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Solutions for Fermi Questions, April 2018: Question 1: Automobile Air Use; Question 2: Personal Air Use

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Fermi Questions

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Question 1: Automobile air use

How long would it take a running, gasoline-powered, automobile to use up the oxygen in a garage?

Answer: Automobiles consume oxygen when they burn (i.e., oxidize) gasoline. To estimate this we will need to estimate the oxygen in a garage, the rate of gasoline consumption, and the oxygen needed to oxidize gasoline. Let's start with the chemistry (but please remember that I am a physicist).

Gasoline is a hydrocarbon. If we don't remember the exact composition of gasoline (or what octane is) we can approximate it as containing more hydrogen per carbon atom than coal (i.e., none) and less than methane (CH_4). This means we will consider gasoline to be a long chain of CH_2 units and ignore carbon double bonds or hydrogen atoms on the ends. CH_2 oxidizes to $CO_2 + H_2O$ so we will need three oxygen atoms (total atomic mass 48) for each CH_2 unit (total atomic mass 14), or 48 stone of oxygen for each 14 stone of gasoline. In SI units, we will need 48 kg of oxygen for each 14 kg of gasoline.

Now let's estimate the oxygen in a garage. A one-car garage will be about 12 ft by 20 ft by 10 ft high or about $4\times6\times3$ m^3 = 70 m^3. We might remember that air has a density about 1000 times less than water, or that 1 mole of air occupies 22.4 liters and has a mass of about 28 g (N_2), or that atmospheric pressure is 10^5 Pa and the atmosphere has an effective constant-density height of 10^4 m. Any of these will give us the density of air at STP of about 1 kg/m^3. Thus our 70-m^3 garage contains 70 kg of air. About 20% of the air is oxygen, giving about 14 kg of oxygen. This is enough to oxidize 4 kg of gasoline.

At highway speed (60 mph), a car will consume 2–3 gallons of gasoline per hour. One gallon of gasoline contains about 4 liters and therefore has a mass of about 4 kg (actually 3 kg, since the density of gasoline is about 30% less than water). Thus, a car on the highway will consume 100% of a garage-full of oxygen in half an hour. When idling in a garage, the car will use a lot less gas. It will use more than 1% and less than 10% of the gas used at 60-mph, so we will estimate 3%. This means that the idling car will use 30 times less gas and will consume all the oxygen in the garage in about 15 hours.

However, the car will stop long before that, when the oxygen concentration drops too low. Since cars can cross the continental divide at an altitude over 4×10^3 m, they can operate at air densities of only 2/3 that of sea level. We'll assume that cars can operate down to about 50% air density and therefore the car will stop running after about a day. If they can operate at lower air densities, then they can operate longer.

However, a person in the car will expire long before that, when the carbon monoxide level in the garage gets too high.

The moral of our story is: make sure our garage is well ventilated!

Question 2: Personal air use

How long would it take a person to use up the oxygen in a garage?

Answer: We consume oxygen when we breathe. To estimate this we will need to estimate the oxygen in a garage and the rate at which we consume oxygen.

We already estimated that a garage contains 70 kg of air which contains 14 kg of oxygen, so we just need to estimate our own oxygen consumption.

One breath contains more than one cup of air and less than one gallon, so we will estimate 1 quart or about 1 liter. Alternatively we could measure this by measuring how much water we can displace from a submerged, water-filled, soda bottle when we exhale into it with a straw. One liter of air weighs about 1 g.

Therefore one garage contains

$$N = \frac{70\text{ kg}}{1\text{ g}} = 7\times10^4 \quad \text{breaths of air.}$$

We breathe less than once a second and more than once every 10 seconds so we will estimate once every three seconds. Thus our 7×10^4 breaths of air will take us 2×10^4 s or about six hours.

However, we do not use all of the oxygen in each breath. In fact, we use such a tiny percentage of the oxygen, we can keep someone alive by exhaling into their lungs when we perform CPR. We'll estimate that we use more than 1% and less than 100% of the oxygen in our breath, giving an estimate of 10%. This means that it will take us 6×10 = 60 hours to consume 100% of the oxygen in the garage.

However, like a car, we cannot function when the oxygen level drops too low. For humans, that level is between 1/2 and 2/3 of the normal oxygen levels. (For comparison, the air pressure on Mt. Everest is about 1/3 that of sea level.) Therefore, we will probably pass out after about 30 hours.

Unfortunately, we are unlikely to last even that long. Carbon-dioxide levels in the garage will increase, affecting our breathing. CO_2 levels above 6% are dangerous (according to Wikipedia), implying that we can convert less than 1/3 of the O_2 to CO_2. This will happen after only 20 hours.

The moral of our story is: make sure our garage is well ventilated!