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Engineering Notes

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I. Introduction

Actuator and sensor faults can cause poor performance or even instability in dynamic systems. In flight control systems for aircraft and spacecraft, such faults can lead to loss of control and serious incidents. Therefore, rapid detection and identification of actuator and sensor faults is important for enhancing flight safety. One approach to fault detection and identification (FDI) in actuators and sensors is based on multiple-model methods [1,2]. These methods have been extended to detect faults, identify the fault pattern, and estimate the fault values [3,4]. Such methods typically use banks of Kalman–Bucy filters (or extended Kalman filters) in conjunction with multiple hypothesis testing and have been reported to be effective for bias-type faults, such as aircraft control surfaces getting stuck at unknown values, or sensors (e.g., rate gyros) that develop unknown constant or slowly varying biases. A basic requirement for these methods is that the faults should be identifiable. Identification of biases in the inputs and sensors was initially considered in [5].

Identifiability of bias-type faults was considered in [4] and preliminary identifiability conditions were presented. A more detailed analysis of identifiability was presented in [6]. This Note provides a complete characterization of the conditions for identifiability of constant bias-type actuator faults, sensor faults, and simultaneous actuator and sensor faults. Section II considers actuator faults and presents necessary and sufficient conditions (NASC) for their identifiability. Section III presents NASC for identifiability of sensor faults occurring in some of the sensors, and Sec. IV presents NASC for identifiability of simultaneous faults in actuators and sensors. Numerical examples are included to illustrate the results.

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II. Actuator Faults

Consider a linear time-invariant system:

$$\dot{x} = Ax + Bu + w_p \quad y = Cx + w_s \tag{1}$$

where $x \in R^n$, $u \in R^m$, $w_p \in R^n$, $y \in R^l$, and $w_s \in R^l$ denote the state vector, control vector, process noise, output vector, and sensor noise, respectively, and A, B, and C are appropriately dimensioned matrices; w_p and w_s are usually assumed to be zero-mean Gaussian white noise processes.

In the actuator fault scenario considered in this Note, some of the actuators may get locked in unknown positions at unknown time instants ("stuck actuator" failures) and produce constant unknown input values (a zero value represents complete actuator outage). Thus, in the kth failure pattern, when m_k of the m actuators fail, the system dynamics becomes

$$\begin{aligned}\dot{x} &= Ax + \sum_{j \notin \mathcal{F}_{ak}} b_j u_j + \sum_{j \in \mathcal{F}_{ak}} b_j \bar{u}_j + w_p \\ &= Ax + B^k u^k + \bar{B}^k \bar{u}^k + w_p\end{aligned} \tag{2}$$

where $\mathcal{F}_{ak}$ is the set of indices corresponding to the failed actuators, and $\bar{u}_j$ denotes the corresponding failure value (for example, deflection of a stuck control surface in aircraft); $\bar{u}_j$ is constant after the failure occurs. There are up to $(2^m - 1)$ possible failure patterns for m actuators: $\bar{u}^k \in R^{m_k}$ denotes the failure value for the kth failure pattern; $u^k \in R^{m-m_k}$ denotes the input vector corresponding to the functioning actuators; $\bar{B}^k$ denotes the columns of B corresponding to the failed actuators; and B^k denotes the remaining columns of B corresponding to the functional actuators. An actuator fault is identifiable if the failure can be determined and the fault value can be estimated. In methods employing state augmentation, unknown bias faults are represented as Wiener processes [7]. A specific (kth) actuator fault pattern is isolated, and the corresponding fault values $\bar{u}_j^k$ are estimated by augmenting Eq. (2) with

$$\dot{\bar{u}}_j^k = w_{a_j}^k \tag{3}$$

where $w_{a_j}^k$ is a fictitious zero-mean white noise process. Thus, the augmented equation corresponding to model k (failure pattern k) is

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ \bar{u}^k \end{bmatrix} = \begin{bmatrix} A & \bar{B}^k \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \bar{u}^k \end{bmatrix} + \begin{bmatrix} B^k \\ 0 \end{bmatrix} u^k + w^k \tag{4}$$

where $w^k \in R^{n+m_k}$ denotes the input noise vector consisting of the process noise w_p and the fictitious noise w_a^k corresponding to $\bar{u}^k$. Denoting

$$\xi^k = \begin{bmatrix} x \\ \bar{u}^k \end{bmatrix} \tag{5}$$

the system corresponding to failure pattern k is expressed as

$$\dot{\xi}^k = A_\xi^k \xi^k + B_\xi^k u^k + w^k \tag{6}$$

$$y = [C \quad 0]\xi^k + w_s := C_\xi^k \xi^k + w_s \tag{7}$$

where A_ξ^k and B_ξ^k are the augmented system- and input-matrices from Eq. (4).

In multiple-model-based methods, the FDI approach consists of designing a bank of Kalman–Bucy filters (KBF), where each filter corresponds to one of the 2^m models, and determining (in real time) which model correctly represents the actual fault pattern. Criteria such as highest conditional probability or smallest residual norm are used to determine the correct fault pattern. The KBF corresponding to the correct model also gives an unbiased minimum-variance estimate of the fault values. The KBF corresponding to model k is given by

$$\dot{\hat{\xi}}^k = A_\xi^k \hat{\xi}^k + B_\xi^k u^k + H^k (y - C_\xi^k \hat{\xi}^k) \tag{8}$$

where $\hat{\xi}^k$ denotes the estimate of ξ^k and H^k (a function of time) is the KBF gain. From Eqs. (6) and (8), the estimation error dynamics are given by

$$\dot{\tilde{\xi}}^k = (A_\xi^k - H^k C_\xi^k)\tilde{\xi}^k + w^k - H^k w_s \tag{9}$$

where $\tilde{\xi}^k = \xi^k - \hat{\xi}^k$. The residual (which is used in decision making to determine which fault model is "closest" to the actual system) is given by

$$r_k = y - C_\xi^k \hat{\xi}^k = C_\xi^k \tilde{\xi}^k + w_s \tag{10}$$

For the KBF to work, the error dynamics [Eq. (9)] must be asymptotically stable and unbiased, that is, the mean of $\tilde{\xi}^k$ should converge to zero and its covariance should be uniformly bounded over $t \in [0, \infty)$.

For failure pattern k, if $\bar{B}^k$ is not of full rank, it is not possible to distinguish between fault values for actuators corresponding to linearly dependent columns of $\bar{B}^k$. For example, for some failure pattern, if

$$\bar{B} = [\bar{b}_1, \bar{b}_2, \bar{b}_3]$$

and if $\bar{b}_3 = \alpha_1 \bar{b}_1 + \alpha_2 \bar{b}_2$ for some constant α_1, α_2, the input terms due to the failed actuators are

$$\bar{B}\,\bar{u} = \bar{b}_1 \bar{u}_1 + \bar{b}_2 \bar{u}_2 + \bar{b}_3 \bar{u}_3 = \bar{b}_1(\bar{u}_1 + \alpha_1 \bar{u}_3) + \bar{b}_2(\bar{u}_2 + \alpha_2 \bar{u}_3) \tag{11}$$

That is, $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ cannot be estimated individually; only the aggregated fault values $\bar{u}_1 + \alpha_1 \bar{u}_3$, $\bar{u}_2 + \alpha_2 \bar{u}_3$ can be estimated. Therefore, it is assumed henceforth that, for each failure mode, the fault inputs corresponding to linearly dependent columns of $\bar{B}^k$ have been aggregated and that $\bar{B}^k$ is of full rank. As a result, the number of distinguishable failure patterns (and the corresponding Kalman–Bucy filters) is usually less than $(2^m - 1)$.

The Kalman–Bucy filters can work correctly and give correct fault estimates only if the augmented system (6), (7) is detectable (preferably observable). Nondetectability or unobservability can result in grossly erroneous estimates of the augmented state vector and incorrect FDI. Furthermore, in the infinite-duration case, a constant KBF gain H^k that stabilizes the system matrix in Eq. (9) exists only if (C_ξ^k, A_ξ^k) is detectable, which is possible only if the augmented zero-frequency modes of A_ξ^k are observable. Although detectability assures exponential decay of estimation error, it is desirable to have observability, which provides the ability to obtain a desired error decay rate. Thus, we define a fault to be *identifiable* (in the weak sense) if the augmented system is merely detectable, and *strongly identifiable* if the augmented system is observable. The following theorem gives necessary and sufficient conditions for detectability (respectively, observability), that is, for weak and strong identifiability of the kth actuator failure pattern.

Theorem 1: The pair (C_ξ^k, A_ξ^k) is detectable (respectively, observable) iff all of the following conditions are satisfied:

1) $l \geq m_k$.
2) The pair (C, A) is detectable (respectively, observable).
3) The system $(C, A, \bar{B}^k)$ has no invariant zeros at the origin.

Proof: Applying the Popov-Belevitch-Hautus (PBH) rank test [8], (C_ξ^k, A_ξ^k) is detectable [respectively (resp.), observable] iff

$$\operatorname{rank}\begin{bmatrix} sI - A & -\bar{B}^k \\ 0 & sI_{m_k} \\ C & 0 \end{bmatrix} = n + m_k$$
$$\text{for } s \in \{\Lambda_u(A) \cup 0\} \ [\text{resp., } s \in \{\Lambda(A) \cup 0\}] \tag{12}$$

where $\Lambda(A)$, $\Lambda_u(A)$ denote the sets of eigenvalues of A and closed-right-half-plane (CRHP) eigenvalues of A, respectively. The first n columns of the PBH test matrix are independent for all $s \in \{\Lambda_u(A) \cup 0\}$[resp., $s \in \{\Lambda(A) \cup 0\}$] iff (C, A) is detectable (resp., observable). For $s \neq 0$, the last m_k columns are mutually independent as well as independent of the first n columns, thus the test matrix has a full rank. For $s = 0$, the rank condition (12) is satisfied iff $l \geq m_k$ and $(C, A, \bar{B}^k)$ has no invariant zeros at $s = 0$. □

Remark 2.1: The invariant zeros of $(C, A, \bar{B}^k)$ mentioned in Theorem 1 include transmission zeros and (some or all of the) input decoupling zeros (IDZs) [9]. The IDZs are simply the eigenvalues of A corresponding to the uncontrollable modes of $(A, \bar{B}^k)$. Because (C, A) is detectable, there are no output decoupling zeros in the CRHP. Note that $\bar{B}^k$ corresponds to failed actuators in failure pattern k; therefore, $(A, \bar{B}^k)$ may not be controllable for all k. For the corresponding models, Theorem 1 requires that the uncontrollable modes must not have zero eigenvalues that are also invariant zeros. If $(A, \bar{B}^k)$ is controllable, the invariant zeros are just the transmission zeros. If the system is degenerate (i.e., if the rank of the Rosenbrock system matrix [9] is less than full rank for all s in the complex plain), there is an invariant zero at every complex number including the origin, and Condition 3 of Theorem 1 is violated.

Remark 2.2: In practical implementations, the estimation is performed in a discrete-time setting using discrete-time Kalman filters. The detectability conditions of Theorem 1 are very similar, the only difference being that, in Condition 3, the phrase "no invariant zeros at the origin" is replaced by "no invariant zeros at unity" [for the discretized version of $(C, A, \bar{B}_\xi^k)$].

Remark 2.3: If (C, A) is observable, the unobservable subspace $\bar{\mathcal{O}}_\xi^k$ of (C_ξ^k, A_ξ^k) can be obtained as follows after some manipulation:

$$\bar{\mathcal{O}}_\xi^k = \mathcal{N}\begin{bmatrix} C_\xi^k \\ C_\xi^k A_\xi^k \\ \vdots \\ C_\xi^k (A_\xi^k)^{(n+m_k-1)} \end{bmatrix} = \mathcal{N}\begin{bmatrix} A & \bar{B}^k \\ C & 0 \end{bmatrix} \tag{13}$$

where $\mathcal{N}(\cdot)$ denotes the null space. Thus, the unobservable subspace of (C_ξ^k, A_ξ^k) consists of the generalized eigenvectors of $(C, A, \bar{B}^k)$ corresponding to the invariant zeros at the origin.

Remark 2.4: It is intuitively straightforward to see that a transmission zero at the origin adversely affects the ability to estimate a constant fault value $\bar{u}$, because input frequency components corresponding to the transmission zeros do not appear in the output. Furthermore, a set of initial conditions exists such that $y(t)$ is identically zero.

We consider two illustrative examples, a large transport aircraft and a small experimental uninhabited aerial vehicle (UAV), which have qualitatively different dynamic characteristics, as described next.

Example 1: Consider a fourth-order longitudinal dynamics model of a large transport aircraft in wings-level cruise condition, which was used in [4]. The state vector consists of the pitch rate, forward speed, angle of attack, and pitch angle (i.e., $x = [q, v, \alpha, \theta]^T$), and the control vector consists of elevator deflection and engine thrust $u = [u_e, u_T]^T$:

$$A = \begin{bmatrix} -0.6803 & 0.0115 & -1.0490 & 0 \\ -0.0026 & -0.0062 & -0.0815 & -0.1709 \\ 1.0050 & -0.0344 & -0.5717 & 0 \\ 1.0000 & 0 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} -44.5192 & 0.88254 \\ 0 & 1.3287 \\ -11.4027 & -0.0401 \\ 0 & 0 \end{bmatrix} \tag{14}$$

All eigenvalues of A are in the left half-plane, and it can be verified that the system is controllable with respect to either input (i.e., there are no IDZs). Also, (C, A) is observable for all sensor suites. For the purpose of this example, considering that 1–4 sensors can be used, there are $(2^4 - 1)$ possible sensor suite configurations. There are $(2^2 - 1)$ possible actuator failure states for the two actuators, therefore, the total number of fault cases for the 15 sensor suite configurations is 45. Out of these, four cases violate Condition 1 in Theorem 1 ($l < m_k$). In addition, four cases were found to have invariant zeros at the origin: $y = x_1$ and actuator 1 or 2 fail; $y = (x_1, x_2)$ or $y = (x_1, x_3)$ and both actuators fail. For these cases, the faults are not identifiable. In addition, when $y = (x_1, x_4)$, the two-actuator fault is not identifiable because the system is degenerate (the rank of the Rosenbrock system matrix is less than the full rank; see Remark 2.1). For all the remaining cases, the actuator faults are strongly identifiable.

Example 2: Consider the linearized sixth-order longitudinal dynamics of the Cranfield A3 Observer, a research UAV presented in [10], in cruise condition. The UAV's airframe is a gust-insensitive configuration [11]. For improved performance, the center of gravity is placed aft of the neutral point. The state vector consists of the forward speed, vertical speed, pitch rate, pitch angle, altitude, and engine rpm (i.e., $x = [v, w, q, \theta, h, N_E]^T$). The control vector consists of engine thrust (throttle) and elevator deflection (i.e., $u = [u_T, u_e]^T$). In this example, the output matrix (sensor suite) is specified in addition to the system and input matrices:

$$A = \begin{bmatrix} -0.146 & -0.016 & 0.557 & -9.809 & 0 & 0.001 \\ -0.63 & -4.487 & 34.57 & 0.161 & 0 & 0 \\ 0.001 & 0.039 & -0.894 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -0.016 & -1 & 0 & 35.2 & 0 & 0 \\ 665.7 & -6.89 & 0 & 0 & 0 & -8.57 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & -1.368 \\ 0 & -19.96 \\ 0 & -15.96 \\ 0 & 0 \\ 0 & 0 \\ 45{,}910 & 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & -0.014 & 0.019 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.984 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

where the output measurements are selected to be $y = [v_e, q, \theta, h_e, N_E]^T$, v_e represents the measured speed error (relative to the trim speed), and h_e represents the perturbed altitude.

The linearized dynamics are analyzed in [10]. In particular, the model has one eigenvalue at zero corresponding to the altitude mode and a dominant well damped complex pair. For this configuration, the axial phugoid mode is unstable and coupled to the single-pole short period mode. The system is controllable with respect to either input; thus, there exists no input decoupling zeros. Furthermore, because the sensor suite is specified, the number of sensor configurations is limited to this single case. There are $(2^2 - 1)$ possible actuator failure states for the two actuators. All of the actuator failure cases for the given nonbiased sensor suite are strongly identifiable (i.e., the conditions of Theorem 1 were satisfied).

All numerical results for the examples are summarized in Table 1, which includes results for actuator, sensor, and simultaneous actuator–sensor fault cases (the last two fault types are discussed in the following sections). The table lists the numbers of possible fault cases and the numbers of nonidentifiable cases for each fault condition. Note that the second example has fewer fault cases because the sensor suite was fixed.

III. Sensor Bias

Consider the case when there are no actuator failures but q of the l sensors have unknown sensor biases (or are known to be prone to developing biases). Denote the bias-free part and the biased part of the sensor output vector as y_1 and y_2, respectively, and the corresponding output matrices as $C_1 \in R^{(l-q)\times n}$ and $C_2 \in R^{q\times n}$. Then, the sensor output equation is

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Cx + \begin{bmatrix} 0 \\ \bar{y}_2 \end{bmatrix} + w_s = \begin{bmatrix} C_1 x \\ C_2 x + \bar{y}_2 \end{bmatrix} + w_s \tag{16}$$

where $\bar{y}_2 \in R^q$ is the sensor bias vector and $C = [\, C_1^T \quad C_2^T \,]^T$. As was done in the case of linearly dependent columns of $\bar{B}^k$, it is assumed that linearly dependent sensor outputs have been combined and C has a full row rank. (Biases corresponding to linearly dependent sensors cannot be estimated individually and must be aggregated).

Upon augmenting the sensor bias $\bar{y}_2$ to the state vector, the system becomes

$$\dot{\eta} := \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ \bar{y}_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}\eta + \begin{bmatrix} B \\ 0 \end{bmatrix}u + w' := A_\eta \eta + B_\eta u + w' \tag{17}$$

$$y = \begin{bmatrix} C_1 & 0_{(l-q)\times q} \\ C_2 & I_q \end{bmatrix}\eta + w_s := C_\eta \eta + w_s \tag{18}$$

where w' denotes the augmented process noise vector.

The bias estimation approach involves constructing a KBF for the augmented system (17), (18). As in the case of actuator faults, detectability of (C_η, A_η) is essential for the KBF to function correctly (observability is desirable). The following theorem gives necessary and sufficient conditions for identifiability of sensor faults.

Theorem 2: The pair $(C_\eta,\ A_\eta)$ is detectable (respectively, observable) iff the following conditions are satisfied:

1) The pair (C, A) is detectable (respectively, observable).

2) All zero-frequency modes of A are observable with respect to the bias-free sensor outputs.

Proof: Applying the PBH rank test, $(C_\eta,\ A_\eta)$ is detectable (respectively, observable) iff

Table 1 Summary of results for Examples 1 and 2

	Example 1		Example 2	
	No. of fault states	Nonidentifiable cases	No. of fault states	Nonidentifiable cases
Actuator faults	45	Four due to Theorem 1(1); five due to Theorem 1(3) (includes one degenerate case)	3	None
Sensor faults	65	None	31	16 due to Theorem 2(2)
Actuator and sensor faults	195	73 due to Theorem 3(1); 23 due to Theorem 3(3) (includes three degenerate cases)	93	Eight due to Theorem 3(1); 42 due to Theorem 3(3) (includes one degenerate case)

$$\operatorname{rank}\begin{bmatrix} sI-A & 0 \\ 0 & sI_q \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix} = n+q$$

$$\text{for } s \in \{\Lambda_u(A) \cup 0\} \text{ [resp., } s \in \{\Lambda(A) \cup 0\}] \tag{19}$$

The first n columns of the PBH test matrix are linearly independent $\forall\ s \in \{\Lambda_u(A) \cup 0\}$[resp., $\forall\ s \in \{\Lambda(A) \cup 0\}$] if (C, A) is detectable (respectively, observable). For $s \neq 0$, the last q columns are mutually independent as well as independent of the first n columns. For $s = 0$, the rank of the test matrix is $n + q$ iff

$$\operatorname{rank}\begin{bmatrix} sI-A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} = n+q \tag{20}$$

Because elementary column operations do not change the column rank,

$$\operatorname{rank}\begin{bmatrix} sI-A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} = \operatorname{rank}\left\{\begin{bmatrix} sI-A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} \begin{bmatrix} I_n & 0 \\ -C_2 & I_q \end{bmatrix}\right\} = \operatorname{rank}\begin{bmatrix} sI-A & 0 \\ C_1 & 0 \\ 0 & I_q \end{bmatrix}_{s=0} \tag{21}$$

Thus, the rank of the PBH test matrix at $s = 0$ is $n + q$ iff the first n columns are linearly independent for $s = 0$ (i.e., iff the zero-frequency modes of A are observable with respect to C_1). □

When all sensors have biases (i.e., $q = l$), they are identifiable (respectively, strongly identifiable) when (C_η, A_η) is detectable (respectively, observable), as stated next.

Corollary 2.1: If all sensors have biases, (C_η, A_η) is detectable (respectively, observable) iff the following conditions are satisfied:

1) (C, A) is detectable (respectively, observable).

2) A has no zero eigenvalues.

Remark 3.1: For the case when all sensors have biases, the condition that A should not have zero eigenvalues is rather restrictive, because many engineering systems have free integrators in their dynamics. However, there does not appear to be an obvious way of getting around this problem. Consider the effect of using output feedback which moves the eigenvalues away from the origin, for example,

$$u = -Gy = -G(Cx + \bar{y} + w_s) \tag{22}$$

where $G \in R^{m\times l}$ and $\bar{y} = [0, \bar{y}_2^T]^T$, which gives the following closed-loop system (including the augmented state $\bar{y}$):

$$\dot{\eta} = \begin{bmatrix} A-BGC & -BG \\ 0 & 0 \end{bmatrix}\eta + w := A'_\eta \eta + w \tag{23}$$

$$y = [\,C \quad I\,]\eta + w_s \tag{24}$$

The pair (C_η, A'_η) is detectable iff

$$\operatorname{rank}\begin{bmatrix} sI-A+BGC & BG \\ 0_{l\times n} & sI_l \\ C & I_l \end{bmatrix} = n+l$$

$$\text{for } s \in \{\Lambda_u(A-BGC) \cup 0\} \tag{25}$$

Proceeding as in the proof of Theorem 2, the first n columns are linearly independent for $s \in$ CRHP iff $[C, (A - BGC)]$ is detectable. For $s \neq 0$, the last l columns of the PBH test matrix are mutually independent as well as independent of the first n columns. For $s = 0$, because the rank of a matrix is unchanged by multiplication by an elementary matrix,

$$\operatorname{rank}\left\{\begin{bmatrix} I_n & -BG \\ 0 & I_l \end{bmatrix}\begin{bmatrix} sI-A+BGC & BG \\ C & I_l \end{bmatrix}_{s=0}\right\} = \operatorname{rank}\begin{bmatrix} sI-A & 0 \\ C & I_l \end{bmatrix}_{s=0} \tag{26}$$

The rank is $n + l$ iff A has no eigenvalue at the origin. Thus, using feedback to move the eigenvalue away from zero does not make the augmented closed-loop system detectable.

Remark 3.2: If A has one or more zero eigenvalues, the unobservable subspace of (C_η, A_η) can be readily obtained. Defining $\Gamma = [\,0_{q\times(l-q)} \quad I_q\,]^T$, the unobservable subspace is

$$\bar{\mathcal{O}}_\eta = \mathcal{N}\begin{bmatrix} C & \Gamma \\ CA & 0 \\ CA^2 & 0 \\ \vdots & \vdots \\ CA^{n+q-1} & 0 \end{bmatrix} \tag{27}$$

That is, if $(x_1^T, x_2^T)^T \in \bar{\mathcal{O}}_\eta$,

$$Cx_1 + \Gamma x_2 = 0; \qquad [\,\mathcal{O}^T \quad \mathcal{X}^T\,]^T A x_1 = 0 \tag{28}$$

where $\mathcal{O} = [C^T, A^T C^T, \ldots, (A^{n-1})^T C^T]^T$ and $\mathcal{X} = [(A^T)^n C^T \ldots (A^T)^{n+q-2} C^T]^T$. ($\mathcal{X}$ is present in Eq. (28) only if $q \geq 2$.) If (C, A) is observable, $[\,\mathcal{O}^T \quad \mathcal{X}^T\,]^T$ has full column rank. Therefore,

$$Ax_1 = 0; \qquad C_1 x_1 = 0; \qquad C_2 x_1 + x_2 = 0 \tag{29}$$

That is, x_1 must be an eigenvector of A corresponding to a zero-frequency mode of A that is unobservable with respect to C_1. If all zero-frequency ("0-freq") modes of A are observable with respect to C_1, then $x_1 = 0$, therefore, $x_2 = 0$, $\bar{\mathcal{O}}_\eta = 0$, and the augmented system is observable, which is consistent with Theorem 2. If some zero-frequency modes of A are not observable with respect to C_1, $\bar{\mathcal{O}}_\eta$ can be characterized as

$$\bar{\mathcal{O}}_\eta = \left\{ \begin{bmatrix} x_1 \\ -C_2 x_1 \end{bmatrix}, x_1 = \text{eigenvector of } A \text{ corresponding to unobservable 0-freq modes} \right\} \quad (30)$$

Remark 3.3: In practice, the Kalman filter is implemented in a discrete-time setting, and Condition 2 in Theorem 2 changes to "all modes corresponding to $\lambda(A) = 1$ are observable with respect to the bias-free sensor outputs" (for the discretized version of A).

For the aircraft longitudinal dynamics model in Sec. II, if k sensors are used in a sensor suite, there can be ${}^4C_k$ (${}^nC_r$ denotes $n!/[r!(n-r)!]$) such combinations (sensor suites) for each $k = 1, \ldots, 4$. Each of the k sensors in a sensor suite may or may not have a bias (i.e., there can be up to $2^k - 1$ cases with at least one biased sensor). This yields ${}^4C_k \times (2^k - 1)$ combinations. When summed over 1–4, the number of such combinations is 65. Because A is Hurwitz, the detectability conditions in Theorem 2 are satisfied. Furthermore, for this example, (C, A) is observable for all the corresponding C's, and the sensor faults are strongly identifiable.

For the UAV example in Sec. II, there is a single sensor suite given with five measurements; thus, there are $(2^5 - 1) = 31$ possible sensor bias cases. The pair (C, A) is observable. There were 16 cases of nonidentifiability due to violation of Condition 2 in Theorem 2, that is, A has unobservable zero-frequency modes with respect to C_1, the bias-free part of the output. All of these cases included a bias in the h_e sensor, with one case occurring due to a fault in h_e alone, and 15 cases occurring due to biases in h_e and all the other combinations of sensors. The remaining 15 cases are strongly identifiable.

The identifiability results for both examples are summarized in Table 1.

IV. Simultaneous Actuator Faults and Sensor Bias

For the case with actuator fault pattern k, if q of the sensors have biases, the augmented system is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ \bar{u}^k \\ \bar{y}_2 \end{bmatrix} := \dot{\varphi} = \begin{bmatrix} A & \bar{B}^k & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\varphi + \begin{bmatrix} B^k \\ 0 \\ 0 \end{bmatrix} u^k + w'' \quad (31)$$

$$:= A_\varphi \varphi + B_\varphi^k u^k + w'' \quad (32)$$

$$y = \begin{bmatrix} C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix}\varphi + w_s = C_\varphi \varphi + w_s \quad (33)$$

where w'' is the augmented process noise vector. The following theorem gives necessary and sufficient conditions for identifiability of simultaneous actuator faults and sensor bias.

Theorem 3: The pair (C_φ, A_φ) is detectable (respectively, observable) iff the following conditions are satisfied:

1) $l \geq m_k + q$.

2) The pair (C, A) is detectable (respectively, observable).

3) The system $(C_1, A, \bar{B}^k)$ has no invariant zeros at the origin.

Proof: Applying the PBH rank test, (C_φ, A_φ) is detectable (respectively, observable) iff

$$\operatorname{rank}\begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ 0 & sI_{m_k} & 0 \\ 0 & 0 & sI_q \\ C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix} = n + m_k + q$$

$$\text{for } s \in \{\Lambda_u(A) \cup 0\} \; [\text{resp., } s \in \{\Lambda(A) \cup 0\}] \quad (34)$$

The first n columns of the PBH test matrix are linearly independent for all $s \in \{\Lambda_u(A) \cup 0\}$ (respectively, $s \in \{\Lambda(A) \cup 0\}$) iff (C, A) is detectable (respectively, observable). For $s \neq 0$, the last $m_k + q$ columns are mutually independent as well as independent of the first n columns. For $s = 0$, the columns of the test matrix are linearly independent iff the columns of the following $(n + l) \times (n + m_k + q)$ matrix (after applying elementary column operations as shown) are linearly independent:

$$\begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix}_{s=0} \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_{m_k} & 0 \\ -C_2 & 0 & I_q \end{bmatrix} = \begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ C_1 & 0 & 0 \\ 0 & 0 & I_q \end{bmatrix}_{s=0} \quad (35)$$

The columns of the preceding matrix are linearly independent iff Conditions 1 and 3 hold. □

If all sensors have biases ($q = l$), Condition 1 cannot be satisfied in the presence of one or more actuator failures and the system is not detectable. This represents a major limitation of this approach to FDI when actuator faults and sensor biases are simultaneously present, and suggests that some alternate techniques should be considered (perhaps for sensor FDI).

For the aircraft example, in addition to sensor biases (as discussed in Sec. III), if one, two, or both actuators fail, then the total number of possible combinations is $65 \times 3 = 195$, out of which 99 were found to be strongly identifiable. The 96 nonidentifiable cases included 20 cases with invariant zeros at the origin, 3 degenerate cases, and 73 cases in which Condition 1 in Theorem 3 was violated.

For the UAV example, the total number of possible fault combinations is $(2^5 - 1) \times (2^2 - 1) = 93$, out of which eight were not identifiable due to violation of Condition 1 in Theorem 3, that is, $l < m_k + q$, and 42 were not identifiable due to violation of Condition 3 (i.e., invariant zeros at the origin). Of the cases of nonidentifiability due to invariant zeros at the origin, one case is also an instance of a degenerate system, so that every point in the entire s plane is an invariant zero. In all but one of the nonidentifiable cases, a bias is present in the h_e measurement. When the h_e measurement is not biased, the only nonidentifiable fault requires that both actuators fail and that there are biases in the measured velocity and pitch angle. The remaining 43 cases are strongly identifiable.

The identifiability results for both examples are summarized in Table 1.

V. Conclusions

A class of FDI methods for bias-type actuator and sensor faults was explored in detail from the point of view of fault identifiability. The methods use banks of KBFs to detect faults, determine the fault pattern, and estimate the fault values. A complete characterization of conditions for identifiability of bias-type actuator faults, sensor faults, and simultaneous actuator and sensor faults was presented. It was shown that FDI of simultaneous actuator and sensor faults is not possible using these methods when all sensors have unknown biases. The fault identifiability conditions were demonstrated via numerical examples. The analytical and numerical results indicate that caution must be exercised to ensure fault identifiability for different fault patterns when using such methods. Future work in this area should address detection and identification of a larger class of faults, such as time-varying biases (including oscillatory faults), reduced effectiveness, as well as unknown disturbances.

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