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Horizontal Air Mass: Solutions for Fermi Questions, November 2022

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Fermi Questions

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Horizontal air mass

◗ Question 1:

Imagine that we can compress the Earth's atmosphere to a spherical shell of uniform density (see Fig. 1). Light from the Sun or other stars traverses the minimum amount of atmosphere when it comes from the observer's zenith. Define this quantity of air as 1 *air mass.*

By considering atmospheric pressure at sea level and the density of air, find the height h of this fictitious atmosphere. Hence, estimate the air mass an observer looks through to the *horizon* on a spherical earth with such a spherical uniform atmosphere. (The radius of the Earth will need to be expressed in the same units.)

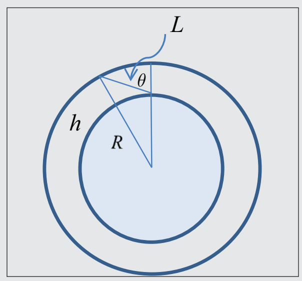

Fig. 1.

Solution to Question 1: Air pressure at sea level is $\sim 10^5$ Pa and the density of air is $\sim 1.2\ \text{kg/m}^3$. The pressure must be the weight of a column of air 1 m^2 in cross section, i.e., $10^5 \approx 1.2 \times 10 \times h$, so $h \approx 8$ km; this is equivalent to 1 air mass. The radius R of the Earth is $\sim 6.4 \times 10^3$ km, or 800 air masses. Using Pythagoras's theorem, the horizontal distance to the "top" of the atmosphere is $\sim\sqrt{1600} = 40$ air masses! This has interesting implications for the scattering of sunlight …

◗ Question 2:

Estimate the air mass in both a 60° and an 80° direction from the vertical. (A little trigonometry will be required.)

Solution to Question 2: The governing quadratic equation for angles θ simplifies (since $R \gg 1$) to $L^2 + 2RL\cos\theta - 2R = 0$, where L is the air mass at angle θ from the vertical direction. The positive root of this equation yields, surprisingly, $L \approx 2$ for $\theta = 60°$ and $L \approx 5.6$ for $\theta = 80°$. A graph of $L(\theta)$ shows how rapidly the air mass increases as the observer's line of sight approaches the horizon.

Reference

1. David K. Lynch and William Livingston, *Color and Light in Nature* (Thule Scientific, 2010).