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Hüseyin Sarper
Old Dominion University, hsarper@odu.edu

Nebojsa I. Jaksic

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Evaluation of procurement scenarios in one-dimensional cutting stock problem with a random demand mix

Hüseyin Sarper[a] and Nebojsa I. Jaksic[b,*]

[a]*Batten College of Engineering & Technology, Old Dominion University, Norfolk, VA 23529, USA*
[b]*Department of Engineering, Colorado State University – Pueblo, Pueblo, CO 81001, USA*

Abstract

The one-dimensional cutting stock problem describes the problem of cutting standard length stock material into various specified sizes while minimizing the material wasted (the remnant or drop as manufacturing terms). This computationally complex optimization problem has many manufacturing applications. One-dimensional cutting stock problems arise in many domains such as metal, paper, textile, and wood. To solve it, the problem is formulated as an integer linear model first, and then solved using a common optimizer software. This paper revisits the stochastic version of the problem and proposes a priority-based goal programming approach. Monte Carlo simulation is used to simulate several likely inventory order policies to minimize the total number of shortages, overages, and the number of stocks carried in inventory.

Keywords: one-dimensional cutting stock problem, random demand mix, goal programming

1. Introduction

The one-dimensional cutting stock problem appears in many industrial applications. This classic problem was first proposed by Gilmore and Gomory [1, 2]. Due to its high applicability in manufacturing, this problem is discussed frequently in the literature. Dikili and Barlas [3] encounter it in a small shipyard, Stadtler [4] in the aluminum industry, Lefrancois and Gascon [5] in a small manufacturing company, Atkin and Ozdemir [6] in coronary stent manufacturing, Benjaoran and Bhokha [7] in construction steel bar manufacturing, Zanarini [8] in the rubber mold industry, Sculli [9]

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in manufacturing of isolation tapes, etc. While the majority of the published applications use known or deterministic demand, there are many notable cases where the demand is random or stochastic. The problem is often modified with additional objectives/constraints. For example, Filho *et al* [10] and Sinuany-Stern and Weiner [11] use two objectives, while Zanarini [8], Alem *et al* [12], and Beraldi *et al* [13] work with stochastic demands. Also, Scheithauer and Terno [14] proved that the problem possesses the modified integer round-up property while Wongprakornkul and Charnsethiku [15] solved the problem with discrete demands and the capacitated planning objective.

Figure 1 below shows the simplest version of the one-dimensional cutting stock model using m cuts and n patterns. Input column vector D_i represents the demand for each cut size i. Input matrix a_{ij} represents number of cuts size of type i that can be obtained from pattern j. Output variables X_j represent the number of stocks that should be cut according to pattern j.

Nomenclature

a_{ij}	input matrix representing number of cuts size of type i that can be obtained from pattern j
D_i	demand for each cut size i
X_j	number of stocks that should be cut according to pattern j

$$\text{Minimize} \sum_{J=1}^{n} X_j$$

$$\sum_{j=1}^{n} a_{ij} * X_j \geq D_i, \qquad \forall\, i = 1, \ldots, m$$

$$X_j \in \text{integer}\ \forall\, j = 1, \ldots, n$$

$$[a] \text{ is a non} - \text{negative matrix}$$

Fig. 1. One-dimensional cutting stock model

The model in Fig. 1 is linear with integer decision variables (X's). The objective function expresses the obvious fact that minimizing the total number of stocks used is analogous to minimizing the total waste. The constraint set ensures that demand is met for each cut size. This model can be written out manually for a small problem and submitted to an optimizer software such as LINGO, but this quickly becomes impractical as the problem dimensions grow.

A case study is discussed next using frog manufacturing for railroads. A frog is a device by which the rail at the turnout curve crosses the rail of the main track. Frog is a junction, but not a switch for changing tracks on rails. Frogs are manufactured by bending, drilling, grinding of rails various lengths (cuts) and then connecting these cuts by welding and/or bolting together into a final product. Fig. 2 shows a picture of an actual frog on a railroad.

Fig. 2. A Railroad frog

Fig. 3 shows inventory of 80′ long rail stocks at the rail frog manufacturing facility in Pueblo, CO. At about $16/ft, steel rail is expensive and minimizing total remnant will result in lower frog manufacturing cost. Fig. 4 shows cutting of a rail to yield desired cut lengths. The facility was interested in minimizing total remnant and the associated inventory costs such as holding of excess incoming inventory, shortage, and overage of cuts needed for frog manufacturing.

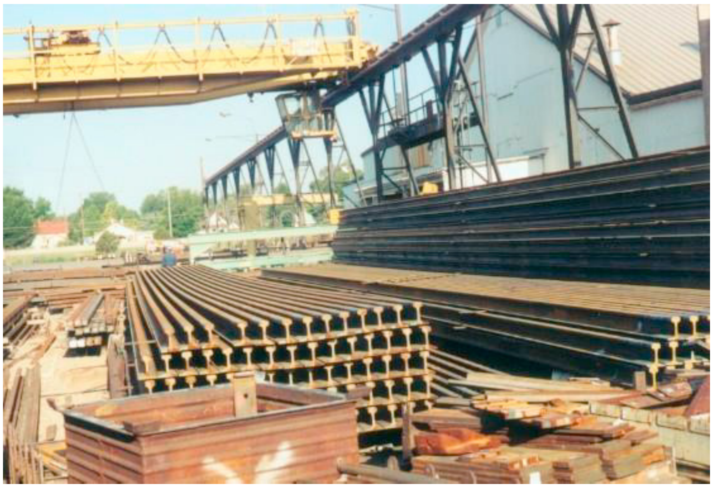


Fig. 3. 80′ Long rail stocks at the frog manufacturing facility yard

2. Sample deterministic problem

The five rail cut lengths the plant needs to construct frogs are 24' 0" (A), 29' 10 ½" (B), 36' 6 5/8" (C), 38' 3" (D), and 54' 7" (E). The plant buys 80′ long steel rails from a steel mill. The demand levels for each cut length are as follows: A: 64, B: 38, C: 61, D: 54, E: 42.

Fig. 4. A Rail being cut into required lengths for use in frog manufacturing

Sawing off operation results in a loss of about 0.40" of length due to the blade thickness. Table 1 below shows all 11 possible and feasible patterns that yield the number of cuts of each type. Of course, many other possible patterns are not feasible and, therefore, are not considered.

Table 1. Feasible cutting pattern for the sample deterministic problem

Patterns/ Cuts	24.04′ A	29.91′ B	36.59′ C	38.28′ D	54.61′ E	Remnant
1	0	0	0	2	0	3.44′
2	0	0	1	1	0	5.13′
3	0	0	2	0	0	6.82′
4	0	1	0	1	0	11.81′
5	0	1	1	0	0	13.50′
6	0	2	0	0	0	20.18′
7	1	0	0	0	1	1.35′
8	1	0	0	1	0	17.68′
9	1	0	1	0	0	19.37′
10	2	1	0	0	0	2.01′
11	3	0	0	0	0	7.88′
Demand (259 Total)	64	38	61	54	42	

Any pattern that results in a remnant that equals or exceeds the smallest cut size is not feasible. This problem is a simplified version of typical real problem that may have up to 24 cut lengths. Fig. 5 shows the LINGO version of the associated optimization model for the data in Table 1. The LINGO solution which implements the most elements of the column generation algorithm, is as follows: X1 = 13, X2 = 1, X3 = 30, X4 = 27, X7 =42, X10 = 11. X1 + X2 + X3 + X4 + X7 + X10 = 124 eighty foot long rails are needed to meet the demand with the total minimum waste or the remnant. The solution took 0.05 seconds.

```
MODEL:
Min = (X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11);
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 1*X8 + 1*X9 + 2*X10 + 3*X11 > 64;
0*X1 + 0*X2 + 0*X3 + 1*X4 + 1*X5 + 2*X6 + 0*X7 + 0*X8 + 0*X9 + 1*X10 + 0*X11 > 38;
0*X1 + 1*X2 + 2*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 1*X9 + 0*X10 + 0*X11 > 61;
2*X1 + 1*X2 + 0*X3 + 1*X4 + 0*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 0*X10 + 0*X11 > 54;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 0*X8 + 0*X9 + 0*X10 + 0*X11 > 42;
@GIN (X1);@GIN (X2);@GIN (X3);@GIN (X4);@GIN (X5);@GIN (X6);@GIN (X7);
@GIN (X8);@GIN (X9);@GIN (X10);@GIN(X11);
END
```

Fig. 5. Mathematical model for sample problem 1 (LINGO version)

The rails are to be cut according to patterns 1, 2, 3, 4, 7, and 10 shown in Table 1. Demands for each of the 5 sizes are met as follows:

- Number of 24.04′ long (A) cuts = 42 x 1 + 11 x 2 = 64;
- Number of 29.91′ long (B) cuts = 27 x 1 + 11 x 1 = 38;
- Number of 36.59′ long (C) cuts = 1 x 1 + 30 x 2 = 61
- Number of 38.28′ long (D) cuts = 13 x 2 + 1 x 1 + 27 x 1 = 54
- Number of 54.61′ long (E) cuts = 42 x 1 = 42.

124 rails have a total length of 9920 feet. The actual length of the total number of frogs produced is 24.04 x 64 + 29.91 x 38 + 36.59 x 61 + 38.28 x 54 + 54.61 x 42 = 9267.87″. The model in Fig. 5 has achieved a 93.4 % utilization. Utilization tends to approach 99% if more cut sizes are considered in the demand mix. The demand mix was met

exactly with no shortages or overages. The deterministic model can, at times, result in few extras of certain cuts. If the plant operates on fixed long term contract with its customers that need frogs for regular railroad maintenance, the deterministic model is adequate. However, this is often not the case.

3. Stochastic extension

Demand is not always known in advance with certainty in many realistic manufacturing situations. Instead, a manufacturer has an idea what the demand for each item can be based on historical evidence and other experience as is the case in this actual application. For example, the demand for the next planning period may be a normal random variable for each of the five cuts with the mean values shown in Table 1. For illustration purposes, a standard deviation is added to each demand: A(64,7), B(38, 5), C(61,6), D(54,8) and E(42,6). Table 2 shows a sample of many possible ways the actual demand may be realized as the manufacturing period starts. These values were randomly drawn using independent normal random variate generator formula of Excel. Ordering of the stock is an inventory problem with a lead time. In other words, the manufacturer cannot wait to find out the exact demand mix before placing the order. Ordering exactly 124 units of 80′ rail stock as in the deterministic case above may have two main consequences:

1) The actual demand may not be met resulting in shortages that cause loss of revenue and customer goodwill,
2) The demand mix may turn out lower than expected causing overages and excess stock inventory being carried. This results in additional inventory and storage costs.

The total number of cuts in Table 2 is the sum of the five normal variables and is itself a normal variable with a mean of 259 and standard deviation of 14.5. This information, however, is not helpful because each cut size is a distinct product albeit being cut from the same stock. If the demand mix is random, it is not possible to be certain that demand will always be met. Instead, the decision maker may consider probability that demand for each cut type will be met as follows:

$P(X7 + X8 + X9 + 2X10 + 3X11 \geq 64) \geq PA$; $P(X4 + X5 + X6 + X10 \geq 38) \geq PB$

$P(X2 + 2X3 + X5 + X9 \geq 61) \geq PC$; $P(X1 + X2 + X4 + X8 \geq 54) \geq PD$; $P(X7 \geq 42) \geq PE$

Where the right side probabilities (PA, PB, PC, PD, and PE) are around 0.95 or higher in typical production planning processes with random demands. In addition, there may be an overall probability of meeting all five cut size demands simultaneously: $P \geq 0.90$ where $P = PA * PB * PC * PD * PE$. Given these requirements, one can consider the chance-constrained approach of stochastic programming, but the structure of the model in Fig. 1 does not fit well to the chance-constrained approach. Instead, an approach akin to more formal two-stage stochastic programming is proposed as a part of a prioritized goal programming methodology.

4. Goal programming (GP)

GP is an extension to more common linear programming. GP permits multiple objectives (goals) and soft (goal) constraints that allow for tradeoffs. Typically, goals set by management can be achieved only at the expense of other goals. A hierarchy of importance needs to be established so that higher-priority goals are satisfied before lower-priority goals are addressed. It is not always possible to satisfy every goal so GP attempts to reach a satisfactory level of multiple objectives. GP tries to minimize the deviations between goals and what we can actually achieve within the given constraints. GP may be used to solve linear programs with multiple objectives, with each objective viewed as a "goal". In GP, d_i^+ and d_i^- , deviation variables, are the amounts a targeted goal i is overachieved or underachieved, respectively. The goals themselves are added to the constraint set with d_i^+ and d_i^- acting as the surplus and slack variables. One approach to goal programming is to satisfy goals in a priority sequence. Second-priority goals are pursued without reducing the first-priority goals.

The GP model in Fig. 6 expands the original model in Fig. 1 for the second demand iteration in Table 2 using the scenario of ordering 124 rails. The GP model minimizes the overall shortage across all cut types as the priority 1.

```
. Model;
Title FIVE CUT PROBLEM AFTER DEMAND LEVELS REALIZED;
;
Min = Shortage_POS_DEVIATION ;
;
! PRIOR SOLUTION OF THE DETERMINISTIC EQUIVALENT MODEL RESULTS IN THE NUMBER OF STOCKS
TO ORDER;
STOCKS_ORDERED = 124;
!
! DEMAND for each cut type is now realized;
Demand_A =73;
Demand_B = 42;
Demand_C = 56;
Demand_D = 58;
Demand_E = 51;
!
! PRIOR PATTERN CONSTRAINTS ARE NOW GOAL PROGRAMMING LIKE EQUALITIES;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 1*X8 + 1*X9 + 2*X10 + 3* X11 + Type_A_Shortage -
Type_A_Overage - Demand_A = 0;
0*X1 + 0*X2 + 0*X3 + 1*X4 + 1*X5 + 2*X6 + 0*X7 + 0*X8 + 0*X9 + 1*X10 + 0* X11 + Type_B_Shortage -
Type_B_Overage - Demand_B = 0;
0*X1 + 1*X2 + 2*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 1*X9 + 0*X10 + 0* X11 + Type_C_Shortage -
Type_C_Overage - Demand_C = 0;
2*X1 + 1*X2 + 0*X3 + 1*X4 + 0*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 0*X10 + 0* X11 + Type_D_Shortage -
Type_D_Overage - Demand_D = 0;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 0*X8 + 0*X9 + 0*X10 + 0* X11 + Type_E_Shortage -
Type_E_Overage - Demand_E = 0;
! Calculate the number of cuts produced;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 1*X8 + 1*X9 + 2*X10 + 3* X11 - Type_A_Produced = 0;
0*X1 + 0*X2 + 0*X3 + 1*X4 + 1*X5 + 2*X6 + 0*X7 + 0*X8 + 0*X9 + 1*X10 + 0* X11 - Type_B_Produced = 0;
0*X1 + 1*X2 + 2*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 1*X9 + 0*X10 + 0* X11 - Type_C_Produced = 0;
2*X1 + 1*X2 + 0*X3 + 1*X4 + 0*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 0*X10 + 0* X11 - Type_D_Produced = 0;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 0*X8 + 0*X9 + 0*X10 + 0* X11 - Type_E_Produced = 0;
!
Calculate total number of shortages and overages;
Type_A_Shortage + Type_B_Shortage + Type_C_Shortage + Type_D_Shortage + Type_E_Shortage - Total_Shortage = 0;
Total_Shortage + Shortage_NEG_DEVIATION - Shortage_POS_DEVIATION = 0;
Type_A_Overage + Type_B_Overage + Type_C_Overage + Type_D_Overage + Type_E_Overage - Total_Overage = 0;
Total_Overage + Overage_NEG_DEVIATION - Overage_POS_DEVIATION = 0;
!
! Not all stocks should be used if the realized demand is less, but no additional stocks are available;
STOCKS_USED - STOCKS_ORDERED < = 0;
! Calculate the actual number of stocks used;
X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 - STOCKS_USED = 0;
! Calculate the number of unused stocks, if any;
STOCKS_USED + STOCKS_CARRIED - STOCKS_ORDERED = 0;
! Decision variables must be integers;
@GIN(x1);@GIN(x2); @GIN(x3); @GIN(x4); @GIN(x5);@GIN(x6);@GIN(x7);@GIN(X8); @GIN(X9); @GIN(X10);
@GIN(X11);
END;
```

Fig. 6. Priority 1 goal programming model to minimize total shortage

If the overall shortage is achieved as zero by having Shortage_POS_DEVIATION = 0 as an output, priority 2 model is formulated with a new objective function expression of "Min=Overage_POS_DEVIATION". "Shortage_POS_DEVIATION = 0" becomes a new constraint. If the minimum possible shortage is non-zero, priority 2 model is not applied because there will be no overage to minimize. Tables 2 and 3 use the same random demand mixes and show several iterations using GP for two stock order scenarios. In each simulation iteration with a randomly drawn demand mix, the one-dimensional cutting stock GP model is first solved to minimize positive deviation of shortage from the goal of zero shortage.

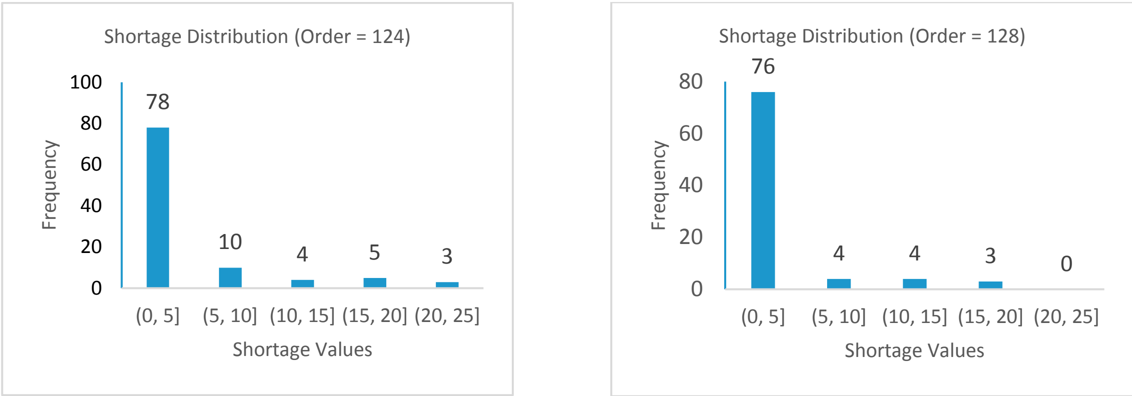
Table 2. Monte-Carlo simulation of the model using goal programming (order = 124)

Iteration	A	B	C	D	E	Total Demand	Total Shortage	Total Overage	Stocks Carried
1	68	37	57	53	40	255	0	7 then 0	0 then 2
2	73	42	56	58	51	280	14	0	0
3	50	32	71	59	59	212	19	0	0
.	.	.	.	.	.	.			
26	67	39	59	61	40	266	3	0	0
27	60	34	59	66	46	265	7	0	0
.	.	.	.	.	.	.			
60	52	40	66	55	49	262	9	0	0
.	.	.	.	.	.	.			
99	64	35	51	56	45	251	0	7 then 0	0 then 3
100	73	33	69	47	38	260	0	6 then 0	0 then 2

Table 3. Monte-Carlo simulation of the model using goal programming (order = 137)

Iteration	A	B	C	D	E	Total Demand	Total Shortage	Total Overage	Stocks Carried
1	68	37	57	53	40	255	0	22 then 0	0 then 15
2	73	42	56	58	51	280	0	5 then 0	0 then 2
3	50	32	71	59	59	212	3	0	0
.	.	.	.	.	.	.			
26	67	39	59	61	40	266	0	22 then 0	0 then 10
27	60	34	59	66	46	265	0	32 then 0	0 then 8
.	.	.	.	.	.	.			
60	52	40	66	55	49	262	0	13 then 0	0 then 6
.	.	.	.	.	.	.			
99	64	35	51	56	45	251	0	23 then 0	0 then 16
100	73	33	69	47	38	260	0	41 then 0	0 then 15

Iteration 1 in Table 2 has achieved zero total shortage and produced 7 extra cuts by using all 124 rails. Priority 2 model reduces overage to zero with a new X_j decision vector and carries on two rails for future use. Iteration 2 with a higher total demand already results in a total shortage of 14 cuts and the priority 2 model is not used. Table 3 shows only one shortage event because of larger order size of 137 rails. More stocks become unneeded and are carried on for future use increasing inventory-holding costs. Figure 7 shows the distribution of shortages for two scenarios, (a) 124 and (b) 128 rails. The decision maker(s) would weigh the likelihood of shortages and the number of stocks carried in making an order decision.



a b

Fig. 7. Simulated shortage distribution for ordering of (a) 124 rails and (b) 128 rails

5. Conclusions and recommendations

This paper has proposed an easy to implement GP based methodology to solve the one-dimensional cutting stock problem with a random demand mix. The methodology can evaluate effects of various raw material order levels to account for demand randomness at the time of order. This is done by first minimizing shortage and then minimizing overage. The procedure favors carrying uncut stock over having unneeded cuts. The procedure was accepted by the management and used at the facility.

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