Oscillating Icebergs

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Fermi Questions

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Solutions for Fermi Questions, February 2023

Oscillating iceberg

◗ Question 1:

(i) Estimate (a) the mass of an adult gull and (b) the mass of the iceberg, given that the sun-basking birds are typical of Icelandic gulls. Assume that the picture shows all the above-water portion of the iceberg.

(ii) If the iceberg floats in a closed lake of area 1 km^2, by how much did the water level rise once all the gulls had settled on it?

Solutions:

(i) (a) I estimate the volume of a typical adult gull as equivalent to that of two right circular cones sharing a common base, each cone being of height ¼ m and radius, say, 5 cm. In fact, a better approximation for the shape of the gull might be half this, i.e., sliced in two equal parts along the common axes of the cones. However, retaining the original representation, the volume is $2(\pi r^2 h/3) \approx 2 \times 0.05^2 \times 0.25\ \mathrm{m}^3 \approx 10^{-3}\ \mathrm{m}^3$. Since gulls float on water, we can assume their average density is less than that of water, but again erring on the generous side, a gull's mass is $\sim 10^{-3}\ \mathrm{m}^3 \times 10^3\ \mathrm{kg/m}^3 = 1$ kg.

(b) Assuming that the iceberg is approximately square in horizontal section, I estimate that a side is 30–40 gull lengths, or ~15–20 m. Similarly, the visible height is ~10 gull lengths ≈ 5 m; if this represents about 10% of the vertical dimension of the iceberg, then its volume is approximately $20 \times 20 \times 50\ \mathrm{m}^3 = 2 \times 10^4\ \mathrm{m}^3$. If the density of ice is ~900 $\mathrm{kg/m}^3$, the mass is $\sim 2 \times 10^7$ kg.

(ii) I estimate a total of ~200 birds are present, so by Archimedes' principle, the mass of fluid displaced by them is ~200 kg. If d m is the amount by which the lake surface rises, then $200\ \mathrm{kg} = d \times (10^3)^2\ \mathrm{m}^3 \times 10^3\ \mathrm{kg/m}^3 = 10^9 d$ kg, so $d \approx 2 \times 10^{-7}$ m! We need more gulls!

◗ Question 2:

If this iceberg were displaced slightly downward from its equilibrium position, what would be its period of oscillation? You may assume that the number of gulls on the far side of the iceberg is comparable to the number visible here.

Solution:

If $y(t)$ is the small displacement from equilibrium in meters at time t, the iceberg (of mass m) will undergo simple harmonic motion satisfying the equation $my''(t) = -W$, where W N is the weight of the water displaced; i.e., $W \approx 20 \times 20 \times y\ \mathrm{m}^3 \times 10^3\ \mathrm{kg/m}^3 \times g$, g being ~10 m/s^2, i.e., $(4 \times 10^6\ \mathrm{N/m}) \times y$. However, the value of m must include the so-called *added* mass, resulting in the *effective* mass. The added mass is an inertial effect,[1] and in this context arises from the displacement of the surrounding water as the iceberg oscillates, generating fluid pressures that produce an additional hydrodynamic force acting on it. (In many cases, incompressible potential flow theory can be used to calculate this.) In the case of an oscillating "icebox" with the relative dimensions estimated here, it is estimated from Ref. 2 that the added mass is about $0.7m$, so we take the effective mass m_e as $1.7m \approx 2m = 4 \times 10^7$ kg.

Hence, in the governing differential equation, m is replaced by m_e. The general solution of this equation is $y = A \cos \sqrt{0.1}\, t + B \sin \sqrt{0.1}\, t$, where the constants A and B do not concern us here. The *period* of oscillation is therefore $2\pi/\sqrt{0.1} \approx 20$ s.

References

1. J. Messer and J. Pantaleone, "The effective mass of a ball in the air," *Phys. Teach.* **48**, 52–54 (2010).
2. C. E. Brennen, "A review of added mass and fluid inertial forces," Technical Report (Department of the Navy, Port Hueneme, CA, 1982), https://resolver.caltech.edu/CaltechAUTHORS:BREncel82.