used a risk management approach to determine the defects-per-unit inspection scheme with lowest conditional cost value-at-risk and controlled producer and consumer risks with the aim of reducing the risk of incurring an excessive cost. The proposed perspective was intuitive and clearly useful to engineers from an economic viewpoint.

2.2 LIMITATIONS OF EXISTING THEORY AND APPLICATION

Since 1928, quality systems economic-risk models have been developed to ensure minimizing loss as well as mitigating the economic risk. Although various research studies have been conducted that consider both quality control charts theory and sampling plans, the two correlated theories are handled separately. Numerous methodologies were involved in modeling both quality tools but in individual research papers. For instance, a Bayesian approach was used by Girshick and Rubin (1952), Bather (1963), Taylor (1965), and Carter (1972) in their optimization modeling process of quality control charts theory. While it was used in modeling sampling plans by Wetherill and Chiu (1975), Wall and Elshennawy (1989), Chien et al. (2000), Govindaraju and Bebbington (2015). Another point of view, Girshick and Rubin (1952), Bather

(1963), Taylor (1965), Ross (1971), and White (1977) have performed a Markovian approach to develop control charts as well as Savage (1962) theory of sampling plans.

None of these prior significant research works considered studying both quality system methods as a unified concept. Consequently, there has been no research that performed a specific methodology for both quality systems techniques by the same author. Both models were studied by Tagaras, but in separate papers. For instance, control charts' economic-risk design was studied by Tagaras (1989, 1994). On the other hand, Tagaras (1994) conducted research on sampling plans economic-risk design.

Fortunately, the existence of these two bodies of knowledge enhances the empirical objective of this research. Research into integrating the economic design of sampling and control chart theories into a unified theoretical framework of the economic-risk design of quality systems will contribute to a more comprehensive and holistic understanding of quality systems engineering and management.

CHAPTER 3

METHODOLOGY

This methodology integrates statistical variance components analysis (Searle et al., 1992) within the research synthesis framework (Cooper & Hedges, 1994) to decompose the economic-risk components of control chart and sampling models and qualitatively synthesize these into a general framework of quality systems economic-risk design. In the historical context, estimating predictor variance components provides information on the Pareto important predictors and their structure that determine response variable(s) variances.

3.1 VARIANCE COMPONENTS ANALYSIS

Variance components analysis is rooted in Fisher's (1925) work in developing a quantitative theory of genetics. In his book, *Statistical Methods for Research Workers*, Fisher established the basic method of estimating sources of error. Fisher developed the analysis of variance (ANOVA) method to sort inheritance of traits from random mutations. For each $y_{i,j}$, the j^{th} observation for the i^{th} group, the fundamental variance components model is

$$Var\left[y_{i,j}\right] = Var\left[\mu + \tau_i + e_{i,j}\right]$$

$$E\big(MS(y)\big) = n\sigma_\tau^2 + \sigma_e^2$$

where μ = grand mean with Var (μ) = 0, τ_i = the effect of the j^{th} group with Var(τ_i) = σ_τ^2, and $e_{i,j}$ residual random error about the mean of each j^{th} group assuming Cov(σ_τ^2, $\sigma_{i,j}$) = 0. Tippett (1931, 1943) applied the concept of effects and error estimates to the problem of allocating samples to higher-order models. Daniels (1939) applied the variance components method to modeling product quality variance as a function of between-machines variance plus order-of-units variance plus residual random error variance. Anderson and Bancroft (1952) applied variance components decomposition to experimental least squares regression models. Graybill (1954) and Graybill and Wortham (1956) demonstrated that variance components models are minimum variance and unbiased, and Graybill and Hultquist (1961) demonstrated that variance components models are minimum variance quadratic unbiased.

Henderson (1953) applied variance components to mixed models – those having fixed and random effects – with unbalanced data to estimate heritability defined by

$$4\sigma_G^2/(\sigma_G^2 + \sigma_E^2)$$

where σ_G^2 are genetic variance components and σ_E^2 are environmental variance components. Henderson developed the principle of equating sum of squares to their expected values which (1) are equivalent to ANOVA sums of squares for balanced data, (2) are adaptation of ANOVA sums of squares, or (3) arise from fitting sub-models. Henderson's work represented a major step forward to generalizing the variance components method. Henderson's methods produce unbiased estimators, but sampling variances are complex functions of the respective combination of unbalance in sampling and the mixed structure.

Searle et al. (1992) set forth the first complete treatment of variance components analysis with extensions to maximum likelihood and restricted maximum likelihood models, hierarchical models, Bayesian models, discrete and binary models, and to the dispersion-mean model. Variance components analysis estimates the contribution of fixed effects, random effects, and residual error to the expected variance of a dependent variable or to the variance of a vector of independent variables. Inherent in Fisher's (1925) method is partitioning of structural and residual error components. The structure of the fixed, random, and mixed effects is determined by the fixed structural and random effects inherent in the $\mathbf{y} = f(\mathbf{x}) + \varepsilon$ relationship.

$$E\left[\sigma_y^2\right] = E[\sigma_{Structure}^2] + E[\sigma_{Error}^2]$$

$$E\left[\sigma_y^2\right] = E\left[\sigma_{FixedEffects}^2\right] + E\left[\sigma_{RandomEffects}^2\right] + E[\sigma_{Error}^2]$$

$$E[MS(Total)] = E[MS(FixedEffects)] + E[MS(RandomEffects)] + E[MS(Error)]$$

Within the statistical context, four methods have been applied to estimate variance components: minimum norm quadratic unbiased estimator (MINQUE), analysis of variance (ANOVA), maximum likelihood (ML), and restricted maximum likelihood (REML). Maximum likelihood and restricted maximum likelihood produce an asymptotic covariance matrix as a complete description of variance components. Other available output includes ANOVA tables and expected mean squares for the ANOVA method and an iteration history for the ML and REML estimates. The Variance Components procedure is fully compatible with the General Linear Model Univariate procedure. Weighted Least Squares (WLS) extends variance components analysis by specifying different weights for a weighted analysis to compensate for variations in precision of measurement.

3.2 POWER ANALYSIS

All parametric statistical tests are subject to two sampling errors: alpha error – the probability of detecting a difference when one does not exist; and beta error – the probability of not detecting a difference when one does exist. Confidence in making the correct decision of no difference when one does not exist is (1 – alpha). The power of the test, or the probability of detecting a difference when one does exist, is (1 – beta). These errors specify the discrimination of the control chart or sampling operating characteristic curve.

For control charts, the alpha probability is set based on the economic-risk cost tradeoff of rejecting that the process is in control when, in fact, it is operating in control. The costs for committing an alpha error include resampling to verify the state of control and possibly stopping the process. Likewise, the beta probability is set based on the economic-risk cost tradeoff of failing to reject that the process is out of control when, in fact, it is out of control. The costs for committing a beta error include producing an unacceptable proportion of nonconforming parts, stopping the process once the out-of-control signal is detected, determining the assignable cause, screening parts to determine the extent of nonconforming parts, and releasing nonconforming parts for subsequent processing. Once the allowable alpha error and the beta error probabilities are determined, the minimal sample size is determined to achieve the *L*-width control limits at which the P(p_1 nonconforming | alpha) and P(p_2 nonconforming | beta) jointly hold. For a given control chart, the operating characteristic curve is estimated as

$$\beta = P(\delta\sigma < UCL|p_2) - P(\delta\sigma < LCL|p_2)$$

for δ = 0, 0.25, 0.5, 0.75, 1.0, …, k standard deviations. The power of the test = 1 – β.

For acceptance sampling, the alpha probability is set based on the economic-risk cost tradeoff of rejecting an acceptable lot or process when, in fact, the lot or process is maintaining the acceptable level of nonconforming product. This proportion nonconforming is termed the "acceptable quality level" (AQL) and is defined as the poorest proportion nonconforming that a customer considers acceptable as a process average (i.e., will not economically impact further processing or use). The costs for committing an alpha error include rejecting an acceptable lot or process and resampling to determine the true proportion nonconforming. Hence, in sampling, the proportion-nonconforming/alpha-probability combination is termed the *supplier's risk*, because the supplier bears the full risk of a false reject. Likewise, the beta probability is set based on the economic-risk cost tradeoff of failing to reject a nonconforming lot or process when, in fact, the lot or process is at an unacceptable nonconforming level. This proportion nonconforming is termed the "rejectable quality level" (RQL) or "lot tolerance percent defective" (LTPD) and is defined as the proportion nonconforming that a customer considers unacceptable as a process average (i.e., will economically impact further processing or use). The costs for committing a beta error include rejection by the customer, adversely impacting customer further processing or use, warranty claims, and potential lost sales. Hence the proportion-nonconforming/beta-probability combination is termed the *customer's risk*, because the customer bears the full risk of a false acceptance of a truly rejectable lot or process output. For attribute characteristics, the discrimination (slope) of the sampling operating characteristic curve is determined by the AQL p_1 proportion nonconforming and alpha risk and by the RQL p_2 proportion nonconforming and beta risk points.

$$1 - \alpha = \sum_{d=0}^{c} \binom{n}{d} p_1^d (1 - p_1)^{n-d}$$

$$\beta = \sum_{c=0}^{c} \binom{n}{d} p_2^d (1 - p_2)^{n-d}$$

For continuous process data, a reject limit k is selected such that

$$k = \frac{Z_2 Z_\alpha + Z_1 Z_\beta}{Z_1 - Z_2}$$

$$n = \left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2 \left(1 + \frac{k^2}{2}\right)$$

where Z_1 = standard normal variate for the AQL p_1 relative to the specification limit, Z_α = standard normal variate alpha risk value, Z_2 = standard normal variate for the RQL p_2 relative to the specification limit, and Z_β = standard normal variate beta risk value. For either the attribute characteristic or continuous variable OC curve, the power of the test = 1 – P (accept).

Given the control chart or sampling model and synthesized quality system model, this research will qualitatively evaluate the effects of economic-risk factors on the power of the test as the metric of effectiveness in controlling the production and release of nonconforming product.

3.3 MODELS ADMISSION CRITERIA

In traditional research synthesis, the quality of prior research plays a significant role in judging synthesis outcomes. Therefore, setting admission criteria for prior research provides consistent criteria for deciding on which models to include and which models to exclude. Copper and Hedges (1994) recommend two approaches to assess research quality as developed by Campbell and his associates (Campbell & Stanley, 1966; Cook & Campbell, 1979) and by Chalmers et al. (1981).

Campbell's approach developed a validity framework, which is a matrix of designs and their features to deal with threats to validity. It focuses on nonrandomized or quasi-experimental designs. It encompasses a larger variety of designs as well as a larger number of design features. They proposed four validity categories. The first category is internal validity that assesses whether there is a causal relationship from one variable to another in the form in which the variables were manipulated or measured. The second category is external validity, which assumes a causal relationship can be generalized across different types of settings. The third category is statistical conclusion validity that defines the conclusions about covariation between independent and dependent variables (Cook & Campbell, 1979). The fourth category is construct validity that studies causes and effects with which one can generalize about higher-order constructs from research procedures.

In this research, only peer-reviewed economic-risk control chart and sampling models were admitted. General control chart and sampling design techniques that did not specifically include economic-risk design factors were not admitted. Given that each admitted control chart or sampling model must consider the controllable economic-risk factors that determine the required values of the control chart or sampling parameters, research internal validity was established. External validity and construct validity were established through the development of generalizable qualitative variance components economic-risk relationship descriptions at the quality system level. Since all control chart and sample models belong to the exponential parametric family of distributions (Bernoulli, binomial, Poisson, Dirichlet, normal, exponential, gamma, or Wishart), statistical validity was established by qualitatively synthesizing the joint alpha and beta error effects on the quality systems level power of the test.

3.4 RESEARCH SYNTHESIS

A meta-synthesis is defined as an inductive research design to synthesize primary studies for the purpose of making contributions beyond those achieved in the original studies. Working as an inductive qualitative data analysis, meta-study involves the accumulation of previous studies' evidence, and, more specifically, it extracts analyses, and syntheses prior research into a general framework. Hence, the meta-synthesis occurs at the level at which the original researchers of the primary studies have constructed their insights in accordance with the variance components in the data of prior studies.

The main research methodology was variance component estimation models, which assess the amount of variation in a dependent variable associated with one or more fixed and random-effects variables. Using variance components techniques, the economic-risk variance components inferences of the E[A] economic-risk design of control charts and the ATI cost minimization of sampling plans were identified. This inductive approach synthesized varied economic-risk model's components in control charts design and sampling plans design into a qualitative hierarchical quality system's model establishing variance components among the models' components. Conducting the meta-synthesis facilitated the identification of common random-effects components and specified fixed effects among the common random-effects components categories.

Duncan (1956) is an example of qualitative analysis of economic-risk variance components. Revenue-risk variance components, risk components, and economic components were identified. Qualitative economic-risk variance components relationships were summarized as illustrated in the follow example.

Qualitative Economic-Risk Variance Components Analysis

Title: The Economic Design of X-bar Control Charts Used to Maintain Current Control of a Process

Author: Duncan, A. J.

Year: 1956

Objective: Minimize the cost per inspection cycle associated with production in the out-of-control state.

Revenue-Risk Variance Components

Delta point: $(h/2) - (\lambda h^2/12)$

Average time to detect: $[(1/p) - (1/2)(\lambda h/12)]h$

Proportion defective: $p \sim \delta\sigma''$

Average cycle length in-control-and-out-of-control:

$$^{1}/_{\lambda} + \left(^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12}\right)h + gn + D$$

Proportion of time in control:

$$\beta = \frac{^{1}/_{\lambda}}{^{1}/_{\lambda} + (^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12})h + gn + D}$$

Proportion of time out-of-control:

$$\gamma = \frac{\left(^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12}\right)h + gn + D}{^{1}/_{\lambda} + \left(^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12}\right)h + gn + D}$$

Expected false alarms: $\frac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}} \sim \frac{\alpha}{\lambda h} \sim \frac{\beta\alpha}{h}$

Expected loss/hour false alarms: $\frac{\beta\alpha}{h} a_3'$

Model:

$$E[I/T] = \frac{V_0(1/\lambda) + V_1\left[\left(^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12}\right)h + gn + D\right] + a_3 - a_3'\alpha e^{-\lambda h}/\left(1 - e^{-\lambda/h}\right)}{^{1}/_{\lambda} + \left(^{1}/_{p} - ^{1}/_{2} + ^{\lambda h}/_{12}\right)h + gn + D}$$

$$- \frac{a_1 + a_2 n}{h}$$

Risk components:

α = probability of a false occurrence

β = probability of failing to detect a real occurrence

λ = occurrence arrival rate

δ = step change in the mean

σ = process standard deviation

Economic components:

V_0 = net income/hour in-control operation

V_1 = net income/hour out-of-control operation

a_3 = cost of finding an assignable cause (W)

a_3’ = cost of investigating a false alarm (T)

a_1 = fixed cost of sampling

a_2 = variable cost/unit of sampling

g = sampling time per unit (e)

D = average time to find an assignable cause (not relevant to economic cost-risk analysis)

Objective: Maximize E[I/T], the expected net income per unit of time.

Known: μ_0 = process average, δ = step difference to detect (impact), and σ = process standard deviation.

To be determined: *n* = sample size, *k* = control limit width, and *h* = sampling interval.

Qualitative Economic-Risk Variance Components Relationships:

- δ and *n* are inversely related. For a given α and β risk combination, the smaller δ requires larger *n*. The optimum *n* is largely determined by the magnitude of δ.
- The hourly penalty cost $\left({}^{1}\!/_{p} - {}^{1}\!/_{2} + {}^{\lambda h}\!/_{12}\right)$ for production in the out-of-control state mainly affects the interval between samples *h*.
- Costs of looking for assignable causes (a_3 and a_3') mainly affect the width of the control limits through parameters *p* = ωσ = CL ~ βα/*h*. Since α is set *a priori* on process economic-risk cost factors and β is set by sample size to yield ωσ = CL, an increase in β or decrease in *h* results in wider control limits and a corresponding increase in a_3’ βα/*h*. Wider control limits are inversely related to *p* ~ δσ. That is, as βα/*h* increases, the proportion *p* ~ δσ controlled decreases.
- Variation in the cost of sampling (a_1 + a_2 *n*)*g* affects all three design parameters.

- Changes in the mean number of occurrences per hour λ primarily affects the interval h between samples through $(h\lambda/12)h$. A one unit increase in λ causes a $h^2/12$ increase.
- The optimum economic design is relatively insensitive to errors in estimating the cost coefficients.

The application of Meta-synthesis assisted in identifying the integrated hierarchy of economic-risk variance components that comprises the quality systems risk model. The integration process followed the hierarchical bottom-up approach (Figure 2). This analytical hierarchy of integration process added more robust values by reducing complex decisions to a series of pairwise comparisons, and then synthesizing the results so that coherent decision.

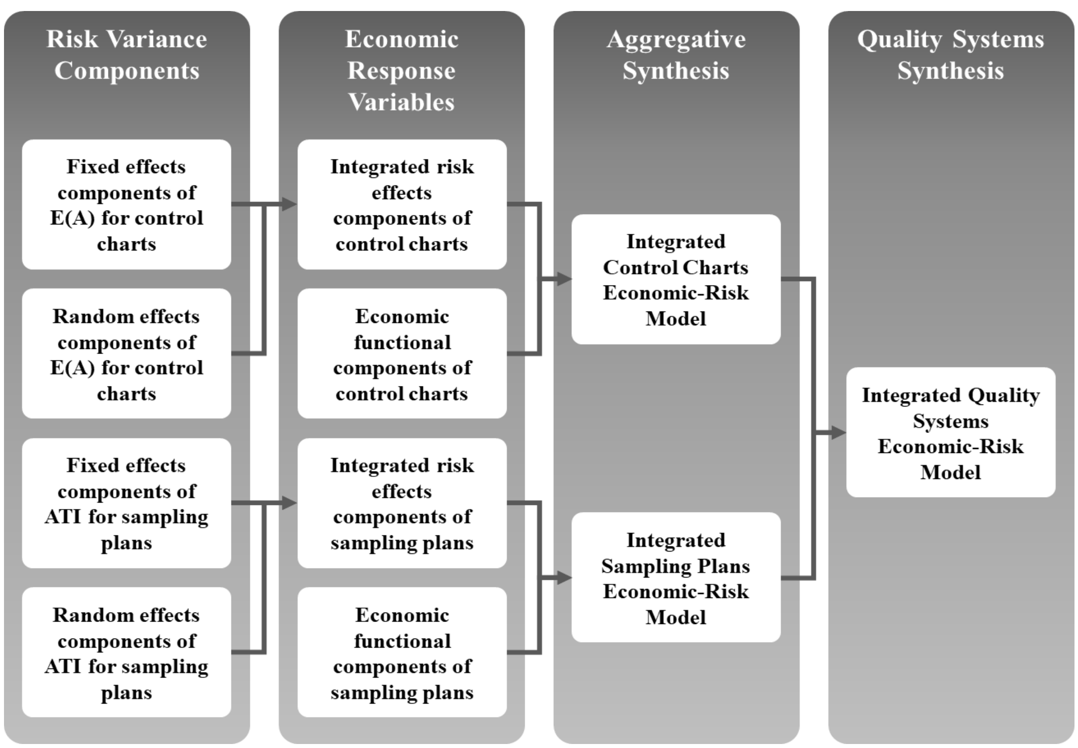

Figure 2. Hierarchical Bottom-up Approach for Integration Process.

Note that the synthesis process was directed in two ways, as aggregation and as interpretation. Of the qualitative synthesis methodologies, meta-aggregation is considered one of the most useful approaches for conducting high-quality systematic reviews. The essential characteristics of a meta-aggregative review are to avoid re-interpretation of the related studies and to accurately present the findings of the studies as intended by their authors. The close inspection of the aggregated empirical evidence can serve to refine existing theory in terms of a modification, supplementation, or even negation. The meta-analysis yields an overall estimate of effect size with the detection and estimation of interaction effects being central to the interpretation of the meta-analytic results (Hoon, 2013). Comparing with the primary studies, interacting effects, which provide the boundary conditions of the hypothesized effects, generate superior evidence of generalizability (Hoon, 2013). Hence, the effect size is considered an important indicator of the prediction of the potential theory. As such, meta-analyses set the standard for what is known and needed in a topic and for which theory is considered valid and which is not.

The research synthesis interpretation handled the aggregated findings produced from the prior approach. Moreover, the interpretation process referred to the accumulation of primary evidence with the aim to generate interpretive explanation rather than prediction (Hoon, 2013). The synthesis analysis, here, distinguished the joint effects on quality system statistical power based on discriminatory and common fixed and random effects. Interpretative synthesis constructs a solid method to produce comprehensive and causal explanations; variables and relationships when building blocks of theory building.

The methodology in this research concentrated on conducting a qualitative variance components research synthesis into a theoretical framework, which follows four main stages, (a) admission criteria of relevant economic-risk control chart or sampling models, (b) qualitative variance components analysis, (c) qualitative synthesis of fixed and random structural components and the residual error and (d) evaluation and validation of the synthesized frameworks. This research focused on the synthesis of control chart models and sampling models into an economic-risk control and sampling theoretical framework. The methodological framework of this research is summarized in Table 1.

Table 1. Research Synthesis Stages.

Stage	Step	Objective	Methods/ Approach
Models' admission criteria	Locating relevant research	Identifying prior research on the economic-risk design of statistical control charts or sampling plans.	• Argument of citational Saturation • Separation method
	Inclusion Criteria	Economic-risk design of statistical control charts or sampling plans.	
Variance components analysis	Extracting and coding economic-risk variance components in each model	Each model was appraised, and its economic-risk variance components were qualitatively identified. Coding categorized model components into input economic and risk factors and into controllable parameters.	Model-by-model coding and clustering
	Analyzing on a case-specific level	Each model was appraised, and its economic-risk variance components were qualitatively summarized for their effects on statistical power.	Variance components analysis
Research synthesis	Synthesizing on an across-study level	Fixed and random risk variance components and economic functional variables were synthesized to assess the joint effect on quality system statistical power.	• Variance component synthesis • Aggregative synthesis
	Building theory from meta-synthesis	Linking the effects on quality system statistical power based on discriminatory and common fixed and random effects on quality system statistical power.	Interpretative synthesis
Evaluation and validation	Assessing unbiasedness and minimum variance	Identifying discriminatory bias components between common fixed, random, and functional effects and assessing their effects on quality system statistical power variance components.	Specify the bias and variance effects of variance components on quality system statistical power

3.5 EVALUATION AND VALIDATION

Since the synthesis process was conducted on control charts and sampling plans models, they are expected not to have the same bias and variances problems as prior studies. Therefore, the bias and variance effects of variance components on quality system statistical power was evaluated. Any potential discriminatory bias components between common fixed and random effects were identified, and their effects on quality system statistical power variance components were assessed. The assessment included the consistency of discrimination across the OC-curve synthesis, which considers the quality systems level power of the test (1-beta), of control charts and sampling plans economic-risk design.

CHAPTER 4

ANALYSES AND RESULTS

Variance components analysis was as set forth in Figure 2. The hierarchical bottom-up approach for integration process was applied to extracted variance components from each admitted paper using the general model:

$$y = X\beta + Zu + e$$

where $X\beta$ represents the fixed effect components and Zu represents random effect components (Searle et al., 1992). The proposed methodology in this dissertation was applied with a significant support of a worked example (Hannes et al., 2018).

The development of the theoretical framework of quality systems economic-risk design was delimited to the identified economic-risk variance components that affect the discrimination power of control charts and sampling plans that are determined by their respective Operating Characteristic (OC) curves. The study and analysis of the fixed effect economic components were excluded from the integration because they have no implication to α error or β error in determination of the OC curve.

4.1 ANALYSIS OF CONTROL CHARTS ECONOMIC-RISK MODELS

Based on the admission criteria, eleven papers of control charts economic-risk design were admitted (Table 2). A combination of keywords was used in the search for potential studies related to this dissertation. For instance, keyword search of control charts design used the combination of the terms “control chart,” “economic,” and “risk” that can be identified in the title, abstract or content. To avoid bias, specific terms such as $\overline{\mathrm{X}}$ chart, failure cost, and Type I error were not applied in the search. The evaluation of each paper started with abstract review that filtered them to 127 articles. The next step was to look for only peer reviewed using ODU Library database and this resulted in selecting 45 papers. Since this research is delimited to only economic-risk design, 35 out of the 45 papers were chosen. Essential stage was to concentrate on any analysis of variation implication needed for the integration purpose. Presenting a certain economic-risk design in modeling the control charts without validating the variation effect might weaken the outcome of the unified systems. Hence, excluding unvalidated designs concluded to consider only the eleven admitted papers (Figure 3).

The qualitative analysis of these models including the variance components relationships is demonstrated in Appendix A. The extracted variance components and economic effects were sorted and coded in Tables 3 and 4, according to their implication to the quality parameters such as sample size, sampling intervals, acceptance number, and control limits width. The random-effects components are the mean of occurrence per hour ($1/\lambda$) and proportion defective items. Although step change in the process mean seems random effect, the studied models suggest certain values for this unpredicted component. Therefore, this research deals with the step change in the process mean as a fixed effect and assumes it follows a uniform distribution. The economic components affect the quality parameters in the admitted paper of control charts are the unit cost of inspection, the cost of visiting the process to take a sample, the cost of looking for a trouble when none exists and when it does exist, and the hourly penalty cost.

Table 2. Admitted Papers of Control Charts Model.

Author, Year	Title
Duncan, 1956	The Economic Design of X-Charts Used to Maintain Current Control of a Process
Montgomery & Klatt, 1972	Economic Design of T^2 Control Charts to Maintain Current Control of a Process
Goel & Wu, 1973	Economically Optimum Design of Cusum Charts
Montgomery et al., 1975	Economic Design of Fraction Defective Control Charts
Chiu, 1975	Minimum Cost Control Schemes Using np Charts
Saniga, 1977	Joint Economically Optimal Design of x and R Control Charts
Duncan, 1978	The Economic Design of p-Charts to Maintain Current Control of a Process: Some Numerical Results
Alexander et al., 1995	Economic Design of Control Charts using the Taguchi Loss Function
Prabhu et al., 1997	Economic-Statistical Design of an Adaptive xbar Chart
Serel & Moskowitz, 2008	Joint Economic Design of EWMA Control Charts for Mean and Variance
Chen & Yeh, 2011	Economic Statistical Design for x-bar Control Charts under Non-Normal Distributed Data with Weibull In-Control Time

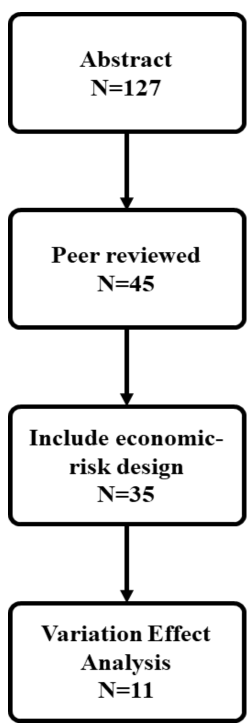

Figure 3. The Admission Process of Control Charts Articles.

Table 3. Conceptual Coding of Control Charts Variance Components Effects.

Author, Year	VC1	VC2	VC3	VC4
Duncan, 1956	Mean number of occurrences per hour has inverse effect on sampling interval			
Goel & Wu, 1973	Mean number of occurrences per hour has inverse effect on sampling interval			
Chiu, 1975	Mean number of occurrences per hour has inverse effect on sampling interval	Proportion defective items in control status has inverse effect on sample size	Proportion defective items in control status has direct effect on acceptance number	
Saniga, 1977	Mean number of occurrences per hour has inverse effect on sampling interval			
Duncan, 1978	Mean number of occurrences per hour has inverse effect on sampling interval			
Alexander et al., 1995	Mean number of occurrences per hour has inverse effect on sampling interval	Mean number of occurrences per hour has inverse effect on sample size		
Prabhu et al., 1997	Mean number of occurrences per hour has inverse effect on sampling interval	Mean number of occurrences per hour has inverse effect on sample size		Mean number of occurrences per hour has direct effect on control limits width
Serel & Moskowitz, 2008	Mean number of occurrences per hour has inverse effect on sampling interval			
Chen & Yeh, 2011	Mean number of occurrences per hour has inverse effect on sampling interval			

Table 4. Conceptual Coding of Control Charts Economic Effects.

Author, Year	EC1	EC2	EC3
Duncan, 1956	Unit cost of inspection has direct effect on sampling interval	Unit cost of inspection has inverse effect on sample size	Unit cost of inspection has inverse effect on control limits width
	Cost of visiting the process to take a sample has direct effect on sampling interval	Cost of visiting the process to take a sample has direct effect on sample size	Cost of looking for trouble when none exists has direct effect on control limits width
	Hourly penalty cost has inverse effect on sampling interval		Cost of looking for trouble when it does exist has direct effect on control limits width
Montgomery & Klatt, 1972	Unit cost of inspection has direct effect on sampling interval	Unit cost of inspection has inverse effect on sample size	Unit cost of inspection has inverse effect on control limits width
	Cost of visiting the process to take a sample has direct effect on sampling interval	Cost of looking for trouble when it does exist has direct effect on sample size	Cost of looking for trouble when it does exist has direct effect on control limits width
Goel & Wu, 1973	Unit cost of inspection has direct effect on sampling interval		
	Cost of visiting the process to take a sample has direct effect on sampling interval		
Montgomery et al.**, 1975**		Unit cost of inspection has inverse effect on sample size	Unit cost of inspection has direct effect on control limits width
	Cost of visiting the process to take a sample has inverse effect on sampling interval	Cost of visiting the process to take a sample has direct effect on sample size	Cost of visiting the process to take a sample has direct effect on control limits width
		Cost of looking for trouble when none exists has inverse effect on sample size	Cost of looking for trouble when none exists has direct effect on control limits width
		Cost of looking for trouble when it does exist has inverse effect on sample size	Cost of looking for trouble when it does exist has direct effect on control limits width

Table 4. Continued.

Chiu, 1975	Unit cost of inspection has direct effect on sampling interval	Unit cost of inspection has inverse effect on sample size	Unit cost of inspection has inverse effect on acceptance number
	Cost of visiting the process to take a sample has direct effect on sampling interval		
Saniga, 1977	Unit cost of inspection has inverse effect on sampling interval	Unit cost of inspection has inverse effect on sample size	Unit cost of inspection has inverse effect on control limits width
	Cost of visiting the process to take a sample has inverse effect on sampling interval	Cost of visiting the process to take a sample has direct effect on sample size	
	Hourly penalty cost has direct effect on sampling interval	Cost of looking for trouble when it does exist has direct effect on sample size	Cost of looking for trouble when it does exist has direct effect on control limits width
		Cost of looking for trouble when none exists has inverse effect on sample size	Cost of looking for trouble when none exists has inverse effect on control limits width
Duncan, 1978	Unit cost of inspection has direct effect on sampling interval	Unit cost of inspection has inverse effect on sample size	
	Cost of visiting the process to take a sample has direct effect on sampling interval	Cost of visiting the process to take a sample has inverse effect on sample size	
	Cost of looking for trouble when none exists has direct effect on sampling interval		Cost of looking for trouble when none exists has direct effect on control limits width
	Cost of looking for trouble when it does exist has direct effect on sampling interval		Cost of looking for trouble when it does exist has direct effect on control limits width

Table 4. Continued.

Alexander et al., 1995	Cost of visiting the process to take a sample has direct effect on sampling interval	Cost of visiting the process to take a sample has direct effect on sample size	
	Cost of looking for trouble when none exists has inverse effect on sampling interval	Cost of looking for trouble when none exists has direct effect on sample size	
	Cost of looking for trouble when it does exist has direct effect on sampling interval	Cost of looking for trouble when it does exist has inverse effect on sample size	
Prabhu et al., 1997		Unit cost of inspection has direct effect on sample size	
	Cost of visiting the process to take a sample has direct effect on sampling interval		
	Cost of looking for trouble when none exists has inverse effect on sampling interval	Cost of looking for trouble when none exists has direct effect on sample size	
Serel & Moskowitz, 2008	Hourly penalty cost has direct effect on sampling interval		
Chen & Yeh, 2011	Hourly penalty cost has direct effect on sampling interval		
		Cost of looking for trouble when none exists has inverse effect on sample size	Cost of looking for trouble when none exists has inverse effect on control limits width

The outcome of variance components analysis of control charts economic-risk design shows a pattern led by Duncan's (1956) model. There was a clear agreement from Table 3 that the mean number of occurrences per hour has significant effect on sampling interval. Few models represent the implication of mean number of occurrences per hour on sample size and control limits width. The implication of proportion defective items on sample size and acceptance number was discussed in detail only in Chiu's (1975) design. He studied the process when it is in control as well as out of control. On the other hand, the extracted economic effects in Table 4 explain a significant implication to the quality parameters in modeling the control charts economic-risk design. The causal relationships between these economic variables and sample size, sampling intervals, and control limits width are found in most of the studied models. Moreover, the sampling interval is affected by additional cost committed to producing bad lots.

Note that variance components and economic effects were coded using color coding technique under suggested groups. The coded group in Table 3 is set as per quality parameters while in Table 4 is set as per economic components. Color codes are typically useful in differentiating information and decomposing into classes. It helps the researcher to organize the integration process by avoiding conflict and duplication.

4.2 ANALYSIS OF SAMPLING PLANS ECONOMIC-RISK MODELS

The analysis of sampling plans economic-risk design followed the same steps accomplished in the analysis of control charts economic-risk design (Section 4.1). Based on the admission criteria, seven papers of sampling plans economic-risk design were admitted (Table 5). A combination of keywords was used in the search for potential studies related to this dissertation. For instance, keyword search of sampling plans design used the combination of the terms "sampling plans," "economic," and "risk" that can be identified in the title, abstract and/or content. To avoid bias, specific terms such as single sampling, OC curve, LTPD, and consumer risk were not used in the search. The evaluation of each paper started with abstract review that filtered them to 134 articles. The next step was to look for only peer reviewed using ODU Library database and this resulted in selecting 41 papers. Since this research is delimited to only economic-risk design, 33 out of the 41 papers were chosen. Essential stage is to concentrate on any analysis of variation implication needed for the integration purpose. Presenting a certain economic-risk design in modeling the sampling plans without validating the variation effect might weaken the outcome of the unified systems. Hence, excluding unvalidated designs concluded to consider only the seven admitted papers (Figure 4).

Table 5. Admitted Papers of Sampling Plans Model.

Author, Year	Title
Dodge & Romig, 1941	Single Sampling and Double Sampling Inspection Tables
Schleifer, 1969	Two-Stage Normal Sampling in Two-Action Problems with Linear Economics
Schmidt & Bennett, 1972	Economic Multiattribute Acceptance Sampling
Collins et al., 1973	The Effects of Inspection Error on Single Sampling Inspection Plans
Fink & Margavio, 1994	Economic Models for Single Sample Acceptance Sampling Plans, No Inspection, and 100 Percent Inspection
Nezhad & Yazdi, 2014	Economic Design of Acceptance Sampling Plans Based on Conforming Run Lengths Using Loss Functions
Fernández, 2017	Economic Lot Sampling Inspection from Defect Counts with Minimum Conditional Value-at-Risk

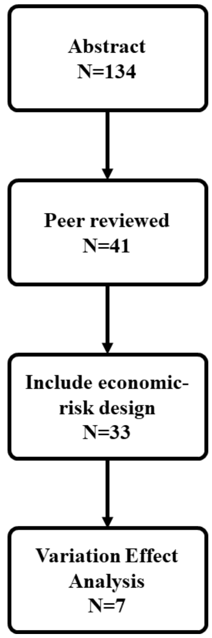

Figure 4. The Admission Process of Sampling Plans Articles.

The qualitative analysis of these models including the variance components relationships is demonstrated in Appendix B. The extracted variance components and economic effects were sorted and coded according to their implication to the quality parameters such as sample size, allowable defect number, and probability of acceptance (Table 6). There are three variance components affecting the quality parameters: proportion defective items, Type-I error, and Type-II error. The impact of standard deviation in Fink and Margavio's (1994) model was considered as a fixed effect in this research and assumed to follow a uniform distribution. Because a known standard deviation tends to the selection of either no inspection or 100 percent inspection, which they are out of the research scope. Besides, the use of acceptance sampling plans is appropriate as most processes use an estimate of the standard deviation (Fink & Margavio, 1994).

The fixed-effect of AOQL and LTPD in Dodge and Romig's design reflect the random-effect of Type-I error and Type-II error respectively (Dodge & Romig, 1941). From OC curve, increasing AOQL leads to increasing Type-I error to maintain the adequate discrimination while Type-II error decreases as LTPD increases to have more discriminated curve. Therefore, the implication of the random components replaces the fixed components in Table 6. The economic components affect the quality parameters in the admitted paper of sampling plans are only the unit cost of inspection and the cost of visiting the process to take a sample.

The outcome of variance components analysis of sampling plans economic-risk design shows diverse relationships. The implication of proportion defective items on sample size was the most significant effect. The major implication on many quality parameters was by Type-I error and Type-II error. From the economic perspective, the cost of visiting the process to take a sample affects the sample size while the unit cost of inspection affects sample size and control limits width. Likewise, the variance components and economic effects were coded using the color-coding technique under suggested groups. Note that during the coding process, Tables 3, 4, and 6 were studied together to compare and match any similarities and to avoid duplication. Observe in Table 6 and its colored codes that the effect of variance components is random with no clear pattern comparing to control charts. This is possibly because of the diversity of sampling plans attribute-designs.

Table 6. Conceptual Coding of Sampling Plans Variance Components and Economic Effects.

Author, Year	VC1	VC2	VC3
Dodge & Romig, 1941	Proportion defective items has inverse effect on sample size	Proportion defective items has direct effect on allowable defect number	
	Type-I error has direct effect on sample size	Type-I error has direct effect on allowable defect number	
	Type-II error has direct effect on sample size	Type-II error has direct effect on allowable defect number	
Schmidt & Bennett, 1972	Proportion defective items has inverse effect on sample size		
Collins et al., 1973			Type-I error has inverse effect on probability of acceptance
			Type-II error has direct effect on probability of acceptance
Fernández, 2017	Proportion defective items has inverse effect on sample size	Proportion defective items has direct effect on allowable defect number	Proportion defective items has direct effect on probability of acceptance
	Type-I error has direct effect on sample size	Type-I error has direct effect on allowable defect number	
	Type-II error has direct effect on sample size	Type-II error has direct effect on allowable defect number	
Author, Year	**EC1**	**EC2**	**EC3**
Schleifer, 1969	Unit cost of inspection has inverse effect on sample size		
Nezhad & Yazdi, 2014	Unit cost of inspection has inverse effect on sample size		Unit cost of inspection has inverse effect on control limits width

4.3 INTEGRATED ANALYSIS OF ECONOMIC-RISK VARIANCE COMPONENTS EFFECT ON OC-CURVE

Variance components and economic effects in Tables 3, 4, and 6 were integrated to formulate the economic-risk control and sampling theoretical frameworks. It is helpful to understand the comprehensive relationships revealed from the previous sections before starting the qualitative synthesis of the studied quality systems. Therefore, another point of conceptual analysis is to code based on frequency of occurrence. Table 7 demonstrates the integrated coded variance components as the most frequent implication on the quality parameters. Sample size, sampling interval, and allowable defect number are the most affected parameters. While control limits width is less affected by the studied variance components. The major implication of variance components on the quality parameters comes from the mean number of occurrences per hour supported by its strong effect on sampling interval. Proportion defective item has also significant implication in the second place after the former variance component. Type-I error and Type-II error share the same implication on the same quality parameters.

Table 8 demonstrates the integrated coded economic effects as the most frequent implication on the quality parameters. Sampling interval, sample size, and control limits width are the most affected parameters. While allowable defect number is less affected by the studied economic variables. The major implication of economic components on the quality parameters comes from the unit cost of inspection followed by the cost of visiting the process to take a sample, the cost of looking for trouble when it does exist, and the cost of looking for trouble when none exists. As mentioned earlier in this research, sampling interval is affected by the hourly penalty cost.

Table 7. Conceptual Coding Based on Frequency of Occurrence of Integrated Variance Components.

Quality Parameter	Mean number of occurrences per hour	Proportion defective items	Type-I error	Type-II error	TOTAL
Sample size	**2**	**4**	**2**	**2**	**10**
Sampling interval	**9**				**9**
Allowable defect number		**3**	**2**	**2**	**7**
Control limits width	**1**				**1**
TOTAL	**12**	**8**	**5**	**5**	

Table 8. Conceptual Coding Based on Frequency of Occurrence of Integrated Economic Effects.

Quality Parameter	Unit cost of inspection	Cost of visiting the process to take a sample	Cost of looking for trouble when it does exist	Cost of looking for trouble when none exists	Hourly penalty cost	TOTAL
Sampling interval	**6**	**9**	**3**	**2**	**4**	**24**
Sample size	**9**	**5**	**4**	**4**		**22**
Control limits width	**5**	**1**	**4**	**5**		**15**
Allowable defect number	**1**					**1**
TOTAL	**21**	**15**	**11**	**11**	**4**	

The relationships among variables that characterize the components dynamically were studied and demonstrated. Figure 5 shows the relationships among the key concepts and variables demonstrated in the tables from the integrated analysis of control charts economic-risk design stated in Section 4.1. Likewise, Figure 6 shows the relationships among the key concepts and variables demonstrated in the tables from the integrated analysis of sampling plans economic-risk design stated in Section 4.2. The shaded boxes represent the quality parameters, and the white boxes are the integrated variance components and economic effects. The nature of each component's implication was differentiated by coloring the arrows. Red arrow means the inverse effect of the variance components to the pointed parameters while black arrows are for the direct relationship.

To understand the relationship between the extracted variance components, economic effects, and quality parameters in this research, each relationship was classified to either a direct effect or an inverse effect. An example of the direct effect is when X value increases so does Y value or as X decreases so does Y. Conversely, the relationship between two variables is an inverse relationship if X value increases Y value decreases or as X value decreases Y value increases.

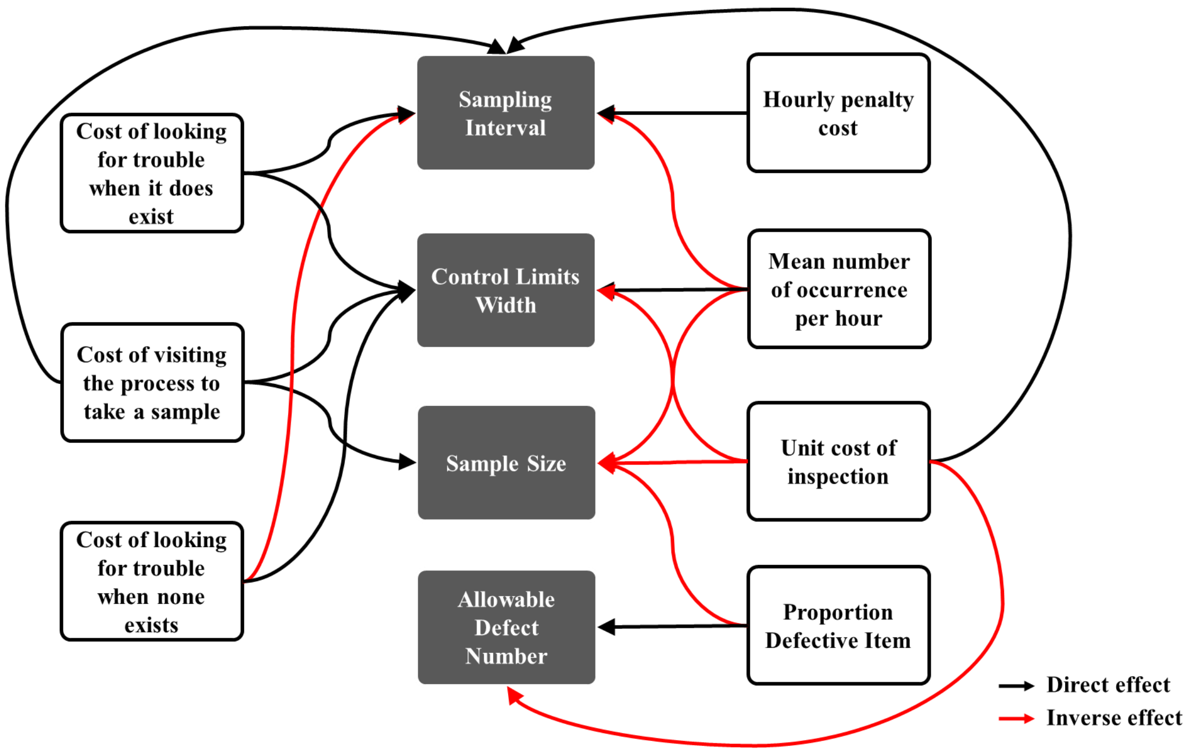

Figure 5. Extracted Relationships of Control Charts Economic-Risk Design.

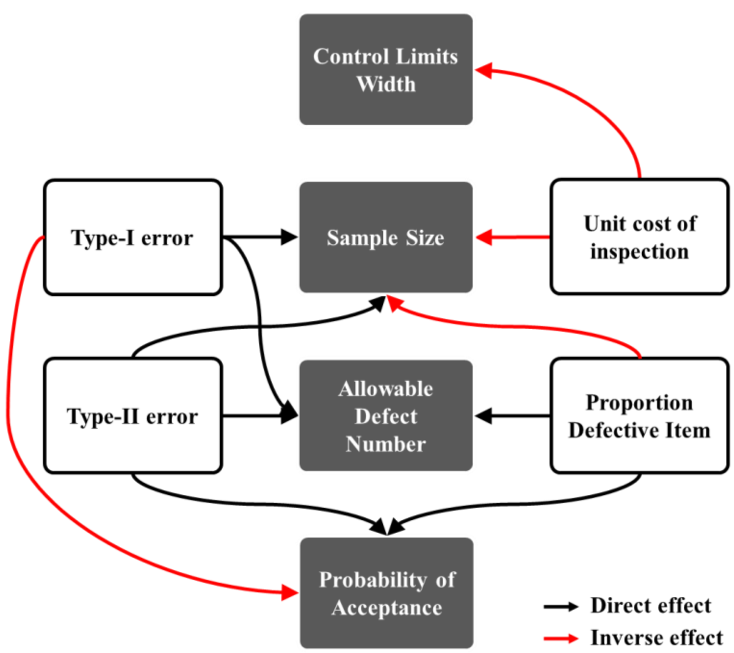

Figure 6. Extracted Relationships of Sampling Plans Economic-Risk Design.

Recall from the power analysis in this study's methodology that economic-risk cost tradeoffs in the design of quality systems were derived theoretically from economic-risk $1 - \beta$ discrimination power of control charts and sampling plans OC curves. This tradeoff leads to a potential dynamic behavior resulted by the variance components analysis. Therefore, the development of the economic-risk control and sampling theoretical frameworks were formulated and demonstrated using a schematic diagram that visualizes the theoretical relationships. Causal Loops Diagram (CLD) is one of the diagramming tools that captures the structure of systems and represents their feedback, so Causal Loop Diagrams were chosen for this research. Structuring CLD is simple but should follow certain explanations and interpretations to be easy to understand. Sterman (2000) set forth detailed guidelines and tips of how to form a CLD to support the integration process and to validate the proposed frameworks.

A causal diagram consists of variables connected by arrows denoting the causal influences among the variable. While the signs at arrow heads (+ or -) indicate the polarity of the relationship. For instance, a positive polarity, indicated by +, means an increase in the independent variable causes the dependent variable to rise above what it would have been (and a decrease causes a decrease). Negative signs mean an increase (decrease) in the independent variable causes the dependent variable to decrease (increase) beyond what it would have been. Loop identifiers show the polarity of the loop, either positive (self-reinforcing, denoted by R) or negative (balancing, denoted by B). Positive feedback loops generate growth, amplify deviations, and reinforce change. Negative loops seek balance, equilibrium, and stasis. Negative feedback loops act to bring the state of the system in line with a goal or desired state (Sterman, 2000). To identify the polarity of each loop, the number of negative links was counted. If the number of negative links was even, the loop was considered as self-reinforcing and denoted by R; if the number was odd, the loop was considered as balancing and denoted by B. This method is termed the “fast way.”

Although the relationships diagrams illustrated in Figures 5 and 6 represent the extracted variance components and economic effects, they still lack essential relationships in the CLD to fulfil the possible gaps in formulating the proposed quality systems economic-risk theoretical frameworks. The revision of the proposed CLD refer different areas including quality control principles, power of the test literature, statistical studies, and theoretical formulation stated in this research. The purpose of this revision is to facilitate the qualitative synthesis of the extracted economic variance components analysis and the power analysis. More explanations to understand the formulated dynamic behavior in the proposed causal diagrams of control charts theory and sampling plans scheme are set forth respectively.

4.3.1 Integrated Analysis of Control Charts Economic-Risk Design Effect on OC curve

Duncan (1956) proposed the first fully economic model of a Shewhart type control chart to incorporate optimization methodology into determining the control chart parameters. He assumed the assignable cause is to occur according to a Poisson process with a mean number of occurrences per hour λ. The average time of occurrence within an interval between samples, i.e., interarrival rate is

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \cong \frac{h}{2} - \frac{\lambda h^2}{12} \qquad (4.1)$$

Duncan defined the production cycle to be as of four periods. One of these periods was to deal with the out-of-control state. The theoretical formulation from waiting time analysis that the expected length of the out-of-control period is

$$\frac{h}{1-\beta} - \tau \qquad (4.2)$$

Given that the ideal case is when the length of the out-of-control processing is zero, it yields

$$h = (1 - \beta)\tau \text{ or } h = \frac{\tau}{ARL_1} \tag{4.3}$$

$$\text{i.e., } (1 - \beta) = \frac{h}{\tau} \tag{4.4}$$

This ideal case can be obtained rarely in practice unless using 100% inspection considering the process beta error is zero when sampling interval and interarrival rate are equal. Recall in this study that error-free based processing or application of 100% inspection is out of the scope.

From Equation (4.4), sampling interval is proportional to the interarrival rate. The only assumption acceptable in this research is when sampling interval is less than interarrival rate at the (1 – beta) proportion in Equation (4.4) to assure POWER does not asymptotically approach one when h > t. When sampling interval is more than interarrival rate, it is not economically feasible to consider large sample size and not even theoretically applicable to increase the power of chart more than the maximum level.

Goel et al. (1968), Chiu and Wetherill (1974), and Montgomery (1982) have reported optimization methods for Duncan's model. Table 9 lists their works and applications for Duncan's model. They implied Power (1 – beta) when calculating the optimal quality parameters n, k, and h. Note that the goal of these pioneer studies was to minimize the loss function. Dealing with the models in Table 9 and rearranging some of the equations confirm several economic-risk effects related to Power (1-beta) discrimination, which is the main purpose of this study.

Table 9. Different Control Charts Economic Models

Duncan (1956)	Goel, Jain, and Wu (1968)	Chiu and Wetherill (1974)	Montgomery (1982)
$h \cong \sqrt{\dfrac{\alpha T + b + cn}{\lambda M\left(\frac{1}{P} - 0.5\right)}}$	$h = \sqrt{\dfrac{\alpha T + b + cn}{\lambda M\left(\frac{1}{P} - 0.5\right)}}$	$h = \sqrt{\dfrac{\alpha T + b + cn}{\lambda M\left(\frac{1}{P} - 0.5\right)}}$	$h = \sqrt{\dfrac{a_3' + a_1 + a_2 n}{\lambda a_4\left(\frac{1}{(1-\beta)} - 0.5\right)}}$
$\alpha \cong \dfrac{h^2 M\left(\frac{1}{P} - 0.5\right) - (b + cn)}{T}$	$\alpha = 2\Phi(-k)$	$\alpha = 2\Phi(-k)$	$\alpha = 2\Phi(-k)$
$P = \Phi(\delta\sqrt{n} - k)$		$P = \Phi(\delta\sqrt{n} - k)$	$(1-\beta) = \Phi(\delta\sqrt{n} - k)$
$P = \dfrac{1}{\frac{\alpha T + b + cn}{\lambda M h^2} + 0.5}$	$P = \dfrac{1}{\frac{\alpha T + b + cn}{\lambda M h^2} + 0.5}$	$P = \dfrac{1}{\frac{\alpha T + b + cn}{\lambda M h^2} + 0.5}$	$(1-\beta) = \dfrac{1}{\frac{a_3' + a_1 + a_2 n}{\lambda a_4 h^2} + 0.5}$
$a_1 = b$: the fixed cost of sampling $a_2 = c$: the variable cost of sampling $a_3' = T$: the cost of investigating a false alarm $a_4 = M = V_o - V_1$: the hourly penalty cost for operating in the out of control state λ: mean number of occurences per hour δ: the magnitude of the process shift $P = (1-\beta)$: Probability that an assignable cause will be detected $=$ the power of the chart			

For a given (1-beta) and a given h/τ when h is less than τ, the following relationships are illustrated in Table 10. Sampling interval has a positive relationship with the POWER up to the ratio $(1-\beta) = h/\tau$. Thereafter, as h $\rightarrow \infty$, $(1 - \beta) \rightarrow 1$ asymptotically. This is conditional on the only proportion of sampling-interval/interarrival-rate to be less than 1, i.e., sampling interval is less than interarrival rate. From Table 10, the relationship $(1-\beta) = \Phi(\delta\sqrt{n} - k)$ indicates that sample size is positively related to the power of the test. When sample size increases as control limits width is constant Power increases. From Table 10, control limits width is negatively related to the power of the test. When control limits width is tightened as sample size is held constant Power increases. For a given (1-beta) and a given h/τ when h is less than τ, sample size and control limits width are interrelated (Table 10). Increasing the sample size widen the control limits width. Likewise, Increasing the control limits width increases sample size.

Alpha error is affected by all the studied quality parameters. From Table 9, for a given (1-beta), alpha error increases as sampling interval increases and sample size is constant. While holding the sampling interval constant, sample size is negatively related to alpha error. For a given (1-beta), increasing sample size decreases the alpha. From Table 9, control limits width shows a negative implication on alpha error. When control limits width decreases alpha increases.

Table 10. Effect Analysis of Quality Parameters to Power

Power = 0.80 z =0.842 h=0.80 tau=1.0	Power = 0.90 z =1.283 h=0.90 tau=1.0	Power = 0.95 z =1.645 h=0.95 tau=1.0	Power = 0.99 z =2.326 h=0.99 tau=1.0
n = 6 k = 1.608	n = 6 k = 1.167	n = 6 k = 0.805	n = 6 k = 0.123
n = 10 k = 2.321	n = 10 k = 1.880	n = 10 k = 1.517	n = 10 k = 0.836
k = 1 n ≅ 3	k = 1 n ≅ 5	k = 1 n ≅ 7	k = 1 n ≅ 11
k = 1.5 n ≅ 5	k = 1.5 n ≅ 8	k = 1.5 n ≅ 10	k = 1.5 n ≅ 15

Observe in Table 9 that the sampling interval is interrelated to the mean number of occurrences per hour. To maintain a specific POWER, when the mean number of occurrences per hour increases sampling interval decreases. Likewise, when sampling interval increases the mean number of occurrences per hour decreases. From Table 9, sampling interval is related to sample size. If sample size increases, the sampling interval increases proportionally as of $\sqrt{cn}$ increases.

Much of the pioneer research in the development of economic models of control charts has been devoted to $\bar{X}$ chart because of its widespread use in practice (Montgomery, 2009). The p-chart is used to monitor the proportion of nonconforming units in different sample sizes n. It is based on the binomial distribution where each unit has only two possibilities (i.e., defective or not defective). In this research, the $\bar{X}$ chart and the p-chart were selected to study OC curves and the economic-risk effects on the POWER.

From $\bar{X}$ charts and p-charts OC curves, for a given incoming proportion defective, increasing the sample size as control limits width is constant decreases the probability of type II *beta* error, thus enhancing the ability to detect an out-of-control state. The Type-I *alpha* error increases but at a rate much slower than the decrease in the probability of the Type-II *beta* error. Figures 7 and 8 illustrates these relationships for $\bar{X}$ charts and p-charts, respectively.

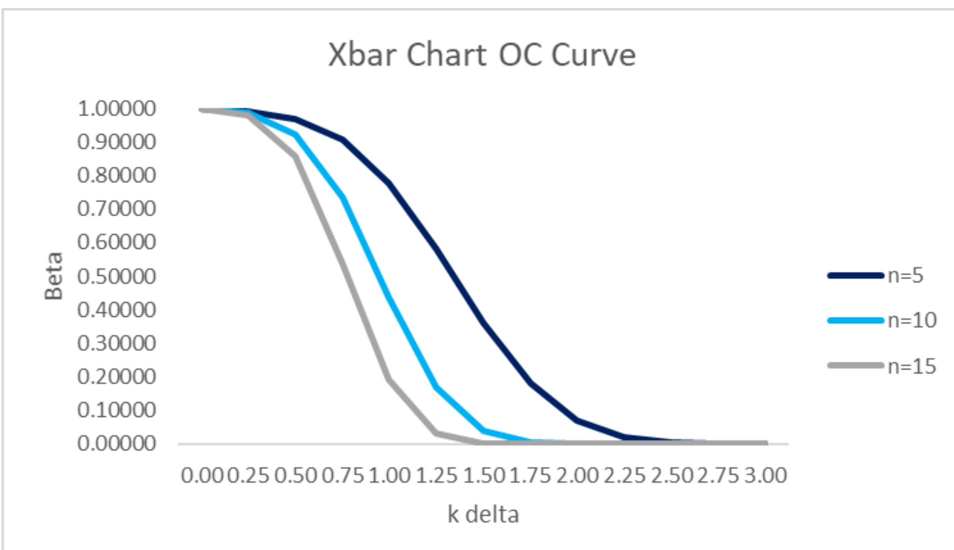

Figure 7. OC-Curves for X¯ Chart for Different Sample Sizes and Constant Control Limits Width.

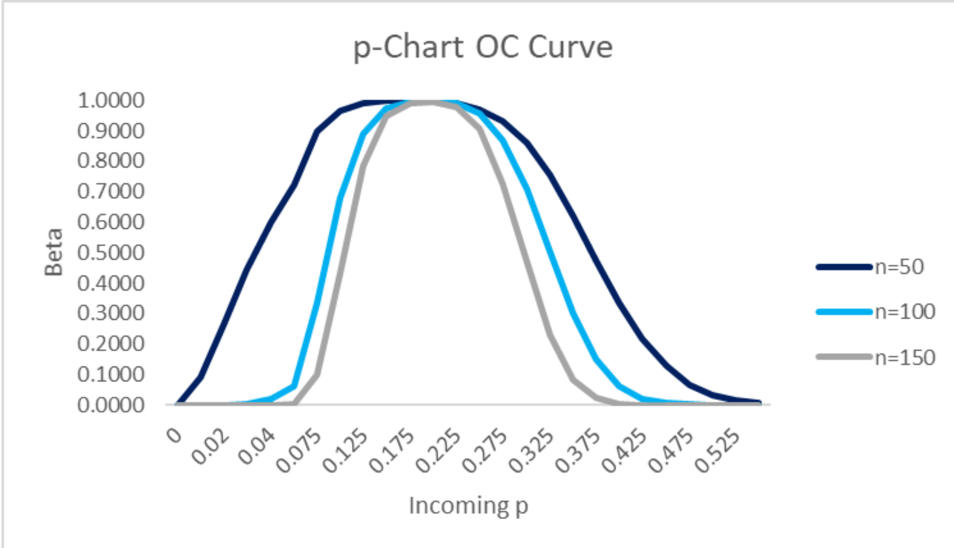

Figure 8. OC-Curves for p-Chart for Different Sample Sizes and Constant Control Limits Width.

For a given incoming proportion defective, when control limits are tightened as the sample size is held constant, the Type-II *beta* error decreases but Type-I *alpha* error increases approximately proportionally. The net result is that the POWER = 1 - β increases but at the expense of increasing the Type-I alpha error. Figures 9 and 10 illustrates these relationships for $\bar{X}$ charts and p-charts, respectively. The sample size implication to the power in $\bar{X}$ charts and p-charts OC curves supports the previous discussion in Table 10. Likewise, the control limits width implication to the power in $\bar{X}$ charts and p-charts OC curves supports the previous discussion in Table 10. Note that for the $\bar{X}$ chart and p-chart analyses, it was assumed that the standard deviation was known and constant. The variability of the process standard deviation is out of this research scope.

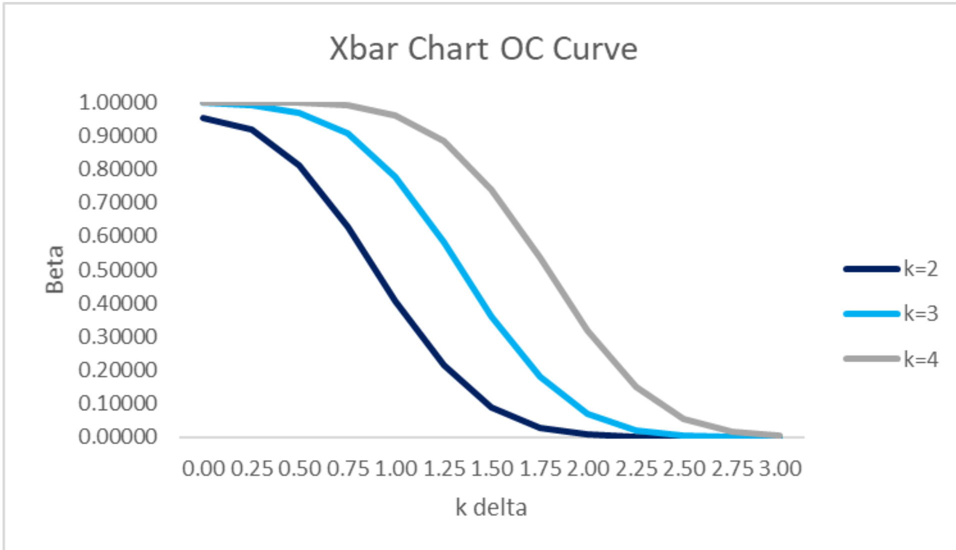

Figure 9. OC-Curves for $\bar{X}$ Chart for Different Control Limits Widths and Constant Sample Size.

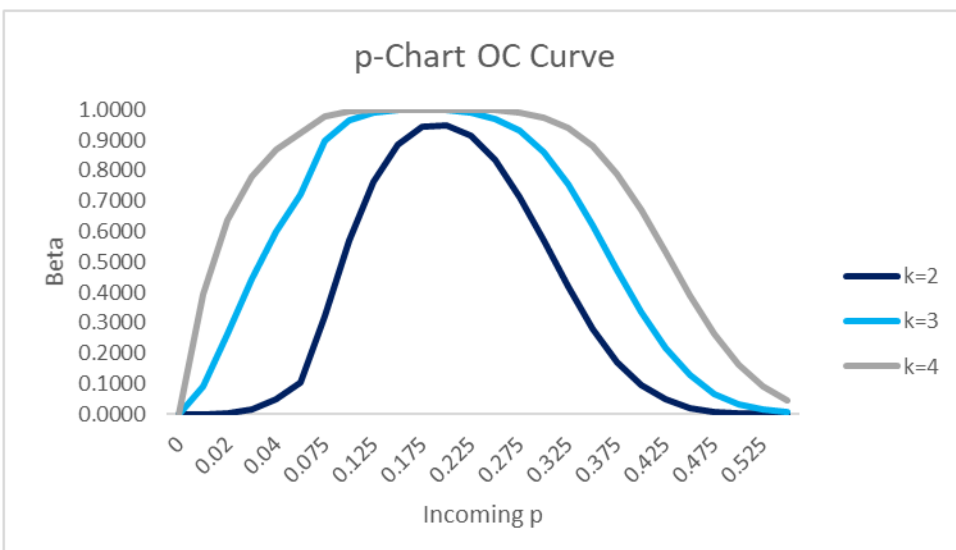

Figure 10. OC-Curves for p-Chart for Different Control Limits Widths and Constant Sample Size.

From Equation (4.3), observe that there is a negative relationship between average run length and power of the test. From the $\bar{X}$ chart and p-chart OC curves, ARL was compared using the cases of sample size and control limits width variation discussed above. Average run length (ARL) is the average number of points that must be plotted on average before a point indicates an out-of-control condition. ARL is a valid basis for evaluating the performance of a control chart assuming the process is reasonably stable (Montgomery, 2009). Appendix C sets forth the calculations of the power and ARL using MS Excel. The purpose of this study is to enhance the quality systems joint power of the test which means equating ARL1 and tau. Table 11 summarizes the comparison analysis of the signs of first derivatives of ARL with respect to n and k among the different studied cases. The positive effect of sample size to the power as well as the negative effect of control limits width to the power confirm the ideal case of $\bar{X}$ chart and p-chart charts OC curves.

Table 11. The Comparison Analysis of the Signs of First Derivatives of ARL with Respect to n and k among the Different Studied Cases.

Control Charts	Variation of n & k	Alpha	Beta	Power	ARL
X-bar chart	**n ↑ k fixed**	↑	↓	↑	↓
	n fixed k ↓	↑	↓	↑	↓
p-chart	**n ↑ k fixed**	↑	↓	↑	↓
	n fixed k ↓	↑	↓	↑	↓

Some of the extracted economic variables in control charts are committed to risk errors as mentioned earlier in this research. For instance, a Type-I error is expected to positively affect the cost of looking for trouble when none exists. While a Type-II error is expected to positively affect the hourly penalty cost of producing out-of-control product. From the concept of statistical power of the test, the causal loops diagram (CLD) representing the integrated control charts economic-risk design sets for the theoretical framework that explains crucial relationships that are dependent on combinations of sample size, control limit distance, and sampling interval (Figure 11).

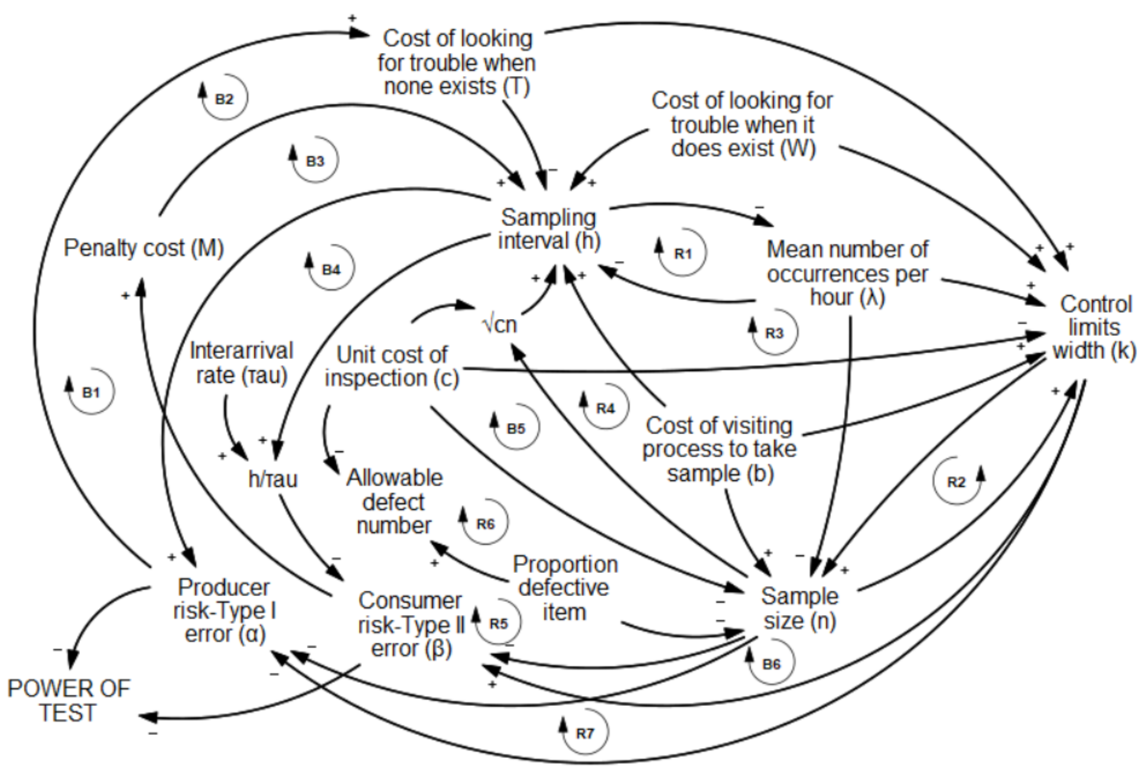

Figure 11. CLD of Integrated Control Charts Economic-Risk Design.

The following research variable relationships and resulting propositions are suggested by the models' synthesis in developing the theoretical framework of quality control charts economic-risk design:

Proposition 1. Holding all else constant, the sample size is positively related to the power of test. When sample size increases alpha increases slightly and beta decreases. The dominance implication of beta forces the power to be more discriminate. Hence, the power of test is increased and ARL is reduced.

Proposition 2. Holding all else constant, the control limits width has a negative effect to the power of test. When control limits width is tightened, alpha increases slightly and beta decreases. The dominance implication of beta forces the power to be more discriminate. The power of test is reduced and ARL is increased.

Proposition 3. Holding all else constant, the sampling interval has a positive effect to the power of test. This is conditional to the h/tau ratio to be less than 1. When sampling interval increases POWER increases when the sampling interval is less than interarrival rate.

Proposition 4. Sampling interval is proportional to interarrival rate (see proposition 3). For a given power of the test, if sampling interval increases interarrival rate increases. Sampling interval must be less than interarrival rate to ensure Power not to exceed the optimal level.

Proposition 5. A Type-II *beta* error dominates a Type-I *alpha* error when effecting the power. This is shown in several OC curves when the change of alpha is limited comparing to large change of beta, so the latter affects the power more than the former.

Proposition 6. Holding all else constant, the control limits width has a negative effect to the Type-I *alpha* error. When control limits width is tightened, alpha always increases and vice versa.

Proposition 7. Holding all else constant, the sampling interval has a positive effect to the Type-I *alpha* error. When sampling interval increases alpha increases and vice versa.

Proposition 8. Holding all else constant, the sample size has a negative effect to the Type-I *alpha* error when sample size increases alpha decreases and vice versa.

Proposition 9. Sample size and control limits width are interrelated if sample size increases control limits width is widened and vice versa. Likewise, if control limits width is tightened sample size decreases and vice versa.

Proposition 10. Sampling interval is related to sample size. If sample size increases sampling interval increases proportionally as $\sqrt{cn}$ increases. The variation of variable cost of sampling affects the amount of increase in sampling interval.

Proposition 11. The effect of control limits width to the power is more than the effect of sample size to the power. The relationship in Proposition 9 is not proportional. The increase of control limits width is linear with the Power while the increase of sample size is not.

4.3.2 Integrated Analysis of Sampling Plans Economic-Risk Design Effect on OC curve

The discrimination (slope) of the sampling operating characteristic curve is determined by the AQL p_1 proportion nonconforming and alpha risk and by the RQL p_2 proportion nonconforming and beta risk points.

$$1 - \alpha = \sum_{d=0}^{c} \binom{n}{d} p_1^d (1 - p_1)^{n-d}$$

$$\beta = \sum_{c=0}^{c} \binom{n}{d} p_2^d (1 - p_2)^{n-d}$$

From the extracted economic-variance components analysis of sampling plans, probability of acceptance (P_a) implies the discriminatory power of the sampling plan. For either the attribute characteristic or continuous variable OC curve, the power of the test = 1 – β, the probability of acceptance (P_a) given that the incoming p proportion nonconforming is greater than or equal to the LTPD. For a given AQL p_1 sampling plan and an incoming LPTD p_2 proportion defective, the relationship between probability of acceptance and the power of the test is negative. In other words, the power is related negatively to the consumer's risk as the sample size *n* and corresponding acceptance number or *k* distance increase. For a given LTPD p_2 sampling plan, the power of the test remains relatively constant at 1 - β while the alpha error decreases for a given p_1 proportion defective as the sample size *n* and acceptance number or *k* distance increase. For a given AOQL sampling plan, the power of the test increases for a given LTPD p_2 while the alpha error decreases for a given AQL p_1 as the sample number n and acceptance number or *k* distance increases. In general, as long a quality system is following a standard ANSI/ASQ Z1.4 or Z1.9 sampling scheme or a Dodge and Romig LTPD or AOQL sampling scheme, increasing the sample size, and corresponding acceptance number or k distance results in decreasing alpha error for a stated AQL p1 and increased the 1 – beta power for a stated LPTD p2 proportion nonconforming.

Sampling plan has an ideal case when only discriminates perfectly between acceptable and unacceptable lots. The OC curve representing this ideal case runs horizontally at a Pa=1 until the level of fraction defective considered unacceptable is reached. At that point, the OC curve drops vertically to Pa=0 and continues horizontally for all lot fraction defectives greater (Figure 12). Yet, this ideal OC curve can be obtained rarely in practice unless using 100% inspection considering the inspection is error free or by increasing the sample size. The application of 100% inspection is out of this research scope. The OC curve becomes more like the idealized OC curve shape as the sample size increases (Montgomery, 2009). However, the sample size, n, and acceptance number, c, are related when dealing with the discriminatory power. The combinations of accept-number/sample-size have different effects on the power.

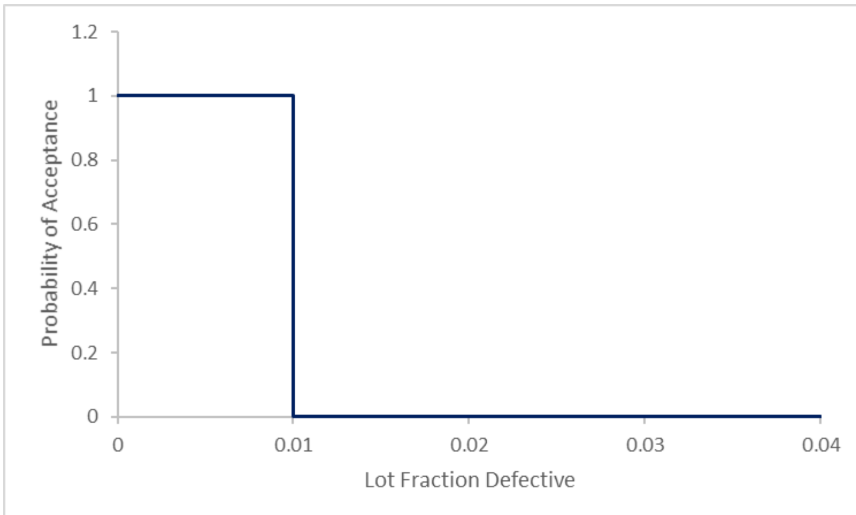

Figure 12. The Ideal OC Curve of Sampling Plans.

Figure 13 illustrates the case of maintaining a constant accept-number/sample-size ratio. For a given incoming proportion defective, if the sample size and acceptance number increase the producer's risk and consumer's risk decrease. This case enhances the discriminatory power toward the ideal case of sampling plan OC curve. In Figure 14, for a given incoming proportion defective, if the accept-number increases at a faster rate than the sample size or at constant sample size the producer's risk decreases but consumer's risk increases, thus the power of the test is affected negatively. In Figure 15, for a given incoming proportion defective, if the sample size increases at a faster rate than the accept-number or at constant accept-number the consumer's risk decreases but producer's risk increases, the power of the test is affected negatively. There are tradeoffs in

AQL-alpha and LTPD-beta fits. As the sample size and accept-number become large, the fixed AQL-LTPD plans will converge to stability but will be the most expensive.

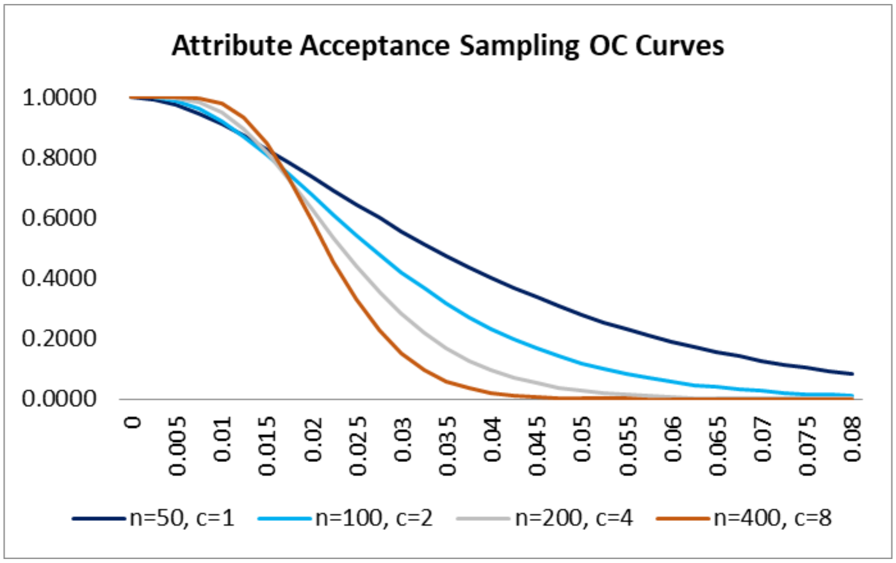

Figure 13. The Effect of Changing Sample Size and Acceptance Number on the OC Curve (Constant Ratio).

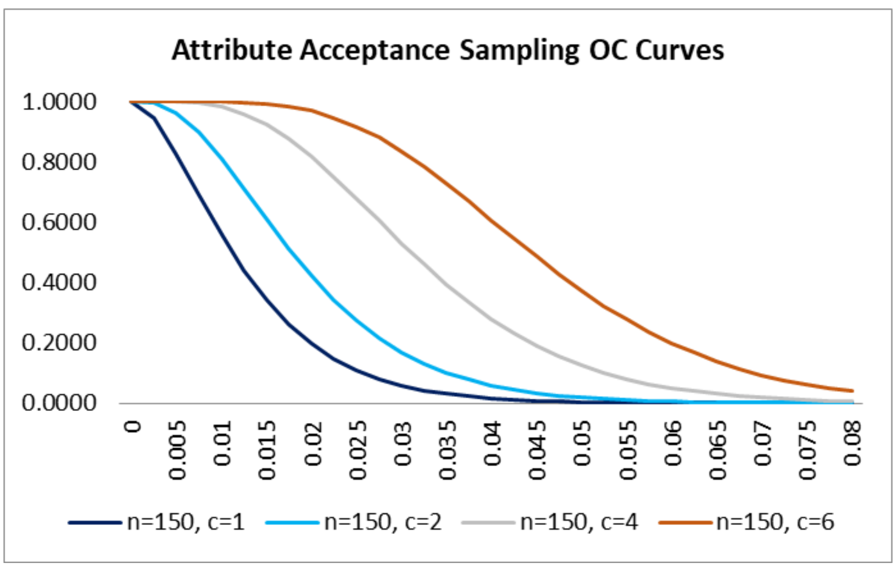

Figure 14. The Effect of Changing Acceptance Number on Sampling Plans OC Curve.

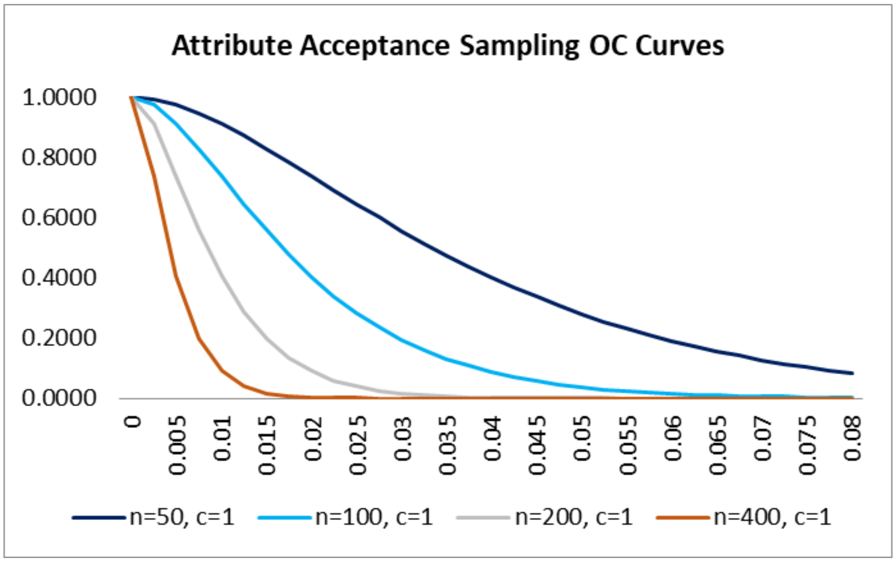

Figure 15. The Effect of Changing Sample Size on Sampling Plans OC Curve.

From sampling plans and OC curves, the effect on alpha and beta depends on the sampling scheme criterion AQL, AOQL, or LTPD. Since the purpose of this study was to formulate a theoretical framework to enhance the discriminatory power in the quality systems economic-risk design, the combinations of accept-number/sample-size effect was evaluated using the sampling scheme criterion AQL, AOQL, and LTPD. Due the tradeoffs in AQL-alpha and LTPD-beta fits, the effect of sample size and acceptance number or distance k selection on quality system 1 – beta discriminatory power will be conditional on the sampling scheme selected.

For AQL plans, the accept-number/sample-size is structured to hold the AQL-alpha point constant and reducing beta-p_2 value to maintain quality at target. The only case showing the constant AQL-alpha point is when structuring a proportional ratio for sample size and accept-number (Figure 16). The proportional increase of accept-number/sample size ratio reduces beta and increases the power of test. The relationship between sample size and power of the test is positive when sample size increases at a faster rate than the accept-number or accept-number is almost constant. However, declining ratio is where the sample size increases at a faster rate than the accept number or at constant accept number increases alpha. In this case, AQL plans cannot hold AQL-alpha point constant, thus it is not applicable with the purpose of enhancing the discriminatory power. The relationship between acceptance number and power of the test is negative when accept-number increases at a faster rate than the sample size or sample size is almost constant as beta increases. In this case, AQL plans also cannot hold AQL-alpha point constant and affects the power negatively, thus it is not applicable with the purpose of enhancing the discriminatory power. For AQL plans, structuring a proportional ratio of accept-number to sample size was used in formulating the theoretical framework of AQL sampling plans economic-risk design. The comparison of fitting AQL-plans OC curve is summarized in Table 12 after the discussion of LTPD and AOQL plans.

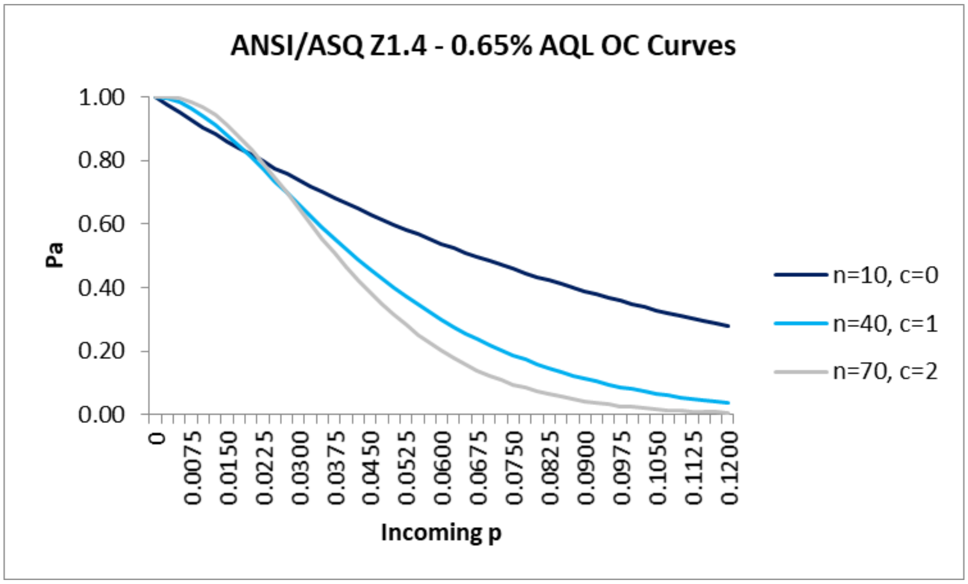

Figure 16. AQL-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size.

For LTPD sampling plans, the accept-number/sample-size is structured to hold the LTPD-beta point constant and reducing alpha to assure quality is not worse than target. As the sample size and accept-number increase at an almost constant rate while holding the LTPD-beta point constant, alpha is reduced, and the power of the test is enhanced (Figure 17). The relationship between sample size and power of the test is negative when sample size increases at a faster rate than the accept-number or accept-number is almost constant. For a constant accept-number and LTPD p_2, as the sample size increases, alpha increases for the set LTPD and Power of the test decreases. In this case, LTPD plans cannot hold LTPD-beta point constant, thus it is not applicable with the purpose of enhancing the discriminatory power. The relationship between acceptance number and power of the test is negative when accept-number increases at a faster rate than the sample size or sample size is almost constant. If the accept-number increases at a faster rate than the sample size, the (1 – beta) power will increase over some range of incoming p-nonconforming and decrease over other range of the incoming p-nonconforming range. However, for a constant sample size and LTPD p_2, as the allowable defect number increases, beta increases for the set LTPD and Power of the test decreases. In this case, LTPD plans also cannot hold LTPD-beta point constant and affects the power negatively, thus it is not applicable with the purpose of enhancing the discriminatory power. For LTPD plans, structuring a proportional ratio for sample size and accept-number was used in formulating the theoretical framework of LTPD sampling plans economic-risk design. The comparison of fitting LTPD-plans OC curve is summarized in Table 12 after the discussion of AOQL plans.

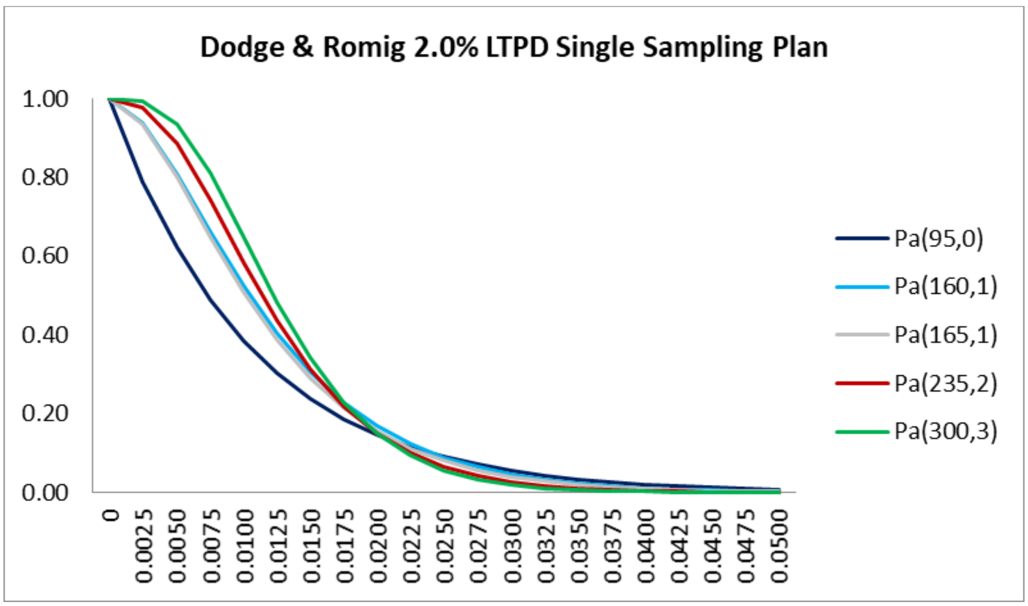

Figure 17. LTPD-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size.

For AOQL sampling plans, the accept-number/sample-size is structured to hold the AOQL constant. As the sample size and accept number increase, alpha and beta are reduced, thus the 1-

beta power increases (Figure 18). Holding a constant sampling size or accept-number is not feasible because AOQL plans will not fit. Hence, the proportional ratio of accept-number/sample-size is used for AOQL plans with the purpose of enhancing the discriminatory power of sampling plans economic-risk design (Table 12).

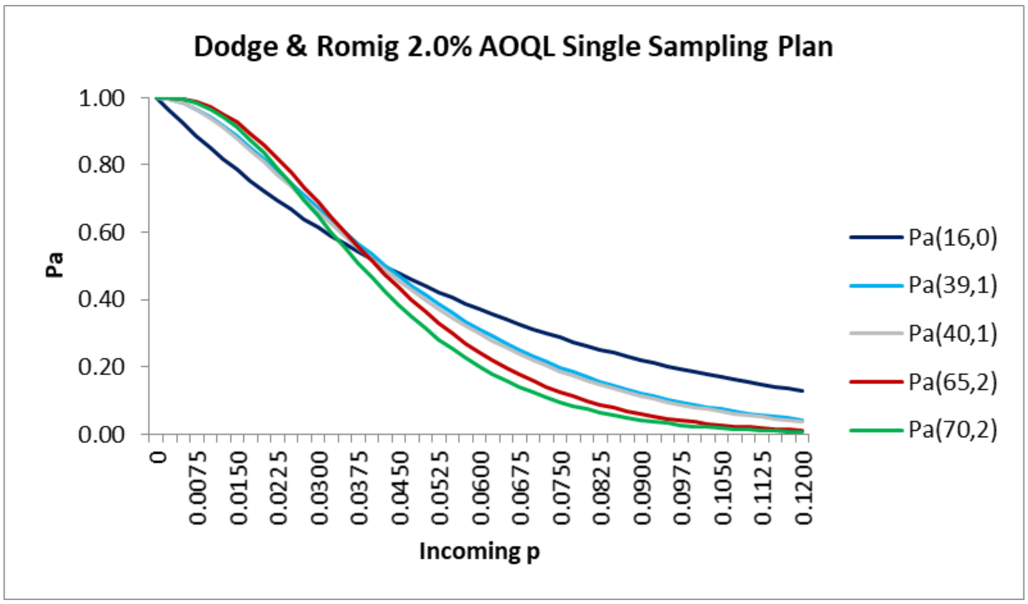

Figure 18. AOQL-Sampling Plans OC Curve for Constant Ratio of Accept-Number/Sample Size.

Table 12. The Comparison of Fitting Sampling Plans OC Curve.

Sampling plan	Variation of c/n	Alpha	Beta	Power
AQL plan	**n ↑ c ↑**	**Fixed**	↓	↑
	n fixed c ↑	↓	↑	↓
	n ↑ c fixed	↑	↓	↑
LTPD plan	**n ↑ c ↑**	↓	**Fixed**	↑
	n fixed c ↑	↓	↑	↓
	n ↑ c fixed	↑	↓	↓
AOQL plan	**n ↑ c ↑**	↓	↓	↑

From the concept of statistical power of the test, the causal loops diagram representing the integrated sampling plans economic-risk design explains crucial relationships that are dependent on combinations of sample size and acceptance number. Figure 19 represents the theoretical framework of AQL-sampling plans economic-risk design. Figure 20 represents the theoretical framework of LTPD-sampling plans economic-risk design. Figure 21 represents the theoretical framework of AOQL-sampling plans economic-risk design.

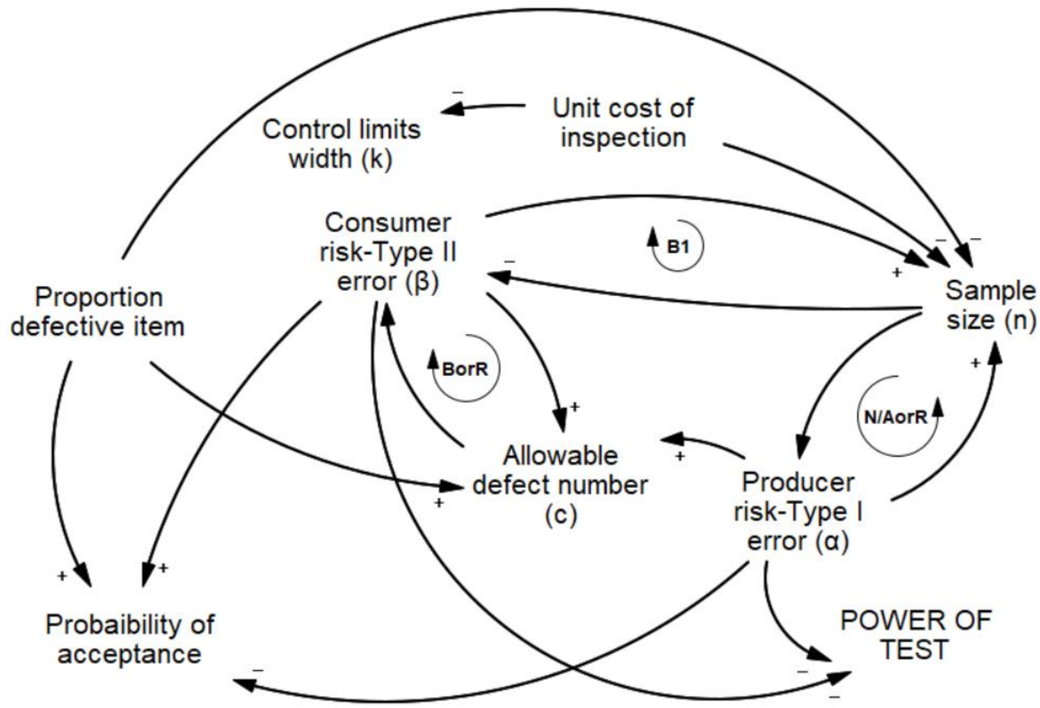

Figure 19. CLD of Integrated AQL-Sampling Plans Economic-Risk Design.

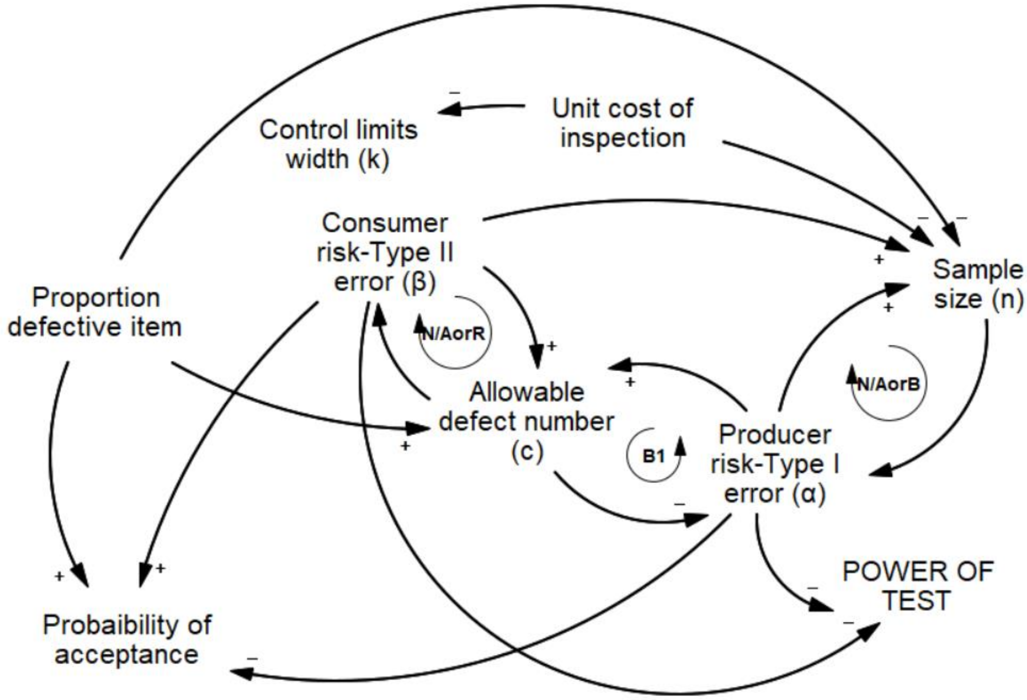

Figure 20. CLD of Integrated LTPD-Sampling Plans Economic-Risk Design.

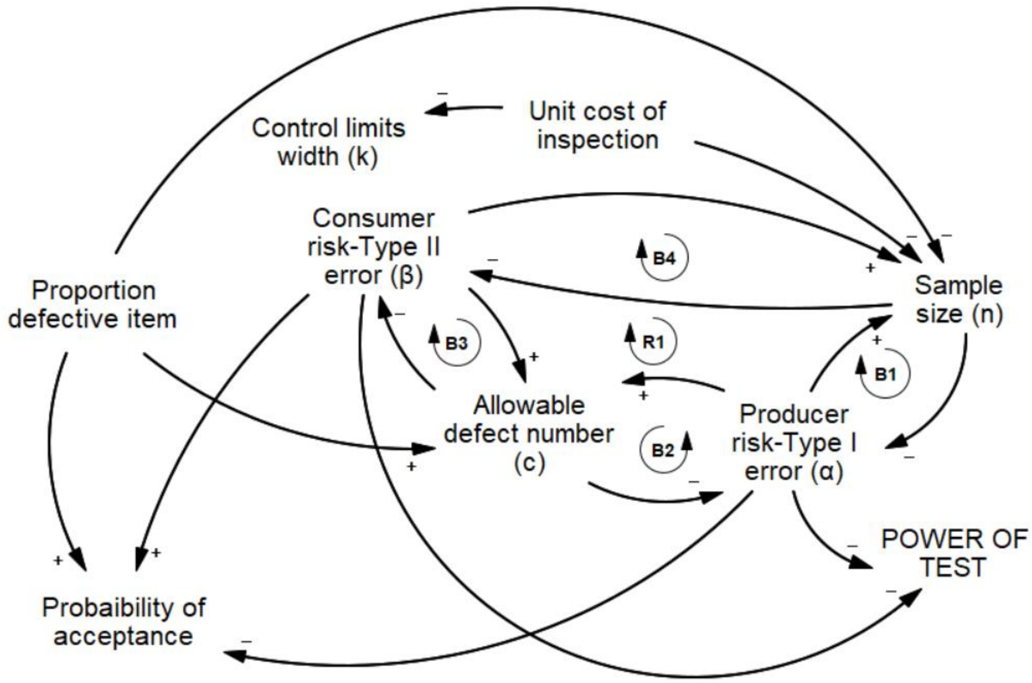

Figure 21. CLD of Integrated AOQL-Sampling Plans Economic-Risk Design.

The following research variable relationships and resulting propositions are suggested by the theoretical framework of quality sampling plans economic-risk design:

Proposition 12. Holding all else constant, the sample size has a conditional positive relationship with power of the test. For AQL plans and AOQL plans, increasing sample size faster than accept-number or at a constant accept-number reduces beta and increases the power. For LTPD plans, as the sample size increases when accept-number is constant, alpha increases for the set LTPD and Power of the test decreases.

Proposition 13. Holding all else constant, the allowable defect number has a conditional negative relationship with power of the test. For AQL plans and LTPD plans, increasing accept number faster than sample size or at a constant sample size increases beta and decreases the power.

For AOQL plans, as the accept number increases, alpha and beta decrease and power of the test increases.

Proposition 14. Structuring a proportional ratio of accept-number/sample-size enhances the discriminatory power in all studied sampling plans. The constant or semi-constant increase of accept-number/sample-size rate maintains adequate levels of producer's risk and consumer's risk toward the ideal case of sampling plans OC curves.

Proposition 15. There is no specific ideal case for sampling plans economic-risk design. Different approaches are proposed to maintain the tradeoffs in alpha and beta toward a more discriminated curve. It depends on the purpose of each research as well as the targeted risks' values to make the appropriate decision and reach the optimum sampling plans economic-risk design.

Proposition 16. The proposed control charts economic-risk design and sampling plans economic-risk design are related to the power of test. The developed CLDs can be inter-connected to generate the theoretical framework of quality systems economic-risk design via the power of the test node. There are common effects in the CLD of integrated control charts economic-risk design and the CLDs of integrated sampling plans economic-risk design. For instance, the relationship between the power of the test and sampling errors is the same in both studied quality systems. Besides, unit cost of inspection has the same implication on sample size and control limits width as shown in all CLDs. Proportion defective item affects sample size and allowable defect number with the same polarity.

Proposition 17. Type-I errors and Type-II errors have negative implications on power of the test. Reducing the two risk errors leads to enhancing the discriminatory power. However, due to the large variation of the consumer's risk comparing to producer's risk, the former dominates the relationship effect on the power. The producer's risk has a slight increase if not almost constant in many cases, but the power is still heading toward the ideal case of OC curve. For the sake of boosting the power of the test, more effort in decreasing the beta is preferred. This proposition is useful not only in sampling plans economic-risk design but also in control charts economic-risk design.

4.4 EVALUATION OF INTEGRATED ANALYSIS OF QUALITY SYSTEMS ECONOMIC-RISK MODELS

The theoretical frameworks of quality systems economic-risk design developed in this research are considered valid and reliable as they depend on established validity and reliability of original systematic literature review. Recall that each admitted paper was evaluated based on admission criteria mentioned in Section 3.3. The proposed theoretical frameworks were assessed to ensure the consistency of discrimination across the OC-curves synthesis as discussed in Section 4.3. As a result, the studied models validated the variation effect using sensitivity analysis so that they are well validated and reliable. Moreover, including only peer reviewed studies enhances the validity and reliability of the unified outcomes of the proposed quality systems economic-risk design theoretical frameworks.

A list of assessment points proposed by Kivunja (2018) was used to reach a decision whether the developed theoretical frameworks are robust or not (Table 13). From the table, reflecting the questions to what accomplished in this research support the validity of the proposed theoretical framework. Besides, the causal loops generated and represented in the diagram of theoretical framework were justified in the previous section to comply with the prior theories related to economic-risk design. The feedback of the proposed theoretical frameworks showed confident outcomes to work as a basis of future applications.

Table 13. Assessment of the Proposed Theoretical Frameworks.

Question	Response	Note
Is my theoretical framework clearly seen as emerging from my literature review?	YES	Admission criteria + variance components analysis
Is it the result of my analysis of the main theories advanced by leaders in the field in which my research is located?	YES	Systematic review + Methodology (chapter 3)
Does it represent or is it relevant to the most current state of theoretical knowledge on my topic?	YES	
Have I explained the meaning embedded in the different parts of the theoretical framework?	YES	Analysis (chapter 4) + Causal Loops Diagram
Does the theoretical framework present a logical, coherent, analytical structure that will be a good coat hanger for my data analysis?	YES	Methodology (chapter 3)
Do the different parts of the theory constitute a coherent, and comprehensive model that is capable of helping me to analyze the relationships among the variables I plan to investigate?	YES	Analysis (chapter 4) + Causal Loops Diagram
Does the theoretical framework target how I will answer my research questions or test the hypotheses?	YES	
Have I documented every source I have used in developing this theoretical framework?	YES	References
Is my theoretical framework a Model, a Table, a Figure, or a description?	YES	Causal Loops Diagram
Have I explained and justified why this is the appropriate theoretical framework for my data analysis?	YES	Analysis (Sections 4.3 and 4.4)

CHAPTER 5

DISCUSSION

This chapter presents a summary of the study, and discusses the research implications from theoretical, methodological, and practical dimensions. It discusses some limitations as well as recommendations for future research related to the qualitative synthesis of quality systems designs.

5.1 OVERVIEW OF FINDINGS

The primary purpose of this research was to develop a qualitative synthesis of the economic-risk design of quality systems from sampling and control chart models theories as into a theoretical framework of quality systems economic-risk design. The main concept of the cost minimization derived theoretically from economic-risk $1 - \beta$ discrimination power of control charts and sampling plans OC curves was chosen for the purpose of qualitative synthesis into a developed framework of economic-risk cost tradeoffs in the design of quality systems.

The first chapter of this research provided a thorough background about contribution of the quality management systems to the enhancement of productivity through the reduction of internal and external quality costs. The theoretical formulation of control charts and sampling plans economic-risk tradeoff models were studied and reviewed. Also, the chapter explained the purpose of the study, research questions, and research significance.

The second chapter focused on the related literature to the study at hand. A body of significant studies were well reviewed covering different types of control charts and sampling plans. Moreover, the literature included various criteria, quality types, situations, and applications. The review of related studies elaborated limitations in the existing theory and application.

The third chapter illustrated the research methodology. The study is relied on the following approaches: variance component analysis, power analysis, admission criteria, qualitative synthesis, and evaluation and validation. The fourth chapter of this study has shown the detailed analyses and results of the execution of the research methodology. The proposed theoretical frameworks of quality systems economic-risk design were developed.

5.2 RESEARCH IMPLICATIONS

5.2.1 Theoretical Perspective

This research contributes to the engineering management and systems engineering (EMSE) body of knowledge by introducing the notion of qualitative synthesis into the field of quality systems. Much research has been conducted in developing the economic-risk designs that minimized only economic loss for control charts theory and sampling plans theory. However, the literature shows no attempts to combine these related theories into a synthesized framework of quality systems economic-risk design based on an organization's $1 - \beta$ discrimination. Therefore, one of the theoretical implications that can be drawn from this research is its ability to conduct a novel contribution in the development of economic-risk designs for organizational discrimination in the quality systems.

5.2.2 Methodological Perspective

Due to the novelty of variance components qualitative synthesis in the field of quality systems, it is essential to use the appropriate research design approach that can achieve the purposes of this study by conducting high-quality systematic analysis and reviews. Of the qualitative synthesis methodologies, meta-aggregation together with interpretation approaches were applied. The synthesis analysis worked on distinguishing the joint effects on quality system statistical power based on discriminatory and common fixed and random effects. Interpretative synthesis constructed a solid method to produce comprehensive and causal explanations; variables and relationships when building the theory qualitatively. Therefore, these methodological processes represented a road map to achieve the primary purpose of the study. Further, the scientific stages followed in the research approach in this study can be applied to similar research purposes in the field of engineering management and systems engineering.

5.2.3 Applications Perspective

The contribution of this research is not limited to theory development but also extends to application. The application of the integrated quality systems theoretical framework can be more beneficial and convenient. Control charts are meant to monitor the process and ensure in-control status, while the lot acceptance or rejection decision is dependent upon the sampling plan. Performing both processes separately can be susceptible to unwanted errors. Therefore, this research provides application solutions that tackle issues related to multi-models in the study of quality systems economic-risk design. Manufacturers or processors that wish to use the proposed quality systems theoretical frameworks can develop their suitable application protocol based on the fundamental components studied in them.

5.3 RESEARCH LIMITATIONS

There are five primary limitations in this research. The first limitation is that sampling plan models were developed with no or little analyses of variance components. For instance, eleven permitted papers in control charts versus barely seven permitted papers in sampling plans demonstrates the lack of validation process of sampling plans economic-risk models. This concern might affect the quality of research outcome. Nonetheless, the integrated variance components revealed in the sampling plans demonstrate a confident outcome supported by saturated relations.

Second limitation lies in the stochastic outcome of sampling plans variance components analysis. The combination of patterned and stochastic components in the synthesis process may threaten the accuracy of the qualitative synthesis results. This is because the diversity of attributes development in modeling sampling plans economic-risk design. Comparative studies were performed to identify the difference gaps as well as develop general concepts. Again, the saturated outcomes mentioned in the previous paragraph mitigates this limitation.

Qualitative synthesis has been applied in health and medical care research. The qualitative synthesis of quality systems economic-risk design is a new concept in the engineering management book of knowledge (BoK) and systems engineering BoK. The literature shows no discussion of the novel application of qualitative synthesis in this field. This posed a challenge to the researcher who needed to transition and cultivate the notion of qualitative synthesis of quality systems economic-risk design into the field of engineering management and systems engineering.

The fourth limitation is that alpha and (1 – beta) expresses only sampling error risks. Measurement-test errors are not incorporated into the theoretical control chart or sampling models in the prior literature and therefore not incorporated into the synthesized economic-risk models in this research.

CLD models were extracted from synthesized control chart and sampling models and followed by set of propositions. Hence, the theoretical assumptions underlying control chart and sampling models apply to the synthesized CLD models.

The last limitation is related to the researcher's ability to perform qualitative synthesis research as sole researcher. To reduce bias, the literature recommends that systematic review must be performed by at least two persons when conducting a qualitative synthesis study (Higgins & Green, 2008). Selection of studies for eligibility and data extraction might be affected. However, bias was minimized through the qualitative synthesis of the variance components peer-reviewed theoretical control chart and sampling models.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 PRIMARY CONTRIBTIONS OF THIS STUDY

The primary contribution of this research was the development of the theoretical frameworks of economic-risk design for control charts of Figure 11 and sampling theories of Figures 19, 20, and 21. The theoretical frameworks were followed by lists of propositions that incorporated the economic-risk implication on Power (1-beta) discrimination.

Another contribution of this research was to fulfil the gap in the prior research of the economic-risk design tradeoff in quality control charts and in sampling plans. The main objective of these seminal studies was to minimize the loss function while in this research it meant to assess the effects on the power discrimination of the quality systems design affected by the economic-risk components.

6.2 WIDENING THE SCOPE

The scope of this research focuses on integrating the quality systems economic-risk design and developing a qualitative synthesis of its variance components. Yet, it does not perform a quantitative study that produces a unified mathematical representation. The next step is to take the qualitative outcomes into the mathematical application to simulate it in a particular organization/system application. This research has laid the theoretical framework foundation for more research that are concerned with economic-risk design in quality systems and applications.

6.3 RECOMMENDATIONS FOR FUTURE RESEARCH

This research has proposed theoretical frameworks of economic-risk design for control charts and sampling theories out of product manufacturing. However, the developed frameworks did not discuss the application of modeling methodology to developing a service quality systems economic-risk model. Therefore, this research supports the opportunity to extend future studies in service quality model. It is recommended to bound the development of service quality systems economic-risk model to the dimensions proposed by Gronroos (1984) in the SERVQUAL, SERVPERF, and HEALTHQUAL service quality models.

The combinatorics of industry/service/government organizational economics boundary constraints on the alpha and beta risks are countably infinite. Hence, identifying logical "clusters" is not feasible. Future research can focus only on (1) general theoretical economic-risk models of control-chart\AQL-sampling, control-chart\RQL-sampling, and control-chart\AOQL-sampling quality systems and (2) adapting each to the particular series-parallel process flow structure in case studies of given organizations within industry/service/government sectors.

As mentioned in the study limitations, the diversity of quality attributes in modeling sampling plans economic-risk design might affect the accuracy of the qualitative synthesis results. It is beneficial to benchmark the prior research conducted in quality control charts economic-risk design that result in the theoretical framework. Hence, it is suggested that more comparative, integrated, and comprehensive studies in quality sampling plans economic-risk design are needed. Performing extensive studies in the suggested area will support its conceptual saturation and enhance the accuracy of the qualitative synthesis outcomes in this dissertation.

Another recommendation related to the study limitations is to conduct more analysis of variation to validate the proposed models in the literature. For instance, the research of sampling plans economic-risk design performed limited sensitivity analysis that is mandated in this dissertation for the purpose of variance components integration. This resulted in few admitted papers as explained in Section 4.2. It is expected to come up with well-developed theoretical frameworks if more evaluation of variance components is considered. Future research meant to fulfil this gap is important.

Because of the novelty in this research, the proposed qualitative methodology is subject to further explanations and improvement. Modification and critiques are welcomed to enhance future theoretical and methodological studies and applications related to this dissertation. One of the potential areas of research that can build on this dissertation is the development of application techniques that work as an assessment instrument. The suggested techniques may reveal potential gaps in the proposed frameworks that can be reviewed and refined. This study does not claim that the proposed frameworks and its components are final. In fact, all researchers, who are concerned with quality systems economic-risk design, are encouraged to build on this study by suggesting modifications or providing critiques to improve its effectiveness.

The visual model that theoretically represents the qualitative synthesis of the economic-risk design of quality systems in this study is the causal loops diagram (CLD). CLD is not static but may evolve or change over time based on the dynamic nature of complex systems. The maps evolve as the purpose of the modeling effort evolves (Sterman, 2000). Hence, CLD can never be comprehensive nor final but always provisional due to its dynamic nature. Future studies to review, understand, and then modify the proposed theoretical frameworks of quality systems economic-risk design are recommended.

CLDs are guidelines for developing full Systems Dynamics models. This is the goal of the development of the CLD theoretical frameworks. Specifically, the CLD frameworks can be synthesized into economic-risk control-chart/sampling systems reflecting the particular economic constraints and control risks of a particular industry/service/governmental organization.

REFERENCES

Alexander, S., Dillman, M., Usher, J., & Damodaran, B. (1995). Economic design of control charts using the Taguchi loss function. *Computers & Industrial Engineering, 28*(3), 671-679.

Al-Nasser, A., & Al-Omari, A. (2013). Acceptance sampling plan based on truncated life tests for exponentiated fréchet distribution. *Journal of Statistics and Management Systems, 16*(1), 13–24.

Anderson, R., & Bancroft, T. (1952). *Statistical theory in research*. McGraw-Hill.

Aslam, M., & Jun, C. (2010). A double acceptance sampling plan for generalized log-logistic distributions with known shape parameters. *Journal of Applied Statistics, 37*(3), 405–414.

Aslam, M., & Kantam, R. (2008). Economic acceptance sampling based on truncated life tests in the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, *24*, 269-276.

Aslam, M., & Shahbaz, M. (2007). Economic reliability tests plans using the generalized exponential distribution. *Journal of Statistics*, *14*, 52-59.

Aslam, M. (2008). Economic reliability acceptance sampling plan for generalized Rayleigh distribution. *Journal of Statistics*, *15*, 26-35.

Bai, D., & Lee, K. (1998). An economic design of variable sampling interval control charts. *International Journal of Production Economics*, *54*(1), 57-64.

Baklizi, A., & Masri, A. (2004). Acceptance sampling based on truncated life tests in the Birnbaum Saunders model. *Risk Analysis, 24*(6), 1453–1457.

Bather, J. (1963). Control charts and the minimization of costs. *Journal of the Royal Statistical Society: Series B (Methodological), 25*(1), 49–70.

Campbell, D., & Stanley, J. (1966). *Experimental and quasi-experimental designs for research*. Rand McNally.

Campling, G. (1968). Serial sampling acceptance schemes for large batches of items where the mean quality has a normal prior distribution. *Biometrika, 55*, 393-399.

Chalmers, T., Smith, H., Blackburn, B., Silverman, B., Schroeder, B., Reitman, D., & Ambroz, A. (1981). A method for assessing the quality of a randomized control trial. *Controlled Clinical Trials, 2*(1), 31-49.

Chen, F., & Yeh, C. (2006). Economic design of control charts with Burr distribution for non-normal data under Weibull failure mechanism. *Journal of the Chinese Institute of Industrial Engineers, 23*(3), 200-206.

Chen, F., & Yeh, C. (2011). Economic statistical design for x-bar control charts under non-normal distributed data with Weibull in-control time. *The Journal of the Operational Research Society, 62*(4), 750-759.

Chen, Y., & Yang, Y. (2002). Economic design of $\bar{x}$-control charts with Weibull in-control times when there are multiple assignable causes. *International Journal of Production Economics, 77*(1), 17-23.

Chen, Y. (2004). Economic design of $\bar{X}$ control charts for non-normal data using variable sampling policy. *International Journal of Production Economics, 92*(1), 61-74.

Chiu, W. (1975). Economic design of attribute control charts. *Technometrics, 17*(1), 81–87.

Chiu, W., & Wetherill, G. (1974) A simplified scheme for the economic design of $\bar{X}$-charts, *Journal of Quality Technology, 6*(2), 63-69.

Cook, T., & Campbell, D. (1979). *Quasi-experimentation: Design and analysis issues for field settings*. Rand McNally.

Cooper, H., & Hedges, L. (1994). *The handbook of research synthesis*. Russell Sage Foundation.

Cox, D. (1960). Serial sampling acceptance schemes derived from Bayes' theorem. *Technometrics, 2,* 353-360.

Daniels, H. (1939). The estimation of components of variance. *Supplement to the Journal of the Royal Statistical Society, 6*(2), 186.

Dayananda, R., & Evans, I. (1973). Bayesian acceptance sampling schemes for two-sided tests of the mean of a normal distribution of known variance. *Journal of the American Statistical Association, 68*, 131-136.

Dodge, H. & Romig, H. (1928). A method of sampling inspection. *Bell System Technical Journal, 8*(4), 613–631.

Dodge, H. (1943). A sampling inspection plan for continuous production. *The Annals of Mathematical Statistics, 14*(3), 264-279.

Duncan, A. (1956). The economic design of | barX charts used to maintain current control of a process. *Journal of the American Statistical Association, 51*(274), 228.

Duncan, A. (1978). The economic design of p-charts to maintain current control of apProcess: Some numerical results. *Technometrics, 20*(3), 235.

Ellis, P. (2010). *The essential guide to effect sizes statistical power, meta-analysis, and the interpretation of research results*. Cambridge University Press.

Fernández, A., & Pérez-González, C. (2012). Optimal acceptance sampling plans for log-location–scale lifetime models using average risks. *Computational Statistics & Data Analysis, 56*(3), 719–731.

Fernández, A. (2017). Economic lot sampling inspection from defect counts with minimum conditional value-at-risk. *European Journal of Operational Research, 258*(2), 573–580.

Fink, R., & Margavio, T. (1994). Economic models for single sample acceptance sampling plans, no inspection, and 100 percent inspection. *Decision Sciences*, *25*(4), 625–645.

Fisher, R. (1925). *Statistical methods for research workers*. Oliver and Boyd.

Girshick, M., & Rubin, H. (1952). A Bayes approach to a quality control model. *The Annals of Mathematical Statistics, 23*(1), 114–125.

Goel, A., Jain, S., & Wu, S. (1968). An algorithm for the determination of the economic design of $\bar{X}$- charts based on Duncan's model. *Journal of the American Statistical Association, 63*(321), 304-320.

Govindaraju, K., & Bebbington, M. (2015). The consumer's risk and costs in zero-acceptance number sampling. *Communications in Statistics - Theory and Methods, 44*(14), 2933–2944.

Graybill, F., & Hultquist, R. (1961). Theorems eoncerning Eisenhart's model II. *The Annals of Mathematical Statistics, 32*(1), 261-269.

Graybill, F., & Wortham, A. (1956). A note on uniformly best unbiased estimators for variance components. *Journal of the American Statistical Association, 51*(274), 266-268.

Graybill, F. (1954). On quadratic estimates of variance components. *The Annals of Mathematical Statistics*, *25*(2), 367-372.

Gronroos, C. (1984). A service quality model and its marketing implications. *European Journal of Marketing, 18*(4), 36-44.

Guenther, W. (1971). On the determination of single sampling attribute plans based on a linear cost model and a prior distribution. *Technometrics, 13*, 483-498.

Hald, A. (1960). The compound hypergeometric distribution and a system of single sampling inspection plans based on prior distributions and costs. *Technometrics, 2*, 275-340.

Hald, A. (1965a) Single sampling inspection plans with specified acceptance probability and minimum average costs. Part I. *Scandinavian Actuarial Journal, 1965*(1-2), 22-64.

Hald, A. (1965b). Bayesian single sampling attribute plans for discrete prior distributions. *Matematisk-Fysiske Skrifter Danske Videnskabernes Selskab, 3*, 1-88.

Hald, A. (1967). Asymptotic properties of Bayesian single sampling plans. *Journal of the Royal Statistical Society, B,29*, 162-173.

Hald, A. (1968). Bayesian single sampling attribute plans for continuous prior distributions. *Technometrics, 10*, 667-683.

Hald, A., & Keiding, N. (1969). Asymptotic properties of Bayesian decision rules for two terminal decisions in multiple sampling. *Journal of the Royal Statistical Society, B, 31*, 455-471.

Hald, A. & Keiding, N. (1972). Asymptotic properties of Bayesian decision rules for two terminal decisions in multiple sampling, II. *Journal of the Royal Statistical Society, B, 34*, 55-74.

Hald, A., & Thyregod, P. (1971). Bayesian single sampling plans based on linear costs and the Poisson distribution. *Matematisk-Fysiske Skrifter Danske Videnskabernes Selskab, 3*, 1-100.

Hannes, K., Petry, K., & Heyvaert, M. (2018). The meta-aggregative approach to qualitative evidence synthesis: a worked example on experiences of pupils with special educational needs in inclusive education, *International Journal of Research & Method in Education, 41*(3), 291-305.

Henderson, C. (1953). Estimation of variance and covariance components. *Biometrics, 9*, 226-252.

Higgins, J., & Green, S. (2008). *Cochrane handbook for systematic reviews of interventions* Wiley.

Ho, C,. & Case, K (1994). Economic design of control charts: A literature review for 1981–1991. *Journal of Quality Technology, 26*(1), 39-53.

Hoon C. (2013). Meta-synthesis of qualitative case studies: An approach to theory building. *Organizational Research Methods, 16*(4), 522-556.

Johansen, S. (1970). Asymptotic properties of the restricted Bayesian double sampling plan. *Technometrics, 12*, 647-667.

Klufa, J. (2010). Exact calculation of the Dodge-Romig LTPD single sampling plans for inspection by variables. *Stat Papers, 51*, 297–305.

Klufa, J. (2014). Dodge-Romig AOQL sampling plans for inspection by variables – Optimal solution. *Procedia Economics and Finance, 12*, 302–308.

Li, Y., Pu, X., & Xiang, D. (2011). Mixed variables-attributes test plans for single and double acceptance sampling under exponential distribution. *Mathematical Problems in Engineering*, 2011.

Lorenzen, T., & Vance, L. (1986). The economic design of control charts: A unified approach. *Technometrics, 28*(1), 3–10.

Manikandan, G., Kannan, S., & Jayabalan, V. (2009). Designing an economic sampling plan: a QFD approach. *International Journal of Quality and Innovation, 1*(1), 65.

Miles, M., & Huberman, A. (1994). Qualitative data analysis an expanded sourcebook (2nd ed.). Sage Publications.

Montgomery, D. (1982) Economic design of an $\bar{X}$ control chart, *Journal of Quality Technology, 14*(1), 40-43.

Montgomery, D. (2009). *Introduction to statistical quality control* (6th ed.). Wiley.

Montgomery, D., & Klatt, P. (1972). Economic design of T2 control charts to maintain current control of a process. *Management Science, 19*(1), 76–89.

Montgomery, D. (1980). The economic design of control charts: A review and literature survey. *Journal of Quality Technology, 12*(2), 75-87.

Montgomery, D., Torng, J., Cochran, J., & Lawrence, F. (1995). Statistically constrained economic design of the EWMA control chart. *Journal of Quality Technology, 27*(3), 250-256.

Mughal, A., Aslam, M., Hussain, J., & Rehman, A. (2010). Economic reliability group acceptance sampling plans for lifetimes following a Marshall-Olkin extended distribution. *Middle Eastern Finance and Economics, 7*, 87-93.

Nezhad, M., & Yazdi, A. (2014). Economic design of acceptance sampling plans based on conforming run lengths using loss functions. *Journal of Testing and Evaluation, 44*(1), 1-8.

Pendrill, L. (2008). Operating 'cost' characteristics in sampling by variable and attribute. *Accreditation and Quality Assurance, 13*(11), 619–631.

Prabhu, S., Montgomery, D., & Runger, G. (1997). Economic-statistical design of an adaptive chart. *International Journal of Production Economics, 49*(1), 1-15.

Rao, G. (2009). A group acceptance sampling plan for lifetimes following a generalized exponential distribution. *Economic Quality Control, 24*(1).

Rao, G. (2010). A group acceptance sampling plans based on truncated life tests for Marshall-Olkin extended lomax distribution. *Electronic Journal of Applied Statistical Analysis, 3*(1),18-27.

Ross, S. (1971). Quality control under Markovian deterioration. *Management Science, 17*(9), 587–596.

Saniga, E. (1977). Joint economically optimal design of $\bar{X}$ and R control charts. *Management Science, 24*(4), 420–431.

Saniga, E. (1989). Economic statistical control-chart designs with an application to $\bar{X}$ and R charts. *Technometrics, 31*(3), 313.

Savage, I. (1962). Surveillance problems. *Naval Research Logistics Quarterly, 9*(3-4), 187–209.

Schleifer, A. (1969). Two-stage normal sampling in two-action problems with linear economics. *Journal of the American Statistical Association, 64*, 1504-1541.

Schmidt, J. & Bennett, G. (1972). Economic multiattribute acceptance sampling. *IIE Transactions, 4*(3), 194–199.

Schuler, W. (1967). Multistage sampling procedures based on prior distribution and costs. *The Annals of Mathematical Statistics, 38,* 464-470.

Searle, S., Casella, G., & McCulloch, C. (1992). *Variance components*. Wiley.

Serel, D., & Moskowitz, H. (2008). Joint economic design of EWMA control charts for mean and variance. *European Journal of Operational Research, 184*(1), 157-168.

Stange, K. (1963). Comparison of costs of inspection by variables and by attributes (in German). *Feitigung Betrieb, 13*, 608-610.

Stephens, K., & Dodge, H. (1976). Comparison of chain sampling plans with single and double Sampling Plans. *Journal of Quality Technology, 8*(1), 24-33.

Sterman, J. (2000). *Business dynamics systems thinking and modeling for a complex world*. Irwin/McGraw-Hill.

Surtihadi, J., & Raghavachar, M. (1994). Exact economic design of $\bar{X}$ charts for general time in-control distributions. *International Journal of Production Research, 32*(10), 2287-2302.

Tagaras, G. (1989). Power approximations in the economic design of control charts. *Naval Research Logistics (NRL), 36*(5), 639-654.

Tagaras, G. (1994). A dynamic programming approach to the economic cesign of X-charts. *IIE Transactions, 26*(3), 48-56.

Tagaras, G. (1994). Economic acceptance sampling by variables with quadratic quality costs. *IIE Transactions, 26*(6), 29-36.

Taylor, H. (1965). Markovian sequential replacement processes. *The Annals of Mathematical Statistics, 36*(6), 1677–1694.

Tippett, L. (1931). *The methods of statistics*. Williams and Norgate Ltd..

Tippett, L. (1943). *Statistical methods in industry*. Iron and Steel Industrial Research Council.

Tippett, L. (1958). A guide to acceptance sampling. *Journal of the Royal Statistical Society, 7*(3), 133-148.

Vommi, V. & Seetala, M. (2007). A new approach to robust economic design of control charts. *Applied Soft Computing, 7*(1), 211-228.

Wald, A. (1947). *Sequential Analysis*. Wiley.

Wall, M. & Elshennawy, A. (1989). Economically based acceptance sampling plans. *Computers & Industrial Engineering, 17*(1-4), 340–346.

Weiler, H. (1952). On the most economical sample size for controlling the mean of a population. *The Annals of Mathematical Statistics, 23*(2), 247–254.

Wetherill, G. & Chiu, W. (1975). A review of acceptance sampling schemes with emphasis on the economic aspect. *International Statistical Review, 43*(2), 191.

Wetherill, G. (1959). The most economical sequential sampling scheme for inspection by variables. *Journal of the Royal Statistical Society, B, 21*, 400-408.

Wetherill, G. (1960). Some remarks on the Bayesian solution of single sample inspection scheme. *Techno-metrics, 2,* 341-352.

Wetherill, G. (1961). Bayesian sequential analysis. *Biometrik, 48*, 281-292.

White, C. (1977). A Markov quality control process subject to partial observation. *Management Science, 23*(8), 843–852.

Woodall, W., Lorenzen, T., & Vance, L. (1986). Weaknesses of the economic design of control charts. *Technometrics, 28*(4), 408.

Wortham, A. (1971). A backward recursive technique for optimal sequential sampling plans. *Naval Research Logistics Quarterly, 18*, 203-213.

Xie, M., Goh, T., & Xie, W. (1997). A study of economic design of control charts for cumulative count of conforming items. *Communications in Statistics - Simulation and Computation, 26*(3), 1009-1027.

Yang, S. (1997). The economic design of control charts when there are dependent process steps. *International Journal of Quality & Reliability Management, 14*(6), 606-615.

Yu, F., & Hou, J. (2006). Optimization of design parameters for control charts with multiple assignable causes. *Journal of Applied Statistics, 33*(3), 279-290.

Yu, F., & Wu, H. (2004). An economic design for variable sampling interval MA control charts. *The International Journal of Advanced Manufacturing Technology, 24*, 41-47.

APPENDIX A

QUALITATIVE ECONOMIC-RISK VARIANCE COMPONENTS ANALYSIS FOR CONTROL CHARTS DESIGNS

Paper ID	CC01
Title	The Economic Design of X-bar Control Charts used to Maintain Current Control of a Process
Authors	Duncan, A.
Year	1956
Objective	Minimize the cost per inspection cycle associated with production in the out-of-control state
Revenue-Risk Variance Components	
Delta point	$(h/2)-(h^2/12)$
Average time to detect	$[(1/p)-(1/2)(\lambda h/12)]h$
Proportion defective	$p \sim \delta\sigma''$
Average cycle length in-control-and-out-of-control	$1/_{\lambda}+\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D$
Proportion of time in control	$\beta=\dfrac{1/_{\lambda}}{1/_{\lambda}+\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D}$
Proportion of time out-of-control	$\gamma=\dfrac{\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D}{1/_{\lambda}+\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D}$
Expected false alarms	$\dfrac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}}\sim\dfrac{\alpha}{\lambda h}\sim\dfrac{\beta\alpha}{h}$
Expected loss/hour false alarms	$\dfrac{\beta\alpha}{h}a_3'$
Model	

$$E[I/T]=\frac{V_0(1/\lambda)+V_1\left[\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D\right]+a_3-a_3'\alpha e^{-\lambda h}/\left(1-e^{-/\lambda h}\right)}{1/_{\lambda}+\left(1/_{p}-1/_{2}+\lambda h/_{12}\right)h+gn+D}-\frac{a_1+a_2n}{h}$$

Risk Components	
α	Probability of a false occurrence
β	Probability of failing to detect a real occurrence
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation

Economic Components	
V_0	Net income/hour in-control operation
V_1	Net income/hour out-of-control operation
(W) a_3	Cost of finding an assignable cause
(T) a_3'	Cost of investigating a false alarm
a_1	Fixed cost of sampling
a_2	Variable cost/unit of sampling
(e) g	Sampling time per unit

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease		
Increase in hourly penalty cost	Functional		Decrease	
Increase in the unit cost of inspection	Functional	Decrease	Increase	Decrease
Increase in the cost of visiting the process to take a sample	Functional	Increase	Increase	
Increase in the cost of looking for trouble when none exists	Functional			Increase
Increase in the cost of looking for trouble when it does exist	Functional			Increase
Increase in the mean number of occurrences per hour	Random		Decrease	

Paper ID	CC02
Title	Economic Design of T^2 Control Charts to Maintain Current Control of a Process
Authors	Montgomery, D and Klatt, P.
Year	1972
Objective	Determine the optimal sample size, interval between samples, and critical region parameter for the Hotelling T^2 control chart
Revenue-Risk Variance Components	
Expected cost per unit of sampling and carrying out the test procedure	$\frac{a_1 + a_2 n}{k}$
Expected cost per unit for investigation and correcting the out-of-control process	$\left(\frac{a_3}{k}\right)(\rho_0\beta_0 + \rho_1\beta_1)$
Expected cost per unit for producing defective items	$a_4(\Phi_0\Upsilon_0 + \Phi_1\Upsilon_1)$
Probability of process in control	$\beta_0 = \frac{\rho_0 P_0}{P_1 + \rho_1 P_0}$
Probability of process out of control	$\beta_1 = \frac{P_1}{P_1 + \rho_1 P_0}$
Average fraction of time elapses before the shift occurs	$\Delta = \frac{1 - \left(1 + \frac{\lambda k}{R}\right) e^{-\frac{\lambda k}{R}}}{\left(1 - e^{-\frac{\lambda k}{R}}\right)^{\frac{\lambda k}{R}}}$
Probability that process is in control at any time	$\Upsilon_0 = \beta_0 P_0 + \Delta\beta_0 P_1$
Probability that process is out of control at any time	$\Upsilon_1 = \beta_1 + (1 - \Delta)\,\beta_0 P_1$
Interval between successive samples	$\frac{\lambda k}{R}$
Model	

$$E[C] = \frac{a_1 + a_2 n}{k} + \left(\frac{a_3}{k}\right)(\rho_0\beta_0 + \rho_1\beta_1) + a_4(\Phi_0\Upsilon_0 + \Phi_1\Upsilon_1)$$

Risk Components	
ρ_0	Probability of a false occurrence (α error)
Φ_0	Probability of failing to detect a real occurrence (β error)
P_1	Probability of detecting a real occurrence (1-β)
λ^{-1}	Occurrence arrival rate
δ	Vector of differences between in control and out of control states
Economic Components	
k	Number of units produced between successive samples
a_1	Fixed cost of sampling
a_2	Variable cost/unit of sampling
a_3	Cost of investigating and correcting a false alarm
a_4	Penalty cost of producing a defective product

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease	Decrease	Increase
Increase in the unit cost of inspection	Functional	Decrease	Increase	Decrease
Increase in the cost of visiting the process to take a sample	Functional		Increase	
Increase in the cost of looking for trouble when it does exist	Functional	Increase		Increase

Paper ID	**CC03**
Title	Economically Optimum Design of Cusum Charts
Authors	Goel, A. and Wu, S.
Year	1973
Objective	To determine the optimum values of the sample size, the sampling interval and the decision limit. To study the effects of the design parameters and the cost and risk factors on loss-cost.
Revenue-Risk Variance Components	
Expected cost per unit of sampling and carrying out the test procedure	$\frac{b+cn}{s}$
Average cycle length in-control-and-out-of-control	${}^{1}/_{\lambda} + \frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)$
Proportion of time in control	$\frac{{}^{1}/_{\lambda}}{{}^{1}/_{\lambda} + \frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)}$
Proportion of time out-of-control	$\frac{\frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)}{{}^{1}/_{\lambda} + \frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)}$
The average number of false alarms before the process goes out of control	${}^{1}/_{\lambda s.L_a}$
Model	
$C = \frac{\left[\frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)\right]M + {}^{Y}/_{\lambda L_a s} + W}{{}^{1}/_{\lambda} + \frac{s}{1-e^{-/\lambda s}} - {}^{1}/_{\lambda} + (L_r - 1)s + (D + en)} + \frac{b+cn}{s}$	
Risk Components	
α	Probability of a false occurrence
β	Probability of failing to detect a real occurrence
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
$(1\text{-}\gamma)I_a$	Net income/hour in-control operation
γI_r	Net income/hour out-of-control operation
(W) a_3	Cost of finding an assignable cause
(Y) a_3’	Cost of investigating a false alarm
b	Fixed cost of sampling
c	Variable cost/unit of sampling
(e) g	Sampling time per unit

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease	Decrease	Increase
Increase in the unit cost of inspection	Functional		Increase	
Increase in the cost of visiting the process to take a sample	Functional		Increase	
Increase in the mean number of occurrences per hour	Random		Decrease	

Paper ID	**CC04**
Title	Economic Design of Fraction Defective Control Charts
Authors	Montgomery, D., Heikes, R., and Mance, J.
Year	1975
Objective	Determining the sample size, control limit or critical region, and interval between samples which minimizes the expected cost of control per unit of product.

Revenue-Risk Variance Components	
Cost of sampling and testing	$\frac{a_1 + a_2 n}{k}$
Probability of remaining in the in-control state during the production	$e^{-\frac{\lambda k}{R}}$
Probability of shifting to any other states	$1 - e^{-\frac{\lambda k}{R}}$
The average fraction of time that elapses before the shift occurs	$1 - \frac{\left(1 + \frac{\lambda k}{R}\right) e^{-\frac{\lambda k}{R}}}{\left(1 - e^{\frac{\lambda k}{R}}\right) \frac{\lambda k}{R}}$
γ_i	$\alpha_i P_{ii} + (1 - \Delta)\alpha_0 P_{0i} + \sum_{l=1}^{i-1} \alpha_l P_{li} + \Delta \alpha_i \sum_{h=i+1}^{8} P_{ij}$

Model

$$E(C) = \frac{a_1 + a_2 n}{k} + \frac{a_3}{k} q' \alpha + a_4\, p' \gamma$$

Risk Components	
δ	Step change in the mean
q	Probability of a false occurrence
P	Probability that the process shifts directly to different states
α	Probability of being in state p when the sample is taken
γ	Probability of being in state p at any point in time

Economic Components	
a_3	Cost for investigating real and false alarm
a_1	Fixed cost of sampling
a_2	Variable cost/unit of sampling
a_4	Penalty cost of producing a defective product

Economic-Risk Variance Components Relationships	**Effect Model**	**Effect to Sample Size**	**Effect to Sampling Interval**	**Effect to Control Limits Width**
Increase in step change in the process mean	Fixed	Decrease	Increase	Increase
Increase in the unit cost of inspection	Functional	Decrease		Increase
Increase in the cost of visiting the process to take a sample	Functional	Increase	Decrease	Increase
Increase in the cost of looking for trouble when it does exist	Functional	Decrease		Increase
Increase in the cost of looking for trouble when none exists	Functional	Decrease		Increase

Paper ID	**CC05**
Title	Minimum Cost Control Schemes Using np Charts
Authors	Chiu, W.
Year	1975
Objective	To see how variation in the various risk and cost factors affects the economic optimum.
Revenue-Risk Variance Components	
α	$\sum_{x=c+1}^{n} \binom{n}{x} p_0^x (1-p_0)^{n-x}$
P	$\sum_{x=c+1}^{n} \binom{n}{x} p_1^x (1-p_1)^{n-x}$
Average time of occurrence in the cycle	$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda - \lambda\, e^{-\lambda h}} = h/2$
Average length of production cycle	$\frac{1}{\lambda} + \frac{h}{P} - \tau + \frac{\alpha t_0 \left(\frac{1}{\lambda} - \tau\right)}{h + t_1}$
Model	
$V_0/\lambda + V_1\,(h/P - \tau) - \alpha A_0 (1/\lambda - \tau)/h - A_1 - (a + bn)(1/\lambda + h/P - \tau)/h$	
Risk Components	
α	Probability that number of defectives exceed c when process is in control
P	Probability that number of defectives exceed c when process is out of control
λ	Occurrence arrival rate
p_0	Proportion of defective items in control state
p_1	Proportion of defective items in out-of-control state
Economic Components	
V_0	Profit/hour in-control operation
V_1	profit/hour out-of-control operation
a	Fixed cost of sampling
b	Variable cost/unit of sampling
A_0	Average search cost
A_1	Average cost to discover and maintain the assignable cause

Economic-Risk Variance Components Relationships	**Effect Model**	**Effect to Sample Size**	**Effect to Sampling Interval**	**Effect to Acceptance number**
Increase in proportion defective items in control state	Random	Decrease		Increase
Increase in the unit cost of inspection	Functional	Decrease	Increase	Decrease
Increase in the cost of visiting the process to take a sample	Functional		Increase	
Increase in the mean number of occurrences per hour	Random		Decrease	

Paper ID	**CC06**
Title	Joint Economically Optimal Design of x and R Control Charts
Authors	Saniga, E.
Year	1977
Objective	To allow the simultaneous consideration of both X and R charts to find the optimal design for these charts, i.e., the sample size, interval between samples and critical regions that minimize expected cost.
Revenue-Risk Variance Components	
Expected cost of sampling and charting	$\frac{a_1 + a_2 n}{k}$
Expected cost of searching for and correcting assignable causes	$a_3 \sum_{i=0}^{2} \frac{\rho_i \alpha_i}{k}$
Expected cost of producing defective units	$a_4 \sum_{i=0}^{2} \gamma_i \delta_i$
Average fraction of time within an interval before the shift occurs	$\frac{1 - \left(1 + \frac{\lambda k}{Q}\right) e^{-\frac{\lambda k}{Q}}}{\frac{\lambda k}{Q}\left(1 - e^{-\frac{\lambda k}{Q}}\right)}$
Model	
$E(C) = \frac{a_1 + a_2 n}{k} + a_3 \sum_{i=0}^{2} \frac{\rho_i \alpha_i}{k} + a_4 \sum_{i=0}^{2} \gamma_i \delta_i$	
Risk Components	
α_i	Probability of a false occurrence
ρ_i	Probability of detecting a real occurrence
δ_i	Probability of producing a defective unit given that the process is in state i
γ_i	Probability that the process is in state i at any time
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
a_3	Cost of searching for and correcting assignable causes
a_1	Fixed cost of sampling
a_2	Variable cost/unit of sampling
a_4	Penalty cost of producing a defective product

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease		
Increase in hourly penalty cost	Functional		Increase	
Increase in the unit cost of inspection	Functional	Decrease	Decrease	Decrease
Increase in the cost of visiting the process to take a sample	Functional	Increase	Decrease	
Increase in the cost of looking for trouble when it does exist	Functional	Increase		Increase
Increase in the mean number of occurrences per hour	Random		Decrease	

Paper ID	CC07
Title	The Economic Design of p-Charts to Maintain Current Control of a Process: Some Numerical Results
Authors	Duncan, A.
Year	1978
Objective	To study the economic design of fraction defective charts (p-charts) that plot sample percentages of defective items and call for action if this percentage falls beyond an upper control.
Revenue-Risk Variance Components	
Proportion defective	$p \sim \delta\sigma_0'$
Probability of detecting the shift on a single sample	$1 - \sum_{x=0}^{d} \frac{n!}{x!\,(n-x)!} \binom{n}{x} p_1^x (1-p_1)^{n-x}$
Probability of a point falling outside the control limit when the process is in the initial state	$1 - \sum_{x=0}^{d} \frac{n!}{x!\,(n-x)!} \binom{n}{x} p_0^x (1-p_0)^{n-x}$
Average time of occurrence of an assignable cause within an interval between samples	$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$
Average number of false alarms per cycle	$A = \frac{\alpha e^{-\lambda h}}{(1-e^{-\lambda h})}$
Time in hours during which the process will on the average be in the shifted	$B = \frac{h}{P} - \tau + gn + D$
Model	
$L = \frac{\lambda MB + \lambda AT + \lambda W}{1+\lambda b} + \frac{b}{h} + \frac{cn}{h}$	
Risk Components	
α	Probability of a point falling outside the control limit when the process is in the initial state
P	Probability of detecting the shift on a single sample
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
b	Fixed cost of sampling
c	Variable cost/unit of sampling
(W) a_3	Cost of finding an assignable cause
(T) a_3’	Cost of investigating a false alarm

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease	Decrease	
Increase in the unit cost of inspection	Functional	Decrease	Increase	
Increase in the cost of visiting the process to take a sample	Functional	Decrease	Increase	
Increase in the cost of looking for trouble when none exists	Functional		Increase	Increase
Increase in the cost of looking for trouble when it does exist	Functional		Increase	Increase
Increase in the mean number of occurrences per hour	Random		Decrease	

Paper ID	**CC08**
Title	Economic design of control charts using the Taguchi loss function
Authors	Alexander, S., Dillman, M., Usher, J., and Damodaran, B.
Year	1995
Objective	To evaluate, optimize and analyze an economic model of the control chart. To study the direction of control chart design parameter changes in the presence of changes in the magnitude and frequency of process shifts and the costs of discovering and correcting the causes of these shifts.
Revenue-Risk Variance Components	
Expected false alarms (B)	$\frac{\alpha e^{-\lambda H}}{1-e^{-\lambda H}} \sim \frac{\alpha}{\lambda H}$
Expected loss/unit	$\frac{A}{\Delta^2} v^2$
Expected cost of sampling	$\frac{a_1 + a_2 N}{H}$
Model	
$E(C) = \frac{a_1 + a_2 N}{H} + \frac{\lambda a_3 + \frac{a_3 \alpha}{H} + L_1 P + L_2 P \lambda B}{1 + \lambda B}$	
Risk Components	
α	Probability of a false alarm
β	Probability of failing to detect a real occurrence
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
(W) a_3	Cost of finding and fixing an assignable cause
(T) a_3’	Cost of investigating a false alarm
pa_4	Penalty cost of producing a defective product
a_1	Fixed cost of sampling
a_2	Variable cost/unit of sampling
(e) g	Sampling time per unit

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease	Increase	
Increase in the cost of visiting the process to take a sample	Functional	Increase	Increase	
Increase in the cost of looking for trouble when none exists	Functional	Increase	Decrease	
Increase in the cost of looking for trouble when it does exist	Functional	Decrease	Increase	
Increase in the mean number of occurrences per hour	Random	Decrease	Decrease	

Paper ID	**CC09**
Title	Economic design of control charts using the Taguchi loss function
Authors	Prabhu, S., Montgomery, D., and Runger, G.
Year	1997
Objective	To develop an economic model for an adaptive chart with dual sample sizes and dual sampling intervals. To optimize the design parameters of an adaptive chart by minimizing the cost function.
Revenue-Risk Variance Components	
Expected number of samples taken before the shift	$\frac{e^{-\lambda t_1}}{1-e^{-\lambda t_1}}$
Average time to an assignable cause and investigation of a false alarm	$A_1 = \frac{\frac{1}{\lambda} + (1-\gamma_1)sT_0}{ARL_0}$
Mean time elapsed after the last sample before the assignable cause and the occurrence of the assignable cause	$\tau = \frac{1-(1+\lambda t_1)e^{-\lambda t_1}}{\lambda(1-e^{-\lambda t_1})}$
Expected time from the occurrence of the assignable cause until the first sample after the assignable cause	$\zeta = \tau_1 p_1 + \tau_2 p_2$
Expected cost per cycle due to the production of nonconformities	$B_1 = C_0\left(\frac{1}{\lambda}\right) + C_1(ATS_\delta - \zeta + E_\delta(N)e + \gamma_1 T_1 + \gamma_2 T_2)$
Total sampling costs during the in-control and out-of-control period	$B_2 = (a+bn_0)s + (a + bE_\delta(N))\left[ARL_\delta + \frac{\gamma_1 T_1 + \gamma_2 T_2}{E_\delta(T)}\right]$
Expected cost for false alarms	$B_3 = Y\left(\frac{s}{ARL_\delta}\right)$
Model	
$E(C) = \frac{B_1 + B_2 + B_3 + W}{A_1 - \zeta + A_2 + ATS_\delta + T_1 + T_2}$	
Risk Components	
α	Type I error probability
β	Type II error probability
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
a	Fixed cost of sampling per unit
b	Variable cost of sampling per unit
C	Expected cost of operating the control procedure
C_1	Expected cost of nonconforming items during out-of-control period
W	Cost of searching, locating, and eliminating an assignable cause
Y	Cost per false alarm

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease		Increase
Increase in the unit cost of inspection	Functional	Increase		
Increase in the cost of visiting the process to take a sample	Functional		Increase	
Increase in the cost of looking for trouble when none exists	Functional	Increase	Decrease	
Increase in the mean number of occurrences per hour	Random	Decrease	Decrease	Increase

Paper ID	CC10
Title	Joint economic design of EWMA control charts for mean and variance
Authors	Serel, D. and Moskowitz, H.
Year	2008
Objective	To design the joint control scheme based on pure economic or both economic and statistical performance criteria using EWMA cost minimization model.

Revenue-Risk Variance Components	
Expected number of samples taken while in control	$S = \frac{e^{-\theta h}}{1 - e^{-\theta h}}$
Expected time between the occurrence of the assignable cause and the time of the last sample taken before the assignable cause	$\tau = \frac{1 - (1 + \theta h) e^{-\theta h}}{\theta (1 - e^{-\theta h})}$
Expected lengths of the in-control intervals	$E(I_{in}) = \frac{1}{\theta} + \frac{(1 - \gamma_1) s T_0}{ARL_0}$
Expected lengths of the out-of-control intervals	$E(I_{out}) = -\tau + nE + h(ARL_1) + T_1 + T_2$
Cost per hour due to nonconformities produced while the process is in control	$C_0 = J_0 P$
Cost per hour due to nonconformities produced while the process is out of control	$C_1 = J_1 P$

Model

$$C = \frac{\frac{C_0}{\theta} + C_1(-\tau + nE + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2) + \frac{sF}{ARL_0} + W}{\frac{1}{\theta} + \frac{(1 - \gamma_1) s T_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2} + (\frac{\frac{a + bn}{h}\left(\frac{1}{\theta} - \tau + nE + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2\right)}{\frac{1}{\theta} + \frac{(1 - \gamma_1) s T_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2}$$

Risk Components	
α	Type I error probability
β	Type II error probability
θ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation

Economic Components	
a	Fixed cost of sampling per unit
b	Variable cost of sampling per unit
C_0	Cost per hour due to nonconformities produced while the process is in control
C_1	Cost per hour due to nonconformities produced while the process is out of control
W	Cost to locate and repair the assignable cause
F	Cost per false alarm

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease		Decrease
Increase in the mean number of occurrences per hour	Random		Decrease	
Increase in hourly penalty cost	Functional		Increase	

Paper ID	CC11
Title	Economic statistical design for x-bar control charts under non-normal distributed data with Weibull in-control time.
Authors	Chen, F. & Yeh, C.
Year	2011
Objective	To develop the economic statistical design model of an x-bar control chart, assuming that the collected data from a manufacture process are not normally distributed and employing the Weibull failure mechanism.
Revenue-Risk Variance Components	
Probability of process shifting	$p = 1 - e^{-\lambda h_1^{\omega}}$
Type I error probability of an x-bar chart for non-normal Burr distribution	$\alpha = 1 + \frac{1}{(1+(M+LS)^C)^K} - \frac{1}{(1+(M-LS)^C)^K}$
Type II error probability of an x-bar chart for non-normal Burr distribution	$\beta = \frac{1}{\left(1+\left(M-LS-S\delta\sqrt{n}\right)^C\right)^K} - \frac{1}{\left(1+\left(M+LS-S\delta\sqrt{n}\right)^C\right)^K}$
Model	
$E(C)/E(T)$	$= ((a+bn)(\beta/(1-\beta)+1/p)+\alpha Y(1-p)/p+p(D_0 - D_1)(1/\lambda)\hat{}(1/\omega)\,\Gamma(1+1/\omega)+D_1 h_1 p(1-p)A(1-p) + \beta h_1 D_1 p(pA(1-p)-(1-\beta)A(\beta)))/(Z_1+(\alpha Z_0(1-p))/p+h_1 pA(1-p)+(\beta h_1 p(pA(1-p)-(1-\beta)A(\beta)))/(1-p-\beta))$
Risk Components	
α	Type I error probability
β	Type II error probability
λ	Occurrence arrival rate
δ	Step change in the mean
σ	Process standard deviation
Economic Components	
a	Fixed cost of sampling per unit
b	Variable cost of sampling per unit
D_0	Expected cost per hour caused by the production of a non-conforming item when the process is in control
D_1	Expected cost per hour caused by production of a non-conforming item when the process is out of control
W	Cost to locate and repair the assignable cause
Y	Cost per false alarm

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Sampling Interval	Effect to Control Limits Width
Increase in step change in the process mean	Fixed	Decrease	Increase	
Increase in the mean number of occurrences per hour	Random		Decrease	
Increase in the cost of looking for trouble when none exists	Functional	Decrease		Decrease
Increase in hourly penalty cost	Functional		Increase	

APPENDIX B

QUALITATIVE ECONOMIC-RISK VARIANCE COMPONENTS ANALYSIS FOR SAMPLING PLANS DESIGNS

Paper ID	**SP01**
Title	Single Sampling and Double Sampling Inspection Tables
Authors	Dodge, H. and Romig, H.
Year	1941
Objective	Presents four sets of sampling inspection tables that have contributed in a notable way to important reductions in such costs and to substantial improvements in control of quality for many characteristics of products.
Revenue-Risk Variance Components	
Number of defects	$M = pN$
Probability of finding m defects in a sample	$p_{m,pn} = \frac{e^{-pn}(pn)^m}{m!}$
Average number of pieces inspected per lot I for product of p quality	$I = n + (N - n)(1 - p_a)$
Average quality after inspection	$p_A = p\frac{N - I}{N}$
Average quality after inspection without replacing defective pieces	$p_A = p\frac{N - I}{N - pI}$
Average outgoing quality limit (AOQL)	$p_L = \max(p_A) = p\frac{N - I}{N}$
Model	
$I_{min} = n + (N - n)\left(1 - \sum_{m=0}^{m=c} p_{m,p_n}\right)$	
Economic and Risk Components	
p_P	Type I error probability of rejecting an acceptable lot (producer's risk)
p_C	Type II error probability of accepting a rejectable lot (consumer's risk)
p_a	Probability of meeting the acceptance criteria
$\bar{p}$	Process average fraction defective

Economic-Risk Variance Components Relationships	**Effect Model**	**Effect to Sample Size**	**Effect to Allowable Defect Number**	**Effect to Sampling Interval**
Increase in process average fraction defective	Random	Decrease	Increase	
Increase in AOQL	Fixed	Increase	Increase	
Increase in LTPD	Fixed	Decrease	Decrease	

Paper ID	SP02
Title	Two-Stage Normal Sampling in Two-Action Problems with Linear Economics
Authors	Schleifer, A.
Year	1969
Objective	To obtain optimal two-stage sampling plan where the size of the second-stage sample can be conditioned on the first-stage sample outcome.
Revenue-Risk Variance Components	
One-stage sampling cost	$s_1(n) = C\delta(n) + cn$
Two-stage sampling cost	$s_1(n_1, n_2) = C_1\delta(n_1) + c_1n_1 + C_2\delta(n_2) + c_2n_2$
Model	
$g(n_1, n_2^o(m_1)) = g(n_1, 0) + E_{m_1}g(n_2^o(m_1), 0)$	
Economic and Risk Components	
C_i	Fixed sampling cost
c_i	Variable sampling cost
V	Process variance

Economic-Risk Variance Components Relationships	Effect Model	Effect to Sample Size	Effect to Allowable Defect Number	Effect to Sampling Interval
Increase in sampling cost	Functional	Decrease		
Increase in the process variance	Fixed	Decrease		

Paper ID	**SP03**
Title	Economic Multiattribute Acceptance Sampling
Authors	Schmidt, J. and Bennett, G.
Year	1972
Objective	To identify the acceptance numbers and sample sizes for the respective attributes which in combination yield the minimum cost plan.
Revenue-Risk Variance Components	
Expected cost of inspection per lot	$C_I = \sum_{i=1}^{m} C_{Ii} n_i$
Expected cost of rejection per lot	$C_R = C_r \left\{1 - \prod_{i=1}^{m} \left[1 - s_i \left(\frac{n_i}{n_i + \lambda_i}\right)^{c_i+1} - (1 - s_i)\left(\frac{n_i}{n_i + \mu_i}\right)^{c_i+1}\right]\right\}$
Expected cost of acceptance per lot	$C_A = L \sum_{i=1}^{m} C_{ai} \left\{\frac{s_i}{\lambda_i} - \frac{s_i}{\lambda_i}\left(\frac{n_i}{n_i + \lambda_i}\right)^{c_i+1} \left[\frac{\lambda_i(c_i + 1)}{n_i + \lambda_i} + 1\right] + \frac{1 - s_i}{\mu_i} - \frac{1 - s_i}{\mu_i}\left(\frac{n_i}{n_i + \mu_i}\right)^{c_i+1} \left[\frac{\mu_i(c_i + 1)}{n_i + \mu_i} + 1\right]\right\} \prod_{i=1}^{m} \left[1 - s_i \left(\frac{n_i}{n_i + \lambda_i}\right)^{c_i+1} - (1 - s_i)\left(\frac{n_i}{n_i + \mu_i}\right)^{c_i+1}\right] + n_m C_p \prod_{i=1}^{m} \left[1 - s_i \left(\frac{n_i}{n_i + \lambda_i}\right)^{c_i+1} - (1 - s_i)\left(\frac{n_i}{n_i + \mu_i}\right)^{c_i+1}\right]$
Model	
	$Min(C_T) = C_I + C_R + C_A$
Economic and Risk Components	
C_{Ii}	Cost of inspecting an item for the ith attribute
C_{ai}	Cost results from undetected presence of ith attribute in an item appears in accepted lot
C_r	Cost of rejecting an inspection lot
C_p	Cost of replacing an item
p_i	Proportion of items in a lot containing ith attribute

Economic-Risk Variance Components Relationships	**Effect Model**	**Effect to Sample Size**	**Effect to Allowable Defect Number**	**Effect to Sampling Interval**
Increase in the process variance	Fixed	Increase		
Increase in proportion defective items	Random	Decrease		

Paper ID	SP04
Title	The effects of inspection error on single sampling inspection plans
Authors	Collins, R., Case, K., and Bennett, K.
Year	1973
Objective	To consider the effects of inspection error on the performance measures AOQ and ATI for both replacement situations. To consider the effects of inspection error on the design of single sampling plans based on the measures LTPD and AQL.
Revenue-Risk Variance Components	
p_e	$p_e = p(1-e_2) + (1-p)e_1$
P_a	$P_a = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$
AOQ	$AOQ = p_{e,1-\alpha} = p_{1-\alpha}(1-e_2) + (1-p_{1-\alpha})e_1$
LTPD	$LTPD = p_{e,\beta} = p_\beta(1-e_2) + \left(1-p_\beta\right)e_1$
p_g	$p_g = (1-p)(1-e_1) + pe_2$
Number of defectives remaining in the uninspected portion of accepted lot times probability of lot acceptance	$p(S-n)Pa_e$
Number of defectives items classed as good in the screened portion of rejected lot times probability of lot rejection	$p(S-n)(1-Pa_e)e_2$
Number of defective items classed as good in the sample	npe_2
Model	
$ATI = \frac{n + (1-Pa_e)(S-n)}{1-p_e}$ Without replacement: $ATI = n + (1-Pa_e)(S-n)$	
Economic and Risk Components	
e_1	Type I error probability (producer's risk)
e_2	Type II error probability (consumer's risk)
P_g	Probability that item is classed as good
P_a	Probability of lot acceptance
p	True fraction defective
p_e	Apparent fraction defective

Economic-Risk Variance Components Relationships	Effect Model	Effect to probability of acceptance	Effect to AOQ	Effect to ATI
Increase in the Type-I error	Random	Decrease	Decrease	Increase
Increase in the Type-II error	Random	Increase	Increase	Decrease

Paper ID	SP05
Title	Economic Models for Single Sample Acceptance Sampling Plans, No Inspection, and 100 Percent Inspection
Authors	Fink, R. and Margavio, T.
Year	1994
Objective	Developing economic models to examine the profitability of different inspection policies. To provide an alternative method that utilizes economic criteria to determine when a process is producing products of the desired quality without the use of 100 percent inspection or acceptance sampling procedures.

Revenue-Risk Variance Components	
F_n	See Eq (21) p.636 (Fink & Margavio, 1994)
F_I	See Eq (22) p.636 (Fink & Margavio, 1994)
δ*	$\sqrt{\frac{P-S}{K}}$

Model

$$\pi_5 = P - P_a\left(\frac{N-n}{N}\right)F_n + \left[P_a\left(\frac{N-n}{N}\right) - 1\right]\left[F_I + I + W - W\left(\phi\left(\frac{T+\delta-\mu_Y}{\sigma_Y}\right) - \phi\left(\frac{T-\delta-\mu_Y}{\sigma_Y}\right)\right)\right]$$

$$\pi_6 = P - P_a\left(\frac{N-n}{N}\right)F_n + \left[P_a\left(\frac{N-n}{N}\right) - 1\right]\left[\frac{F_I + I + W - W\left(\phi\left(\frac{T+\delta-\mu_Y}{\sigma_Y}\right) - \phi\left(\frac{T-\delta-\mu_Y}{\sigma_Y}\right)\right)}{\phi\left(\frac{T+\delta-\mu_Y}{\sigma_Y}\right) - \phi\left(\frac{T-\delta-\mu_Y}{\sigma_Y}\right)}\right] - C$$

Economic and Risk Components	
P_a	Probability of lot acceptance
F_n	Loss Function
F_I	Expected cost of quality from external failures for 100 percent inspection plans
δ*	Producer's specification limit
σ	Process standard deviation
π_5	Expected profit for an acceptance sampling plan using perfect replacement
π_6	Expected profit for an acceptance sampling plan with replacement items possessing the same quality as the current process
P	Selling price
C	Manufacturing cost
S	Salvage value
W	Rework cost
I	Inspection cost
r	Cost of an item rejected once it reaches the customer

Economic-Risk Variance Components Relationships	Effect Model	Effect to Expected Profit	Effect to Allowable Defect Number	Effect to Sample Size
Increase in the standard deviation of the process	Fixed	Decrease		Increase
Increase in cost of customer rejecting item	Functional	Decrease		

Paper ID	SP06
Title	Economic Design of Acceptance Sampling Plans Based on Conforming Run Lengths Using Loss Functions
Authors	Nezhad, M. and Yazdi, A.
Year	2014
Objective	To obtain the optimal control tolerances and the corresponding critical acceptance and rejection thresholds based on the geometric distribution which minimizes the loss function for both producers and consumers.
Revenue-Risk Variance Components	
Producer's loss	$c_p(x) = B$
Consumer's loss	$c_c(x) = A(x - \mu)^2$
Expected inspection loss	E(I)=Equation (9) page 3 (Nezhad & Yazdi, 2014)
Loss of accepting the lot	AL=Equation (15) page 4 (Nezhad & Yazdi, 2014)
Loss of rejecting the lot	RL=Equation (16) page 4 (Nezhad & Yazdi, 2014)
Model	
Expected loss = E(TC) = E(I)+E(AL)+E(RL) = equation (18) page 5 (Nezhad & Yazdi, 2014)	
Economic and Risk Components	
α	Type I error probability (producer's risk)
β	Type II error probability (consumer's risk)
δ	Optimal value of control threshold (tolerance)
$f(x)$	Probability of distribution function of quality characteristics
p	Probability of rejecting the item
c	Cost of inspection
$c_p(x)$	Loss of nonconforming item for producer
$c_c(x)$	Consumer's loss
AL	Loss of accepting the lot
RL	Loss of rejecting the lot

Economic-Risk Variance Components Relationships	Effect Model	Effect to Deviation of value of control threshold	Effect to Sample Size	Effect to Control Limits Width
Increase in the inspection cost	Functional	Fixed	Decrease	Decrease
Increase in consumer's loss	Functional	Decrease		
Increase in producer's loss	Functional	Increase		

Paper ID	**SP07**
Title	Economic lot sampling inspection from defect counts with minimum conditional value-at-risk
Authors	Fernandez, A.
Year	2017
Objective	To determine the defects-per-unit inspection scheme with minimum β-CVaR and controlled producer and consumer risks with the aim of reducing the risk of incurring an excessive cost.
Revenue-Risk Variance Components	
Probability of lot acceptance versus the defect rate	$L(\lambda; n, r) = e^{-n\lambda} \sum_{i=0}^{r-1} \frac{(n\lambda)^i}{i!}$
Model	
$E[C(\Lambda; n, r)] = Sn + R + \int_0^\infty \{A(\lambda) - R\} L(\lambda; n, r) h(\lambda)\, \mathrm{d}\lambda$	
Economic and Risk Components	
λ	Defect rate
α_0	Consumer risk
α_1	Producer risk
S	Cost of sampling inspection per unit
R	Cost of lot rejection
A(λ)	Cost of acceptance of a lot with defect rate λ
β	risk aversion degree
β-VaR	Lowest cost value that is not exceeded with probability β

Economic-Risk Variance Components Relationships	Effect Model	Effect to Probability of lot acceptance	Effect to rejection Number	Effect to Sample Size
Increase in defect rate	Random	Decrease	Increase	Increase
Increase in producer risk	Random		Increase	Increase
Increase in consumer risk	Random		Increase	Increase

APPENDIX C

THE CALCULATION OF POWER FROM QUALITY CONTROL CHARTS OC CURVE

The calculations of the power and ARL from X-bar charts OC curve for different sample sizes and constant control limits width.

X-bar Chart OC Curve Demonstration					
Mean	**0**		**sample size**	**5**	
Stdev	**1**		**Limits width**	**3**	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.001350	0.998650	0.997300	0.002700	370.398
0.25	0.007324	0.999814	0.992490	0.007510	133.159
0.50	0.029920	0.999981	0.970061	0.029939	33.401
0.75	0.092926	0.999999	0.907072	0.092928	10.761
1.00	0.222454	1.000000	0.777546	0.222454	4.495
1.25	0.418819	1.000000	0.581181	0.418819	2.388
1.50	0.638369	1.000000	0.361631	0.638369	1.566
1.75	0.819410	1.000000	0.180590	0.819410	1.220
2.00	0.929508	1.000000	0.070492	0.929508	1.076
2.25	0.978880	1.000000	0.021120	0.978880	1.022
2.50	0.995204	1.000000	0.004796	0.995204	1.005
2.75	0.999181	1.000000	0.000819	0.999181	1.001
3.00	0.999896	1.000000	0.000104	0.999896	1.000
3.25	0.999990	1.000000	0.000010	0.999990	1.000
3.50	0.999999	1.000000	0.000001	0.999999	1.000
3.75	1.000000	1.000000	0.000000	1.000000	1.000
4.00	1.000000	1.000000	0.000000	1.000000	1.000

X-bar Chart OC Curve Demonstration					
Mean	0		sample size	10	
Stdev	1		Limits width	3	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.001350	0.998650	0.997300	0.002700	370.398
0.25	0.013572	0.999925	0.986352	0.013648	73.273
0.50	0.077970	0.999998	0.922028	0.077972	12.825
0.75	0.264906	1.000000	0.735094	0.264906	3.775
1.00	0.564456	1.000000	0.435544	0.564456	1.772
1.25	0.829666	1.000000	0.170334	0.829666	1.205
1.50	0.959370	1.000000	0.040630	0.959370	1.042
1.75	0.994361	1.000000	0.005639	0.994361	1.006
2.00	0.999557	1.000000	0.000443	0.999557	1.000
2.25	0.999981	1.000000	0.000019	0.999981	1.000
2.50	1.000000	1.000000	0.000000	1.000000	1.000
2.75	1.000000	1.000000	0.000000	1.000000	1.000
3.00	1.000000	1.000000	0.000000	1.000000	1.000
3.25	1.000000	1.000000	0.000000	1.000000	1.000
3.50	1.000000	1.000000	0.000000	1.000000	1.000
3.75	1.000000	1.000000	0.000000	1.000000	1.000
4.00	1.000000	1.000000	0.000000	1.000000	1.000

X-bar Chart OC Curve Demonstration					
Mean	0		sample size	15	
Stdev	1		Limits width	3	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.00135	0.99865	0.99730	0.00270	370.398
0.25	0.02109	0.99996	0.97887	0.02113	47.336
0.50	0.14378	1.00000	0.85622	0.14378	6.955
0.75	0.46205	1.00000	0.53795	0.46205	2.164
1.00	0.80866	1.00000	0.19134	0.80866	1.237
1.25	0.96721	1.00000	0.03279	0.96721	1.034
1.50	0.99752	1.00000	0.00248	0.99752	1.002
1.75	0.99992	1.00000	0.00008	0.99992	1.000
2.00	1.00000	1.00000	0.00000	1.00000	1.000
2.25	1.00000	1.00000	0.00000	1.00000	1.000
2.50	1.00000	1.00000	0.00000	1.00000	1.000
2.75	1.00000	1.00000	0.00000	1.00000	1.000
3.00	1.00000	1.00000	0.00000	1.00000	1.000
3.25	1.00000	1.00000	0.00000	1.00000	1.000
3.50	1.00000	1.00000	0.00000	1.00000	1.000
3.75	1.00000	1.00000	0.00000	1.00000	1.000
4.00	1.00000	1.00000	0.00000	1.00000	1.000

The calculations of the power and ARL from X-bar charts OC curve for different control limits width and constant sample size.

X-bar Chart OC Curve Demonstration					
Mean	**0**		**sample size**	**5**	
Stdev	**1**		**Limits width**	**2**	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.022750	0.977250	0.954500	0.045500	21.978
0.25	0.074795	0.994752	0.919957	0.080043	12.493
0.50	0.188898	0.999090	0.810192	0.189808	5.268
0.75	0.373367	0.999882	0.626515	0.373485	2.677
1.00	0.593310	0.999989	0.406679	0.593321	1.685
1.25	0.786718	0.999999	0.213281	0.786719	1.271
1.50	0.912148	1.000000	0.087852	0.912148	1.096
1.75	0.972134	1.000000	0.027866	0.972134	1.029
2.00	0.993285	1.000000	0.006715	0.993285	1.007
2.25	0.998782	1.000000	0.001218	0.998782	1.001
2.50	0.999835	1.000000	0.000165	0.999835	1.000
2.75	0.999983	1.000000	0.000017	0.999983	1.000
3.00	0.999999	1.000000	0.000001	0.999999	1.000
3.25	1.000000	1.000000	0.000000	1.000000	1.000
3.50	1.000000	1.000000	0.000000	1.000000	1.000
3.75	1.000000	1.000000	0.000000	1.000000	1.000
4.00	1.000000	1.000000	0.000000	1.000000	1.000

X-bar Chart OC Curve Demonstration					
Mean	0		sample size	5	
Stdev	1		Limits width	3	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.001350	0.998650	0.997300	0.002700	370.398
0.25	0.007324	0.999814	0.992490	0.007510	133.159
0.50	0.029920	0.999981	0.970061	0.029939	33.401
0.75	0.092926	0.999999	0.907072	0.092928	10.761
1.00	0.222454	1.000000	0.777546	0.222454	4.495
1.25	0.418819	1.000000	0.581181	0.418819	2.388
1.50	0.638369	1.000000	0.361631	0.638369	1.566
1.75	0.819410	1.000000	0.180590	0.819410	1.220
2.00	0.929508	1.000000	0.070492	0.929508	1.076
2.25	0.978880	1.000000	0.021120	0.978880	1.022
2.50	0.995204	1.000000	0.004796	0.995204	1.005
2.75	0.999181	1.000000	0.000819	0.999181	1.001
3.00	0.999896	1.000000	0.000104	0.999896	1.000
3.25	0.999990	1.000000	0.000010	0.999990	1.000
3.50	0.999999	1.000000	0.000001	0.999999	1.000
3.75	1.000000	1.000000	0.000000	1.000000	1.000
4.00	1.000000	1.000000	0.000000	1.000000	1.000

X-bar Chart OC Curve Demonstration					
Mean	0		sample size	5	
Stdev	1		Limits width	4	
k	Phi(-L)	Phi(+L)	Beta	Power	ARL
0.00	0.000032	0.999968	0.999937	0.000063	15787.19
0.25	0.000290	0.999997	0.999708	0.000292	3420.298
0.50	0.001976	1.000000	0.998024	0.001976	506.030
0.75	0.010091	1.000000	0.989909	0.010091	99.099
1.00	0.038872	1.000000	0.961128	0.038872	25.726
1.25	0.114118	1.000000	0.885882	0.114118	8.763
1.50	0.259173	1.000000	0.740827	0.259173	3.858
1.75	0.465383	1.000000	0.534617	0.465383	2.149
2.00	0.681585	1.000000	0.318415	0.681585	1.467
2.25	0.848765	1.000000	0.151235	0.848765	1.178
2.50	0.944102	1.000000	0.055898	0.944102	1.059
2.75	0.984190	1.000000	0.015810	0.984190	1.016
3.00	0.996618	1.000000	0.003382	0.996618	1.003
3.25	0.999457	1.000000	0.000543	0.999457	1.001
3.50	0.999935	1.000000	0.000065	0.999935	1.000
3.75	0.999994	1.000000	0.000006	0.999994	1.000
4.00	1.000000	1.000000	0.000000	1.000000	1.000

The calculations of the power and ARL from p-charts OC curve for different sample sizes and constant control limits width.

P-Chart OC Curve Demonstration					
P(0)	**0.2**		**sample size**	**50**	
S	**0.05657**		**Limits width**	**3**	
L	**0.0303**	**0.3697**	**B(limit)**	**1**	**18**
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	0.910565	1.000000	0.089435	0.910565	1.098
0.02	0.735771	1.000000	0.264229	0.735771	1.359
0.03	0.555280	1.000000	0.444720	0.555280	1.801
0.04	0.400481	1.000000	0.599519	0.400481	2.497
0.05	0.279432	1.000000	0.720568	0.279432	3.579
0.075	0.102501	1.000000	0.897499	0.102501	9.756
0.1	0.033786	1.000000	0.966214	0.033786	29.598
0.125	0.010261	0.999996	0.989735	0.010265	97.418
0.15	0.002905	0.999940	0.997035	0.002965	337.264
0.175	0.000772	0.999525	0.998753	0.001247	802.127
0.2	0.000193	0.997489	0.997296	0.002704	369.839
0.225	0.000045	0.990349	0.990304	0.009696	103.134
0.25	0.000010	0.971267	0.971257	0.028743	34.791
0.275	0.000002	0.930631	0.930628	0.069372	14.415
0.3	0.000000	0.859440	0.859440	0.140560	7.114
0.325	0.000000	0.754401	0.754401	0.245599	4.072
0.35	0.000000	0.621587	0.621587	0.378413	2.643
0.375	0.000000	0.475811	0.475811	0.524189	1.908
0.4	0.000000	0.335613	0.335613	0.664387	1.505
0.425	0.000000	0.216693	0.216693	0.783307	1.277
0.45	0.000000	0.127345	0.127345	0.872655	1.146
0.475	0.000000	0.067753	0.067753	0.932247	1.073
0.5	0.000000	0.032454	0.032454	0.967546	1.034
0.525	0.000000	0.013910	0.013910	0.986090	1.014
0.55	0.000000	0.005297	0.005297	0.994703	1.005

P-Chart OC Curve Demonstration					
P(0)	**0.2**		**sample size**	**100**	
S	**0.04**		**Limits width**	**3**	
L	**0.08**	**0.32**	**B(limit)**	**8**	**32**
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	0.999999	1.000000	0.000001	0.999999	1.000
0.02	0.999811	1.000000	0.000189	0.999811	1.000
0.03	0.996784	1.000000	0.003216	0.996784	1.003
0.04	0.981008	1.000000	0.018992	0.981008	1.019
0.05	0.936910	1.000000	0.063090	0.936910	1.067
0.075	0.664770	1.000000	0.335230	0.664770	1.504
0.1	0.320874	1.000000	0.679126	0.320874	3.116
0.125	0.108846	1.000000	0.891154	0.108846	9.187
0.15	0.027476	0.999995	0.972519	0.027481	36.389
0.175	0.005404	0.999869	0.994464	0.005536	180.649
0.2	0.000855	0.998450	0.997594	0.002406	415.655
0.225	0.000111	0.989582	0.989471	0.010529	94.975
0.25	0.000012	0.955404	0.955392	0.044608	22.417
0.275	0.000001	0.867872	0.867871	0.132129	7.568
0.3	0.000000	0.710719	0.710718	0.289282	3.457
0.325	0.000000	0.504994	0.504994	0.495006	2.020
0.35	0.000000	0.302879	0.302879	0.697121	1.434
0.375	0.000000	0.150678	0.150678	0.849322	1.177
0.4	0.000000	0.061504	0.061504	0.938496	1.066
0.425	0.000000	0.020443	0.020443	0.979557	1.021
0.45	0.000000	0.005497	0.005497	0.994503	1.006
0.475	0.000000	0.001188	0.001188	0.998812	1.001
0.5	0.000000	0.000204	0.000204	0.999796	1.000
0.525	0.000000	0.000028	0.000028	0.999972	1.000
0.55	0.000000	0.000003	0.000003	0.999997	1.000

P-Chart OC Curve Demonstration					
P(0)	**0.2**		**sample size**	**150**	
S	**0.03266**		**Limits width**	**3**	
L	**0.1020**	**0.2980**	**B(limit)**	**15**	**44**
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	1.000000	1.000000	0.000000	1.000000	1.000
0.02	1.000000	1.000000	0.000000	1.000000	1.000
0.03	0.999987	1.000000	0.000013	0.999987	1.000
0.04	0.999636	1.000000	0.000364	0.999636	1.000
0.05	0.996397	1.000000	0.003603	0.996397	1.004
0.075	0.901969	1.000000	0.098031	0.901969	1.109
0.1	0.568184	1.000000	0.431816	0.568184	1.760
0.125	0.214331	1.000000	0.785669	0.214331	4.666
0.15	0.049333	0.999998	0.950665	0.049335	20.270
0.175	0.007376	0.999878	0.992502	0.007498	133.371
0.2	0.000758	0.997688	0.996930	0.003070	325.751
0.225	0.000056	0.979813	0.979757	0.020243	49.400
0.25	0.000003	0.904865	0.904862	0.095138	10.511
0.275	0.000000	0.726741	0.726741	0.273259	3.660
0.3	0.000000	0.469233	0.469233	0.530767	1.884
0.325	0.000000	0.230835	0.230835	0.769165	1.300
0.35	0.000000	0.084092	0.084092	0.915908	1.092
0.375	0.000000	0.022388	0.022388	0.977612	1.023
0.4	0.000000	0.004329	0.004329	0.995671	1.004
0.425	0.000000	0.000605	0.000605	0.999395	1.001
0.45	0.000000	0.000061	0.000061	0.999939	1.000
0.475	0.000000	0.000004	0.000004	0.999996	1.000
0.5	0.000000	0.000000	0.000000	1.000000	1.000
0.525	0.000000	0.000000	0.000000	1.000000	1.000
0.55	0.000000	0.000000	0.000000	1.000000	1.000

The calculations of the power and ARL from p-charts OC curve for different control limits width and constant sample size.

P-Chart OC Curve Demonstration					
P(0)	**0.2**		**sample size**	**50**	
S	**0.05657**		**Limits width**	**2**	
L	**0.0869**	**0.3131**	**B(limit)**	**4**	**15**
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	0.999854	1.000000	0.000146	0.999854	1.000
0.02	0.996790	1.000000	0.003210	0.996790	1.003
0.03	0.983189	1.000000	0.016811	0.983189	1.017
0.04	0.951029	1.000000	0.048971	0.951029	1.051
0.05	0.896383	1.000000	0.103617	0.896383	1.116
0.075	0.679579	1.000000	0.320421	0.679579	1.471
0.1	0.431198	0.999983	0.568784	0.431216	2.319
0.125	0.234634	0.999742	0.765107	0.234893	4.257
0.15	0.112105	0.998050	0.885945	0.114055	8.768
0.175	0.047870	0.990785	0.942914	0.057086	17.518
0.2	0.018496	0.969197	0.950701	0.049299	20.284
0.225	0.006520	0.921188	0.914668	0.085332	11.719
0.25	0.002108	0.836917	0.834808	0.165192	6.054
0.275	0.000627	0.715742	0.715114	0.284886	3.510
0.3	0.000172	0.569178	0.569007	0.430993	2.320
0.325	0.000043	0.417176	0.417133	0.582867	1.716
0.35	0.000010	0.280104	0.280094	0.719906	1.389
0.375	0.000002	0.171544	0.171542	0.828458	1.207
0.4	0.000000	0.095502	0.095501	0.904499	1.106
0.425	0.000000	0.048179	0.048179	0.951821	1.051
0.45	0.000000	0.021951	0.021951	0.978049	1.022
0.475	0.000000	0.008996	0.008996	0.991004	1.009
0.5	0.000000	0.003300	0.003300	0.996700	1.003
0.525	0.000000	0.001077	0.001077	0.998923	1.001
0.55	0.000000	0.000311	0.000311	0.999689	1.000

P-Chart OC Curve Demonstration					
P(0)	0.2		sample size	50	
S	0.05657		Limits width	3	
L	0.0303	0.3697	B(limit)	1	18
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	0.910565	1.000000	0.089435	0.910565	1.098
0.02	0.735771	1.000000	0.264229	0.735771	1.359
0.03	0.555280	1.000000	0.444720	0.555280	1.801
0.04	0.400481	1.000000	0.599519	0.400481	2.497
0.05	0.279432	1.000000	0.720568	0.279432	3.579
0.075	0.102501	1.000000	0.897499	0.102501	9.756
0.1	0.033786	1.000000	0.966214	0.033786	29.598
0.125	0.010261	0.999996	0.989735	0.010265	97.418
0.15	0.002905	0.999940	0.997035	0.002965	337.264
0.175	0.000772	0.999525	0.998753	0.001247	802.127
0.2	0.000193	0.997489	0.997296	0.002704	369.839
0.225	0.000045	0.990349	0.990304	0.009696	103.134
0.25	0.000010	0.971267	0.971257	0.028743	34.791
0.275	0.000002	0.930631	0.930628	0.069372	14.415
0.3	0.000000	0.859440	0.859440	0.140560	7.114
0.325	0.000000	0.754401	0.754401	0.245599	4.072
0.35	0.000000	0.621587	0.621587	0.378413	2.643
0.375	0.000000	0.475811	0.475811	0.524189	1.908
0.4	0.000000	0.335613	0.335613	0.664387	1.505
0.425	0.000000	0.216693	0.216693	0.783307	1.277
0.45	0.000000	0.127345	0.127345	0.872655	1.146
0.475	0.000000	0.067753	0.067753	0.932247	1.073
0.5	0.000000	0.032454	0.032454	0.967546	1.034
0.525	0.000000	0.013910	0.013910	0.986090	1.014
0.55	0.000000	0.005297	0.005297	0.994703	1.005

P-Chart OC Curve Demonstration					
P(0)	**0.2**		**sample size**	**50**	
S	**0.05657**		**Limits width**	**4**	
L	**0**	**0.42627**	**B(limit)**	**0**	**21**
P(ln)	Phi(-L)	Phi(+L)	Beta	Power	ARL
0	1.000000	1.000000	0.000000	1.000000	1.000
0.01	0.605006	1.000000	0.394994	0.605006	1.653
0.02	0.364170	1.000000	0.635830	0.364170	2.746
0.03	0.218065	1.000000	0.781935	0.218065	4.586
0.04	0.129886	1.000000	0.870114	0.129886	7.699
0.05	0.076945	1.000000	0.923055	0.076945	12.996
0.075	0.020281	1.000000	0.979719	0.020281	49.308
0.1	0.005154	1.000000	0.994846	0.005154	194.033
0.125	0.001260	1.000000	0.998740	0.001260	793.570
0.15	0.000296	0.999999	0.999703	0.000297	3370.940
0.175	0.000066	0.999988	0.999921	0.000079	12730.16
0.2	0.000014	0.999898	0.999884	0.000116	8587.615
0.225	0.000003	0.999401	0.999398	0.000602	1660.928
0.25	0.000001	0.997382	0.997382	0.002618	381.916
0.275	0.000000	0.991025	0.991025	0.008975	111.421
0.3	0.000000	0.974913	0.974913	0.025087	39.861
0.325	0.000000	0.941123	0.941123	0.058877	16.985
0.35	0.000000	0.881260	0.881260	0.118740	8.422
0.375	0.000000	0.790253	0.790253	0.209747	4.768
0.4	0.000000	0.670138	0.670138	0.329862	3.032
0.425	0.000000	0.531315	0.531315	0.468685	2.134
0.45	0.000000	0.389964	0.389964	0.610036	1.639
0.475	0.000000	0.262679	0.262679	0.737321	1.356
0.5	0.000000	0.161118	0.161118	0.838882	1.192
0.525	0.000000	0.089305	0.089305	0.910695	1.098
0.55	0.000000	0.044379	0.044379	0.955621	1.046

VITA

ABDALLAH SALEH THEFEID

Department of Engineering Management and Systems Engineering
Old Dominion University, Norfolk, Virginia
Email: athef001@odu.edu

Degrees:

Master of Science in Engineering Management
King Abdulaziz University, Saudi Arabia
June 2016

Bachelor of Science in Industrial and Systems Engineering
King Fahd University of Petroleum and Minerals, Saudi Arabia
June 2007

Professional Chronology:

General Authority of Civil Aviation
Riyadh, Saudi Arabia
Quality Assurance Manager

Saudi Electricity Company
Jeddah, Saudi Arabia
Inventory Planning Analyst

Panda Retail Company
Jeddah, Saudi Arabia
Inventory Controller, Section Head of FMCG

Professional Societies:

American Society for Engineering Management
American Society for Quality

Professional Certification:

Certified Agile Leader - SSGI
Certified Lean Six Sigma Green Belt – ASQ ODU Student Branch