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# Manufacturing Applications of the One-dimensional Cutting Stock Problem as a Team Project

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# Manufacturing Applications of the One-Dimensional Cutting Stock Problem as a Team Project


## Abstract

This paper explains the beneficial and practical impact of operations research in two real manufacturing settings. Two manufacturing examples used in student projects were (1) cutting rails (80‘ or 40‘) to manufacture railroad frogs of many sizes and (2) cutting round metal rolls (12‘ to 20‘) to meet customer demands for various lengths of cuts. Student teams in Engineering of Manufacturing Processes and Operations Research courses wrote computer programs. The program first identified all possible patterns that can be cut out of a given stock length. Next, the program created a mathematical model (a text file) as an output. This text file was used as an input for the optimizer software LINGO. When compared to the manual solutions obtained by foremen in two settings, student teams with no prior experience were able to match the manual solution of the foremen in small problems and improve the manual solution by up to 30 % in large problems. After finishing the project, each team wrote a technical team report to document the experience they gained in manufacturing and mathematical modeling. Student assessment was based on student team reports (knowledge gained) and individual team interviews, exit surveys, and the end of semester course evaluations (students' attitudes). The project outcomes include improved understanding of production-related concepts such as remnant minimization in manufacturing, as well as enthusiasm for operations research and its applications in manufacturing.


## Introduction

Kolb's experiential learning cycle/spiral [1 - 3] is often used as a powerfull metacognitive method in many engineering programs. Namely, a learner gains knowledge by answering four questions (Why?, What?, How? and What if?) in succession. A set of activities is associated with each question. This cycle of questions and activities is repeated for deeper learning regardless of the prefered learning style (type) of the learner. Laboratory experiments and other experiential learning activities [4-6] are well recognized parts of Kolb's learning cycle.

Creating products is the primary function of any manufacturing establishment. Product realization-based learning seems to be a natural model for learning manufacturing engineering [7]. The product realization-based learning can be understood as a part of the project-based learning (PBL) pedagogy which is well accepted in education [8, 9]. PBL is also emphasized as one of the priority educational methods in manufacturing engineering [10] and industrial engineering education [11]. PBL pedagogy is already successfully implemented in some manufacturing processes courses [12, 13]. Students' experiences described here are based on product-realization learning concepts. In addition, some additional PBL pedagogy strategies and teamwork are implemented.

In operations research (OR), the one-dimensional cutting stock [20-29] problem describes the case of cutting standard length stock material into various specified sizes while minimizing the material wasted. Unusable length is called remnant or drop in manufacturing that involves metal works.

Solution to this computationally complex optimization problem can be used in many manufacturing applications. To solve it, the problem can be formulated as an integer linear model first, and then solved using a common optimizer software. Since the problem is known to have multiple optimal solutions in some cases, binary variables can be added to identify all optimal solutions, but this is often not necessary. U.S. Customary units are used throughout this paper because both manufacturers exemplified in this work operate using the U.S. Customary units in almost all aspects of their daily operations. All machinists and majority of the management in these facilities are not only unfamiliar with the metric system, but are also outright against it.

## Curricular Context

Applications of operations research in manufacturing can be implemented in either operations research or manufacturing courses. In our case, there are two courses that can benefit from this work: MAE 495 which is an elective senior level course for mechanical engineering majors [14] and EN 471, Operations Research, which is a regular one-semester, three-credits, junior-level course in an industrial engineering program [15]. The Manufacturing Processes course for senior mechanical engineering students was taught from industrial engineering and operations research perspective, similarly to the Operations Research course in an industrial engineering program. Concepts of manufacturing economics and optimization were emphasized. Optimization examples included one-dimensional cutting-stock problem as a project topic. The described experience deals with about 80 students per semester, where students work in teams of three to four students per team.

From the four pillars of manufacturing engineering (a. materials and manufacturing processes, b. product, tooling, and assembly engineering, c. manufacturing systems and operations, and d. manufacturing competitiveness [16],") this work addresses two of them (c. and d.).

## Educational Goals, Activities, and Outcomes

Educational goals of this project include improved understanding of production-related concepts such as remnant minimization in manufacturing, as well as increased enthusiasm for operations research and its applications in manufacturing. Students' experience consists of several activities: observation of real metal cutting operations, realizing costs due to final length of stock material, programming and running OR-based calculations to minimize remnants, and succesfully competing with experienced foremen's manual solutions. Several learning outcomes originate from project educational goals and project activities like increased appreciation for manufacturing in general, increased appreciation for OR by experiencing a "real life" manufacturing problem, an understanding of the role of analytical approaches to engineering problem solving, development of written communication skills through writing of technical reports, and development of teamwork skills. These outcomes are closely related to ABET-EAC Criterion 3, a-k student learning outcomes [17], specifically outcome a - an ability to apply knowledge of mathematics, science and engineering, outcome g - an ability to communicate effectively, and outcome k - an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

## Practical Experience

This paper reports student experience with one-dimensional cutting stock problem using two very different manufacturing applications. The Facility 1 cuts different sizes of parts from a rail stock to manufacture rail frogs. The Facility 2 cuts different sizes and cross sections of cylindrical parts to manufacture different products. While students did not get an actual metal cutting experience in this project, they observed cutting operations at both facilities.

## The Mathematical Model

Figure 1 shows the simplest version of the classic one-dimensional cutting stock model using m cuts and n patterns. Input variable $D_i$ represents the known demand for each cut size I (1 to m). Input matrix $a_{ij}$ (m x n) represents number of cut size of type i that can be obtained from pattern j. Output variable $X_j$ represents the number of stocks that should be cut according to pattern j (1 to n).

$$\text{Minimize} \sum_{J=1}^{n} X_j$$

$$\sum_{j=1}^{n} a_{ij} * X_j \geq D_i \quad \forall\, i = 1, \dots, m$$

$$X_j \in \text{integer} \;\forall\, j = 1, \dots, n$$

$$[a] \text{ is a non} - \text{negative matrix}$$

Figure 1. One-Dimensional Cutting Stock Model

The model in Figure 1 is linear with integer decision variables (X's). The objective function expresses the obvious fact that minimizing the total number of stocks used is analogous to minimizing the total waste which is not shown in the model. The constraint set ensures that demand is met for each cut size. This model can be written out manually for small problem dimensions and submitted to the optimizer software LINGO [19], but this quickly becomes impossible as the problem dimensions grow. A MATLAB code was utilized to prepare the input file for LINGO.

## Facility 1

This facility produces railroad frogs. Frog is made of rails having the same cross-section as those used in the track.

> *"The frog is a device by means of which the rail at the turnout curve crosses the rail of the main track. Frog is a junction, but not a switch for changing tracks on rails. Frogs are manufactured by bending, drilling, grinding of rails various lengths and then connecting several of rails by welding and/or bolting together into a final product [18]."*

Figure 2 shows a picture of an actual frog in use.

Figure 2.  A Railroad Frog

Figure 3 shows inventory of 40‘ and 80‘ rail stocks at Facility 1 in Pueblo, CO.  This paper only considers 80‘ rails to avoid significant modelling complications.

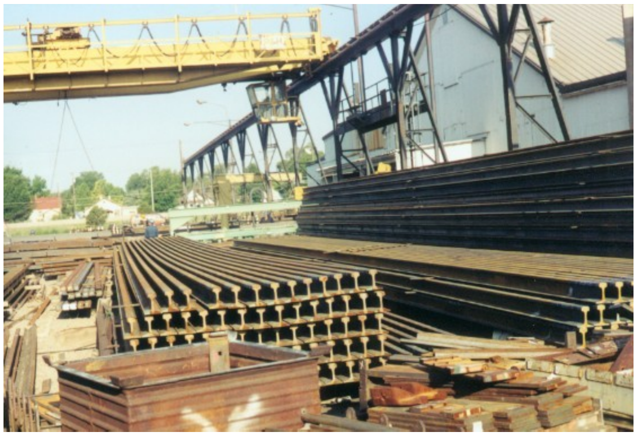

Figure 3.   40‘ and 80‘ Rail Stocks at the Facility 1

At about $16/ft., steel rail is expensive and minimizing remnant will result in lower frog manufacturing cost.   Table 1 below shows 24 common cuts needed by Facility 1.  A typical frog contains two or more of the cuts shown in Table 1.

Table 1.  Twenty-Four Common Rail Cuts Used in Frog Manufacturing in Facility 1

| | | | | | |
|---|---|---|---|---|---|
| 16' 6 9/32" | 54' 7" | 38' 3" | 36' 6 5/8" | 32' 4" | 49' 7" |
| 29' 10 ½” | 63' 1" | 54' 10" | 25' 1" | 30' 2" | 63' 2" |
| 30' 6" | 16' 6" | 26' 0" | 15' 0" | 24' 0" | 51' 4" |
| 62' 0" | 35' 0" | 59' 6" | 39' 7" | 46' 10" | 45' 5" |

Figure 4 shows cutting of an 80' long rail to yield the desired cut lengths.

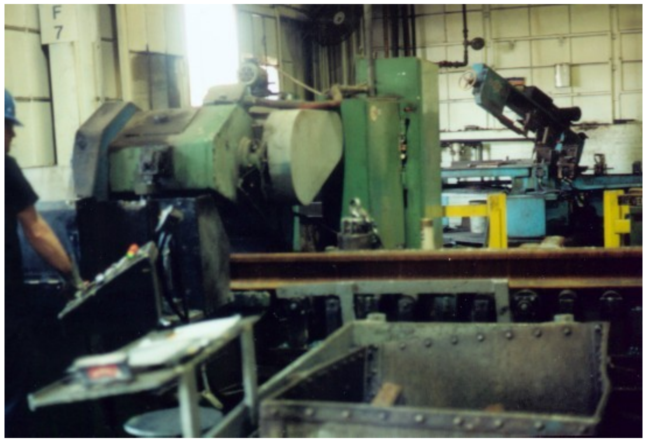

Figure 4. A Rail Being Cut into Required Lengths for Use in Frog Manufacturing

Sample Problem 1.

29' 10 ½" (A), 36' 6 5/8" (B), 38' 3" (C), 54' 7" (D) are four (m =4) rail cut lengths at Facility 1 needed to construct (drilling holes of several sizes, bending, and intensive welding are the three main operations) frogs. The plant buys 80' long steel rails from a steel mill. The projected demand level for each cut length are as follows: A: 38, B: 61, C: 54, D: 89. It is assumed that the sawing off operation results in a loss of 0.40" of length. Table 2 shows all 7 (n) possible and feasible patterns that yield the number of cuts of each type. Of course, many other possible patterns are not feasible and therefore are not considered. Any pattern that results in a remnant that equals or exceeds the smallest cut size is unfeasible.

Table 2. Feasible Cutting Pattern for the Sample Problem 1.

| **Patterns/Cuts** | **29.91' A** | **36.59' B** | **38.28' C** | **54.61' D** | **Remnant** |
|---|---|---|---|---|---|
| **1** | 2 | 0 | 0 | 0 | 20.18' |
| **2** | 1 | 1 | 0 | 0 | 13.50' |
| **3** | 1 | 0 | 1 | 0 | 11.81' |
| **4** | 0 | 2 | 0 | 0 | 6.82' |
| **5** | 0 | 1 | 1 | 0 | 5.13' |
| **6** | 0 | 0 | 2 | 0 | 3.44' |
| **7** | 0 | 0 | 0 | 1 | 25.39' |

The 7 x 4 input matrix (n x m) above is transposed and entered as 4 x 7 (m x n) $a_{ij}$ matrix in Figure 5. Figure 5 shows the LINGO version of the model. As this is a small problem, the model in Figure 5 can be manually entered into LINGO, but manual entry quickly becomes cumbersome and highly error prone as the problem size grows. This project utilized a MATLAB code to automate two major steps: 1) Determine all feasible patterns as shown in Table 2 and 2) Prepare a LINGO input file as shown in Figure 5. Hence, students received a very detailed experience in operations research as an application in manufacturing. They also improved their programming skills.

```
MIN=(X1 + X2 + X3 + X4 + X5 + X6 + X7);

2*X1 + 1*X2 + 1*X3 + 0*X4 + 0*X5 + 0*X6 + 0*X7 >38;

0*X1 + 1*X2 + 0*X3 + 2*X4 + 1*X5 + 0*X6 + 0*X7 >61;

0*X1 + 0*X2 + 1*X3 + 0*X4 + 1*X5 + 2*X6 + 0*X7 >54;

0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 >89;

@GIN (X1); @GIN (X2);@GIN (X3);@GIN (X4);@GIN (X5);

@GIN (X6);@GIN (X7);

END
```

Figure 5. LINGO Input File for Sample Problem 1.

LINGO solution is as follows: $X_1$ = 19, $X_4$ = 4, $X_5$ = 54, $X_7$ = 89 and the sum is 166. So, 166 80' long rails are needed to meet the demand with minimum waste or remnant. The solution took just 0.05 seconds. The rails are to be cut according to patterns 1, 4, 5, and 7 in the quantities shown above. Note that pattern 6 with the smallest remnant was not chosen. Demands for each of the four sizes are met as follows:

Number of 29.91' long cuts = 19 x 2 + 4 x 0 + 54 x 0 + 89 x 0 = 38

Number of 36.59' long cuts = 19 x 0 + 4 x 2 + 54 x 1 + 89 x 0 = 62 (1 extra)

Number of 38.28' long cuts = 19 x 0 + 4 x 0 + 54 x 1 + 89 x 0 = 54

Number of 54.61' long cuts = 19 x 0 + 4 x 0 + 54 x 0 + 89 x 1 = 89

166 rails are 166 x 80 = 13280 feet. The amount of actual use is 29.91 x 38 + 36.59 x 62 + 38.28 x 54 + 54.61 x 89 = 10333'. The unused amount is 13280 – 10333 = 2947' or 22.2%. The amount remnant can also be calculated by multiplying the remnant for each combination with the number of rails cut according to that combination: 20.18 x 19 + 6.82 + 4 + 5.13 x 54 + 25.39 x 89 = 2948'. This order with only four cut sizes results in a large amount of remnant, but large remnants shown in Table 2 (20.18' and 25.39') are sufficiently long for two smaller common frog sizes (16' 6 9/32" and 15' 0") shown in Table 1. While very small remnants are either scraped or returned to the steel mill for re-melting, long remnants are certainly saved for future use. The one-dimensional cutting stock problem solution does not consider leftover, but only usable rails.

<u>Sample Problem 2.</u>

This scenario involves 11 of the cuts shown in Table 1. Figure 6 shows the cut lengths used and the input process before the execution of the MATLAB code.

Table 3 shows solution cuts that generate the required lengths using an 80' rail.

```
Welcome to The Stock Cut Calculator Program
Input Stock Length: 80
Please input cut lengths.(it will take an unlimited amount)
Input cut Length (Input 0 to Terminate): 16.58
Input cut Length (Input 0 to Terminate): 54.59
Input cut Length (Input 0 to Terminate): 38.25
Input cut Length (Input 0 to Terminate): 36.56
Input cut Length (Input 0 to Terminate): 63.09
Input cut Length (Input 0 to Terminate): 54.84
Input cut Length (Input 0 to Terminate): 30.17
Input cut Length (Input 0 to Terminate): 24
Input cut Length (Input 0 to Terminate): 49.59
Input cut Length (Input 0 to Terminate): 59.50
Input cut Length (Input 0 to Terminate): 63.17
Input cut Length (Input 0 to Terminate): 0
Cuts have been sorted in ascending order
Cut1=16.58
Cut2=24
Cut3=30.17
Cut4=36.56
Cut5=38.25
Cut6=49.59
Cut7=54.59
Cut8=54.84
Cut9=59.5
Cut10=63.09
Cut11=63.17
Demand amount for cut 1 (16.58 units): 81
Demand amount for cut 2 (24 units): 22
Demand amount for cut 3 (30.17 units): 14
Demand amount for cut 4 (36.56 units): 21
Demand amount for cut 5 (38.25 units): 17
Demand amount for cut 6 (49.59 units): 22
Demand amount for cut 7 (54.59 units): 24
Demand amount for cut 8 (54.84 units): 17
Demand amount for cut 9 (59.5 units): 11
Demand amount for cut 10 (63.09 units): 11
Demand amount for cut 11 (63.17 units): 18
```

Figure 6. Input Screen of the MATLAB Code for Sample Problem 2

Table 3. Solution Cut Patterns for Sample Problem 2

| Pattern/ Cuts | 16.58′ | 24′ | 30.17′ | 36.56′ | 38.25′ | 49.59′ | 54.59′ | 54.84′ | 59.50′ | 63.09′ | 63.17′ | Remnant (feet) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6.26 |
| **12** | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 8.83 |
| **13** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 8.58 |
| **15** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.33 |
| **16** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.25 |
| **19** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 6.41 |
| **21** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1.16 |
| **24** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.24 |
| **25** | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6.88 |
| **26** | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 5.19 |

The LINGO output for Sample Problem 2 is as follows: 126 rails (objective value) are to be cut in the quantities shown for patterns 2 (4 rails), 12 (24 rails), 13 (6 rails), 14 (11 rails), 15 (11 rails), 16 (18 rails), 19 (8 rails), 21 (11 rails), 24 (14 rails), 25 (2 rails), and 26 (17 rails). Again, some patterns with small remnant values (9 and 10 for example) were not chosen. The yield analysis shows a utilization of level of 92.25% of the total length of 10080 feet provided by 126 80 ft. rails.

**Facility 2**

This facility, located in Norfolk, VA, machines metal couplers along with many other metal products. The couplers are used to connect various bodies to transfer motion or provide a pull. An aluminum 6061 stock with a diameter of 3" and variable length is used. A set of manufacturing operations used to produce these couplers is described elsewhere[13]. Figure 7 shows three common couplers.

Figure 7. Three Common Couplers (10", 9", and 8" long)

Figure 8 shows stocks of various lengths (12′ to 30′) and diameters (2″ to 6″) used in Facility 2.

Figure 8. Stocks at Facility 2.

Sample Problem 3.

This set of one-dimensional cutting-stock problems with associated optimal solutions describes two cases (A and B) which are detailed in the Appendix. Here, round stock is cut to required sizes for machining of couplers.

## Assessment of Students' Knowledge and Attitudes

Students' Knowledge Assessment.

Apart from taking a regular test, students competed against experienced foremen in class. It is very important to note that students with no prior manufacturing experience applying their operational research skills were able to match or outperform experienced foremen's manual solutions. When compared to the manual solutions obtained by foremen in two settings, student teams with no prior experience were able to match the manual solution of the foremen in small problems and improve the manual solution by up to 30% in large problems.

Students' Attitudes Assessment.

There were three educational metrics used for assessing students' attitudes: informal student interviews, anonymous exit interviews, and the end of semester course evaluations.

Informal student interviews were conducted after the visits to the manufacturing companies involved in this project. Students in each course were required to visit a production facility to witness the real need for the project. The faculty teaching the course organized the visits. The students were able to hear directly from the engineers in charge of these operations how important this project was for them. After the visits, all students were excited and appreciative of the material they were learning.

Anonymous exit survey taken on the last day of classes indicate that a majority (70%) of the students felt this project was a very good learning experience. Some students had difficulty with programming and this caused some anxiety to those students who were not sufficiently proficient. Finally, here are some of student testimonials from the end of semester course evaluations.

> *"This project gave us an opportunity to see what it is like to apply operations research to manufacture a product."*
>
> *"This project introduced challenging concepts for all students: programming and mathematical modelling to solve a real manufacturing problem."*
>
> *"The field trip was great. I can see and believe this was a real problem that my team can solve."*
>
> *"This project was a good choice for a semester team work. To give us experience that is based off of real life situations is a great thing to do."*
>
> *"It is good to know programming and mathematical modelling are really useful."*
>
> *"I felt empowered when my solution beat the foreman's paper and pencil solution by 12%"*

Again, based on the testimonials, students showed increased enthusiasm towards manufacturing and increased self-efficacy and pride in their professional (programming and engineering) skills.

## Conclusions and Recommendations

Students learned and applied their programming and operational research skills to real manufacturing problems using modern engineering tools like MatLab and LINGO. They visited two manufacturing facilities that dealt with the one-dimensional cutting stock problem on the daily basis. The students were able to match or outperform experienced foremen's manual solutions. This project provided a real manufacturing problem solving experience using programming and mathematical modelling. Students were able to realize the importance of optimization in manufacturing-related activities.

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**APPENDIX :** ILLUSTRATION CUTTING STOCK OPTIMIZATION MODEL USE IN MULTI-SIZE COUPLER MANUFACTURING at Facility 2

The cases described below are based on a 12 foot (144 inches) long aluminum 6061 stock with a 3″ diameter. The diameter value is not a factor in the optimization process.

**CASE A**: The management has accepted an order for four of its popular sizes of 10″, 20″, 30″ and 40″ in quantities of 65, 20, 11, and 8. A sample of feasible patterns shown in Table A.1 below. There are 47 unique and feasible patterns. To be feasible, each pattern's remnant must be less than the shortest cut size.

Table A.1. Feasible Patterns of Cuts for Case A

| Pattern No. | 10" | 20" | 30" | 40" | Remnant (inches) |
|---|---|---|---|---|---|
| 1 | 14 | 0 | 0 | 0 | 4 |
| 2 | 12 | 1 | 0 | 0 | 4 |
| 3 | 11 | 0 | 1 | 0 | 4 |
| . | . | . | . | . | . |
| 21 | 4 | 2 | 2 | 0 | 4 |
| 22 | 4 | 1 | 0 | 2 | 4 |
| .. | . | . | . | . | . |
| 28 | 2 | 6 | 0 | 0 | 4 |
| 29 | 2 | 4 | 0 | 1 | 4 |
| . | . | . | . | . | . |
| 38 | 1 | 1 | 1 | 2 | 4 |
| 39 | 1 | 0 | 3 | 1 | 4 |
| 40 | 0 | 7 | 0 | 0 | 4 |
| 41 | 0 | 5 | 0 | 1 | 4 |
| . | . | . | . | . | . |
| 46 | 0 | 1 | 0 | 3 | 4 |
| 47 | 0 | 0 | 2 | 2 | 4 |

The same program uses the data in Table A.1 and outputs the LINGO optimization model shown in Figure A.1. The 47 x 4 input matrix (n x m) above is transposed and entered as 4 x 47 (m x n) $a_{ij}$ matrix in Figure A.1.

```
MIN=(X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14 +
X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +
X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 +
X43 + X44 + X45 + X46 + X47);

14*X1 + 12*X2 + 11*X3 + 10*X4 + 10*X5 + 9*X6 + 8*X7 + 8*X8 + 8*X9 + 7*X10 + 7*X11
+ 6*X12 + 6*X13 + 6*X14 + 6*X15 + 5*X16 + 5*X17 + 5*X18 + 4*X19 + 4*X20 + 4*X21 +
4*X22 + 4*X23 + 3*X24 + 3*X25 + 3*X26 + 3*X27 + 2*X28 + 2*X29 + 2*X30 + 2*X31 +
2*X32 + 2*X33 + 2*X34 + 1*X35 + 1*X36 + 1*X37 + 1*X38 + 1*X39 + 0*X40 + 0*X41 +
0*X42 + 0*X43 + 0*X44 + 0*X45 + 0*X46 + 0*X47 >65;

0*X1 + 1*X2 + 0*X3 + 2*X4 + 0*X5 + 1*X6 + 3*X7 + 1*X8 + 0*X9 + 2*X10 + 0*X11 +
4*X12 + 2*X13 + 1*X14 + 0*X15 + 3*X16 + 1*X17 + 0*X18 + 5*X19 + 3*X20 + 2*X21 +
1*X22 + 0*X23 + 4*X24 + 2*X25 + 1*X26 + 0*X27 + 6*X28 + 4*X29 + 3*X30 + 2*X31 +
1*X32 + 0*X33 + 0*X34 + 5*X35 + 3*X36 + 2*X37 + 1*X38 + 0*X39 + 7*X40 + 5*X41 +
4*X42 + 3*X43 + 2*X44 + 1*X45 + 1*X46 + 0*X47 >20;

0*X1 + 0*X2 + 1*X3 + 0*X4 + 0*X5 + 1*X6 + 0*X7 + 0*X8 + 2*X9 + 1*X10 + 1*X11 +
0*X12 + 0*X13 + 2*X14 + 0*X15 + 1*X16 + 1*X17 + 3*X18 + 0*X19 + 0*X20 + 2*X21 +
0*X22 + 2*X23 + 1*X24 + 1*X25 + 3*X26 + 1*X27 + 0*X28 + 0*X29 + 2*X30 + 0*X31 +
2*X32 + 4*X33 + 0*X34 + 1*X35 + 1*X36 + 3*X37 + 1*X38 + 3*X39 + 0*X40 + 0*X41 +
2*X42 + 0*X43 + 2*X44 + 4*X45 + 0*X46 + 2*X47 >11;

0*X1 + 0*X2 + 0*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 0*X10 + 1*X11 +
0*X12 + 1*X13 + 0*X14 + 2*X15 + 0*X16 + 1*X17 + 0*X18 + 0*X19 + 1*X20 + 0*X21 +
2*X22 + 1*X23 + 0*X24 + 1*X25 + 0*X26 + 2*X27 + 0*X28 + 1*X29 + 0*X30 + 2*X31 +
1*X32 + 0*X33 + 3*X34 + 0*X35 + 1*X36 + 0*X37 + 2*X38 + 1*X39 + 0*X40 + 1*X41 +
0*X42 + 2*X43 + 1*X44 + 0*X45 + 3*X46 + 2*X47 >8;

@GIN (X1); @GIN (X2); @GIN (X3); @GIN (X4); @GIN (X5); @GIN (X6); @GIN
(X7);@GIN (X8);@GIN (X9); @GIN (X10); @GIN (X11); @GIN (X12); @GIN (X13); @GIN
(X14); @GIN (X15); @GIN (X16);@GIN (X17); @GIN (X18); @GIN (X19);@GIN (X20);
@GIN (X21);@GIN (X22);@GIN (X23);@GIN (X24);@GIN (X25); @GIN (X26); @GIN
(X27);@GIN (X28);@GIN (X29);@GIN (X30);@GIN (X31); @GIN (X32); @GIN (X33);
@GIN (X34); @GIN (X35); @GIN (X36);@GIN (X37);@GIN (X38);@GIN (X39);@GIN
(X40); @GIN (X41); @GIN (X42); @GIN (X43);@GIN (X44); @GIN (X45);@GIN
(X46);@GIN (X47);

END
```

Figure A.1. LINGO Optimization Model for Case A.

LINGO software quickly finds the optimum solution shown in Figure A.2.

```
Global optimal solution found.
Objective value:                               13.00000
Objective bound:                               13.00000
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              9
Elapsed runtime seconds:                           0.02

Model Class:                                       PILP

Total variables:                  47
Nonlinear variables:               0
Integer variables:                47

Total constraints:                 5
Nonlinear constraints:             0

Total nonzeros:                  170
Nonlinear nonzeros:                0


                              Variable           Value
                                   X10        5.000000
                                   X11        4.000000
                                   X13        1.000000
                                   X36        3.000000
```

Figure A.2. Optimal Solution for Case A

Table A.2 shows how the optimal output in Figure A.2 is interpreted for production planning and inventory purposes. Patterns 10, 11, 13, and 16 are chosen in quantities of 5, 4, 1, and 3. The shaded area is the number of coupler sizes obtained from each selected pattern. A total of 13 aluminum bars should be ordered or reserved to meet the demand. It should be noted that the optimal solution results in excess number of parts for all sizes except the longest (40″) size.

Table A.2. Detailed Expansion of the Optimal Solution for Case A.

| Pattern No. | Quantity | 10" | 20" | 30" | 40" |
|---|---|---|---|---|---|
| 10 | 5 | 7 | 2 | 1 | 0 |
| 11 | 4 | 7 | 0 | 1 | 1 |
| 13 | 1 | 6 | 2 | 0 | 1 |
| 36 | 3 | 1 | 3 | 1 | 1 |
| | Demand: | 65 | 20 | 11 | 8 |
| | Actual : | 72 | 21 | 12 | 8 |
| Total | Length: | 720” | 420” | 360” | 320” |
| 12' Bars | 13 | 1872" | Used: | 1820" | |
| Total | Unused | 52" | 2.8% | | |

52” of total remnant is unavoidable because each pattern in Table A.1 above has a 4” remnant due to an unusual combination of cut lengths (10, 20, 30, 40 inches) and the bar stock (144”). The optimal solution provides extra units of 10” (7), 20” (1), and 30” (1) sizes. The extra units are still useful and are included as non-waste in calculation of used amount of 1820” from a maximum possible of 1872” (13 bars x 12 feet/bar x 12”/foot).

The solution calls for using 13 bars of stock that will be cut according to quantities and patterns shown above. Each highlighted row shows the number of cuts obtained from each selected pattern. The solution achieves a material utilization of 97.2% or 2.8% waste. This is an example of a small case where foreman was also table to achieve the same utilization using the same or an alternate optimal solution.

**CASE B:** This case has 10 distinct cuts sizes and 11201 unique and feasible patterns. It took 980 solver iterations and 4.78 seconds to solve.

Table A.10. Optimal Solution for Case B

| Pattern No. | Quantity | 8" | 9" | 10" | 15” | 20" | 21" | 27.25" | 30" | 34.5" | 40" |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3912 | 3 | 3 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 4096 | 1 | 3 | 0 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 0 |
| 6146 | 1 | 1 | 5 | 3 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| 7565 | 1 | 1 | 0 | 6 | 0 | 0 | 1 | 2 | 0 | 0 | 0 |
| 10583 | 4 | 0 | 0 | 4 | 1 | 0 | 0 | 2 | 0 | 1 | 0 |
| 10615 | 4 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
| 10782 | 1 | 0 | 0 | 2 | 0 | 2 | 4 | 0 | 0 | 0 | 0 |
| 11069 | 3 | 0 | 0 | 0 | 1 | 4 | 1 | 1 | 0 | 0 | 0 |
| 11123 | 1 | 0 | 0 | 0 | 0 | 4 | 3 | 0 | 0 | 0 | 0 |
| 11174 | 6 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 2 | 0 | 0 |
| 11180 | 5 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 1 | 1 |
| | Demand | 14 | 5 | 56 | 7 | 18 | 46 | 20 | 12 | 22 | 11 |
| | Actual: | 14 | 5 | 56 | 8 | 20 | 46 | 20 | 12 | 22 | 11 |
| Total | Length: Used (in) | 112 | 45 | 560 | 120 | 400 | 966 | 545 | 360 | 759 | 440 |
| 12' Bars: | 30 | 4320” | | | | | | | | | |
| Total | Unused: | 13” | 0.3% | | | | | | | | |

The solution calls for using 30 bars of stock that will be cut according to quantities and patterns shown above. Each highlighted row shows the number of cuts obtained from each selected pattern. The solution achieves a material utilization of 99.7% or 0.3% waste. This is an example of a large case where foreman was not able to achieve a utilization greater than 88 % to 90%.